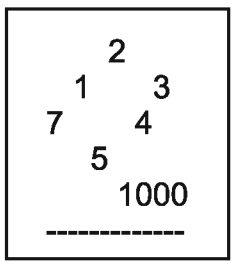
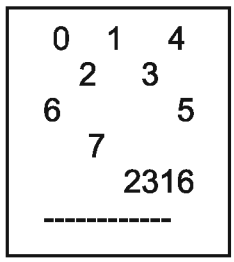
Chapter 4 Rational Numbers

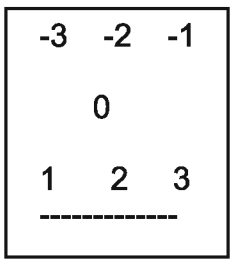
4.1 We started learning the numbers by counting the things around us. The numbers used for counting are called Natural numbers. We got the Whole numbers by including 0 in Natural numbers 1, 2, 3, 4, 5, ... Thereafter by including the negative of Natural numbers in Whole numbers 0, 1, 2, 3, 4 ..... we got Integers ..... -3, -2, -1, 0, 1, 2, 3,..... Thus we have extended the number system up to Integers.



Natural Numbers



Whole Numbers



Integers

We have introduced fractions in previous chapters. We will further extend the number system in this chapter. We will learn the concepts of rational numbers, representation of rational numbers on number line, their comparison and finding the rational numbers lying between two rational numbers.

4.2 Need of Rational Numbers

We have studied that the integers are used for representing the opposite situations.

Example 1 If the profit of Rs. 250 is expressed as +250 then the loss of Rs. 250 is expressed as -250.

Example 2 If the height of any place 800 m above sea level is expressed as $\frac{4}{5}$ km then depth of 800 m below sea level can be expressed as $-\frac{4}{5}$ km. We could understand that $-\frac{4}{5}$ is neither an integer nor a fraction. In order to define such numbers we need to extend the number system.

4.3 What are Rational Numbers?

The word rational originated from ratio. We know that the ratio 2:5 can also be written as $\frac{2}{5}$, here 2 and 5 are natural numbers. But $\frac{-2}{5}$ cannot be expressed in -2:5. Any two integers p and q (where $q \neq 0$) can be written as $\frac{p}{q}$ Rational numbers are expressed in this form.

A rational number can be defined in the form of a number, which can be expressed as $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

Thus, $\frac{3}{7}$ is a rational number, Here, $p = 3$ and $q = 7$.

Think and tell Is $\frac{-3}{7}$ a rational number?

4.4 Fractions and Rational Numbers

Write different fractions such as $\frac{3}{8}, \frac{7}{11}, \frac{4}{9}, 1\frac{3}{5}$ etc.
Compare each fraction with $\frac{p}{q}$.

In $\frac{3}{8}$, $p = 3$ and $q = 8$

In $\frac{7}{11}$, $p = 7$ and $q = 11$.

Take other examples of fractions and compare with $\frac{p}{q}$ We find that every fraction is of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$. We could now say that all the fractions are rational numbers.

Do and learn

Write the rational numbers in which

1. Numerator is a negative integer and denominator is a positive integer.
2. Numerator is a positive integer and denominator is a negative integer.
3. Both numerator and denominator are positive integers.
4. Both numerator and denominator are negative integers.

- Are all integers also rational numbers?

Any integer can be considered as a rational number. For example -3 is a rational number, because we can write it as $\frac{-3}{1}$. Integer 0 can also be written in $\frac{0}{1}$ or $\frac{0}{2}$ etc. Hence 0 is also a rational number.

- Zero is a rational number.
- Zero is neither a positive rational number nor a negative rational number.
- Rational numbers include integers and fractions.

Think! Is $\frac{-3}{-5}$ a rational number?

All rational numbers are not fractions but all fractions are rational numbers.

Rational number $\frac{-2}{-9}$ is not a fraction, whereas alternative form of $\frac{-2}{-9}$ is $\frac{2}{9}$, which is a fraction number.

4.5 Equivalent Rational Numbers

By multiplying or dividing the numerator and denominator of a rational number by same number it can be converted in to desired numerator or desired denominator.

Think of rational number $\frac{-5}{7}$

$$\frac{-5}{7}=\frac{(-5)\times 2}{7\times 2}=\frac{-10}{14}$$

$$\frac{-5}{7}=\frac{(-5)\times 3}{7\times 3}=\frac{-15}{21}$$

$$\frac{-5}{7}=\frac{(-5)\times(-2)}{7\times(-2)}=\frac{10}{-14}$$

So $\frac{-5}{7}=\frac{-10}{14}=\frac{-15}{21}=\frac{10}{-14}$

The rational numbers which are mutually equal are called equivalent rational numbers.

$$\frac{10}{-15}=\frac{10\div 5}{-15\div 5}=\frac{2}{-3}$$

$$\frac{10}{-15}=\frac{10\div(-5)}{(-15)\div(-5)}=\frac{-2}{3}$$

Hence, $\frac{10}{-15}=\frac{2}{-3}=\frac{-2}{3}$ are equivalent.

Do and learn Fill in the blanks.

$$\frac{2}{3}=\frac{4}{\ldots\ldots}=\frac{\ldots\ldots}{12}=\frac{10}{\ldots\ldots}=\frac{\ldots\ldots}{24}$$

$$\frac{5}{7}=\frac{\ldots\ldots}{14}=\frac{25}{\ldots\ldots}=\frac{\ldots\ldots}{63}=\frac{100}{\ldots\ldots}$$

$$\frac{25}{50}=\frac{\ldots\ldots}{10}=\frac{1}{\ldots\ldots}=\frac{\ldots\ldots}{150}=\frac{250}{\ldots\ldots}$$

4.6 Positive and Negative Rational Numbers

Both numerator and denominator in rational numbers $\frac{2}{3}, \frac{3}{7}, \frac{5}{8}$ and $\frac{2}{9}$ are positive. Such rational numbers are called positive rational numbers.

Rational numbers in which either numerator or denominator is a negative integer are called negative rational numbers. For example: $\frac{-3}{7}, \frac{4}{-5}, -\frac{1}{3}$ etc.

What do you think about $\frac{-5}{-7}$

$$\frac{-5}{-7} = \frac{5\times(-1)}{7\times(-1)} = \frac{5}{7}$$

So, $\frac{-5}{-7}$ is a positive rational number.

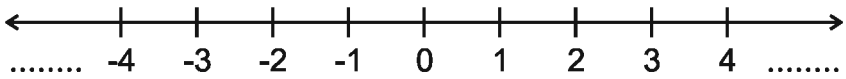
Do and learn:

1. Write three positive rational numbers.
2. Write two negative numbers.
3. Is $\frac{-15}{-1}$ a positive rational number? (Justify the answer)
4. Is -7 a negative rational number? (Justify the answer)
5. Which of the following numbers are positive rational number?

(i) $\frac{-4}{5}$ (ii) $\frac{-7}{-9}$ (iii) $1\frac{2}{3}$ (iv) $\frac{3}{-7}$ (v) $\frac{1}{3}$

4.7 Rational Numbers on a Number Line

We have learnt to represented integers on a number line. Let us see such number line

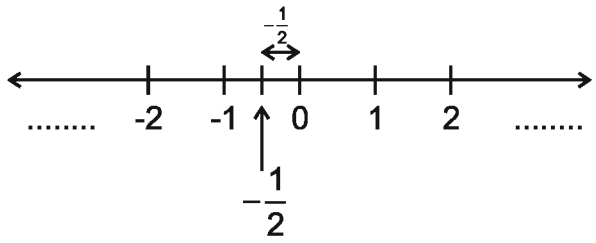


On number line positive integers lie on right hand side of 0 and are denoted by '+' sign. Negative integers lie on the left hand side of 0 and are denoted by '-' sign.

In previous classes, we have denoted fractions on number line.

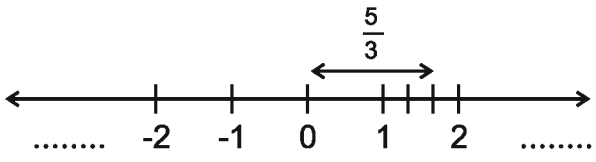
Let us denote the rational number $-\frac{1}{2}$ on number line.

Since $-\frac{1}{2}$ is a negative rational number, it will lie on the left of 0. $-\frac{1}{2}$ lies between 0 and -1.

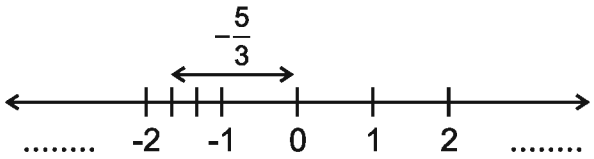


Divide space between 0 and -1 into two equal half and then locate $-\frac{1}{2}$ exactly in the middle of 0 and -1.

We know that how $\frac{5}{3}$ is denoted on number line.
We make three equal parts between 1 and 2 on the right of 0. Second part on the right of 1 denote $\frac{5}{3}$.



Let us now denote $\frac{-5}{3}$ on number line. It will be denoted at a distance on the left hand side of 0 equal to the point where $\frac{5}{3}$ is denoted on the right hand side of 0



Do and learn

Denote the following rational numbers on number line:

(i) $-\frac{5}{4}$ (ii) $-\frac{7}{2}$ (iii) $-\frac{11}{3}$ (iv) $\frac{2}{5}$ (v) $\frac{4}{3}$

4.8 Simplest form of Rational Number

Look at the following rational numbers carefully.

$\frac{1}{3}, \frac{3}{5}, \frac{-2}{7}, \frac{5}{8}, \frac{-9}{11}$

In all the rational numbers given above

(i) Denominator is positive integer, and

(ii) The only one common factor between numerator and denominator is '1'.

Such rational numbers are called rational numbers in simplest form.

Every rational number can be expressed in simplest form.

Example 3 Express $\frac{-36}{24}$ in the simplest form.

Solution

$$\frac{-36}{24}=\frac{-36\div 3}{24\div 3}=\frac{-12}{8}=\frac{-12\div 4}{8\div 4}=\frac{-3}{2}$$

$$\frac{-36}{24}=\frac{-36\div 12}{24\div 12}=\frac{-3}{2}$$

So, $\frac{-36}{24}$ is the simplest form of $\frac{-3}{2}$

Do and learn

Convert the following in to the simplest form

(i) $\frac{3}{15}$ (ii) $\frac{-6}{20}$ (iii) $\frac{10}{-35}$ (iv) $\frac{-45}{30}$ (v) $\frac{18}{-45}$

4.9 Comparison of Rational Numbers

We know how to compare two integers or two fractions and also the order relation, i.e., which of the given numbers is greater or smaller. Let us now compare two rational numbers.

Two rational numbers $\frac{5}{7}$ and $\frac{7}{9}$ can be compared in the same manner as we compared fractions.

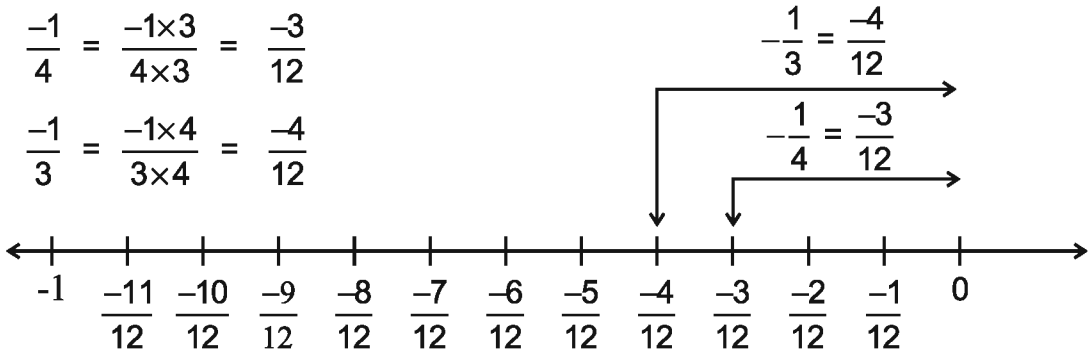
Let us compare two negative rational numbers $\frac{-1}{4}$ and $\frac{-1}{3}$ on the number line.

We have seen in reference to comparison of integers that integers on the right hand side are greater than the integer on the left hand side of the number line. Similarly $\frac{-1}{4}$ and $\frac{-1}{3}$ can also be compared by denoting them on number line. We take two equivalent rational numbers of both which has similar denominations.

For example -

$\frac{-1}{4} = \frac{-1\times3}{4\times3} = \frac{-3}{12}$

$\frac{-1}{3} = \frac{-1\times4}{3\times4} = \frac{-4}{12}$



Since $\frac{-1}{4}$ is on the right hand side of $\frac{-1}{3}$, $\frac{-1}{4}$ will be greater than $\frac{-1}{3}$

$-\frac{1}{4} > -\frac{1}{3}$

But we have learnt from the study of fractions that

$\frac{1}{4} < \frac{1}{3}$

Do and learn

Compare the numbers $\frac{-3}{4}$ & $-\frac{2}{3}$ and $-\frac{1}{3}$ & $-\frac{1}{5}$

Pairs of negative rational numbers are also treated in similar manner. In order to compare two negative rational numbers, we neglect the negative sign and compare them and then reverse the sign of inequality.

For example: In order to compare the numbers $-\frac{3}{7}$ & $-\frac{5}{9}$ we first compare the pair $\frac{3}{7}$ & $\frac{5}{9}$

$\frac{3 \times 9}{7 \times 9} = \frac{27}{63}$, $\frac{5 \times 7}{9 \times 7} = \frac{35}{63}$ $\therefore$ $\frac{27}{63} < \frac{35}{63}$

Or $\frac{3}{7} < \frac{5}{9}$ From this we conclude that $-\frac{3}{7} > -\frac{5}{9}$

Do and learn Which rational number is greater?

1. $-\frac{3}{8}$ Or $-\frac{2}{7}$
2. $-\frac{7}{5}$ Or $-\frac{5}{3}$
3. $-\frac{5}{6}$ Or $-\frac{7}{8}$

Comparison of a negative rational number and a positive rational number is quite obvious. A negative rational number lies on the left hand side of zero and a positive rational number lies on the right hand side of zero. A negative rational number is always less than a positive rational number.

For example $-\frac{1}{2} < \frac{1}{2}$

$-\frac{3}{5} < \frac{1}{5}$

$-\frac{9}{4} < \frac{3}{2}$

In order to compare the rational numbers $\frac{-4}{-7}$ and $\frac{-3}{-5}$ we first convert them in to standard form and then compare.

The standard forms of $\frac{-4}{-7}$ and $\frac{-3}{-5}$ are $\frac{4}{7}$ and $\frac{3}{5}$ respectively. This suggests that

$\frac{4}{7} < \frac{3}{5}$

Do and learn

Do $\frac{4}{-9}$ and $\frac{-20}{45}$ denote same rational number?

4.10 Rational Numbers between two Rational Numbers

We know that the integers between 5 and 12 are 6, 7, 8, 9, 10, and 11. The integers between -3 and 3 are -2, -1, 0, 1, 2. This shows that the number of integers between two integers is finite.

Does this happen in case of rational numbers also? Let us see it with following example:

Kiran considered two rational numbers $-\frac{4}{3}$ and $-\frac{1}{2}$

She converted them in to the numbers with common denominator.

So $-\frac{4}{3} = -\frac{8}{6}$ and $-\frac{1}{2} = -\frac{3}{6}$

She wrote the rational numbers between $-\frac{8}{6}$ and $-\frac{3}{6}$ as follows:

$$-\frac{7}{6} < -\frac{6}{6} < -\frac{5}{6} < -\frac{4}{6}$$

Thus, she got the rational numbers $-\frac{7}{6}, -\frac{6}{6}, -\frac{5}{6}, -\frac{4}{6}$ between $-\frac{4}{3}$ and $-\frac{1}{2}$

Think! Are $-\frac{7}{6}, -\frac{1}{1}, -\frac{5}{6}, -\frac{2}{3}$ the only rational numbers between $-\frac{4}{3}$ and $-\frac{1}{2}$

Let us see.

$$-\frac{4}{3} = -\frac{8}{6} = -\frac{16}{12} \text{ and } -\frac{1}{2} = -\frac{3}{6} = -\frac{6}{12}$$

Now, the rational numbers between $-\frac{16}{12}$ and $-\frac{6}{12}$

$$-\frac{15}{12} < -\frac{14}{12} < -\frac{13}{12} < -\frac{12}{12} < -\frac{11}{12} < -\frac{10}{12} < -\frac{9}{12} < -\frac{8}{12} < -\frac{7}{12}$$

Or $-\frac{5}{4} < -\frac{7}{6} < -\frac{13}{12} < -\frac{1}{1} < -\frac{11}{12} < -\frac{5}{6} < -\frac{3}{4} < -\frac{2}{3} < -\frac{7}{12}$

Thus, we are successful in finding five more rational numbers $-\frac{5}{4}, -\frac{13}{12}, -\frac{11}{12}, -\frac{3}{4}$ $-\frac{7}{12}$ between $-\frac{4}{3}$ and $-\frac{1}{2}$

Using this method we can find as many (infinite) rational numbers as we want between two rational numbers.

Do and learn

(i) Find five rational numbers between $-\frac{5}{7}$ and $-\frac{3}{8}$

(ii) Find five rational numbers between $-\frac{5}{3}$ and $-\frac{8}{7}$

Example 5 Write two rational numbers between the two rational numbers -2 and -1.

Solution First of all we write -2 and -1 in the form of rational numbers having common denominator.

$-2 = -\frac{10}{5}$ and $-1 = -\frac{5}{5}$

Now, the number of rational numbers between $-\frac{10}{5}$ and $-\frac{5}{5}$ are $-\frac{9}{5} < -\frac{8}{5} < -\frac{7}{5} < -\frac{6}{5}$

So, the rational numbers between-2 and -1 are $-\frac{8}{5}$ and $-\frac{7}{5}$

(We can take any two rational numbers from $-\frac{9}{5}$, $-\frac{8}{5}$, $-\frac{7}{5}$, $-\frac{6}{5}$)

Exercise 4

1. Write five rational numbers equivalent to the following rational numbers:

(i) $-\frac{2}{3}$ (ii) $\frac{1}{5}$ (iii) $\frac{-5}{3}$ (iv) $\frac{4}{-9}$

2. Write three such rational numbers equivalent to $\frac{-5}{12}$ in which the denominator is 60, -96 and 108.

3. Write three such rational numbers equivalent to $\frac{-3}{7}$ in which the numerator is 24, -60 and 75.

4. Write the following rational numbers in the simplest form (standard forms):

(i) $\frac{-18}{30}$ (ii) $\frac{44}{-72}$ (iii) $\frac{55}{22}$ (iv) $\frac{-16}{20}$

5. Represent the following numbers on number line:

(i) $\frac{3}{5}$ (ii) $\frac{7}{8}$ (iii) $\frac{-8}{3}$ (iv) $-2\frac{1}{2}$ (v) $\frac{5}{7}$

6. Choose the correct sign from <, >, = and fill in the blanks:

(i) $\frac{2}{3}$ ☐ $\frac{-5}{7}$ (ii) $\frac{-1}{4}$ ☐ $\frac{1}{-3}$ (iii) $\frac{-3}{5}$ ☐ $\frac{-1}{3}$

(iv) $\frac{2}{7}$ ☐ $\frac{1}{2}$ (v) $\frac{-1}{2}$ ☐ $\frac{1}{-2}$ (vi) $\frac{-5}{4}$ ☐ $\frac{3}{5}$

7. Write five rational numbers between following rational numbers:

(i) -3 and -1 (ii) 0 and -1 (iii) $\frac{-4}{5}$ and $\frac{-5}{7}$

(iv) $\frac{1}{2}$ and $\frac{1}{4}$ (v) $\frac{2}{5}$ and $\frac{-4}{5}$ (vi) -2 and 0

8. Write three more rational numbers in each of the following:

(i) $\frac{-2}{5}, \frac{-4}{10}, \frac{-6}{15}$, — — — (ii) $\frac{2}{-3}, \frac{4}{-6}, \frac{6}{-9}$, — — —

(iii) $\frac{1}{-3}, \frac{2}{-6}, \frac{3}{-9}$, — — — (iv) $\frac{1}{-5}, \frac{2}{-10}, \frac{3}{-15}$, — — —

9. Write the following rational numbers in increasing order:

(i) $\frac{1}{2}, \frac{-1}{2}, \frac{-3}{4}, \frac{3}{4}$ (ii) $\frac{-3}{4}, \frac{-3}{7}, \frac{-3}{2}$ (iii) $\frac{-7}{11}, \frac{7}{15}, 0, -2, \frac{-2}{15}$ (iv) $\frac{2}{5}, \frac{4}{7}, \frac{1}{6}, \frac{5}{9}$

10. Write the following rational numbers in decreasing order:

(i) $\frac{9}{-24}, \frac{-3}{4}, \frac{5}{-12}, \frac{-7}{16}$ (ii) $\frac{-5}{6}, \frac{1}{6}, \frac{-8}{9}, \frac{-11}{12}$ (iii) $\frac{1}{3}, \frac{-2}{3}, \frac{-5}{6}, \frac{4}{-3}$ (iv) $\frac{3}{5}, \frac{-17}{-30}, \frac{-7}{10}, \frac{8}{-15}$

We Learnt

1. Rational numbers are denoted by $\frac{p}{q}$ where p and q are integers and $q \neq 0$.
2. All the fractions and whole numbers are rational numbers. $\frac{7}{8}, \frac{-2}{3}, 5$ are rational numbers.
3. If the numerator and denominator in a rational number are multiplied or divided by a same integer (other than zero) then the rational number so obtained is known as equivalent rational number. For example: $\frac{5}{6} = \frac{5\times3}{6\times3} = \frac{15}{18}$
4. Rational numbers are classified as positive and negative rational numbers. When both numerator and denominator are either positive or negative, the rational numbers is called as positive rational number. When either of the numerator and denominator are negative, then the rational number is called as negative rational number. For example: $\frac{2}{3}$ is a positive rational number and $-\frac{2}{3}$ a negative rational number.
5. Zero is a rational number but it is neither positive rational number nor negative rational number.
6. There lie infinite number of rational numbers between two rational numbers.