Definition: If f is continuous in (a, b) and bounded then $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$ is called the definite integral of f(x) between the limits a and b.

where $\frac{d}{dx}(F(x)) = f(x)$

Very Important:

1. If $\int_a^b f(x)\,dx = 0$, then the equation f (x) = 0 has atleast one root in (a, b) provided f is continuous in (a, b).

Note that the converse is not true.

e.g. If 2a + 3b + 6c = 0 then the QE $ax^2 + bx + c = 0$ must have a root in (0, 1)

2. (a) If f (x) > 0 in (a, b) then $\int_a^b f(x)\,dx > 0$ provided a < b.

(b) $\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$

3. $\int_a^a f(x)\,dx = 0$ only if f (a) is defined.

e.g. $\int_0^0 \frac{dt}{t} \ne 0$

4. $\int_a^b f(x)\cdot d(g(x)) = \int_{g^{-1}(a)}^{g^{-1}(b)} f(x)\cdot g'(x)\,dx$. [a and b are limit of g (x)]

Note that when g (x) = b then $x = g^{-1}(b)$ and g(x) = a then $x = g^{-1}(a)$;

5. $\int_a^b \left(\frac{d}{dx} f(x)\right) dx = [f(x)]_a^b$ if f(x) is continuous in (a, b) however if f(x) is discontinuous at x = c ∈ (a, b) then $\int_a^b \left(\frac{d}{dx} f(x)\right) dx = [f(x)]_a^{c^-} + [f(x)]_{c^+}^b$ [convergent and divergent]

6. $\lim_{n\to\infty}\left(\int_a^b f_n(x)\,dx\right) = \int_a^b \left(\lim_{n\to\infty} f_n(x)\right) dx$;

7. If g (x) is the inverse of f (x) and f (x) has domain x ∈ [a, b] where f (a) = c and f (b) = d then the value of $\int_a^b f(x)\,dx + \int_c^d g(y)\,dy = (bd - ac)$

8. Remember the values of the following def. integral

(a) $\int_0^{\pi/2} \sin x\,dx = \int_0^{\pi/2} \cos x\,dx = 1$ (b) $\int_0^{\pi/2} \sin^2 x\,dx = \int_0^{\pi/2} \cos^2 x\,dx = \frac{\pi}{4}$

(c) $\int_0^{\pi/2} \sin^3 x\,dx = \int_0^{\pi/2} \cos^3 x\,dx = \frac{2}{3}$ (d) $\int_0^{\pi/2} \sin^4 x\,dx = \int_0^{\pi/2} \cos^4 x\,dx = \frac{3\pi}{16}$

PROPERTIES OF DEFINITE INTEGRAL
WITH ILLUSTRATIONS

(A) PROPERTIES:

P – 1 $\int_a^b f(x)\,dx = \int_a^b f(t)dt$; **P – 2** $\int_a^b f(x)\,dx = -\int_b^a f(x)dx$

P – 3 $\int_a^b f(x) = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

provided f has a piece wise continuity when f is not uniformly defined in (a, b)

P – 4 $\int_{-a}^{a} f(x)\,dx = \int_0^a \left(f(x)+f(-x)\right)dx = \begin{cases} 0 & \text{if } f(x) \text{ is odd} \\ 2\int_0^a f(x)dx & \text{if } f(x) \text{ is even} \end{cases}$

P–5 $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ or $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

P–6 $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx \Rightarrow \begin{cases} 0 & \text{if } f(2a-x) = -f(x) \\ 2\int_0^a f(x)\,dx & \text{if } f(2a-x) = f(x) \end{cases}$

P–7 $\int_0^{na} f(x)\,dx = n\int_0^a f(x)dx$ where $f(a + x) = f(x)$ $n \in I$

Note: $\int_{ma}^{na} f(x)\,dx = (n - m)\int_0^a f(x)\,dx$, f (x) is periodic with period = a (n, m $\in$ N, n > m)

(B) DERIVATIVES OF ANTIDERIVATIVES (LEIBNITZ RULE)

If f is continuous and g(x) and h(x) are differentiable function.

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))\cdot h'(x) - f(g(x)).\,g'(x)$$ (integrand of a continuous function is always differentiable)

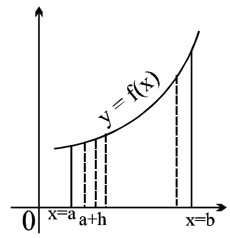
Note: $$\frac{d}{dx}\int_{g(x)}^{h(x)} f(x,t)\,dt = f\left(x,h(x)\right)h'(x) - f\left(x,g(x)\right)g'(x) + \int_{g(x)}^{h(x)}\left(\frac{\partial}{\partial x}f(x,t)\right)dt$$

(C) DEFINITE INTEGRAL AS A LIMIT OF SUM

Fundamental theorem of integral calculus

$$\int_a^b f(x)\,dx = \underset{\substack{h \to 0 \\ n \to \infty}}{\text{Limit}}\; h\,[f(a) + f(a+h) + f(a+2h) + \ldots + f(a + \overline{n-1}\,h)]$$

or $$\int_a^b f(x)\,dx = \underset{\substack{h \to 0 \\ n \to \infty}}{\text{Limit}}\; h \sum_{r=0}^{n-1} f(a + rh) \quad \text{where} \quad b - a = nh$$

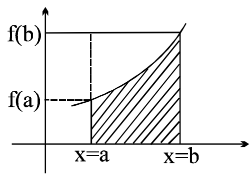


(D) ESTIMATION OF DEFINITE INTEGRAL AND GENERAL INEQUALITIES IN INTEGRATION:

Not all integers can be evaluated using the technique discussed so far.

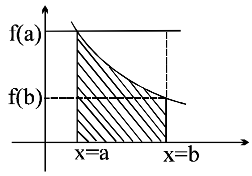
(a) For a monotonic increasing function in (a, b)

$$(b-a)\,f(a) < \int_a^b f(x)\,dx < (b-a)\,f(b)$$



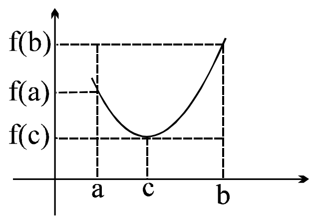
(b) Foa a monotonic decreasing function in (a, b)

$$f(b).\,(b-a) < \int_a^b f(x)\,dx < (b-a)\,f(a)$$



(c) For a non monotonic function in (a, b)

$$f(c)\,(b-a) < \int_a^b f(x)\,dx < (b-a)\,f(b)$$



(d) In addition to this note that

$$\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$$ equality holds when f (x) lies completely above the x-axis

(E) WALLI'S THEORM & REDUCTION FORMULA

$$\int_0^{\pi/2} \sin^n x \cos^m x\;dx = \frac{[(n-1)(n-3)\ldots.1 \text{ or } 2]\,[(m-1)(m-3)\ldots 1 \text{ or } 2]}{(m+n)(m+n-2)\ldots 1 \text{ or } 2}\;K$$

(m, n are non-negative integer)

(F) DIFFERENTIATING AND INTEGRATING SERIES

1. If $\int_1^x \frac{dt}{|t|\sqrt{t^2-1}} = \frac{\pi}{6}$, then x can be equal to

(A) $\frac{2}{\sqrt{3}}$ (B) $\sqrt{3}$

(C) 2 (D) None of these

2. $\int_0^{\pi/4} \frac{x\sin x}{\cos^3 x}$ dx equals to :

(A) $\frac{\pi}{4}+\frac{1}{2}$ (B) $\frac{\pi}{4}-\frac{1}{2}$

(C) $\frac{\pi}{4}$ (D) None of these

3. If f(0) = 1, f(2) = 3, f '(2) = 5 and f '(0) is finite, then $\int_0^1 x . f''(2x)$ dx is equal to

(A) zero (B) 1

(C) 2 (D) None of these

4. If $I_1 = \int_e^{e^2} \frac{dx}{\ln x}$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$, *then*

(A) $I_1 = I_2$

(B) $2 I_1 = I_2$

(C) $I_1 = 2 I_2$ (D) None of these

5. $\int_0^{(\pi/2)^{1/3}} x^5 . \sin x^3 \, dx$ equals to

(A) 1 (B) 1/2

(C) 2 (D) 1/3

6. If $A = \int_0^{\pi} \frac{\cos x}{(x+2)^2} dx$, then $\int_0^{\pi/2} \frac{\sin 2x}{x+1} dx$ is equal to

(A) $\frac{1}{2} + \frac{1}{\pi+2} - A$ (B) $\frac{1}{\pi+2} - A$

(C) $1 + \frac{1}{\pi+2} - A$ (D) $A - \frac{1}{2} - \frac{1}{\pi+2}$

7. If $f(x) = \begin{cases} 0 \text{ , where } x = \frac{n}{n+1}, n = 1,2,3..... \\ 1 \text{ , else where} \end{cases}$, then the value of $\int_0^2 f(x)\,dx$

(A) 1 (B) 0

(C) 2 (D) ∞

8. For $0 < x < \frac{\pi}{2}$, $\int_{1/\sqrt{2}}^{1/2} \cot x \; d(\cos x)$ equals to

(A) $\frac{\sqrt{3}-\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}-\sqrt{3}}{2}$

(C) $\frac{1-\sqrt{3}}{2}$ (D) None of these

9. The value of $\int_{\pi/4}^{\pi/3} \text{cosec } x \; d(\sin x)$ for $0 < x < \pi/2$ is

(A) $\ell n\, 2$ (B) $\frac{1}{2}\, \ell n\, \frac{3}{2}$

(C) $\ell n\left(\frac{\sin 1/2}{\sin 1/\sqrt{2}}\right)$ (D) None of these

10. If $x \in (0, 2)$ then the value of $\int_0^1 e^{2x-[2x]} d(x-[x])$ is (where [$*$] denotes the greatest integer function)

(A) e + 1 (B) e

(C) 2e – 2 (D) None of these

11. $\int_{\log\pi-\log 2}^{\log \pi} \frac{e^x}{1-\cos\left(\frac{2}{3}e^x\right)}$ dx is equal to

(A) $\sqrt{3}$ (B) $-\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$

12. If $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, then $\int_0^{\infty} e^{-ax^2} dx$ where a > 0 is

(A) $\frac{\sqrt{\pi}}{2}$ (B) $\frac{\sqrt{\pi}}{2a}$

(C) $2\frac{\sqrt{\pi}}{a}$ (D) $\frac{1}{2}\sqrt{\frac{\pi}{a}}$

13. $\int_{5/2}^{5} \frac{\sqrt{(25-x^2)^3}}{x^4}$ dx equals to

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$ (D) None of these

14. If $f(x) = \begin{cases} x & ; x < 1 \\ x-1 & ; x \geq 1 \end{cases}$, then $\int_0^2 x^2 f(x)\,dx$ is equal to

(A) 1 (B) $\frac{4}{3}$

(C) $\frac{5}{3}$ (D) $\frac{5}{2}$

15. Suppose for every integer n, $\int_n^{n+1} f(x)\,dx = n^2$. The value of $\int_{-2}^{4} f(x)\,dx$ is

(A) 16 (B) 14
(C) 19 (D) 21

16 The value of $\int_{-2}^{3} \left|1-x^2\right| dx$ is-

(A) 28/3 (B) 14/3
(C) 7/3 (D) 1/3

17. . $\int_0^{\infty} [2e^{-x}]\,dx$ is equal to (where [∗] denotes the greatest integer function)

(A) 0 (B) $\ell n\ 2$
(C) e^2 (D) $2e^{-1}$

18. $\int_0^{\sqrt{2}} [x^2]\,dx =$

(A) $\sqrt{2} - 1$ (B) $2\ (\sqrt{2} - 1)$

(C) $\sqrt{2}$ (D) None of these

19. If $f(x) = \int_0^x \sin[2x]\,dx$ then $f(\pi/2)$ is (where [∗] denotes greatest integer function)

(A) $\frac{1}{2}$ $\{\sin 1 + (\pi - 2) \sin 2\}$

(B) $\frac{1}{2}$ $\{\sin 1 + \sin 2 + (\pi - 3) \sin 3\}$

(C) 0

(D) $\sin 1 + \left(\frac{\pi}{2} - 2\right) \sin 2$

20 $f(x)$ = Minimum $\{\tan x, \cot x\}$ $\forall\ x \in \left(0, \frac{\pi}{2}\right)$.

Then $\int_0^{\pi/3} f(x)\,dx$ is equal to

(A) $\ell n\left(\frac{\sqrt{3}}{2}\right)$ (B) $\ell n\left(\sqrt{\frac{3}{2}}\right)$

(C) $\ell n(\sqrt{2})$ (D) $\ell n(\sqrt{3})$

21. The value of $\int_1^2 ([x^2] - [x]^2)\,dx$ is equal to (where [∗] denotes the greatest integer function)

(A) $4 + \sqrt{2} - \sqrt{3}$ (B) $4 - \sqrt{2} + \sqrt{3}$

(C) $4 - \sqrt{3} - \sqrt{2}$ (D) None of these

22. If $f(x) = \begin{cases} e^{\cos x} \sin x & , |x| \leq 2 \\ 2 & , \text{otherwise} \end{cases}$, then $\int_{-2}^{3} f(x)\,dx =$

(A) 0 (B) 1
(C) 2 (D) 3

23. If f(x) and g(x) are continuous functions satisfying $f(x) = f(a - x)$ and $g(x) + g(a - x) = 2$, then $\int_0^a f(x)\,g(x)\,dx$ is equal to

(A) $\int_0^a g(x)dx$ (B) $\int_0^a f(x)dx$
(C) 0 (D) None of these

24. If $\int_0^{\pi} x\, f(\sin x)\,dx = A \int_0^{\pi/2} f(\sin x)\,dx$, then A is -

(A) 0 (B) π
(C) $\pi/4$ (D) 2π

25. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} x\,g\,\{x\,(1-x)\}\,dx$ and

$I_2 = \int_{f(-a)}^{f(a)} g\,\{x\,(1-x)\}\,dx$, then the value of $\frac{I_2}{I_1}$ is-

(A) 2 (B) – 3
(C) – 1 (D) 1

26. $\int_{-1}^{1} \frac{\sin x + x^2}{3-|x|} dx$

(A) 0 (B) $2\int_0^1 \frac{\sin x}{3-|x|} dx$

(C) $2\int_0^1 \frac{x^2}{3-|x|} dx$ (D) $2\int_0^1 \frac{\sin x + x^2}{3-|x|} dx$

27. $\int_{-\pi/4}^{\pi/4} \frac{e^x \sec^2 x}{e^{2x}-1} dx =$

(A) 0 (B) $\frac{\pi}{2}$

(C) $2e^{\pi/4}$ (D) None of these

28. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} =$

(A) π^2 (B) $\pi^2/4$

(C) $\pi/8$ (D) $\pi^2/8$

29. If $\int_0^{11} \frac{11^x}{11^{[x]}} dx = \frac{k}{\log 11}$ then value of k is

(where [*] denotes greatest integer function)

(A) 11 (B) 101

(C) 110 (D) None of these

30. Let T > 0 be a fixed real number. Suppose f is a continuous function such that for all $x \in R$, $f(x+T)=f(x)$. If $I = \int_0^T f(x)\,dx$ then the value of $\int_3^{3+3T} f(2x)\,dx$ is

(A) $\frac{3}{2}I$ (B) 2 I

(C) 3 I (D) 6 I

31. Let f : R→ R be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then $\lim_{x\to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals

(A) 18 (B) 12

(C) 36 (D) 24

32. The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\, dt}{x \sin x}$ is

(A) 3 (B) 2

(C) 1 (D) –1

33. $\lim_{x\to 0} \frac{\int_0^{x^2} \cos t^2\, dt}{x \sin x}$ is equal to

(A) –1 (B) 1

(C) 2 (D) –2

34. $\lim_{n\to\infty} \left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + ... + \frac{1}{n}\sec^2 1\right]$ equals

(A) $\frac{1}{2}$ sec 1 (B) $\frac{1}{2}$ cosec 1

(C) tan 1 (D) $\frac{1}{2}$ tan 1

35. If $\int_0^{x^2} f(t)dt = x\cos \pi x$, then the value of f(4) is

(A) 1 (B) 1/4

(C) – 1 (D) –1/4 (E) –4

36. $\lim_{n\to\infty} \frac{1^P + 2^P + 3^P + ... + n^P}{n^{P+1}}$ equals-**[AIEEE-2002]**

(A) 1 (B) $\frac{1}{P+1}$

(C) $\frac{1}{P+2}$ (D) P^2

37. $\lim_{x\to\infty} \frac{1+2^4+3^4+....+n^4}{n^5} + \lim_{n\to\infty} \frac{1+2^3+3^3+....+n^3}{n^5}$ is equal to **[AIEEE-2003]**

(A) 1/5 (B) 1/30

(C) zero (D) 1/4

38. Let f(x) be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and g(x) be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$ is **[AIEEE 2003]**

(A) $e + \frac{e^2}{2} + \frac{5}{2}$ (B) $e - \frac{e^2}{2} - \frac{5}{2}$

(C) $e + \frac{e^2}{2} - \frac{3}{2}$ (D) $e - \frac{e^2}{2} - \frac{3}{2}$

39. If $f(x) = \frac{e^x}{1+e^x}, I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}\,dx$ and

$I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}\,dx$, then the value of $\frac{I_2}{I_1}$ is-

[AIEEE 2004]

(A) 2 (B) –3

(C) –1 (D) 1

40. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx$, $a > 0$, is -

[AIEEE-2005 IIT-97, 2000]

(A) $a\pi$ (B) $\pi/2$

(C) π/a (D) 2π

41. $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)\,dx$ is equal to

[AIEEE 2006]

(A) $(\pi^4/32) + (\pi/2)$ (B) $\pi/2$

(C) $(\pi/4)-1$ (D) $\pi^4/32$

42. The value of $\int_1^a [x]\, f'[x]\,dx$, $a > 1$, where $[x]$ denotes the greatest integer not exceeding x is

[AIEEE 2006]

(A) $[a]\, f(a) - \{f(1) + f(2) + ... + f([a])\}$

(B) $[a]\, f([a]) - \{f(1) + f(2) + ... + f(a)\}$

(C) $a\, f([a]) - \{(1) + f(2) + ... + f(a)\}$

(D) $a\, f(a) - \{f(1) + f(2) + ... + f([a])\}$

43. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t}\,dt$.

Then F(e) equals **[AIEEE 2007]**

(A) 1/2 (B) 0

(C) 1 (D) 2

44. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}\,dx$. Then which one of the following is true? **[AIEEE 2008]**

(A) $I < \frac{2}{3}$ amd $J < 2$ (B) $I < \frac{2}{3}$ and $J > 2$

(C) $I > \frac{2}{3}$ and $J < 2$ (D) $I > \frac{2}{3}$ and $J > 2$

45. Let p (x) be a function defined on R such that $p'(x) = p'(1-x)$, for all $x \in [0,1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals **[AIEEE 2010]**

(A) $\sqrt{41}$ (B) 21

(C) 41 (D) 42

46. Let $T > 0$ be a fixed real number. Suppose f is a continuous function such that for all $x \in R$, $f(x + T) = f(x)$. If $I = \int_0^T f(x)\,dx$ then the value of $\int_3^{3+3T} f(2x)\,dx$ is **[JEE 2002 (Scr.)]**

(A) $\frac{3}{2}I$ (B) 2I

(C) 3I (D) 6 I

47. If f is an even function then prove that $\int_0^{\pi/2} f(\cos 2x) \cos\,dx = \sqrt{2}\int_0^{\pi/4} f(\sin 2x)\cos x\,dx$

[JEE 2003 (Mains, 2]

48. If $\int_{\sin x}^{1} t^2 (f(t))\,dt = (1-\sin x)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is

[JEE 2005 (Scr.)]

(A) 1/3 (B) $1/\sqrt{3}$

(C) 3 (D) $\sqrt{3}$

(b) Evaluate :

$$\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right) + 3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x\,dx.$$

49. Let f be a non-negative function defined on the interval [0,1]. If $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt$, $0 \le x \le 1$, and $f(0) = 0$, then

[JEE 2009,3+4+,]

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

(D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

50. If $\int_0^{\pi/3} \frac{\cos x}{3+4\sin x}dx = k\log\left(\frac{3+2\sqrt{3}}{3}\right)$ then k is-

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{8}$

51. $\int_{e^{e^e}}^{e^{e^{e^e}}} \frac{dx}{x\ell nx.\ell n(\ell nx).\ell n(\ell n(\ell nx))}$ equals -

(A) 1 (B) 1/e

(C) e – 1 (D) 1 + e

52. The value of the definite integral $\int_1^{\infty}(e^{x+1}+e^{3-x})^{-1}\,dx$ is

(A) $\frac{\pi}{4e^2}$ (B) $\frac{\pi}{4e}$

(C) $\frac{1}{e^2}\left(\frac{\pi}{2}-\tan^{-1}\frac{1}{e}\right)$ (D) $\frac{\pi}{2e^2}$

53. Let a, b, c be non-zero real numbers such that ;

$\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx = \int_0^2 (1+\cos^8 x)(ax^2+bx+c)dx$

then the quadratic equation $ax^2 + bx + c = 0$ has -

(A) no root in (0,2)
(B) atleast one root in (0,2)
(C) a double root in (0,2)
(D) none

54. $\int_0^{\infty}\frac{x\tan^{-1}x}{(1+x^2)^2}dx$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{8}$

55. $\int_{1/2}^{2}\frac{1}{x}\sin\left(x-\frac{1}{x}\right)dx$ has the value equal to -

(A) 0 (B) $\frac{3}{4}$

(C) $\frac{5}{4}$ (D) 2

56. $\int_2^4\left[\log_x 2-\frac{(\log_x 2)^2}{\ell n2}\right]dx =$

(A) 0 (B) 1
(C) 2 (D) 4

57 Suppose that F(x) is an antiderivative of $f(x) = \frac{\sin x}{x}$, x > 0 then $\int_1^3 \frac{\sin 2x}{x}dx$ can be expressed as

(A) F(6) – F(2) (B) $\frac{1}{2}$(F(6) – F(2))

(C) $\frac{1}{2}$(F(3) – F(1)) (D) 2(F(6) – F(2))

58. $\int_0^{\infty} f\left(x+\frac{1}{x}\right).\frac{\ell nx}{x}dx$

(A) is equal to zero (B) is equal to one

(C) is equal to $\frac{1}{2}$ (D) can not be evaluated

59. Integral $\int_0^1 |\sin 2\pi x|\,dx$ is equal to -

(A) 0 (B) $-\frac{1}{\pi}$

(C) $\frac{1}{\pi}$ (D) $\frac{2}{\pi}$

60. $\int_2^3 \frac{\sqrt{x}}{\sqrt{(5-x)}+\sqrt{x}}dx =$

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{5}$ (D) none