LCM AND HCF

LEAST COMMON MULTIPLE (LCM)

LCM of two or more numbers is the least among the numbers which are common multiples of the given numbers. In other words, LCM of given numbers is the smallest number which is exactly divisible by each of them. In the above examples, the LCM for

2 and 5 is 10
2 and 3 is 6
4 and 6 is 12

We can find LCM by two methods.

Method 1:

Step I : Write the numbers as product of prime factors.

Step II : Find the product of the highest powers of the prime factors, which will be the LCM

Note : Do not repeat any factor while writing the product in Step II.

Example : Find the LCM of 36, 56, 105 and 108.

Step I : $36 = 2^2 \times 3^2$
$56 = 2^3 \times 7$
$105 = 3 \times 5 \times 7$
$108 = 2^2 \times 3^3$

Step II: The LCM must contain every prime factor of each of the numbers. Also it must include the highest power of each prime factor which appears in any of them. So, it must contain 2 or it would not be a multiple of 56, it must contain 3 or it would not be a multiple of 108, it must contain 5 or it would not be a multiple of 105, and it must contain 7 or it would not be a multiple of 56 or of 105.

Therefore, the LCM = $2^3 \times 3^3 \times 5 \times 7 = 7560$

Method 2 :

This is quicker method to find the prime factors and hence LCM In this method there can be more than one arrangement for the same numbers.

Step I : Write the numbers in a row and strike out those numbers which are factors of any other number in the set.

Step II : Write the factor on the left hand side which can divide maximum of the numbers.

Step III : Write in the next row the quotients obtained and also those numbers (as they are) which are not divisible by that factor. You can strike out from any row 1, if it appears.

Step IV : Repeat steps II and III until we get a set where no two numbers have a common factor or divisor, *i.e.,* all the numbers in the row are prime to each other, though individually they may not be prime numbers.

Step V : Multiply all the factors or divisors and the numbers left in the last row. The product gives the LCM of the given numbers.

Let us now see how it works and how simple it is.

Example : Find LCM of 48, 108 and 140.

Method 1: Factorization Method

Factors of 48 $= 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$
Factors of 108 $= 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$
Factors of 140 $= 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

LCM = Highest power of 2 × Highest power of 3 × Highest power of 5 × Highest power of 7
$= 2^4 \times 3^3 \times 5 \times 7 = 15120$

Method 2 : By Division Method

2	48, 108,140
2	24, 54, 70
3	12, 27, 35
	4, 9, 35

So, LCM = $2 \times 2 \times 3 \times 4 \times 9 \times 35 = 15120$.

L.C.M of Decimals

To find the LCM of decimal numbers first of all we find out the LCM of numbers without decimal. And then we see the number in which the decimal is given in the minimum digits from right to left. We put the decimal in our result which is equal to that number of digits.

Example : Find the LCM of 0.16, 5.4 and .0098.

First of all we find out the LCM of 16, 54, 98.

Here, LCM of 16, 54, 98 is 21168.

In numbers 0.16, 5.4, 0.0098, the minimum digits from right to left is 5.4.

Here, in 5.4 the decimal is given of one digit from right to left is 5.4.

So, we put decimal in our result such that: = 21168 = 2116.8.

Example : Find the LCM of 48, 10.8 and 0.140.

LCM of 48, 108 and 140 = 15120

So, LCM of 48, 10.8, and 0.140 = 1.5120.

L.C.M of Fractions

If *a/b, c/d, e/f* be the proper fractions then their LCM is given by

$$= \frac{\text{L.C.M of numerators } a, c, e}{\text{H.C.F of denominators } b, d, f}$$

Example : Find the LCM of 3^5, 3^8 ,3^{12} , 3^{15} ,3^{20}

If the base of these numbers is same then LCM of these numbers will be equal to maximum power of these numbers.

So, LCM = 3^{20}

Imp : If A and B be the two numbers then the product of their LCM and HCF is equal to the product of the two numbers. *i.e.*,

LCM × HCF = $A \times B$

So, L.C.M = $\dfrac{A \times B}{\text{H.C.F}}$

To Find LCM By Multiples

If we want to find LCM of 3 and 4 then first of all we find the multiples of 3 and 4. Then the lowest common multiples of both of them is their LCM

Multiples of 3: 3, 6, 9, 12, 15, 18, ...

Multiples of 4: 4, 8, 12 , 16, 20, ...

Here, lowest common multiple is 12 which is our LCM.

HIGHEST COMMON FACTOR (HCF)

A number which is a factor of two or more numbers is said to be a common factor or common measure of the numbers. We exclude unity which is common measure of all numbers. The greatest number which will divide each of two or more numbers is called their Highest Common Factor or Greatest Common Measure and is denoted by the letters HCF or GCM(Greatest Common Measure).

Example: Find the HCF of 8 and 12.

Factors of 8 are 1, 2, 4, 8 and

Factors of 12 are 1, 2, 3, 4, 6, 12

The common factors are 1, 2, 4 but highest of these is 4. Hence, 4 is the HCF.

By Factorization Method

Factor method has discussed above or, we express each given number as the product of primes. Now, we take the product of common factors which is our required HCF.

Example : Find the HCF of 144, 336 and 2016.

Factors of 144 = $2^4 \times 3^2$

Factors of 336 = $2^4 \times 3 \times 7$

Factors of 2016 = $2^5 \times 7 \times 3^2$

So, HCF of given numbers = $2^4 \times 3 = 48$.

By Division Method

Step I : We divide the greater number by the smaller and find out the remainder.

Step II : Then divide the first divisor by remainder and find the second remainder.

Step III : Then divide the second divisor by the second remainder.

Step IV : We repeat this process till no remainder is left. The last divisor is our required HCF.

Example : HCF of 513 and 783.

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513 ) 783 ( 1
      513
      270 ) 513 ( 1
             270
             243) 270 (1
                  243
                   27 ) 243 ( 9
                        243
                         ×
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So, HCF = 27.

HCF of Decimals

Here, first of all we find HCF of the given numbers without decimals and then put decimal.

Example : Find the HCF of 0.0012, 1.6, and 28.

Here, HCF of 12, 16 and 28 is 4.

The decimal is given at maximum digits from right to left.

So, HCF = 0.0004.

HCF of Fractions

If *a/b, c/d, e/f* be the proper fractions then their HCF is given by

$$= \frac{\text{H.C.F of numerators } a, c, e\ldots}{\text{L.C.M of denominators } b, d, f\ldots}$$

Example : Find the HCF of 2/5, 8/35, 4/15 and 6/25.

$$\text{HCF} = \frac{\text{H.C.F. of } 2, 8, 4, 6}{\text{L.C.M. of } 5, 35, 15, 25} = \frac{2}{525}.$$

EXERCISE

1. HCF of 11, 0.121, 0.1331 is :
(*a*) 0.0011 (*b*) 0.121
(*c*) 0.1331 (*d*) 12.21
(*e*) None of these

2. The L.C.M of 22, 54, 108, 135 and 198 is :
(*a*) 330 (*b*) 1980
(*c*) 5940 (*d*) 11880
(*e*) None of these

3. HCF of 8^{-2}, 8^{-3}, 8^{-4}, 8^{-5} is :
(*a*) 8^{-2} (*b*) 8^{-3}
(*c*) 8^{-4} (*d*) 8^{-5}
(*e*) None of these

4. The sum of two numbers is 528, and their HCF is 33. How many pairs of such numbers can be formed?
(*a*) 4 (*b*) 5
(*c*) 8 (*d*) 2
(*e*) None of these

5. HCF of 4^5, 4^{11} and 4^{15} is :
(*a*) 4^5 (*b*) 4^{11}
(*c*) 4^{15} (*d*) 4
(*e*) None of these

6. The HCF of 2^3, 3^2, 4 and 15 is :
(*a*) 2^3 (*b*) 3^2
(*c*) 1 (*d*) 360
(*e*) None of these

7. HCF of 15, 45, 90 is :
(*a*) 12 (*b*) 13
(*c*) 13 (*d*) 15
(*e*) None of these

8. The GCM of 9/45, 15/20, 16/20 and 15/25 is :
(*a*) 1/20 (*b*) 1/40
(*c*) 1/60 (*d*) 1/15
(*e*) None of these

9. GCM of 3556 and 3444 is :
(*a*) 25 (*b*) 26
(*c*) 27 (*d*) 28
(*e*) None of these

EXPLANATORY ANSWERS

1. (*a*) : HCF of 11, 121, 1331 is 11.
So, HCF of 11, 0.121 and 0.1331 is = 0.0011.

2. (*c*) : Method 1

2	22, 54, 108, 135, 198
11	11, 27, 54, 135, 99
9	1, 27, 54, 135, 9
3	1, 3, 6, 15, 1
	1, 1, 2, 5, 1

So, LCM = 2 × 11 × 9 × 3 × 2 × 5 = 5940

Method 2

Factors of 22 = 2 × 11

Factors of 54 = 2 × 3 × 3 × 3 = 2×3^3

Factors of 108 = 2 × 2 × 3 × 3 × 3 = $2^2 \times 3^3$

Factors of 135 = 5 × 3 × 3 × 3 = 5×3^3

Factors of 198 = 2 × 3 × 3 × 11 = $2^1 \times 3^2 \times 11^1$

So, LCM = Max. power of 2 × Max. power of 3 × Max. power of 5 × Max. power of 11
= $2^2 \times 3^3 \times 5 \times 11$ = 5940

3. (*d*) : HCF of the given numbers = 8^{-5}

4. (*a*) : Trick :

Let the numbers be 33 a and 33 b

Now, 33 a + 33b = 528

$\Rightarrow$ 33 ($a + b$) = 528 $a + b = 16$

The possible values of a and b are (1, 15); (3, 13) ; (5, 11) ; and (7, 9).

So, the possible pairs of numbers are (33, 495); (99, 429); (165, 363); (231, 297).

5. (*a*) : HCF of the given numbers = 4^5

Minimum power of 4.

6. (*c*) : Trick :

HCF of 2^3, 3^2, 4 and 15

Here by factorization method we see that 1 is the HCF of given numbers

$$\left.\begin{array}{ll} 2^3 & = 2^3 \\ 3^2 & = 3^2 \\ 4 & = 2^2 \\ 15 & = 3 \times 5 \end{array}\right] = 1$$

7. (*d*) : By Factorization Method

Factors of 15 = 3×5

Factors of 45 = $3^2 \times 5$

Factors of 90 = $3^2 \times 5 \times 2$

So, HCF = $3 \times 5 = 15$.

8. (*a*) : GCM of the given fractions

$$= \frac{\text{G.C.M of } 9, 15, 16, 15}{\text{L.C.M of } 45, 20, 20, 25} = \frac{1}{900}$$

9. (*d*) : Trick :

HCF = 3444) 3556 (1
- 3444
112) 3444 (30
–360
84) 112 (1
–84
28) 84 (3
84
×

HCF = 28.