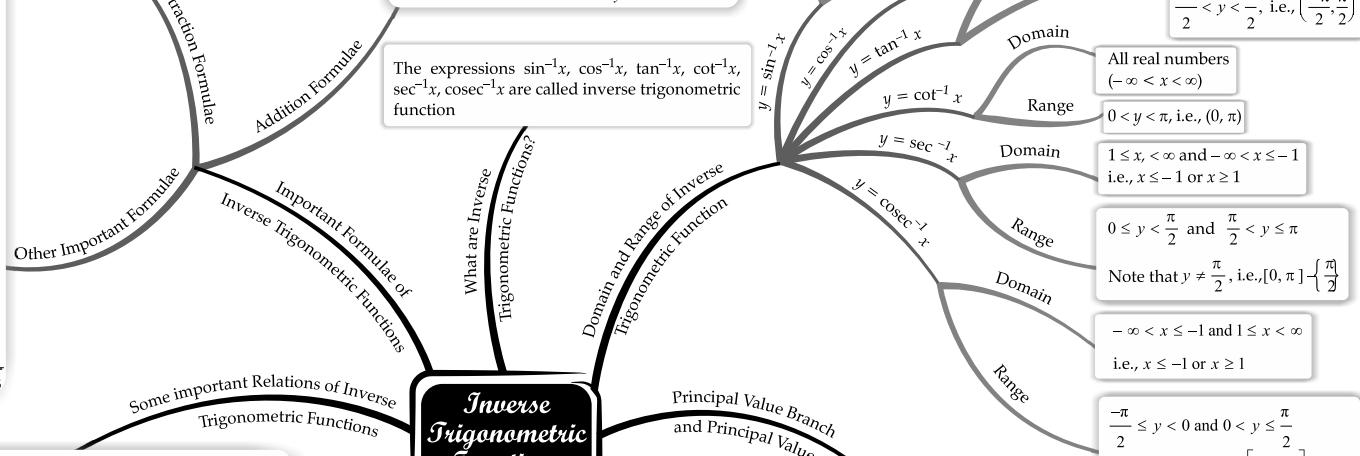


(i)  $\sin^{-1}(-x) = -\sin^{-1}x, x \in [-1, 1]$   
(ii)  $\cos^{-1}(-x) = \pi - \cos^{-1}x, x \in [-1, 1]$   
(iii)  $\tan^{-1}(-x) = -\tan^{-1}x, x \in \mathbb{R}$   
(iv)  $\cot^{-1}(-x) = \pi - \cot^{-1}x, x \in \mathbb{R}$   
(v)  $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x, x \in \mathbb{R} - (-1, 1)$   
(vi)  $\sec^{-1}(-x) = \pi - \sec^{-1}x, x \in \mathbb{R} - (-1, 1)$   
(vii)  $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, x \in [-1, 1]$   
(viii)  $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}, x \in \mathbb{R} - (-1, 1)$   
(ix)  $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, x \in \mathbb{R}$   
(x)  $2\sin^{-1}x = \sin^{-1}[2x\sqrt{1-x^2}], -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$   
(xi)  $2\cos^{-1}x = \cos^{-1}[2x^2-1], 0 \leq x \leq 1$   
 $= 2\pi - \cos^{-1}(2x^2-1), -1 \leq x \leq 0$   
 $= \begin{cases} \sin^{-1}\frac{2x}{1+x^2}, & x \in [-1, 1] \\ \cos^{-1}\frac{1-x^2}{1+x^2}, & x \geq 0 \end{cases}$   
(xii)  $2\tan^{-1}x = \begin{cases} \tan^{-1}\frac{2x}{1-x^2}, & -1 < x < 1 \\ \tan^{-1}\frac{-2x}{1-x^2}, & -1 < x < 1 \end{cases}$   
(xiii)  $3\sin^{-1}x = \sin^{-1}[3x-4x^3], -\frac{1}{2} \leq x \leq \frac{1}{2}$   
(xiv)  $3\cos^{-1}x = \cos^{-1}[4x^3-3x], -\frac{1}{2} \leq x \leq 1$   
(xv)  $3\tan^{-1}x = \tan^{-1}\left(\frac{3x-x^3}{1-x^2}\right), \text{ if } -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

(i)  $\sin^{-1}x - \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} - y\sqrt{1-x^2}],$   
if  $-1 \leq x, y \leq 1 \text{ and } x^2 + y^2 \leq 1$   
(ii)  $\cos^{-1}x - \cos^{-1}y = \cos^{-1}[xy + \sqrt{1-x^2}\sqrt{1-y^2}],$   
if  $-1 \leq x, y \leq 1 \text{ and } x \leq y$   
(iii)  $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}, \text{ if } xy > -1$

(i)  $\sin^{-1}x + \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} + y\sqrt{1-x^2}],$   
if  $-1 \leq x, y \leq 1 \text{ and } x^2 + y^2 \leq 1$   
(ii)  $\cos^{-1}x + \cos^{-1}y = \cos^{-1}[xy - \sqrt{1-x^2}\sqrt{1-y^2}],$   
if  $-1 \leq x, y \leq 1 \text{ and } x+y \geq 0$   
(iii)  $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}, \text{ if } xy < 1$

The expressions  $\sin^{-1}x, \cos^{-1}x, \tan^{-1}x, \cot^{-1}x, \sec^{-1}x, \operatorname{cosec}^{-1}x$  are called inverse trigonometric function



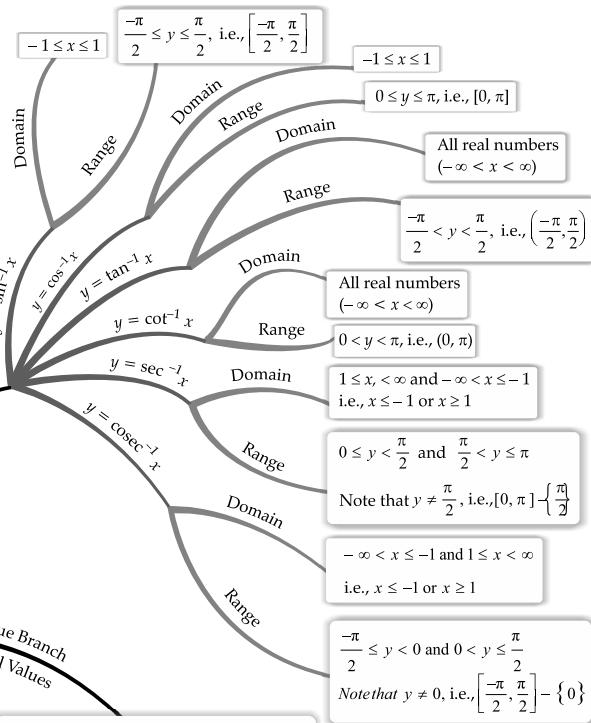
(i)  $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \sec^{-1}\frac{1}{\sqrt{1-x^2}} = \operatorname{cosec}^{-1}\frac{1}{x}$   
(ii)  $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}} = \sec^{-1}\frac{1}{x} = \operatorname{cosec}^{-1}\frac{1}{\sqrt{1-x^2}}$   
(iii)  $\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \cot^{-1}\frac{1}{x} = \operatorname{cosec}^{-1}\frac{\sqrt{1+x^2}}{x} = \sec^{-1}\sqrt{1+x^2}$

•  $\sin^{-1}(\sin x) = x, \text{ for all } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$   
•  $\sin(\sin^{-1}x) = x, \text{ for all } x \in [-1, 1]$

•  $\cos^{-1}(\cos x) = x, \text{ for all } x \in [0, \pi]$   
•  $\cos(\cos^{-1}x) = x, \text{ for all } x \in [-1, 1]$

•  $\tan^{-1}(\tan x) = x, \text{ for all } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$   
•  $\tan(\tan^{-1}x) = x, \text{ for all } x \in \mathbb{R}$

•  $\cot^{-1}(\cot x) = x, \text{ for all } x \in (0, \pi)$   
•  $\cot(\cot^{-1}x) = x, \text{ for all } x \in \mathbb{R}$



The range of an inverse trigonometric function is the principle value branch and those values which lie in the principal value branch is called the principal value of inverse trigonometric function

•  $\operatorname{cosec}^{-1}(\operatorname{cosec}x) = x, \text{ for all } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], x \neq 0$   
•  $\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x,$   
for all  $x \in [-\infty, -1] \cup (1, \infty)$ , i.e.,  $R - (-1, 1)$

•  $\sec^{-1}(\sec x) = x, \text{ for all } x \in [0, \pi], x \neq \frac{\pi}{2}$   
•  $\sec(\sec^{-1}x) = x, \text{ for all } x \in (-\infty, -1) \cup (1, \infty)$ , i.e.,  $R - (-1, 1)$

**Trace the Mind Map** ↗  
First Level → Second Level → Third Level