

Total No. of Questions—24

Total No. of Printed Pages—4 Regd. No.

Part III

MATHEMATICS, Paper - II(B)

(English Version)

Time : 3 Hours]

[Max. Marks : 75

Note :—This question paper consists of **THREE** sections A, B and C.

SECTION A

10×2=20

I. Very short answer type questions :

(i) Attempt **ALL** questions.

(ii) Each question carries **TWO** marks.

1. Find the centre and radius of the circle :

$$x^2 + y^2 - 4x - 8y - 41 = 0.$$

2. If the length of the tangent from (5, 4) to the circle

$$x^2 + y^2 + 2ky = 0$$

is 1, then find k .

3. Find the equation of the common chord of the circles :

$$x^2 + y^2 - 4x - 4y + 3 = 0 \text{ and}$$

$$x^2 + y^2 - 5x - 6y + 4 = 0.$$

4 Find the equation of the tangent to the parabola $y^2 = 6x$ at the positive end of the latus rectum.

5 If the angle between the asymptotes of the hyperbola is 30° , then find its eccentricity.

6. Evaluate the integral :

$$\int \frac{\sin^2 x}{1 + \cos 2x} dx$$

on $I \subset \mathbb{R} \setminus \{(2n \pm 1)\pi : n \in \mathbb{Z}\}$.

7. Evaluate the integral :

$$\int \frac{2x + 1}{x^2 + x + 1} dx, \quad x \in \mathbb{R}.$$

8. Evaluate the definite integral :

$$\int_2^3 \frac{2x}{1 + x^2} dx.$$

9. Find the area of the region enclosed by $y = x^3 + 3$, $y = 0$, $x = -1$, $x = 2$.

10. Form the differential equation corresponding to :

$$y = A \cos 3x + B \sin 3x,$$

where A and B are parameters.

SECTION B

5×4=20

- II. Short answer type questions :

(i) Attempt **ANY FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. Find the length of the chord intercepted by the circle :

$$x^2 + y^2 - x + 3y - 22 = 0$$

on the line $y = x - 3$.

12. Show that the angle between the circles $x^2 + y^2 = a^2$,

$$x^2 + y^2 = ax + ay \text{ is } \frac{3\pi}{4}.$$

13. Find the length of latus rectum, eccentricity, foci and the equations of directrices of the ellipse :

$$9x^2 + 16y^2 = 144.$$

14. Find the equation of tangent and normal to the ellipse $x^2 + 8y^2 = 33$ at $(-1, 2)$.
15. Find the equations of the tangents to the hyperbola :

$$3x^2 - 4y^2 = 12$$

which are :

- (i) Parallel and
- (ii) Perpendicular to the line $y = x - 7$.
16. Evaluate :

$$\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx.$$

17. Solve the differential equation :

$$(xy^2 + x) dx + (yx^2 + y) dy = 0.$$

SECTION C

5×7=35

- III. Long answer type questions :

- (i) Attempt **ANY FIVE** questions.
- (ii) Each question carries **SEVEN** marks.

18. Find the equation of a circle which passes through $(2, -3)$ and $(-4, 5)$ and having the centre on $4x + 3y + 1 = 0$.
19. Show that :

$$x^2 + y^2 - 6x - 9y + 13 = 0,$$

$$x^2 + y^2 - 2x - 16y = 0$$

touch each other. Find the point of contact and the equation of common tangent at their point of contact.

20. Derive the equation of a parabola in standard form.

21. Evaluate :

$$\int \frac{dx}{(1+x)\sqrt{3+2x-x^2}} \text{ on } (-1, 3).$$

22. Obtain the reduction formula for

$$\int \sin^n x \, dx$$

for an integer $n \geq 2$ and deduce the value of :

$$\int \sin^4 x \, dx.$$

23. Evaluate :

$$\int_0^1 \frac{\log(1+x)}{1+x^2} \, dx.$$

24. Solve the differential equation :

$$\sin^{-1}\left(\frac{dy}{dx}\right) = x + y.$$