Chapter 1 TRIGONOMETRIC FUNCTIONS

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. The ratio of the greatest value of $2 - \cos x + \sin^2 x$ is to its least value is

(a) $\frac{7}{4}$ (b) $\frac{11}{4}$

(c) $\frac{13}{4}$ (d) none of these

2. If $a\cos^2 3\alpha + b\cos^4 \alpha = 16\cos^6 \alpha + 9\cos^2 \alpha$ is identity, then

(a) $a = 1, b = 24$ (b) $a = 3, b = 24$

(c) $a = 4, b = 2$ (d) $a = 7, b = 18$

3. If $\sin x + \cos y = a$ and $\cos x + \sin y = b$, then $\tan\frac{x-y}{2}$ is equal to

(a) $a + b$ (b) $a - b$

(c) $\frac{a+b}{a-b}$ (d) $\frac{a-b}{a+b}$

4. Number of ordered pairs (a, x) satisfying the equation $\sec^2 (a+2)x + a^2 - 1 = 0; -\pi < x < \pi$ is

(a) 2 (b) 1

(c) 3 (d) infinite

5. If $\sin\alpha$, $\sin\beta$ and $\cos\alpha$ are in G.P. Then roots of the equation $x^2 + 2x\cot\beta + 1 = 0$ are always

(a) equal (b) real

(c) imginary (d) greater than 1

6. The expression $(x\tan\alpha + y\cot\alpha)(x\cot\alpha + y\tan\alpha) - 4xy\cot^2 2\alpha$ is

(a) independent of x (b) independent of y

(c) independent of α (d) independent of x, y, α

7. If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then g of $\left(\frac{\pi}{8}\right) =$

(a) $\frac{5}{4}$ (b) 1

(c) 2 (d) π

8. If $u_n = \sin n\theta \sec^n \theta$, $v_n = \cos n\theta \sec^n \theta$ $n \neq 1$ then $\frac{v_n - v_{n-1}}{u_{n-1}} + \frac{1}{n}\frac{u_n}{v_n}$ is equal to

(a) 0 (b) $\tan\theta$

(c) $-\tan\theta + \frac{\tan n\theta}{n}$ (d) $\tan\theta + \frac{\tan n\theta}{n}$

9. Let n be a fixed positive integer such that $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$, then

(a) $n = 4$ (b) $n = 5$

(c) $n = 6$ (d) none of these

10. If $\alpha = \frac{2\pi}{7}$, then the value of $\tan\alpha . \tan 2\alpha + \tan 2\alpha \tan 4\alpha + \tan 4\alpha \tan\alpha$ is

(a) -7 (b) -4

(c) 0 (d) 4

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ

11. If $a\sin x + b\cos(x+\theta) + b\cos(x-\theta) = d$ for some real x, then the minimum value of $|\cos\theta|$ is equal to

(a) $\frac{1}{2|b|}\sqrt{d^2-a^2}$ (b) $\frac{1}{2|a|}\sqrt{d^2-a^2}$

(c) $\frac{1}{2|d|}\sqrt{d^2-a^2}$ (d) $\frac{1}{|a|}\sqrt{d^2-a^2}$

12. In ΔABC, if $\cot\theta = \cot A + \cot B + \cot C$, then $\sin(A-\theta).\sin(B-\theta).\sin(C-\theta) =$

(a) $\sin^3\theta$ (b) $\sin A\sin B\sin C$

(c) $3\sin\theta$ (d) 1

13. If $K_1 = \tan 27\theta - \tan\theta$ and

$K_2 = \frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}$, then

(a) $K_1 = 2K_2$ (b) $K_1 = K_2 + 2$

(c) $K_1 = K_2$ (d) none of these

14. The value of $\sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$ is

(a) 1 (b) $\frac{\sqrt{7}}{2}$

(c) $\frac{3\sqrt{3}}{4}$ (d) $\frac{\sqrt{15}}{4}$

15. If $\tan\alpha = \frac{x^2-x}{x^2-x+1}$ and $\tan\beta = \frac{1}{2x^2-2x+1}$,

$0 < \alpha, \beta < \frac{\pi}{2}$ then $\alpha + \beta =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{3\pi}{4}$

16. If $\alpha, \beta, \gamma, \delta$ are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive quantity k, then the value of

$4\sin\frac{\alpha}{2} + 3\sin\frac{\beta}{2} + 2\sin\frac{\gamma}{2} + \sin\frac{\delta}{2}$ is equal to

(a) $2\sqrt{1-k}$ (b) $2\sqrt{1+k}$

(c) $2\sqrt{k}$ (d) none of these

17. If $\tan\theta = n\tan\phi$, then minimum value of $\tan^2(\theta-\phi)$ is :

(a) $\frac{(n+1)^2}{4n}$ (b) $\frac{(n-1)^2}{4n}$

(c) $\frac{(2n+1)^2}{4n}$ (d) $\frac{(2n-1)^2}{4n}$

18. The acute angle of a rhombus whose side is a mean proportional between its diagonals is

(a) 15° (b) 20°

(c) 30° (d) 80°

19. The summation of the series

$\tan\alpha\tan 2\alpha + \tan 2\alpha\tan 3\alpha + \tan 3\alpha\tan 4\alpha + ... + \tan n\alpha\ \tan(n+1)\alpha$ is

(a) $\cot\alpha\tan(n+1)\alpha - n - 1$ (b) $\tan\alpha\tan(n+1)\alpha - 1$

(c) $n\cot\alpha - \tan(n+1)\alpha$ (d) none of these

20. If $a_1\cos\alpha_1 + a_2\cos\alpha_2 + ... + a_n\cos\alpha_n = 0$ and

$a_1\cos(\alpha_1+\theta) + a_2\cos(\alpha_2+\theta) + ... + a_n\cos(\alpha_n+\theta) = 0$

for all values of θ, then

$a_1\cos(\alpha_1+\lambda) + a_2\cos(\alpha_2+\lambda) + ... + a_n\cos(\alpha_n+\lambda) =$

(a) 0 (b) λ

(c) $\theta+\lambda$ (d) $\theta-\lambda$

21. If $\cos\alpha = \frac{2\cos\beta - 1}{2-\cos\beta}$ $(0 < \alpha < \beta < \pi)$, then $\frac{\tan\alpha/2}{\tan\beta/2}$ is equal to

(a) 1 (b) $\sqrt{2}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

22. If ABC is an acute angle triangle, then the least value of tan A tan B tan C is

(a) 3 (b) $3\sqrt{3}$

(c) 1 (d) $3^{-3/2}$

23. If $x\sin a + y\sin 2a + z\sin 3a = \sin 4a$

$x\sin b + y\sin 2b + z\sin 3b = \sin 4b$

$x\sin c + y\sin 2c + z\sin 3c = \sin 4c$

Then, the roots of the equation

$t^3 - \frac{z}{2}t^2 - \frac{y+2}{4}t + \frac{z-x}{8} = 0$, $a, b, c \neq n\pi$, are

(a) sin a, sin b, sin c (b) cos a, cos b, cos c

(c) sin $2a$, sin $2b$, sin $2c$ (d) cos $2a$, cos $2b$, cos $2c$

MARK YOUR RESPONSE					
	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ
	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ
	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ		

24. If $xy + yz + zx = 1$, then $\frac{x}{1-x^2} + \frac{y}{1-y^2} + \frac{z}{1-z^2} =$

(a) $\frac{xyz}{(1-x^2)(1-y^2)(1-z^2)}$

(b) $\frac{2xyz}{(1-x^2)(1-y^2)(1-z^2)}$

(c) $\frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}$

(d) none of these

25. If $a_{r+1} = \sqrt{\frac{1}{2}(1+a_r)}$, then $\cos\left(\frac{\sqrt{1-a_0^2}}{a_1\, a_2\, a_3 ....to\ \infty}\right) =$

(a) 1 (b) 0

(c) a_0 (d) $\frac{1}{2}$

26. Let A, B, C be three angles such that $A = \frac{\pi}{4}$ and tan B tan $C = p$. The set of all possible values of p such that A, B, C are three angles of a triangle is

(a) $(-\infty, 3-2\sqrt{2}] \cup [3+2\sqrt{2}, \infty)$

(b) $[3-2\sqrt{2}, 3+2\sqrt{2}]$

(c) $(-\infty, \sqrt{2}-1] \cup [2, \infty)$

(d) $(-\infty, 0) \cup [3+2\sqrt{2}, \infty)$

27. If the mapping $f(x) = ax + b, a < 0$ maps [– 1, 1] onto [0, 2], then for all values of θ, $A = \cos^2\theta + \sin^4\theta$ is such that

(a) $f\left(\frac{1}{4}\right) \le A \le f(0)$ (b) $f(0) \le A \le f(-2)$

(c) $f\left(\frac{1}{3}\right) \le A \le f(0)$ (d) $f(-1) < A \le f(-2)$

28. If $0 < \theta < \pi$, then

(a) $1 + \cot\theta \le \cot\frac{\theta}{2}$ (b) $1 + \cot\theta \ge \cot\frac{\theta}{2}$

(c) $1 + \cot\frac{\theta}{2} \ge \cot\theta$ (d) $1 + \cot\frac{\theta}{2} \le \cot\theta$

29. If $A + B + C = \pi$, then the greatest value of $\cos A + \cos B + \cos C$ is

(a) 2 (b) 3

(c) $\frac{3}{2}$ (d) 1

30. The minimum value of $3\tan^2\theta + 12\cot^2\theta$ is

(a) 6 (b) 15

(c) 24 (d) none of these

31. If $\sin x + \cos x + \tan x + \cot x + \sec x + \text{cosec } x = 7$ and $\sin 2x = a - b\sqrt{7}$, then ordered pair (a, b) can be,

(a) $(6, 2)$ (b) $(8, 3)$

(c) $(22, 8)$ (d) $(11, 4)$

32. If $\tan x - \tan^2 x = 1$, then the value of $\tan^4 x - 2\tan^3 x - \tan^2 x + 2\tan x + 1$ is

(a) 1 (b) 2

(c) 3 (d) 4

33. The value of $\cot^2 36° \cot^2 72°$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

34. The minimum value of the expression is $3^{\sin^6 x} + 3^{\cos^6 x}$

(a) $2.3^{1/8}$ (b) $2.3^{7/8}$

(c) $3.2^{1/8}$ (d) 6

35. If $T_n = \sin^n\theta + \cos^n\theta$, and $\frac{T_6 - T_4}{T_6} = m$ then

(a) $m \in \left[-1, \frac{1}{3}\right]$ (b) $m \in \left[0, \frac{1}{3}\right]$

(c) $m \in [-1, 0]$ (d) $m \in [0, 1]$

MARK YOUR RESPONSE					
	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ
	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ
	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ			

36. If $\sin\theta + \sin\phi = \sqrt{3}(\cos\phi - \cos\theta)$, then $\sin 3\theta + \sin 3\phi$ is equal to

(a) $\sqrt{3}$ (b) 0
(c) 1 (d) -1

37. $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$ equals

(a) $\frac{1}{4}$ (b) $-\frac{1}{8}$
(c) $\frac{1}{8}$ (d) $\frac{1}{64}$

38. If $\cos x - \frac{\cot\beta \sin x}{2} = \frac{\sqrt{3}}{2}$, then the value of $\tan\frac{x}{2}$ is

(a) $\tan\frac{\beta}{2}\tan 15°$ (b) $\tan\frac{\beta}{2}$
(c) $\tan 15°$ (d) none of these

39. If the expression $n\sin^2\theta + 2n\cos(\theta+\alpha)\sin\alpha\sin\theta + \cos 2(\alpha+\theta)$ is independent of 'θ', then the value of n is

(a) 1 (b) 2
(c) 3 (d) 4

40. $\sec\theta$ and $\text{cosec}\,\theta\left(0<\theta<\frac{\pi}{2}\right)$ may be the roots of the equation

(a) $2x^2 - x + 1 = 0$
(b) $3x^2 - 8x + 5 = 0$
(c) $3x^2 - 10x + 6 = 0$
(d) $x^2 - \sqrt{15}x + 3 = 0$

41. If $A + B = \frac{\pi}{4}$, then maximum value of $\cos A\cos B$ is equal to

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1+\sqrt{2}}{2\sqrt{2}}$
(c) $\frac{\sqrt{2}-1}{2\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

42. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then $\cos\theta$, $\sin\theta$ are roots of the equation

(a) $4x^2 - 4x - 1 = 0$ (b) $4x^2 - 2x - 1 = 0$
(c) $8x^2 - 4x - 3 = 0$ (d) $x^2 - x - 1 = 0$

43. Value of $\tan^6 40° - 33\tan^4 40° + 27\tan^2 40°$ is

(a) 1 (b) 2
(c) 3 (d) -2

44. If $\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)} + \frac{\cos(\theta_3+\theta_4)}{\cos(\theta_3-\theta_4)} = 0$, then $\tan\theta_1\tan\theta_2\tan\theta_3\tan\theta_4$ equal to

(a) 0 (b) 1
(c) -1 (d) 2

45. If $\cos(\theta-\phi), \cos\theta, \cos(\theta+\phi)$ are in H.P., then $\cos\theta\sec\frac{\phi}{2}$ is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) 1 (d) -1

46. The product $\cos\left\{\frac{2\pi}{2^{64}-1}\right\}\cos\left\{\frac{2^2\pi}{2^{64}-1}\right\}\ldots\cos\left\{\frac{2^{64}\pi}{2^{64}-1}\right\}$

(a) $\frac{1}{16^{16}}$ (b) $\frac{1}{8^8}$
(c) $\frac{1}{32^{32}}$ (d) $\frac{1}{64^{64}}$

47. If $\frac{\tan(\alpha+\beta-\gamma)}{\tan(\alpha-\beta+\gamma)} = \frac{\tan\gamma}{\tan\beta}$, then $\sin 2\alpha + \sin 2\beta + \sin 2\gamma$ is equal to

(a) 0 (b) 1
(c) 3 (d) -1

MARK YOUR RESPONSE					
	36.ⓐⓑⓒⓓ	37.ⓐⓑⓒⓓ	38.ⓐⓑⓒⓓ	39.ⓐⓑⓒⓓ	40.ⓐⓑⓒⓓ
	41.ⓐⓑⓒⓓ	42.ⓐⓑⓒⓓ	43.ⓐⓑⓒⓓ	44.ⓐⓑⓒⓓ	45.ⓐⓑⓒⓓ
	46.ⓐⓑⓒⓓ	47.ⓐⓑⓒⓓ			

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

Complex numbers can be used for finding out the sums of the two series of the form

$C = a_0 \cos\theta + a_1 \cos(\theta+\phi) + a_2 \cos(\theta+2\phi) + ...$ and

$S = a_0 \sin\theta + a_1 \sin(\theta+\phi) + a \sin(\theta+2\phi) + ...$

provided the sum of the series

$f(x) = a_0 + a_1 x + a_0 x^2 + ...$ is known

Suppose that series from C and S are infinite, then we have

$C + iS = a_0(\cos\theta + i\sin\theta) + a_1\{\cos(\theta+\phi) + i\sin(\theta+\phi)\} + ...$

$= a_0 e^{i\theta} + a_1 e^{i(\theta+\phi)} + a_2 e^{i(\theta+2\phi)} + ...$

$= e^{i\theta}\left[a_0 + a_1 e^{i\phi} + ae^{2i\phi} + ...\right] = e^{i\theta}[a_0 + a_1 x + a_2 x^2 + ...]$

$= e^{i\theta} f(e^{i\phi})$

Similarly $C - iS = e^{-i\theta} f(e^{-i\phi})$

$\therefore\ C = \frac{1}{2}\left[e^{i\theta} f(e^{i\theta}) + e^{-i\theta} f(e^{-i\phi})\right]$

$S = \frac{1}{2i}\left[e^{i\theta} f(e^{i\phi}) - e^{-i\theta} f(e^{-i\phi})\right]$

1. If $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ then

   $1 + \frac{1}{2}\cos 2\alpha - \frac{1}{2\cdot 4}\cos 4\alpha + \frac{1.3}{2\cdot 4\cdot 6}\cos 6\alpha - .....$ to inf is equal to

   (a) $\sqrt{\cos\alpha(1+\cos\alpha)}$ (b) $\sqrt{\cos\alpha\cos\frac{\alpha}{2}}$

   (c) $\sqrt{2\cos\alpha\cos\frac{\alpha}{2}}$ (d) $\sqrt{2\cos\alpha\sin\frac{\alpha}{2}}$

2. If $a < c$ in a ΔABC, then the sum to infinite terms of the series $n.\frac{a}{c}\sin B + \frac{n(n+1)}{2!}\frac{a^2}{c^2}\sin 2B + ...$ is

   (a) $\frac{c^n}{b^n}\sin nA$ (b) $\frac{a^n}{b^n}\sin nA$

   (c) $\frac{b^n}{c^n}\sin nA$ (d) $\sin nA$

3. The sum to infinite terms of the series

   $\cos\frac{\pi}{3} + \frac{1}{2}\cos\frac{2\pi}{3} + \frac{1}{3}\cos\frac{3\pi}{3} + ...$ is

   (a) $-\log 2$ (b) 1

   (c) -1 (d) 0

PASSAGE-2

For each natural number k, let C_k denote the circle with radius k centimeter and centre at the origin. On the circle C_k, a particle moves k centimeter in the counter clockwise direction. After completing its motion on C_k, the particle moves to C_{k+1}, in the radial direction.The motion of the particle continues in this manner. The particle starts at (1, 0).

4. If the particle crosses the positve direction of the y-axis for the first time on the circle C_m then m equals to
   (a) 1
   (b) 2
   (c) 3
   (d) It jumps from C_2 to C_3 at y axis

5. If the particle crosses the positive direction of the x-axis for the first time on the circle C_n then n equals to
   (a) 4 (b) 6
   (c) 7 (d) It jumps from C_7 to C_8

6. When the particle crosses the positive direction of the x-axis for the first time, the distance travelled by it in centimeter is
   (a) 27 (b) 14π
   (c) $27 + \frac{\pi}{4}$ (d) $14\pi - 15$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ				

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options:

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is true but Statement-2 is false.
(d) Statement-1 is false but Statement-2 is true.

1. Statement-1 : $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7} = -\frac{1}{8}$

Statement-2 : $\cos\theta\cos 2\theta\cos 2^2\theta$.............

$$\cos 2^{n-1}\theta = \frac{-1}{2^n} \text{ if } \theta = \frac{\pi}{2^n - 1}$$

2. Statement-1 : $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\tan 8\alpha - 16\cot 16\alpha = \cot\alpha$

Statement-2 : $\cot\alpha - \tan\alpha = 2\cot 2\alpha$

3. Statement-1 : The maximum and minimum values of the function $f(x) = \frac{1}{6\sin x - 8\cos x + 5}$ does not exist

Statement-2 : The given function is an unbounded function.

4. Statement-1 : There exist no real x and y such that $\sec^2\theta = \frac{4xy}{(x+y)^2}$ for any θ

Statement-2 : For all θ, $\sec^2\theta \geq 1$

5. Statement-1 : Let $\tan\theta = \sqrt{n}$, where n is a positive integer. $\sec 2\theta$ is a rational number only if n is a perfect square.

Statement-2 : If n is a perfect square then $\tan\theta$ is rational.

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. If $\frac{\tan 3A}{\tan A} = k, (k \neq 1)$, then

(a) $\frac{\cos A}{\cos 3A} = \frac{k^2 - 1}{2k}$ (b) $\frac{\sin 3A}{\sin A} = \frac{2k}{k-1}$

(c) $k < \frac{1}{3}$ (d) $k > 3$

2. Let $f_n(\theta) = \tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)$, ...$(1+\sec 2^n\theta)$ then

(a) $f_2\left(\frac{\pi}{16}\right) = 1$ (b) $f_3\left(\frac{\pi}{32}\right) = 1$

(c) $f_4\left(\frac{\pi}{64}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ			

(a) 1 (b) – 1
(c) 2 (d) – 2

3. Given that $\sin\beta = \frac{12}{13}, 0 < \beta < \pi$, then $\{5\sin(\alpha+\beta) - 12\cos(\alpha+\beta)\}\operatorname{cosec}\alpha$ is equal to :

(a) 13 if $\tan\beta > 0$
(b) 13 if $\tan\beta < 0$
(c) $\frac{119 + 120\cot\alpha}{13}$ if $\tan\beta < 0$
(d) $\frac{119 + 120\cot\alpha}{13}$ if $\tan\beta > 0$

4. If $(a-b)\sin(\theta+\phi) = (a+b)\sin(\theta-\phi)$ and $a\tan\frac{\theta}{2} - b\tan\frac{\phi}{2} = c$, then

(a) $b\tan\phi = a\tan\theta$ (b) $a\tan\phi = b\tan\theta$
(c) $\sin\phi = \frac{2bc}{a^2 - b^2 - c^2}$ (d) $\sin\theta = \frac{2ac}{a^2 - b^2 + c^2}$

5. In a triangle ABC

(a) $\sin A\sin B\sin C \le \frac{3\sqrt{3}}{8}$
(b) $\sin^2 A + \sin^2 B + \sin^2 C \le \frac{9}{4}$
(c) $\sin A \sin B \sin C$ is always positive
(d) $\sin^2 A + \sin^2 B \le 1 + \cos C$

6. $\frac{\sin 3\alpha}{\cos 2\alpha}$ is

(a) negative if $\alpha \in \left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$
(b) negative if $\alpha \in \left(\frac{14\pi}{48}, \frac{18\pi}{48}\right)$
(c) Positive if $\alpha \in \left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$
(d) positive if $\alpha \in \left(\frac{14\pi}{48}, \frac{18\pi}{48}\right)$

7. If $\cos^4\theta + \alpha, \sin^4\theta + \alpha$ are the roots of the equation $x^2 + b(2x+1) = 0$ and $\cos^2\theta + \beta, \sin^2\theta + \beta$ are the roots of the equation $x^2 + 4x + 2 = 0$ then b is equal to

8. If α, β, γ and δ are four solutions of the equation $\tan\left(\theta + \frac{\pi}{4}\right) = 3\tan 3\theta$, then

(a) $\sum \tan\alpha = 0$
(b) $\sum \tan\alpha\tan\beta = -2$
(c) $\sum \tan\alpha\tan\beta\tan\gamma = \frac{-8}{3}$
(d) $\tan\alpha\tan\beta\tan\gamma\tan\delta = -3$

9. Integral values of a satisfying the equation $[\sin x]^2 + \sin x - 2a = 0$ are (where [.] denotes the greatest integer function)

(a) –1 (b) 0
(c) 1 (d) 2

10. If $A + B = \frac{\pi}{3}$ and $\cos A + \cos B = 1$ then which of the following is/are true.

(a) $\cos(A-B) = \frac{1}{\sqrt{3}}$
(b) $|\cos A - \cos B| = \sqrt{\frac{2}{3}}$
(c) $\cos(A-B) = -\frac{1}{3}$
(d) $|\cos A - \cos B| = \frac{1}{2\sqrt{3}}$

11. If $\sqrt{1-\sin A} = \sin\frac{A}{2} - \cos\frac{A}{2}$, then $\frac{A}{2} - \frac{\pi}{4}$ can lie in

(a) first quadrant (b) second quadrant
(c) third quadrant (d) fourth quadrant

12. If $\sin x + \cos x = \sqrt{y + \frac{1}{y}}$ for $x \in [0, \pi]$ and $y > 0$, then

(a) $x = \frac{\pi}{4}$ (b) $y = 0$
(c) $y = 1$ (d) $x = \frac{3\pi}{4}$

MARK YOUR RESPONSE					
	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ
	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ

13. Which of the following number (s) is/are irrational

(a) $\sin 15°$ (b) $\cos 15°$

(c) $\sin 15° \cos 15°$ (d) $\cos 15° \sin 75°$

14. The graph of the function $\cos x \cos(x+2) - \cos^2(x+1)$ is

(a) A straight line passing through (0, 0)

(b) A straight line passing through $\left(\frac{\pi}{2}, -\sin^2 1\right)$ and paralles to x-axis

(c) A straight line passing through $(0, -\sin^2 1)$

(d) not a straight line

MARK YOUR RESPONSE	13. (a)(b)(c)(d)	14. (a)(b)(c)(d)			

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

1. Observe the following columns :

Column-I	**Column-II**
(A) $\cos^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$ equals	p. 1
(B) $\cos^2 \frac{3\pi}{5} + \cos^2 \frac{4\pi}{5}$ equals	q. $\frac{3-\sqrt{3}}{4\sqrt{2}}$
(C) $\sin 24° + \cos 6°$ equals	r. $\frac{3}{4}$
(D) $\sin^2 50° + \cos^2 130°$ equals	s. $\frac{\sqrt{15}+\sqrt{3}}{4}$

2. Observe the following columns :

Column-I	**Column-II**
(A) If $f(\theta) = (\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$, then $f(\theta)$ cannot be less than	p. 1
(B) If $\sin\alpha - \sin\beta = a$ and $\cos\alpha + \cos\beta = b$ then $a^2 + b^2$ cannot exceed	q. 2
(C) If $A + B = \frac{\pi}{2}$, where A and B are positive then $(\sin A + \sin B) \cos\frac{\pi}{4}$ is always less than	r. 4
(D) If $2\cos x + \sin x = 1$, then the value of $7\cos x + 6\sin x$ is equal to	s. 6

MARK YOUR RESPONSE

1.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

2.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

3. Observe the following columns :

Column-I	Column-II
(A) If sinθ and cosθ are the roots of the equation $ax^2 - bx + c = 0$, then $\cos^{-1}(a^2 - b^2 + 2ac)$ is equal to	p. $\frac{\pi}{6}$
(B) The expression $\cos y \cos\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-y\right)\cos x+\sin y\cos\left(\frac{\pi}{2}-x\right)+\cos x\sin\left(\frac{\pi}{2}-y\right)$ vanishes if $\lvert x-y\rvert$ is equal to	q. $\frac{\pi}{4}$
(C) If A and B are positive angles such that $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$ then $A + 2B$ is equal to	r. $\frac{\pi}{2}$
(D) If A and B are acute angles and $A + B$ and $A - B$ satisfy the equation $\tan^2\theta - 4\tan\theta + 1 = 0$ then B is equal to	s. $\frac{3\pi}{4}$

4. Observe the following columns :

Column-I	Column-II
(A) The values of $\cos^2\theta + \sin^4\theta$ for all θ	p. belong to (0, 1]
(B) In a ΔABC if $\tan A < 0$ then values of $\tan B \tan C$	q. belong to $\left[\frac{3}{4},1\right]$
(C) For any real $\theta \neq n\pi,\ n \in I$ then values of $\frac{\cos^2\theta - 1}{\cos^2\theta + \cos\theta}$	r. are less than 0 or greater than 2
(D) If $A > 0, B > 0$ and $A + B = \frac{\pi}{3}$ then the values of $3\tan A\tan B$	s. belong to (0, 1)

MARK YOUR RESPONSE

3.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

4.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

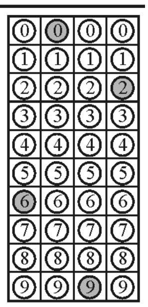
F NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010 etc.) or a single-digit integer, ranging from 0 to 9.

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to a question is 6092, then the correct darkening of bubbles will look like the given.

For single digit integer answer darken the extreme right bubble only.



1. If $\cot(\theta-\alpha), 3\cot\theta, \cot(\theta+\alpha)$ are in A.P., and θ is not an integral multiple of $\frac{\pi}{2}$, then $\frac{2\sin^2\theta}{\sin^2\alpha}$ is equal to

2. If ABC is a triangle then the least value of $\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}$ is equal to

3. If in the triangle ABC, $\tan\frac{A}{2}$, $\tan\frac{B}{2}$ and $\tan\frac{C}{2}$ are in harmonic progression then the least value of $\cot^2\frac{B}{2}$ is equal to

4. If a $\tan\alpha+\sqrt{a^2-1}\tan\beta+\sqrt{a^2+1}\tan\gamma=2a$, where a is constant and α, β, γ are variable angles. Then the least value of $3(\tan^2\alpha+\tan^2\beta+\tan^2\gamma)$ is equal to

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MARK YOUR RESPONSE

1. 2. 3. 4.

Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(c)	**9**	(c)	**17**	(b)	**25**	(c)	**33**	(d)	**41**	(b)
2	(a)	**10**	(a)	**18**	(c)	**26**	(d)	**34**	(a)	**42**	(c)
3	(d)	**11**	(a)	**19**	(a)	**27**	(a)	**35**	(c)	**43**	(c)
4	(c)	**12**	(a)	**20**	(a)	**28**	(a)	**36**	(b)	**44**	(c)
5	(b)	**13**	(a)	**21**	(c)	**29**	(c)	**37**	(d)	**45**	(b)
6	(c)	**14**	(b)	**22**	(b)	**30**	(d)	**38**	(a)	**46**	(a)
7	(b)	**15**	(a)	**23**	(b)	**31**	(c)	**39**	(b)	**47**	(a)
8	(c)	**16**	(b)	**24**	(c)	**32**	(d)	**40**	(d)		

B COMPREHENSION TYPE

1	(a)	**2**	(a)	**3**	(d)	**4**	(b)	**5**	(c)	**6**	(d)

C REASONING TYPE

1	(a)	**2**	(a)	**3**	(a)	**4**	(d)	**5**	(d)

D MULTIPLE CORRECT CHOICE TYPE

1	(b,c,d)	**4**	(b, c, d)	**7**	(b,c)	**10**	(b,c)	**13**	(a,b,d)
2	(a,b,c,d)	**5**	(a,b,c,d)	**8**	(a, b,c)	**11**	(a,b)	**14**	(b,c)
3	(a,c)	**6**	(a, c) (i)	**9**	(b,c)	**12**	(a,c)		

E MATRIX-MATCH TYPE

1. **A - q; B - r; C - s; D - p**

2. **A - p,q,r,s; B - r,s; C - q,r,s; D - q, s**

3. **A - r; B - s; C - r; D - p**

4. **A - q; B - p, s; C - r; D - p**

F NUMERIC/INTEGER ANSWER TYPE

1	3	**2**	1	**3**	3	**4**	4

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(c)** Let $P = 2 - \cos x + \sin^2 x$
$= 2 - \cos x + 1 - \cos^2 x = 3 - (\cos^2 x + \cos x)$

$$= 3 - \left\{\left(\cos x + \frac{1}{2}\right)^2 - \frac{1}{4}\right\} = \frac{13}{4} - \left(\cos x + \frac{1}{2}\right)^2$$

$P_{\max} = \frac{13}{4}$, when $\cos x = \frac{1}{2}$

$P_{\min} = \frac{13}{4} - \frac{9}{4} = 1$, when $\cos x = 1$

$\therefore \frac{P_{\max}}{P_{\min}} = \frac{13}{4}$

2. **(a)** Given $a \cos^2 3\alpha + b \cos^4 \alpha = 16 \cos^6 \alpha + 9\cos^2 \alpha$

$\Rightarrow 9\cos^2\alpha + 16\cos^6\alpha$

$$= a(4\cos^3\alpha - 3\cos\alpha)^2 + b\cos^4\alpha$$

$\Rightarrow 16(a-1)\cos^6\alpha + (b - 24a)\cos^4\alpha$

$$+ 9(a-1)\cos^2\alpha = 0$$

Above is an identity, so

$a = 1,\ b - 24a = 0 \Rightarrow a = 1, b = 24$

3. **(d)** From the given relation we have

$\sin x + \sin\left(\frac{\pi}{2} - y\right) = a$ and $\cos x + \cos\left(\frac{\pi}{2} - y\right) = b$

$\Rightarrow 2\sin\frac{x + (\pi/2) - y}{2}\cos\frac{x - (\pi/2) + y}{2} = a$ and

$2\cos\frac{x + (\pi/2) - y}{2}\cos\frac{x - (\pi/2) + y}{2} = b$

Dividing we get

$$\tan\left(\frac{\pi}{4} + \frac{x-y}{2}\right) = \frac{a}{b} \Rightarrow \frac{1 + \tan\frac{x-y}{2}}{1 - \tan\frac{x-y}{2}} = \frac{a}{b}$$

or $\tan\frac{x-y}{2} = \frac{a-b}{a+b}$

4. **(c)** We have $\sec^2(a+2)x = 1 - a^2$

$\because \sec^2 x \ge 1 \Rightarrow (1 - a^2) \ge 1 \Rightarrow a = 0$

So, $\sec^2(a+2)x = 1 \Rightarrow \sec^2 2x = 1$

$2x = n\pi \Rightarrow x = \frac{n\pi}{2}$ or $x = -\frac{\pi}{2}, 0, \frac{\pi}{2}$

5. **(b)** $\sin\alpha, \sin\beta, \cos\alpha$ are in G.P.

$\Rightarrow \sin^2\beta = \sin\alpha\cos\alpha \Rightarrow \cos 2\beta = 1 - \sin 2\alpha \ge 0$

Now, the discriminant of the given equation is

$4\cot^2\beta - 4 = 4\cos 2\beta.\operatorname{cosec}^2\beta \ge 0 \Rightarrow$ Roots are always real

Product of roots = 1 $\Rightarrow$ Both roots $\not> 1$

6. **(c)** The given expression is equal to

$$x^2 + xy(\tan^2\alpha + \cot^2\alpha) + y^2 - 4xy\frac{\cos^2 2\alpha}{\sin^2 2\alpha}$$

$$= x^2 + y^2 + xy\left[\frac{\sin^4\alpha + \cos^4\alpha}{\sin^2\alpha\cos^2\alpha} - \frac{4(\cos^2\alpha - \sin^2\alpha)^2}{4\sin^2\alpha\cos^2\alpha}\right]$$

$= x^2 + y^2 + xy\left[\frac{2\sin^2\alpha\cos^2\alpha}{\sin^2\alpha\cos^2\alpha}\right] = (x+y)^2$ which is independent of α.

7. **(b)** $f(x) = \frac{1}{2}(1 - \cos 2x) + \frac{1}{2}\left\{1 - \cos\left(2x + \frac{2\pi}{3}\right)\right\}$

$$+ \frac{1}{2}\left[\cos\left(2x + \frac{\pi}{3}\right) + \cos\frac{\pi}{3}\right]$$

$$= \frac{5}{4} - \frac{1}{2}\left\{\cos 2x + \cos\left(2x + \frac{2\pi}{3}\right) - \cos\left(2x + \frac{\pi}{3}\right)\right\}$$

$$= \frac{5}{4} - \frac{1}{2}\left\{2\cos\left(2x + \frac{\pi}{3}\right)\cos\frac{\pi}{3} - \cos\left(2x + \frac{\pi}{3}\right)\right\} = \frac{5}{4}$$

for all x

$\therefore f\left(\frac{\pi}{8}\right) = \frac{5}{4}$ and therefore g of $\left(\frac{\pi}{8}\right) = g\left(\frac{5}{4}\right) = 1$

8. **(c)** We have $\frac{u_n}{v_n} = \tan n\theta$ and

$$\frac{v_n - v_{n-1}}{u_{n-1}} = \frac{\cos n\theta \sec^n\theta - \cos(n-1)\theta\sec^{n-1}\theta}{\sin(n-1)\theta\sec^{n-1}\theta}$$

$$= \frac{\cos n\theta \sec\theta - \cos(n-1)\theta}{\sin(n-1)\theta} = \frac{\cos n\theta - \cos(n-1)\theta\cos\theta}{\cos\theta\sin(n-1)\theta}$$

$$= \frac{\cos(n-1)\theta\cos\theta - \sin(n-1)\theta\sin\theta - \cos(n-1)\theta\cos\theta}{\cos\theta\sin(n-1)\theta}$$

$= -\tan\theta$

so that $\frac{v_n - v_{n-1}}{u_{n-1}} + \frac{1}{n}\frac{u_n}{v_n} = -\tan\theta + \frac{\tan n\theta}{n} \neq 0$

9. **(c)** $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right)$

$\Rightarrow \frac{\sqrt{n}}{2} = \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right)$

so for $n > 1$, $\frac{\sqrt{n}}{2\sqrt{2}} = \sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) > \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

then $n > 4$

Since $\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) < 1$ for all $n > 2$, we get $\frac{\sqrt{n}}{2\sqrt{2}} < 1$

or $n < 8$

so that $4 < n < 8$. By acual verification we find that only $n = 6$ satisfies the given relation.

10. **(a)** We have $\tan\alpha\tan2\alpha + \tan2\alpha\tan4\alpha + \tan4\alpha\tan\alpha$

$$= \frac{\sin\alpha\sin2\alpha\cos4\alpha + \sin2\alpha\sin4\alpha\cos\alpha + \sin4\alpha\sin\alpha\cos2\alpha}{\cos\alpha\cos2\alpha\cos4\alpha}$$

$$= \frac{\cos\alpha\cos2\alpha\cos4\alpha - \cos(\alpha+2\alpha+4\alpha)}{\cos\alpha\cos2\alpha\cos4\alpha}$$

[Using formula of $\cos(A+B+C)$]

$$= 1 - \frac{\cos7\alpha}{\cos\alpha\cos2\alpha\cos4\alpha}$$

$$= 1 - \frac{(\cos2\pi)(2\sin\alpha)}{2\sin\alpha\cos\alpha\cos2\alpha\cos4\alpha} \quad \left[\because \alpha = \frac{2\pi}{7}\right]$$

$$= 1 - \frac{4\sin\alpha}{2\sin2\alpha\cos2\alpha\cos4\alpha} = 1 - \frac{8\sin\alpha}{2\sin4\alpha\cos4\alpha}$$

$$= 1 - \frac{8\sin\alpha}{\sin8\alpha} = 1 - \frac{8\sin\alpha}{\sin(2\pi+\alpha)} = -7$$

11. **(a)** $a\sin x + b\{\cos(x+\theta) + \cos(x-\theta)\} = d$

$\Rightarrow a\sin x + 2b\cos x\cos\theta = d$

$\Rightarrow |d| \leq \sqrt{a^2 + 4b^2\cos^2\theta}$

$\Rightarrow \cos^2\theta \geq \frac{d^2 - a^2}{4b^2} \Rightarrow |\cos\theta| \geq \frac{\sqrt{d^2 - a^2}}{2|b|}$

12. **(a)** Given $\cot\theta = \cot A + \cot B + \cot C$

$\Rightarrow \cot\theta - \cot A = \cot B + \cot C$

$\Rightarrow \frac{\cos\theta\sin A - \sin\theta\cos A}{\sin\theta\sin A} = \frac{\cos B\sin C + \sin B\cos C}{\sin B\sin C}$

$\Rightarrow \sin(A-\theta) = \frac{\sin A\sin\theta\sin(B+C)}{\sin B\sin C} = \frac{\sin^2 A\sin\theta}{\sin B\sin C}$

[$\because A + B + C = \pi$]

Similarly, $\sin(B-\theta) = \frac{\sin^2 B\sin\theta}{\sin A\sin C}$ and

$\sin(C-\theta) = \frac{\sin^2 C\sin\theta}{\sin A\sin B}$

$\therefore \sin(A-\theta)\sin(B-\theta).\sin(C-\theta) = \sin^3\theta$

13. **(a)** We have $K_1 = \tan27\theta - \tan\theta = (\tan27\theta - \tan9\theta) + (\tan9\theta - \tan3\theta) + (\tan3\theta - \tan\theta)$

Now, $\tan3\theta - \tan\theta = \frac{\sin2\theta}{\cos3\theta\cos\theta} = \frac{2\sin\theta}{\cos3\theta}$

Similarly, $\tan9\theta - \tan3\theta = \frac{2\sin3\theta}{\cos9\theta}$ and

$\tan27\theta - \tan9\theta = \frac{2\sin9\theta}{\cos27\theta}$

$\therefore K_1 = 2\left[\frac{\sin9\theta}{\cos27\theta} + \frac{\sin3\theta}{\cos9\theta} + \frac{\sin\theta}{\cos3\theta}\right] = 2K_2$

14. **(b)** Let $S = \sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$ and

$C = \cos\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$

Then $C + iS = \alpha + \alpha^2 + \alpha^4$ ...(i)

where $\alpha = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$ is complex 7th root of unity

and

$C - iS = \alpha^6 + \alpha^5 + \alpha^3$ ...(ii)

$\because \alpha^6 = \overline{\alpha}$, etc

Add (i) and (ii)

$2C = \alpha + \alpha^2 + \alpha^3 + ... + \alpha^6 = \frac{\alpha^7 - \alpha}{\alpha - 1} = -1 \quad (\because \alpha^7 = 1)$

$\therefore C = -\frac{1}{2}$

Also Multiple (i) and (ii), $C^2 + S^2 = 2 \Rightarrow S = \frac{\sqrt{7}}{2}$

15. (a) $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\dfrac{\dfrac{a}{a+1}+\dfrac{1}{2a+1}}{1-\dfrac{a}{(a+1)(2a+1)}}$;

where $x^2-x=a$

$$=\frac{\dfrac{a}{a+1}+\dfrac{1}{2a+1}}{1-\left\{\dfrac{1}{a+1}-\dfrac{1}{2a+1}\right\}}=\frac{\dfrac{a}{a+1}+\dfrac{1}{2a+1}}{\dfrac{a}{a+1}+\dfrac{1}{2a+1}}=1$$

Hence $\alpha+\beta=\dfrac{\pi}{4}$ (Note : $\alpha+\beta$ can not be $\dfrac{5\pi}{4}$)

16. (b) $\alpha<\beta<\gamma<\delta$ and $\sin\alpha=\sin\beta=\sin\gamma=\sin\delta=k$

$\Rightarrow\beta=\pi-\alpha,\ \gamma=2\pi+\alpha,\ \delta=3\pi-\alpha$

So that the given expression is equal to

$$4\sin\frac{\alpha}{2}+3\sin\left(\frac{\pi-\alpha}{2}\right)+2\sin\frac{2\pi+\alpha}{2}+\sin\frac{3\pi-\alpha}{2}$$

$$=4\sin\frac{\alpha}{2}+3\cos\frac{\alpha}{2}-2\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}$$

$$=2\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)=2\sqrt{1+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}=2\sqrt{1+k}$$

17. (b) Given $\tan\theta=n\tan\phi$

Now $\tan(\theta-\phi)=\dfrac{\tan\theta-\tan\phi}{1+\tan\theta\tan\phi}=\dfrac{(n-1)\tan\phi}{1+n\tan^2\phi}$

$$\Rightarrow\ \tan(\theta-\phi)=\frac{n-1}{\cot\phi+n\tan\phi}$$

$$\Rightarrow\ \tan^2(\theta-\phi)=\frac{(n-1)^2}{\cot^2\phi+n^2\tan^2\phi+2n}$$

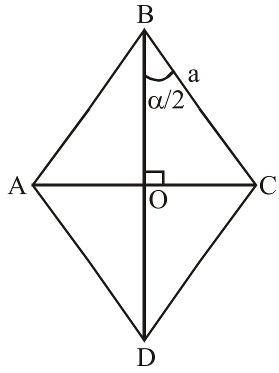
$$=\frac{(n-1)^2}{(\cot\phi-n\tan\phi)^2+4n}$$

Denominator is minimum at $\tan^2\phi=\dfrac{1}{n}$

So, maximum value of

$$\tan^2(\theta-\phi)=\frac{(n-1)^2}{0+4n}=\frac{(n-1)^2}{4n}$$

18. (c) Let a denotes the side of the rhombus and α its acute angle.



Then $OC=a\sin\dfrac{\alpha}{2}$

$\Rightarrow\ AC=2OC=2a\sin\dfrac{\alpha}{2}$

and $OB=a\cos\dfrac{\alpha}{2}$

$\Rightarrow\ BD=2a\cos\dfrac{\alpha}{2}$

Since it is given that $BC=\sqrt{AC\times BD}$

$\Rightarrow a=\sqrt{4a^2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2}}\Rightarrow 1=\sqrt{2\sin\alpha}$ or

$\sin\alpha=\dfrac{1}{2}\Rightarrow\alpha=30^\circ$. Since $0<\alpha<90^\circ$

19. (a) We have, $\tan k\alpha\tan(k+1)\alpha+1$

$$=\frac{\sin k\alpha\sin(k+1)\alpha+\cos k\alpha\cos((k+1)\alpha}{\cos k\alpha\cos(k+1)\alpha}$$

$$=\frac{\cos\alpha}{\cos k\alpha\cos(k+1)\alpha}=\cot\alpha.\frac{\sin\alpha}{\cos k\alpha\cos(k+1)\alpha}$$

$$=\frac{\sin\{(k+1)\alpha-k\alpha\}}{\cos k\alpha\cos(k+1)\alpha}.\cot\alpha$$

$$=\frac{\sin(k+1)\alpha\cos k\alpha-\cos(k+1)\alpha\sin k\alpha}{\cos k\alpha\cos(k+1)\alpha}\cot\alpha$$

$=\{\tan(k+1)\alpha-\tan k\alpha\}\cot\alpha$

$\therefore\ \tan k\alpha\tan(k+1)\alpha$

$=-1+\{\tan(k+1)\alpha-\tan k\alpha\}\cot\alpha$

$\therefore\ \displaystyle\sum_{k=1}^{n}\tan k\alpha\tan(k+1)\alpha$

$=-n+\{\tan(n+1)\alpha-\tan\alpha\}\cot\alpha$

$=-n-1+\tan(n+1)\alpha\cot\alpha$

20. (a) $a_1\cos(\alpha_1+\theta)+a_2\cos(\alpha_2+\theta).....$

$+a_n\cos(\alpha_n+\theta)=0$

$\Rightarrow\ (a_1\cos\alpha_1+a_2\cos\alpha_2+...+a_n\cos\alpha_n)\cos\theta$

$-(a_1\sin\alpha_1+a_2\sin\alpha_2+...+a_n\sin\alpha_n)\sin\theta=0$

$\Rightarrow\ a_1\sin\alpha_1+a_2\sin\alpha_2+...+a_n\sin\alpha_n=0$

(since $\sin\theta\neq 0$)

and $a_1\cos\alpha_1+a_2\cos\alpha_2+...+a_n\cos\alpha_n=0$

Now,

$$a_1 \cos(\alpha_1 + \lambda) + a_2 \cos(\alpha_2 + \lambda) + ... + a_n \cos(\alpha_n + \lambda)$$

$$= (a_1 \cos\alpha_1 + a_2 \cos\alpha_2 + ... + a_n \cos\alpha_n)\cos\lambda$$

$$- (a_1 \sin\alpha_1 + a_2 \sin\alpha_2 + ... + a_n \sin\alpha_n)\sin\lambda = 0$$

21. (c) $1 + \cos\alpha = 1 + \frac{2\cos\beta - 1}{2 - \cos\beta} = \frac{2 - \cos\beta + 2\cos\beta - 1}{2 - \cos\beta}$

$$= \frac{1 + \cos\beta}{2 - \cos\beta}$$

$$\Rightarrow \cos^2\frac{\alpha}{2} = \frac{\cos^2\frac{\beta}{2}}{1 + 2\sin^2\frac{\beta}{2}} \quad ...(1)$$

$$\Rightarrow 1 - \cos^2\frac{\alpha}{2} = 1 - \frac{\cos^2\frac{\beta}{2}}{1 + 2\sin^2\frac{\beta}{2}}$$

$$= \frac{1 + 2\sin^2\frac{\beta}{2} - \cos^2\frac{\beta}{2}}{1 + 2\sin^2\frac{\beta}{2}} = \frac{1 + 2\sin^2\frac{\beta}{2} - [1 - \sin^2\frac{\beta}{2}]}{1 + 2\sin^2\frac{\beta}{2}}$$

$$\Rightarrow \sin^2\frac{\alpha}{2} = \frac{3\sin^2\frac{\beta}{2}}{1 + 2\sin^2\frac{\beta}{2}} \quad ...(2)$$

Divide equation (2) by (1)

$$\tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2} \Rightarrow \frac{\tan\frac{\alpha}{2}}{\tan\frac{\beta}{2}} = \sqrt{3}$$

22. (b) As the triangle is acute, tan$A$, tan$B$ and tan$C$ are positive.

$$AM \geq GM$$

$$\Rightarrow \frac{\tan A + \tan B + \tan C}{3} \geq \sqrt[3]{\tan A \tan B \tan C}$$

equality holding if the numbers are equal, i.e., $\tan A = \tan B = \tan C$

Now, $A + B + C = \pi$; $\therefore B + C = \pi - A$

$\therefore \tan(B + C) = \tan(\pi - A)$ or

$$\frac{\tan B + \tan C}{1 - \tan B.\tan C} = -\tan A \text{ or}$$

$\tan A + \tan B + \tan C = \tan A.\tan B.\tan C = x$ (say)

$\therefore$ from, (1), $\frac{x}{3} \geq \sqrt[3]{x}$; or $\frac{x^3}{27} \geq x$

As $x > 0$, $x^2 \geq 27$; $\quad \therefore x \geq \sqrt{27} = 3\sqrt{3}$

$\therefore \tan A.\tan B.\tan C \geq 3\sqrt{3}$, equality holding when $\tan A = \tan B = \tan C$

$\therefore A = B = C$, i.e., the triangle is equilateral.

23. (b) The first equation can be written as

$$x\sin a + y \times 2\sin a\cos a + z\sin a(3 - 4\sin^2 a)$$

$$= 2 \times 2\sin a\cos a\cos 2a$$

$$\Rightarrow x + 2y\cos a + z(3 + 4\cos^2 a - 4)$$

$$= 4\cos a(2\cos^2 a - 1) \; as \sin a \neq 0$$

$$\Rightarrow 8\cos^3 a - 4z\cos^2 a - (2y + 4)\cos a + (z - x) = 0$$

$$\Rightarrow \cos^3 a - \frac{z}{2}\cos^2 a - \frac{y+2}{4}\cos a + \frac{z - x}{8} = 0$$

which shows that cos a is root of the equation

$$t^3 - \frac{z}{2}t^2 - \frac{y+2}{4}t + \frac{z-x}{8} = 0$$

Similarly from second and third equations we can varify that cos b and cos c are the roots of the above equation

24. (c) Let $x = \tan A$, $y = \tan B$ and $z = \tan C$, then

$xy + yz + zx = 1$

$\Rightarrow \tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

$\Rightarrow 2A + 2B + 2C = \pi$

$\therefore \tan(2A + 2B) = \tan(\pi - 2C) = -\tan 2C$

$$\Rightarrow \frac{\tan 2A + \tan 2B}{1 - \tan 2A\tan 2B} = -\tan 2C$$

$$\Rightarrow \tan 2A + \tan 2B + \tan 2C = \tan 2A\tan 2B\tan 2C$$

$$\Rightarrow \frac{2\tan A}{1 - \tan^2 A} + \frac{2\tan B}{1 - \tan^2 B} + \frac{2\tan C}{1 - \tan^2 C}$$

$$= \frac{2\tan A}{1 - \tan^2 A}.\frac{2\tan B}{1 - \tan^2 B}.\frac{2\tan C}{1 - \tan^2 C}$$

$$\Rightarrow \frac{x}{1 - x^2} + \frac{y}{1 - y^2} + \frac{z}{1 - z^2} = \frac{4xyz}{(1 - x^2)(1 - y^2)(1 - z^2)}$$

25. (c) Let $a_0 = \cos\theta$, then $a_{r+1} = \sqrt{\frac{1}{2}(1 + a_r)}$ gives

$$a_1 = \sqrt{\frac{1}{2}(1 + a_0)} = \sqrt{\frac{1}{2}(1 + \cos\theta)} = \cos\frac{\theta}{2}$$

$$a_2 = \sqrt{\frac{1}{2}(1 + a_1)} = \sqrt{\frac{1}{2}(1 + \cos\frac{\theta}{2})} = \cos\frac{\theta}{2^2}$$

$$a_3 = \sqrt{\frac{1}{2}(1 + a_2)} = \sqrt{\frac{1}{2}(1 + \cos\frac{\theta}{2^2})} = \cos\frac{\theta}{2^3} \text{....etc}$$

$\therefore a_1 a_2 a_3 .... a_n =$

$$\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}\cos\frac{\theta}{2^3}...\cos\frac{\theta}{2^n} = \frac{\sin\theta}{2^n \sin\frac{\theta}{2^n}}$$

$$\therefore a_1\ a_2\ a_3....to\infty = \lim_{n\to\infty} a_1 a_2\ ....a_n$$

$$= \lim_{n\to\infty}\frac{\sin\theta}{2^n\sin\frac{\theta}{2^n}} = \lim_{n\to\infty}\frac{\sin\theta}{\left(\frac{\sin\theta/2^n}{\theta/2^n}\right)\theta} = \frac{\sin\theta}{\theta}$$

$$\therefore \frac{\sqrt{1-a_0^2}}{a_1\ a_2\ a_3....to\ \infty} = \frac{\sqrt{1-\cos^2\theta}}{\frac{\sin\theta}{\theta}} = \theta$$

$$\Rightarrow \cos\left(\frac{\sqrt{1-a_0^2}}{a_1\,a_2...to\ \infty}\right) = \cos\theta = a_0$$

26. (d) Given $\tan B\tan C = p$

$$\Rightarrow \frac{\sin B\sin C}{\cos B\cos C} = p \quad \Rightarrow \frac{\cos(B-C)}{\cos(B+C)} = \frac{1+p}{1-p}$$

$$\Rightarrow \cos(B-C) = \left(\frac{1+p}{1-p}\right)\left(-\frac{1}{\sqrt{2}}\right)$$

$$\because\ 0 \le B-C < \frac{3\pi}{4} \Rightarrow -\frac{1}{\sqrt{2}} < \cos(B-C) \le 1$$

$$\therefore -\frac{1}{\sqrt{2}} < \left(\frac{1+p}{1-p}\right)\left(-\frac{1}{\sqrt{2}}\right) \le 1 \Rightarrow -\sqrt{2} \le \frac{1+p}{1-p} < 1$$

$$\Rightarrow \frac{1+p}{1-p} \ge -\sqrt{2} \text{ and } \frac{1+p}{1-p} < 1$$

$$\Rightarrow p < 1 \text{ or } p \ge (\sqrt{2}+1)^2 \text{ and } p < 0 \text{ or } p > 1$$

$$\therefore p \in (-\infty, 0)\cup[(\sqrt{2}+1)^2, \infty)$$

27. (a) Given $f(x) = ax + b \therefore f'(x) = a$
Since $a < 0, f(x)$ is a decreasing function
$\therefore f(-1) = 2$ and $f(1) = 0$
$\Rightarrow -a + b = 2$ and $a + b = 0 \therefore a = -1$ and $b = 1$.
Thus $f(x) = -x + 1$

Clearly $f(0) = 1,\ f\left(\frac{1}{4}\right) = \frac{3}{4},\ f(-2) = 3,$

$f\left(\frac{1}{3}\right) = \frac{2}{3},\ f(-1) = 2$

Also, $A = \frac{1+\cos 2\theta}{2} + \left(\frac{1-\cos 2\theta}{2}\right)^2$

$$= \frac{1}{2} + \frac{1}{2}\cos 2\theta + \frac{1}{4} - \frac{1}{2}\cos 2\theta + \frac{1}{4}\cos^2 2\theta$$

$$= \frac{3}{4} + \frac{1}{4}\left(\frac{1+\cos 4\theta}{2}\right) = \frac{7}{8} + \frac{1}{8}\cos 4\theta$$

$$\therefore \frac{3}{4} \le A \le 1 \Rightarrow f\left(\frac{1}{4}\right) \le A \le f(0)$$

28. (a) $1+\cot\theta-\cot\frac{\theta}{2} = 1+\frac{\cot^2\frac{\theta}{2}-1}{2\cot\frac{\theta}{2}} - \cot\frac{\theta}{2}$

$$= \frac{2\cot\frac{\theta}{2}+\cot^2\frac{\theta}{2}-1-2\cot^2\frac{\theta}{2}}{2\cot\frac{\theta}{2}} = \frac{-\left(\cot\frac{\theta}{2}-1\right)^2}{2\cot\frac{\theta}{2}} \le 0$$

$$\left[\because 0 < \frac{\theta}{2} < \frac{\pi}{2}\right]$$

$$\Rightarrow 1+\cot\theta \le \cot\frac{\theta}{2}$$

29. (c) $\cos A+\cos B+\cos C$

$$= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} + 1 - 2\sin^2\frac{C}{2}$$

$$= 2\sin\frac{C}{2}.\left(\cos\frac{A-B}{2} - \sin\frac{C}{2}\right) + 1\ (\because A+B = \pi - C)$$

$$\le 2\sin\frac{C}{2}\left(1-\sin\frac{C}{2}\right)+1$$

$$\left(\because \text{ the greatest value of } \cos\frac{A-B}{2} \text{ is } 1\right)$$

and equality holds when $\cos\frac{A-B}{2} = 1$ ...(1)

$$\therefore \cos A+\cos B+\cos C \le 1-2\left(\sin^2\frac{C}{2}-\sin\frac{C}{2}\right)$$

$$= 1-2\left(\sin^2\frac{C}{2}-\sin\frac{C}{2}+\frac{1}{4}\right)+2.\frac{1}{4}$$

$$= \frac{3}{2}-2\left(\sin\frac{C}{2}-\frac{1}{2}\right)^2 \le \frac{3}{2}$$

equality holding when $\sin\frac{C}{2} = \frac{1}{2}$

Thus, $\cos A+\cos B+\cos C \le \frac{3}{2}$

equality holding when (1) and (2) both hold, i.e.,

when $A=B=C=\frac{\pi}{3}$

30. (d) Using $A.M. \ge G.M$

$\Rightarrow \frac{1}{2}(3\tan^2\theta+12\cot^2\theta) \ge 6$

$\Rightarrow 3\tan^2\theta+12\cot^2\theta$ has minimum value 12.

31. (c) $\sin x+\cos x+\tan x+\cot x+\sec x+\operatorname{cosec} x = 7$

$\Rightarrow (\sin x+\cos x)+\left(\frac{1}{\sin x\cos x}\right)+\frac{(\sin x+\cos x)}{\sin x.\cos x}=7$

$\Rightarrow (\sin x+\cos x)\left(1+\frac{2}{\sin 2x}\right)=\left(7-\frac{2}{\sin 2x}\right)$

$\Rightarrow (1+\sin 2x)\left(1+\frac{4}{\sin^2 2x}+\frac{4}{\sin 2x}\right)$

$=49+\frac{4}{\sin^2 2x}-\frac{28}{\sin 2x}$ (squaring both sides)

$\Rightarrow \sin^3 2x-44\sin^2 2x+36\sin 2x=0$

$\Rightarrow \sin 2x = 22-8\sqrt{7}.$

32. (d) $\tan^4 x-2\tan^3 x-\tan^2 x+2\tan x+1$

$=\tan^4 x+\tan^2 x-2\tan^3 x+2\tan x-2\tan^2 x+1$

$=(\tan^2-\tan x)^2+2(\tan x-\tan^2 x)+1=4.$

33. (d) $\cot^2 36°\cot^2 72° = (\cot^2 36°\cot^2 72° - 1)+1$

$=\left(\frac{\cos^2 36°\cos^2 72°-\sin^2 36°\sin^2 72°}{\sin^2 36°\sin^2 72°}\right)+1$

$=\left(\frac{(1+\cos 144°)(1+\cos 72°)-(1-\cos 144°)(1-\cos 72°)}{4\sin^2 36°\sin^2 72°}\right)$

$+1=\frac{\cos 144°+\cos 72°}{2\sin^2 36°\sin^2 72°}+1$

$=\frac{\cos 108°\cos 36°}{\sin^2 36°\sin^2 72°}+1=\frac{-\cos 72°\cos 36°}{\sin^2 36°\sin^2 72°}+1$

$=1-\frac{(\cot 36°\cot 72°)^2}{\cos 36°\cos 72°}$

$=1-\frac{2\sin 36°(\cot 36°\cot 72°)^2}{2\sin 36°\cos 36°\cos 72°}$

$=1-\frac{4\sin 36°(\cot 36°\cot 72°)^2}{\sin 144°}$

$\Rightarrow \cot^2 36°\cot^2 72° = 1-4\cot^2 36°\cot^2 72°$

$\therefore 5\cot^2 36°\cot^2 72° = 1$

$\Rightarrow \cot^2 36°\cot^2 72° = \frac{1}{5}.$

ALTERNATIVELY:

$\cot A.\cot B = \frac{\cos(A+B)}{\sin A.\sin B}+1$

$\Rightarrow \cot 36°.\cot 72° = \frac{\cos 108°}{\sin 36°.\sin 72°}+1$

$\Rightarrow \frac{-\sin 18°}{\sin 18°.\cos 18°.\cos 18°}+1=1-\frac{1}{2\cos^2 18°}$

$=1-\frac{1}{1+\cos 36°}$

$=\frac{\cos 36°}{1+\cos 36°}=\frac{\frac{\sqrt{5}+1}{4}}{1+\frac{\sqrt{5}+1}{4}}=\frac{\sqrt{5}+1}{\sqrt{5}(1+\sqrt{5})}=\frac{1}{\sqrt{5}}$

$=\cot^2 36°.\cot^2 72° = \frac{1}{5}$

34. (a) $3^{\sin^6 x}$ and $3^{\cos^6 x}$ are positive numbers.

And A.M $\ge$ G.M.

$\Rightarrow \frac{3^{\sin^6 x}+3^{\cos^6 x}}{2} \ge \sqrt{3^{\sin^6 x+\cos^6 x}}$ ...(1)

$=2\sqrt{3^{1-\frac{3}{4}\sin^2 2x}}$

$\Rightarrow 3^{\sin^6 x}+3^{\cos^6 x} \ge 2.\sqrt{3^{\frac{1}{4}}} = 2.3^{1/8}$

ALTERNATIVELY:

Clearly the equality (1) holds for $3^{\sin^6 x}=3^{\cos^6 x}$

$\Rightarrow \sin^6 x=\cos^6 x=\left(\frac{1}{\sqrt{2}}\right)^6=\frac{1}{8}$

35. (c) $T_6=\sin^6\theta+\cos^6\theta=\left(\sin^2\theta+\cos^2\theta\right)^3$

$-3\sin^2\theta\cos^2\theta\left(\sin^2\theta+\cos^2\theta\right)$

$=1-3\sin^2\theta\cos^2\theta$

$T_4=\sin^4\theta+\cos^4\theta=\left(\sin^2\theta+\cos^2\theta\right)^2$

$-2\sin^2\theta\cos^2\theta=1-2\sin^2\theta\cos^2\theta$

$$\therefore \frac{T_6 - T_4}{T_6} = \frac{\left(1-3\sin^2\theta\cos^2\theta\right)-\left(1-2\sin^2\theta\cos^2\theta\right)}{1-3\sin^2\theta\cos^2\theta}$$

$$= \frac{-\sin^2\theta\cos^2\theta}{1-3\sin^2\theta\cos^2\theta} = \frac{-\sin^2 2\theta}{4-3\sin^2 2\theta} = m$$

$$\Rightarrow \sin^2 2\theta = \frac{4m}{3m-1}$$

$$\because 0 \le \sin^2 2\theta \le 1 \Rightarrow 0 \le \frac{4m}{3m-1} \le 1 \Rightarrow m \in [-1, 0]$$

36. (b) $\sin\theta + \sin\phi = \sqrt{3}(\cos\phi - \cos\theta)$

$$\Rightarrow \sin\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} = \sqrt{3}\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$$

$$\Rightarrow \sin\frac{\theta+\phi}{2}\left[\cos\frac{\theta-\phi}{2} - \sqrt{3}\sin\frac{\theta-\phi}{2}\right] = 0$$

$$\Rightarrow \sin\frac{\theta+\phi}{2} = 0 \text{ or } \tan\frac{\theta-\phi}{2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta+\phi = 2n\pi \text{ or } \theta-\phi = 2n\pi + \frac{\pi}{3}$$

Now, $\sin 3\theta + \sin 3\phi = \sin 3\theta - \sin 3\theta = 0$

or $\sin 3\theta + \sin 3\phi = \sin 3\theta - \sin(180° - 3\theta) = 0$

37. (d) Given expression

$$= \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)\left(\sin\frac{\pi}{2}\right)$$

$$\left(\sin\left(\pi - \frac{\pi}{14}\right)\sin\left(\pi - \frac{3\pi}{14}\right)\sin\left(\pi - \frac{5\pi}{14}\right)\right)$$

$$= \sin^2\frac{\pi}{14}\sin^2\frac{3\pi}{14}\sin^2\frac{5\pi}{14}$$

$$= \left[\sin\left(\frac{\pi}{2} - \frac{6\pi}{14}\right)\sin\left(\frac{\pi}{2} - \frac{4\pi}{14}\right)\sin\left(\frac{\pi}{2} - \frac{2\pi}{14}\right)\right]^2$$

$$= \left(\cos^2\frac{3\pi}{7}\cos^2\frac{2\pi}{7}\cos^2\frac{\pi}{7}\right)\frac{4\sin^2\frac{\pi}{7}}{4\sin^2\frac{\pi}{7}}$$

$$= \frac{1}{4\sin^2\frac{\pi}{7}}\sin^2\frac{2\pi}{7}\cos^2\frac{2\pi}{7}\cos^2\frac{3\pi}{7}$$

$$= \frac{1}{16\sin^2\frac{\pi}{7}}\sin^2\frac{4\pi}{7}\cos^2\left(\pi - \frac{4\pi}{7}\right)$$

$$= \frac{1}{64\sin^2\frac{\pi}{7}}\sin^2\frac{8\pi}{7}$$

$$= \frac{1}{64\sin^2\frac{\pi}{7}}\sin^2\left(\pi + \frac{\pi}{7}\right) = \frac{1}{64}$$

ALTERNATIVELY

$$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$$

$$= \cos\frac{6\pi}{14}\cos\frac{4\pi}{14}\cos\frac{2\pi}{14}\cos\frac{2\pi}{14} \times 1 \times \cos\frac{2\pi}{14} \times \cos\frac{4\pi}{14} \times \cos\frac{6\pi}{14}$$

$$= \left(\cos\frac{2\pi}{14} \times \cos\frac{4\pi}{14} \times \cos\frac{6\pi}{14}\right)^2$$

$$= \left(\frac{\sin 2^3.\frac{2\pi}{14}}{2^3.\sin\frac{2\pi}{14}}\right)^2 \quad \text{(by formula)}$$

$$= \frac{1}{64}.$$

38. (a) $\cos x - \dfrac{\cot\beta\sin x}{2} = \dfrac{\sqrt{3}}{2}$

$$\Rightarrow \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} - \frac{\cot\beta\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow 1 - \tan^2\frac{x}{2} - \cot\beta\tan\frac{x}{2} = \frac{\sqrt{3}}{2}\left(1+\tan^2\frac{x}{2}\right)$$

$$\Rightarrow \left(2+\sqrt{3}\right)\tan^2\frac{x}{2} + 2\cot\beta\tan\frac{x}{2} + \left(\sqrt{3}-2\right) = 0$$

$$\Rightarrow \tan\frac{x}{2} = \frac{-2\cot\beta \pm \sqrt{4\cot^2\beta + 4}}{2\left(2+\sqrt{3}\right)}$$

$$= \frac{-2\cot\beta \pm 2\,\text{cosec}\,\beta}{2\left(2+\sqrt{3}\right)}$$

$$\Rightarrow \tan\frac{x}{2} = \frac{-\cot\beta + \text{cosec}\beta}{\left(2+\sqrt{3}\right)}$$

$$\text{or } \tan\frac{x}{2} = \frac{-\cot\beta - \text{cosec}\beta}{\left(2+\sqrt{3}\right)}$$

$$\Rightarrow \tan\frac{x}{2} = \tan\frac{\beta}{2}\tan 15° \text{ or } \tan\frac{x}{2} = -\cot\frac{\beta}{2}\tan 15°$$

39. **(b)** $n\sin^2\theta + 2n\cos(\theta+\alpha)\,\sin\alpha\sin\theta + \cos 2(\alpha+\theta)$

$$= n\sin^2\theta + n\cos(\theta+\alpha)\{\cos(\theta-\alpha)-\cos(\theta+\alpha)\}$$
$$+2\cos^2(\theta+\alpha)-1$$
$$= n\sin^2\theta + n(\cos^2\theta - \sin^2\alpha)$$
$$-n\cos^2(\theta+\alpha)+2\cos^2(\alpha+\theta)-1$$
$$= n\sin^2\theta + n\cos^2\theta - n\sin^2\alpha + (2-n)\cos^2(\theta+\alpha)-1$$
$$= (n-1) - n\sin^2\alpha + (2-n)\cos^2(\theta+\alpha)$$

It is independent of θ if $n = 2$

40. **(d)** We have $\sec\theta\,\text{cosec}\theta = \dfrac{2}{\sin 2\theta}$

$\because\ 0 < \theta < \pi \Rightarrow 0 < \sin 2\theta \le 1$

$\Rightarrow\ \sec\theta\,\text{cosec}\theta \ge 2$

Thus the product of roots ≥ 2

Again $\sec\theta + \text{cosec}\theta \ge 2\sqrt{\sec\theta\text{cosec}\theta} \ge 2\sqrt{2}$

$(\because \sec\theta > 0, \text{cosec}\theta > 0)$

equality holds if $\theta = \dfrac{\pi}{4}$

Thus the product of roots $\ge 2\sqrt{2}$

Also, $\sec\theta + \text{cosec}\theta = \dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$

$$\Rightarrow\ (\sec\theta+\text{cosec}\theta)^2 = \frac{1+2\sin\theta\cos\theta}{\sin^2\theta\cos^2\theta}$$

$$\Rightarrow\ S^2 = \frac{1+\frac{2}{P}}{\frac{1}{P^2}} = P^2 + 2P$$

(S = sum of roots and P = product of roots)

Now for the option (A) roots are non-real

For the option (B) sum of roots $= \dfrac{8}{3}, < 2\sqrt{2}$

For the option (C) sum of roots $= \dfrac{10}{3} > 2\sqrt{2}$

product of roots $= \dfrac{6}{3} = 2$

But $S^2 = \dfrac{100}{9}$ and $P^2 + 2P = 4 + 4 = 8$

$\therefore\ S^2 \ne P^2 + 2P$

Finally for the option (D) $S = \sqrt{15} > 2\sqrt{2}, P = 3 > 2$

Also, $P^2 + 2P = 9 + 6 = 15 = S^2$

41. **(b)** Given that $A + B = \dfrac{\pi}{4}$

$$\Rightarrow\ \cos A\cos B = \cos A\cos\left(\frac{\pi}{4} - A\right)$$
$$= \cos A\left[\cos\frac{\pi}{4}\cos A + \sin\frac{\pi}{4}\sin A\right]$$
$$= \frac{\cos A}{\sqrt{2}}(\cos A + \sin A)$$
$$= \frac{\cos^2 A}{\sqrt{2}} + \frac{\sin A\cos A}{\sqrt{2}}$$
$$= \frac{1+\cos 2A}{2\sqrt{2}} + \frac{\sin 2A}{2\sqrt{2}}$$
$$= \frac{1}{2\sqrt{2}} + \frac{\cos 2A + \sin 2A}{2\sqrt{2}}$$
$$= \frac{1}{2\sqrt{2}} + \frac{1}{2}\sin\left(\frac{\pi}{4} + 2A\right).$$

$\Rightarrow$ maximum value of $\cos A\cos B$ is

$$\frac{1}{2\sqrt{2}} + \frac{1}{2} = \frac{1+\sqrt{2}}{2\sqrt{2}}$$

42. **(c)** $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$

$$\Rightarrow\ \pi\cos\theta = \left(\frac{\pi}{2} - \pi\sin\theta\right) + n\pi$$
$$\Rightarrow\ \cos\theta + \sin\theta = n + \frac{1}{2},\ n \in I$$
$$\Rightarrow\ \sin\theta + \cos\theta = \frac{1}{2},\ \text{or}\ -\frac{1}{2}$$

$(\because -\sqrt{2} \le \cos\theta + \sin\theta \le \sqrt{2})$

$$\Rightarrow\ \sin\theta\cos\theta = -\frac{3}{8}$$

$\Rightarrow\ \cos\theta, \sin\theta$ are roots of the equation

$$\Rightarrow\ 8x^2 \pm 4x - 3 = 0$$

43. **(c)** We know $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Putting $A = 40^\circ$ and squaring we get

$$(1 - 3\tan^2 40^\circ)^2\, 3 = \tan^2 40^\circ (3 - \tan^2 40^\circ)^2$$
$$\Rightarrow\ \tan^6 40^\circ - 33\tan^4 40^\circ + 27\tan^2 40^\circ = 3$$

44. **(c)** On expansion we get

$$\cos(\theta_1-\theta_2)\cos(\theta_3-\theta_4)+\cos(\theta_1+\theta_2)\cos(\theta_3+\theta_4)=0$$

$$\Rightarrow (\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)(\cos\theta_3\cos\theta_4+\sin\theta_3\sin\theta_4)+(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2)(\cos\theta_3\cos\theta_4-\sin\theta_3\sin\theta_4)=0$$

$$\Rightarrow 2\cos\theta_1\cos\theta_2\cos\theta_3\cos\theta_4+2\sin\theta_1\sin\theta_2\sin\theta_3\sin\theta_4=0$$

$$\therefore \tan\theta_1\tan\theta_2\tan\theta_3\tan\theta_4=-1$$

45. **(b)** We have $\dfrac{2}{\cos\theta}=\dfrac{1}{\cos(\theta-\phi)}+\dfrac{1}{\cos(\theta+\phi)}$

$$=\frac{\cos(\theta+\phi)+\cos(\theta-\phi)}{\cos(\theta+\phi)\cos(\theta-\phi)}$$

$$\Rightarrow \cos 2\theta+\cos 2\phi=2\cos^2\theta\cos\phi$$

$$\Rightarrow 2\cos^2\theta-2\sin^2\phi=2\cos^2\theta\cos\phi$$

$$\Rightarrow 2\cos^2\theta(1-\cos\phi)=2\sin^2\phi$$

$$\Rightarrow \cos^2\theta.\,2\sin^2\frac{\phi}{2}=4\sin^2\frac{\phi}{2}\cos^2\frac{\phi}{2}$$

$$\Rightarrow \cos\theta\sec\frac{\phi}{2}=\pm\sqrt{2}$$

46. (a) Let $P=\cos\left\{\dfrac{2\pi}{2^{64}-1}\right\}\cos\left\{\dfrac{2^2\pi}{2^{64}-1}\right\}...\cos\left\{\dfrac{2^{64}\pi}{2^{64}-1}\right\}$

$$\Rightarrow P=\frac{1}{2\sin\left(\frac{2\pi}{2^{64}-1}\right)}\sin\left(\frac{2^2\pi}{2^{64}-1}\right)\cos\left(\frac{2^2\pi}{2^{64}-1}\right)...\cos\left(\frac{2^{64}\pi}{2^{64}-1}\right)$$

$$\Rightarrow P=\frac{1}{2\sin\left(\frac{2\pi}{2^{64}-1}\right)}\sin\left(\frac{2^3\pi}{2^{64}-1}\right)...\cos\left(\frac{2^{64}\pi}{2^{64}-1}\right)$$

$$\Rightarrow P=\frac{1}{2^{64}\sin\left(\frac{2\pi}{2^{64}-1}\right)}\sin\left(\frac{2^{65}\pi}{2^{64}-1}\right)=\frac{1}{2^{64}}$$

47. **(a)** The given equation may be written as

$$\frac{\sin(\alpha+\beta-\gamma)\cos(\alpha-\beta+\gamma)}{\tan(\alpha-\beta+\gamma)\cos(\alpha+\beta-\gamma)}=\frac{\sin\gamma\cos\beta}{\sin\beta\cos\gamma}$$

$$\Rightarrow \frac{\sin 2\alpha+\sin(2\beta-2\gamma)}{\sin 2\alpha-\sin(2\beta-2\gamma)}=\frac{\sin\gamma\cos\beta}{\sin\beta\cos\gamma}$$

Using componedo and divendendo, we get

$$\frac{\sin 2\alpha}{\sin(2\beta-2\gamma)}=\frac{\sin\gamma\cos\beta+\sin\beta\cos\gamma}{\sin\gamma\cos\beta-\sin\beta\cos\gamma}=\frac{\sin(\beta+\gamma)}{\sin(\gamma-\beta)}.$$

$$\Rightarrow \frac{\sin 2\alpha}{2\sin(\beta-\gamma)\cos(\beta-\gamma)}=\frac{\sin(\beta+\gamma)}{\sin(\gamma-\beta)}$$

$$\Rightarrow \sin 2\alpha+2\sin(\beta+\gamma)\cos(\beta-\gamma)=0$$

$$\Rightarrow \sin 2\alpha+\sin 2\beta+\sin 2\gamma=0.$$

B COMPREHENSION TYPE

1. **(a)** Let $C=1+\dfrac{1}{2}\cos 2\alpha-\dfrac{1}{2.4}\cos 4\alpha+\dfrac{1.3}{2.4.6}\cos 6\alpha-....$

and $S=\dfrac{1}{2}\sin 2\alpha-\dfrac{1}{2.4}\sin 4\alpha+\dfrac{1.3}{2.4.6}\sin 6\alpha-...$

$$\therefore C+iS=1+\frac{1}{2}e^{i2\alpha}-\frac{1}{2.4}e^{i4\alpha}+\frac{1.3}{2.4.6}e^{i6\alpha}-....$$

$$=\left(1+e^{2i\alpha}\right)^{1/2}=(1+\cos 2\alpha+i\sin 2\alpha)^{1/2}$$

$$=\left\{2\cos^2\alpha+i2\sin\alpha\cos\alpha\right\}^{1/2}$$

$$=\sqrt{2\cos\alpha}(\cos\alpha+i\sin\alpha)^{1/2}\qquad\left[\therefore -\frac{\pi}{2}<\alpha<\frac{\pi}{2}\right]$$

$$=\sqrt{2\cos\alpha}\left(\cos\frac{\alpha}{2}+i\sin\frac{\alpha}{2}\right)$$

Equating real part

$$C=\sqrt{2\cos\alpha}\cos\frac{\alpha}{2}=\sqrt{\cos\alpha(1+\cos\alpha)}$$

2. **(a)** Let $C=1+n\dfrac{a}{c}\cos B+\dfrac{n(n+1)}{2!}\dfrac{a^2}{c^2}\cos 2B+...$

$$S=n.\frac{a}{c}\sin B+\frac{n(n+1)}{2!}\frac{a^2}{c^2}\sin 2B+....$$

$\therefore\ C+iS = 1 + n\frac{a}{c}e^{iB} + \frac{n(n+1)}{2!}\frac{a^2}{c^2}e^{2iB} + ...$

$= \left(1-\frac{a}{c}e^{iB}\right)^{-n} = \left[1-\frac{\sin A}{\sin C}(\cos B + i\sin B)\right]^{-n}$

$= \sin^n C[\sin C - \sin A\cos B - i\sin A\sin B]^{-n}$

$= \sin^n C[\sin(B+A) - \sin A\cos B - i\sin A\sin B]^{-n}$

$= \sin^n C[\sin B\cos A - i\sin A\sin B]^{-n}$

$= \frac{\sin^n C}{\sin^n B}(\cos A - i\sin A)^{-n} = \frac{c^n}{b^n}(\cos nA + i\sin nA)$

Equating imaginary parts $S = \frac{c^n}{b^n}\sin nA$

3. **(d)** Let $C = \cos\frac{\pi}{3} + \frac{1}{2}\cos\frac{2\pi}{3} + \frac{1}{3}\cos\frac{3\pi}{3} + ...$

and $S = \sin\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3} + \frac{1}{3}\sin\frac{3\pi}{3} + ...$

$\therefore C + iS = e^{i\frac{\pi}{3}} + \frac{1}{2}e^{i\frac{2\pi}{3}} + \frac{1}{3}e^{i\frac{3\pi}{3}} + ...$

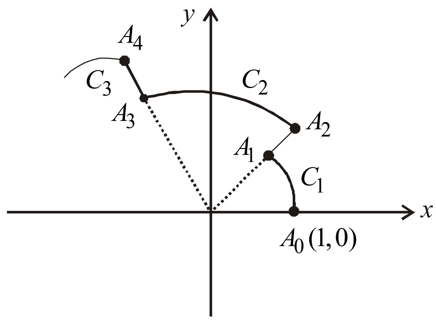
$= -\log(1-e^{i\pi/3}) = -\log(1-\cos\frac{\pi}{3} - i\sin\frac{\pi}{3})$

$= -\log\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = -\left[\frac{1}{2}\log\left(\frac{1}{4}+\frac{3}{4}\right) - i\tan^{-1}\sqrt{3}\right]$

Equating real parts, $C = -\frac{1}{2}\log 1 = 0$

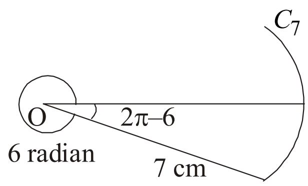
4. **(b)** As the particle moves k centimeters along the arc of a circle of radius k, hence in one rotation it covers 1 radian. The first time it crosses positive y axis then it covers $\frac{\pi}{2}$ radian

$\because\ 1 < \frac{\pi}{2} < 2 \Rightarrow$ Particle is travelling on the circle C_2



5. **(c)** When the particle first time crosses postive x-axis then it covers 2π radian
$\because\ 6 < 2\pi < 7 \Rightarrow$ Particle is travelling on the circle C_7

6. **(d)** Particle travels 1 cm at C_1, 2 cm at C_2, 3cm at C_3, 4 cm at C_4, 5 cm at C_5, 6 cm at C_6 and $(2\pi - 6) \times 7$ cm at C_7. In addition it travels 1 cm each along radial direction C_1 to C_2, C_2 to C_3, C_3 to C_4, C_4 to C_5, C_5 to C_6 and C_6 to C_7. Total distance travelled
$= (1+2+3+4+5+6) + 7 \times (2\pi - 6) + 6 = 14\pi - 15$



C REASONING TYPE

1. **(a)** $\cos\theta\cos 2\theta\cos 2^2\theta...\cos(2^{n-1}\theta) = \frac{\sin(2^n\theta)}{2^n\sin\theta}$

$= \frac{\sin\left(\frac{2^n\pi}{2^n-1}\right)}{2^n\sin\left(\frac{\pi}{2^n-1}\right)}$ if $\theta = \frac{\pi}{2^n-1}$

$= \frac{\sin\left(\pi + \frac{\pi}{2^n-1}\right)}{2^n\sin\left(\frac{\pi}{2^n-1}\right)} = -\frac{1}{2^n}$.

Also, $\frac{\pi}{7} = \frac{\pi}{2^3-1}$

2. **(a)** $\cot\alpha - \tan\alpha = \frac{\cos^2\alpha - \sin^2\alpha}{\sin\alpha\cos\alpha} = \frac{\cos 2\alpha}{\frac{1}{2}\sin 2\alpha} = 2\cot 2\alpha$

Now

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\tan 8\alpha - 16\cot 16\alpha$

$= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\tan 8\alpha$

$+8(\cot 8\alpha - \tan 8\alpha) = \tan\alpha + 2\tan 2\alpha$

$+4\tan 4\alpha + 4(\cot 4\alpha - \tan 4\alpha) = ... = \cot\alpha$

3. **(a)** Let $g(x) = 6\sin x - 8\cos x + 5$

Max. value of $g(x) = \sqrt{6^2 + 8^2} + 5 = 5 + 10 = 15$

Min. value of $g(x) = -\sqrt{6^2 + 8^2} + 5 = 5 - 10 = -5$

$\therefore$ The range of $f(x) = \frac{1}{g(x)}$ is $R - \left(-\frac{1}{5}, \frac{1}{15}\right)$

$\Rightarrow$ it is an unbounded function $\Rightarrow f(x)$ has no maximum and no minimum values.

4. **(d)** $\sec^2\theta \ge 1 \Rightarrow \frac{4xy}{(x+y)^2} \ge 1$

$\Rightarrow (x-y)^2 \le 0 \Rightarrow x = y \ne 0$

5. **(d)** $\sec 2\theta = \frac{1+\tan^2\theta}{1-\tan^2\theta} = \frac{1+n}{1-n}$, which is always rational provided $n \ne 1$.

D MULTIPLE CORRECT CHOICE TYPE

1. **(b,c,d)** Given : $\frac{\tan 3A}{\tan A} = k$ ...(1)

$\Rightarrow \frac{\tan 3A - \tan A}{\tan A} = k - 1 \Rightarrow \frac{\sin 2A}{\cos 3A \sin A} = k - 1$

$\Rightarrow \frac{2\cos A}{\cos 3A} = k - 1 \Rightarrow \frac{\cos A}{\cos 3A} = \frac{k-1}{2}$

$\Rightarrow$ (a) is incorrect

Again $\frac{\tan 3A}{\tan A} = k \Rightarrow \frac{\sin 3A}{\cos 3A} \cdot \frac{\cos A}{\sin A} = k$

$\Rightarrow \frac{\sin 3A}{\sin A} = k \cdot \frac{2}{k-1} = \frac{2k}{k-1}$

$\Rightarrow \frac{3\sin A - 4\sin^3 A}{\sin A} = \frac{2k}{k-1}$

$\Rightarrow 3 - 4\sin^2 A = \frac{2k}{k-1}$ or $4\sin^2 A = \frac{k-3}{k-1}$

$\Rightarrow 0 < \frac{k-3}{k-1} < 4$ [$\sin A \ne 0$ or 1]

Now, $\frac{k-3}{k-1} > 0$ is $k < 1$ or $k > 3$ ...(ii)

and $\frac{k-3}{k-1} < 4$

$\Rightarrow \frac{3k-1}{k-1} > 0 \Rightarrow k < \frac{1}{3}$ or $k > 1$ ...(iii)

(ii) and (iii) simultaneously hold if $k < \frac{1}{3}$ or $k > 3$

2. **(a,b,c,d)** We have, $1 + \sec\theta = \frac{1+\cos\theta}{\cos\theta} = \frac{2\cos^2\frac{\theta}{2}}{\cos\theta}$,

similarly for others.

$f_n(\theta) = \tan\frac{\theta}{2} \cdot \frac{2\cos^2\frac{\theta}{2}}{\cos\theta} \cdot \frac{2\cos^2\theta}{\cos 2\theta} \cdots\cdots \frac{2\cos^2 2^{n-1}\theta}{\cos 2^n\theta}$

$= \tan\frac{\theta}{2} \cdot 2^{n+1} \frac{[\cos\theta \cdot \cos 2\theta \ldots\ldots \cos 2^{n-1}\theta]\cos^2\frac{\theta}{2}}{\cos 2^n\theta}$

$= \frac{\sin\theta}{\cos 2^n\theta} \cdot 2^n \cdot \frac{\sin 2^n\theta}{2^n \sin\theta} = \tan 2^n\theta$

$\therefore f_2\left(\frac{\pi}{16}\right) = \tan\left(4 \cdot \frac{\pi}{16}\right) = 1$, .

$f_3\left(\frac{\pi}{32}\right) = \tan\left(8 \cdot \frac{\pi}{32}\right) = 1$

Similarly others are also true.

3. **(a,c)** $\sin\beta = \frac{12}{13} \Rightarrow \cos\beta = \pm\frac{5}{13}$

according as $\tan\beta > 0$ or < 0

$\therefore 5\sin(\alpha+\beta) - 12\cos(\alpha+\beta)$

$= 5[\sin\alpha\cos\beta + \cos\alpha\sin\beta]$

$-12[\cos\alpha\cos\beta - \sin\alpha\sin\beta]$

$= (5\cos\beta + 12\sin\beta)\sin\alpha$

$+ (5\sin\beta - 12\cos\beta)\cos\alpha$

$$=\left(\frac{25}{13}+\frac{144}{13}\right)\sin\alpha+\left(\frac{60}{13}-\frac{60}{13}\right)\cos\alpha$$

$$=13\sin\alpha \text{ if } \tan\beta>0$$

$$\Rightarrow \{(5\sin(\alpha+\beta)-12\cos(\alpha+\beta)\}\operatorname{cosec}\alpha=13$$

If $\tan\beta<0$ then $5\sin(\alpha+\beta)-12\cos(\alpha+\beta)$

$$=\frac{119}{13}\sin\alpha+\frac{120}{13}\cos\alpha$$

$$\Rightarrow [5\sin(\alpha+\beta)-12\cos(\alpha+\beta)]\operatorname{cosec}\alpha$$

$$=\frac{119}{13}+\frac{120}{13}\cot\alpha$$

4. **(b, c, d)** From the first relation we have

$$a[\sin(\theta+\phi)-\sin(\theta-\phi)]=b[\sin(\theta-\phi)+\sin(\theta+\phi)]$$

$$\Rightarrow 2a\sin\phi\cos\theta=2b\sin\theta\cos\phi$$

$$\Rightarrow a\tan\phi=b\tan\theta \Rightarrow \text{(b) is correct}$$

$$\Rightarrow \frac{2a\tan\frac{\phi}{2}}{1-\tan^2\frac{\phi}{2}}=\frac{2b\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

From the second relation replacing

$\tan\frac{\theta}{2}=\frac{1}{a}[b\tan\frac{\phi}{2}+c]$ we have

$$\frac{a\tan\frac{\phi}{2}}{1-\tan^2\frac{\phi}{2}}=\frac{\left(b(b\tan\frac{\phi}{2}+c)\right)}{a\left[1-\left\{\frac{1}{a}(b\tan\frac{\phi}{2}+c\right\}^2\right]}$$

$$\Rightarrow \tan\frac{\phi}{2}\left[a^2-(b\tan\frac{\phi}{2}+c)^2\right]$$

$$=b\left(b\tan\frac{\phi}{2}+c\right)\left(1-\tan^2\frac{\phi}{2}\right)$$

$$\Rightarrow \tan\frac{\phi}{2}(a^2-b^2-c^2)=bc\left(1+\tan^2\frac{\phi}{2}\right)$$

$$\Rightarrow \frac{2\tan\frac{\phi}{2}}{1+\tan^2\frac{\phi}{2}}=\frac{2bc}{a^2-b^2-c^2}$$

$$\Rightarrow \sin\phi=\frac{2bc}{a^2-b^2-c^2}$$

Similarly we get $\sin\theta=\dfrac{2ac}{a^2-b^2+c^2}$

5. **(a,b,c,d)** $\sin^2 A+\sin^2 B+\sin^2 C=\frac{1}{2}(1-\cos 2A)$

$$+\frac{1}{2}(1-\cos 2B)+\sin^2 C$$

$$=1-\frac{1}{2}(\cos 2A+\cos 2B)+\sin^2 C$$

$$=1-\frac{1}{2}\{2\cos(A+B).\cos(A-B)\}+\sin^2 C$$

$$=1+\cos C.\cos(A-B)+1-\cos^2 C$$

$$=2-\cos^2 C+\cos C.\cos(A-B)$$

$\therefore \sin^2 A+\sin^2 B+\sin^2 C\le 2-\cos^2 C+\cos C$

{$\because$ the greatest value of cos $(A-B)=1$}

$$=2-(\cos^2 C-\cos C)$$

$$=2-\left(\cos^2 C-\cos C+\frac{1}{4}\right)+\frac{1}{4}$$

$$=\frac{9}{4}-\left(\cos C-\frac{1}{2}\right)^2$$

$\therefore \sin^2 A+\sin^2 B+\sin^2 C\le\frac{9}{4}$ .....(1)

Now, for positive quantities, $AM\ge GM$

$$\therefore \frac{\sin^2 A+\sin^2 B+\sin^2 C}{3}\ge\sqrt[3]{\sin^2 A\sin^2 B\sin^2 C}$$

$\therefore$ from (2), $\dfrac{\frac{9}{4}}{3}\ge\dfrac{\sin^2 A+\sin^2 B+\sin^2 C}{3}$

$$\ge(\sin A\sin B\sin C)^{2/3}$$

$\frac{3}{4}\ge(\sin A\sin B\sin C)^{2/3}$ or

$\sin A\sin B\sin C\le\left(\frac{3}{4}\right)^{\frac{3}{2}}$, *i.e.*, $\frac{3\sqrt{3}}{8}$.

Also,

$\sin^2 A + \sin^2 B + \sin^2 C \le 2 - \cos^2 C + \cos C$

$\Rightarrow \sin^2 A + \sin^2 B \le 1 + \cos C$

6. (a, c) (i) $\alpha \in \left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$

$\Rightarrow 3\alpha \in \left(\frac{13\pi}{16}, \frac{14\pi}{16}\right) = \left(\pi - \frac{3\pi}{16}, \pi - \frac{2\pi}{16}\right)$

i.e., $3\alpha \in$ II quadrant $\Rightarrow \sin 3\alpha > 0$

Again $2\alpha \in \left(\frac{13\pi}{24}, \frac{14\pi}{24}\right) = \left(\pi - \frac{11\pi}{24}, \pi - \frac{10\pi}{24}\right)$

i.e., $2\alpha \in$ II quadrant $\Rightarrow \cos 2\alpha < 0$

So that $\frac{\sin 3\alpha}{\cos 2\alpha}$ is negative

(ii) $\alpha \in \left(\frac{14\pi}{48}, \frac{18\pi}{48}\right) \Rightarrow 3\alpha \in \left(\frac{14\pi}{16}, \frac{18\pi}{16}\right)$

$3\alpha \in \left(\pi - \frac{2\pi}{16}, \pi + \frac{2\pi}{16}\right)$ i.e., II or III quadrant

so that $\sin 3\alpha$ can be positive or negative and

$2\alpha \in \left(\pi - \frac{10\pi}{24}, \pi - \frac{6\pi}{14}\right)$ i.e., II quadrant

$\Rightarrow \cos 2\alpha$ is negative and hence $\frac{\sin 3\alpha}{\cos 2\alpha}$ can be positive or negative.

(iii) $\alpha \in \left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$

$3\alpha \in \left(\pi + \frac{2\pi}{16}, \pi + \frac{7\pi}{16}\right) \Rightarrow 3\alpha \in$ III quadrant

so that $\sin 3\alpha$ is negative

and $2\alpha \in \left(\pi - \frac{6\pi}{24}, \pi - \frac{\pi}{24}\right) \Rightarrow 2\alpha \in$ II quadant

$\Rightarrow \cos 2\alpha$ is negative $\Rightarrow \frac{\sin 3\alpha}{\cos 2\alpha}$ is positive.

7. (b,c) We have

$\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta = \cos 2\theta$

$\Rightarrow (-2b)^2 - 4b = (-4)^2 - 4 \times 2$

$\Rightarrow b^2 - b - 2 = 0 \Rightarrow b = -1, 2$

8. (a, b,c) Since $\tan\left(\theta + \frac{\pi}{4}\right) = 3\tan 3\theta$

$\Rightarrow \frac{1 + \tan\theta}{1 - \tan\theta} = \frac{3.(3\tan\theta - \tan^3\theta)}{1 - 3\tan^2\theta}$

$\Rightarrow 3\tan^4\theta - 6\tan^2\theta + 8\tan\theta - 1 = 0$

If $\tan\alpha, \tan\beta, \tan\gamma, \tan\delta$ are the roots of this equation, then

$\sum \tan\alpha =$ sum of roots $= 0$

$\sum \tan\alpha \tan\beta = \frac{-6}{3} = -2$

$\sum \tan\alpha \tan\beta \tan\gamma = -\frac{8}{3}$ and

$\tan\alpha \tan\beta \tan\gamma \tan\delta = -\frac{1}{3}$

9. (b,c) $2a = [\sin x]^2 + \sin x \Rightarrow \sin x \in I (\text{as } a \in I)$

$\Rightarrow [\sin x] = \sin x \Rightarrow 2a = \sin x(\sin x + 1).$

Also, $\sin x$ can take the values – 1, 0 and 1 only

$\Rightarrow$ a can take two values 0 and 1.

10. (b,c) $\cos A + \cos B = 1$

$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 1$

$\Rightarrow \cos\left(\frac{A-B}{2}\right) = \frac{1}{\sqrt{3}}$

$\Rightarrow 2\cos^2\left(\frac{A-B}{2}\right) - 1 = \frac{1}{3}$

$\Rightarrow \cos(A-B) = -\frac{1}{3}$

$|\cos A - \cos B| = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$

$= 2 \times \frac{1}{2}\sqrt{1 - \frac{1}{3}} = \sqrt{\frac{2}{3}}$

11. **(a,b)** We can write the given equation as

$$\sqrt{\left(\sin\frac{A}{2}-\cos\frac{A}{2}\right)^2}=\left(\sin\frac{A}{2}-\cos\frac{A}{2}\right)$$

or $\left|\sin\frac{A}{2}-\cos\frac{A}{2}\right|=\left(\sin\frac{A}{2}-\cos\frac{A}{2}\right)$

$\Rightarrow\ \sin\frac{A}{2}\ge\cos\frac{A}{2}$

or $\sin\left(\frac{A}{2}-\frac{\pi}{4}\right)\ge 0 \quad \Rightarrow\ 0\le\frac{A}{2}-\frac{\pi}{4}\le\pi.$

12. **(a,c)** $y+\frac{1}{y}\ge 2\ \Rightarrow\ \sqrt{y+\frac{1}{y}}\ge\sqrt{2}.$

But $\sin x+\cos x\le\sqrt{2}$

$\Rightarrow\ y+\frac{1}{y}=2$ and $\sin x+\cos x=\sqrt{2}$

$\Rightarrow\ y=1$ and $\sin\left(x+\frac{\pi}{4}\right)=1$

or $y=1$ and $x=\frac{\pi}{4}.$

13. **(a,b,d)** $\sin 15°=\sqrt{\frac{1-\cos 30°}{2}}=\sqrt{\frac{2-\sqrt{3}}{4}}$

= an irrational number

$\cos 15°=\sqrt{\frac{1+\cos 30°}{2}}=\sqrt{\frac{2+\sqrt{3}}{4}}$

= an irrational number

$\sin 15°\cos 15°=\frac{1}{2}\sin 30°$

$=\frac{1}{4}$ = a rational number

$\sin 15°\cos 75°=\sin^2 15°$

$=\frac{2-\sqrt{3}}{4}$ = an irrational number

14. **(b,c)** $y=\cos x\cos(x+2)-\cos^2(x+1)$

$=\frac{1}{2}[\cos(2x+2)+\cos 2-(1+\cos(2x+2))]$

$=\frac{1}{2}(\cos 2-1)=-\sin^2 1$

$\therefore$ Graph is a straight line $y=-\sin^2 1$, which is parallel to x-axis

E MATRIX-MATCH TYPE

1. **A - q; B - r; C - s; D - p**

(A) $\cos^2 52\frac{1}{2}-\sin^2 22\frac{1}{2}=\cos 75°\cos 30°$

$$=\frac{\sqrt{3}-1}{2\sqrt{2}}\times\frac{\sqrt{3}}{2}=\frac{3-\sqrt{3}}{4\sqrt{2}}$$

(B) $\cos^2\frac{3\pi}{5}+\cos^2\frac{4\pi}{5}=\left(\frac{\sqrt{5}-1}{4}\right)^2+\left(\frac{\sqrt{5}+1}{4}\right)^2=\frac{3}{4}$

(C) $\sin 24°+\cos 6°=2\sin 54°\cos 30°=\frac{\sqrt{15}+\sqrt{3}}{4}$

(D) $\sin^2 50°+\cos^2 130°=1$

2. **A - p,q,r,s; B - r,s; C - q,r,s; D - q, s**

(A) $f(\theta)=(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2$

$=\sin^2\theta+\cos^2\theta+\sec^2\theta+\operatorname{cosec}^2\theta+4$

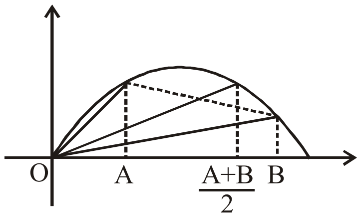
$=5+1+\cot^2\theta+1+\tan^2\theta=9+(\tan\theta-\cot\theta)^2\ge 9$

(B) $\sin\alpha-\sin\beta=a,\ \cos\alpha+\cos\beta=b$

$\Rightarrow a^2+b^2=2+2\cos(\alpha+\beta)=4\cos^2\frac{\alpha+\beta}{2}\le 4$

(C) $\frac{\sin A+\sin B}{2}\le\sin\left(\frac{A+B}{2}\right)$

$\therefore \sin A+\sin B\le 2\sin\frac{\pi}{4}$



or $\frac{1}{\sqrt{2}}(\sin A + \sin B) \le 1$

(D) Let $A = 7\cos x + 6\sin x = 6\,(2\cos x + \sin x) - 5\cos x$
$= 6 - 5\cos x$

Now, $2\cos x + \sin x = 1 \Rightarrow \sin x = 1 - 2\cos x$

$\Rightarrow \sin^2 x = 1 - \cos^2 x = 1 - 4\cos x + 4\cos^2 x$

$\therefore \cos x = 0$ or $\frac{4}{5}$. So, $A = 6$ or 2

3. A - r; B - s; C - r; D - p

(A) $\sin\theta + \cos\theta = \frac{b}{a}$ and $\sin\theta\cos\theta = \frac{c}{a}$

Now $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

$\Rightarrow \frac{b^2}{a^2} = 1 + \frac{2c}{a} \Rightarrow a^2 - b^2 + 2ac = 0$

(B) The expression is

$\cos y \sin x - \sin y \cos x + \sin y \sin x + \cos x \cos y$

$= \sin(x - y) + \cos(x - y)$

$= \sqrt{2}\sin\left\{\frac{\pi}{4} + (x - y)\right\}$ which vanishes if

$\frac{\pi}{4} + (x - y) = n\pi$ or $(x - y) = n\pi - \frac{\pi}{4}$

(C) We have $\sin 2B = \frac{3}{2}\sin 2A$ and $3\sin^2$

$A = 1 - 2\sin^2 B = \cos 2B$

Now $\cos(A + 2B) = \cos A.3\sin^2$

$A - \sin A\frac{3}{2}\sin 2A = 0$

$\therefore A + 2B = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

(D) We have $\tan(A + B) + \tan(A - B) = 4$;

$\tan(A + B)\tan(A - B) = 1 \Rightarrow 2A = \frac{\pi}{2}$ or $A = \frac{\pi}{4}$

Also, $\tan\left(\frac{\pi}{4} + B\right) + \tan\left(\frac{\pi}{4} - B\right) = 4$

$\Rightarrow \frac{1 + \tan B}{1 - \tan B} + \frac{1 - \tan B}{1 + \tan B} = 4 \Rightarrow \frac{1 - \tan^2 B}{1 + \tan^2 B} = \frac{1}{2}$

or $\cos 2B = \frac{1}{2}$

4. A - q; B - p, s; C - r; D - p

(A) $y = \cos^2\theta + \sin^4\theta = \cos^2\theta + \sin^2\theta(1 - \cos^2\theta)$

$= 1 - \frac{1}{4}\sin^2 2\theta \Rightarrow \frac{3}{4} \le A \le 1$

(B) $\tan A < 0 \Rightarrow A > \frac{\pi}{2} \Rightarrow 0 < B + C < \frac{\pi}{2}$

$\Rightarrow \tan(B + C) > 0 \Rightarrow \frac{\tan B + \tan C}{1 - \tan B \tan C} > 0$

$\Rightarrow 0 < \tan B \tan C < 1$

(C) Let $y = \frac{\cos^2\theta - 1}{\cos^2 + \cos\theta}$

$\Rightarrow (y - 1)\cos^2\theta + y\cos\theta + 1 = 0$

$\Rightarrow \cos\theta = -1$ or $\cos\theta = \frac{1}{1 - y}$

$-1 < \frac{1}{1 - y} < 1$

$\Rightarrow y < 0$ or $y > 2$

(D) $y = \tan A \tan B = \tan A \tan\left(\frac{\pi}{3} - A\right)$

$= x\left(\frac{\sqrt{3} - x}{1 + \sqrt{3}x}\right)$, where $x = \tan A$

$\Rightarrow x^2 + \sqrt{3}x(y - 1) + y = 0$

$\because x \in R \Rightarrow 3(y - 1)^2 - 4y \ge 0 \Rightarrow y \le \frac{1}{3}$ or $y \ge 3$

Also, $0 < A, B < \frac{\pi}{3} \Rightarrow 0 < \tan A, \tan B < \sqrt{3}$

$\Rightarrow 0 < \tan A \tan B < 3$

$\therefore 0 < y \le \frac{1}{3}$

F NUMERIC/INTEGER ANSWER TYPE

1. Ans : 3

Given, $6\cot\theta = \cot(\theta-\alpha)+\cot(\theta+\theta)$

$$=\frac{\cos(\theta-\alpha)}{\sin(\theta-\alpha)}+\frac{\cos(\theta+\alpha)}{\sin(\theta+\alpha)}$$

$$=\frac{\sin(\theta+\alpha)\cos(\theta-\alpha)+\cos(\theta+\alpha)\sin(\theta-\alpha)}{\sin(\theta+\alpha)\sin(\theta-\alpha)}$$

$$=\frac{\sin 2\theta}{\sin(\theta+\alpha)\sin(\theta-\alpha)}$$

$$\Rightarrow \frac{3\cos\theta}{\sin\theta}=\frac{2\sin\theta\cos\theta}{\cos 2\alpha-\cos 2\theta} \quad \text{Since, } \cos\theta\neq 0$$

$$\therefore\ 3(\cos 2\alpha-\cos 2\theta)=2\sin^2\theta$$

$$\Rightarrow 3\{(1-2\sin^2\alpha)-(1-2\sin^2\theta)\}=2\sin^2\theta$$

$$\Rightarrow 3(2\sin^2\theta-2\sin^2\alpha)=2\sin^2\theta$$

$$\Rightarrow 4\sin^2\theta=6\sin^2\alpha \Rightarrow \sin^2\theta=\frac{3}{2}\sin^2\alpha$$

$$\Rightarrow \frac{2\sin^2\theta}{\sin^2\alpha}=3$$

2. Ans : 1

Here $A+B+C=\pi$

So, $\tan\frac{A}{2}.\tan\frac{B}{2}+\tan\frac{B}{2}.\tan\frac{C}{2}+\tan\frac{C}{2}.\tan\frac{A}{2}=1 \quad \ldots(1)$

Now, $\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}-1$

$$\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}$$

$$-\left(\tan\frac{A}{2}.\tan\frac{B}{2}+\tan\frac{B}{2}.\tan\frac{C}{2}+\tan\frac{C}{2}.\tan\frac{A}{2}\right) \text{ using (1)}$$

$$=\frac{1}{2}\left\{\left(\tan\frac{A}{2}-\tan\frac{B}{2}\right)^2+\left(\tan\frac{B}{2}-\tan\frac{C}{2}\right)^2+\left(\tan\frac{C}{2}-\tan\frac{A}{2}\right)^2\right\}\geq 0$$

Equality holds if and only if

$\tan\frac{A}{2}=\tan\frac{B}{2}=\tan\frac{C}{2}$ i.e., $A=B=C$

Thus, $\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}-1\geq 0$ i.e.,

$$\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}\geq 1$$

equality holding when $A = B = C$, i.e., the triangle is equilateral.

3. Ans : 3

Given $A+B+C=\pi$

$$\Rightarrow \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}=\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2} \quad \ldots(i)$$

But $\tan\frac{A}{2}$, $\tan\frac{B}{2}$, $\tan\frac{C}{2}$ are in H.P.

$\Rightarrow \cot\frac{A}{2},\cot\frac{B}{2},\cot\frac{C}{2}$ are in A.P.

$$\Rightarrow \cot\frac{A}{2}+\cot\frac{C}{2}=2\cot\frac{B}{2} \quad \ldots(ii)$$

From (i) and (ii), we get $\cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2}=3\cot\frac{B}{2}$

$$\therefore\ \cot\frac{A}{2}.\cot\frac{C}{2}=3$$

Now, $\dfrac{\cot\frac{A}{2}+\cot\frac{C}{2}}{2}\geq\sqrt{\cot\frac{A}{2}\cot\frac{C}{2}}$

$$\Rightarrow \frac{2\cot\frac{B}{2}}{2}\geq\sqrt{3} \quad \text{[From (ii) an (iii)]}$$

$$\therefore\ \cot\frac{B}{2}\geq\sqrt{3}$$

4. Ans : 4

We have

$$\left(a\tan\beta-\sqrt{a^2-1}\tan\alpha\right)^2+\left(\sqrt{a^2+1}\tan\beta-\sqrt{a^2-1}\tan\gamma\right)^2$$

$$+\left(a\tan\gamma-\sqrt{a^2+1}\tan\alpha\right)^2\geq 0$$

$$\Rightarrow \{a^2+a^2-1+a^2+1\}(\tan^2\alpha+\tan^2\beta+\tan^2\gamma)$$

$$-\left\{a\tan\alpha+\sqrt{a^2-1}\tan\beta+\sqrt{a^2+1}\tan\gamma\right\}^2\geq 0$$

$$\Rightarrow \tan^2\alpha+\tan^2\beta+\tan^2\gamma\geq\frac{4a^2}{3a^2}\Rightarrow 3\sum\tan^2\alpha\geq 4$$

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