# CHAPTER 7

# Capacitor

## LEVEL 1

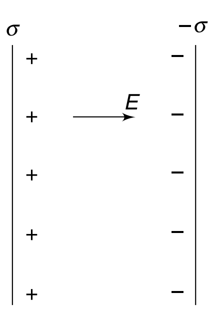
**Q. 1:** Two students *A* and *B* were taught that electric field near a uniformly charged large surface is normal to the surface and is equal to $\frac{\sigma}{2\epsilon_0}$. They were also told that field near the surface of a conductor is $\frac{\sigma}{\epsilon_0}$ normal to the conductor where $\sigma$ is charge density on the conductor surface. Now both of them were asked to write field between the plates of an ideal parallel plate capacitor having charge density $\sigma$ and $-\sigma$ on its plates. Student *A* said that field can be seen as superposition of field due to two large charged surfaces. He wrote the answer as

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Student *B* thought that a capacitor has conducting plates and therefore field due to each plate

Must be $\frac{\sigma}{\epsilon_0}$. He wrote his answer as $E = \frac{\sigma}{\epsilon_0} + \frac{\sigma}{\epsilon_0} = \frac{2\sigma}{\epsilon_0}$

Who is wrong and where is the flaw in thinking?




**Q. 2:** A large parallel plate capacitor has vertical plates with a potential difference of 2000 V between them. Oil drops are sprayed between the plates. Few drops are observed to move with uniform velocities in directions inclined at 45°, 33.7° and 18.4° to the vertical. The space between the plates has air and mass of each drop is $m = 3.3 \times 10^{-15}$ kg. Separation between the plates is 3 cm.

(a) Explain the observations.

(b) From the above observations estimate the magnitude of smallest charge in nature.

tan (33.7°) = 0.67 and tan (18.4°) = 0.333

**Q. 3:** Electrical susceptibility ($\chi$) of a dielectric material is defined as $\chi = \epsilon_r - 1$ where $\epsilon_r$ is its relative permittivity. An isolated parallel plate capacitor carries some charge and the field in the dielectric present between its plates is *E*. Express the electric field due to induced charge on dielectric surface in terms of $\chi$ and *E*.

**Q. 4:** Imagin a parallel plate capacitor with a charge $+Q$ on one plate and $-Q$ on the other. Initially, the plates are almost, but not quite, touching. The plates are gradually pulled apart to make the separation *d*. The separation *d* is small compared to dimensions of the plates and we can maintain that field between the plates is uniform. Area of each plate is *A*.

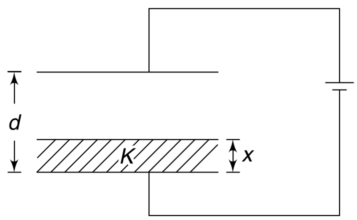
(a) Write the energy stored in the electric field between the plates when separation between then is *d*.

(b) Assuming that the energy calculated in part (a) can be attributed to the work done by the external agent in pulling the plates apart, calculate the electrostatic attraction force between the plates.

**Q. 5:** Two conducting plates each having area *A* are kept at a separation *d* parallel to each other. The two plates are connected to a battery of emf *V*. The space between the plates is filled with a liquid of dielectric constant *K*. The height (*x*) of the liquid between the plates increases at a uniform rate from zero to *d* in a time interval $t_0$.

(a) Write the capacitance of the system as a function of *x* as the liquid beings to fill the space between the plates.

(b) Write the current through the cell at time $t$.



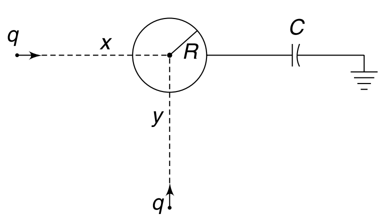

**Q. 6:** You have been given a parallel plate air capacitor having capacitance $C$, a battery of emf $\varepsilon$ and three dielectric blocks having dielectric constants $K_1$, $K_2$ and $K_3$ such that $K_1 > K_2 > K_3$. Describe a sequence of steps such as connecting or disconnecting the capacitor to the battery, inserting or taking out of one of the dielectrics etc – so that the capacitor ends up having maximum possible energy stored in it. [Each dielectric block fills completely the space between the plates]. Write this maximum energy.

**Q. 7:** A parallel plate capacitor has two plates of area $A$ separated by a small distance $d$. The capacitor is charged to a potential difference of $V$ and the battery is disconnected. A metal plate with area $A$ and thickness $\frac{d}{2}$ is fully inserted between the plates, so that it always remains parallel to the plates.

(a) Calculate the work done on the metal slab while it was inserted.

(b) Does the two plates of the capacitor attract of repel the metal plate that is being inserted. Does the answer obtained in part (a) help you in answering this?

**Q. 8:** A neutral conducting ball of radius $R$ is connected to one plate of a capacitor (Capacitance = $C$), the other plate of which is grounded. The capacitor is at a large distance from the ball. Two point charges, $q$ each, begin to approach the ball from infinite distance. The two point charges move in mutually perpendicular directions. Calculate the charge on the capacitor when the two point charges are at distance $x$ and $y$ form the centre of the sphere.



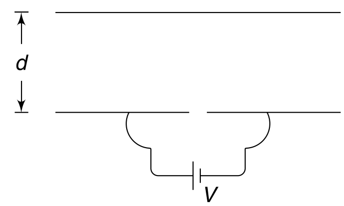

**Q. 9:** A parallel plate capacitor of capacitance $C_0$ is charged using a cell of emf $V_0$. Calculate the work done in reducing the separation between the plates to half its original value if

(a) The cell is disconnected before you start decreasing the plate separation.

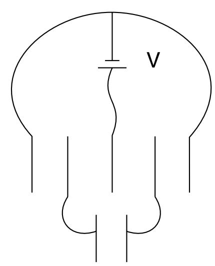
(b) The cell remains connected while you are reducing the separation.

Assume that the plates are moved very slowly.

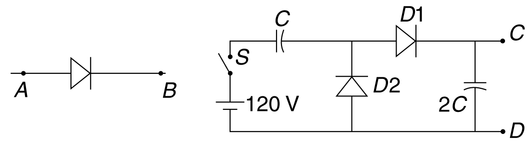
**Q. 10:** Two large conducting plates, identical in size, are placed parallel to each other at a separation $d$. Each plate has area $A$. One of the plates is cut into two equal parts and then a battery of emf $V$ is connected across these two pieces. Find the work done by the battery in supplying charge to the plates.




**Q. 11:** Seven identical plates, each of area $A$, are placed as shown. Any two adjacent plates are at separation $d$. Conducting wires have been used to connect the plates and a cell of emf $V$ volt as shown in the figure. How much charge does the cell supply?

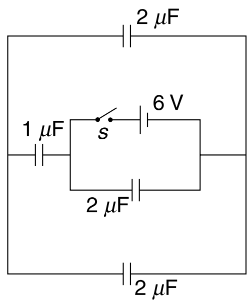



**Q. 12:** A diode is a device that conducts in one direction only. Figure (a) shows the symbol for a diode. When terminal $A$ is at higher potential than $B$, the diode conducts; it means current flows from $A$ to $B$. No current flows if $B$ is kept at higher potential. Find the potential difference between terminals $C$ and $D$ after the switch ($S$) is closed in the circuit shown in Figure (b).

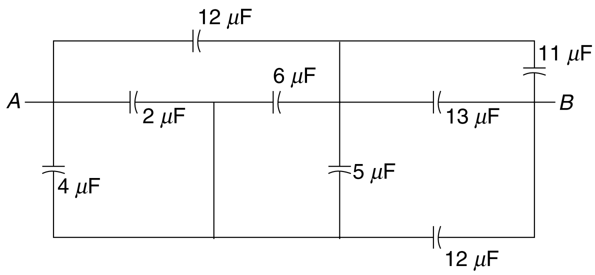



(a) (b)

**Q. 13:** Find charge supplied by the cell after the switch is closed.



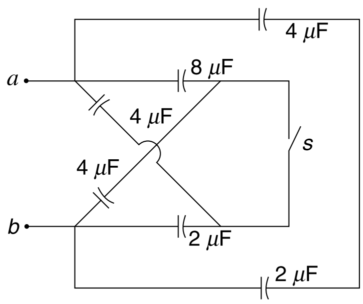

**Q. 14:** Find the equivalent capacitance across points $A$ and $B$.



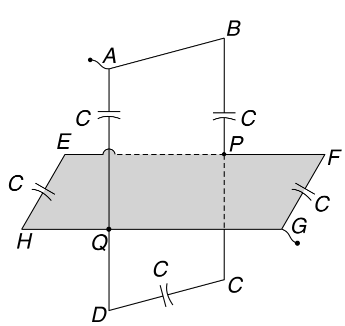

**Q. 15:** In the circuit shown in the Figure find the equivalent capacitance between points $a$ and $b$ when–

(a) Switch $S$ is open.

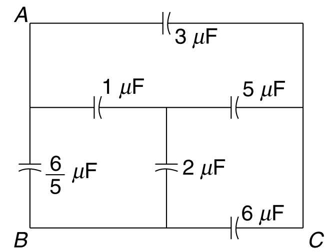
(b) Switch $S$ is closed.




**Q. 16:** In the circuit shown in the Figure $ABCD$ is a rectangular and vertical frame of conducting wires having three capacitors. $EFGH$ is in horizontal plane having two capacitors. The two rectangular frames are connected at $P$ and $Q$ only.Find equivalent capacitance between $A$ and $G$ if each capacitor has capacitance $C$.

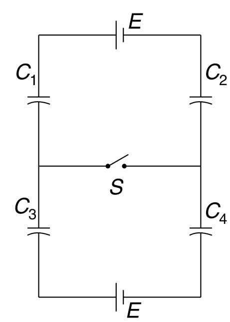



**Q. 17:** In the circuit shown in the Figure find the ratio of equivalent capacitance between $A$ and $B$ to that between $A$ and $C$.

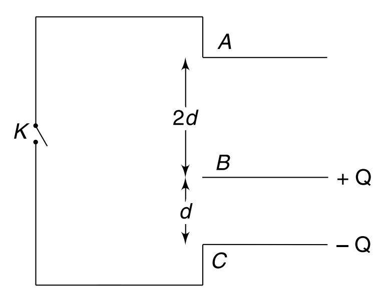



**Q. 18:** In the circuit shown in Figure $E = 12$ V, $C_1 = 4\ \mu F$, $C_2 = 2$ μF, $C_3 = 6$ μF and $C_4 = 3\ \mu F$.

Find the heat produced in the circuit after switch $S$ is shorted.




**Q. 19:** Three identical large metal plates each of area $S$ are at distance $d$ and $2d$ from each other as shown. Metal plate $A$ is uncharged, while metal plates $B$ and $C$ have charges $+Q$ and $-Q$ respectively. Metal plates $A$ and $C$ are connected by a conducting wire through a switch $K$. How much electrostatic energy is lost when the switch is closed?

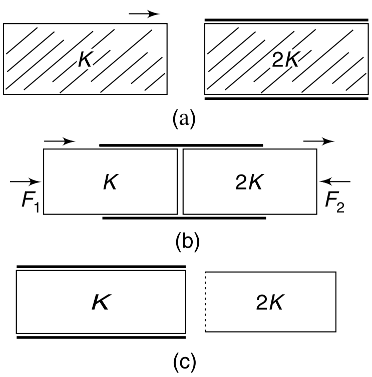



**Q. 20:** An air core parallel plate capacitor has capacitance $C$. It is completely filled with a dielectric slab having dielectric constant $2K$. The capacitor is now connected to a battery of emf $V$. It was planned to replace the dielectric slab of the capacitor while it remains connected to the battery. Another dielectric slab (which fits exactly between the plates) is inserted slowly so as to push out the earlier slab. The new slab has a dielectric constant K [see Figure (a) to (c)]

(a) By energy considerations calculate the mechanical work that must be done against the electric forces in order to complete the process.

(b) Looking at the expression of mechanical work obtained in (a), tell what was the direction of force applied by the external agent - from left to right (as indicated by $F_1$ in Figure) or from right to left (as indicated by $F_2$).

(c) Which dielectric slab experienced higher force of attraction from the capacitor plates during the process?

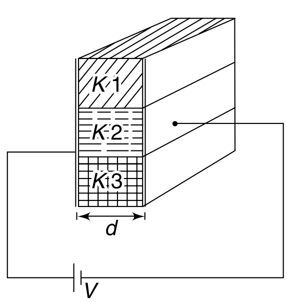



**Q. 21:** A parallel plate capacitor of plate area $A$ and spacing between the plates $d$ is filled with three dielectrics as shown in the Figure. The dielectric constants of the three dielectrics are $K_1 = K$, $K_2 = 2K$, $K_3 = 3K$. The capacitor is connected to a cell of emf V.

(a) Write the ratio of maximum to minimum charge density on the surface of the capacitor plate.

(b) Calculate the surface charge density of bound (induced) charge on the middle dielectric.

(c) If the three dielectrics occupy equal volume between the plates, calculate the capacitance of the capacitor.



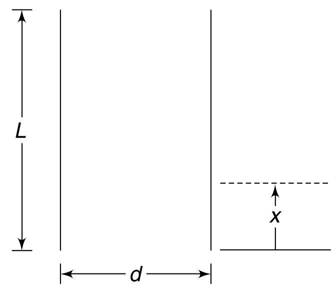

## LEVEL 2

**Q. 22:** A parallel plate capacitor has square plates of side length $L$. Plates are kept vertical at separation $d$ between them. The space between the plates is filled with a dielectric whose dielectric constant ($K$) changes with height ($x$) from the lower edge of the plates as $K = e^{\beta x}$ where $\beta$ is a positive constant. $A$ potential difference of $V$ is applied across the capacitor plates.

(i) Plot the variation of surface charge density ($\sigma$) on the positive plate of the capacitor versus $x$.

(ii) Plot the variation of electric field between the plates as a function of $x$.

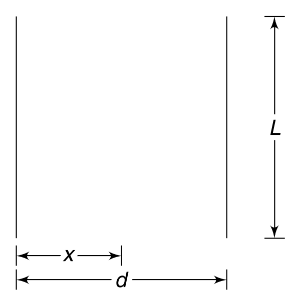
(iii) Calculate the capacitance of the capacitor.




**Q. 23:** A parallel plate capacitor has square plates of side length $L$ kept at a separation $d$. The space between them is filled with a dielectric whose dielectric constant changes as $K = e^{\beta x}$ where $x$ is distance measured from the left plate towards the right plate, and $\beta$ is a positive constant. A potential difference of $V$ volt is applied with left plate positive.

(i) What happens to capacitance of the capacitor if $d$ is increased? What is the smallest possible capacitance that can be obtained by changing $d$?

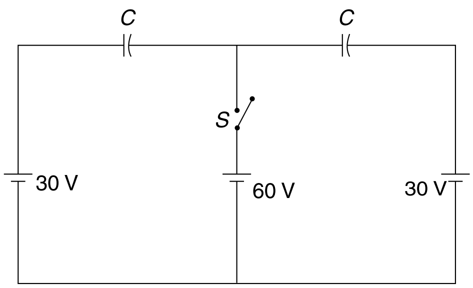
(ii) Write the expression of electric field between the plates as a function of $x$.




**Q. 24:** A parallel plate capacitor is to be constructed which can store $q = 10\ \mu C$ charge at $V = 1000$ volt. The minimum plate area of the capacitor is required to be $A_1$ when space between the plates has air. If a dielectric of constant $K = 3$ is used between the plates, the minimum plate area required to make such a capacitor is $A_2$. The breakdown field for the dielectric is 8 times that of air. Find $\frac{A_1}{A_2}$.

**Q. 25:** The electric field between the plates of a parallel plate capacitor is $E_0$. The space between the plates is filled completely with a dielectric. There are $n$ molecules in unit volume of the dielectric and each molecules is like a dumb – bell of length $L$ with its ends carrying charge $+q$ and $-q$. Assume that all molecular dipoles get aligned along the field between the plates. Find the electric field between the plates after insertion of the dielectric.

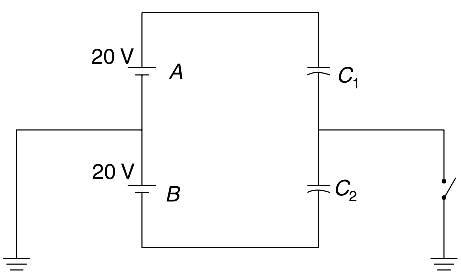
**Q. 26:** Find heat dissipated in the circuit after switch $S$ in closed. $C = 2\ \mu$F.




## LEVEL 2

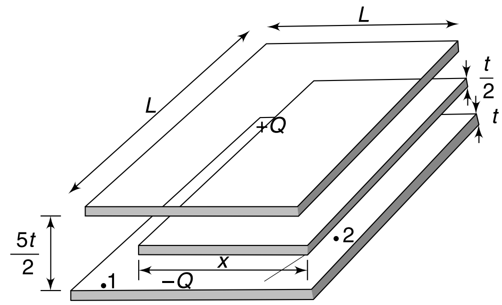
**Q. 27:** In the circuit shown in the Figure calculate the quantity of charge that flows through the switch after it is closed. Give your answer for following two cases-

(a) $C_1 = C_2 = 2\ \mu$F (b) $C_1 = 2\ \mu$F; $C_2 = 1\ \mu$F




**Q. 28:** In a parallel plate capacitor the separation between the two plates is maintained by a dielectric of dielectric constant $K$ and thickness $d$. The dielectric material is not rigid and has a young's modulus of $Y$. Capacitance of the capacitor is $C_0$ if applied potential difference is nearly zero. At higher potentials the attractive force between the plates compresses the dielectric (by a small amount) and reduces the gap between the plates. Change in $K$ can be neglected due to compression in the dielectric. Find the change in capacitance when a battery of $V$ volt is connected across it

**Q. 29:** Two square metal plates have sides of length $L$ and thickness $t$ ($<< L$). They are arranged parallel to each other with their inner faces at a separation of $\frac{5}{2}t$. One of the plates is given a charge $-Q$ and the other one is given a charge $+Q$. A third rectangular metal plate of sides $L$ and $x$, having thickness $\frac{t}{2}$ is inserted between the plates as shown. The third plate is equidistant from the two plates and parallel to them. Neglect edge effects.




(a) Find the charge density on lower plate at points 1 and 2 shown in Figure.

(b) Find potential difference between the upper plate and the middle plate.

(c) Find electric field between the two outer plates in space where the third plate is not present (i.e., at a point above point 1.)

(d) Find the capacitance of the system across two outer plates.

**Q. 30:** A hollow spherical conductor of radius $R$ has a charge $Q$ on it. A small dent on the surface decreases the volume of the spherical conductor by 2%. Assume that the charge density on the surface does not change due to the dent and the electric field in the dent region remains same as other points on the surface.

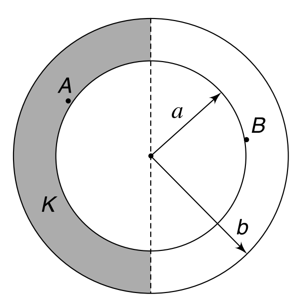
(a) $\Delta E$ is the electrostatic energy stored in the electric field in the shallow dent region and $E$ is the total electrostatic energy of the spherical shell. Find the ratio $\frac{\Delta E}{E}$

(b) Using the ratio obtained in part (a) calculate the percentage change in capacitance of the sphere due to the dent.

**Q. 31:** The space between the conductors of a spherical capacitor is half filled with a dielectric as shown is Figure. The dielectric constant is $K$.

(a) If a charge is given to the capacitor write the ratio of free charge density on the inner sphere at point $A$ and $B$.

(b) Write the ratio of capacitance with dielectric and without dielectric.



**Q. 32:** Two concentric spherical shells have radii $a$ and $b$ $(>a)$. Write the capacitance of the system in following cases.

(a) Positive terminal of the battery is connected to the outer shell and its other terminal and the inner shell are grounded.

(b) Positive terminal of the battery is connected to the inner shell and its negative terminal is grounded.

(c) A terminal of the battery is connected to the inner shell and the other terminal along with the outer shell is grounded.

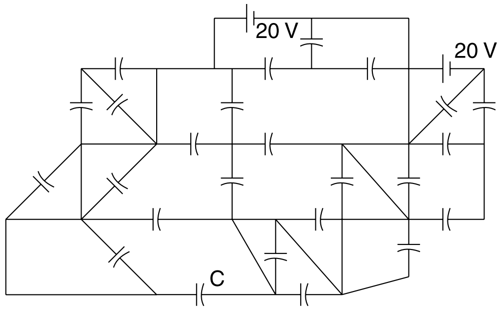
(d) A terminal of the battery is connected to the outer shell and the other terminal is grounded.

**Q. 33:** Two conducting sphere of radii $a$ and $b$ are placed at separation $d$. It is given that $d >> a$ and $d >> b$ so that charge distribution on both the sphere remains spherically symmetric. Assume that a charge $+q$ is given to the sphere of radius $a$ and $-q$ is given to the sphere of radius $b$.

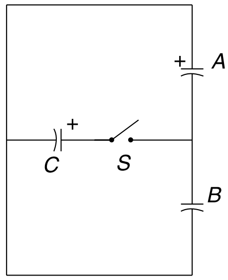
(i) Write the electrostatic energy ($U$) of the system and calculate the capacitance of the system using the expression of $U$.

(ii) Prove that the capacitance of the system is given by $\frac{1}{C} = \left(\frac{1}{4\pi\epsilon_0 a} + \frac{1}{4\pi\epsilon_0 b}\right)$ if $d \to \infty$.

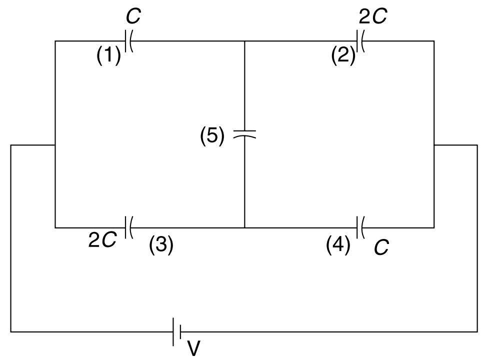
**Q. 34:** All capacitors in the network given below are identical with capacitance of each being 1 $\mu$F. Find the charge on the capacitor marked as $C$.



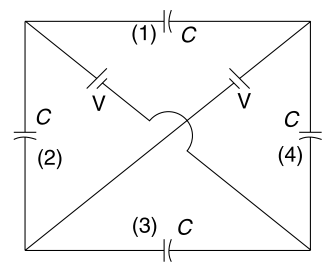

**Q. 35:** In the given circuit it is known that the capacitor $A$ has a capacitance of 2 $\mu$F and carries a charge of 40 $\mu$C. Capacitor $C$ has a capacitance of 6 $\mu$F and carries a charge of 180 $\mu$C. The positive plate of both capacitors has been indicated in the Figure. Capacitance of capacitor $B$ is 3 $\mu$F. Calculate charge on $B$ after the switch $S$ is closed.




**Q. 36:** In the circuit shown in the Figure, find the ratio of potential difference across capacitor 1 and 2. The capacitance values are as indicated in the Figure.



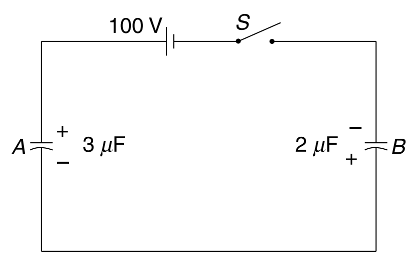

**Q. 37:** In the circuit shown in the figure, find charge on each capacitor.




**Q. 38:** Two capacitors $A$ and $B$ with capacitors 3 $\mu$F and 2 $\mu$F are charged to a potential difference of 100 V and 180 V respectively. One plate of two capacitors are connected as shown. Now switch $S$ is closed so as to connect a cell of 100 V to the free plates of two capacitors.

(a) Find charge on the two capacitors after the switch is closed.

(b) Calculate heat generated in the circuit.



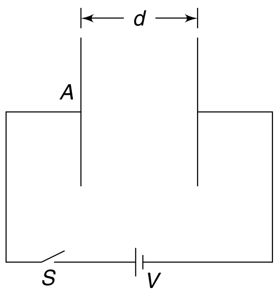

**Q. 39:** A parallel plate air capacitor has plate area $A$ and separation between the plates $d$. Switch $S$ is closed to connect the capacitors to a cell of emf $V$.

(a) Calculate the amount of heat generated in the circuit as the capacitor gets charged.

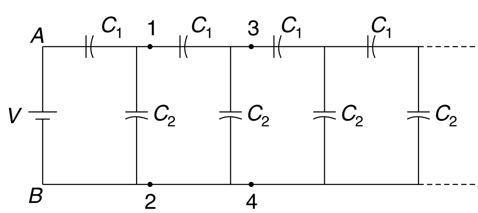
(b) Calculate the force ($F$) that one capacitor plate exerts on the other.

(c) The distance between the plates is slowly reduced to $\frac{d}{2}$. Calculate the work done by the external agent in the process. For your calculation use the basic definition of work as work = force × displacement.

(d) How much energy is dissipated in the circuit as the distance between the plates is reduced from $d$ to $\frac{d}{2}$? Try to give answer without any calculations. Give reasons. Now use work energy theorem to show that your answer is right.




**Q. 40:** The circuit shown in the figure continues to infinity. The potential difference between points 1 and 2 is $\frac{V}{2}$, that between points 3 & 4 is $\frac{V}{4}$ and so on ; i.e., the potential difference becomes $\frac{1}{2}$ after every step of the ladder. Find the ratio $\frac{C_1}{C_2}$

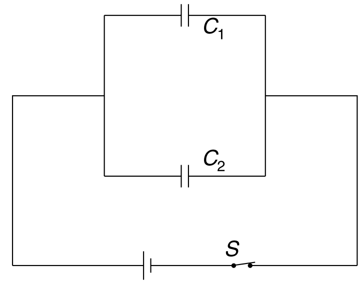



**Q. 41:** The parallel plate capacitors shown in the Figure have capacitance $C_1 = C$ and $C_2 = 2C$. The switch $S$ is opened. Energy stored in the capacitor system is $U_1$. Now the separation between the plates of $C_1$ is slowly reduced to half its original value. Energy stored in the capacitor system now changes to $U_2$.

(a) Which will be larger - $U_1$ or $U_2$? Why?

(b) Calculate work done by the external agent in slowly reducing the distance between the plates of $C_1$.

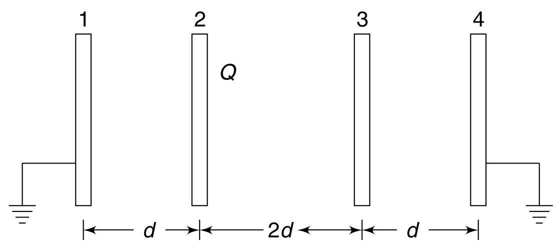
(c) If the plate separation of $C_1$ is reduced to half and simultaneously the separation between plates of $C_2$ is doubled, will the energy stored in the capacitor system increase or decrease? Quantify the change in energy.




**Q. 42:** Four large identical metallic plates are placed as shown in the Figure. Plate 2 is given a charge $Q$. All other plates are neutral. Now plates 1 and 4 are earthed. Area of each plate is $A$.

(a) Find charge appearing on right side of plate 3.

(b) Find potential difference between plates 1 and 2.

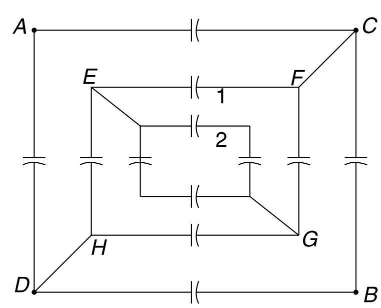



**Q. 43:** In the circuit shown in the figure all the capacitors have capacitance $C$.

(a) Find the charge on capacitors marked as 1 and 2 when a battery of emf $V$ is connected across points $A$ and $B$.

(b) Find the equivalent capacitance across points $C$ and $D$ marked in the Figure.

(c) Find the equivalent capacitance across points $E$ and $G$.

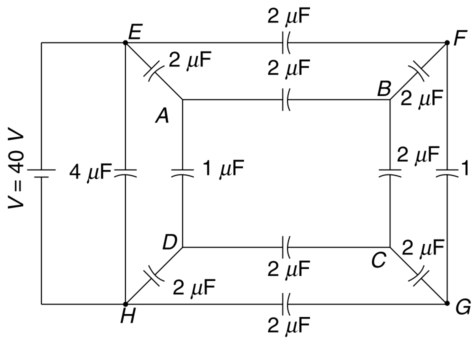



**Q. 44:** There are nine 2 $\mu$F capacitors, two 1 $\mu$F capacitors and one 4 $\mu$F capacitor in the circuit shown in the Figure.

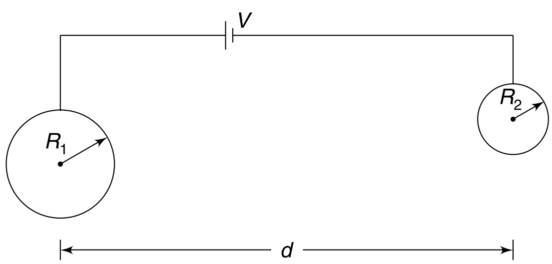
(a) Identify a point in the circuit where potential is same as that of point $A$.

(b) Identify another pair of points which are having equal potential.

(c) Calculate the charge supplied by the cell to the network of capacitors.




**Q. 45:** Two solid conducting spheres of radii $R_1$ and $R_2$ are kept at a distance $d$ (>> $R_1$ and $R_2$) apart. The two spheres are connected by thin conducting wires to the positive and negative terminals of a battery of emf $V$. Find the electrostatic force between the two spheres




## LEVEL 3

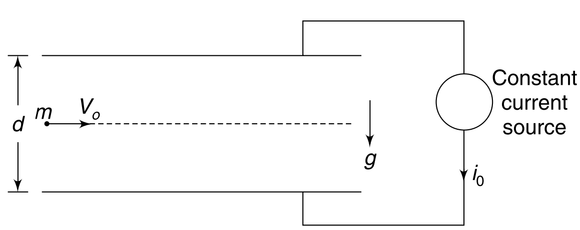
**Q. 46:** Two identical long metal wires having radius $a$ are held parallel to each other at a separation $d$ (>> $a$). Calculate the capacitance of the system per unit length.

**Q. 47:** A particle of mass m and charge $+q$ enters horizontally with speed $V_0$ midway between the horizontal plates of a parallel plate capacitor at time $t = 0$. Separation between the capacitor plates is '$d$' and it starts getting charged, by a constant current source, at time $t = 0$. Plate area of the capacitor is $A$. It was found that the particle just misses (to hit) the lower plate. Assume that the plates are quite long and acceleration due to gravity is $g$.

(a) Give a rough sketch of the path of the particle.

(b) Find the constant current ($i_0$) supplied by the source to the capacitor.

Consider no magnetic force on the charge.



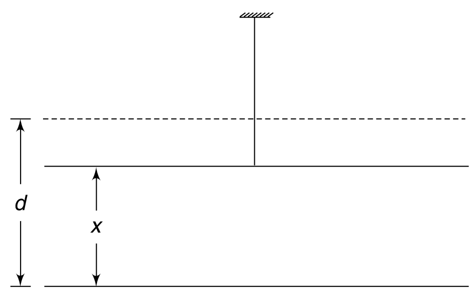

**Q. 48:** Lower plate of a parallel plate capacitor is fixed on a horizontal insulating surface. The upper plate is suspended above it using on elastic cord of force constant $K$. The upper plate has negligible mass and area of each plate is $A$. When there is no charge on the plates the equilibrium separation between them is $d$. When a potential difference $= V$ is applied between the plates the equilibrium separation changes to $x$.

(a) Calculate $V$ as a function of $x$.

(b) Find the value of $x$ for which $V$ is maximum. Calculate the maximum value of $V$ $(= V_{max})$

(c) What will happen if $V > V_{max}$?

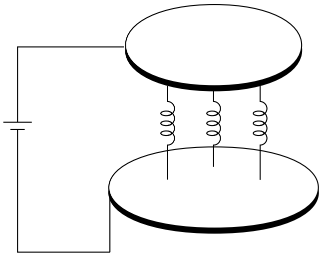
(d) Plot a rough graph showing variation of equilibrium separation ($x$) with $V$.




**Q. 49:** Two identical metal plates with area $A$ and mass m are kept separated by help of three insulating springs as shown in the Figure. The equilibrium separation between the plates is $d_0$ $(<< \sqrt{A})$ and force constant of each spring is $K$. When a constant voltage source having emf $V$ is connected to the plates, the equilibrium separation changes to $d$. Assume that the lower plate is fixed and the upper plate is free to move.

(a) Find $V$ in terms of given parameters.

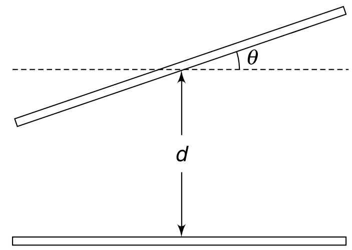
(b) If the upper disc is slightly displaced from its equilibrium position and released, calculate the time period of its oscillation.



**Q. 50:** One plate of a parallel plate capacitor is tilted by a small angle about its central line as shown in the Figure. The tilt angle $\theta$ is small. Both the plates are square in shape with side length $a$ and separation between their centers is $d$. Find the capacitance of the capacitor.

Given: $\ell n(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} + ...$

$\ell n(1 - x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + ...\right)$




## ANSWERS

**1.** $B$ is wrong

**2.** (b) $1.62 \times 10^{-19}$ C

**3.** $\chi E$

**4.** (a) $\frac{Q^2 d}{2\epsilon_0 A}$ (b) $\frac{Q^2}{2\epsilon_0 A}$

**5.** (a) $C = \frac{K\epsilon_0 A}{Kd - (K-1)x}$

(b) $V \frac{K\epsilon_0 A d t_0 (K-1)}{[K d t_0 - (K-1)\, dt]^2}$

**6.** Insert dielectric with constant $k_1$, connect the battery, disconnect the battery and remove the dielectric.

$U_{\max} = \frac{1}{2} K_1^2 V^2 C$

**7.** (a) $-\frac{\epsilon_0 A}{4d} V^2$ (b) Attract. Yes

**8.** $Q = \frac{qRC}{(4\pi\epsilon_0 R + C)}\left(\frac{1}{x} + \frac{1}{y}\right)$

**9.** (a) $-\frac{1}{4} C_0 V_0^2$ (b) $-\frac{1}{2} C_0 V_0^2$

**10.** $\frac{\epsilon_0 A}{4d} V^2$

**11.** $Q = \frac{\epsilon_0 A}{d} V$

**12.** 40 V

**13.** 16.8 $\mu$c

**14.** 12 $\mu$F

**15.** (a) $\frac{16}{3}$ $\mu$F (b) $\frac{16}{3}$ $\mu$F

**16.** $C$

**17.** $\frac{4}{5}$

**18.** 240 $\mu$J

**19.** $\frac{Q^2 d}{6\epsilon_0 S}$

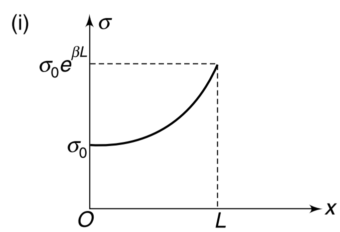
**20.** (a) $\frac{1}{2} KCV^2$

(b) In the direction indicated by $\vec{F}_1$

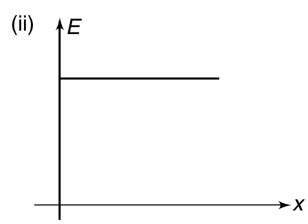
(c) Dielectric having constant $2K$ experienced higher force.

**21.** (a) $\frac{\sigma_{\max}}{\sigma_{\min}} = \frac{3}{1}$ (b) $\sigma_b = \frac{\epsilon_0 V}{d}(2K - 1)$ (c) $C = \frac{2K\epsilon_0 A}{d}$

**22.** (i)




(ii)




(iii) $C = \frac{\epsilon_0 L}{\beta d}[e^{\beta L} - 1]$

**23.** (i) Capacitance decreases, $C_{\min} = \epsilon_0 L^2 \beta$

(ii) $E = \frac{\beta V e^{-\beta x}}{1 - e^{-\beta d}}$

**24.** 24

**25.** $E = E_0 - \frac{qLn}{\epsilon_0}$

**26.** 1.8 mJ

**27.** (a) Zero (b) 20 $\mu$C

**28.** $\Delta C \simeq \frac{C_0 K^2 V^2 \epsilon_0}{2d^2 Y}$

**29.** (a) $\sigma_1 = \frac{4Q}{L(4L + x)}$; $\sigma_2 = \frac{5Q}{L(4L + x)}$

(b) $\frac{5Qt}{\epsilon_0 L(4L + x)}$

(c) $\dfrac{4Q}{\epsilon_0 L(4L + x)}$

(d) $\dfrac{\epsilon_0 L(4L + x)}{10t}$

**30.** (a) $\dfrac{1}{150}$ (b) 0.67%

**31.** (a) $\dfrac{\sigma_A}{\sigma_B} = K$ (b) $\dfrac{1 + K}{2}$

**32.** (a) $4\pi\epsilon_0\left(\dfrac{b^2}{b - a}\right)$ (b) $4\pi\epsilon_0 a$

(c) $4\pi\epsilon_0\left(\dfrac{ab}{b - a}\right)$ (d) $4\pi\epsilon_0 b$

**33.** (i) $U = \dfrac{q^2}{8\pi\epsilon_0}\left(\dfrac{1}{a} + \dfrac{1}{b} - \dfrac{2}{d}\right)$; $C = \dfrac{4\pi\epsilon_0}{\dfrac{1}{a} + \dfrac{1}{b} - \dfrac{2}{d}}$

**34.** 20 $\mu$F

**35.** $\dfrac{240}{11}$ $\mu$C

**36.** $\dfrac{V_1}{V_2} = \dfrac{3}{2}$

**37.** $q_1 = q_3 = 0$; $q_2 = q_4 = CV$

**38.** (a) $q_A = 84$ $\mu$C; $q_B = 144$ $\mu$C

(b) 19.44 mJ

**39.** (a) $\dfrac{1}{2}CV^2$ (b) $\dfrac{\epsilon_0 AV^2}{2d^2}$

(c) $-\dfrac{1}{2}CV^2$ (d) zero

**40.** $\dfrac{C_1}{C_2} = \dfrac{2}{1}$

**41.** (a) $U_1 > U_2$ (b) $W_{ext} = -\dfrac{3}{8}CV^2$

(c) No change.

**42.** (a) $\dfrac{Q}{4}$ (b) $\dfrac{3}{4}\dfrac{Qd}{\epsilon_0 A}$

**43.** (a) Zero on both (b) 2 C

(c) 2 C

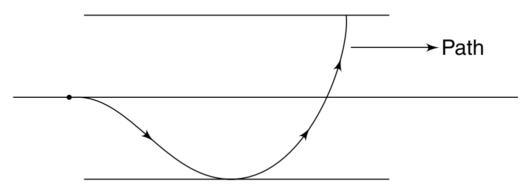
**44.** (a) $F$ (b) $D$ and $G$

(c) 208 $\mu$C

**45.** $\dfrac{1}{K}\left(\dfrac{R_1R_2V}{d(R_1 + R_2)}\right)^2$

**46.** $\dfrac{\pi\epsilon_0}{\ln\left(\dfrac{d}{a}\right)}$

**47.** (a)



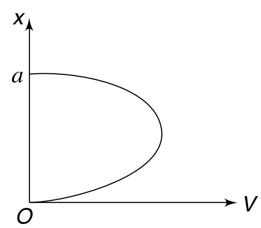

(b) $i_0 = \dfrac{2mgA\epsilon_0}{q}\sqrt{\dfrac{g}{3d}}$

**48.** (a) $V = \sqrt{\dfrac{2K(d - x)\,x^2}{\epsilon_0 A}}$

(b) $V_{max} = \sqrt{\dfrac{8Kd^3}{27\epsilon_0 A}}$ at $x = \dfrac{2d}{3}$

(c) System cannot remain in equilibrium.

(d)

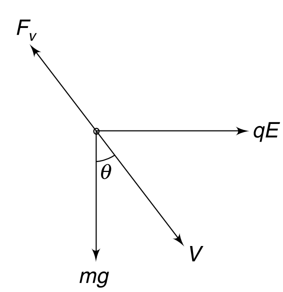



**49.** (a) $V = \sqrt{\dfrac{6Kd^2(d_0 - d)}{\epsilon_0 A}}$ (b) $T = 2\pi\sqrt{\dfrac{m \cdot d}{3K(3d - 2d_0)}}$

**50.** $\dfrac{\epsilon_0 a^2}{d}\left[1 + \dfrac{a^2\theta^2}{12d^2}\right]$

## SOLUTIONS

**2.** Electric field between the plates is $E = \dfrac{V}{d} = \dfrac{2000}{3}\dfrac{\text{V}}{\text{cm}}$.




If a drop has charge $q$, it experiences three force–

(i) Electric force, $qE$

(ii) Weight, $mg$

(iii) Viscous force, $F_v$

$F_v$ is opposite to velocity ($V$). The resultant force perpendicular to the velocity ($V$) must be zero.

$\therefore$ $$mg\sin\theta = qE\cos\theta \qquad \therefore \tan\theta = \frac{qE}{mg}$$

$$\Rightarrow \qquad q = \frac{mg}{E}\tan\theta$$

for $\theta = 45°$

$$q_1 = \frac{mg}{E} \times 1 = \frac{3.3 \times 10^{-15} \times 9.8}{\frac{2000}{300}\ \frac{V}{m}} = 4.85 \times 0^{-19}\ \text{C}$$

For $\theta = 33.7°$

$$q_2 = \frac{mg}{E} \times 0.67 = 4.85 \times 0.67 \times 10^{-19} = 3.25 \times 10^{-19}\ \text{C}$$

For $\theta = 18.4°$

$$q_3 = 4.85 \times 10^{-19} \times 0.333 = 1.62 \times 10^{-19}\ \text{C}$$

Therefore, the oil drops have acquired different charges during spraying and move in different directions. Smallest charge in nature is LCM of $q_1$, $q_2$ and $q_3$.

$\therefore$ Answer is $\simeq 1.62 \times 10^{-19}$ C

**3.** If field between the plates is $E_0$ without any dielectric between them then

$$E = \frac{E_0}{\epsilon_r}$$

But $E = E_0 - E_{\text{in}}$

$$\therefore \qquad E_{\text{in}} = E_0 - E = \epsilon_r E - E = (\epsilon_r - 1)\ E = \chi E$$

**4.** (a) Energy stored $U = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2 d}{2\epsilon_0 A}$

(b) If the required force is $F$, then $Fd = U$

$$Fd = \frac{Q^2 d}{2\epsilon_0 A} \qquad \therefore \quad F = \frac{Q^2}{2\epsilon_0 A}$$

5. (a) Height of liquid at time $t$ is $x = \frac{d}{t_0}t$

We can take the system as two capacitors in series

$$C_1 = \frac{K\epsilon_0 A}{x} \quad \text{and} \quad C_2 = \frac{\epsilon_0 A}{d - x}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{x}{K\epsilon_0 A} + \frac{d-x}{\epsilon_0 A} = \frac{Kd - (K-1)\ x}{K\epsilon_0 A}$$

$$\Rightarrow \qquad C = \frac{K\epsilon_0 A}{Kd - (K-1)\ x}$$

(b)

$$i = \frac{dq}{dt} = V\frac{dC}{dt} = V\frac{K\epsilon_0 A\,(K-1)}{[Kd - (K-1)\ x]^2}\frac{dx}{dt}$$

$$i = V\frac{K\epsilon_0 A\,(K-1)}{\left[Kd - (K-1)\ \frac{dt}{t_0}\right]^2}\frac{d}{t_0} = V\frac{K\epsilon_0 A d t_0\,(K-1)}{[Kdt_0 - (K-1)\ dt]^2}$$

**6.**

$$U = \frac{Q^2}{2C}$$

For maximum $U$, we must have maximum $Q$ and minimum $C$.

$Q$ will be maximum when battery is connected with capacitance at maximum possible value.

$$C_{max} = K_1 C \qquad \text{[when dielectric with constant } K_1 \text{ is inserted]}$$

If battery is connected to this capacitor, charge on it

$$Q_{max} = VC_{max} = K_1 VC$$

Now battery is disconnected and dielectric is removed.

$$\therefore \qquad U_{max} = \frac{Q_{max}^2}{2C} = \frac{K_1^2 V^2 C^2}{2C} = \frac{1}{2} K_1^2 V^2 C$$

**7.** (a) Capacitance $C_0 = \dfrac{\in_0 A}{d}$

Charge $Q = C_0 V$

Energy in capacitor $U_1 = \dfrac{1}{2} C_0 V^2$

After insertion of the metal plate, capacitance becomes

$$C = \frac{\in_0 A}{d - \frac{d}{2}} = \frac{2\in_0 A}{d} = 2C_0$$

$\therefore$ Energy stored in the capacitor becomes

$$U_2 = \frac{Q^2}{2C} = \frac{C_0^2 V^2}{2.2C_0} = \frac{1}{4} C_0 V^2$$

$$\therefore \qquad W = U_2 - U_1$$

$$= -\frac{1}{4} C_0 V^2 = -\frac{\in_0 A}{4d} V^2$$

(b) Since the energy stored in the system has decreased, the electrostatic force must have performed positive work. It means the metal plate being inserted was sucked into the gap between the capacitor plates.

**8.** Let a charge $Q$ be induced on capacitor plate connected to the ball. Induced charge on the ball is $-Q$.

$\therefore$ Potential at the centre of the ball will be

$$V = K\frac{q}{x} + K\frac{q}{y} + K\frac{-Q}{R} = \frac{q}{4\pi\in_0}\left(\frac{1}{x} + \frac{1}{y}\right) - \frac{1}{4\pi\in_0}\frac{Q}{R}$$

This is the potential of the entire ball and this is the potential of the capacitor plate connected to the sphere. The other plate of the capacitor is at zero potential.

$\therefore$ Potential difference across capacitor plates = $V$

$$V = \frac{Q}{C} = \frac{q}{4\pi\in_0}\left(\frac{1}{x} + \frac{1}{y}\right) - \frac{1}{4\pi\in_0}\frac{Q}{R}$$

$$\Rightarrow \qquad Q\left(\frac{1}{C} + \frac{1}{4\pi\in_0 R}\right) = \frac{q}{4\pi\in_0}\left(\frac{1}{x} + \frac{1}{y}\right)$$

$$\Rightarrow \qquad Q = \frac{qRC}{(4\pi\in_0 R + C)}\left(\frac{1}{x} + \frac{1}{y}\right)$$

**9.** (a) $Q_0 = C_0 V_0$

$$U_0 = \frac{1}{2} C_0 V_0^2$$

After the separation is made half new capacitance is

$$C = 2C_0$$

But charge remains fixed.

$$\therefore \qquad U = \frac{Q_0^2}{2(C)} = \frac{C_0^2 V_0^2}{2(2C_0)} = \frac{1}{4} C_0 V_0^2$$

Work done by the external agent

$$W = \Delta U = U - U_0 = -\frac{1}{4} C_0 V_0^2$$

***Note:*** The two plate plates attract each other. The external agent applies force that is opposite to the displacement. Hence, work done is negative.

(b) This time the p.d across the capacitor remains constant.

$$U_0 = \frac{1}{2} C_0 V_0^2$$

$$U = \frac{1}{2} (2C_0)\, V_0^2$$

$$\therefore \qquad \Delta U = \frac{1}{2} C_0 V_0^2$$

Battery supplies an additional charge $\Delta Q = 2C_0V_0 - C_0V_0 = C_0V_0$

$$\therefore \qquad W_{\text{batt}} = \Delta Q V_0 = C_0 V_0^2$$

$$\therefore \qquad W + W_{\text{batt}} = \Delta U$$

$$\therefore \qquad W = \frac{1}{2} C_0 V_0^2 - C_0 V_0^2 = -\frac{1}{2} C_0 V_0^2$$

**10.** There are two capacitors in series.
Equivalent capacitance is

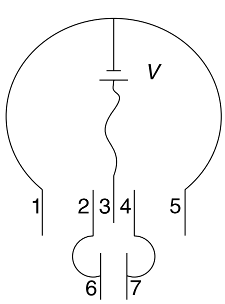
$$C_0 = \frac{1}{2}\left(\frac{\epsilon_0 A/2}{d}\right) = \frac{\epsilon_0 A}{4d}$$

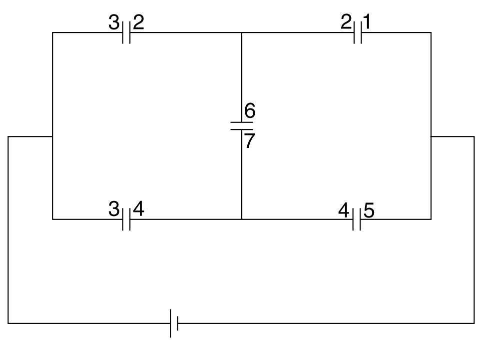
$\therefore$ Charge supplied by the cell

$$Q = C_0 V = \frac{\epsilon_0 A}{4d} V$$

$$\therefore \qquad W_{\text{batt}} = QV = C_0 V^2 = \frac{\epsilon_0 A}{4d} V^2$$

**11.** There are five capacitors each of capacitance $C = \dfrac{\epsilon_0 A}{d}$




The arrangement is a Wheatstone bridge as shown in figure.
It is a balanced bridge and equivalent capacitance is

$$C_0 = C = \frac{\epsilon_0 A}{d}$$

$$\therefore \qquad Q = C_0 V = \frac{\epsilon_0 A}{d} V$$

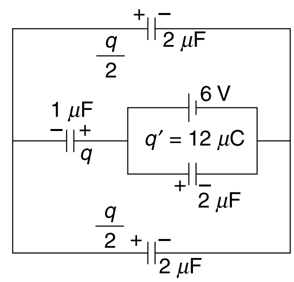
**12.** The diode $D2$ will never conduct as its end $B$ is at higher potential. It is like an infinite resistance. $D1$ will conduct and the two capacitors will get charged. Both will have same charge.

$$\frac{q}{C} + \frac{q}{2C} = 120$$

$$\therefore \quad \frac{3q}{2C} = 120$$

$$\Rightarrow \quad \frac{q}{2C} = 40 \text{ volt}$$

**13.** The potential difference across 1 $\mu$F and 2 $\mu$F (uppermost) capacitor must sum up to 6 V



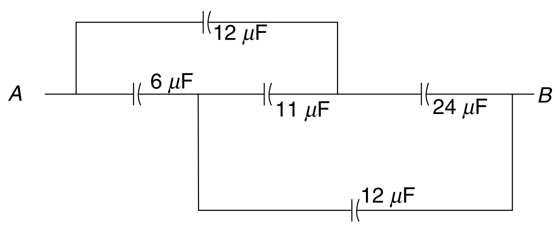

$$\frac{q}{1} + \frac{q/2}{2} = 6 \quad \Rightarrow \quad q = \frac{24}{5}\,\mu\text{C}$$

$\therefore$ Answer is $12 + \frac{24}{5} = 16.8\ \mu$C

**14.** 11 $\mu$F and 13 $\mu$F are in parallel.

6 $\mu$F and 5 $\mu F$ are in parallel.

2 $\mu$F and 4 $\mu$F are in parallel.



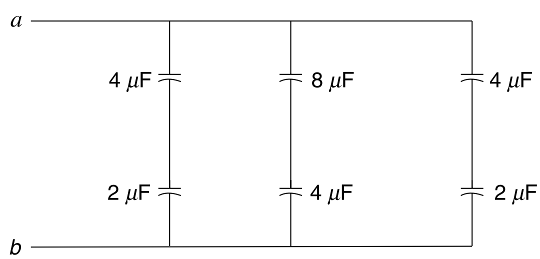

This is a balanced whetstone bridge. 11 $\mu$F capacitor can be removed.

12 $\mu$F and 24 $\mu$F in series = 8 $\mu$F.

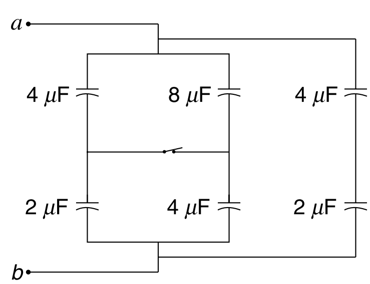
6 $\mu$F and 12 $\mu$F in series = 4 $\mu$F.

Equivalent = 8 + 4 = 12 $\mu$F

15. (a) With switch open the circuit is as shown in the figure. Equivalent capacitance is $C = \frac{16}{3}\ \mu$F




(b) With switch closed the circuit is as shown below.




Equivalent is $C = \frac{16}{3}\ \mu$F

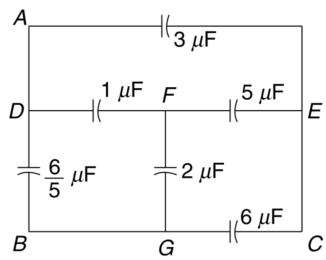
**16.** Hint: Balanced Wheatstone Bridge.

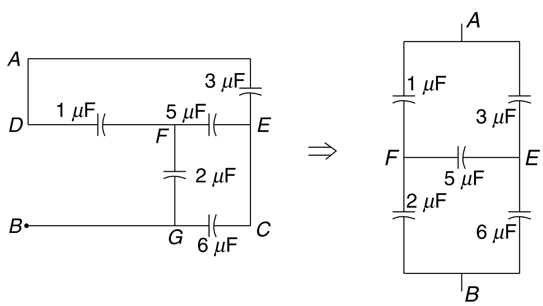
**17.** **Equivalent across *A* & *B***

Remove the $\frac{6}{5}$ $\mu$F capacitor between *B* and *D* and the remaining circuit becomes a balanced Wheatstone bridge. [Note *A* & *D* are same points]

The equivalent of Wheatstone bridge is $\frac{8}{3}$ $\mu$F. Add to this the removed capacitor of $\frac{6}{5}$ $\mu$F in parallel and the equivalent becomes.

$$C_{AB} = \frac{8}{3} + \frac{6}{5} = \frac{58}{15}\ \mu\text{F}$$




**Equivalent across *A* & *C***

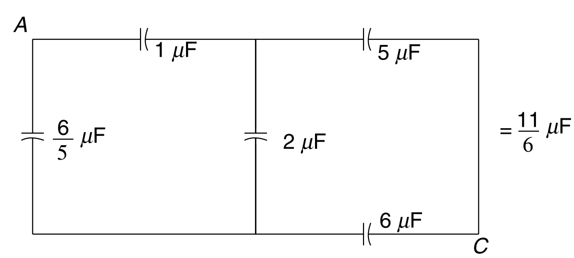
Remove the 3 $\mu$F capacitor between *A* and *E* (*E* and *C* are same points)

Now remaining circuit is a balanced Wheatstone bridge between *A* and *C* [with the 2 $\mu$F capacitor between *F* & *G* being charge less]

The equivalent of Wheatstone bridge is = $\frac{11}{6}$ $\mu$F

Add the removed capacitor and the equivalent becomes

$$C_{AC} = \frac{11}{6} + 3 = \frac{29}{6}\ \mu\text{F}$$




$\therefore$ Required ratio is $\frac{58}{15} \times \frac{6}{29} = \frac{4}{5}$

**18.** **With *S* open**

Net emf = 0

$\therefore$ Charge on all capacitors = 0

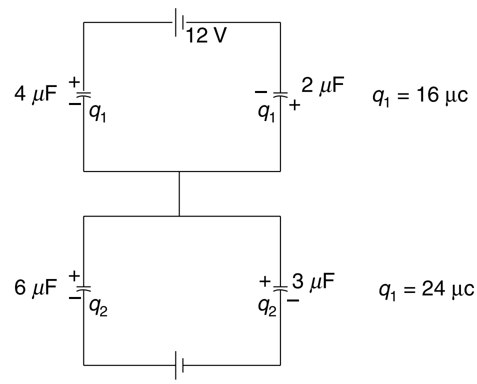
**With *S* closed**

Energy stored in capacitors

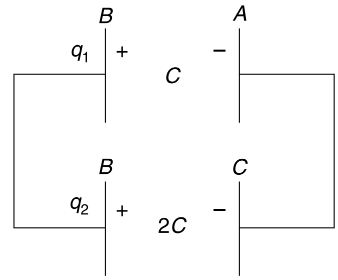
$$U = 240\ \mu\text{J}$$

Work done by cells = 480 $\mu$J

Heat produced = 240 $\mu$J



**19.** Capacitance between $A$ and $B$ is $C = \dfrac{\epsilon_0 S}{2d}$.




Capacitance between $B$ and $C$ is $\dfrac{\epsilon_0 S}{d} = 2C$.

Initial energy stored $U_i = \dfrac{Q^2}{2(2C)} = \dfrac{Q^2}{4C}$.

After the switch is closed, we have two capacitors in parallel, as shown in Figure

$$q_1 + q_2 = Q \quad \text{...(i)}$$

and $$\frac{q_1}{C} = \frac{q_2}{2C}$$

$\Rightarrow$ $$q_1 = \frac{q_2}{2} \quad \text{...(ii)}$$

Solving (i) and (ii) we get $q_1 = \dfrac{Q}{3};\ q_2 = \dfrac{2Q}{3}$

Final energy stored in the system

$$U_f = \frac{1}{2}\frac{q_1^2}{C} + \frac{1}{2}\frac{q_2^2}{2C} = \frac{Q^2}{6C}$$

$\therefore$ Loss in energy $$\Delta U = U_i - U_f$$

$$= \frac{Q^2}{4C} - \frac{Q^2}{6C} = \frac{Q^2}{12C}$$

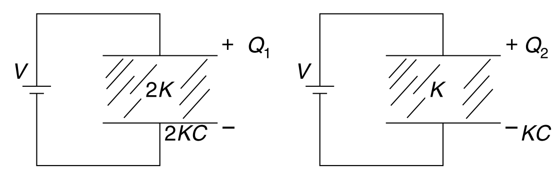
$$= \frac{Q^2}{12\dfrac{\epsilon_0 S}{2d}} = \frac{Q^2 \cdot d}{6\epsilon_0 \cdot S}$$

**20.** (a) Initially, capacitance $= 2KC$

Charge on capacitor $Q_1 = 2KC \cdot V$

Finally, capacitance $= KC$

Charge on capacitor $= Q_2 = KCV$




$\therefore$ Charge $\Delta q = 2\,KCV - KCV = KCV$ was pushed back from the capacitor plates into the battery. Battery gained energy.

Work done by battery $W_{\text{batt}} = -\Delta qV = -KCV^2$.

Change in electrostatic energy of the capacitor is

$$\Delta U = U_f - U_i = \frac{1}{2}(KC)V^2 - \frac{1}{2}(2\,KC)V^2 = -\frac{1}{2}KCV^2$$

Negative sign shows that the capacitor lost energy. Overall, the capacitor lost $\frac{1}{2}KCV^2$ energy and the battery gained $KCV^2$ energy ! The gap $\frac{1}{2}KCV^2$ was fulfilled by the work done by the external agent.

Work done by the external agent $= \frac{1}{2}KCV^2$.

(b) Because work done by agent is positive, the force applied must be in the direction of displacement.

$\therefore$ Force was in the direction $F_1$ indicated in the Figure.

(c) The dielectric slab with constant 2 $K$ experienced higher attractive force. To overcome it the external agent had to apply a rightward force.

**21.** (a) Electric field between plates is same inside all the dielectrics.

$$E = \frac{V}{d}$$

If $\sigma$ = free charge density on capacitor plate then $E = \dfrac{\sigma}{K\in_0}$

$$\therefore \qquad \frac{\sigma}{K\in_0} = \frac{V}{d} \Rightarrow \sigma = \frac{\in_0 V}{d} K$$

$\therefore$ $\sigma$ is maximum when $K_3 = 3K$ and it is minimum when $K_1 = K$

$$\therefore \qquad \sigma_{\max} = \frac{3K\in_0 V}{d}$$

$$\therefore \qquad \sigma_{\min} = \frac{K\in_0 V}{d}$$

$$\therefore \qquad \frac{\sigma_{\max}}{\sigma_{\min}} = \frac{3}{1}$$

(b) $$\sigma_b = \sigma\left[1 - \frac{1}{2K}\right] = \frac{2\in_0 VK}{d}\left[1 - \frac{1}{2K}\right] = \frac{\in_0 V}{d}(2K - 1)$$

(c) We can consider the capacitor as three parallel plate capacitors placed in parallel.

$$C = C_1 + C_2 + C_3 = \frac{K\in_0 A/3}{d} + \frac{2K\in_0 A/3}{d} + \frac{3K\in_0 A/3}{d}$$

$$\Rightarrow \qquad C = \frac{2K\in_0 A}{d}$$

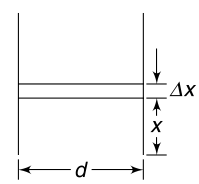
**Level 2**

**22.** (i) Consider a plate width of $dx$ at a height $x$ (see Figure)

Capacitance of this segment is

$$\Delta C = \frac{\in_0 K \Delta A}{d} = \frac{\in_0 e^{\beta x} L \Delta x}{d} \qquad \text{...(i)}$$

Potential difference across this small capacitor is $V$.



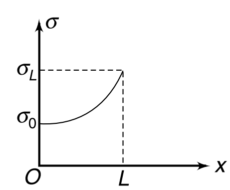

$$\therefore \qquad \Delta q = V\Delta C$$

$$\therefore \qquad \sigma = \frac{\Delta q}{\Delta A} = \frac{V\Delta C}{L\Delta x} = \frac{V\in_0 e^{\beta x}}{d}$$

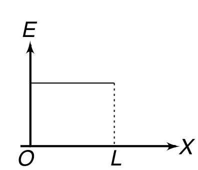
Graph is as shown

$$\sigma_0 = \frac{V\in_0}{d}$$

And $$\sigma_L = \frac{V\in_0}{d} e^{\beta L} = \sigma_0 e^{\beta L}$$




(ii) Electric field remains constant at $E = \dfrac{V}{d}$




(iii) Writing equation (i) for infinitesimal width $dx$

$$dC = \frac{\in_0 L}{d} e^{\beta x} dx$$

There will be infinite number of such capacitance all in parallel.

$$\therefore \qquad C = \int dC = \frac{\in_0 L}{d} \int_0^L e^{\beta x} \cdot dx = \frac{\in_0 L}{\beta d}[e^{\beta L} - 1]$$

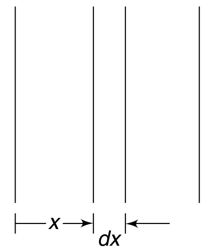
**23.** (i) We can calculate the capacitance by considering large number of capacitors assembled in series. One such capacitor having plate separation $dx$ is shown in the Figure

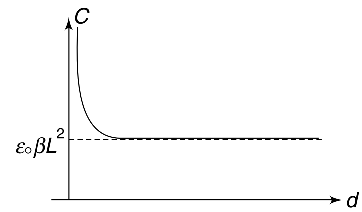
$$dC = \frac{\in_0 K L^2}{dx} = \frac{\in_0 e^{\beta x} L^2}{dx}$$

Capacitance will be given by

$$\frac{1}{C} = \int \frac{1}{dC} = \frac{1}{\in_0 L^2} \int_0^d \frac{dx}{e^{\beta x}}$$

$$= -\frac{1}{\in_0 \beta L^2} [e^{-\beta x}]_0^d = \frac{1}{\in_0 \beta L^2} [1 - e^{-\beta d}]$$




$\therefore$ $$C = \frac{\in_0 \beta L^2}{1 - e^{-\beta d}}$$

Graph is as shown.

When $d \to \infty$; $C \to \in_0 \beta L^2$

(ii) $$Q = CV = \frac{\in_0 \beta L^2 V}{1 - e^{-\beta d}}$$

$$\sigma = \frac{Q}{L^2} = \frac{\in_0 \beta V}{1 - e^{-\beta d}}$$

$\therefore$ $$E = \frac{\sigma}{\in_0 K} = \frac{\beta V}{e^{\beta x}[1 - e^{-\beta d}]} = \frac{\beta V e^{-\beta x}}{[1 - e^{-\beta d}]}$$

**24.** $$C = \frac{K \in_0 A}{d} = \text{a constant}$$

For $A$ to be minimum, $d$ must be minimum. The separation between the plates is limited by the breakdown strength of the dielectric.

For air capacitor $$\frac{V}{d_{\min}} = E_{\text{air}}$$ [$E_{\text{air}}$ = Breakdown field for air]

$\therefore$ $$d_{\min} = \frac{V}{E_{\text{air}}}$$

Now $$\frac{\in_0 A_{\min}}{d_{\min}} = C$$

$\Rightarrow$ $$A_{\min} = \frac{C}{\in_0} \frac{V}{E_{\text{air}}}$$

$\therefore$ $$A_1 = \frac{CV}{\in_0 E_{\text{air}}}$$

With dielectric, similar calculation gives

$$A_2 = \frac{CV}{K \in_0 E_{\text{dielec}}}$$

$\therefore$ $$\frac{A_1}{A_2} = \frac{K E_{\text{dielec}}}{E_{\text{air}}} = 3 \times 8 = 24$$

**25.** Dipole moment per unit volume (after perfect alignment) in the dielectric = $qLn$. Net dipole moment induced in dielectric slab is

$$= (qLn)\ (A \cdot d)$$

$\therefore$ Charge induced on walls of the dielectric

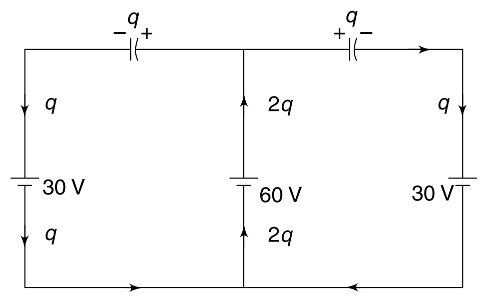
$$Q_{\text{in}} = \pm \frac{qLnAd}{d} = \pm qLnA$$

$\therefore$ Field due to induced charge

$$E_{in} = \frac{Q_{in}}{\epsilon_0 A} = \frac{qLn}{\epsilon_0}$$

$\therefore$
$$E = E_0 - E_{in} = E_0 - \frac{qLn}{\epsilon_0}$$

**26.** With switch $S$ open the potential difference across the group of capacitors is zero. There is no charge on both of them.




With $S$ closed, potential difference across both of them is 30 V .Charge on each of them is

$$q = (2\ \mu F)\ (30\ V) = 60\ \mu C$$

Polarity is as shown. The charge flow in various path has also been shown.

Energy supplied by 60 V cell = $60 \times 2q = 60 \times 120\ \mu J$

Energy absorbed by each of 30 V cell = $30 \times q = 30 \times 60\ \mu J$

Energy stored in each capacitor = $\frac{1}{2}\ CV^2 = \frac{1}{2} \times 2 \times 30^2\ \mu J$

$\therefore$ Heat dissipated $= 60 \times 120 - 2 \times 30 \times 60 - 2 \times 30^2$

$= 60\ [120 - 60 - 30]$

$= 60 \times 30 = 180\ \mu J = 1.8\ mJ$

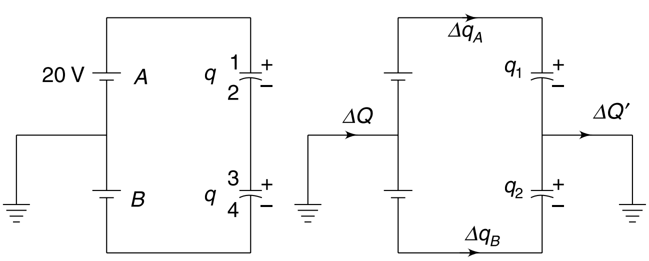
**27.** (a) When $C_1$ and $C_2$ both are equal, potential difference across each of them is 20 V. The point between the two capacitors is at zero potential and connecting it to earth by closing the switch will make no difference to the circuit. There will be no charge flow.

(b) Potential difference across two capacitors is

$$V_1 = 40 \times \frac{C_2}{C_1 + C_2} = \frac{40}{3}\ V$$

And
$$V_2 = 40 \times \frac{2}{2 + 1} = \frac{80}{3}\ V$$

Charge on them is
$$q = \frac{80}{3}\ \mu C$$




After the switch is closed, potential difference across both capacitors becomes 20 V. Charges on them are $q_1 = 40\ \mu C$ and $q_2 = 20\ \mu C$

Looking at change in charge on plate 1, it is easy to conclude that a charge

$$\Delta q_A = 40 - \frac{80}{3} = \frac{40}{3}\ \mu C \text{ flows through cell } A.$$

Similarly looking at charge on plate 4, we can conclude that charge flow $\Delta q_B$ (see Figure) is

$$\Delta q_B = \frac{80}{3} - 20 = \frac{20}{3}\ \mu C$$

$\therefore$
$$\Delta Q = \Delta q_A + \Delta q_B = \frac{60}{3} = 20\ \mu C$$

Sum of charge on plate 2 and 3 before closing the switch was zero. After closing the switch the charge on these two plates is $= -40 + 20 = -20\ \mu C$

$\therefore$ $\Delta Q'$ (in direction shown) $= 20\ \mu C$

28. (a) $$C_0 = \frac{K\in_0 A}{d}$$

When connected to a $V$ volt cell, the plate separation reduces by $x$.

$$C = \frac{K\in_0 A}{d-x} \qquad ...(1)$$

$$\therefore \quad \Delta C = C - C_0 = \frac{K\in_0 A}{d}\left[\frac{1}{1-\frac{x}{d}} - 1\right] = C_0\left[\left(1-\frac{x}{d}\right)^{-1} - 1\right]$$

$$= C_0\left[1 + \frac{x}{d} - 1\right] \qquad [\because \ x << d]$$

$$\therefore \quad \Delta C = C_0\frac{x}{d} \qquad ...(2)$$

Charge on capacitor plates $Q \simeq C_0 V$

Force between plates $F = \frac{Q^2}{2A\in_0} = \frac{C_0^2V^2}{2A\in_0}$

Stress on dielectric $\frac{F}{A} = \frac{C_0^2V^2}{2A^2\in_0}$

$\therefore$ Strain is $\frac{x}{d} = \frac{\text{stress}}{Y} = \frac{C_0^2V^2}{2A^2\in_0 Y}$

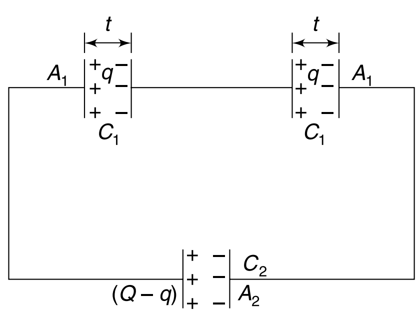
Putting $C_0 = \frac{K\in_0 A}{d}$ we get

$$\frac{x}{d} = \frac{K^2\in_0 V^2}{2d^2Y}$$

$$\therefore \quad \Delta C \simeq \frac{C_0K^2V^2\in_0}{2d^2Y}$$

**29.** The given system is equivalent to the one shown in the Figure. The distance between the plates of $C_2$ is $\frac{5}{2}t$

Plate area of capacitors represented by $C_1$ and $C_2$ are $A_1 = xL$ and $A_2 = L(L-x)$




$$C_1 = \frac{\in_0 A_1}{t} = \frac{\in_0 Lx}{t}$$

$$C_2 = \frac{\in_0 A_2}{\frac{5t}{2}} = \frac{2\in_0 \cdot L(L-x)}{5t}$$

Let charge on capacitors having capacitance $C_1$ be $q$ and that on capacitor of capacitance $C_2$ be $Q - q$.

The equivalent of two $C_1$ in series is $\frac{C_1}{2}$.

$\therefore$ Potential difference across $\frac{C_1}{2}$ = p.d across $C_2$.

$$\therefore \quad \frac{q}{C_{1/2}} = \frac{Q-q}{C_2}$$

$$\frac{2q}{\frac{\in_0 Lx}{t}} = \frac{(Q-q)}{\frac{2\in_0 L(L-x)}{5t}}$$

$$\Rightarrow \quad \frac{2q}{x} = \frac{5(Q-q)}{2(L-x)}$$

$$\Rightarrow \quad 4q(L-x) = 5(Q-q)\,x$$

$$\therefore \quad q[4L-4x+5x] = 5Qx$$

$$\therefore \quad q = \frac{5Qx}{4L+x}$$

And $$Q-q = \frac{4Q(L-x)}{4L+x}$$

(a) $$\sigma_1 = \frac{Q-q}{L(L-x)} = \frac{4Q}{L(4L+x)}$$

$$\sigma_2 = \frac{q}{L\cdot x} = \frac{5Q}{L(4L+x)}$$

(b) Required p.d = p.d. across $C_1 = \dfrac{q}{C_1}$

$$= \frac{5Qx}{4L+x}\cdot\frac{t}{\epsilon_0 Lx} = \frac{5Qt}{\epsilon_0 L(4L+x)}$$

(c) $$E = \frac{\sigma_1}{\epsilon_0} = \frac{4Q}{\epsilon_0 L(4L+x)}$$

(d) $$C = C_2 + \frac{C_1}{2} = \frac{2\epsilon_0 L(L-x)}{5t} + \frac{\epsilon_0 Lx}{2t}$$

$$= \frac{\epsilon_0 L(4L+x)}{10t}$$

**30.** (a) Energy stored in the electrostatic field produced by the spherical conductor.

$$E = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{8\pi\epsilon_0 R}$$

The electric field on the surface $= \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{R^2}$

$\therefore$ Energy stored in the electric field in the dent region is

$$\Delta E = \frac{1}{2}\epsilon_0\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\right)^2 \Delta V = \frac{Q^2\Delta V}{32\pi^2\epsilon_0 R^4} \qquad [\Delta V = \text{Volume of dent}]$$

$$\therefore \quad \frac{\Delta E}{E} = \frac{\Delta V}{4\pi R^3} = \frac{\Delta V}{3\left(\frac{4}{3}\pi R^3\right)} = \frac{\Delta V}{3V} \qquad [V = \text{Volume of the sphere}]$$

$$\therefore \quad \frac{\Delta E}{E} = \frac{1}{3}\times 0{\cdot}02 = \frac{1}{150}$$

(b) $$E = \frac{Q^2}{2C}$$

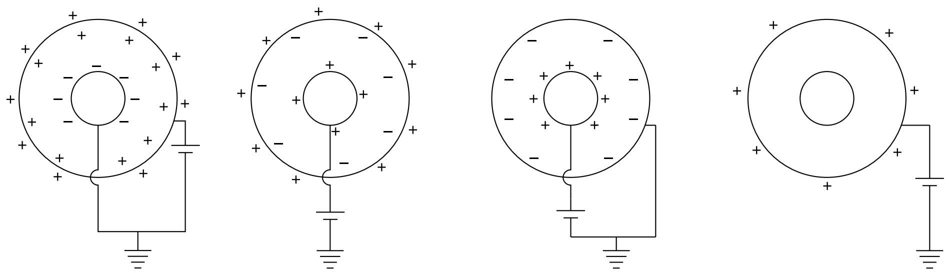
$$\therefore \quad \Delta E = -\frac{Q^2}{2C^2}\,\Delta C \qquad [\text{–sign tells that if } \Delta E \text{ is + ve them } \Delta C \text{ is –ve}]$$

Fractional Decrease in capacitance

$$\frac{\Delta C}{C} = \Delta E \cdot \frac{2C}{Q^2} = \frac{\Delta E}{E} = \frac{1}{150}$$

$$\therefore \quad \frac{\Delta C}{C} \times 100 = \frac{1}{150} \times 100 = \frac{2}{3} = 0.67\%$$

**32.** The charge distribution on the surfaces of shells in four cases is as shown in the Figures.



(a) $$C = C_1 + C_2$$

$$= 4\pi\epsilon_0\left(\frac{ab}{b-a}\right) + 4\pi\epsilon_0 b = 4\pi\epsilon_0\left(\frac{b^2}{b-a}\right)$$

(b) In this case $C_1$ and $C_2$ are in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \quad \frac{1}{C} = \frac{b-a}{ab}\frac{1}{4\pi\epsilon_0} + \frac{1}{4\pi\epsilon_0 b}$$

$$= \frac{1}{4\pi\epsilon_0 b}\left(\frac{b-a}{a} + 1\right) = \frac{1}{4\pi\epsilon_0 b}\left(\frac{b}{a}\right) = \frac{1}{4\pi\epsilon_0 a}$$

$$\therefore \quad C = 4\pi\epsilon_0 a$$

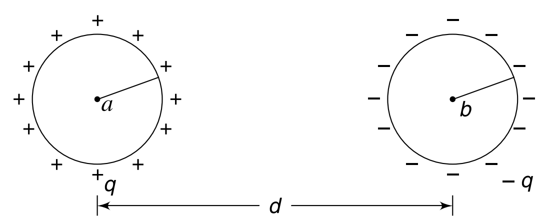
(c) $$C = C_1 = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$$

(d) $$C = C_2 = 4\pi\epsilon_0 b$$

**33.** (i) The electrostatic self energy of two conductors are

$$U_1 = \frac{q^2}{8\pi\epsilon_0 a} \quad \text{and} \quad U_2 = \frac{q^2}{8\pi\epsilon_0 b}$$

The interaction energy of two conductors can be written assuming them to be point charges.




$$U_{12} = \frac{q(-q)}{4\pi\epsilon_0 d} = \frac{-q^2}{4\pi\epsilon_0 d}$$

$\therefore$ Total energy of the system is

$$U = U_1 + U_2 + U_{12}$$

$$= \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right)$$

Now $$\frac{q^2}{2C} = U \quad \Rightarrow \quad C = \frac{4\pi\epsilon_0}{\frac{1}{a} + \frac{1}{b} - \frac{2}{d}}$$

(ii) If $d \to \infty$

$$C = \frac{4\pi\epsilon_0}{\frac{1}{a} + \frac{1}{b}}$$

$$\therefore \quad \frac{1}{C} = \frac{1}{4\pi\epsilon_0 a} + \frac{1}{4\pi\epsilon_0 b}$$

**34.** Look at the given figure carefully to find that two plates of the capacitor marked as $C$ are directly connected to the terminals of the battery.

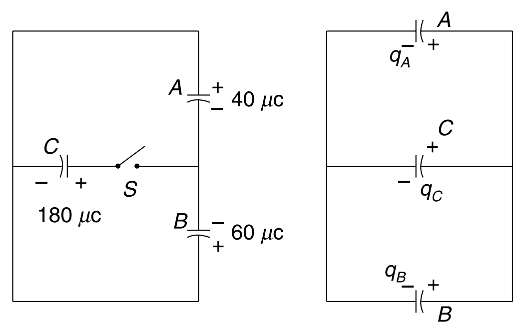
Potential difference across the marked capacitor = 20 V.

$\therefore$ Charge on it = 20 $\mu$C

**35.** Initially $B$ must also have some charge. Potential difference across plates of $A$ is

$$V_A = \frac{40\ \mu\text{C}}{2\ \mu\text{F}} = 20\ \text{V}$$

$$\therefore \quad V_B = 20\ \text{V}$$




Charge on $B$ is (3 $\mu$F) (20 V) = 60 $\mu$C

Charge on $C$ is 180 $\mu$F. Polarity of charges is as shown in the Figure.

After the switch is closed, the three capacitors will be in parallel and they have a total charge of 180 – 40 – 60 = 80 $\mu$C to be shared among them.

The charge will be shared in the ratio of capacitance

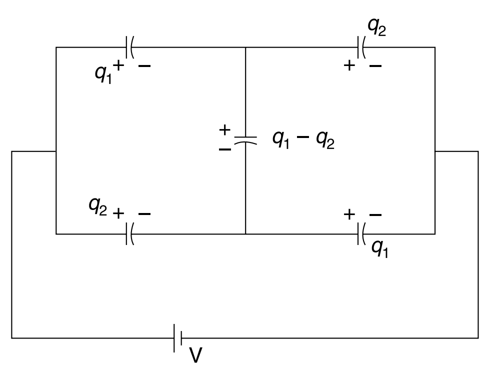
$$\therefore \quad q_A : q_B : q_C = 2:3:6$$

$$\therefore \quad q_A = \frac{2 \times 80}{2+3+6}\ \mu\text{C} = \frac{160}{11}\ \mu\text{C}$$

$$q_B = \frac{3 \times 80}{2+3+6} = \frac{240}{11}\ \mu\text{C}$$

$$q_C = \frac{6 \times 80}{2+3+6} = \frac{480}{11}\ \mu\text{F}$$

**36.** Due to symmetry charge on capacitor 1 and 4 will be same. Similarly, charge on 2 and 3 will be same. Charge distribution is shown in the Figure.




***Note:*** that charge on capacitor 5 has been written using the fact that sum of charges on three plates connected together must be zero.

Using Kirchhoff's voltage law in loop having capacitors 1, 5 and 3 –

$$\frac{q_1}{C} + \frac{q_1 - q_2}{C} = \frac{q_2}{2C}$$

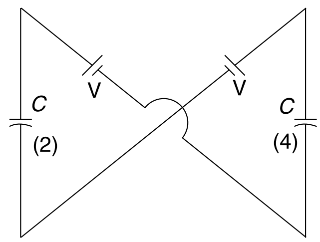
$$\Rightarrow \quad 2q_1 = \frac{3q_2}{2} \quad \Rightarrow \quad \frac{q_1}{q_2} = \frac{3}{4}$$

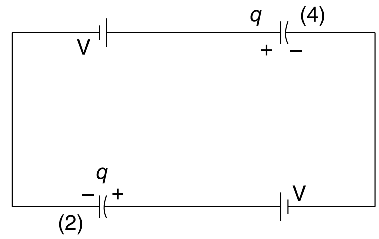
Potential difference across capacitor 1 and 2 will be in ratio

$$\frac{V_1}{V_2} = \frac{q_1/C_1}{q_2/C_2} = \frac{q_1}{q_2}\frac{C_2}{C_1} = \frac{3}{4}\cdot\frac{2C}{C} = \frac{3}{2}$$

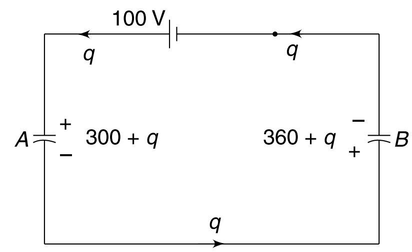
**37.** The terminals of capacitor 1 are in identical position and cannot have different potential. Similarly, there cannot be a potential difference across the terminals of capacitor 3. We can remove capacitors 1 and 3 from the circuit. The effective circuit is as shown in Figure.

Charge on (2) and (4) is $q = CV$




**38.** Charges on two capacitors are




$$q_A = 100 \times 3 = 300\ \mu\text{C}$$

And $$q_B = 180 \times 2 = 360\ \mu\text{C}.$$

After the switch $S$ is closed let charge $q$ flow in the circuit as shown in the Figure.

Using Kirchhoff's voltage law we can write–

$$\frac{360 + q}{2} + \frac{300 + q}{3} = 100$$

$$\Rightarrow \quad 360 \times 3 + 3q + 600 + 2q = 600$$

$$\Rightarrow \quad q = -216\ \mu\text{C}$$

$\therefore$ Final charge on the two capacitors is

$$q'_A = 300 + q = 84\ \mu\text{C}$$

$$q'_B = 360 + q = 144\ \mu\text{C}$$

Change in electrostatic energy of the capacitor system is

$$\Delta U = \left(\frac{(q'_A)^2}{2 \times 3} + \frac{(q'_B)^2}{2 \times 2}\right) - \left(\frac{q_A^2}{2 \times 3} + \frac{q_B^2}{2 \times 2}\right)$$

$$= \left(\frac{84^2}{6} + \frac{144^2}{4}\right) - \left(\frac{300^2}{6} + \frac{360^2}{4}\right)$$

$$= 1176 + 5184 - 15000 - 32400$$

$$= -41040\ \mu\text{J} = -41.04\ \text{mJ}$$

$\therefore$ Capacitor system has lost 41.04 mJ energy.

Energy gained by the cell $\quad U_{\text{cell}} = 100 \times 216\ \mu\text{J} = 21.6\ \text{mJ}$

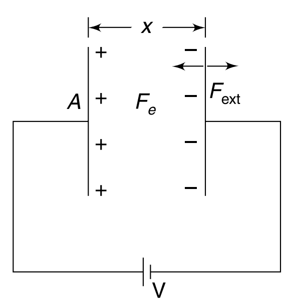
$\therefore \quad$ Heat dissipated $= 41.04 - 21.60 = 19.44$ mJ

**39.** (a) Charge on capacitor is $\quad Q = CV \quad \left[C = \dfrac{\epsilon_0 A}{d}\right]$

$$W_{\text{battery}} = Q \cdot V = CV^2$$

Energy stored in the capacitor $\quad U = \dfrac{1}{2} CV^2$

$\therefore \quad$ Heat generated $= W_{\text{batt}} - U = \dfrac{1}{2} CV^2$




(b) Let the distance between the plates be $x$ at some intermediate stage.

$$C = \frac{\epsilon_0 A}{x}$$

$$q = CV = \frac{\epsilon_0 AV}{x}$$

Field due to charge on one plate is $E = \dfrac{q}{2A\epsilon_0}$

$\therefore$ Force between plates $\quad F = Eq = \dfrac{q^2}{2A\in_0} = \dfrac{\in_0 AV^2}{2x^2}$

(c) Assuming that the positive plate is fixed and the negative plate moves slowly towards it, work done by external agent in displacement $dx$ is

$$dW_{ext} = -|F_{ext}||dx| = -|F_e||dx| = \frac{\in_0 AV^2}{2}\frac{dx}{x^2}$$

$\therefore$ Work done in reducing the distance from $d$ to $\dfrac{d}{2}$ is

$$W_{ext} = +\frac{\in_0 AV^2}{2}\int_d^{d/2}\frac{dx}{x^2} = -\frac{\in_0 AV^2}{2d} = -\frac{1}{2}CV^2$$

(e) Energy dissipated must be zero as the plates are moved slowly. Charge flow from battery to the capacitor plates is very slow.

$$W_{ext} = -\frac{1}{2}CV^2$$

Charge in $PE$ stored in the capacitor

$$\Delta U = \frac{1}{2}2CV^2 - \frac{1}{2}CV^2 = \frac{1}{2}CV^2$$

[Capacitance doubled when $d$ was halved]

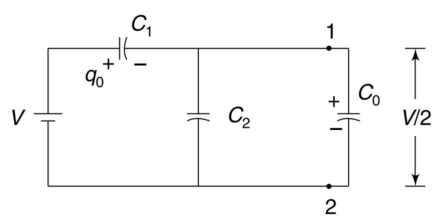
$$W_{batt} = [(2CV) - CV]\ V = CV^2$$

$\therefore \quad W_{batt} + W_{ext} = \Delta U + \text{Heat generated}$

$\therefore$ Heat generated = 0

**40.** Let the equivalent capacitance be $C_0$. Even if we remove one ladder, the equivalent will remain $C_0$.

The potential drop across $C_1$ in figure shown is $\dfrac{V}{2}$.




$\therefore \quad \dfrac{q_0}{C_1} = \dfrac{V}{2} \Rightarrow q_0 = \dfrac{C_1 V}{2}$ ...(1)

Also $\quad q_0 = C_0 V$ ...(2)

$\therefore \quad \dfrac{C_1}{2} = C_0 \Rightarrow C_1 = 2C_0$ ...(3)

From the given circuit

$$C_0 = \frac{(C_0 + C_2)\ C_1}{C_0 + C_2 + C_1}$$

$\Rightarrow \quad C_0^2 + C_0C_2 + C_0C_1 = C_0C_1 + C_1C_2$

$\Rightarrow \quad C_0^2 + C_2C_0 - C_1C_2 = 0$

$$C_0 = \frac{-C_2 \pm \sqrt{C_2^2 + 4C_1C_2}}{2}$$

Only positive sign is acceptable.

$$\therefore \quad C_0 = \frac{-C_2 + \sqrt{C_2^2 + 4C_1C_2}}{2}$$

$\therefore \quad \dfrac{C_1}{2} + \dfrac{C_2}{2} = \dfrac{\sqrt{C_2^2 + 4C_1C_2}}{2} \qquad \left[\because \dfrac{C_1}{2} = C_0\right]$

$$(C_1 + C_2)^2 = C_2^2 + 4C_1C_2$$

$$C_1^2 = 2C_1C_2 \quad \Rightarrow \quad C_1 = 2C_2$$

**41.** (a) $U_1$ is larger. The external agent does a negative work as the capacitor plates attract each other. Therefore the energy of the system must decrease.

(b) $$U_1 = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{3C}{2}V^2$$

Charge on capacitors are $\quad Q_1 = C_1V = CV$ and $Q_2 = C_2V = 2CV$

After $S$ is opened the total charge on connected plates of the two capacitors will remain $\pm(Q_1 + Q_2)$.

After separation between plates is halved, the capacitance $C_1$ becomes $2C$.

$\therefore$ Final charge on both capacitors will be

$$q = \frac{Q_1 + Q_2}{2} = \frac{3CV}{2}$$

Potential difference across both will be

$$V' = \frac{q}{2C} = \frac{3V}{4}$$

$\therefore$ $$U_2 = \frac{1}{2}(2C)\left(\frac{3\text{ V}}{4}\right)^2 + \frac{1}{2}(2C)\left(\frac{3\text{ V}}{4}\right)^2 = \frac{9}{8}CV^2.$$

$\therefore$ Loss in energy $$U_1 - U_2 = \left(\frac{3}{2} - \frac{9}{8}\right)CV^2 = \frac{3}{8}CV^2$$

$\therefore$ $$W_{\text{ext}} = -\frac{3}{8}CV^2$$

(c) In this case $C_1$ becomes $2C$ and $C_2$ becomes $C$.

Final common potential difference will be

$$V' = \frac{Q_1 + Q_2}{2C + C} = \frac{CV + 2CV}{3C} = V.$$

This is expected since the situation remains practically same as original, with capacitance getting exchanged.

$\therefore$ Final energy $\quad U_3 = U_1$

$\therefore$ No work is done by the external agent.

***Note:*** In a slow process no heat will be dissipated.

**42.** The opposite faces must have equal and opposite charge. The outer surface of plate 1 & 4 will have no charge. Let charge on right face of plate 2 be $q$. Charges on other faces are as shown.

$$E_1 = \frac{Q - q}{\epsilon_0 A};\ E_2 = \frac{q}{\epsilon_0 A};\ E_3 = \frac{q}{\epsilon_0 A}$$

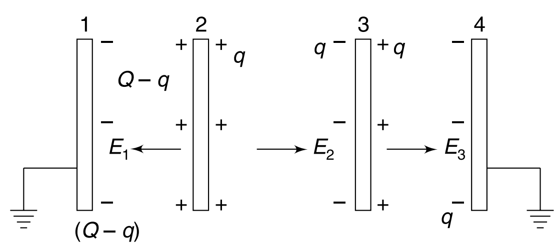
$$V_1 + E_1d - E_2(2d) - E_3d = V_4$$

But $$V_1 = V_4 = 0$$

$\Rightarrow$ $$E_1 - 2E_2 - E_3 = 0$$

$\Rightarrow$ $$Q - q - 2q - q = 0$$

$\Rightarrow$ $$q = \frac{Q}{4}$$




Potential difference between plate 1 and 2 is

$$V = E_1d = \frac{3Qd}{4\epsilon_0 A}$$

**43.** (a) When a battery is connected across points $A$ and $B$ the circuit is a balanced Wheatstone bridge, with the inner network connected to $C$ and $D$ receiving no charge. Hence charge on 1 and 2 is zero.

(b) When a battery is connected across $C$ and $D$, the innermost loop will have no charge because the 4 capacitors in loop $EFGH$ and the innermost loop (acting as fifth capacitor of bridge) forms a balanced Wheatstone bridge.

The innermost loop can be removed. It leaves us with 4 parallel branches between $C$ and $D$ each having two capacitors in series.

$\therefore$ Equivalent capacitance $= 4 \cdot \frac{C}{2} = 2C$

(d) In this case the innermost loop (having capacitance $C_1 = C$) is a parallel path across $EG$. The loop $EFGH$ forms a balanced Wheatstone bridge with the outer loop acting as fifth capacitance between $F$ and $H$.

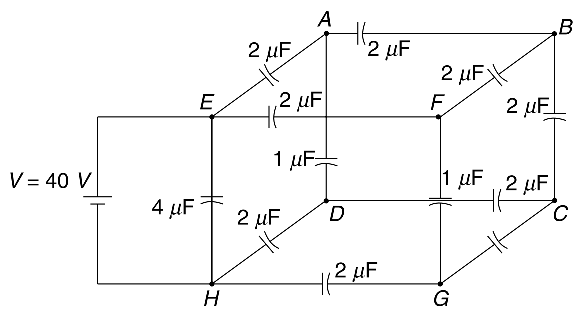
Equivalent of the bridge is $C_2 = C$. This is connected in parallel to $C_1$

$$\therefore \quad C_{eq} = C_1 + C_2 = 2C$$

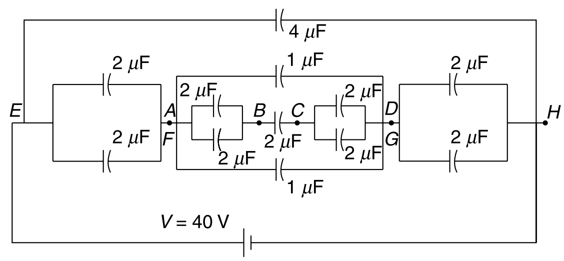
**44.** Convince yourself that the network obtained by placing 12 capacitors on the edges of a cube (see Figure) is same as the given circuit. Yes, I am asking you to convert a two dimensional circuit into a three dimensional one!

The symmetry becomes obvious.

Point $A$ and $F$ cannot have different potentials and similarly, $D$ and $G$ are equipotential.




Points $A$ and $F$ can be joined together and points $D$ and $G$ can be put together. We can redraw the circuit as shown below.




Equivalent capacitance can be shown to be

$$C_{eq} = \frac{26}{5}\ \mu\text{F}$$

$$\therefore \quad Q = \frac{26}{5} \times 40 = 208\ \mu\text{C}$$

**45.** let's first calculate the capacitance of the system.

Assuming that the two spheres have charge $Q$ and $-Q$, we can write their potentials as

$$V_1 = \frac{KQ}{R_1} \quad \text{and} \quad V_2 = \frac{K(-Q)}{R_2}$$

Note that charge on one sphere does not have any significant effect on the potential of other since $d >> R_1$ and $R_2$.

$\therefore$ Potential difference between the two spheres

....

$$V = V_1 - V_2 = \frac{KQ}{R_1} + \frac{KQ}{R_2}$$

$\therefore$ Capacitance $C = \frac{Q}{V} = \frac{R_1 R_2}{K(R_1 + R_2)}$

Charge on the spheres $Q = CV = \frac{R_1 R_2 V}{K(R_1 + R_2)}$

$\therefore$ Force between the spheres

$$F = K\frac{Q^2}{d^2} = \frac{K}{d^2}\left[\frac{R_1 R_2 V}{K(R_1 + R_2)}\right]^2$$

$$= \frac{R_1^2 R_2^2 V^2}{Kd^2 (R_1 + R_2)^2}$$

**46.** The electric field outside a long wire having linear charge density $\lambda$ is

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \text{ where } r = \text{distance from axis}$$

Electric potential at a point can be written as

$$V = -\int E dr + C \qquad [C = \text{a constant}]$$

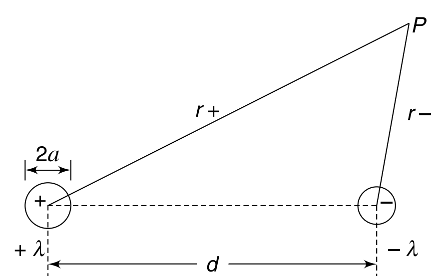
$$= -\frac{\lambda}{2\pi\epsilon_0}\ell n r + \frac{\lambda}{2\pi\epsilon_0}\ell n r_0 \qquad \left[\text{put } C = \frac{\lambda}{2\pi\epsilon_0}\ell n r_0\right]$$

$$\therefore \quad V = \frac{\lambda}{2\pi\epsilon_0}\ell n\frac{r_0}{r}$$

Potential at any point $P$ is presence of both the wires – one having charge $+\lambda\ Cm^{-1}$ and the other having charge $-\lambda\ Cm^{-1}$ is

$$V = \frac{\lambda}{2\pi\epsilon_0}\ell n\left(\frac{r_0}{r_+}\right) - \frac{\lambda}{2\pi\epsilon_0}\ell n\left(\frac{r_0}{r_-}\right)$$

$$\therefore \quad V = \frac{\lambda}{2\pi\epsilon_0}\ell n\left(\frac{r_-}{r_+}\right)$$




Potential on the surface of the left wire is

$$V_1 = \frac{\lambda}{2\pi\epsilon_0}\ell n\left(\frac{d}{a}\right)$$

Potential on the surface of the right wire is

$$V_2 = \frac{\lambda}{2\pi\epsilon_0}\ell n\left(\frac{a}{d}\right)$$

$\therefore$ Potential difference $V = V_1 - V_2 = \frac{\lambda}{\pi\epsilon_0}\ell n\left(\frac{d}{a}\right)$

$\therefore$ Capacitance per unit length is

$$C = \frac{\lambda}{V} = \frac{\pi\epsilon_0}{\ell n\left(\frac{d}{a}\right)}$$

**47.** In time '$t$' the capacitor acquires a charge $Q$ given by $Q = i_0 t$

Electric field between the plates is $E = \frac{Q}{A\epsilon_0} = \frac{i_0 t}{A\epsilon_0}$

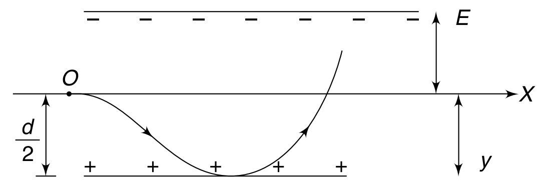
Electric force on the charge $$F_e = qE = \frac{qi_0 t}{A\epsilon_0}$$

Force on the particle in $y$ direction is $F_y = mg - \frac{qi_0 t}{A\epsilon_0}$

Acceleration in $y$ direction

$$a_y = g - \frac{qi_0 t}{mA\epsilon_0} = g - kt \qquad \left[\text{where } k = \frac{qi_0}{mA\epsilon_0}\right]$$

The particle initially experiences a downward acceleration. After some time when $kt > g$, its acceleration changes direction. Hence, path of the particle is as shown below.




For $y$ component of motion of the particle

$$\frac{dv_y}{dt} = g - kt$$

$$\therefore \quad \int_0^{V_y} dv_y = g\int_0^t dt - k\int_0^t t\,dt$$

$$v_y = gt - \frac{1}{2}kt^2 \qquad \text{...(1)}$$

$$v_y = 0 \quad \text{when} \quad t_0 = \frac{2g}{k} \qquad \text{...(2)}$$

At time $t_0$, $y$ – co-ordinate of the particle is $\frac{d}{2}$,

$$v_y = gt - \frac{1}{2}kt^2$$

$$\Rightarrow \quad \frac{dy}{dt} = gt - \frac{1}{2}kt^2$$

$$\Rightarrow \quad \int_0^{\frac{d}{2}} dy = g\int_0^{t_0} t\,dt - \frac{1}{2}k\int_0^{t_0} t^2\,dt$$

$$\therefore \quad \frac{d}{2} = g\frac{t_0^2}{2} - \frac{kt_0^3}{6}$$

$$\Rightarrow \quad \frac{d}{2} = g\frac{2g^2}{k^2} - \frac{k}{6}\frac{8g^3}{k^3} \qquad \left[\because \ t_0 = \frac{2g}{k}\right]$$

$$\Rightarrow \quad \frac{d}{2} = \frac{2}{3}\frac{g^3}{k^2}$$

$$\therefore \quad k = \sqrt{\frac{4}{3}\frac{g^3}{d}}$$

$$\Rightarrow \quad \frac{qi_o}{mA\epsilon_0} = \sqrt{\frac{4}{3}\frac{g^3}{d}} \quad \Rightarrow \quad i_o = \frac{mA\epsilon_0}{q}\sqrt{\frac{4}{3}\frac{g^3}{d}}$$

**48.** (a) Capacitance when separation between the plates is $x$ is given as

$$C = \frac{\epsilon_0 A}{x}$$

Change on plates when applied potential difference is $V$ is

$$Q = CV = \frac{\epsilon_0 AV}{x}$$

Force between the plates

$$F = \frac{1}{2} EQ = \frac{Q}{2\epsilon_0 A} \cdot Q = \frac{Q^2}{2\epsilon_0 A}$$

$\Rightarrow$ $$F = \frac{1}{2} \frac{\epsilon_0 A}{x^2} V^2$$

For equilibrium

$$K(d - x) = \frac{1}{2} \frac{\epsilon_0 A}{x^2} V^2$$

$\Rightarrow$ $$V^2 = \frac{2K(d-x)\, x^2}{\epsilon_0 A}$$

$$V = \sqrt{\frac{2K(d-x)\, x^2}{\epsilon_0 A}}$$

(b) $V$ is maximum when $(d - x)\, x^2$ is maximum. It means when $\frac{d}{dx}[(d - x)\, x^2] = 0$

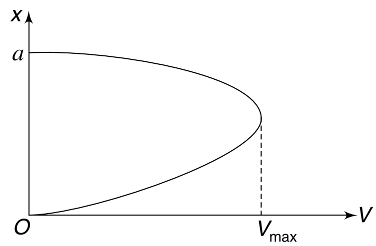
$\Rightarrow$ $$2xd - 3x^2 = 0$$

$\Rightarrow$ $$x = \frac{2d}{3}$$

$\therefore$ $$V_{max} = \sqrt{\frac{2k\left(d - \frac{2d}{3}\right)\left(\frac{2d}{3}\right)^2}{\epsilon_0 A}} = \sqrt{\frac{8Kd^3}{27\epsilon_0 A}}$$

(c) If $V > V_{max}$, the system cannot remain in equilibrium.

(d)




**49.** (a) With no voltage source

$$mg = F_{spring} \quad \text{...(1)}$$

With voltage source connected, let the electrostatic force between the two plates in equilibrium be $F_0$. The rise in spring force balances this $F_0$.

$$F_0 = 3K(d_0 - d) \quad \text{...(2)}$$

Electric force between plates, $$F_0 = Q \cdot \left(\frac{\sigma}{2\epsilon_0}\right)$$

$$= \frac{Q^2}{2\epsilon_0 A} = \frac{C^2 V^2}{2\epsilon_0 A} = \frac{\epsilon_0 A V^2}{2d^2}$$

$\therefore$ $$\frac{\epsilon_0 A V^2}{2d^2} = 3K(d_0 - d) \quad \text{...(1)}$$

$\therefore$ $$V = \sqrt{\frac{6Kd^2(d_0 - d)}{\epsilon_0 A}}$$

(c) If plate moves by a small distance $\Delta x$ the electrostatic force changes by

$$\Delta F_e = \frac{\epsilon_0 AV^2}{2(d+\Delta x)^2} - \frac{\epsilon_0 AV^2}{2d^2} = \frac{\epsilon_0 AV^2}{2d^2}\cdot\left[\left(1+\frac{\Delta x}{d}\right)^{-2} - 1\right]$$

$$= \frac{\epsilon_0 AV^2}{2d^2}\left(-2\frac{\Delta x}{d}\right) = -\frac{\epsilon_0 AV^2}{d^3}\cdot\Delta x$$

Substituting for $V$ we get

$$|\Delta F_e| = \frac{6K(d_0 - d)}{d}\Delta x$$

The spring force changes by

$$|\Delta F_s| = 3K\,\Delta x$$

The two forces $\Delta F_e$ & $\Delta F_s$ have opposite directions. While approaching the spring pushes the plates away and the electric force causes them to attract more strongly.

$$\therefore \quad m\frac{d^2x}{dt^2} = -3K\Delta x + \frac{6K(d_0-d)}{d}\Delta x$$

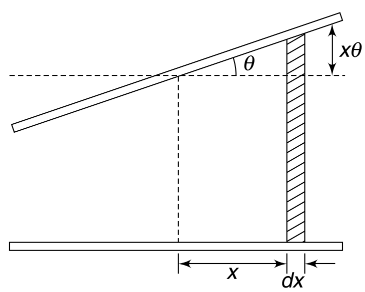
$$\frac{d^2x}{dt^2} = -\frac{3K}{m}\left[\frac{3d-2d_0}{d}\right]\Delta x$$

$$\therefore \quad \omega = \sqrt{\frac{3K}{m}\frac{(3d-2d_0)}{d}}$$

$$T = 2\pi\sqrt{\frac{m\cdot d}{3K(3d-2d_0)}}$$

50. Consider a strip of width dx in the two plates as shown in the Figure

Capacitance of two parallel strips (nearly parallel !) is




$$dC = \frac{\epsilon_0 a\,dx}{d + x\theta}$$

Capacitance of the capacitor can be obtained by summing up the capacitance of all such small capacitor.

$$\therefore \quad C = \int dC = \epsilon_0 a \int_{-\frac{a}{2}}^{+\frac{a}{2}} \frac{dx}{d+\theta x}$$

$$\therefore \quad C = \frac{\epsilon_0 a}{\theta}\left[\ell n(d+\theta x)\right]_{-a/2}^{a/2}$$

$$= \frac{\epsilon_0 a}{\theta}\left[\ell n\left(d + \frac{a\theta}{2}\right) - \ell n\left(d - \frac{a\theta}{2}\right)\right]$$

$$= \frac{\epsilon_0 a}{\theta}\left[\ell n d + \ell n\left(1+\frac{a\theta}{2d}\right) - \ell n d - \ell n\left(1-\frac{a\theta}{2d}\right)\right]$$

$$= \frac{\epsilon_0 a}{\theta}\left[\ell n\left(1+\frac{a\theta}{2d}\right) - \ell n\left(1-\frac{a\theta}{2d}\right)\right]$$

$$= \frac{\epsilon_0 a}{\theta}\left[\frac{a\theta}{2d} - \frac{1}{2}\left(\frac{a\theta}{2d}\right)^2 + \frac{1}{3}\left(\frac{a\theta}{2d}\right)^3 + \ldots\right]$$

$$+ \frac{\epsilon_0 a}{\theta}\left[\frac{a\theta}{2d} + \frac{1}{2}\left(\frac{a\theta}{2d}\right)^2 + \frac{1}{3}\left(\frac{a\theta}{2d}\right)^3 + \ldots\right]$$

$$\simeq \frac{\epsilon_0 a}{\theta}\left[\frac{a\theta}{d} + \frac{2}{3}\left(\frac{a\theta}{2d}\right)^3\right]$$

$$= \frac{\epsilon_0 a}{\theta}\left[\frac{a\theta}{d} + \frac{1}{12}\,\frac{a^3\theta^3}{d^3}\right]$$

$$= \frac{\epsilon_0 a^2}{d}\left[1 + \frac{a^2\theta^2}{12d^2}\right]$$