Level-I

Chapter 12

Three Dimensional Geometry

Solutions (Set-1)

Very Short Answer Type Questions :

1. Find the vector joining the points $P(2, 3, 0)$ and $Q(-1, -2, -4)$ directed from P to Q. Also find direction ratio and direction cosine.

Sol. $\overrightarrow{PQ} = -3\hat{i} - 5\hat{j} - 4\hat{k}$

Direction ratio is < –3, –5, –4 >

Direction cosine is $< \frac{-3}{5\sqrt{2}}, \frac{-1}{\sqrt{2}}, \frac{-4}{5\sqrt{2}} >$

2. The Cartesian equation of a line is $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$. Find the vector equation of the line.

Sol. $\vec{r} = (-3\hat{i} + 5\hat{j} - 6\hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 2\hat{k})$

3. Find the distance between the two parallel planes

$\pi_1 : x + 2y - 5z = 2$ ...(i)

$\pi_2 : x + 2y - 5z = 11$ ...(ii)

Sol. $\frac{9}{\sqrt{30}}$

4. Find the equation of line through the origin and orthogonal to the plane $2x - y + 3z = 1$.

Sol. Direction ratios are < 2, – 1, 3 >

Line passes through (0, 0, 0)

$\frac{x}{2} = \frac{y}{-1} = \frac{z}{3}$

5. Find the angle between lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$, $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$.

Sol. $\cos\theta = \left|\frac{8+2+8}{3\times 9}\right| = \frac{2}{3}$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

6. Find the coordinates of the foot of perpendicular drawn from the point (2, 5, 7) on the x-axis.

Sol. (2, 0, 0)

7. For any two lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \mu\vec{d}$. Find the condition that these two lines are coplanar.

Sol. $(\vec{a} - \vec{c})\cdot(\vec{b} \times \vec{d}) = 0$

8. Find a point through which line $\vec{r} = (1+t)\hat{i} + (2+3t)\hat{j} + (3-2t)\hat{k}$ passes.

Sol. (1, 2, 3)

9. Find the vector equation of a plane which is at a distance of $\frac{6}{\sqrt{33}}$ units from origin and its normal vector from origin is $2\hat{i} + 2\hat{j} - 5\hat{k}$.

Sol. Direction cosine of normal vector is $\frac{2\hat{i}}{\sqrt{33}} + \frac{2\hat{j}}{\sqrt{33}} - \frac{5\hat{k}}{\sqrt{33}}$

$$\vec{r}\cdot\left(\frac{2\hat{i} + 2\hat{j} - 5\hat{k}}{\sqrt{33}}\right) = \frac{6}{\sqrt{33}}$$

$$\boxed{\vec{r}\cdot(2\hat{i} + 2\hat{j} - 5\hat{k}) = 6}$$

10. Planes are drawn through the points (4, 0, 2) and (2, 3, –2) parallel to coordinate planes. Find the lengths of the edges of the rectangular parallelopiped so formed.

Sol. So, length of edges will be |4 – 2| = 2

|0 – 3| = 3

(2 – (–2)) = 4

∴ Length of rectangular parallelopiped are 2, 3, 4.

Short Answer Type Questions :

11. Find the angle between the two lines which are in the same direction as the two vectors $\vec{c} = 3\hat{i} + 3\hat{j} + 5\hat{k}$, $\vec{d} = -4\hat{i} - 3\hat{j} - 7\hat{k}$.

Sol. $\cos\theta = \left(\frac{+12+9+35}{\sqrt{74}\sqrt{43}}\right)$

$$\theta = \cos^{-1}\left(\frac{56}{\sqrt{74}\sqrt{43}}\right)$$

12. Find the vector and Cartesian equation of the line through the point (5, 2, –4) and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.

Sol. $\vec{r} = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$ [Vector form]

$\frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$ [Cartesian form]

13. Find the shortest distance between the l_1 and l_2 whose vector equations are

$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

Sol. Shortest distance $= \left|\dfrac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$= \left|\frac{(\hat{i} - \hat{k})\cdot(3\hat{i} - \hat{j} - 7\hat{k})}{\sqrt{59}}\right|$$

$$\vec{b}_1 \times \vec{b}_2 = 3\hat{i} - \hat{j} - 7\hat{k}$$

$$= \frac{3+7}{\sqrt{59}}$$

$$= \frac{10}{\sqrt{59}}$$

14. Show that the lines $\dfrac{x-5}{7} = \dfrac{y+2}{-5} = \dfrac{z}{1}$ and $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$ are perpendicular to each others.

Sol. For lines to be perpendicular $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$7(1) - 5(2) + 3(1) = 0$

15. Find the vector equation of the plane passing through the points $R(2, 5, -3)$, $Q(-2, -3, 5)$ and $T(5, 3, -3)$.

Sol. $\overrightarrow{RQ} \times \overrightarrow{RT} = 16\hat{i} + 24\hat{j} + 32\hat{k}$

$[\vec{r} - (2\hat{i} + 5\hat{j} - 3\hat{k})]\cdot(16\hat{i} + 24\hat{j} + 32\hat{k}) = 0$

$\boxed{\vec{r}\cdot(2\hat{i} + 3\hat{j} + 4\hat{k}) = 7}$

16. Find the Cartesian equation of the plane $\vec{r}\cdot(\hat{i} + \hat{j} - \hat{k}) = 2$.

Sol. $x + y - z = 2$

17. Find the shortest distance between the lines $\dfrac{x+1}{7} = \dfrac{y+1}{-6} = \dfrac{z+1}{1}$ and $\dfrac{x-3}{1} = \dfrac{y-5}{-2} = \dfrac{z-7}{1}$.

Sol. Shortest distance $= \left|\dfrac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$= \left|\frac{(4\hat{i} + 6\hat{j} + 8\hat{k})\cdot(-4\hat{i} - 6\hat{j} - 8\hat{k})}{\sqrt{4^2 + 6^2 + 8^2}}\right|$$

$$= \frac{4^2 + 6^2 + 8^2}{\sqrt{4^2 + 6^2 + 8^2}}$$

$$= 2\sqrt{29}$$

18. Find the vector and Cartesian equation of the plane which passes through the point (5, 2, –4) and perpendicular to the line with direction ratios (2, 3, –1).

Sol. $[\vec{r}-(5\hat{i}+2\hat{j}-4\hat{k})]\cdot(2\hat{i}+3\hat{j}-\hat{k})=0$

$\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})=20$ [Vector form]

$2x + 3y - z = 20$ [Cartesian form]

19. Find the parametric form of the line given by the equations $2x - y + 3z = 1$, $x + 5y - 2z = 0$.

Sol. $2x - y + 3z = 1$, $x + 5y - 2z = 0$

$\boxed{x = 2z - 5y}$

Substituting x in first equation

$7z - 11y = 1$

$y = k$

$z=\dfrac{11k+1}{7}$

$x=\dfrac{2}{7}-\dfrac{13}{7}k$

$$\boxed{\therefore \frac{x-\frac{2}{7}}{-\frac{13}{7}}=\frac{y}{1}=\frac{z-\frac{1}{7}}{\frac{11}{7}}}$$

20. Find the foot of perpendicular drawn from the point $P(2, -3, 4)$ to the plane $x + 2y + 2z = 13$.

Sol. $P(2, -3, 4)$ to plane $x + 2y + 2z = 13$

Direction ratio perpendicular to plane are < 1, 2, 2 >

Let Q be perpendicular to plane (x, y, z)

$$\frac{x-2}{1}=\frac{y+3}{2}=\frac{z-4}{2}=\lambda$$

Any general point on line is $(\lambda + 2, 2\lambda - 3, 2\lambda + 4)$

This satisfy the plane $x + 2y + 2z = 13$

$\lambda + 2 + 2(2\lambda - 3) + 2(2\lambda + 4) = 13$

$9\lambda + 4 = 13$

$\boxed{\lambda = 1}$

$x = 3$, $y = -1$, $z = 6$

Point Q be $(3, -1, 6)$

21. Find the equation of plane that passes through the point (1, –1, 1) and is perpendicular to the plane $3x + 4y - 2z = 3$ and $x - y + 2z = 4$.

Sol. Let equation of plane is

$ax + by + cz = d$ ...(i)

As the plane is perpendicular to $3x + 4y - 2z = 3$, $x - y + 2z = 4$

$\therefore \quad 3a + 4b - 2c = 0$

$\quad a - b + 2c = 0$

Adding both

$4a + 3b = 0$

$\boxed{b = \frac{-4a}{3}}$ $\boxed{c = \frac{-7a}{6}}$

Substituting in equation (i)

$$ax - \frac{4a}{3}y - \frac{7a}{6}z = d$$

$$a\left(x - \frac{4}{3}y - \frac{7}{6}z\right) = d$$

Plane passes through (1, –1, 1)

$$a\left(1 + \frac{4}{3} - \frac{7}{6}\right) = d$$

$$\boxed{a\left(\frac{7}{6}\right) = d}$$

$\therefore$ Equation of plane is $x - \frac{4}{3}y - \frac{7}{6}z = \frac{7}{6}$

$$\boxed{6x - 8y - 7z = 7}$$

22. Find the coordinates of the point where the line through the points $A(3, 4, -1)$ and $B(2, -4, -3)$ crosses the zx plane.

Sol. $A(3, 4, -1)$ and $B(2, -4, -3)$

$$\frac{x-3}{1} = \frac{y-4}{8} = \frac{z+1}{2} = \lambda$$

Any general point on plane is $(\lambda + 3, 8\lambda + 4, 2\lambda - 1)$

As plane passes through XZ plane, $y = 0$

$$\boxed{\lambda = \frac{-1}{2}}$$

Point is $\left(\frac{5}{2}, 0, -2\right)$

23. Find the shortest distance between the lines l_1 and l_2 given by

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k}).$$

Sol. Shortest distance $= \left|\dfrac{(\vec{a}_2 - \vec{a}_1)\times\vec{b}}{|\vec{b}|}\right|$

$$= \left|\frac{[(2\hat{i}+\hat{j}-\hat{k})\times(2\hat{i}+3\hat{j}+6\hat{k})}{7}\right| \qquad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \left|\frac{9\hat{i}-14\hat{j}+4\hat{k}}{7}\right|$$

$$= \frac{\sqrt{293}}{7}$$

24. Find the vector and Cartesian equation of the plane which passes through the point (5, 2, –4) and perpendicular to the line with direction ratios (2, 3, –1).

Sol. $[\vec{r}-(5\hat{i}+2\hat{j}-4\hat{k})]\cdot[2\hat{i}+3\hat{j}-\hat{k}]=0$

$\boxed{\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k}]=20}$ [Vector equation]

$\boxed{2x+3y-z=20}$ [Cartesian equation]

25. Find the Cartesian equation of the plane $\vec{r}\cdot[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}]=15$.

Sol. $(5 - 2t)x + (3 - t)y + (25 + t)z = 15$

26. Find the point of intersection of the lines

$$\frac{x-\alpha}{\alpha'}=\frac{y-\beta}{\beta'}=\frac{z-\gamma}{\gamma'} \text{ and } \frac{x-\alpha'}{\alpha}=\frac{y-\beta'}{\beta}=\frac{z-\gamma'}{\gamma} \text{ (here } \alpha'\beta\neq\alpha\beta'\text{)}$$

Sol. $\dfrac{x-\alpha}{\alpha'}=\dfrac{y-\beta}{\beta'}=\dfrac{z-\gamma}{\gamma'}=\lambda$

Any general point is $(\alpha'\lambda+\alpha,\ \beta'\lambda+\beta,\ \gamma\lambda+\gamma)$ ...(i)

$\dfrac{x-\alpha'}{\alpha}=\dfrac{y-\beta'}{\beta}=\dfrac{z-\gamma'}{\gamma}=\mu$

Any general points $(\alpha\mu+\alpha',\ \beta\mu+\beta',\ \gamma\mu+\gamma')$

Now equating these points

$\alpha'\lambda+\alpha=\alpha\mu+\alpha'$ and $\beta'\lambda+\beta=\beta\mu+\beta'$

$\lambda=\dfrac{\alpha\mu+\alpha'-\alpha}{\alpha'}$ $\lambda=\dfrac{\beta\mu+\beta'-\beta}{\beta'}$ ...(ii)

$\dfrac{\alpha\mu+\alpha'-\alpha}{\alpha'}=\dfrac{\beta\mu+\beta'-\beta}{\beta'}$

$\alpha\beta'\mu+\alpha'\beta'-\alpha\beta'=\beta\alpha'\mu+\beta'\alpha'-\beta\alpha'$

$(\alpha\beta'-\beta\alpha')\mu=\alpha\beta'-\beta\alpha'$

$\boxed{\mu=1}$

Put $\mu = 1$ in equation in (ii), we get $\lambda = 1$

$\therefore$ Point of intersection is $(\alpha+\alpha',\ \beta+\beta',\ \gamma+\gamma')$

27. Find the foot of the perpendicular drawn from the point (6, –2, 5) to the straight line passing through the point (4, –1, 2) having direction ratios 1, 4, –3.

Sol. $\frac{x-4}{1}=\frac{y+1}{4}=\frac{z-2}{-3}=\lambda$

Any general point on line is $(\lambda + 4, 4\lambda - 1, -3\lambda + 2)$

Direction ratio of line $(\lambda - 2, 4\lambda + 1, -3\lambda - 3)$

As this is perpendicular to first line

$(\lambda - 2) + 4(4\lambda + 1) + 3(3\lambda + 3) = 0$

$26\lambda + 11 = 0$

$\lambda = \frac{-11}{26}$

Point (foot of perpendicular) is = $\left(\frac{93}{26}, \frac{-35}{13}, \frac{85}{26}\right)$

28. A line $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z-\frac{9}{2}}{3}$ cuts the *y-z* plane and *x-y* plane at *A* and *B* respectively. Find $\angle AOB$, *O* being origin.

Sol. General point is $\left(\lambda-2,\ 2\lambda+3,\ 3\lambda+\frac{9}{2}\right)$

Cuts *YZ* plane *i.e.*, $x = 0$ $\therefore$ $\lambda = 2$

$A=\left(0,\ 7,\ \frac{21}{2}\right)$

Cuts the *XY* plane $z = 0$ $\therefore$ $\lambda=\frac{-9}{6}=\frac{-3}{2}$

$B=\left(\frac{-7}{2},\ 0,\ 0\right)$

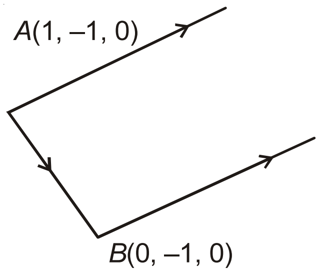
$\angle AOB$ $\cos\theta = 0$

$\boxed{\theta = 90^\circ}$

29. Find the equation of plane containing the two lines $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{3}$ and $\frac{x}{2}=\frac{y+1}{-1}=\frac{z}{3}$.

Sol. $A = (1, -1, 0)$ Direction ratio L < 2, –1, 3 >

$B = (0, -1, 0)$



$\overrightarrow{AB}=\hat{i}$

$$\overrightarrow{AB}\times\vec{L}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & 0\\ 2 & -1 & 3\end{vmatrix}=-3\hat{j}-\hat{k}$$

$[\vec{r}-(\hat{i}-\hat{j})]\cdot[-3\hat{j}-\hat{k}]=0$

$\boxed{\vec{r}\cdot(3\hat{j}+\hat{k})=-3}$

30. Find the equation of plane which passes through the point (3, 2, 0) and containing the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$.

Sol. A = (3, 6, 4), B = (3, 2, 0)

$\vec{b}=\hat{i}+5\hat{j}+4\hat{k}$

$$\overrightarrow{AB}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 4 & 4\\ 1 & 5 & 4\end{vmatrix}=-4\hat{i}+4\hat{j}-4\hat{k}$$

$[\vec{r}-(3\hat{i}+2\hat{j})]\cdot(-4\hat{i}+4\hat{j}-4\hat{k})=0$

$\boxed{\vec{r}(\hat{i}-\hat{j}+\hat{k})=1}$

Long Answer Type Questions :

31. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point (2, 2, 1).

Sol. $(3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0$

As plane passes through (2, 2, 1)

$2 + \lambda(3) = 0$

$\boxed{\lambda=\frac{-2}{3}}$

Equation of plane is $7x - 5y + 4z - 8 = 0$

32. If the points (1, 1, p) and (–3, 0, 1) be equidistant from the plane $\vec{r}\cdot(3\hat{i}+4\hat{j}-12\hat{k})+13=0$, then find the value of p.

Sol. $3x + 4y - 12z + 13 = 0$

$$\left|\frac{3+4-12p+13}{\sqrt{3^2+4^2+12^2}}\right|=\left|\frac{-9-12+13}{\sqrt{3^2+4^2+12^2}}\right|$$

$|20-12p|=|-8|$

$20-12p=\pm 8$

$\boxed{p=1,\ p=\frac{7}{3}}$

33. (i) Find the point where the plane $\vec{r}\cdot(\hat{i}+2\hat{j}+\hat{k})=4$ meets the line $\vec{r}=(2\hat{i}+\hat{j}-\hat{k})+t(\hat{i}+\hat{j}+2\hat{k})$.

(ii) Find the angle that the line makes with the plane.

Sol. (i) General point on line $(2 + t, 1 + t, -1 + 2t)$

It satisfy $x + 2y + z = 4$

$(2 + t) + 2(1 + t) + (2t - 1) = 4$

$5t = 1$

$t = \frac{1}{5}$

Point $\left(\frac{7}{5}, \frac{6}{5}, \frac{-3}{5}\right)$

(ii) $\sin\theta = \left(\frac{1+2+2}{\sqrt{6}\sqrt{6}}\right)$

$\sin\theta = \frac{5}{6}$

$\theta = \sin^{-1}\left(\frac{5}{6}\right)$

34. Show that the line $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar.

Sol. Two lines are coplanar if $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 0$

$$\begin{vmatrix} b+d-(a+c) & b-a & (b+c)-(a+d) \\ \alpha-\delta & \alpha & \alpha+\delta \\ \beta-\gamma & \beta & \beta+\gamma \end{vmatrix}$$

$C_1 \to C_1 + C_3$

$$= \begin{vmatrix} 2(b-a) & b-a & (b+c)-(a+d) \\ 2\alpha & \alpha & \alpha+\delta \\ 2\beta & \beta & \beta+\gamma \end{vmatrix}$$

As C_1 and C_2 are proportional so determinant is zero.

35. Find the equations of straight line perpendicular to the lines

$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$, and passing through their point of intersection.

Sol. Normal vector perpendicular to both the lines is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} = 7\hat{i} + 7\hat{j} + 7\hat{k}$$

Point of intersection of two lines

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1} = 1$$

Any general point is $(-3\lambda - 1, 2\lambda + 3, \lambda - 2)$

$$\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2} = \mu$$

Any general point is $(\mu, 7 - 3\mu, 2\mu - 7)$

$\mu = -3\lambda - 1$ and $7 - 3\mu = 2\lambda + 3$

$\mu + 3\lambda = -1$ ...(i)

$3\mu + 2\lambda = 4$ ...(ii)

Solving $\lambda = -1, \mu = 2$

So, point is (2, 1, –3)

$$\frac{x-2}{7} = \frac{y-1}{7} = \frac{z+3}{7}$$

or $\boxed{\frac{x-2}{1} = \frac{y-1}{1} = \frac{z+3}{1}}$

36. Prove that the line passing through (1, –2, 0) having direction ratios 2, 3, 6 and the line $3(x + 3) = 2(y - 5) = z + 1$ are parallel. Find the distance between the lines.

Sol. $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{6}$ ...(i)

$$\frac{x+3}{\frac{1}{3}} = \frac{y-5}{\frac{1}{2}} = \frac{z+1}{1}$$

or $\frac{x+3}{2} = \frac{y-5}{3} = \frac{z+1}{6}$ ...(ii)

Shortest distance $= \left|\frac{(\vec{a}_2 - \vec{a}_1)\times\vec{b}}{|\vec{b}|}\right|$ $\quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -7 & 1 \\ 2 & 3 & 6 \end{vmatrix}$

Shortest distance $= \left|\frac{-45\hat{i} - 22\hat{j} + 26\hat{k}}{\sqrt{2^2+3^2+6^2}}\right|$

$$= \frac{\sqrt{45^2+22^2+26^2}}{\sqrt{49}}$$

$$= \frac{7\sqrt{65}}{7} = \sqrt{65}$$

37. Find the shortest distance between the lines $\frac{x-3}{2} = \frac{y-4}{1} = \frac{z+1}{-3}$ and $\frac{x-1}{-1} = \frac{y-3}{3} = \frac{z-1}{2}$ also find the equation of line of shortest distance.

Sol. Shortest distance $= \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$$= \left|\frac{(2\hat{i}+\hat{j}-2\hat{k})\cdot(11\hat{i}-\hat{j}+7\hat{k})}{\sqrt{11^2+1^2+7^2}}\right| \qquad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ -1 & 3 & 2 \end{vmatrix} = 11\hat{i} - \hat{j} + 7\hat{k}$$

$$= \frac{7}{\sqrt{171}} = \frac{7}{3\sqrt{19}} = \frac{7\sqrt{19}}{57}$$

38. Prove that the straight lines $\frac{x}{\alpha}=\frac{y}{\beta}=\frac{z}{\gamma}$, $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ and $\frac{x}{a\alpha}=\frac{y}{b\beta}=\frac{z}{c\gamma}$ are coplanar if

$\frac{l}{\alpha}(b-c)+\frac{m}{\beta}(c-a)+\frac{n}{\gamma}(a-b)=0$.

Sol. $\begin{vmatrix} \alpha & \beta & \gamma \\ l & m & n \\ a\alpha & b\beta & c\gamma \end{vmatrix}$

Equation through second row

$l(c\beta\gamma - b\beta\gamma) - m(c\alpha\gamma - a\alpha\gamma) + n(2b\beta - a\alpha\beta) = 0$

$l(\beta\gamma)(c-b) - m(c-a)\alpha\gamma + n(b-a)\alpha\beta$

Divide by $\alpha\beta\gamma$

$\frac{l}{\alpha}(c-b)-\frac{m}{\beta}(c-a)+\frac{n}{\gamma}(b-a)=0$

39. Find the equation of line through the intersection of two planes $x + y - 2z = 1$, and $x + 3y - z = 4$, and also find its perpendicular distance from $c(1, 0, 1)$.

Sol. $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 3 & -1 \end{vmatrix} = 5\hat{i} - \hat{j} + 2\hat{k}$

$x + y - 2z = 1$

$\underline{x + 3y - z = 4}$

$\underline{-2y - z = -3}$

$\boxed{2y + z = 3}$

Put $z = 1$, so $y = 1$, hence $x = 2$

Point on line is (2, 1, 1)

$\therefore$ Equation of line is $\frac{x-2}{5}=\frac{y-1}{-1}=\frac{z-1}{2}$

Shortest distance from $C(1, 0, 1)$ is $\left|\frac{(\vec{a}_2-\vec{a}_1)\times(\vec{b})}{|\vec{b}|}\right| = \frac{(\hat{i}+\hat{j})\times(5\hat{i}-\hat{j}+2\hat{k})}{\sqrt{25+1+4}}$ $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 5 & -1 & 2 \end{vmatrix}$

$= \frac{|2\hat{i}-2\hat{j}-6\hat{k}|}{\sqrt{30}}$

$= \frac{\sqrt{44}}{\sqrt{30}}$

$\boxed{\text{Shortest distance} = \sqrt{\frac{22}{15}}}$

40. Find the linear equation describing the plane perpendicular to the line of intersection of the planes $x + y - 2z = 4$ and $3x - 2y + z = 1$ and which passes through (6, 0, 2), also find foot of perpendicular from origin to this plane.

Sol. So line perpendicular to plane is

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 3 & -2 & 1 \end{vmatrix} = -3\hat{i} - 7\hat{j} - 5\hat{k}$$

$$[\vec{r} - (6\hat{i} + 2\hat{k})] \cdot [-3\hat{i} - 7\hat{j} - 5\hat{k}] = 0$$

Solving it

$$\vec{r} \cdot (3\hat{i} + 7\hat{j} + 5\hat{k}) = 28$$

$3x + 7y + 5z = 28$

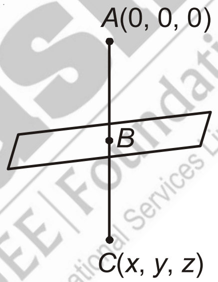
Equation of perpendicular line is

$$\frac{x}{3} = \frac{y}{7} = \frac{z}{5} = \lambda$$

Any general point on line is

$B = (3\lambda, 7\lambda, 5\lambda)$

Now the point satisfy the plane



$\therefore \quad 9\lambda + 49\lambda + 25\lambda = 28$

$83\lambda = 28$

$$\lambda = \frac{28}{83}$$

Now point $B = \left(\frac{84}{83}, \frac{196}{83}, \frac{140}{83}\right)$

Now, $\frac{x+0}{2} = \frac{84}{83} \Rightarrow x = \frac{168}{83}$

$\frac{y+0}{2} = \frac{196}{83} \Rightarrow y = \frac{392}{83}$

$\frac{z+0}{2} = \frac{140}{83} \Rightarrow z = \frac{280}{83}$

$$\boxed{\text{Point is } \left(\frac{168}{83}, \frac{392}{83}, \frac{280}{83}\right)}$$

41. The points A (1, 1, 5), B(2, 2, 1), C(1, –2, 2) and D(–2, 1, 2) are vertices of tetrahedron. Find the equation of the line through A perpendicular to face BCD and distance of A from this face. Also find the shortest distance between skew lines AD and BC.

Sol. Equation of plane BCD

$$(\vec{r} - \vec{B}) \cdot (\overrightarrow{BC} \times \overrightarrow{BD}) = 0$$

$$\overrightarrow{BC} \times \overrightarrow{BD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -4 & 1 \\ -4 & -1 & 1 \end{vmatrix}$$

$$[\vec{r} - (2\hat{i} + 2\hat{j} + \hat{k})] \cdot [-3\hat{i} - 3\hat{j} - 15\hat{k}] = 0$$

$$= -3\hat{i} - 3\hat{j} - 15\hat{k}$$

$\boxed{\vec{r}\cdot[\hat{i}+\hat{j}+3\hat{k}]=9}$

Equation of line through A perpendicular to $(\hat{i}+\hat{j}+5\hat{k})$

$$\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-5}{5}$$

Distance from A is = $\frac{18}{\sqrt{27}}=2\sqrt{3}$

Line AD and Line BC

$$\text{Shortest distance} = \left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$$= \left|\frac{(\hat{i}+\hat{j}-4\hat{k})\cdot(-12\hat{i}+6\hat{j}+12\hat{k})}{6(\sqrt{9})}\right|$$

$$\overrightarrow{AD}\times\overrightarrow{BC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -3 & 0 & -3\\ -1 & -4 & 1\end{vmatrix}$$

$$= \left|\frac{-12+6-48}{6\times3}\right|$$

$$= -12\hat{i}+6\hat{j}+12\hat{k}$$

$$= \frac{54}{6\times3}$$

$\boxed{\text{Shortest distance} = 3}$

42. Find the equations of two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle of $\frac{\pi}{3}$.

Sol. $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ is equation of line through origin

$l^2+m^2+n^2=1$ ...(i)

Now as the two lines are coplanar so $\begin{vmatrix} l & m & n\\ 3 & 3 & 0\\ 2 & 1 & 1\end{vmatrix}=0$

$3l - 3m - 3n = 0$

$\boxed{l=m+n}$ ...(ii)

$$\cos\theta=\left|\frac{2l+m+n}{\sqrt{6}\sqrt{1}}\right|$$

$$\frac{1}{2}=\left|\frac{2l+m+n}{\sqrt{6}}\right|$$

Substituting $m + n$ from (ii)

$$3l = \pm\frac{\sqrt{6}}{2}$$

$$l = \pm\frac{\sqrt{6}}{6} = \pm\frac{1}{\sqrt{6}}$$

From (i) $\Rightarrow$ $m^2 + n^2 = \frac{5}{6}$

From (ii) $\Rightarrow$ $(m+n) = \frac{1}{\sqrt{6}}$

$\Rightarrow$ $mn = \frac{-1}{3}$

$\Rightarrow$ $m - n = \sqrt{\frac{3}{2}} = \pm\frac{3}{\sqrt{6}}$

$$l = \frac{1}{\sqrt{6}}$$

$$m = \frac{2}{\sqrt{6}} \text{ or } \frac{-1}{\sqrt{6}}$$

$$n = \frac{-1}{\sqrt{6}} \text{ or } \frac{2}{\sqrt{6}}$$

Equation is $\boxed{\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}}$ Or $\boxed{\frac{x}{1} = \frac{y}{-2} = \frac{z}{2}}$

43. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Sol. Any general point on line is (λ, $2\lambda + 1$, $3\lambda + 2$)

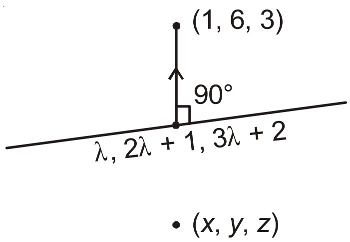
Direction ratio of perpendicular line is ($\lambda - 1$, $2\lambda - 5$, $3\lambda - 1$)

$1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

$\lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

$14\lambda - 14 = 0$

$\boxed{\lambda = 1}$



Point (1, 3, 5)

Now this point is mid-point as shown in figure

Therefore, $\frac{x+1}{2} = 1$, $\frac{y+6}{2} = 3$, $\frac{z+3}{2} = 5$

$x = 1$ $y = 0$ $z = 7$

$\boxed{\text{Point } (1, 0, 7)}$

44. By computing the shortest distance, determine whether the following pairs of line intersect or not :

(i) $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$, $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} - \hat{k})$

(ii) $\dfrac{x-5}{4} = \dfrac{y-7}{-5} = \dfrac{z+3}{-5}$, $\dfrac{x-8}{7} = \dfrac{y-7}{1} = \dfrac{z-5}{3}$

Sol. (i) $\left|\dfrac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$ $\qquad \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -1 & -1 \end{vmatrix}$

$= \dfrac{(\hat{i})\cdot(\hat{i} + 3\hat{j} - 2\hat{k})}{\sqrt{14}}$

$= \dfrac{1}{\sqrt{14}} \neq 0$

Do not intersect.

(ii) $\dfrac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{(\vec{b}_1 \times \vec{b}_2)}$ $\qquad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix}$

$= \dfrac{(3\hat{i} - 8\hat{k})\cdot(-10\hat{i} - 47\hat{j} + 39\hat{k})}{\sqrt{10^2 + 47^2 + 39^2}}$

$= \left|\dfrac{-30 - 8(39)}{\sqrt{10^2 + 47^2 + 39^2}}\right| \neq 0$

Hence do not intersect.

❑ ❑ ❑

Chapter 12

Three Dimensional Geometry

Solutions (Set-2)

[**Direction Ratios and Cosines**]

1. Angle between diagonals of a cube can be

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$ (3) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $\cos^{-1}\left(\frac{1}{3}\right)$

Sol. Answer (4)

Angle between diagonals of a cube

$$\theta = \cos^{-1}\frac{a^2}{\sqrt{3}a \times \sqrt{3}a} = \cos^{-1}\left(\frac{1}{3}\right)$$

2. The angle between diagonal of a cube and diagonal of a face of the cube will be

(1) $\cos^{-1}\sqrt{\frac{2}{3}}$ (2) $\sin^{-1}\sqrt{\frac{2}{3}}$ (3) $\sin^{-1}\left(\frac{2}{3}\right)$ (4) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Sol. Answer (1)

3. A line makes equal angles with the coordinate axis. The direction cosines of this line are

(1) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (2) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (3) $\left(\frac{1}{\sqrt{3}}, \frac{1}{3}, \frac{1}{3}\right)$ (4) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Sol. Answer (2)

4. The angle between the two lines whose d.c. are given by equations $3l + m + 5n = 0$ and $6mn - 2nl + 5ml = 0$

(1) $\cos^{-1}\left(\frac{1}{3}\right)$ (2) $\cos^{-1}\left(\frac{1}{6}\right)$ (3) $\cos^{-1}\left(\frac{2}{3}\right)$ (4) $\cos^{-1}\left(\frac{1}{4}\right)$

Sol. Answer (2)

$3l + m + 5n = 0$

$6mn - 2nl + 5ml = 0$

Similar to (37)

$$\theta = \cos^{-1}\left(\frac{1}{6}\right)$$

5. If direction cosines of a line are $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$ then

(1) $a > 0$ (2) $0 < a < 1$ (3) $a = \pm\sqrt{3}$ (4) $a > 2$

Sol. Answer (3)

$\frac{3}{a^2} = 1 \Rightarrow a = \pm\sqrt{3}$

6. Let $A \equiv (1, 2, 3)$, $B \equiv (3, 4, 5)$ then

(1) Direction ratios of AB line cannot be (2, 2, 2)

(2) Direction ratios of $\overrightarrow{AB}$ are (–1, 1, –1)

(3) Direction ratios of $\overrightarrow{AB}$ are (1, 1, 1)

(4) Direction cosine of AB line may be $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}},\right)$

Sol. Answer (3)

(1) Direction ratios of AB = (2, 2, 2)

(3) Direction ratios of $\overrightarrow{AB}$ = (1, 1, 1)

And direction cosine of line AB = $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ or $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

7. The distance of point $P(a, b, c)$ from the x-axis is

(1) $\sqrt{b^2+c^2}$ (2) $\sqrt{a^2+c^2}$ (3) $\sqrt{a^2+b^2}$ (4) a

Sol. Answer (1)

The distance of (a, b, c) from x-axis = $\sqrt{b^2+c^2}$

8. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(1) 0º (2) 90º (3) 45º (4) 30º

Sol. Answer (2)

$2x = 3y = -z$ and $6x = -y = -4z$

$$\cos\theta = \frac{1}{12} - \frac{1}{3} + \frac{1}{4} = 0$$

$\theta = 90º$.

9. If the lines $\frac{x-0}{1} = \frac{y+1}{2} = \frac{z-1}{-1}$ and $\frac{x+1}{k} = \frac{y-3}{-2} = \frac{z-2}{1}$ are at right angles, then the value of k is

(1) 5 (2) 0 (3) 3 (4) –1

Sol. Answer (1)

Lines are perpendicular

$\therefore$ $1 \times k + 2 \times (-2) + (-1)(1) = 0$

$k - 4 - 1 = 0$

$k = 5$

10. The lines $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{-2k}$ and $\frac{x-1}{k}=\frac{y-2}{3}=\frac{z-6}{1}$ are coplanar if

(1) $k = 0,\ k = 2$ (2) $k=\frac{5+\sqrt{15}}{2},\ k=3$ (3) $k=\frac{5\pm\sqrt{15}}{2}$ (4) $k = 3,\ k = 5$

Sol. Answer (3)

The lines are coplanar if

$$\begin{vmatrix} 1 & 1 & -2 \\ 1 & 2 & -2k \\ k & 3 & 1 \end{vmatrix}=0$$

$1(2 + 6k) - 1\ (1 + 2k^2) - 2\ (3 - 2k) = 0$

$-5 + 10k - 2k^2 = 0$

$2k^2 - 10k + 5 = 0$

$$k=\frac{10\pm\sqrt{100-40}}{4}=\frac{10\pm 2\sqrt{15}}{4}=\frac{5\pm\sqrt{15}}{2}$$

11. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{5t-1}$ and $\frac{x+1}{2s+1}=\frac{y}{2}=\frac{z}{4}$ are parallel to each other, then value of s, t will be

(1) $6,\ \frac{5}{7}$ (2) $\frac{1}{6},\ \frac{7}{5}$ (3) $3,\ \frac{5}{7}$ (4) $4,\ \frac{7}{10}$

Sol. Answer (2)

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{5t-1} \text{ and } \frac{x+1}{2s+1}=\frac{y}{2}=\frac{z}{4}$$

Or

$$\frac{x-1}{\frac{2}{3}}=\frac{y+1}{1}=\frac{z}{\frac{5t-1}{3}} \text{ and } \frac{x+1}{\frac{2s+1}{2}}=\frac{y}{1}=\frac{z}{2}$$

Now comparing direction ratios of two lines as lines are parallel.

$\frac{2s+1}{2}=\frac{2}{3}$ $\qquad \frac{5t-1}{3}=2$

$6s + 3 = 4$ $\qquad 5t = 7$

$6s = 1 \Rightarrow s=\frac{1}{6}$ $\qquad t=\frac{7}{5}$

12. Find the distance of plane through (1, 1, 1) and perpendicular to the line $\frac{x-1}{3}=\frac{y-1}{0}=\frac{z-1}{4}$ from the origin is

(1) $\frac{3}{4}$ (2) $\frac{4}{3}$ (3) $\frac{7}{5}$ (4) 1

Sol. Answer (3)

Equation of plane will be

$[\vec{r} - (\hat{i} + \hat{j} + \hat{k})] \cdot (3\hat{i} + 4\hat{j}) = 0$

$\vec{r} \cdot \dfrac{(3\hat{i} + 4\hat{j})}{5} = \dfrac{+3+4}{5}$

$\vec{r} \cdot \hat{n} = \left|\dfrac{+7}{5}\right|$

13. The equation of line through the point (2, –1, 3) and which is equally inclined to axis

(1) $\dfrac{x-2}{-\frac{1}{\sqrt{3}}} = \dfrac{y+1}{\frac{1}{\sqrt{3}}} = \dfrac{z-3}{\frac{1}{\sqrt{3}}}$

(2) $\dfrac{x-2}{-1} = \dfrac{y+1}{1} = \dfrac{z-3}{-1}$

(3) $\dfrac{x-2}{-1} = \dfrac{y+1}{-1} = \dfrac{z-3}{1}$

(4) $\dfrac{x-2}{\sqrt{3}} = \dfrac{y+1}{\sqrt{3}} = \dfrac{z-3}{\sqrt{3}}$

Sol. Answer (4)

$\dfrac{x-2}{l} = \dfrac{y+1}{m} = \dfrac{z-3}{n}$

$l^2 + m^2 + n^2 = 1$

$3l^2 = 1 \Rightarrow l = \pm\dfrac{1}{\sqrt{3}}$

$\dfrac{x-2}{\frac{1}{\sqrt{3}}} = \dfrac{y+1}{\frac{1}{\sqrt{3}}} = \dfrac{z-3}{\frac{1}{\sqrt{3}}} \qquad \Rightarrow \dfrac{x-2}{\sqrt{3}} = \dfrac{y+1}{\sqrt{3}} = \dfrac{z-3}{\sqrt{3}}$

14. A point at a distance of $\sqrt{6}$ from the origin which lies on the straight line $\dfrac{x-1}{1} = \dfrac{y-2}{2} = \dfrac{z+1}{3}$ will be

(1) (1, –1, 2) (2) (1, 2, –1) (3) $\left(\dfrac{5}{7}, \dfrac{10}{7}, \dfrac{-6}{7}\right)$ (4) $\left(\dfrac{5}{7}, \dfrac{2}{7}, \dfrac{-6}{7}\right)$

Sol. Answer (2)

$x = 1 + \lambda,\ y = 2 + 2\lambda,\ z = 3\lambda - 1$

Distance from origin

$= \sqrt{(1+\lambda)^2 + 4(1+\lambda)^2 + (3\lambda-1)^2} = \sqrt{6}$

$5(1 + \lambda^2 + 2\lambda) + (9\lambda^2 + 1 - 6\lambda) = 6$

$14\lambda^2 + 4\lambda = 0$

$\lambda(14\lambda + 4) = 0 \qquad \Rightarrow \lambda = 0,\ \lambda = \dfrac{-2}{7}$

Point can be (1, 2, –1)

15. The equation of a line passing through (*a*, *b*, *c*) and parallel to *z*-axis is

(1) $\dfrac{x-a}{1} = \dfrac{y-b}{1} = \dfrac{z-c}{0}$

(2) $\dfrac{x-a}{0} = \dfrac{y-b}{1} = \dfrac{z-c}{1}$

(3) $\dfrac{x-a}{1} = \dfrac{y-b}{1} = \dfrac{z-c}{1}$

(4) $\dfrac{x-a}{0} = \dfrac{y-b}{0} = \dfrac{z-c}{1}$

Sol. Answer (4)

Direction ratios of the line are (0, 0, 1)

Hence equation = $\dfrac{x-a}{0}=\dfrac{y-b}{0}=\dfrac{z-c}{1}$

16. The equation of the line passing through the point (1, 2, –4) and perpendicular to lines

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5} \text{ is}$$

(1) $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z+4}{6}$

(2) $\dfrac{x-1}{-2}=\dfrac{y-2}{3}=\dfrac{z+4}{8}$

(3) $\dfrac{x-1}{3}=\dfrac{y-2}{2}=\dfrac{z+4}{8}$

(4) $\dfrac{x+1}{-3}=\dfrac{y+2}{-2}=\dfrac{z-4}{8}$

Sol. Answer (1)

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -16 & 7\\ 3 & 8 & -5\end{vmatrix}=12\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$

So line

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$$

[**Plane**]

17. Let $\vec{a}=\hat{i}+\hat{j}$ and $\vec{b}=2\hat{i}-\hat{k}$, then the point of intersection of the lines $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}$ and $\vec{r}\times\vec{b}=\vec{a}\times\vec{b}$ is

(1) (–1, 1, 1) (2) (3, –1, 1) (3) (3, 1, –1) (4) (1, –1, –1)

Sol. Answer (3)

$\vec{r}\times\vec{a}=\vec{b}\times\vec{a}$

$\Rightarrow$ $(\vec{r}-\vec{b})\times\vec{a}=0$

$\vec{r}-\vec{b}$ is parallel to $\vec{a}$

$\Rightarrow$ $\vec{r}-\vec{b}=t_1$

$\Rightarrow$ $\vec{r}=\vec{b}+\vec{a}t_1$ … (i)

Similarly other line like

$\vec{r}=\vec{a}+\vec{b}t_2$ … (ii)

By (i) and (ii)

$=\vec{b}+\vec{a}t_1=\vec{a}+\vec{b}t_2$

$\Rightarrow$ Clearly $t_1=t_2=1$

Hence point of intersection in $\vec{a}+\vec{b}=(\hat{i}+\hat{j})+(2\hat{i}-\hat{k})$

$=3\hat{i}+\hat{j}-\hat{k}$

$=(3, 1, -1)$

18. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{7}{2}$ (4) $\frac{9}{2}$

Sol. Answer (3)

$2x + y + 2z = 8$

$4x + 2y + 4z = -5 \Rightarrow 2x + y + 2z = -\frac{5}{2}$

Distance between these two planes $= \dfrac{8+\frac{5}{2}}{3}$

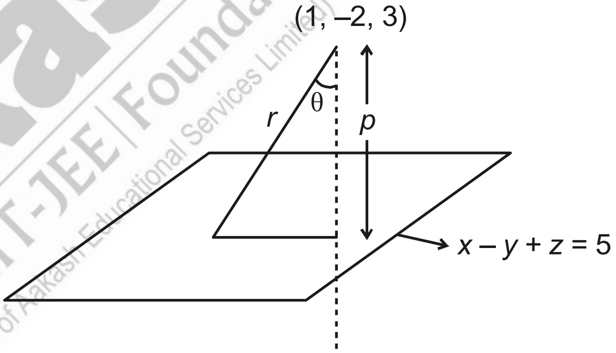
$= \frac{21}{6} = \frac{7}{2}$

19. The distance of a point (1, –2, 3) from the plane $x - y + z = 5$ and parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

(1) 7 (2) 1 (3) 3 (4) 13

Sol. Answer (2)

$p = \dfrac{1+2+3-5}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$



Distance along given line $= \dfrac{p}{\cos\theta}$

$\cos\theta = \dfrac{2-3-6}{\sqrt{3}\,\sqrt{49}} = \left|\dfrac{1}{\sqrt{3}}\right|$

Distance along given line $= \dfrac{1}{\sqrt{3}} \times \dfrac{\sqrt{3}}{1} = 1$

20. The plane passing through the point (–2, –2, –3) and containing the line joining the points (1, 1, 1) and (1, –1, 2) makes an intercept a, b, c along x, y and z axis respectively, then $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ will be

(1) 3 (2) 4 (3) 6 (4) 1

Sol. Answer (4)

$$\begin{vmatrix} x+2 & y+2 & z+3 \\ 3 & 3 & 4 \\ 3 & 1 & 5 \end{vmatrix} = 0$$

$11(x + 2) - 3(y + 2) - 6(z + 3) = 0$

$11x - 3y - 6z - 2 = 0$

$11x - 3y - 6z = 2$

$a = \frac{2}{11}, b = \frac{-2}{3}, c = \frac{-2}{6}$

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$

21. Equation of plane parallel to $3x + 4y + 5z - 6 = 0$, $6x + 8y + 10z - 16 = 0$ and equidistant from them is

(1) $3x + 4y + 5z = 7$

(2) $3x + 4y + 5z = 10$

(3) $6x + 8y + 10z = 0$

(4) $6x + 8y + 10z = 3$

Sol. Answer (1)

Equation of two given planes

$3x + 4y + 5z = 6$

$3x + 4y + 5z = 8$

Distance between two parallel planes $= \frac{2}{\sqrt{50}}$

Equation of plane which is equidistant to the two given planes

$\boxed{3x + 4y + 5z = 7}$

22. If the plane $3x - 4y + 5z = 0$ is parallel to $\frac{2x-1}{4} = \frac{1-y}{-3} = \frac{z-2}{a}$, then the value of a is

(1) $\frac{6}{4}$ (2) $\frac{6}{5}$ (3) 1.4 (4) $\frac{3}{4}$

Sol. Answer (2)

$3 \times 2 - 4 \times 3 + 5a = 0$

$-6 + 5a = 0$

$a = \frac{6}{5}$

23. Lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + s\vec{b}_2$ will lie in a plane if

(1) $\vec{a}_1 \times \vec{a}_2 = 0$ (2) $(\vec{a}_2 - \vec{a}_1) \times \vec{b}_1 \times \vec{b}_2 = 0$

(3) $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$ (4) $\vec{b}_1 \times \vec{b}_2 = 0$

Sol. Answer (3)

24. The equation of a plane through the point (2, 3, 1) and (4, –5, 3) and parallel to x-axis

(1) $x + y + 4z = 7$ (2) $y + 4z = 7$ (3) $x - 4z = 7$ (4) $y + 4x = -7$

Sol. Answer (2)

25. The line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies exactly in the plane $2x - 4y + z = 7$, then value of k is

(1) 7 (2) –7 (3) 1 (4) No real value

Sol. Answer (1)

Point should lie in the plane

$k = 7$

26. Find the equation of plane containing the line $x + y - z = 0 = 2x - y + z$ and passing through the point (1, 2, 1)

(1) $3x + 2y + z = 1$ (2) $x - y + z = 0$ (3) $x + y + z = 0$ (4) $2x + 3y + z = 2$

Sol. Answer (2)

$(x + y - z) + \lambda (2x - y + z) = 0$

$x (1 + 2\lambda) + y (1 - \lambda) + z (\lambda - 1) = 0$

Passes through point (1, 2, 1)

$1 + 2\lambda + 2 - 2\lambda + \lambda - 1 = 0$

$\lambda = -2$

$-3x + 3y - 3z = 0$

$\boxed{x - y + z = 0}$

27. The direction ratios of the normal to the plane passing through origin and line of intersection of the planes $x + 2y + 3z = 4$ and $4x + 3y + 2z = 1$ can be

(1) (1, 2, 3) (2) (3, 2, 1) (3) (2, 3, 1) (4) (3, 1, 2)

Sol. Answer (2)

$(x + 2y + 3z - 4) + \lambda (4x + 3y + 2z - 1) = 0$

Passes through origin so

$-4 - \lambda = 0$

$\lambda = -4$

$(x + 2y + 3z - 4) - 4 (4x + 3y + 2z - 1) = 0$

$-15x - 10y - 5z = 0$

$3x + 2y + z = 0$

Direction ratios (3, 2, 1)

28. A unit vector normal to the plane through the points $\hat{i}$, $2\hat{j}$ and $3\hat{k}$ is

(1) $6\hat{i} + 3\hat{j} + 2\hat{k}$ (2) $\hat{i} + 2\hat{j} + 3\hat{k}$ (3) $\dfrac{6\hat{i} + 3\hat{j} + 2\hat{k}}{7}$ (4) $\dfrac{\hat{i} + 2\hat{j} + 3\hat{k}}{7}$

Sol. Answer (3)

29. The lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ will be perpendicular if and only if

(1) $aa' + cc' + 1 = 0$ (2) $aa' + cc' = 0$ (3) $aa' + bb' = 0$ (4) $aa' + bb' + cc' = 0$

Sol. Answer (1)

30. The planes $x - 3y + 4z - 1 = 0$ and $kx - 4y + 3z - 5 = 0$ are perpendicular then value of k is

(1) 24 (2) –24 (3) 12 (4) 0

Sol. Answer (2)

Planes are perpendicular

$\therefore\ k + 12 + 12 = 0$

$k = -24$

31. The equation of a plane passing through the intersection of planes $x + 2y + 3z - 5 = 0$ and $3x - 2y - z + 1 = 0$ axes and cutting equal intercepts on the ox and oz axes.

(1) $5x + 3y + 4z - 3 = 0$ (2) $5x + 2y + 5z - 9 = 0$

(3) $3x + 2y + 4z - 9 = 0$ (4) $2x - 3y + 4z + 3 = 0$

Sol. Answer (2)

$x(1 + 3\lambda) + y(2 - 2\lambda) + z(3 - \lambda) - 5 + \lambda = 0$

$x(3\lambda + 1) + y(2 - 2\lambda) + z(3 - \lambda) = 5 - \lambda$

$\frac{5-\lambda}{3\lambda+1} = \frac{5-\lambda}{3-\lambda} \Rightarrow 3\lambda + 1 = 3 - \lambda$

$4\lambda = 2 \Rightarrow \lambda = \frac{1}{2}$

$\boxed{5x + 2y + 5z - 9 = 0}$

32. The locus represented by $xy + yz = 0$ is

(1) Pair of perpendicular lines (2) Pair of parallel lines

(3) Pair of parallel planes (4) Pair of perpendicular planes

Sol. Answer (4)

$y(x + z) = 0$

Pair of perpendicular planes.

33. The reflection of point (α, β, γ) in the xy-plane is

(1) $(\alpha, \beta, 0)$ (2) $(0, 0, \gamma)$ (3) $(-\alpha, -\beta, \gamma)$ (4) $(\alpha, \beta, -\gamma)$

Sol. Answer (4)

Reflection of (α, β, γ) in x-y plane = $(\alpha, \beta, -\gamma)$

34. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and plane $2x - 2y + z - 5 = 0$ is

(1) $\frac{10}{6\sqrt{5}}$ (2) $\frac{4}{5\sqrt{2}}$ (3) $\frac{2\sqrt{3}}{5}$ (4) $\frac{\sqrt{2}}{10}$

Sol. Answer (4)

$\theta = \sin^{-1}\frac{6-8+5}{3\times 5\sqrt{2}} = \sin^{-1}\left(\frac{1}{5\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{2}}{10}\right)$

35. If the line $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-5}{-1}$ is parallel to the plane $px + 3y - z + 5 = 0$, then the value of p

(1) 2 (2) –2 (3) $\frac{1}{2}$ (4) $\frac{1}{3}$

Sol. Answer (2)

$2p + 3 + 1 = 0 \Rightarrow p = -2$

36. If distances of (–1, 2, 3) from x, y, z axis are d_1, d_2, d_3 respectively and the distances from xy, yz, zx planes are d_4, d_5, d_6 then the value of $\sum_{i=1}^{6} d_i$ is

(1) $\sqrt{5}+\sqrt{13}+\sqrt{10}+6$ (2) $\sqrt{2}+\sqrt{3}+6$ (3) $\sqrt{5}+\sqrt{13}+7$ (4) $\sqrt{4}+\sqrt{13}+7$

Sol. Answer (1)

$d_1 = \sqrt{4+9} = \sqrt{13}$

Similarly, $d_2 = \sqrt{1+9} = \sqrt{10}$, $d_3 = \sqrt{1+4} = \sqrt{5}$, $d_4 = 3$, $d_5 = 1$, $d_6 = 2$.

Hence sum = $6 + \sqrt{10} + \sqrt{5} + \sqrt{13}$

37. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the points of intersection is

(1) $(2a, 3a, 3a), (2a, a, a)$

(2) $(3a, 2a, 3a), (a, a, a)$

(3) $(3a, 2a, 3a), (a, a, 2a)$

(4) $(3a, 3a, 3a), (a, a, a)$

Sol. Answer (2)

$$\frac{x}{1} = \frac{y+a}{1} = \frac{z}{1} = t_1 \quad \ldots \text{(i)}$$

$\Rightarrow$ $x = t_1$, $y = t_1 - a$, $z = t_1$

$$\frac{x+a}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{1}{2}} = t_2 \quad \ldots \text{(ii)}$$

$x = t_2 - a$, $y = \frac{t_2}{2}$, $z = \frac{t_2}{2}$

Let the line with direction ratio (2, 1, 2) meets (i) and (ii) at $A(t_1, t_1 - a, t_1)$ and $B\left(t_2 - a, \frac{t_2}{2}, \frac{t_2}{2}\right)$ respectively.

Direction ratios of AB are proportional to (2, 1, 1)

$$\Rightarrow \frac{t_2 - a - t_1}{2} = \frac{\frac{t_2}{2} - t_1 + a}{1} = \frac{\frac{t_2}{2} - t_1}{2}$$

$\Rightarrow$ $t_2 - a - t_1 = t_2 - 2t_1 + 2a$

or $t_1 = 3a$

$\Rightarrow$ $A \equiv (3a, 2a, 3a)$

Again by $\dfrac{\frac{t_2}{2} - t_1 + a}{1} = \dfrac{\frac{t_2}{2} - t_1}{2}$

$t_2 - 2t_1 + 2a = \frac{t_2}{2} - t_1$

$\Rightarrow$ $\frac{t_2}{2} = t_1 - 2a$

$t_2 = 2(t_1 - 2a) = 2(3a - 2a)$

$t_2 = 2a$

$\Rightarrow$ $B \equiv (a, a, a)$

38. The equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0, 7, –7) is

(1) $x + y + z = 1$ (2) $x + y + z = 2$ (3) $x + y + z = 0$ (4) $x + y + z = 3$

Sol. Answer (3)

Equation of the plane

$$\begin{vmatrix} x+1 & y-3 & z+2 \\ -3 & 2 & 1 \\ -1 & -4 & 5 \end{vmatrix} = 0$$

$(x + 1)(10 + 4) - (y - 3)(-15 + 1) + (z + 2)(12 + 2) = 0$

$14x + 14 + 14y - 42 + 14z + 28 = 0$

$14x + 14y + 14z = 0$

$x + y + z = 0$

[**Miscellaneous**]

39. A plane $x - 3y + 5z = d$ passes through the point (1, 2, 4). Intercepts on the axes are

(1) 15, –5, 3 (2) 1, –5, 3 (3) –15, 5, –3 (4) 1, –6, 20

Sol. Answer (1)

$x - 3y + 5z = d$

$\Rightarrow\ 1 - 3 \times 2 + 5 \times 4 = d$

$\Rightarrow\ 1 - 6 + 20 = d \quad \Rightarrow\ d = 15$

Hence the plane is $x - 3y + 5z = 15$ ... (i)

$\Rightarrow\ \frac{x}{15} + \frac{y}{-5} + \frac{z}{3} = 1$

Intercepts on the axis are 15, –5 and 3 respectively.

40. The position vectors of points A and B are $\hat{i} - \hat{j} + 3\hat{k}$ and $3\hat{i} + 3\hat{j} - 3\hat{k}$ respectively. The equation of a plane is $\vec{r}.(5\hat{i} + 2\hat{j} - 7\hat{k}) = 0$. The points A and B

(1) Lie on the plane

(2) Are on the same side of the plane

(3) Are on the opposite sides of the plane

(4) Nothing can be said

Sol. Answer (3)

$\vec{r}_1 \cdot \left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) = 5 - 2 - 21 = -\text{ve}$

$\vec{r}_2 \cdot \left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) = 15 + 6 + 21 = +\text{ve}$

So points lie on opposite sides of the plane.

41. The vector equation of the plane through the point (2, 1, –1) and passing through the line of intersection of the plane $\vec{r}.(\hat{i} + 3\hat{j} - \hat{k}) = 0$ and $\vec{r}.(\hat{j} + 2\hat{k}) = 0$ is

(1) $\vec{r}.(\hat{i} + 9\hat{j} + 11\hat{k}) = 0$ (2) $\vec{r}.(\hat{i} + 9\hat{j} + 11\hat{k}) = 6$ (3) $\vec{r}.(\hat{i} - 3\hat{j} - 13\hat{k}) = 0$ (4) $\vec{r}.(2\hat{i} - 3\hat{j} - 4\hat{k}) = 0$

Sol. Answer (1)

The required equation of the plane is

$\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})+\lambda(\vec{r}.(\hat{j}+2\hat{k}))=0$, where $\lambda \in R$

But $\vec{r} = 2\hat{i}+\hat{j}-\hat{k}$, satisfies the equation

$\Rightarrow (2\hat{i}+\hat{j}-\hat{k}).(\hat{i}+3\hat{j}-\hat{k})+\lambda(2\hat{i}+\hat{j}-\hat{k}).(\hat{j}+2\hat{k})=0$

$\Rightarrow (2+3+1)+\lambda(1-2)=0$

$6-\lambda=0$

$\lambda=6$

Hence the equation of the plane is

$\vec{r}.(\hat{i}+3\hat{j}-\hat{k})+6\vec{r}(\hat{j}+2\hat{k})$

$\Rightarrow \vec{r}.(\hat{i}+9\hat{j}+11\hat{k})=0$

42. The lines $x + y + z - 3 = 0 = 2x - y + 5z - 6$ and $x - y - z + 1 = 0 = 2x + 3y + 7z - k$ are coplanar then k equals

(1) 10 (2) 11 (3) 12 (4) 13

Sol. Answer (3)

$x + y + z - 3 = 0$ … (i)

$2x - y + 5z - 6 = 0$ … (ii)

If $z = 0$

$x + y = 3$

$2x - y = 6$

$\Rightarrow x = 3, y = 0$

Point on the line is $(3, 0, 0) = \vec{a}_1$

$\vec{n}_1 = \hat{i}+\hat{j}+\hat{k}$

$\vec{n}_2 = 2\hat{i}-\hat{j}+5\hat{k}$

$$\vec{b}_1 = \vec{n}_1\times\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 5 \end{vmatrix}$$

$= \hat{i}(5+1) - \hat{j}(5-2) + \hat{k}(-1-2)$

$= \vec{b}_1 = 6\hat{i}-3\hat{j}-3\hat{k}$

$x - y - z = -1$ … (iii)

$2x + 3y + 7z = k$ … (iv)

At $z = 0$,

We get, $x - y = -1$

$2x + 3y = k$

On solving $x = \dfrac{k-3}{5}, y = \dfrac{k+2}{5}$

Hence point on the second line is $\left(\frac{k-3}{5}, \frac{k+2}{5}, 0\right) = \vec{a}_2$

The vector parallel to this line is

$$\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 2 & 3 & 7 \end{vmatrix}$$

$$= \hat{i}\,(-7+3) - \hat{j}\,(7+2) + \hat{k}\,(3+2)$$

$$= -4\hat{i} - 9\hat{j} + 5\hat{k}$$

If both lines are coplanar then the vector

$\vec{a}_2 - \vec{a}_1$, $\vec{b}_1$, $\vec{b}_2$ will be coplanar

$$\Rightarrow \begin{vmatrix} \frac{k-3}{5} - 3 & \frac{k+2}{5} & 0 \\ 6 & -3 & -3 \\ -4 & -9 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} k-18 & k+2 & 0 \\ 2 & -1 & -1 \\ -4 & -9 & 5 \end{vmatrix} = 0$$

$\Rightarrow$ $(k-18)(-5-9) - (k+2)(10-4) = 0$

$\Rightarrow$ $(k-18)(-5-9) - (k+2)\,6 = 0$

$\Rightarrow$ $(k-18)(-14) - (k+2)\,6 = 0$

$\Rightarrow$ $-7k + 126 - 3k - 6 = 0$

$\Rightarrow$ $-10k = -120$

$\Rightarrow$ $k = 12$

43. The line $\vec{r} = \vec{a} + \lambda\vec{b}$ will not meet the plane $\vec{r}\cdot\vec{n} = q$, if

(1) $\vec{b}\cdot\vec{n} = 0, \vec{a}\cdot\vec{n} = q$ (2) $\vec{b}\cdot\vec{n} \neq 0, \vec{a}\cdot\vec{n} \neq q$ (3) $\vec{b}\cdot\vec{n} = 0, \vec{a}\cdot\vec{n} \neq q$ (4) $\vec{b}\cdot\vec{n} \neq 0, \vec{a}\cdot\vec{n} = q$

Sol. Answer (3)

If $\vec{r} = \vec{a} + \lambda\vec{b}$ will not meet the plane

$\vec{r}\cdot\vec{n} = q$, then

Line must be parallel to the plane but not lies in the plane

$\Rightarrow$ $\vec{b}\cdot\vec{n} = 0$ and $\vec{a}\cdot\vec{n} \neq q$

44. The plane $\vec{r}\cdot\vec{n} = q$ will contain the line $\vec{r} = \vec{a} + \lambda\vec{b}$ if

(1) $\vec{b}\cdot\vec{n} \neq 0, \vec{a}\cdot\vec{n} = q$ (2) $\vec{b}\cdot\vec{n} \neq 0, \vec{a}\cdot\vec{n} \neq q$ (3) $\vec{b}\cdot\vec{n} = 0, \vec{a}\cdot\vec{n} \neq q$ (4) $\vec{b}\cdot\vec{n} = 0, \vec{a}\cdot\vec{n} = q$

Sol. Answer (4)

If a line lies in the plane then one point of the line must satisfy the equation of plane and the line must parallel to the plane.

$\Rightarrow$ $\vec{b}\cdot\vec{n} = 0$ and $\vec{a}\cdot\vec{n} = q$

45. The projection of the line segment joining the points (1, 2, 3) and (4, 5, 6) on the plane $2x + y + z = 1$ is

(1) 1 (2) $\sqrt{3}$ (3) 5 (4) 6

Sol. Answer (2)

$\vec{n} = 2\hat{i} + \hat{j} + \hat{k}$,

$\vec{b} = (4\hat{i} + 5\hat{j} + 6\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = 3\hat{i} + 3\hat{j} + 3\hat{k}$

Let $l = |\vec{b}|\cos\theta$ where θ is the angle between $\vec{n}$ and $\vec{b}$

$$= \frac{\vec{n}\cdot\vec{b}}{|\vec{n}|} = \frac{6+3+3}{\sqrt{6}} = \frac{12}{\sqrt{6}} = 2\sqrt{6}$$

Length of line segment = $\sqrt{b^2 - \left(2\sqrt{6}\right)^2} = \sqrt{27-24} = \sqrt{3}$

46. The line $x + 2y - z - 3 = 0 = x + 3y - z - 4$ is parallel to

(1) x-y plane (2) y-z plane (3) z-x plane (4) z-axis

Sol. Answer (3)

The vector parallel to the plane is

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 1 & 3 & -1 \end{vmatrix} = \hat{i}(-2+3) - \hat{j}(-1+1) + \hat{k}(3-1) = \hat{i} + \hat{k}$$

This is parallel to z-x plane.

47. The shortest distance between the lines $x = y = z$ and the line $2x + y + z - 1 = 0 = 3x + y + 2z - 2$ is

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$ (3) $\frac{3}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

Sol. Answer (1)

$x = y = z$ … (i)

$2x + y + z = 1$ … (ii)

$3x + y + 2z = 2$ … (iii)

Normal vector to plane (ii) is $\vec{n}_2 = 2\hat{i} + \hat{j} - \hat{k}$

Similarly normal vector to the plane (iii) is $\vec{n}_3 = 3\hat{i} + \hat{j} + 2\hat{k}$

Vector parallel to the line of intersection is $\vec{n}_2 \times \vec{n}_3$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$= \hat{i}\,(2-1) - \hat{j}\,(4-3) + \hat{k}\,(2-3)$

$= (\hat{i} - \hat{j} - \hat{k})$, hence DR's of the line are (1, –1, –1)

To find point on the line of intersection, we put $z = 0$ in (ii) and (iii)

$\Rightarrow\ 2x + y = 1$ … (iv)

$3x + y = 2$ … (v)

Solving (iv) and (v)

$x = 1,\ y = -1,\ z = 0$

Equation of line is

$$\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-0}{-1} \quad \dots \text{(vi)}$$

Let one point on the line (i) is $\vec{a}_1 = (0\hat{i} + 0\hat{j} + 0\hat{k})$

Similarly the point on the line is $\vec{a}_2 = (\hat{i} - \hat{j} + 0\hat{k})$

Vector parallel to line (i) is $\vec{b}_1 = \hat{i} + \hat{j} + \hat{k}$

Similarly the vector parallel to the line (vi) is $\vec{b}_2 = (1, -1, -1)$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 1 & -1 & -1 \end{vmatrix}$$

$\Rightarrow\ \hat{i}\,(0) - \hat{j}\,(-2) + \hat{k}\,(-2) = 2 - 2$

Shortest distance $= \left|\dfrac{(\hat{i} - \hat{j}) \cdot (2\hat{j} - 2\hat{k})}{2\sqrt{2}}\right| = \left|\dfrac{2}{2\sqrt{2}}\right| = \dfrac{1}{\sqrt{2}}$

48. The co-ordinates of the point P on the line $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(-\hat{i} + \hat{j} - \hat{k})$ which is nearest to the origin is

(1) $\left(\frac{2}{3}, \frac{4}{3}, \frac{2}{3}\right)$ (2) $\left(-\frac{2}{3}, -\frac{4}{3}, \frac{2}{3}\right)$ (3) $\left(\frac{2}{3}, \frac{4}{3}, -\frac{2}{3}\right)$ (4) $\left(\frac{2}{3}, \frac{2}{3}, \frac{4}{3}\right)$

Sol. Answer (1)

Let the point nearest to origin is $P\ (\hat{i} + \hat{j} + \hat{k}) + \lambda(-\hat{i} + \hat{j} - \hat{k})$

If O is origin, then $\overline{OP} = \hat{i} + \hat{j} + \hat{k} + \lambda(-\hat{i} + \hat{j} - \hat{k}) = \hat{i}(1-\lambda) + \hat{j}(1+\lambda) + \hat{k}(1-\lambda)$

$\overline{OP}$ is perpendicular to $-\hat{i} + \hat{j} - \hat{k}$

$\Rightarrow\ [\hat{i}(1-\lambda) + \hat{j}(1+\lambda) + \hat{k}(1-\lambda)] \cdot (-\hat{i} + \hat{j} - \hat{k}) = 0$

$\Rightarrow\ -1 + \lambda + 1 + \lambda - 1 + \lambda = 0$

$\Rightarrow\ 3\lambda = 1$

$\Rightarrow\ \lambda = \dfrac{1}{3}$

$$\vec{P} = \left(\frac{2}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$$

$\Rightarrow$ Coordinates $= \left(\dfrac{2}{3}, \dfrac{4}{3}, \dfrac{2}{3}\right)$

❑ ❑ ❑