Angle and Cyclic Properties of a Circle 17

STUDY NOTES

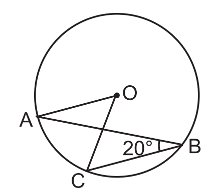
- The angle subtended by an arc of a circle at the centre is twice the angle subtended by it at any point on the remaining part of the circumference.
- Angles in the same segment of a circle are equal. Or in other words, angles subtended by an arc of a circle at any point on the remaining part of the circumference are equal.
- The angle in a semi-circle is a right angle.
- If an arc of a circle substends a right angle at any point on the remaining part of the circle, then the arc is a semi circle.
- If a quadrilateral is inscribed in a circle, then the quadrilateral is known as a cyclic quadrilateral. Thus, the vertices of a cyclic quadrilateral lie on the circumference of a circle.
- The points which lie on the circumference of the same circle are known as concyclic points.
- The opposite angles of a cyclic quadrilateral are supplementary.
- The converse of this theorem is also true, i.e., if the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.
- The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- Every cyclic parallelogram is a rectangle.
- An isosceles trapezium is always cyclic.
- A cyclic trapezium is isosceles and its diagonals are equal.
- Any four vertices of a regular pentagon lie on a circle.
- If two chords of a circle bisect each other, they are diameters of the circle.
- Quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.
- The mid-point of the hypotenuse of a right triangle is equidistant from its vertices.

QUESTION BANK

A. Multiple Choice Questions [1 Mark]

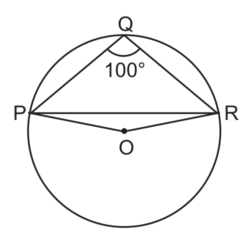
Choose the correct option:

1. In the figure, O is the centre of the circle. If ∠ABC = 20°, then ∠AOC is equal to :



(a) 20°
(b) 40°
(c) 60°
(d) 10°

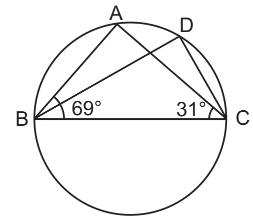
2. In the figure, ∠PQR = 100°, where P, Q and R are points on a circle with centre O, then ∠OPR =



(a) 15°
(b) 12°
(c) 10°
(d) 8°

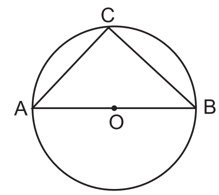
3. In the figure, if ∠ABC = 69°, ∠ACB = 31°, then ∠BDC =

(a) 60°
(b) 70°
(c) 80°
(d) 100°



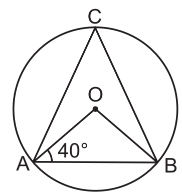
4. In the figure, if AOB is a diameter of the circle and AC = BC, then ∠CAB is equal to :

(a) 30°
(b) 60°
(c) 90°
(d) 45°



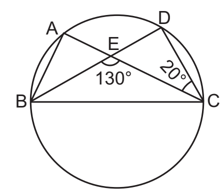
5. In the figure, O is the centre of the circle. If ∠OAB = 40°, then ∠ACB is equal to :

(a) 50° (b) 40°
(c) 60° (d) 70°



6. In the figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°, then ∠BAC =

(a) 100° (b) 102°
(c) 50° (d) 110°

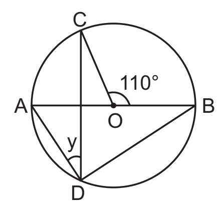


7. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠DBC = 70°, and ∠BAC = 30°, then ∠BCD=

(a) 60° (b) 70° (c) 80° (d) 75°

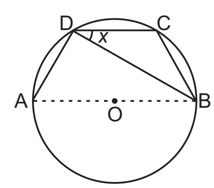
8. The value of y in the given figure, where O is the centre of the circle, is:

(a) 50° (b) 45°
(c) 40° (d) 35°



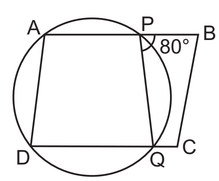
9. In the given figure, O is the centre of the circle. If ∠ADC = 118°, then the value of x is :

(a) 18° (b) 28°
(c) 32° (d) 46°



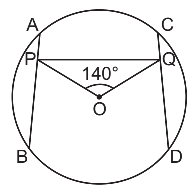
10. ABCD is a parallelogram. A circle passes through A and D, cuts AB at P and DC at Q. If ∠BPQ = 80°, then ∠ABC is equal to :

(a) 60° (b) 75°
(c) 80° (d) 105°

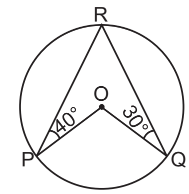


11. In the given figure, AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chord AB and CD respectively. If ∠POQ = 140°, then ∠APQ is equal to :

(a) 70° (b) 80°
(c) 95° (d) 105°

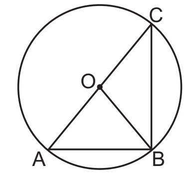


12. In the given figure, O is the centre of the circle. If ∠OPR and ∠OQR are 40° and 30° respectively then, the measure of ∠POQ is :



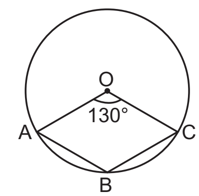
(a) 160° (b) 150°
(c) 140° (d) 130°

13. In the figure, O is the centre of the circle. If ∠OAB = 40°, then the measure of ∠ACB is:



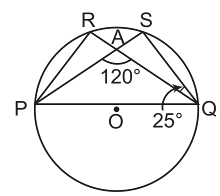
(a) 40° (b) 50°
(c) 60° (d) 75°

14. In the given figure, if O is the centre of the circle and ∠AOC = 130°, then ∠ABC is equal to :



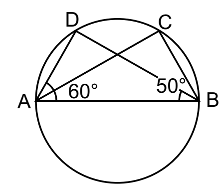
(a) 115° (b) 120°
(c) 135° (d) 150°

15. In the given figure, O is the centre of the circle. If ∠PAQ = 120° and ∠RQS = 25°, then the measure of ∠PRQ is equal to :



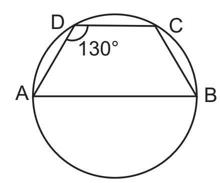
(a) 95° (b) 105°
(c) 115° (d) 135°

16. In the figure, if ∠DAB = 60°, ∠ABD = 50°, then ∠ACB is equal to :



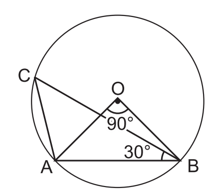
(a) 60° (b) 50°
(c) 70° (d) 80°

17. ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and ∠ADC = 130°, then ∠BAC is equal to:



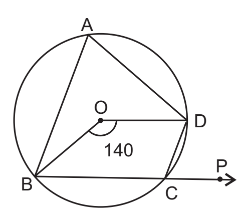
(a) 80° (b) 50°
(c) 40° (d) 30°

18. In the figure, if ∠AOB = 90° and ∠ABC = 30°, then ∠CAO is equal to :



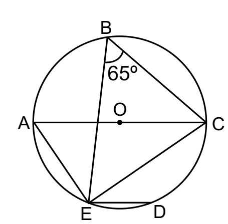
(a) 30° (b) 45°
(c) 90° (d) 60°

19. In the figure, O is the centre of the circle. The angle subtended by the arc BCD at the centre is 140°, BC is produced to P, then ∠BAD + ∠BCD =



(a) 160° (b) 170°
(c) 180° (d) 210°

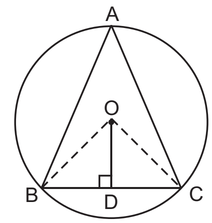
20. In the figure, chord ED is parallel to the diameter AC of the circle. Given, ∠CBE = 65°, then ∠DEC =

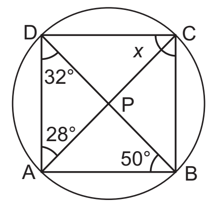


(a) 35° (b) 30°
(c) 25° (d) 20°

21. If O is the circumcentre of ΔABC and $OD \perp BC$, then $\angle BOD$ =

(a) $\angle A$
(b) $\angle B$
(c) $\angle C$
(d) none of these





22. In the given figure, the value of x is :

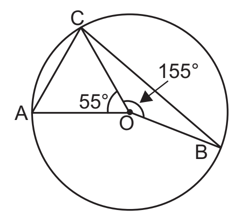
(a) 86° (b) 84°
(c) 82° (d) 80°

A. Answers

1. (b) **2.** (c) **3.** (c) **4.** (d) **5.** (a) **6.** (d) **7.** (c) **8.** (d) **9.** (b) **10.** (c)
11. (a) **12.** (c) **13.** (b) **14.** (a) **15.** (a) **16.** (c) **17.** (c) **18.** (d) **19.** (c) **20.** (c)
21. (a) **22.** (c)

B. Short Answer Type Questions [3 Marks]

1. In the given figure, angles subtended by arcs AC and BC at the centre O of the circle are 55° and 155° respectively. Find $\angle ACB$.

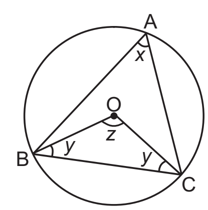


Sol. $OA = OC$ [Radii]
In ΔAOC,
$\angle OAC = \angle OCA = x$
Now, $\angle OAC + \angle AOC + \angle ACO = 180°$
$\Rightarrow x + 55° + x = 180° \Rightarrow 2x = 180° - 55° \Rightarrow 2x = 125° \Rightarrow x = \dfrac{125}{2} = 62.5°.$

$OB = OC$ [Radii]
In ΔBOC, $\angle OBC = \angle OCB = y$
Now, $\angle BOC + \angle OBC + \angle OCB = 180°$
$\Rightarrow 155° + y + y = 180° \Rightarrow 2y = 180° - 155° \Rightarrow y = \dfrac{25°}{2} = 12.5°$

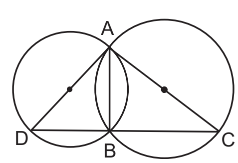
But, $\angle ACB = \angle ACO + \angle BCO = 62.5 + 12.5 = 75°$

2. BC is a chord of a circle with centre O. A is a point on major arc BC as shown in the figure. Prove that $\angle BAC + \angle OBC = 90°$.



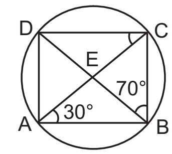
Sol. $\angle BOC = 2\angle BAC$
$\Rightarrow z = 2x$
In ΔBOC,
Also, $\angle BOC + \angle OBC + \angle OCB = 180°$
$\Rightarrow z + y + y = 180° \Rightarrow 2x + y + y = 180° \Rightarrow 2(x + y) = 180° \Rightarrow x + y = \dfrac{180°}{2} = 90°$
$\Rightarrow \angle BAC + \angle OBC = 90°$ **Proved.**

3. Two circles intersect at two points A and B. AD and AC are diameters of the two circles. Prove that B lies on the line segment DC.



Sol. Join AB,
$\angle ABD = 90°$ [Angle in a semi-circle]
$\angle ABC = 90$ [Angle in a semi-circle]
$\Rightarrow \angle ABD + \angle ABC = 180°$
$\Rightarrow$ DBC is a line, or B lies on the segment DC. **Proved.**

4. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC = 30°$, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.



Sol. $\angle DAC = \angle CBD = 70°$ [Angles in the same segment]

$\angle BAD = \angle BAC + \angle DAC = 30° + 70° = 100°$

$\angle BCD + \angle BAD = 180°$ [Opposite angles of a cyclie quad. are supplementary]

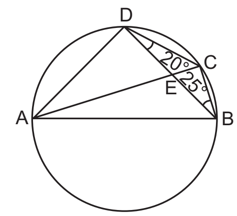
$\Rightarrow \angle BCD + 100° = 180° \Rightarrow \angle BCD = 180° - 100° = 80°$

AB = BC

$\Rightarrow \angle BAC = \angle BCA = 30°$

$\angle ECD = \angle BCD - \angle BCE = 80° - 30° = 50°$.

5. In the given figure, AB is a diameter of the circle. $\angle BDC = 20°$ and $\angle CBD = 25°$. Find $\angle AED$ and $\angle ACD$.



Sol. $\angle BCD + \angle BDC + \angle CBD = 180°$

$\Rightarrow \angle BCD + 20° + 25° = 180° \Rightarrow \angle BCD + 45° = 180°$

$\Rightarrow \angle BCD = 180° - 45° \Rightarrow \angle BCD = 135°$

$\angle BCA = 90°$ [Angle in a semi-circle]

$\angle ACD = \angle BCD - \angle BAC = 135° - 90° = 45°$

$\angle EBC + \angle BCE + \angle BEC = 180° \Rightarrow 25° + 90° + \angle BEC = 180°$

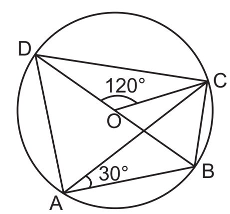
$\Rightarrow 115° + \angle BEC = 180° \Rightarrow \angle BEC = 180° - 115° = 65°$

$\angle BEC = 65°$

$\angle AED = \angle BEC = 65°$ [Vertically Opposite Angles]

So, $\angle AED = 65°$, $\angle ACD = 45°$.

6. In the given figure, ABCD is a cyclic quadrilateral. O is the centre of the circle. If $\angle COD = 120°$ and $\angle BAC = 30°$, find $\angle BOC$ and $\angle BCD$.



Sol. $\angle DAC = \frac{1}{2} \angle COD = \frac{1}{2} \times 120° = 60°$ [On the same segment]

$\angle BAD = \angle DAC + \angle BAC = 60° + 30° = 90°$

$\angle BCD + \angle BAD = 180°$ [Opposite angles of a cyclic quad.]

$\Rightarrow \angle BCD + 90° = 180°$

$\Rightarrow \angle BCD = 180° - 90° = 90°$

$\angle BOC + \angle COD = 180°$ [Linear Pair]

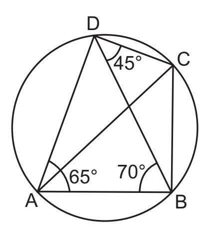
$\Rightarrow \angle BOC + 120° = 180°$

$\Rightarrow \angle BOC = 180° - 120° = 60°$.

7. In the given figure, $\angle BAD = 65°$, $\angle ABD = 70°$ and $\angle BDC = 45°$. Find

(a) $\angle BCD$

(b) $\angle ADB$. Hence, show that AC is a diameter of the circle.



Sol. (a) $\angle BAD + \angle BCD = 180°$ [Opposite angles of a cyclic quad.]

$\Rightarrow 65° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 180° - 65° = 115°$

(b) $\angle ADB + \angle BAD + \angle ABD = 180°$

$\Rightarrow \angle ADB + 65° + 70° = 180°$

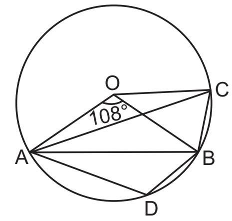
$\Rightarrow \angle ADB + 135° = 180°$

$\Rightarrow \angle ADB = 180° - 135° = 45°$

Since, $\angle ADB + \angle BDC = 45° + 45° = 90°$

So, AC is a diameter of the circle.

8. In the figure, A, D, B, C are four points on the circumference of a circle with centre O. arc AB = 2 arc BC and $\angle AOB = 108°$. Calculate in degrees (a) $\angle ACB$ (b) $\angle CAB$ (c) $\angle ADB$.



Sol. Let $\angle BOC = x°$, then $\angle AOB = 2x°$

$\Rightarrow 2x = 108° \Rightarrow x = 54°$

$\angle BOC = 54°$ and $\angle DOB = 108°$

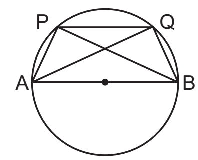
(a) $\angle ACB = \frac{1}{2}\angle AOB$

$\Rightarrow \angle ACB = \frac{1}{2}\times 108° = 54°$

(b) $\angle CAB = \frac{1}{2}\angle COB = \frac{1}{2}\times 54° = 27°$

(c) $\angle ADB = \frac{1}{2}$ reflex $\angle AOB = \frac{1}{2}\times(360°-108°) = \frac{1}{2}\times 252° = 126°$.

9. In the given figure, AB is a diameter of the circle. PQ is a chord such that $\angle BAP = \angle ABQ$. Prove that ABQP is a cyclic trapezium.



Sol. ABQP is a cyclic quadrilateral.

$\angle BAP + \angle BQP = 180°$ ...(i)

$\angle ABQ + \angle APQ = 180°$ ...(ii)

From (i) and (ii)

$\angle BAP + \angle BQP = \angle ABQ + \angle APQ$

$\Rightarrow \angle BAP + \angle BQP = \angle BAP + \angle APQ$ [$\angle BAP = \angle ABQ$ given]

$\Rightarrow \angle BQP = \angle APQ$

From (i), we have

$\angle BAP + \angle APQ = 180°$

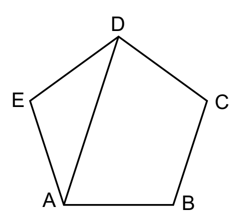
Sum of the adjacent angles on the same side of the transversal AP of the line segment AB and PQ is 180°.

$\therefore$ AB and PQ are parallel

$\therefore$ ABQP is cyclic trapezium. **Proved.**

C. Long Answer Type Questions [4 Marks]

1. Prove that any four vertices of a regular pentagon are concyclic.



Sol. Given, ABCDE is a regular pentagon

AB = BC = CD = DE = AE

Sum of all interior angles = 540º

One interior angle = $\frac{540}{5} = 108°$

In ΔADE

AE = DE

$\Rightarrow \angle ADE = \angle DAE$

$\angle ADE + \angle DAE + \angle AED = 180°$.

$\Rightarrow \angle ADE + \angle ADE + 108° = 180°$

$\Rightarrow 2\angle ADE = 180° - 108°$

$\Rightarrow 2\angle ADE = 72° \Rightarrow \angle ADE = \frac{72º}{2} = 36°$

$\angle ADE = \angle DAE = 36°$

$\angle DAB = 108° - 36° = 72°$

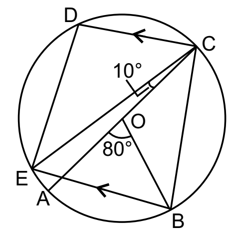
In quadrilateral ABCD

$\angle DAB + \angle C = 72° + 108° = 180°$.

The sum of the opposite angles of a quadrilateral is 180°.

ABCD is a cyclic quadrilateral

So, A, B, C and D are concyclic. **Proved.**



2. In the given figure, AC is the diameter of the circle with centre O. CD and BE are parallel, $\angle AOB = 80°$ and $\angle ACE = 10°$.
Calculate (a) $\angle BEC$ (b) $\angle BCD$ (c) $\angle CED$.

Sol. (a) $\angle AOB + \angle BOC = 180°$

$\Rightarrow 80° + \angle BOC = 180° \Rightarrow \angle BOC = 180° - 80° = 100°$

And $\angle BEC = \frac{1}{2} \angle BOC$ [On the same segment]

$= \frac{1}{2} \times 100° \Rightarrow \angle BEC = 50°$

(b) DC || EB

$\angle DCE = \angle BEC = 50°$

And $\angle AOB = 80°$

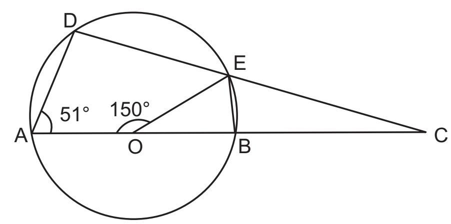
$\angle ACB = \frac{1}{2} \angle AOB$

$\Rightarrow \angle ACB = \frac{1}{2} \times 80° = 40°$

$\angle BCD = \angle ACB + \angle ACE + \angle DCE = 40° + 10° + 50° = 100°$

(c) $\angle BED = 180° - \angle BCD = 180° - 100° = 80°$

$\angle CED = \angle BED - \angle BEC = 80° - 50° = 30°$.



3. In the figure, O is the centre of the circle.
$\angle AOE = 150°$, $\angle DAO = 51°$.
Calculate the measures of $\angle BEC$ and $\angle EBC$.

Sol. $\angle DAB + \angle BED = 180°$ [sum of opp. angles of a cyclic quadrilateral]

$\angle BED = 180° - 51° = 129°$

$\angle CEB = 180° - \angle BED = 180° - 129°$

$\Rightarrow \angle BEC = 51°$

$\Rightarrow \angle ADE = \frac{1}{2}$ reflex $\angle AOE = \frac{1}{2}(360° - 150°) = \frac{1}{2} \times 210° = 105°$

$\angle ABE + \angle ADE = 180°$ [Sum of opp. angles of a cyclic quadrilateral]

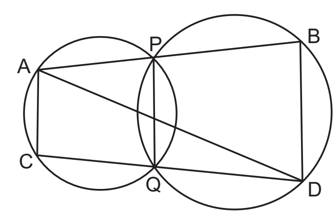
$\Rightarrow \angle ABE + 105° = 180°$

$\Rightarrow \angle ABE = 180° - 105° = 75°$

$\angle CBE + \angle ABE = 180°$

$\Rightarrow \angle CBE + 75° = 180°$

$\Rightarrow \angle CBE = 180° - 75° = 105°$.



4. Two circles intersect at P and Q. Through P, a straight line APB is drawn to meet the circles in A and B. Through Q, a straight line is drawn to meet the circles at C and D. Prove that AC is parallel to BD.

Sol. ACQP is a cyclic quadirlateral

So, $\angle CAP + \angle PQC = 180°$ ...(i)

PQDB is a cyclic quadirlateral

So, $\angle PQD + \angle DBP = 180°$ ...(ii)

$\angle PQC + PQD = 180°$ [Linear pair] ...(iii)

From (i), (ii) and (iii), we have

$\angle CAP + 180^o - \angle PQD = 180°$

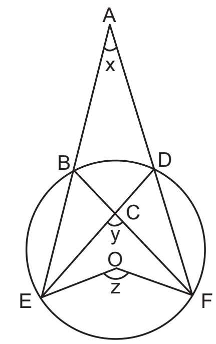
$\Rightarrow \angle CAP + 180^o - (180° - \angle DBP) = 180°$

$\Rightarrow \angle CAP + 180° - 180° + \angle DBP = 180°$

$\Rightarrow \angle CAP + \angle DBP = 180°$

$\Rightarrow \angle CAB + \angle DBA = 180°$

AC || BD [Co-interior angles are supplementary] **Proved.**



5. In the figure, O is the centre of the circle. Prove that $x + y = z$.

Sol. $z = 2 \times \angle 3$ [On the same segment]

$z = 2 \times \angle 4$ [On the same segment]

$\therefore \angle 3 = \angle 4$

$\Rightarrow z = 2\angle 3 = \angle 3 + \angle 3 = \angle 3 + \angle 4$ ...(i)

$y = \angle 1 + \angle 3$ [Exterior angle = sum of interior opposite angles]

$\Rightarrow \angle 3 = y - \angle 1$ ...(ii)

From (i) and (ii), we have

$z = y - \angle 1 + \angle 4$ ...(iii)

$\angle 4 = \angle 1 + x$

$\Rightarrow \angle 4 - \angle 1 = x$ ...(iv)

From (iii) and (iv), we have

$z = y + x$ **Proved.**

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