Permutations and Combinations

17

STANDARD THEORY

Factorial Notation! Or $\lfloor\ $

$\lfloor n = n(n-1)(n-2)\ldots3.2.1$

$n! = \underline{n}(n-1)(n-2)\ldots3.2.1$

= Product of n consecutive integers starting from 1.

1. $0! = 1$
2. Factorials of only Natural numbers are defined.
   $n!$ is defined only for $n \geq 0$
   $n!$ is not defined for $n < 0$
4. ${}^nC_r = 1$ when $n = r$.
5. Combinations (represented by ${}^nC_r$) can be defined as the number of ways in which r things at a time can be **SELECTED** from amongst n things available for selection.

   The key word here is **SELECTION**. Please understand here that the order in which the r things are selected has no importance in the counting of combinations.

   ${}^nC_r$ = Number of combinations (selections) of n things taken r at a time.

   ${}^nC_r = n!/[r!\,(n-r)!]$; where $n \geq r$ (n is greater than or equal to r).

Some typical situations where selection/combination is used:

(a) Selection of people for a team, a party, a job, an office etc. (e.g. Selection of a cricket team of 11 from 16 members)

(b) Selection of a set of objects (like letters, hats, points pants, shirts, etc) from amongst another set available for selection.

In other words any selection in which the order of selection holds no importance is counted by using combinations.

6. Permutations (represented by ${}^nP_r$) can be defined as the number of ways in which r things at a time can be **SELECTED** & **ARRANGED** at a time from amongst n things.

   The key word here is **ARRANGEMENT**. Hence please understand here that the order in which the r things are arranged has critical importance in the counting of permutations.

   In other words permutations can also be referred to as an **ORDERED SELECTION**.

   ${}^nP_r$ = number of permutations (arrangements) of n things taken r at a time.

   ${}^nP_r = n!/(n-r)!$; $n \geq r$

Some typical situations where **ordered selection/permutations** are used:

(a) Making words and numbers from a set of available letters and digits respectively

(b) Filling posts with people

(c) Selection of batting order of a cricket team of 11 from 16 members

(d) Putting distinct objects/people in distinct places, e.g. making people sit, putting letters in envelopes, finishing order in horse race, etc.)

The exact difference between selection and arrangement can be seen through the illustration below:

Selection

Suppose we have three men A, B and C out of which 2 men have to be selected to two posts.

This can be done in the following ways: AB, AC or BC (These three represent the basic selections of 2 people out of three which are possible. Physically they can be counted as 3 distinct selections. This value can also be got by using $3C_2$.

Note here that we are counting AB and BA as one single selection. So also AC and CA and BC and CB are considered to be the same instances of selection since the order of selection is not important.

Arrangement

Suppose we have three men A, B and C out of which 2 men have to be selected to the post of captain and vice captain of a team.

In this case we have to take AB and BA as two different instances since the order of the arrangement makes a difference in who is the captain and who is the vice captain.

Similarly, we have BC and CB and AC and CA as 4 more instances. Thus in all there could be 6 arrangements of 2 things out of three.

This is given by ${}^3P_2 = 6$.

7. **The Relationship Between Permutation & Combination:**

   When we look at the formulae for Permutations and Combinations and compare the two we see that,

   $${}^nP_r = r! \times {}^nC_r$$
   $$= {}^nC_r \times {}^rP_r$$

   This in words can be said as:

   The permutation or arrangement of r things out of n is nothing but the selection of r things out of n followed by the arrangement of the r selected things amongst themselves.

8. **MNP Rule:** If there are three things to do and there are M ways of doing the first thing, N ways of doing the second thing and P ways of doing the third thing then there will be **M** × **N** × **P** ways of doing all the three things together.The works are mutually inclusive.

   This is used to for situations like:

   The numbers 1, 2, 3, 4 and 5 are to be used for forming 3 digit numbers without repetition. In how many ways can this be done?

   Using the MNP rule you can visualise this as: There are three things to do→ The first digit can be selected in 5 distinct ways, the second can be selected in 4 ways and the third can be selected in 3 different ways. Hence, the total number of 3 digit numbers that can be formed are 5 × 4 × 3 = 60

9. When the pieces of work are mutually exclusive, there are M+N+P ways of doing the complete work.

Important Results

The following results are important as they help in problem solving.

1. Number of permutations (or arrangements) of n different things taken all at a time = $n!$
2. Number of permutations of n things out of which P_1 are alike and are of one type, P_2 are alike and are of a second type and P_3 are alike and are of a third type and the rest are all different = $n! / P_1! \, P_2! \, P_3!$

   Illustration: The number of words formed with the letters of the word Allahabad.

   Solution: Total number of Letters = 9 of which A occurs four times, L occurs twice and the rest are all different.

   Total number of words formed = 9! / (4! 2! 1!)
3. Number of permutations of n different things taken r times when repetition is allowed = $n \times n \times n \times \ldots$ (r times) = n^r.

   Illustration: In how many ways can 4 rings be worn in the index, ring finger and middle finger if there is no restriction of the number of rings to be worn on any finger?

   Solution: Each of the 4 rings could be worn in 3 ways either on the index, ring or middle finger. So, four rings could be worn in $3 \times 3 \times 3 \times 3 = 3^4$ ways.
4. Number of selections of r things out of n identical things = 1

   Illustration: In how many ways 5 marbles can be chosen out of 100 identical marbles?

   Solution: Since, all the 100 marbles are identical Hence, Number of ways to select 5 marbles = 1
5. Total number of selections of zero or more things out of k identical things = $k + 1$.

   This includes the case when zero articles are selected.
6. Total number of selections of zero or more things out of n different things =

   ${}^nC_o + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$

   ${}^nC_o + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

   Corollary: The number of selections of 1 or more things out of n different things $= {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - 1$
7. Number of ways of distributing n identical things among r persons when each person may get any number of things = ${}^{n+r-1}C_{r-1}$

   Imagine a situation where 27 marbles have to be distributed amongst 4 people such that each one of them can get any number of marbles (including zero marbles). Then for this situation we have, n = 27(no. of identical objects), r = 4 (no. of people) and the answer of the number of ways this can be achieved is given by:

   ${}^{n+r-1}C_{r-1} = {}^{30}C_3$.
8. Corollary: No. of ways of dividing n non distinct things to r distinct groups are:

   ${}^{n-1}C_{r-1}$ → For non-empty groups only

Also, the number of ways in which n distinct things can be distributed to r different persons:

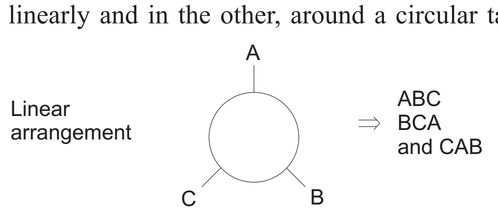
$$= r^n$$

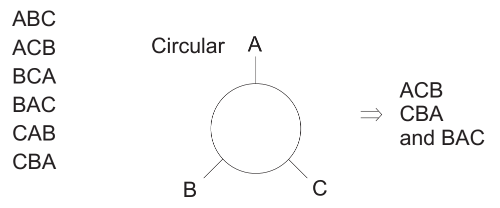
9. Number of ways of dividing $m + n$ different things in two groups containing m and n things respectively = ${}^{m+n}C_n \times {}^mC_m =$
   $= (m + n)!\ /m!\ n!$
   Or, ${}^{m+n}C_m \times {}^nC_n = (m + n)!\ /\ n!\ m!$
10. Number of ways of dividing $2n$ different things in two groups containing n things = $2n!\ /\ n!\ n!\ 2!$
11. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
12. ${}^nC_x = {}^nC_y \Rightarrow x = y$ or $x + y = n$
13. ${}^nC_r = {}^nC_{n-r}$
14. $r\ .\ {}^nC_r = n\ .\ {}^{n-1}C_{r-1}$
15. ${}^nC_r\ /\ (r + 1) = {}^{n+1}C_{r+1}\ /(\ n + 1)$
16. For ${}^nC_r$ to be greatest,
    (a) if n is even, $r = n/2$
    (b) if n is odd, $r = (n + 1)/2$ or $(n - 1)/2$
17. Number of selections of r things out of n different things
    (a) When k particular things are always included $= {}^{n-k}C_{r-k}$
    (b) When k particular things are excluded $= {}^{n-k}C_r$
    (c) When all the k particular things are not together in any selection
    $$= {}^nC_r - {}^{n-k}C_{r-k}$$
    No. of ways of doing a work with given restriction = total no. of ways of doing it — no. of ways of doing the same work with opposite restriction.
18. The total number of ways in which 0 to n things can be selected out of n things such that p are of one type, q are of another type and the balance r of different types is given by: $(p + 1)(q + 1)(2^r - 1)$.
19. Total number of ways of taking some or all out of $p + q + r$ things such that p are of one type and q are of another type and r of a third type
    $= (p + 1)(q + 1)(r + 1) - 1$
    [Only non-empty sets]
20. $$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$
21. Number of selections of k consecutive things out of n things in a row = $n - k + 1$

Circular Permutations

Consider two situations:

There are three A, B and C. In the first case, they are arranged linearly and in the other, around a circular table –





For the linear arrangement, each arrangement is a totally new way. For circular arrangements, three linear arrangements are represented by one and the same circular arrangement.

So, for six linear arrangements, there correspond only 2 circular arrangements. This happens because there is no concept of a starting point on a circular arrangement. (i.e., the starting point is not defined.)

Generalising the whole process, for $n!$, there corresponds to be $(1/n)\ n!$ ways.

Important Results

1. Number of ways of arranging n people on a circular track (circular arrangement) = $(n - 1)!$
2. When clockwise and anti-clockwise observation are not different then number of circular arrangements of n different things = $(n - 1)!\ /2$
   e.g. the case of a necklace with different beads, the same arrangement when looked at from the opposite side becomes anti-clockwise.
3. Number of selections of k consecutive things out of n things in a circle
   $= n$ when $k < n$
   $= 1$ when $k = n$

Some More Results

1. Number of terms in $(a_1 + a_2 + \ldots + a_n)^m$ is ${}^{m+n-1}C_{n-1}$

   Illustration: Find the number of terms in $(a + b + c)^2$.

   Solution: $n = 3$, $m = 2$

   ${}^{m+n-1}C_{n-1} = {}^4C_2 = 6$

   Corollary: Number of terms in
   $(1 + x\ + x^2 + \ldots x^n)^m$ is $mn + 1$
2. Number of zeroes ending the number represented by
   $n! = [n\ /5] + [n\ /5^2] + [n\ /5^3] + \ldots[n\ /5^x]$
   [] Shows greatest integer function where $5^x \le n$

   Illustration: Find the number of zeroes at the end of 1000!

   Solution: $[1000/5] + [1000/5^2] + [1000/5^3] + [1000/5^4]$

   $200 + 40 + 8 + 1 = 249$

   Corollary: Exponent of 3 in $n! = [n!/3] + [n!/32] + [n!/3^3] + \ldots[n!/3^x]$ where $3^x \le n$
   [] Shows greatest integer *fn*.

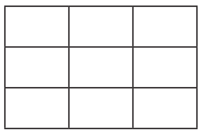
   Illustration: Find how many exponents of 3 will be there in 24!.

   Solution: $[24/3] + [24/3^2] = 8 + 2 = 10$

3. Number of squares in a square of $n \times n$ side $= 1^2 + 2^2 + 3^2 + 4^2 + \ldots.\ n^2$

   Number of rectangles in a square of $n \times n$ side $= 1^3 + 2^3 + 3^3 + 4^3 + \ldots.\ n^3$. (This includes the number of squares.)

   Thus the number of squares and rectangles in the following figure are given by:



Number of squares $= 1^2 + 2^2 + 3^2 = 14 = \in n^2$

Number of rectangles $= 1^3 + 2^3 + 3^3 = 36\ (\in n)^2 = \in n^3$ for the rectangle.

A rectangle having m rows and n columns:

The number of squares is given by: $m.n + (m - 1)(n - 1) + (m - 2)(n - 2) + \ldots..$ until any of $(m - x)$ or $(n - x)$ comes to 1.

The number of rectangles is given by: $(1 + 2 + \ldots\ldots + m)(1 + 2 + \ldots\ldots + n)$

Space for Notes

WORKED-OUT PROBLEMS

In the following examples the solution is given upto the point of writing down the formula that will apply for the particular question. The student is expected to calculate the values after understanding the solution.

Problem 17.1 Find the number of permutations of 6 things taken 4 at a time.

Solution The answer will be given by $^6P_4$.

Problem 17.2 How many 3-digit numbers can be formed out of the digits 1, 2, 3, 4 and 5?

Solution Forming numbers requires an ordered selection. Hence, the answer will be $^5P_3$.

Problem 17.3 In how many ways can the 7 letters *M*, *N*, *O*, *P*, *Q*, *R*, *S* be arranged so that *P* and *Q* occupy continuous positions?

Solution For arranging the 7 letters keeping *P* and *Q* always together we have to view *P* and *Q* as one letter. Let this be denoted by $\underline{PQ}$.

Then, we have to arrange the letters *M*, *N*, *O*, $\underline{PQ}$, *R* and *S* in a linear arrangement. Here, it is like arranging 6 letters in 6 places (since 2 letters are counted as one). This can be done in 6! ways.

However, the solution is not complete at this point of time since in the count of 6! the internal arrangement between *P* and *Q* is neglected. This can be done in 2! ways. Hence, the required answer is $6! \times 2!$.

Task for the student: What would happen if the letters *P*, *Q* and *R* are to be together? (Ans: $5! \times 3!$)

What if *P* and *Q* are never together? (Answer will be given by the formula: Total number of ways – Number of ways they are always together)

Problem 17.4 Of the different words that can be formed from the letters of the words BEGINS how many begin with *B* and end with *S*?

Solution *B* & *S* are fixed at the start and the end positions. Hence, we have to arrange *E*, *G*, *I* and *N* amongst themselves. This can be done in 4! ways.

Task for the student: What will be the number of words that can be formed with the letters of the word BEGINS which have *B* and *S* at the extreme positions? (Ans: $4! \times 2!$)

Problem 17.5 In how many ways can the letters of the word VALEDICTORY be arranged, so that all the vowels are adjacent to each other?

Solution There are 4 vowels and 7 consonants in Valedictory. If these vowels have to be kept together, we have to consider AEIO as one letter. Then the problem transforms itself into arranging 8 letters amongst themselves (8! ways). Besides, we have to look at the internal arrangement of the 4 vowels amongst themselves. (4! ways)

Hence Answer = $8! \times 4!$.

Problem 17.6 If there are two kinds of hats, red and blue and at least 5 of each kind, in how many ways can the hats be put in each of 5 different boxes?

Solution The significance of at least 5 hats of each kind is that while putting a hat in each box, we have the option of putting either a red or a blue hat. (If this was not given, there would have been an uncertainty in the number of possibilities of putting a hat in a box.)

Thus in this question for every task of putting a hat in a box we have the possibility of either putting a red hat or a blue hat. The solution can then be looked at as: there are 5 tasks each of which can be done in 2 ways. Through the MNP rule we have the total number of ways = 2^5 (Answer).

Problem 17.7 In how many ways can 4 Indians and 4 Nepalese people be seated around a round table so that no two Indians are in adjacent positions?

Solution If we first put 4 Indians around the round table, we can do this in 3! ways.

Once the 4 Indians are placed around the round table, we have to place the four Nepalese around the same round table. Now, since the Indians are already placed we can do this in 4! ways (as the starting point is defined when we put the Indians. Try to visualize this around a circle for placing 2 Indians and 2 Nepalese.)

Hence, Answer = $3! \times 4!$

Problem 17.8 How many numbers greater than a million can be formed from the digits 1, 2, 3, 0, 4, 2, 3?

Solution In order to form a number greater than a million we should have a 7 digit number. Since we have only seven digits with us we cannot take 0 in the starting position. View this as 7 positions to fill:

_ _ _ _ _ _ _

To solve this question we first assume that the digits are all different. Then the first position can be filled in 6 ways (0 cannot be taken), the second in 6 ways (one of the 6 digits available for the first position was selected. Hence, we have 5 of those 6 digits available. Besides, we also have the zero as an additional digit), the third in 5 ways (6 available for the 2nd position – 1 taken for the second position.) and so on. Mathematically this can be written as:

$$6 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6 \times 6!$$

This would have been the answer had all the digits been distinct. But in this particular example we have two 2's and two 3's which are identical to each other. This complication is resolved as follows to get the answer:

$$\frac{6 \times 6}{2! \times 2!}$$

Problem 17.9 If there are 11 players to be selected from a team of 16, in how many ways can this be done?

Solution $^{16}C_{11}$.

Problem 17.10 In how many ways can 18 identical white and 16 identical black balls be arranged in a row so that no two black balls are together?

Solution When 18 identical white balls are put in a straight line, there will be 19 spaces created. Thus 16 black balls will have 19 places to fill in. This will give an answer of: $^{19}C_{16}$. (Since, the balls are identical the arrangement is not important.)

Problem 17.11 A mother with 7 children takes three at a time to a cinema. She goes with every group of three that she can form. How many times can she go to the cinema with distinct groups of three children?

Solution She will be able to do this as many times as she can form a set of three distinct children from amongst the seven children. This essentially means that the answer is the number of selections of 3 people out of 7 that can be done.

Hence, Answer = $^7C_3$.

Problem 17.12 For the above question, how many times will an individual child go to the cinema with her before a group is repeated?

Solution This can be viewed as: The child for whom we are trying to calculate the number of ways is already selected. Then, we have to select 2 more children from amongst the remaining 6 to complete the group. This can be done in $^6C_2$ ways.

Problem 17.13 How many different sums can be formed with the following coins:
5 rupee, 1 rupee, 50 paisa, 25 paisa, 10 paisa and 1 paisa?

Solution A distinct sum will be formed by selecting either 1 or 2 or 3 or 4 or 5 or all 6 coins.

But from the formula we have the answer to this as : $2^6 - 1$.

[Task for the student: How many different sums can be formed with the following coins:

5 rupee, 1 rupee, 50 paisa, 25 paisa, 10 paisa, 3 paisa, 2 paisa and 1 paisa?

Hint: You will have to subtract some values for double counted sums.]

Problem 17.14 A train is going from Mumbai to Pune and makes 5 stops on the way. 3 persons enter the train during the journey with 3 different tickets. How many different sets of tickets may they have had?

Solution Since the 3 persons are entering during the journey they could have entered at the:

1^{st} station (from where they could have bought tickets for the 2^{nd}, 3^{rd}, 4^{th} or 5^{th} stations or for Pune $\rightarrow$ total of 5 tickets.)

2^{nd} station (from where they could have bought tickets for the 3^{rd}, 4^{th} or 5^{th} stations or for Pune $\rightarrow$ total of 4 tickets.)

3^{rd} station (from where they could have bought tickets for the 4th or 5th stations or for Pune $\rightarrow$ total of 3 tickets.)

4^{th} station (from where they could have bought tickets for the 5^{th} station or for Pune $\rightarrow$ total of 2 tickets.)

5^{th} station (from where they could have bought a ticket for Pune $\rightarrow$ total of 1 ticket.)

Thus, we can see that there are a total of 5 + 4 + 3 + 2 + 1 = 15 tickets available out of which 3 tickets were selected. This can be done in $^{15}C_3$ ways (Answer).

Problem 17.15 Find the number of diagonals and triangles formed in a decagon.

Solution A decagon has 10 vertices. A line is formed by selecting any two of the ten vertices. This can be done in $^{10}C_2$ ways. However, these $^{10}C_2$ lines also count the sides of the decagon.

Thus, the number of diagonals in a decagon is given by: $^{10}C_2 - 10$ (Answer)

Triangles are formed by selecting any three of the ten vertices of the decagon. This can be done in $^{10}C_3$ ways (Answer).

Problem 17.16 Out of 18 points in a plane, no three are in a straight line except 5 which are collinear. How many straight lines can be formed?

Solution If all 18 points were non-collinear then the answer would have been $^{18}C_2$. However, in this case $^{18}C_2$ has double counting since the 5 collinear points are also amongst the 18. These would have been counted as $^5C_2$ whereas they should have been counted as 1. Thus, to remove the double counting and get the correct answer we need to adjust by reducing the count by ($^5C_2 - 1$).

Hence, Answer = $^{18}C_2 - (^5C_2 - 1) = {^{18}C_2} - {^5C_2} + 1$

Problem 17.17 For the above situation, how many triangles can be formed?

Solution The triangles will be given by $^{18}C_3 - {^5C_3}$.

Problem 17.18 A question paper had ten questions. Each question could only be answered as True (T) or False (F). Each candidate answered all the questions. Yet, no two candidates wrote the answers in an identical sequence. How many different sequences of answers are possible?

(a) 20 (b) 40
(c) 512 (d) 1024

Solution 2^{10} = 1024 unique sequences are possible. Option (d) is correct.

Problem 17.19 When ten persons shake hands with one another, in how many ways is it possible?

(a) 20 (b) 25
(c) 40 (d) 45

Solution For n people there are always ${}^nC_2$ shake hands. Thus, for 10 people shaking hands with each other the number of ways would be ${}^{10}C_2 = 45$.

Problem 17.20 In how many ways can four children be made to stand in a line such that two of them, A and B are always together?

(a) 6 (b) 12
(c) 18 (d) 24

Solution If the children are A, B, C, D we have to consider A & B as one child. This, would give us 3! ways of arranging AB, C and D. However, for every arrangement with AB, there would be a parallel arrangement with BA. Thus, the correct answer would be $3! \times 2! = 12$ ways. Option (b) is correct.

Problem 17.21 Each person's performance compared with all other persons is to be done to rank them subjectively. How many comparisons are needed to total, if there are 11 persons?

(a) 66 (b) 55
(c) 54 (d) 45

Solution There would be ${}^{11}C_2$ combinations of 2 people taken 2 at a time for comparison. ${}^{11}C_2 = 55$.

Problem 17.22 A person X has four notes of Rupee 1, 2, 5 and 10 denomination. The number of different sums of money she can form from them is

(a) 16 (b) 15
(c) 12 (d) 8

Solution $2^4 - 1 = 15$ sums of money can be formed. Option (b) is correct.

Problem 17.23 A person has 4 coins each of different denomination. What is the number of different sums of money the person can form (using one or more coins at a time)?

(a) 16 (b) 15
(c) 12 (d) 11

Solution $2^4 - 1 = 15$. Hence, option (b) is correct.

Problem 17.24 How many three-digit numbers can be generated from 1, 2, 3, 4, 5, 6, 7, 8, 9, such that the digits are in ascending order?

(a) 80 (b) 81
(c) 83 (d) 84

Solution Numbers starting with 12 – 7 numbers

Numbers starting with 13 – 6 numbers; 14 – 5, 15 – 4, 16 – 3, 17 – 2, 18 – 1. Thus total number of numbers starting from 1 is given by the sum of 1 to 7 = 28.

Number of numbers starting from 2- would be given by the sum of 1 to 6 = 21

Number of numbers starting from 3- sum of 1 to 5 = 15

Number of numbers starting from 4 – sum of 1 to 4 = 10

Number of numbers starting from 5 – sum of 1 to 3 = 6

Number of numbers starting from 6 = 1 + 2 = 3

Number of numbers starting from 7 = 1

Thus a total of: 28 + 21 + 15 + 10 + 6 + 3 + 1 = 84 such numbers. Option (d) is correct.

Problem 17.25 In a carrom board game competition, m boys n girls ($m > n > 1$) of a school participate in which every student has to play exactly one game with every other student. Out of the total games played, it was found that in 221 games one player was a boy and the other player was a girl.

Consider the following statements:

I. The total number of students that participated in the competition is 30.

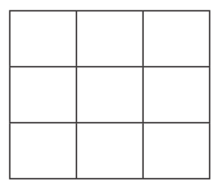
II. The number of games in which both players were girls is 78.

Which of the statements given above is/are correct?

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II.

Solution The given condition can get achieved if we were to use 17 boys and 13 girls. In such a case both statement I and II are correct. Hence, option (c) is correct.

Problem 17.26



In how many different ways can all of 5 identical balls be placed in the cells shown above such that each row contains at least 1 ball?

(a) 64 (b) 81
(c) 84 (d) 108

Solution The placement of balls can be 3, 1, 1 and 2, 2, 1. For 3, 1, 1- If we place 3 balls in the top row, there would be ${}^3C_1$ ways of choosing a place for the ball in the second row and ${}^3C_1$ ways of choosing a place for the ball in the third row. Thus, ${}^3C_1 \times {}^3C_1 = 9$ ways. Similarly there would be 9 ways each if we were to place 3 balls in the second row and 3 balls in the third row. Thus, with the 3, 1, 1 distribution of 5 balls we would get 9 + 9 + 9 = 27 ways of placing the balls.

We now need to look at the 2, 2, 1 arrangement of balls. If we place 1 ball in the first row, we would need to place 2 balls each in the second and the third rows. In such a case, the number of ways of arranging the balls would be ${}^3C_1 \times {}^3C_2 \times {}^3C_2 = 27$ ways. (choosing 1 place out of 3 in the first row, 2 places out of 3 in the second row and 2 places out of 3 in the third row).

Similarly if we were to place 1 ball in the second row and 2 balls each in the first and third rows we would get 27 ways of placing the balls and another 27 ways of placing the balls if we place 1 ball in the third row and 2 balls each in the other two rows.

Thus with a 2, 2, 1 distribution of the 5 balls we would get 27 + 27 + 27 = 81 ways of placing the balls.

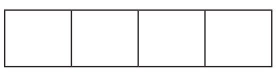
Hence, total number of ways = Number of ways of placing the balls with a 3,1,1 distribution of balls + number of ways of placing the balls with a 2, 2, 1 distribution of balls = 27 + 81 = 108.

Hence, option (d) is correct.

Problem 17.27 There are 6 different letter and 6 correspondingly addressed envelopes. If the letters are randomly put in the envelopes, what is the probability that exactly 5 letters go into the correctly addressed envelopes?

(a) Zero (b) 1/6
(c) 1/2 (d) 5/6

Solution If 5 letters go into the correct envelopes the sixth would automatically go into it's correct envelope. Thus, there is no possibility when exactly 5 letters are correct and 1 is wrong. Hence, option (a) is correct.

Problem 17.28 

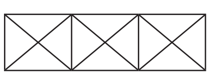
There are two identical red, two identical black and two identical white balls. In how many different ways can the balls be placed in the cells (each cell to contain one ball) shown above such that balls of the same colour do not occupy any two consecutive cells?

(a) 15 (b) 18
(c) 24 (d) 30

Solution In the first cell, we have 3 options of placing a ball. Suppose we were to place a red ball in the first cell- then the second cell can only be filled with either black or white – so 2 ways. Subsequently there would be 2 ways each of filling each of the cells (because we cannot put the colour we have already used in the previous cell).

Thus, the required number of ways would be 3 × 2 × 2 × 2 = 24 ways.

Hence, option (c) is correct.

Problem 17.29 

How many different triangles are there in the figure shown above?

(a) 28 (b) 24
(c) 20 (d) 16

Solution Look for the smallest triangles first—there are 12 of them.

Then, look for the triangles which are equal to half the rectangle—there are 12 of them.

Besides, there are 4 bigger triangles (spanning across 2 rectangles).

Thus a total of 28 triangles can be seen in the figure.

Hence, option (a) is correct.

Problem 17.30 A teacher has to choose the maximum different groups of three students from a total of six students. Of these groups, in how many groups there will be included a particular student?

(a) 6 (b) 8
(c) 10 (d) 12

Solution If the students are *A*, *B*, *C*, *D*, *E* and *F*- we can have $^6C_3$ groups in all. However, if we have to count groups in which a particular student (say *A*) is always selected- we would get $^5C_2$ = 10 ways of doing it. Hence, option (c) is correct.

Problem 17.31 Three dice (each having six faces with each face having one number from 1 to 6) are rolled. What is the number of possible outcomes such that at least one dice shows the number 2?

(a) 36 (b) 81
(c) 91 (d) 116

Solution **All 3 dice have twos** – 1 case.

Two dice have twos:

This can principally occur in 3 ways which can be broken into:

If the first two dice have 2- the third dice can have 1, 3, 4, 5 or 6 = 5 ways.

Similarly, if the first and third dice have 2, the second dice can have 5 outcomes → 5 ways and if the second and third dice have a 2, there would be another 5 ways. Thus a total of 15 outcomes if 2 dice have a 2.

With only 1 dice having a two- If the first dice has 2, the other two can have 5 × 5 = 25 outcomes.

Similarly 25 outcomes if the second dice has 2 and 25 outcomes if the third dice has 2. A total of 75 outcomes. Thus, a total of 1 + 15 + 75 = 91 possible outcomes with at least.

Hence, option (c) is correct.

Problem 17.32 All the six letters of the name SACHIN are arranged to form different words without repeating any letter in any one word. The words so formed are then arranged as in a dictionary. What will be the position of the word SACHIN in that sequence?

(a) 436 (b) 590
(c) 601 (d) 751

Solution All words staring with *A*, *C*, *H*, *I* and *N* would be before words starting with S. So we would have 5! Words (= 120 words) each starting with *A*, *C*, *H*, *I* and *N*. Thus, a total of 600 words would get completed before we start off with *S*. SACHIN would be the first word starting with *S*, because *A*, *C*, *H*, *I*, *N* in that order is the correct alphabetical sequence. Hence, Sachin would be the 601st word. Hence, option (c) is correct.

Problem 17.33 Five balls of different colours are to be placed in three different boxes such that any box contains at least one 1 ball. What is the maximum number of different ways in which this can be done?

(a) 90 (b) 120
(c) 150 (d) 180

Solution The arrangements can be [3 & 1 & 1 or 1 & 3 & 1 or 1 & 1 & 3] or 2 & 2 & 1 or 2 & 1 & 2 or 1 & 2 and 2.

Total number of ways = $3 \times {}^5C_3 \times {}^2C_1 \times {}^1C_1 + 3 \times {}^5C_2 \times {}^3C_2 \times {}^1C_1 = 60 + 90 = 150$ ways

Hence, option (c) is correct.

Problem 17.34 Amit has five friends: 3 girls and 2 boys. Amit's wife also has 5 friends : 3 boys and 2 girls. In how many maximum number of different ways can they invite 2 boys and 2 girls such that two of them are Amit's friends and two are his wife's?

(a) 24 (b) 38
(c) 46 (d) 58

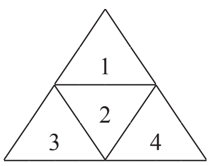
Solution The selection can be done in the following ways:

2 boys from Amit's friends and 2 girls from his wife's friends OR 1 boy & 1 girl from Amit's friends and 1 boy and 1 girl from his wife's friends OR 2 girls from Amit's friends and 2 boys from his wife's friends.

The number of ways would be:

${}^2C_2 \times {}^2C_2 + {}^3C_1 \times {}^2C_1 \times {}^3C_1 \times {}^2C_1 + {}^3C_2 \times {}^3C_2 = 1 + 36 + 9 = 46$ ways.

Problem 17.35



In the given figure, what is the maximum number of different ways in which 8 identical balls can be placed in the small triangles 1, 2, 3 and 4 such that each triangle contains at least one ball?

(a) 32 (b) 35
(c) 44 (d) 56

Solution The ways of placing the balls would be 5, 1, 1, 1 (4!/3! = 4 ways); 4, 2, 1 & 1 (4!/2! = 12 ways); 3, 3, 1, 1 (4!/2! × 2! = 6 ways); 3, 2, 2, 1 (4!/2! = 12 ways) and 2, 2, 2, 2 (1 way). Total number of ways = 4 + 12 + 6 + 12 + 1 = 35 ways. Hence, option (b) is correct.

Problem 17.36 6 equidistant vertical lines are drawn on a board. 6 equidistant horizontal lines are also drawn on the board cutting the 6 vertical lines, and the distance between any two consecutive horizontal lines is equal to that between any two consecutive vertical lines. What is the maximum number of squares thus formed?

(a) 37 (b) 55
(c) 91 (d) 225

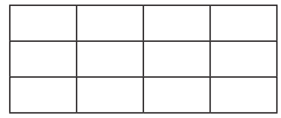
Solution The number of squares would be $1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$. Hence, option (b) is correct.

Problem 1.37 Groups each containing 3 boys are to be formed out of 5 boys—*A*, *B*, *C*, *D* and *E* such that no group contains both *C* and *D* together. What is the maximum number of such different groups?

(a) 5 (b) 6
(c) 7 (d) 8

Solution All groups – groups with *C* and *D* together = ${}^5C_3 - {}^3C_1 = 10 - 3 = 7$

Problem 17.38



In how many maximum different ways can 3 identical balls be placed in the 12 squares (each ball to be placed in the exact centre of the squares and only one ball is to be placed in one square) shown in the figure given above such that they do not lie along the same straight line?

(a) 144 (b) 200
(c) 204 (d) 216

Solution The thought process for this question would be:

All arrangements (${}^{12}C_3$) – Arrangements where all 3 balls are in the same row ($3 \times {}^4C_3$)– arrangements where all 3 balls are in the same straight line diagonally (4 arrangements) – arrangements where all 3 balls are in the same column (4 arrangements) = ${}^{12}C_3 - 3 \times {}^4C_3 - 4 - 4 = 220 - 12 - 4 - 4 = 200$ ways.

Hence, option (b) is correct.

Problem 17.39 How many numbers are there in all from 6000 to 6999 (Both 6000 and 6999 included) having at least one of their digits repeated?

(a) 216 (b) 356
(c) 496 (d) 504

Solution All numbers – numbers having no numbers repeated = 1000 – 9 × 8 × 7 = 1000 – 504 = 496 numbers. Hence, option (c) is correct.

Problem 17.40 Each of two women and three men is to occupy one chair out of eight chairs, each of which is numbered from one to eight. First, women are to occupy any two chairs from\those numbered one to four; and then the three men would occupy any three chairs out of the remaining six chairs. What is the maximum number of different ways in which this can be done?

(a) 40 (b) 132
(c) 1440 (d) 3660

Solution ${}^4C_2 \times 2! \times {}^6C_3 \times 3! = 6 \times 2 \times 20 \times 6 = 1440$. Hence, option (c) is correct.

Problem 17.41 A box contains five set of balls while there are three balls in each set. Each set of balls has one ball, whose colour is different from every other ball in that set and also from every other ball in any other set. What is the least number of balls that must be removed from the box in order to claim with certainty that a pair of balls of the same colour has been removed?

(a) 6 (b) 7
(c) 9 (d) 11

Solution Let *C*1, *C*2, *C*3, *C*4 and *C*5 be the 5 distinct colours which have no repetition. For being definitely sure that we have picked up 2 balls of the same colour we need to consider the worst case situation.

Consider the following scenario:

Set 1	*Set 2*	*Set 3*	*Set 4*	*Set 5*
C1	C2	C3	C4	C5
C6	C8	C7	C9	C9
C7	C6	C10	C10	C8

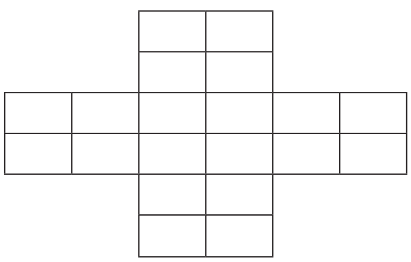
In the above distribution of balls each set has exactly 1 ball which is unique in it's colour while the colours of the other two balls are shared at least once in one of the other sets. In such a case, the worst scenario would be if we pick up the first 10 balls and they all turn out to be of different colours. The 11^{th} ball has to be of a colour which has already been taken. Thus,if we were to pick out 11 balls we would be sure of having at least 2 balls of the same colour. Hence, option (d) is correct.

Problem 17.42 In a question paper, there are four multiple-choice questions. Each question has five choices with only one choice as the correct answer. What is the total number of ways in which a candidate will not get all the four answers correct?

(a) 19 (b) 120
(c) 624 (d) 1024

Solution 5^4 would be the total number of ways in which the questions can be answered. Out of these there would be only 1 way of getting all 4 correct. Thus, there would be 624 ways of not getting all answers correct.

Problem 17.43



Each of 8 identical balls is to be placed in the squares shown in the figure given in a horizontal direction such that one horizontal row contains 6 balls and the other horizontal row contains 2 balls. In how many maximum different ways can this be done?

(a) 38 (b) 28
(c) 16 (d) 14

Solution The 6 balls must be on either of the middle rows. This can be done in 2 ways. Once, we put the 6 balls in their single horizontal row- it becomes evident that for placing the 2 remaining balls on a straight line there are 2 principal options:

1. Placing the two balls in one of the four rows with two squares. In this case the number of ways of placing the balls in any particular row would be 1 way (since once you were to choose one of the 4 rows, the balls would automatically get placed as there are only two squares in each row.) Thus the total number of ways would be $2 \times 4 \times 1 = 8$ ways.
2. Placing the two balls in the other row with six squares. In this case the number of ways of placing the 2 balls in that row would be ${}^6C_2$. This would give us ${}^2C_1 \times 1 \times {}^6C_2 = 30$ ways. Total is $30 + 8 = 38$ ways.

Hence, option (a) is correct.

Problem 17.44 In a tournament each of the participants was to play one match against each of the other participants. 3 players fell ill after each of them had played three matches and had to leave the tournament. What was the total number of participants at the beginning, if the total number of matches played was 75?

(a) 8 (b) 10
(c) 12 (d) 15

Solution The number of players at the start of the tournament cannot be 8, 10 or 12 because in each of these cases the total number of matches would be less than 75 (as ${}^8C_2$, ${}^{10}C_2$ and ${}^{12}C_2$ are all less than 75.) This only leaves 15 participants in the tournament as the only possibility.

Hence, option (d) is correct.

Problem 17.45 There are three parallel straight lines. Two points *A* and *B* are marked on the first line, points *C* and *D* are marked on the second line and points *E* and *F* are marked on the third line. Each of these six points can move to any position on its respective straight line.

Consider the following statements:

I. The maximum number of triangles that can be drawn by joining these points is 18.
II. The minimum number of triangles that can be drawn by joining these points is zero.

Which of the statements given above is/are correct?

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

Solution The maximum triangles would be in case all these 6 points are non-collinear. In such a case the number of triangles is ${}^6C_3 = 20$. Statement I is incorrect.

Statement II is correct because if we take the position that *A* and *B* coincide on the first line, *C* & *D* coincide on the second line, *E* & *F* coincide on the third line and all these coincidences happen at 3 points which are on the same straight line- in such a case there would be 0 triangles formed. Hence, option (b) is correct.

Problem 17.46 A mixed doubles tennis game is to be played between two teams (each team consists of one male and one female). There are four married couples. No team is to consist of a husband and his wife. What is the maximum number of games that can be played?

(a) 12 (b) 21
(c) 36 (d) 42

Solution First select the two men. This can be done in ${}^4C_2$ ways. Let us say the men are *A*, *B*, *C* and *D* and their respective wives are *a*, *b*, *c* and *d*.

If we select A and B as the two men then while selecting the women there would be two cases as seen below:

Case 1:

A	If b is selected to partner A
B	There will be 3 choices for choosing B's partner – viz a, c and d

Thus, total number of ways in this case = ${}^4C_2 \times 1 \times {}^3C_1$ = 18 ways.

Case 2:

A	If either c or d is selected to partner A
B	There will be 2 choices for choosing B's partner – viz a and any one of c and d

Total number of ways of doing this = ${}^4C_2 \times 2 \times {}^2C_1$ = 24 ways.

Hence, the required answer is 18 + 24 = 42 ways.

Hence, option (d) is correct.

Space for Rough Work

LEVEL OF DIFFICULTY (I)

1. How many numbers of 3-digits can be formed with the digits 1, 2, 3, 4, 5 (repetition of digits not allowed)?
   (a) 125 (b) 120
   (c) 60 (d) 150
2. How many numbers between 2000 and 3000 can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7 (repetition of digits not allowed?
   (a) 42 (b) 210
   (c) 336 (d) 440
3. In how many ways can a person send invitation cards to 6 of his friends if he has four servants to distribute the cards?
   (a) 6^4 (b) 4^6
   (c) 24 (d) 120
4. In how many ways can 5 prizes be distributed to 8 students if each student can get any number of prizes?
   (a) 40 (b) 5^8
   (c) 8^5 (d) 120
5. In how many ways can 7 Indians, 5 Pakistanis and 6 Dutch be seated in a row so that all persons of the same nationality sit together?
   (a) 3! (b) 7!5!6!
   (c) 3!.7!.5!.6! (d) 182
6. There are 5 routes to go from Allahabad to Patna & 4 ways to go from Patna to Kolkata, then how many ways are possible for going from Allahabad to Kolkata via Patna?
   (a) 20 (b) 5^4
   (c) 4^5 (d) $5^4 + 4^5$
7. There are 4 qualifying examinations to enter into Oxford University: RAT, BAT, SAT, and PAT. An Engineer cannot go to Oxford University through BAT or SAT. A CA on the other hand can go to the Oxford University through the RAT, BAT & PAT but not through SAT. Further there are 3 ways to become a CA(viz., Foundation, Inter & Final). Find the ratio of number of ways in which an Engineer can make it to Oxford University to the number of ways a CA can make it to Oxford University.
   (a) 3:2 (b) 2:3
   (c) 2:9 (d) 9:2
8. How many straight lines can be formed from 8 non-collinear points on the X-Y plane?
   (a) 28 (b) 56
   (c) 18 (d) 19860
9. If ${}^nC_3 = {}^nC_8$, find n.
   (a) 11 (b) 12
   (c) 14 (d) 10
10. In how many ways can the letters of the word DELHI be arranged?
    (a) 119 (b) 120
    (c) 60 (d) 24
11. In how many ways can the letters of the word PATNA be rearranged?
    (a) 60 (b) 120
    (c) 119 (d) 59
12. For the arrangements of the letters of the word PATNA, how many words would start with the letter P?
    (a) 24 (b) 12
    (c) 60 (d) 120
13. In Question no.11, how many words will start with P and end with T?
    (a) 3 (b) 6
    (c) 11 (d) 12
14. If ${}^nC_4 = 70$, find n.
    (a) 5 (b) 8
    (c) 4 (d) 7
15. If ${}^{10}P_r = 720$, find r.
    (a) 4 (b) 5
    (c) 3 (d) 6
16. How many numbers of four digits can be formed with the digits 0, 1, 2, 3 (repetition of digits is not allowed?
    (a) 18 (b) 24
    (c) 64 (d) 192
17. How many numbers of four digits can be formed with the digits 0, 1, 2, 3 (repetition of digits being allowed?
    (a) 12 (b) 108
    (c) 256 (d) 192
18. How many numbers between 200 and 1200 can be formed with the digits 0, 1, 2, 3 (repetition of digits not allowed?
    (a) 6 (b) 6
    (c) 2 (d) 14
19. For the above question, how many numbers can be formed with the same digits if repetition of digits is allowed?
    (a) 48 (b) 63
    (c) 32 (d) 14

20. If $(2n + 1)P_{(n-1)} : (2n - 1)P_n = 7:10$ find n

(a) 4 (b) 6
(c) 3 (d) 7

21. If $({}^{28}C_{2r} : {}^{24}C_{2r-4}) = 225:11$ Find the value of r.

(a) 10 (b) 11
(c) 7 (d) 9

22. Arjit being a party animal wants to hold as many parties as possible amongst his 20 friends. However, his father has warned him that he will be financing his parties under the following conditions only:

(a) The invitees have to be amongst his 20 best friends.
(b) He cannot call the same set of friends to a party more than once.
(c) The number of invitees to every party have to be the same.

Given these constraints Arjit wants to hold the maximum number of parties. How many friends should he invite to each party?

(a) 11 (b) 8
(c) 10 (d) 12

23. In how many ways can 10 identical presents be distributed among 6 children so that each child gets at least one present?

(a) ${}^{15}C_5$ (b) ${}^{16}C_6$
(c) ${}^{9}C_5$ (d) 6^{10}

24. How many four digit numbers are possible, criteria being that all the four digits are odd?

(a) 125 (b) 625
(c) 45 (d) none of these

25. A captain and a vice-captain are to be chosen out of a team having eleven players. How many ways are there to achieve this?

(a) 10.9 (b) ${}^{11}C_2$
(c) 110 (d) 10.9!

26. There are five types of envelopes and four types of stamps in a post office. How many ways are there to buy an envelope and a stamp?

(a) 20 (b) 45
(c) 54 (d) 9

27. In how many ways can Ram choose a vowel and a consonant from the letters of the word ALLAHABAD?

(a) 4 (b) 6
(c) 9 (d) 5

28. There are three rooms in a motel: one single, one double and one for four persons. How many ways are there to house seven persons in these rooms?

(a) 7!/1!2!4! (b) 7!
(c) 7!/3 (d) 7!/3!

29. How many ways are there to choose four cards of different suits and different values from a deck of 52 cards?

(a) 13.12.11.10 (b) ${}^{52}C_4$
(c) 134 (d) 52.36.22.10

30. How many new words are possible from the letters of the word PERMUTATION?

(a) 11!/2! (b) (11!/2!) – 1
(c) 11! – 1 (d) None of these

31. A set of 15 different words are given. In how many ways is it possible to choose a subset of not more than 5 words?

(a) 4944 (b) 4^{15}
(c) 15^4 (d) 4943

32. In how many ways can 12 papers be arranged if the best and the worst paper never come together?

(a) 12!/2! (b) 12! – 11!
(c) (12! – 11!)/2 (d) 12! – 2.11!

33. In how many ways can the letters of the word 'EQUATION' be arranged so that all the vowels come together?

(a) ${}^{9}C_4 . {}^{9}C_5$ (b) 4!.5!
(c) 9!/5! (d) 9! – 4!5!

34. A man has 3 shirts, 4 trousers and 6 ties. What are the number of ways in which he can dress himself with a combination of all the three?

(a) 13 (b) 72
(c) 13!/3!.4!.6! (d) 3!.4!.6!

35. How many motor vehicle registration number of 4 digits can be formed with the digits 0, 1, 2, 3, 4, 5? (No digit being repeated.)

(a) 1080 (b) 120
(c) 300 (d) 360

36. How many motor vehicle registration number plates can be formed with the digits 1, 2, 3, 4, 5 (No digits being repeated) if it is given that registration number can have 1 to 5 digits?

(a) 100 (b) 120
(c) 325 (d) 205

Directions for Question 37 to 39: There are 25 points on a plane of which 7 are collinear. Now solve the following:

37. How many straight lines can be formed?

(a) 7 (b) 300
(c) 280 (d) none of these

38. How many triangles can be formed from these points?

(a) 453 (b) 2265
(c) 755 (d) none of these

39. How many quadrilaterals can be formed from these points?

(a) 5206 (b) 2603
(c) 13015 (d) None of these

40. There are ten subjects in the school day at St.Vincent's High School but the sixth standard students have only 5 periods in a day. In how many ways can we form a time table for the day for the sixth standard students if no subject is repeated?

(a) 510 (b) 105
(c) 252 (d) 30240

41. There are 8 consonants and 5 vowels in a word jumble. In how many ways can we form 5-letter words having three consonants and 2 vowels?

(a) 67200 (b) 8540
(c) 720 (d) None of these

42. How many batting orders are possible for the Indian cricket team if there is a squad of 15 to choose from such that Sachin Tendulkar is always chosen?

(a) 1001.11! (b) 364.11!
(c) 11! (d) 15.11!

43. There are 5 blue socks, 4 red socks and 3 green socks, all different in Debu's wardrobe. He has to select 4 socks from this set. In how many ways can he do so?

(a) 245 (b) 120
(c) 495 (d) 60

44. A class prefect goes to meet the principal every week. His class has 30 people apart from him. If he has to take groups of three every time he goes to the principal, in how many weeks will he be able to go to the principal without repeating the group of same three which accompanies him?

(a) $^{30}P_3$ (b) $^{30}C_3$
(c) 30!/3 (d) None of these

45. For the above question if on the very first visit the principal appoints one of the boys accompanying him as the head boy of the school and lays down the condition that the class prefect has to be accompanied by the head boy every time he comes then for a maximum of how many weeks (including the first week) can the class prefect ensure that the principal sees a fresh group of three accompanying him?

(a) $^{30}C_2$ (b) $^{29}C_2$
(c) $^{29}C_3$ (d) None of these

46. How many distinct words can be formed out of the word PROWLING which start with R & end with W?

(a) 8!/2! (b) 6!2!
(c) 6! (d) None of these

47. How many 7-digit numbers are there having the digit 3 three times & the digit 5 four times?

(a) 7!/(3!)(5!) (b) $3^3 \times 5^5$
(c) 77 (d) 35

48. How many 7-digit numbers are there having the digit 3 three times & the digit 0 four times?

(a) 15 (b) $3^3 \times 4^4$
(c) 18 (d) None of these

49. From a set of three capital consonants, five small consonants and 4 small vowels how many words can be made each starting with a capital consonant and containing 3 small consonants and two small vowels.

(a) 3600 (b) 7200
(c) 21600 (d) 28800

50. Several teams take part in a competition, each of which must play one game with all the other teams. How many teams took part in the competition if they played 45 games in all?

(a) 5 (b) 10
(c) 15 (d) 20

51. In how many ways a selection can be made of at least one fruit out of 5 bananas, 4 mangoes and 4 almonds?

(a) 129 (b) 149
(c) 139 (d) 109

52. There are 5 different Jeffrey Archer books, 3 different Sidney Sheldon books and 6 different John Grisham books. The number of ways in which at least one book can be given away is

(a) $2^{10} – 1$ (b) $2^{11} – 1$
(c) $2^{12} – 1$ (d) $2^{14} – 1$

53. In the above problem, find the number of ways in which at least one book of each author can be given.

(a) $(2^5 – 1)(2^3 – 1)(2^8 – 1)$
(b) $(2^5 – 1)(2^3 – 1)(2^3 – 1)$
(c) $(2^5 – 1)(2^3 – 1)(2^3 – 1)$
(d) $(2^5 – 1)(2^3 – 1)(2^6 – 1)$

54. There is a question paper consisting of 15 questions. Each question has an internal choice of 2 options. In how many ways can a student attempt one or more questions of the given fifteen questions in the paper?

(a) 3^7 (b) 3^8
(c) 3^{15} (d) $3^{15} – 1$

55. How many numbers can be formed with the digits 1, 6, 7, 8, 7, 6, 1 so that the odd digits always occupy the odd places?

(a) 15 (b) 12
(c) 18 (d) 20

56. There are five boys of McGraw-Hill Mindworkzz and three girls of I.I.M Lucknow who are sitting together to discuss a management problem at a round table. In how many ways can they sit around the table so that no two girls are together?

(a) 1220 (b) 1400
(c) 1420 (d) 1440

57. Amita has three library cards and seven books of her interest in the library of Mindworkzz. Of these books she would not like to borrow the D.I. book,

unless the Quants book is also borrowed. In how many ways can she take the three books to be borrowed?

(a) 15 (b) 20
(c) 25 (d) 30

58. From a group of 12 dancers, five have to be taken for a stage show. Among them Radha and Mohan decide either both of them would join or none of them would join. In how many ways can the 5 dancers be chosen?

(a) 190 (b) 210
(c) 278 (d) 372

59. Find the number of 6-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 once such that the 6-digit number is divisible by its unit digit. (The unit digit is not 1.)

(a) 620 (b) 456
(c) 520 (d) 528

60. An urn contains 5 boxes. Each box contains 5 balls of different colours red, yellow, white, blue and black. Rangeela wants to pick up 5 balls of different colours, a different coloured ball from each box. If from the first box in the first draw, he has drawn a red ball and from the second box he has drawn a black ball, find the maximum number of trials that are needed to be made by Rangeela to accomplish his task if a ball picked is not replaced.

(a) 12 (b) 11
(c) 20 (d) 60

61. How many rounds of matches does a knock-out tennis tournament have if it starts with 64 players and every player needs to win 1 match to move at the next round?

(a) 5 (b) 6
(c) 7 (d) 64

62. There are N men sitting around a circular table at N distinct points. Every possible pair of men except the ones sitting adjacent to each other sings a 2 minute song one pair after other. If the total time taken is 88 minutes, then what is the value of N?

(a) 8 (b) 9
(c) 10 (d) 11

63. In a class with boys and girls a chess competition was played wherein every student had to play 1 game with every other student. It was observed that in 36 matches both the players were boys and in 66 matches both were girls. What is the number of matches in which 1 boy and 1 girl play against each other?

(a) 108 (b) 189
(c) 210 (d) 54

64. Zada has to distribute 15 chocolates among 5 of her children Sana, Ada, Jiya, Amir and Farhan. She has to make sure that Sana gets at least 3 and at most 6 chocolates. In how many ways can this be done if each child gets at least one chocolate?

(a) 495 (b) 77
(c) 417 (d) 425

65. Mr Shah has to divide his assets worth ₹ 30 crores in 3 parts to be given to three of his sons Ajay, Vijay and Arun ensuring that every son gets assets atleast worth ₹ 5 crores. In how many ways can this be done if it is given that the three sons should get their shares in multiples of ₹ 1 crore?

(a) 136 (b) 152
(c) 176 (d) 98

66. Three variables x, y, z have a sum of 30. All three of them are non-negative integers. If any two variables don't have the same value and exactly one variable has a value less than or equal to three, then find the number of possible solutions for the variables.

(a) 98 (b) 285
(c) 68 (d) 252

67. The letters of the word ALLAHABAD are rearranged to form new words and put in a dictionary. If the dictionary has only these words and one word on every page in alphabetical order then what is the page number on which the word LABADALAH comes?

(a) 6269 (b) 6268
(c) 6087 (d) 6086

68. If x, y and z can only take the values 1, 2, 3, 4, 5, 6, 7 then find the number of solutions of the equation $x + y + z = 12$.

(a) 36 (b) 37
(c) 38 (d) 31

69. There are nine points in a plane such that no three are collinear. Find the number of triangles that can be formed using these points as vertices.

(a) 81 (b) 90
(c) 9 (d) 84

70. There are nine points in a plane such that exactly three points out of them are collinear. Find the number of triangles that can be formed using these points as vertices.

(a) 81 (b) 90
(c) 9 (d) 83

71. If xy is a 2-digit number and u, v, x, y are digits, then find the number of solutions of the equation: $(xy)^2 = u! + v$

(a) 2 (b) 3
(c) 0 (d) 5

72. Ten points are marked on a straight line and eleven points are marked on another straight line. How many triangles can be constructed with vertices from among the above points?

(a) 495 (b) 550
(c) 1045 (d) 2475

73. For a scholarship, at the most n candidates out of $2n + 1$ can be selected. If the number of different ways of selection of at least one candidate is 63, the maximum number of candidates that can be selected for the scholarship is
(a) 3 (b) 4
(c) 2 (d) 5

74. One red flag, three white flags and two blue flags are arranged in a line such that,
(a) no two adjacent flags are of the same colour
(b) the flags at the two ends of the line are of different colours.
In how many different ways can the flags be arranged?
(a) 6 (b) 4
(c) 10 (d) 2

75. Sam has forgotten his friend's seven-digit telephone number. He remembers the following: the first three digits are either 635 or 674, the number is odd, and the number nine appears once. If Sam were to use a trial and error process to reach his friend, what is the minimum number of trials he has to make before he can be certain to succeed?
(a) 1000 (b) 2430
(c) 3402 (d) 3006

76. There are three cities A, B and C. Each of these cities is connected with the other two cities by at least one direct road. If a traveler wants to go from one city (origin) to another city (destination), she can do so either by traversing a road connecting the two cities directly, or by traversing two roads, the first connecting the origin to the third city and the second connecting the third city to the destination. In all there are 33 routes from A to B (including those via C). Similarly there are 23 routes from B to C (including those via A). How many roads are there from A to C directly?
(a) 6 (b) 3
(c) 5 (d) 10

77. Let n be the number of different 5-digit numbers, divisible by 4 that can be formed with the digits 1, 2, 3, 4, 5 and 6, with no digit being repeated. What is the value of n?
(a) 144 (b) 168
(c) 192 (d) none of these

78. How many numbers greater than 0 and less than a million can be formed with the digits 0, 7 and 8?
(a) 486 (b) 1086
(c) 728 (d) none of these

79. In how many ways is it possible to choose a white square and a black square on a chess board so that the squares must not lie in the same row or column?
(a) 56 (b) 896
(c) 60 (d) 768

Directions for Questions 80 and 81: Answer these questions based on the information given below.

Each of the 11 letters A, H, I, M, O, T, U, V, W, X and Z appear same when looked at in a mirror. They are called symmetric letters. Other letters in the alphabet are asymmetric letters.

80. How many four-letter computer passwords can be formed using only the symmetric letters (no repetition allowed)?
(a) 7920 (b) 330
(c) 14640 (d) 419430

81. How many three-letter computer passwords can be formed (no repetition allowed) with at least one symmetric letter?
(a) 990 (b) 2730
(c) 12870 (d) 15600

82. Twenty seven persons attend a party. Which one of the following statements can never be true?
(a) There is a person in the party who is acquainted with all the twenty six others.
(b) Each person in the party has a different number of acquaintances.
(c) There is a person in the party who has an odd number of acquaintances.
(d) In the party, there is no set of three mutual acquaintances.

83. There are 6 boxes numbered 1, 2, ...6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is
(a) 5 (b) 21
(c) 33 (d) 60

84. How many numbers can be formed with odd digits 1, 3, 5, 7, 9 without repetition?
(a) 275 (b) 325
(c) 375 (d) 235

85. In how many ways five chocolates can be chosen from an unlimited number of Cadbury, Five-star, and Perk chocolates?
(a) 81 (b) 243
(c) 21 (d) 31

Directions for Questions 86 and 87: In a chess tournament there were two women participating and every participant played two games with the other participants. The number of games that the men played among themselves exceeded the number of games that the men played with the women by 66.

86. The number of participants in the tournament were?

(a) 12 (b) 13
(c) 15 (d) 11

87. The total number of games played in the tournament were?

(a) 132 (b) 110
(c) 156 (d) 210

Space for Rough Work

LEVEL OF DIFFICULTY (II)

1. How many even numbers of four digits can be formed with the digits 1, 2, 3, 4, 5, 6 (repetitions of digits are allowed)?
   (a) 648 (b) 180
   (c) 1296 (d) 600
2. How many 4 digit numbers divisible by 5 can be formed with the digits 0, 1, 2, 3, 4, 5, 6 and 6?
   (a) 220 (b) 249
   (c) 432 (d) 288
3. There are 6 pups and 4 cats. In how many ways can they be seated in a row so that no cats sit together?
   (a) 6^4 (b) 10!/(4!).(6!)
   (c) $6! \times {}^7P_4$ (d) None of these
4. How many new words can be formed with the word MANAGEMENT all ending in *G*?
   (a) $10!/(2!)^4 – 1$ (b) $9!/(2!)^4$
   (c) $10!/(2!)^4$ (d) None of these
5. Find the total numbers of 9-digit numbers that can be formed all having different digits.
   (a) ${}^{10}P_9$ (b) 9!
   (c) 10!–9! (d) 9.9!
6. There are *V* lines parallel to the *x*-axis and '*W*' lines parallel to *y*-axis. How many rectangles can be formed with the intersection of these lines?
   (a) ${}^vP_2.{}^wP_2$ (b) ${}^vC_2.{}^wC_2$
   (c) ${}^{v-2}C_2.\ {}^{w-2}C_2$ (d) None of these
7. From 4 gentlemen and 4 ladies a committee of 5 is to be formed. Find the number of ways of doing so if the committee consists of a president, a vice-president and three secretaries?
   (a) ${}^8P_5$ (b) 1120
   (c) ${}^4C_2 \times {}^4C_3$ (d) None of these
8. In the above question, what will be the number of ways of selecting the committee with at least 3 women such that at least one woman holds the post of either a president or a vice-president?
   (a) 420 (b) 610
   (c) 256 (d) None of these
9. Find the number of ways of selecting the committee with a maximum of 2 women and having at the maximum one woman holding one of the two posts on the committee.
   (a) 16 (b) 512
   (c) 608 (d) 324
10. The crew of an 8 member rowing team is to be chosen from 12 men, of which 3 must row on one side only and 2 must row on the other side only. Find the number of ways of arranging the crew with 4 members on each side.
    (a) 40,320 (b) 30,240
    (c) 60,480 (d) None of these
11. In how many ways 5 MBA students and 6 Law students can be arranged together so that no two MBA students are side by side?
    (a) $\dfrac{7!6!}{2!}$ (b) 6!.6!
    (c) 5!.6! (d) ${}^{11}C_5$
12. The latest registration number issued by the Delhi Motor Vehicle Registration Authority is DL-5S 2234. If all the numbers and alphabets before this have been used up, then find how many vehicles have a registration number starting with DL-5?
    (a) 1,92,234 (b) 1,92,225
    (c) 1,72,227 (d) None of these
13. There are 100 balls numbered $n_1, n_2, n_3, n_4, \ldots n_{100}$. They are arranged in all possible ways. How many arrangements would be there in which n_{28} ball will always be before n_{29} ball and the two of them will be adjacent to each other?
    (a) 99!/2! (b) 99!.2!
    (c) 99! (d) None of these
14. Find the sum of the number of sides and number of diagonals of a hexagon.
    (a) 210 (b) 15
    (c) 6 (d) 9
15. A tea party is arranged for 2*M* people along two sides of a long table with *M* chairs on each side. *R* men wish to sit on one particular side and *S* on the other. In how many ways can they be seated (provided $R, S \le M$)
    (a) ${}^Mp_R.{}^Mp_S$
    (b) ${}^Mp_R.{}^Mp_S\,({}^{2M-R-S}p_{2M-R-S})$
    (c) ${}^{2M}P_R.\ {}^{2M-R}P_S$
    (d) None of these
16. In how many ways can '*mn*' things be distributed equally among *n* groups?
    (a) ${}^{mn}P_m.\ {}^{mn}P_n$ (b) ${}^{mn}C_m.\ {}^{mn}C_n$
    (c) $(mn)!/(m!)(.n!)$ (d) None of these
17. In how many ways can a selection be made of 5 letters out of 5As, 4Bs, 3Cs, 2Ds and 1E?
    (a) 70 (b) 71
    (c) ${}^{15}C_5$ (d) None of these

18. Find the number of ways of selecting 'n' articles out of $3n + 1$, out of which n are identical.
(a) 2^{2n-1} (b) ${}^{3n+1}C_n/n!$
(c) ${}^{3n+1}P_n/n!$ (d) None of these
19. The number of positive numbers of not more than 10 digits formed by using 0, 1, 2, 3 is
(a) $4^{10} - 1$ (b) 4^{10}
(c) $4^9 - 1$ (d) None of these
20. There is a number lock with four rings with each ring having digits 0 to 9. How many attempts at the maximum would have to be made before getting the right number?
(a) 10^4 (b) 255
(c) $10^4 - 1$ (d) None of these
21. Find the number of numbers that can be formed using all the digits 1, 2, 3, 4, 3, 2, 1 only once so that the odd digits occupy odd places only.
(a) $4!/(2!)^2$ (b) $7!/(2!)^3$
(c) 1!.3!.5!.7! (d) None of these
22. There is a 7-digit telephone number with all different digits. If the digit at extreme right and extreme left are 5 and 6 respectively, find how many such telephone numbers are possible.
(a) 120 (b) 1,00,000
(c) 6720 (d) None of these
23. If a team of four persons is to be selected from 8 males and 8 females, then in how many ways can the selections be made to include at least one male.
(a) 3500 (b) 875
(c) 1200 (d) None of these
24. In the above question, in how many ways can the selections be made if it has to contain at the maximum three women?
(a) 1750 (b) 1200
(c) 875 (d) None of thes
25. How many figures are required to number a book containing 150 pages?
(a) 450 (b) 360
(c) 262 (d) None of these
26. There are 8 orators A, B, C, D, E, F, G and H. In how many ways can the arrangements be made so that A always comes before B and B always comes before C.
(a) 8!/3! (b) 8!/6!
(c) 5!.3! (d) 8!/(5!.3!)
27. There are 4 letters and 4 envelopes. In how many ways can wrong choices be made?
(a) 4^3 (b) $4! - 1$
(c) 16 (d) $4^4 - 1$
28. In the question above, find the number of ways in which only one letter goes in the wrong envelope?
(a) 4^3 (b) ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$
(c) ${}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3$
(d) 0
29. In question 27, find the number of ways in which only two letters go in the wrong envelopes?
(a) 4 (b) 5
(c) 6 (d) 3
30. A train is running between Patna to Howrah. Seven people enter the train somewhere between Patna and Howrah. It is given that nine stops are there in between Patna and Howrah. In how many ways can the tickets be purchased if no restriction is there with respect to the number of tickets at any station? 2 people do not buy the same ticket.
(a) ${}^{45}C_7$ (b) ${}^{63}C_7$
(c) ${}^{56}C_7$ (d) ${}^{52}C_7$
31. There are seven pairs of black shoes and five pairs of white shoes. They all are put into a box and shoes are drawn one at a time. To ensure that at least one pair of black shoes are taken out, what is the number of shoes required to be drawn out?
(a) 12 (b) 13
(c) 7 (d) 18
32. In the above question, what is the minimum number of shoes required to be drawn out to get at least 1 pair of correct shoes (either white or black)?
(a) 12 (b) 7
(c) 13 (d) 18
33. In how many ways one white and one black rook can be placed on a chessboard so that they are never in an attacking position?
(a) 64×50 (b) 64×49
(c) 63×49 (d) None of these
34. How many 6-digit numbers have all their digits either all odd or all even?
(a) 31,250 (b) 28,125
(c) 15,625 (d) None of these
35. How many 6-digit numbers have at least 1 even digit?
(a) 884375 (b) 3600
(c) 880775 (d) 15624
36. How many 10-digit numbers have at least 2 equal digits?
(a) $9 \times {}^{10}C_2 \times 8!$ (b) $9.10^9 – 9. 9!$
(c) $9 \times 9!$ (d) None of these
37. On a triangle ABC, on the side AB, 5 points are marked, 6 points are marked on the side BC and 3 points are marked on the side AC (none of the points being the vertex of the triangle). How many triangles can be made by using these points?
(a) 90 (b) 333
(c) 328 (d) None of these

38. If we have to make 7 boys sit with 7 girls around a round table, then the number of different relative arrangements of boys and girls that we can make so that there are no two boys nor any two girls sitting next to each other is

(a) $2 \times (7!)^2$ (b) $7! \times 6!$
(c) $7! \times 7!$ (d) None of these

39. If we have to make 7 boys sit alternately with 7 girls around a round table which is numbered, then the number of ways in which this can be done is

(a) $2 \times (7!)^2$ (b) $7! \times 6!$
(c) $7! \times 7!$ (d) None of these

40. In the Suniti Building in Mumbai there are 12 floors plus the ground floor. 9 people get into the lift of the building on the ground floor. The lift does not stop on the first floor. If 2, 3 and 4 people alight from the lift on its upward journey, then in how many ways can they do so? (Assume they alight on different floors.)

(a) ${}^{11}C_3 \times {}^3P_3$ (b) ${}^{11}P_3 \times {}^9C_4 \times {}^5C_3$
(c) ${}^{10}P_3 \times {}^9C_4 \times {}^5C_3$ (d) ${}^{12}C_3$

Directions for Questions 41 and 42. There are 40 doctors in the surgical department of the AIIMS. In how many ways can they be arranged to form a team with:

41. 1 surgeon and an assistant

(a) 1260 (b) 1320
(c) 1440 (d) 1560

42. 1 surgeon and 4 assistants

(a) $40 \times {}^{39}C_4$ (b) $41 \times {}^{39}C_4$
(c) $41 \times {}^{40}C_4$ (d) None of these

43. In how many ways can 10 identical marbles be distributed among 6 children so that each child gets at least 1 marble?

(a) ${}^{15}C_5$ (b) ${}^{15}C_9$
(c) ${}^{10}C_5$ (d) ${}^{9}C_5$

44. Seven different objects must be divided among three people. In how many ways can this be done if one or two of them can get no objects?

(a) 15120 (b) 2187
(c) 3003 (d) 792

45. How many 6-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 and 7 so that the digits should not repeat?

(a) 720 (b) 1440
(c) 2160 (d) 6480

46. How many 6-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 and 7 so that the digits should not repeat and the second last digit is even?

(a) 720 (b) 320
(c) 2160 (d) 1440

47. How many 5-digit numbers that do not contain identical digits can be written by means of the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9?

(a) 6048 (b) 7560
(c) 5040 (d) 15,120

48. How many different 4-digit numbers are there which have the digits 1, 2, 3, 4, 5, 6, 7 and 8 such that the digit 1 appears exactly once.

(a) $7.\,{}^8P_4$ (b) ${}^8P_4$
(c) 4.7^3 (d) 7^3

49. How many different 7-digit numbers can be written using only three digits 1, 2 and 3 such that the digit 3 occurs twice in each number?

(a) ${}^7C_2.2^5$ (b) $7!/(2!)$
(c) $7!/(2!)^3$ (d) None of these

50. How many different 4-digit numbers can be written using the digits 1, 2, 3, 4, 5, 6, 7 and 8 only once such that the number 2 is contained once.

(a) 360 (b) 840
(c) 760 (d) 1260

Space for Rough Work

LEVEL OF DIFFICULTY (III)

1. The number of ways in which four particular persons A, B, C, D and six more persons can stand in a queue so that A always stands before B, B always before C and C always before D is
   (a) 10!/4! (b) $^{10}P_4$
   (c) $^{10}C_4$ (d) None of these
2. The number of circles that can be drawn out of 10 points of which 7 are collinear is
   (a) 130 (b) 85
   (c) 45 (d) Cannot be determined
3. How many different 9-digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?
   (a) 120 (b) $9!/(2!)^3.3!$
   (c) $(4!)(2!)^3.3!$ (d) None of these
4. How many diagonals are there in an n-sided polygon ($n > 3$)?
   (a) $(^nC_2 - n)$ (b) $^nC_2$
   (c) $n(n - 1)/2$ (d) None of these
5. A polygon has 54 diagonals. Find the number of sides.
   (a) 10 (b) 14
   (c) 12 (d) 9
6. The number of natural numbers of two or more than two digits in which digits from left to right are in increasing order is
   (a) 127 (b) 128
   (c) 502 (d) 512
7. In how many ways a cricketer can score 200 runs with fours and sixes only?
   (a) 13 (b) 17
   (c) 19 (d) 16
8. A dices is rolled six times. One, two, three, four, five and six appears on consecutive throws of dices. How many ways are possible of having 1 before 6?
   (a) 120 (b) 360
   (c) 240 (d) 280
9. The number of permutations of the letters a, b, c, d, e, f, g such that neither the pattern '*beg*' nor '*acd*' occurs is
   (a) 4806 (b) 420
   (c) 2408 (d) None of these
10. In how many ways can the letters of the English alphabet be arranged so that there are seven letters between the letters A and B?
   (a) 31!.2! (b) $^{24}P_7.18!.2$
   (c) 36.24! (d) None of these
11. There are 20 people among whom two are sisters. Find the number of ways in which we can arrange them around a circle so that there is exactly one person between the two sisters?
   (a) 18! (b) 2!.19!
   (c) 19! (d) None of these
12. There are 10 points on a straight line AB and 8 on another straight line, AC none of them being A. How many triangles can be formed with these points as vertices?
   (a) 720 (b) 640
   (c) 816 (d) None of these
13. In an examination, the maximum marks for each of the three papers is 50 each. The maximum marks for the fourth paper is 100. Find the number of ways with which a student can score 60% marks in aggregate.
   (a) 3,30,850 (b) 2,33,551
   (c) 1,10,551 (d) None of these
14. How many rectangles can be formed out of a chess-board?
   (a) 204 (b) 1230
   (c) 1740 (d) None of these
15. On a board having 18 rows and 16 columns, find the number of squares.
   (a) $^{18}C_2.\ ^{16}C_2$
   (b) $^{18}P_2.\ ^{16}P_2$
   (c) 18.16 + 17.15 + 16.14 + 15.13 + 14.12 +…….+ 4.2 + 3.1
   (d) None of these
16. In the above question, find the number of rectangles.
   (a) $^{18}C_2.\ ^{16}C_2$ (b) $^{18}P_2.\ ^{16}P_2$
   (c) 171.136 (d) None of these

Directions for Questions 17 and 18: Read the passage below and answer the questions.

In the famous program *Kaun Banega Crorepati*, the host shakes hand with each participant once, while he shakes hands with each qualifier (amongst participant) twice more. Besides, the participants are required to shake hands once with each other, while the winner and the host each shake hands with all the guests once.

17. How many handshakes are there if there are 10 participants in all, 3 finalists and 60 spectators?
   (a) 118 (b) 178
   (c) 181 (d) 122
18. In the above question, what is the ratio of the number of handshakes involving the host to the number of handshakes not involving the host?

(a) 43 : 75 (b) 76 : 105
(c) 46 : 75 (d) None of these

19. What is the percentage increase in the total number of handshakes if all the guests are required to shake hands with each other once?
(a) 82.2% (b) 822%
(c) 97.7% (d) None of these

20. Two variants of the CAT paper are to be given to twelve students. In how many ways can the students be placed in two rows of six each so that there should be no identical variants side by side and that the students sitting one behind the other should have the same variant?
(a) $2 \times {}^{12}C_6 \times (6!)^2$ (b) $6! \times 6!$
(c) $7! \times 7!$ (d) None of these

21. For the above question, if there are now three variants of the test to be given to the twelve students (so that each variant is used for four students) and there should be no identical variants side by side and that the students sitting one behind the other should have the same variant. Find the number of ways this can be done.
(a) $6!^2$ (b) $6 \times 6! \times 6!$
(c) $6!^3$ (d) None of these

22. Five boys and three girls are sitting in a row of eight seats. In how many ways can they be seated so that not all girls sit side by side?
(a) 36,000 (b) 45,000
(c) 24,000 (d) None of these

23. How many natural numbers are there that are smaller than 10^4 and whose decimal notation consists only of the digits 0, 1, 2, 3 and 5, which are not repeated in any of these numbers?
(a) 32 (b) 164
(c) 31 (d) 212

24. Seven different objects must be divided among three people. In how many ways can this be done if one or two of them must get no objects?
(a) 381 (b) 36
(c) 84 (d) 180

25. Seven different objects must be divided among three people. In how many ways can this be done if at least one of them gets exactly 1 object?
(a) 2484 (b) 1218
(c) 729 (d) None of these

26. How many 4-digit numbers that are divisible by 4 can be formed from the digits 1, 2, 3, 4 and 5?
(a) 36 (b) 72
(c) 24 (d) None of these

27. How many natural numbers smaller than 10,000 are there in the decimal notation of which all the digits are different?
(a) 2682 (b) 4474
(c) 5274 (d) 1448

28. How many 4-digit numbers are there whose decimal notation contains not more than two distinct digits?
(a) 672 (b) 576
(c) 360 (d) 448

29. How many different 7-digit numbers are there the sum of whose digits are odd?
(a) 45.10^5 (b) 24.10^5
(c) 224320 (d) None of these

30. How many 6-digit numbers contain exactly 4 different digits?
(a) 4536 (b) 2,94,840
(c) 1,91,520 (d) None of these

31. How many numbers smaller than 2.10^8 and are divisible by 3 can be written by means of the digits 0, 1 and 2 (exclude single digit and double digit numbers)?
(a) 4369 (b) 4353
(c) 4373 (d) 4351

32. Six white and six black balls of the same size are distributed among ten urns so that there is at least one ball in each urn. What is the number of different distributions of the balls?
(a) 25,000 (b) 26,250
(c) 28,250 (d) 13,125

33. A bouquet has to be formed from 18 different flowers so that it should contain not less than three flowers. How many ways are there of doing this in?
(a) 5,24,288 (b) 2,62,144
(c) 2,61,972 (d) None of these

34. How many different numbers which are smaller than 2.10^8 can be formed using the digits 1 and 2 only?
(a) 766 (b) 94
(c) 92 (d) 126

35. How many distinct 6-digit numbers are there having 3 odd and 3 even digits?
(a) 55 (b) $(5.6)^3.(4.6)^3.3$
(c) 281250 (d) None of these

36. How many 8-digit numbers are there the sum of whose digits is even?
(a) 14400 (b) 4.5^5
(c) 45.10^6 (d) None of these

Directions for Questions 37 and 38: In a chess tournament there were two women participating and every participant played two games with the other participants. The number of games that the men played among themselves exceeded the number of games that the men played with the women by 66.

37. The number of participants in the tournament were:
(a) 12 (b) 13
(c) 15 (d) 11

38. The total number of games played in the tournament were:

(a) 132 (b) 110
(c) 156 (d) 210

39. There are 5 bottles of sherry and each have their respective caps. If you are asked to put the correct cap to the correct bottle then how many ways are there so that not a single cap is on the correct bottle?
(a) 44 (b) $5^5 - 1$
(c) 5^5 (d) None of these

40. Amartya Banerjee has forgotten the telephone number of his best friend Abhijit Roy. All he remembers is that the number had 8-digits and ended with an odd number and had exactly one 9. How many possible numbers does Amartya have to try to be sure that he gets the correct number?
(a) 104.9^5 (b) 113.9^5
(c) 300.9^5 (d) $764.9^5.6!$

41. In Question 40, if Amartya is reminded by his friend Sharma that apart from what he remembered there was the additional fact that the last digit of the number was not repeated under any circumstance then how many possible numbers does Amartya have to try to be sure that he gets the correct number?
(a) $200.8^5 + 72.9^5$ (b) 8.96
(c) 36.85 + 7.96 (d) $36.8^5 + 8.9^6$

42. How many natural numbers not more than 4300 can be formed with the digits 0,1, 2, 3, 4 (if repetitions are allowed)?
(a) 574 (b) 570
(c) 575 (d) 569

43. How many natural numbers less than 4300 can be formed with the digits 0, 1, 2, 3, 4 (if repetitions are not allowed)?
(a) 113 (b) 158
(c) 154 (d) 119

44. How many even natural numbers divisible by 5 can be formed with the digits 0, 1, 2, 3, 4, 5, 6 (if repetitions of digits not allowed)?
(a) 1957 (b) 1956
(c) 1236 (d) 1235

45. There are 100 articles numbered $n_1, n_2, n_3, n_4, \ldots n_{100}$. They are arranged in all possible ways. How many arrangements would be there in which n_{28} will always be before n_{29}.
(a) $5050 \times 99!$ (b) $5050 \times 98!$
(c) $4950 \times 98!$ (d) $4950 \times 99!$

46. The letters of the word PASTE are written in all possible orders and these words are written out as in a dictionary. Then the rank of the word SPATE is
(a) 432 (b) 86
(c) 59 (d) 446

47. The straight lines S_1, S_2, S_3 are in a parallel and lie in the same plane. A total number of A points on S_1; B points on S_2 and C points on S_3 are used to produce triangles. What is the maximum number of triangles formed?
(a) ${}^{A+B+C}C_3 - {}^{A}C_3 - {}^{B}C_3 - {}^{C}C_3 + 1$
(b) ${}^{A+B+C}C_3$
(c) ${}^{A+B+C}C_3 + 1$
(d) $({}^{A+B+C}C_3 - {}^{A}C_3 - {}^{B}C_3 - {}^{C}C_3)$

48. The sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The total number of triangles that can be constructed by using these points as vertices are
(a) 212 (b) 210
(c) 205 (d) 190

49. A library has 20 copies of CAGE; 12 copies each of RAGE Part1 and Part 2; 5 copies of PAGE Part 1, Part 2 and Part 3 and single copy of SAGE, DAGE and MAGE. In how many ways can these books be distributed?
(a) $62!/(20!)(12!)(5!)$ (b) $62!$
(c) $62!/(37)^3$ (d) $62!/(20!)(12!)^2(5!)^3$

50. The AMS MOCK CAT test CATALYST 19 consists of four sections. Each section has a maximum of 45 marks. Find the number of ways in which a student can qualify in the AMS MOCK CAT if the qualifying marks is 90.
(a) 36,546 (b) 6296
(c) 64906 (d) None of these

Space for Rough Work

ANSWER KEY

Level of Difficulty (I)

1. (c)	2. (b)	3. (b)	4. (c)
5. (c)	6. (a)	7. (b)	8. (a)
9. (a)	10. (b)	11. (d)	12. (b)
13. (a)	14. (b)	15. (c)	16. (a)
17. (d)	18. (d)	19. (b)	20. (c)
21. (c)	22. (c)	23. (c)	24. (b)
25. (c)	26. (a)	27. (a)	28. (a)
29. (a)	30. (b)	31. (a)	32. (d)
33. (b)	34. (b)	35. (d)	36. (c)
37. (c)	38. (b)	39. (d)	40. (d)
41. (a)	42. (a)	43. (c)	44. (b)
45. (b)	46. (c)	47. (d)	48. (a)
49. (c)	50. (b)	51. (b)	52. (d)
53. (d)	54. (d)	55. (c)	56. (d)
57. (c)	58. (d)	59. (d)	60. (a)
61. (b)	62. (d)	63. (a)	64. (d)
65. (a)	66. (d)	67. (a)	68. (b)
69. (d)	70. (d)	71. (b)	72. (c)
73. (a)	74. (a)	75. (c)	76. (a)
77. (c)	78. (c)	79. (d)	80. (a)
81. (c)	82. (b)	83. (b)	84. (b)
85. (b)	86. (b)	87. (c)	

Level of Difficulty (II)

1. (a)	2. (b)	3. (c)	4. (b)
5. (d)	6. (b)	7. (b)	8. (d)
9. (b)	10. (c)	11. (a)	12. (d)
13. (c)	14. (b)	15. (b)	16. (d)
17. (b)	18. (d)	19. (a)	20. (c)
21. (d)	22. (c)	23. (d)	24. (a)
25. (d)	26. (a)	27. (b)	28. (d)
29. (c)	30. (a)	31. (d)	32. (c)
33. (b)	34. (b)	35. (a)	36. (b)
37. (b)	38. (b)	39. (a)	40. (b)
41. (d)	42. (a)	43. (d)	44. (b)
45. (c)	46. (a)	47. (d)	48. (c)
49. (a)	50. (b)		

Level of Difficulty (III)

1. (a)	2. (b)	3. (d)	4. (a)
5. (c)	6. (c)	7. (b)	8. (b)
9. (a)	10. (c)	11. (d)	12. (b)
13. (c)	14. (d)	15. (c)	16. (c)
17. (c)	18. (b)	19. (d)	20. (a)
21. (d)	22. (a)	23. (b)	24. (a)
25. (b)	26. (c)	27. (c)	28. (b)
29. (a)	30. (b)	31. (c)	32. (b)
33. (c)	34. (a)	35. (c)	36. (c)
37. (b)	38. (c)	39. (a)	40. (c)
41. (a)	42. (c)	43. (b)	44. (b)
45. (c)	46. (b)	47. (d)	48. (c)
49. (d)	50. (c)		

Hints

Level of Difficulty (III)

1. ${}^{10}C_4 \times 6! = 10!/4!$
2. For drawing a circle we need 3 non collinear points. This can be done in:
   $${}^3C_3 + {}^3C_2 \times {}^7C_1 + {}^3C_1 \times {}^7C_2 = 1 + 21 + 63 = 85$$
3. The odd digits have to occupy even positions. This can be done in $\frac{4!}{2!2!} = 6$ ways.

   The other digits have to occupy the other positions. This can be done in $\frac{5!}{3!2!} = 10$ ways.

   Hence total number of rearrangements possible = 6 × 10 = 60.
4. The number of straight lines is ${}^nC_2$ out of which there are n sides. Hence, the number of diagonals is ${}^nC_2 - n$.
5. ${}^nC_2 - n = 54$.
6. We cannot take '0' since the smallest digit must be placed at the left most place. We have only 9 digits from which to select the numbers. First select any number of digits. Then for any selection there is only one possible arrangement where the required condition is met. This can be done in ${}^9C_1 + {}^9C_2 + {}^9C_3 + \ldots + {}^9C_9$ ways $= 2^9 - 1 = 511$ ways.

   But we can't take numbers which have only one digit, hence the required answer is 511 – 9.
7. 200 runs can be scored by scoring only fours or through a combination of fours and sixes.

   Possibilities are 50 × 4, 47 × 4 + 2 × 6, 44 × 4 + 4 × 6 … A total of 17 ways.
8. Of the total arrangements possible (6!) exactly half would have 1 before 6. Thus, 6!/2 = 360.
9. Total number of permutations without any restrictions – Number of permutations having the '*acd*' pattern – Number of permutations having the '*beg*' pattern + Number of permutations having both the '*beg*' and '*acd*' patterns.
10. *A* and *B* can occupy the first and the ninth places, the second and the tenth places, the third and the eleventh place and so on… This can be done in 18 ways.

    A and *B* can be arranged in 2 ways.

    All the other 24 alphabets can be arranged in 24! ways.

    Hence the required answer = 2 × 18 × 24!
11. First arrange the two sisters around a circle in such a way that there will be one seat vacant between them. [This can be done in 2! ways since the arrangement of the sisters is not circular.]

    Then, the other 18 people can be arranged on 18 seats in 18! ways.

12. ${}^{10}C_2 \times {}^{8}C_1 + {}^{10}C_1 \times {}^{8}C_2 = 360 + 280 = 640$

14. A chess board consists 9 parallel lines × 9 parallel lines. For a rectangle we need to select 2 parallel lines and two other parallel lines that are perpendicular to the first set. Hence, ${}^{9}C_2 \times {}^{9}C_2$

15-16. Based on direct formulae.

15. This is a direct result based question. Option (c) is correct. Refer to result no. 6 in Important Results 2.

16. $(1 + 2 + 3 + \ldots + 18)\ (1 + 2 + 3 + \ldots + 16)$

17-19. Based on simple counting according to the conditions given in the passage

17. ${}^{10}C_1 + 3 \times 2 + {}^{10}C_2 + 60 + 60 = 181$

18. Handshakes involving host = 76

    Hence, the required ratio is 76: 105.

19. The guests (Spectators) would shake hands ${}^{60}C_2$ times = 1770.

    Required percentage increase = 977.9%.

20. First select six people out of 12 for the first row. The other six automatically get selected for the second row. Arrange the two rows of people amongst themselves. Besides, the papers can be given in the pattern of 121212 or 212121. Hence the answer is $2 \times ({}^{12}C_6 \times 6! \times 6!)$.

21. The difference in this question from the previous question is the number of ways in which the papers can be distributed. This can be done by either distributing three different variants in the first three places of each row or by repeating the same variant in the first and the third places.

22. Required permutations = Total permutations with no condition – permutations with the conditions which we do not have to count.

23. We have to count natural numbers which have a maximum of 4 digits. The required answer will be given by:

    Number of single digit numbers + Number of two digit numbers + Number of three digit numbers + Number of four digit numbers.

24. Let the three people be *A*, *B* and *C*.

    If 1 person gets no objects, the 7 objects must be distributed such that each of the other two get 1 object at least.

    This can be done as 6 & 1, 5 & 2, 4 & 3 and their rearrangements.

    The answer would be

    $$({}^{7}C_6 + {}^{7}C_5 + {}^{7}C_4) \times 3! = 378$$

    Also, two people getting no objects can be done in 3 ways.

    Thus, the answer is 378 + 3 = 381

25. If only one gets 1 object

    The remaining can be distributed as: (6,0), (4, 2), (3, 3).

    $$({}^{7}C_1 \times {}^{6}C_6 \times 3! + {}^{7}C_1 \times {}^{6}C_4 \times 3! + {}^{7}C_1 \times {}^{6}C_3 \times 3!/2!) = 42 + 630 + 420 + 1092.$$

    If 2 people get 1 object each:

    $${}^{7}C_1 \times {}^{6}C_1 \times {}^{5}C_5 \times 3!/2! = 126.$$

    Thus, a total of 1218.

26. Natural numbers which consist of the digits 1, 2, 3, 4. and 5 and are divisible by 4 must have either 12, 24, 32 or 52 in the last two places. For the other two places we have to arrange three digits in two places.

27. No. of 1 digit nos = 9

    No. of 2 digit nos = 81

    No. of 3 digit nos = $9 \times 9 \times 8 = 648$

    No. of 4 digit nos = $9 \times 9 \times 8 \times 7 = 4536$

    Total nos = 9 + 81 + 648 + 4536 = 5274

28. If the two digits are *a* and *b* then 4 digit numbers can be formed in the following patterns.

    aabb; *aaab* or *aaaa*.

    You will have to take two situations in each of the cases- first when the two digits are non zero digits and second when the two digits are zero.

29. For the total of the digits to be odd one of the following has to be true:

    The number should contain 1 odd + 6 even or 3 odd + 4 even or 5 odd + 2 even or 7 odd digits. Count each case separately.

33. ${}^{18}C_4 + {}^{18}C_5 + {}^{18}C_6 + \ldots + {}^{18}C_{17} + {}^{18}C_{18}$

    $= [{}^{18}C_0 + {}^{18}C_4 + \ldots + {}^{18}C_{18}] - [{}^{18}C_0 + {}^{18}C_1 + {}^{18}C_2 + {}^{18}C_3]$

    $= 2^{18} - [1 + 18 + 153 + 816]$

    $= 261158$

35. Total number of 6 digit numbers having 3 odd and 3 even digits (including zero in the left most place) $= 5^3 \times 5^3$.

    From this subtract the number of 5 digit numbers with 2 even digits and 3 odd digits (to take care of the extra counting due to zero)

36. There will be 5 types of numbers, viz. numbers which have

    All eight digits even or six even and two odd digits or four even and four odd digits or two even and six odd digits or all eight odd digits. This will be further solved as below:

    Eight even digits $\rightarrow 5^8 - 5^7 = 4 \times 5^7$

    Six even and two odd digits →

    when the left most digit is even $\rightarrow 4 \times {}^{7}C_5 \times 5^5 \times 5^5$

    when the left most digit is odd $\rightarrow 5 \times {}^{7}C_6 \times 5^6 \times 5^1$

    Four even and four odd digits →

    when the left most digit is even $\rightarrow 4 \times {}^{7}C_5 \times 5^5 \times 5^4$

    when the left most digit is odd $\rightarrow 5 \times {}^{7}C_4 \times 5^4 \times 5^3$

    Two even and six odd digits →

    when the left most digit is even $\rightarrow 4 \times {}^{7}C_1 \times 5 \times 5^6$

when the left most digit is odd $\rightarrow 5 \times {}^7C_2 \times 5^2 \times 5^5$
Eight odd digits $\rightarrow 5^8$

37–38. Solve through options.

39. This question is based on a formula: The condition is that 'n' things (each thing belonging to a particular place) have to be distributed in 'n' places such that no particular thing is arranged in its correct place.

$n! - \frac{n!}{1!} + \frac{n!}{2!} - \frac{n!}{3!}$ sign of the terms will be alternate and the last term will be $\frac{n!}{n!}$.

However, this can also be solved through logic.

40. The possible cases for counting are:
Number of numbers when the units digit is nine + the number of numbers when neither the units digit nor the left most is nine + number of numbers when the left most digit is nine.

42. The condition is that we have to count the number of natural numbers not more than 4300.
The total possible numbers with the given digits = $5 \times 5 \times 5 \times 5 = 625 - 1 = 624$.
Subtract form this the number of natural number greater than 4300 which can be formed from the given digits = $1 \times 2 \times 5 \times 5 - 1 = 49$.
Hence, the required number of numbers = 624 – 49 = 575.

43. The required answer will be given by
The number of one digit natural number + number of two digit natural numbers + the number of three digit natural numbers + the number of four digit natural number starting with 1, 2, or 3 + the number of four digit natural numbers starting with 4.

46. The following words will appear before SPATE. All words starting with A + All words starting with E + All words starting with P + All words starting with SA + All words starting with SE + SPAET

47. For the maximum possibility assume that no three points other than given in the question are in a straight line.
Hence, the total number of Δ's = ${}^{A+B+C}C_3 - {}^AC_3 - {}^BC_3 - {}^CC_3$

48. Use the formula $\frac{n!}{p!q!r!}$.

$n(E) = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times {}^7C_2$
$n(S) = 7^7$
$n(E) = 8 \times 7 + 8 \times 7 = 112$

$$n(S) = 64C_2 = \frac{2 \times 8 \times 7 \times 2}{64 \times 63} = \frac{1}{18}$$

49. Use the formula $\frac{n!}{p!q!r!}$

Solutions and Shortcuts

Level of Difficulty (I)

1. The number of numbers formed would be given by $5 \times 4 \times 3$ (given that the first digit can be filled in 5 ways, the second in 4 ways and the third in 3 ways – MNP rule).
2. The first digit can only be 2 (1 way), the second digit can be filled in 7 ways, the third in 6 ways and the fourth in 5 ways. A total of $1 \times 7 \times 6 \times 5 = 210$ ways.
3. Each invitation card can be sent in 4 ways. Thus, $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$.
4. In this case since nothing is mentioned about whether the prizes are identical or distinct we can take the prizes to be distinct (the most logical thought given the situation). Thus, each prize can be given in 8 ways — thus a total of 8^5 ways.
5. We need to assume that the 7 Indians are 1 person, so also for the 6 Dutch and the 5 Pakistanis. These 3 groups of people can be arranged amongst themselves in 3! ways. Also, within themselves the 7 Indians the 6 Dutch and the 5 Pakistanis can be arranged in 7!, 6! And 5! ways respectively. Thus, the answer is $3! \times 7! \times 6! \times 5!$.
6. Use the MNP rule to get the answer as $5 \times 4 = 20$.
7. An engineer can make it through in 2 ways, while a CA can make it through in 3 ways. Required ratio is 2:3. Option (b) is correct.
8. For a straight line we just need to select 2 points out of the 8 points available. ${}^8C_2$ would be the number of ways of doing this.
9. Use the property ${}^nC_r = {}^nC_{n-r}$ to see that the two values would be equal at $n = 11$ since ${}^{11}C_3 = {}^{11}C_8$.
10. There would be 5! ways of arranging the 5 letters. Thus, 5! = 120 ways.
11. Rearrangements do not count the original arrangements. Thus, 5!/2! – 1 = 59 ways of rearranging the letters of PATNA.
12. We need to count words starting with P. These words would be represented by P _ _ _ _.
The letters ATNA can be arranged in 4!/2! ways in the 4 places. A total of 12 ways.
13. P _ _ _ T. Missing letters have to be filled with A,N,A. 3!/2!.= 3 ways.
14. Trial and error would give us ${}^8C_4$ as the answer. ${}^8C_4 = 8 \times 7 \times 6 \times 5/4 \times 3 \times 2 \times 1 = 70$.
15. ${}^{10}P_3$ would satisfy the value given as ${}^{10}P_3 = 10 \times 9 \times 8 = 720$.
16. $3 \times 3 \times 2 \times 1 = 18$
17. $3 \times 4 \times 4 \times 4 = 192$
18. Divide the numbers into three-digit numbers and 4-digit numbers—Number of 3 digit numbers = 2 ×

$3 \times 2 = 12$. Number of 4-digit numbers starting with $10 = 2 \times 1 = 2$. Total = 14 numbers.

19. 3-digit numbers $= 2 \times 4 \times 4 - 1 = 31$ (-1 is because the number 200 cannot be counted); 4-digit numbers starting with $10 = 4 \times 4 = 16$, Number of 4 digit numbers starting with $11 = 4 \times 4 = 16$. Total numbers $= 31 + 16 + 16 = 63$.
20. At $n = 3$, the values convert to ${}^7P_2$ and ${}^5P_3$ whose values respectively are 42 & 60 giving us the required ratio.
21. At $r = 7$, the value becomes $(28!/14! \times 14!)/(24!/10! \times 14!) \to 225:11$.
22. The maximum value of ${}^nC_r$ for a given value of n, happens when r is equal to the half of n. So if he wants to maximise the number of parties given that he has 20 friends, he should invite 10 to each party.
23. This is a typical case for the use of the formula ${}^{n-1}C_{r-1}$ with $n = 10$ and $r = 6$. So the answer would be given ${}^9C_5$.
24. For each digit there would be 5 options (viz 1, 3, 5, 7, 9). Hence, the total number of numbers would be $5 \times 5 \times 5 \times 5 = 625$.
25. ${}^{11}C_1 \times {}^{10}C_1 = 110$. Alternately, ${}^{11}C_2 \times 2!$
26. $5 \times 4 = 20$.
27. In the letters of the word ALLAHABAD there is only 1 vowel available for selection (A). Note that the fact that A is available 4 times has no impact on this fact. Also, there are 4 consonants available — viz: L, H, B and D. Thus, the number of ways of selecting a vowel and a consonant would be $1 \times {}^4C_1 = 4$.
28. Choose 1 person for the single room & from the remaining choose 2 people for the double room & from the remaining choose 4 people for the 4 persons room $\to {}^7C_1 \times {}^6C_2 \times {}^4C_4$.
29. From the first suit there would be 13 options of selecting a card. From the second suite there would be 12 options, from the third suite there would be 11 options and from the fourth suite there would be 10 options for selecting a card. Thus, $13 \times 12 \times 11 \times 10$.
30. Number of 11 letter words formed from the letters P, E, R, M, U, T, A, T, I, O, N = 11!/2!.

    Number of new words formed = total words -1 = $11!/2! - 1$.
31. ${}^{15}C_0 + {}^{15}C_1 + {}^{15}C_2 + {}^{15}C_3 + {}^{15}C_4 + {}^{15}C_5 = 1 + 15 + 105 + 455 + 1365 + 3003 = 4944$
32. All arrangements – Arrangements with best and worst paper together $= 12! - 2! \times 11!$.
33. The vowels EUAIO need to be considered as 1 letter to solve this. Thus, there would be 4! ways of arranging Q, T and N and the 5 vowels taken together. Also, there would be 5! ways of arranging the vowels amongst themselves. Thus, we have $4! \times 5!$.
34. ${}^3C_1 \times {}^4C_1 \times {}^6C_1 = 72$.
35. 4-digit Motor vehicle registration numbers can have 0 in the first digit. Thus, we have $6 \times 5 \times 4 \times 3 = 360$ ways.
36. Single digit numbers = 5

    Two digit numbers $= 5 \times 4 = 20$

    Three digit numbers $= 5 \times 4 \times 3 = 60$

    Four digit numbers $= 5 \times 4 \times 3 \times 2 = 120$

    Five digit numbers $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

    Total $= 5 + 20 + 60 + 120 + 120 = 325$
37. ${}^{25}C_2 - {}^7C_2 + 1 = 280$
38. ${}^{25}C_3 - {}^7C_3 = 2265$
39. ${}^{25}C_4 - {}^7C_4 - {}^7C_3 \times {}^{18}C_1 = 11985$
40. ${}^{10}C_5 \times 5! = 30240$
41. ${}^8C_3 \times {}^5C_2 \times 5! = 67200$
42. The selection of the 11 player team can be done in ${}^{14}C_{10}$ ways. This results in the team of 11 players being completely chosen. The arrangements of these 11 players can be done in 11!.

    Total batting orders $= {}^{14}C_{10} \times 11! = 1001 \times 11!$

    (**Note:** Arrangement is required here because we are talking about forming batting orders).
43. ${}^{12}C_4 = 495$
44. ${}^{30}C_3$
45. ${}^{29}C_2$
46. R _ _ _ _ _ _ W. The letters to go into the spaces are P, O, L, I, N, G. Since all these letters are distinct the number of ways of arranging them would be 6!.
47. $7!/3! \times 4! = 35$
48. The number has to start with a 3 and then in the remaining 6 digits it should have two 3's and four 0's. This can be done in $6!/2! \times 4! = 15$ ways.
49. ${}^3C_1 \times {}^5C_3 \times {}^4C_2 \times 5! = 21600$
50. If the number of teams is n, then ${}^nC_2$ should be equal to 45. Trial and error gives us the value of n as 10.
51. From 5 bananas we have 6 choices available (0, 1, 2, 3, 4 or 5). Similarly 4 mangoes and 4 almonds can be chosen in 5 ways each.

    So total ways $= 6 \times 5 \times 5 = 150$ possible selections. But in this 150, there is one selection where no fruit is chosen.

    So required no. of ways $= 150 - 1 = 149$

    Hence Option (b) is correct.
52. For each book we have two options, give or not give. Thus, we have a total of 2^{14} ways in which the 14 books can be decided upon. Out of this, there would be 1 way in which no book would be given. Thus, the number of ways is $2^{14} - 1$.

    Hence, Option (d) is correct.
53. The number of ways in which at least 1 Archer book is given is (2^5-1). Similarly, for Sheldon and Grisham we have (2^3-1) and (2^6-1). Thus required answer would be the multiplication of the three. Hence, Option (d) is the correct answer.

54. For each question we have 3 choices of answering the question (2 internal choices + 1 non-attempt). Thus, there are a total of 3^{15} ways of answering the question paper. Out of this there is exactly one way in which the student does not answer any question. Thus there are a total of $3^{15} - 1$ ways in which at least one question is answered.

Hence, Option (d) is correct.

55. The digits are 1, 6, 7, 8, 7, 6, 1. In this seven-digit no. there are four odd places and three even places—OEOEOEO. The four odd digits 1, 7, 7, 1 can be arranged in four odd places in [4!/2! × 2] = 6 ways [as 1 and 7 are both occurring twice].

The even digits 6, 8, 6 can be arranged in three even places in 3!/2! = 3 ways.

Total no. of ways = 6 × 3 = 18.

Hence, Option (c) is correct.

56. We have no girls together, let us first arrange the 5 boys and after that we can arrange the girls in the spaces between the boys.

Number of ways of arranging the boys around a circle = [5 – 1]! = 24.

Number of ways of arranging the girls would be by placing them in the 5 spaces that are formed between the boys. This can be done in ${}^5P_3$ ways = 60 ways.

Total arrangements = 24 × 60 = 1440.

Hence, Option (d) is correct.

57. Books of interest = 7, books to be borrowed = 3

Case 1— Quants book is taken. Then D.I book can also be taken.

So Amita is to take 2 more books out of 6 which she can do in ${}^6C_2$ = 15 ways.

Case 2— If Quants book has not been taken, the D.I book would also not be taken.

So Amita will take three books out of 5 books. This can be done in ${}^5C_3$ = 10 ways.

So total ways = 15 + 10 = 25 ways.

Hence Option (c) is correct.

58. We have to select 5 out of 12.

If Radha and Mohan join- then we have to select only 5 – 2 = 3 dancers out of 12 – 2 = 10 which can be done in ${}^{10}C_3$ = 120 ways.

If Radha and Mohan do not join, then we have to select 5 out of 12 – 2 = 10-> ${}^{10}C_5$ = 252 ways.

Total number of ways = 120 + 252 = 372.

Hence, Option (d) is correct.

59. The unit digit can either be 2, 3, 4, 5 or 6.

When the unit digit is 2, the number would be even and hence will be divisible by 2. Hence all numbers with unit digit 2 will be included which is equal to 5! Or 120.

When the unit digit is 3, then in every case the sum of the digits of the number would be 21 which is a multiple of 3. Hence all numbers with unit digit 3 will be divisible by 3 and hence will be included. Total number of such numbers is 5! or 120.

Similarly for unit digit 5 and 6, the number of required numbers is 120 each.

When the unit digit is 4, then the number would be divisible by 4 only if the ten's digit is 2 or 6. Total number of such numbers is 2 × 4! or 48.

Hence total number of required numbers is (4 × 120) + 48 = 528.

Hence, Option (d) is the answer.

60. As we need to find the maximum number of trials, so we have to assume that the required ball in every box is picked as late as possible. So in the third box, first two balls will be red and black. Hence third trial will give him the required ball. Similarly, in fourth box, he will get the required ball in fourth trial and in the fifth box, he will get the required ball in fifth trial. Hence maximum total number of trials required is 3 + 4 + 5 = 12.

Hence, Option (a) is the answer.

61. Since every player needs to win only 1 match to move to the next round, therefore the 1st round would have 32 matches between 64 players out of which 32 will be knocked out of the tournament and 32 will be moved to the next round. Similarly in 2nd round 16 matches will be played, in the 3rd round 8 matches will be played, in 4th round 4 matches, in 5th round 2 matches and the 6th round will be the final match. Hence total number of rounds will be 6 (2^6 = 64).

Hence, option (b) is the answer.

62. Total number of pairs of men that can be selected if the adjacent ones are also selected is ${}^NC_2$. Total number of pairs of men selected if only the adjacent ones are selected is N. Hence total number of pairs of men selected if the adjacent ones are not selected is ${}^NC_2 - N$.

Since the total time taken is 88 min, hence the number of pairs is 44.

Hence, ${}^NC_2 - N = 44 \rightarrow N = 11$.

Hence, Option (d) is the answer.

63. Let the number of boys be B. Then ${}^BC_3 = 36 \rightarrow B = 9$

Let the number of girls be G. Then ${}^GC_2 = 66 \rightarrow G = 12$.

Therefore total number of students in the class = 12 + 9 = 21. Hence total number of matches = ${}^{21}C_2$ = 210. Hence, number of matches between 1 boy and 1 girl = 210 – (36 + 66) = 108.

Hence, Option (a) is the answer.

64. With 3 chocolates to Sana, the remaining 12 chocolates, would then get divided among 4 children, with each child getting minimum 1 chocolate in ${}^{11}C_3$ ways.

With 4 chocolates to Sana, the remaining 11 chocolates, would then get divided among 4 children, with each child getting minimum 1 chocolate in ${}^{10}C_3$ ways. With 5 chocolates to Sana, the remaining 10 chocolates, would then get divided among 4 children, with each child getting minimum 1 chocolate in ${}^{9}C_3$ ways. With 6 chocolates to Sana, the remaining 9 chocolates, would then get divided among 4 children, with each child getting minimum 1 chocolate in ${}^{8}C_3$ ways. The total number of distributions = 165 + 120 + 84 + 56 = 425. Hence, option (d) is correct.

65. Firstly we will give 5 crores each to the three sons. That will cover 15 crores out of 30 crores leaving behind 15 crores. Now 15 crores can be distributed in three people in 17!15!2! ways or 136 ways.

    Hence, Option (a) is the answer.

66. Let $x = 3$. Then $y + z = 27$. For the conditions given in the question, no. of solutions is 20.

    Similarly for $x = 2$ there will be 20 solutions, for $x = 1$ there will be 22 solutions and for $x = 0$, there will be 22 solutions. Therefore total 84 solutions are possible.

    Similarly for $y = 3$ to 0, there will be 84 solutions and for $z = 3$ to 0, there will be 84 solutions.

    Hence there will be total of 252 solutions. Hence, Option (d) is the answer.

67. No. of words starting with A = 8!/ 2!3! = 3360.

    No. of words starting with B = 8!/ 2!4! = 840

    No. of words starting with D = 8!/2!4! = 840

    No. of words starting with H = 8!/2!4! = 840

    Now words with L start.

    No. of words starting with LAA = 6!/2! = 360

    Now LAB starts and first word starts with LABA.

    No. of words starting with LABAA = 4! = 24

    After this the next words will be LABADAAHL, LABADAALH, LABADAHAL, LABADAHLA and hence, Option (a) is the answer.

68. We will consider $x = 7$ to $x = 1$.

    For $x = 7$, $y + z = 5$. No. of solutions = 4

    For $x = 6$, $y + z = 6$. No. of solution = 5

    For $x = 5$, $y + z = 7$. No. of solutions = 6

    For $x = 4$, $y + z = 8$. No. of solutions = 7

    For $x = 3$, $y + z = 9$. No. of solutions = 6

    For $x = 2$, $y + z = 10$. No. of solutions = 5

    For $x = 1$, $y + z = 11$. No. of solutions = 4

    Hence number of solutions = 37

    Hence, Option (b) is the answer.

69. As no three points are collinear, therefore every combination of 3 points out of the nine points will give us a triangle. Hence, the answer is ${}^{9}C_3$ or 84.

    Hence, Option (d) is correct.

70. The number of combinations of three points picked from the nine given points is ${}^{9}C_3$ or 84. All these combinations will result in a triangle except the combination of the three collinear points. Hence number of triangles formed will be 84 – 1 = 83.

    Hence, Option (d) is the answer.

71. $(xy)^2 = u! + v$

    Here xy is a two-digit number and maximum value of its square is 9801. 8! is a five-digit number => u is less than 8 and 4! is 24 which when added to a single digit will never give the square of a two-digit number. Hence u is greater than 4. So, possible values of u can be 5, 6 and 7.

    If $u = 5$, $u! = 120 \Rightarrow (xy)^2 = u! + v \Rightarrow (xy)^2 = 120 + v = 120 + 1 = 121 = 11^2$

    If $u = 6$, $u! = 720 \Rightarrow (xy)^2 = u! + v \Rightarrow (xy)^2 = 720 + v = 720 + 9 = 729 = 27^2$

    If $u = 7$, $u! = 5040 \Rightarrow (xy)^2 = u! + v \Rightarrow (xy)^2 = 5040 + v = 5040 + 1 = 5041 = 71^2$

    So there are three cases possible. Hence, 3 solutions exist for the given equation.

    Hence, Option (b) is the correct answer.

72. In order to form triangles from the given points, we can either select 2 points from the first line and 1 point from the second OR select one point from the first line and 2 from the second.

    This can be done in:

    ${}^{10}C_2 \times {}^{11}C_1 + {}^{10}C_1 \times {}^{11}C_2 = 495 + 550 = 1045$

73. If we have 'n' candidates who can be selected at the maximum, naturally, the answer to the question would also represent 'n'.

    Hence we check for the first option. If $n = 3$, then $2n + 1 = 7$ and it means that there are 7 candidates to be chosen from. Since it is given that the number of ways of selection of at least 1 candidate is 63, we should try to see, whether selecting 1, 2 or 3 candidates from 7 indeed adds up to 63 ways. If it does this would be the correct answer.

    ${}^{7}C_1 + {}^{7}C_2 + {}^{7}C_3 = 7 + 21 + 35 = 63$. Thus, the first Option fits the situation and is hence correct.

74. This problem can be approached by putting the white flags in their possible positions. There are essentially 4 possibilities for placing the 3 white flags based on the condition that two flags of the same color cannot be together:

    1, 3, 5; 1, 3, 6; 1, 4, 6 and 2, 4, 6.

    Out of these 4 possible arrangements for the 3 white flags we cannot use 1, 3, 6 and 1, 4, 6 as these have the same color of flag at both ends- something which is not allowed according to the question. Thus there are only 2 possible ways of placing the white flags— 1, 3, 5 OR 2, 4, 6. In each of these 2 ways, there are a further 3 ways of placing the 1 red flag and the 2 blue flags. Thus we get a total of 6 ways. Option (a) is correct.

75. The possible numbers are:

635_ _ _9	9 in the units place	$9 \times 9 \times 9 = 729$ numbers
635 _ _ _ _	9 used before the units place	$3 \times 9 \times 9 \times 4 = 972$ numbers
674_ _ _9	9 in the units place	$9 \times 9 \times 9 = 729$ numbers
674 _ _ _ _	9 used before the units place	$3 \times 9 \times 9 \times 4 = 972$ numbers
Total		3402 numbers

76. We need to go through the options and use the MNP rule tool relating to Permutations and Combinations. We can draw up the following possibilities table for the number of routes between each of the three towns. If the first option is true, i.e., there are 6 routes between *A* to *C*:

A-C	*Possibilities for C-B*	*Possibilities for total routes A-C-B (Say X)*	*Possibilities for Total routes A–B (Y)*
6	5, 4, 3, 2, 1	30, 24, 18, 12, 6	3, 9, 15, 21, 27 Note: these values are derived based on the logic that X + Y = 33

We further know that there are 23 routes between *B* to *C*.

From the above combinations the possibilities for the routes between *B* to *C* are:

B–A (Y in the table above)	*A–C*	*B–A–C*	*B–C*	*Total*
3	6	18	5	23
9	6	54 not possible	4	
15	6	90 not possible	3	
21	6	126 not possible	2	
27	6	162 not possible	1	

It is obvious that the first possibility in the table above satisfies all conditions of the given situation. Option (a) is correct.

77. With the digits 1, 2, 3, 4, 5 and 6 the numbers divisible by 4 that can be formed are numbers ending in: 12, 16, 24, 32, 36, 52, 56 and 64.

Number of numbers ending in 12 are: $4 \times 3 \times 2 = 24$

Thus the number of numbers is $24 \times 8 = 192$

Option (c) is correct.

78. A million is 1000000 (i.e. the first seven digit number). So we need to find how many numbers of less than 7 digits can be formed using the digits 0,7 and 8.

Number of 1 digit numbers = 2

Number of 2 digit numbers = $2 \times 3 = 6$

Number of 3 digit numbers = $2 \times 3 \times 3 = 18$

Number of 4 digit numbers = $2 \times 3 \times 3 \times 3 = 54$

Number of 5 digit numbers = $2 \times 3 \times 3 \times 3 \times 3 = 162$

Number of 6 digit numbers = $2 \times 3 \times 3 \times 3 \times 3 \times 3 = 486$

Total number of numbers = 728. Option (c) is correct.

79. The white square can be selected in 32 ways and once the white square is selected 8 black squares become ineligible for selection. Hence, the black square can be selected in 24 ways. $32 \times 24 = 768$. Option (d) is correct.

80. Since there are 11 symmetric letters, the number of passwords that can be formed would be $11 \times 10 \times 9 \times 8 = 7920$. Option (a) is correct.

81. This would be given by the number of passwords having:

1 symmetric and 2 asymmetric letters + 2 symmetric and 1 asymmetric letter + 3 symmetric and 0 asymmetric letters

${}^{11}C_1 \times {}^{15}C_2 \times 3! + {}^{11}C_2 \times {}^{15}C_1 \times 3! + {}^{11}C_3 \times 3! = 11 \times 105 \times 6 + 55 \times 15 \times 6 + 11 \times 10 \times 9 = 6930 + 4950 + 990 = 12870$. Option (c) is correct.

82. Each of the first, third and fourth options can be obviously seen to be true— no mathematics needed there. Only the second option can never be true.

In order to think about this mathematically and numerically— think of a party of 3 persons say *A*, *B* and *C*. In order for the second condition to be possible, each person must know a different number of persons. In a party with 3 persons this is possible only if the numbers are 0, 1 and 2. If *A* knows both *B* and *C* (2), *B* and *C* both would know at least 1 person— hence it would not be possible to create the person knowing 0 people. The same can be verified with a group of 4 persons i.e., the minute you were to make 1 person know 3 persons it would not be possible for anyone in the group to know 0 persons and hence you would not be able to meet the condition that every person knows a different number of persons. Option (b) is correct.

83. With one green ball there would be six ways of doing this. With 2 green balls 5 ways, with 3 green balls 4 ways, with 4 green balls 3 ways, with 5 green balls 2 ways and with 6 green balls 1 way. So a total of $1 + 2 + 3 + 4 + 5 + 6 = 21$ ways. Option (b) is correct.

84. One digit no. = 5; Two digit nos = $5 \times 4 = 20$; Three digit no = $5 \times 4 \times 3 = 60$; four digit no = $5 \times 4 \times 3 \times 2 = 120$; Five digit no.= $5 \times 4 \times 3 \times 2 \times 1 = 120$ Total number of nos = 325. Hence Option (b) is correct.

85. For each selection there are 3 ways of doing it. Thus, there are a total of $3 \times 3 \times 3 \times 3 \times 3 = 243$. Hence, Option (b) is correct.

86. Solve this one through options. If you pick up option (a) it gives you 12 participants in the tournament.

This means that there are 10 men and 2 women. In this case there would be $2 \times {}^{10}C_2 = 90$ matches amongst the men and $2 \times {}^{10}C_1 \times {}^2C_1 = 40$ matches between 1 man and 1 woman. The difference between number of matches where both participants are men and the number of matches where 1 participant is a man and one is a woman is 90 – 40 = 50 – which is not what is given in the problem.

With 13 participants → 11 men and 2 women.

In this case there would be $2 \times {}^{11}C_2 = 110$ matches amongst the men and $2 \times {}^{11}C_1 \times {}^2C_1 = 44$ matches between 1 man and 1 woman. The difference between number of matches where both participants are men and the number of matches where 1 participant is a man and one is a woman is 110 – 44 = 66 – which is the required value as given in the problem. Thus, option (b) is correct.

87. Based on the above thinking we get that since there are 13 players and each player plays each of the others twice, the number of games would be $2 \times {}^{13}C_2 = 2 \times 78 = 156$.

Level of Difficulty (II)

1. Number of even numbers = $6 \times 6 \times 6 \times 3$
2. We need to think of this as: Number with two sixes or numbers with one six or number with no six.

   0, 1, 2, 3, 4, 5, 6 and 6

   Numbers with 2 sixes:

   Numbers ending in zero $\quad {}^5C_1 \times 3!/2! = 15$

   Numbers Ending in 5 and

   (a) Starting with 6 $\quad {}^5C_1 \times 2! = 10$

   (b) Not starting with 6 $\quad {}^4C_1$ (as zero is not allowed) = 4

   Number with 1 six or no sixes.

   Numbers ending in 0 $\quad {}^6C_3 \times 3! = 120$

   Numbers ending in 5 $\quad {}^5C_1 \times {}^5C_2 \times 2! = 100$

   Thus a total of 249 numbers.
3. First arrange 6 pups in 6 places in 6! ways.

   This will leave us with 7 places for 4 cats. Answer $= 6! \times 7p_4.$
4. Arrangement of M, A, N, A, E, M, E, N, T is $\dfrac{9!}{2!\times 2!\times 2!\times 2!}$.
5. For nine places we have following number of arrangements.

   $9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$
6. For a rectangle, we need two pair of parallel lines which are perpendicular to each other. We need to select two parallel lines from '*v*' lines and 2 parallel lines from '*w*' lines. Hence required number of parallel lines is ${}^VC_2 \times {}^WC_2$.
7. From 8 people we have to *arrange* a group of 5 in which three are similar $\dfrac{8P_5}{3!}$ or $\dfrac{8C_5 \times 5!}{3!}$.
8. $\dfrac{4C_4 \times 4C_1 \times 5!}{3!} + \dfrac{4C_2 \times 4C_3 \times 5!}{3!} - 4C_2 \times 4C_3 \times 2C_2 \times 2!$
9. Since the number of men and women in the question is the same, there is no difference in solving this question and solving the previous one (question number 8) as committees having a maximum of 2 women would mean committees having a minimum of 3 men and committees having at maximum one woman holding the post of either president or vice president would mean at least 1 man holding one of the two posts.

   Thus, the answer would be:

   Number of committees with 4 men and 1 woman (including all arrangements of the committees) + Number of committees with 3 men and 2 women (including all arrangements of the committees)–Number of committees with 3 men and 2 women where both the women are occupying the two posts.

   $= ({}^4C_4 \times {}^4C_1 \times 5!)/3! + ({}^4C_3 \times {}^4C_2 \times 5!)/3! - ({}^4C_3 \times {}^4C_2 \times {}^2C_2 \times 2!) = 80 + 480 - 48 = 512$
10. ${}^7C_1 \times {}^6C_2 \times 4! \times 4! = 60480$
11. First make the six law students sit in a row. This can be done in 6! Ways. Then, there would be 7 places for the MBA students. We need to select 5 of these 7 places for 5 MBA students and then arrange these 5 students in those 5 places. This can be done in ${}^7C_5 \times 5!$ Ways.

    Thus, the answer is:

    $6! \times {}^7C_5 \times 5! = 7! \times 6!/2!$
12. The required answer will be given by counting the total number of registration numbers starting with DL-5A to DL-5R and the number of registration numbers starting with DL-5S that have to be counted.
13. Out of 100 balls arrange 99 balls (except n_{28}) amongst themselves. Now put n_{28} just before n_{29} in the above arrangement.
14. ${}^6C_2 = 15$.
15. We need to arrange *R* people on *M* chairs, *S* people on another set of *M* chairs and the remaining people on the remaining chairs. ${}^MP_R \times {}^MP_S \times {}^{2M-R-S}P_{2M-R-S.}$
16. Each group will consists of m things. This can be done in: ${}^{mn}C_m \cdot {}^{mn-m}C_m \cdot {}^{mn-2m}C_{m\ldots\ldots} {}^mC_m$

    $$= \frac{mn!}{(mn-m)!m!} \cdot \frac{(mn-m)!}{(mn-2m)!m!} \cdots \frac{m!}{0!m!} = \frac{mn!}{(m!)^n}$$

    Divide this by $n!$ since arrangements of the n groups amonst themselves is not required.

    Required number of ways = $\dfrac{mn!}{(m!)^n \cdot n!}$
17. Number of ways of selecting 5 different letters = ${}^5C_5 = 1$

Number of ways of selecting 2 similar and 3 different letters = ${}^4C_1 \times {}^4C_3 = 16$

Number of ways of selecting 2 similar letters + 2 more similar letters and 1 different letter = ${}^4C_2 \times {}^3C_1 = 18$

Number of ways of selecting 3 similar letters and 2 different letters = ${}^3C_1 \times {}^4C_2 = 18$

Number of ways of selecting 3 similar letters and another 2 other similar letters = ${}^3C_1 \times {}^3C_1 = 9$

Number of ways of selecting 4 similar letters and 1 different letter = ${}^2C_1 \times {}^4C_1 = 8$

Number of ways of selecting 5 similar letters = ${}^1C_1 = 1$

Total number of ways = 1 + 16 + 18 + 18 + 9 + 8 + 1 = 71.

18. Divide $3n + 1$ articles in two groups.
 (i) n identical articles and the remaining
 (ii) $2n + 1$ non-identical articles

 We will select articles in two steps. Some from the first group and the rest from the second group.

Number of articles from first group	*Number of articles from second group*	*Number of ways.*
0	n	$1 \times {}^{2n+1}C_n$
1	$n - 1$	$1 \times {}^{2n+1}C_{n-1}$
2	$n - 2$	$1 \times {}^{2n+1}C_{n-2}$
3	$n - 3$	$1 \times {}^{2n+1}C_{n-3}$
..	..	…….........
$n - 1$	1	$1 \times {}^{2n+1}C_1$
n	0	$1 \times {}^{2n+1}C_0$

Total number of ways = ${}^{2n+1}C_n + {}^{2n+1}C_{n-1} + {}^{2n+1}C_{n-2} + \ldots {}^{2n+1}C_1 + {}^{2n+1}C_0 = \dfrac{2^{2n+1}}{2} = 2^{2n}$.

19. We have four options for every place including the left most.

 So the total number of numbers = $4 \times 4 \times 4 \times \ldots = 4^{10}$.

 We have to consider only positive numbers, so we don't consider one number in which all ten digits are zeroes.

20. Total number of attempts = 10^4 out of which one is correct.

21. For odd places, the number of arrangements = $\dfrac{4!}{2!2!}$

 For even places, the number of arrangements = $\dfrac{3!}{2!}$

 Hence the total number of arrangements = $\dfrac{4! \times 3!}{2! \times 2! \times 2!}$

22. The number would be of the form _6_ _____ 5

 The 5 missing digits have to be formed using the digits 0, 1, 2, 3, 4, 7, 8, 9 without repetition.

 Thus, ${}^8C_5 \times 5! = 6720$

23. $1m + 3f = {}^8C_1 \times {}^8C_3 = 8 \times 56 = 448$
 $2m + 2f = {}^8C_2 \times {}^8C_2 = 28 \times 28 = 784$
 $3m + 1f\ {}^8C_3 \times {}^8C_1 = 56 \times 8 = 448$
 $4m + 0f = {}^8C_4 \times {}^8C_0 = 70 \times 1 = 70$
 Total = 1750

24. Solve this by dividing the solution into,
 3 women and 1 man or
 2 women and 2 men or
 1 woman and 3 men or
 0 woman and 4 men.
 This will give us:
 ${}^8C_3 \times {}^8C_1 + {}^8C_2 \times {}^8C_2 + {}^8C_1 \times {}^8C_3 + {}^8C_0 \times {}^8C_4$
 = 448 + 784 + 448 + 70 = 1750

25. For 1 to 9 we require 9 digits
 For 10 to 99 we require 90×2 digits
 For 100 to 150 we require 51×3 digits

26. Select any three places for A, B and C. They need no arrangement amongst themselves as A would always come before B and B would come before C.

 The remaining 5 people have to be arranged in 5 places.

 Thus, ${}^8C_3 \times 5! = 56 \times 120 = 6720$ OR $8!/3!$

27. Total number of choices = 4! out of which only one will be right.

28. At least two letters have to interchange their places for a wrong choice.

29. Select any two letters and interchange them (${}^4C_2$).

30. ${}^{45}C_7$ (refer to solved example 16.14).

31. For one pair of black shoes we require one left black and one right black. Consider the worst case situation:
 $7LB + 5LW + 5RW + 1RB$ or
 $7RB + 5LW + 5RW + 1LB$ = 18 shoes

32. For one pair of correct shoes one of the possible combinations is $7LB + 5LW + 1R$ (B or W) = 13

 Some other cases are also possible with at least 13 shoes.

33. The first rook can be placed in any of the 64 squares and the second rook will then have only 49 places so that they are not attacking each other.

34. When all digits are odd.
 $$5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$$
 When all digits are even
 $$4 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 4 \times 5^5$$
 $$5^6 + 4 \times 5^5 = 28125$$

35. All six digit numbers – Six digit numbers with only odd digits.
 $= 900000 - 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 884375.$

36. "Total number of all 10-digits numbers – Total number of all 10-digits numbers with no digit repeated"

will give the required answer.

$$= 9 \times 10^9 - 9 \times {}^9P_8$$

37. There will be two types of triangles

The first type will have its vertices on the three sides of the ΔABC.

The second type will have two of it's vertices on the same side and the third vertex on any of the other two sides.

Hence, the required number of triangles

$= 6 \times 5 \times 3 + {}^6C_2 \times 8 + {}^5C_2 \times 9 + {}^3C_2 \times 11$

$= 90 + 120 + 90 + 33$

$= 333$

38. First step – arrange 7 boys around the table according to the circular permutations rule. i.e. in 6! ways.

Second step – now we have 7 places and have to arrange 7 girls on these places. This can be done in ${}^7P_7$ ways. Hence, the total number of ways = 6! × 7!

39. 2 × 7! × 7! (Note: we do not need to use circular arrangements here because the seats are numbered.)

40. We just need to select the floors and the people who get down at each floor.

The floors selection can be done in ${}^{11}C_3$ ways.

The people selection is ${}^9C_4 \times {}^5C_3$.

Also, the floors need to be arranged using 3!

Thus, ${}^{11}C_3 \times {}^9C_4 \times {}^5C_3 \times 3!$ or ${}^{11}P_3 \times {}^9C_4 \times {}^5C_3$

41. To arrange a surgeon and an assistant we have ${}^{40}P_2$ ways.

42. To arrange a surgeon and 4 assistants we have or $40 \times {}^{39}C_4$ ways.

43. Give one marble to each of the six children. Then, the remaining 4 identical marbles can be distributed amongst the six children in ${}^{(4+6-1)}C_{(6-1)}$ ways.

44. Since it is possible to give no objects to one or two of them we would have 3 choices for giving each item.

Thus, 3 × 3 × 3 × 3 × 3 × 3 = 2187.

45. For an even number the units digit should be either 2, 4 or 6. For the other five places we have six digits. Hence, the number of six digit numbers = ${}^6P_5 \times 3$ = 2160.

46. Visualize the number as:

_ _ _ _ _ _

This number has to have the last two digits even. Thus, ${}^3C_2 \times 2!$ will fill the last 2 digits.

For the remaining places : ${}^5C_4 \times 4!$

Thus, we have ${}^5C_4 \times 4! \times {}^3C_2 \times 2! = 720$

47. ${}^9C_5 \times 5! = 15120$

48. ${}^4C_1 \times 7 \times 7 \times 7 = {}^4C_1 \times 7^3$

49. Select the two positions for the two 3's. After that the remaining 5 places have to be filled using either 1 or 2.

Thus, ${}^7C_2 \times 2^5$

50. ${}^4C_1 \times {}^7C_3 \times 3! = 840$