

Wave Motion

- Energy can be transmitted from one point to another in two ways.
 - (a) When a particle travels while carrying energy (**PARTICLE MOTION**).
 - (b) When energy is transmitted without the transfer of particle (**WAVE MOTION**)

Thus we can understand that

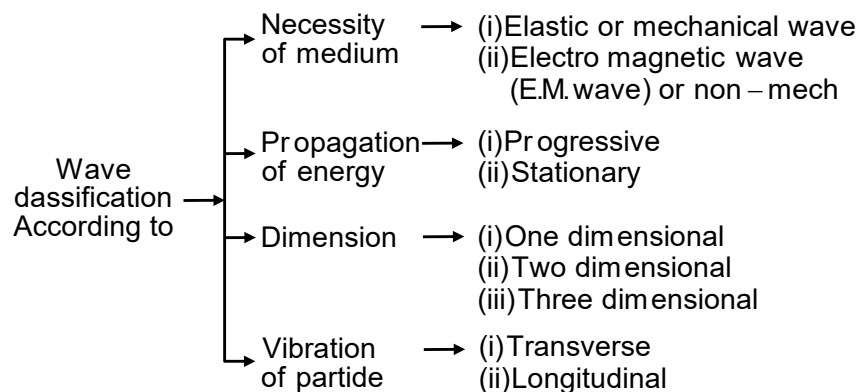
A wave is a disturbance that propagates in space, transports energy and momentum from one point to another without the actual transportation of medium.

In wave motion

- (a) Energy & momentum get transferred
- (b) There is no mass transfer
- (c) Energy is transmitted through vibrations/ oscillations

Note : Here vibrations/oscillations denote the periodic variation of any physical quantity associated with the wave.

1. Classification of Waves



1.1 Based on Medium Necessity

A wave may or may not require a medium for its propagation. The waves which do not require medium for their propagation are called **non-mechanical**, e.g. light, heat (infrared), radio waves etc.

On the other hand, the waves which require medium for their propagation are called **mechanical waves**. In the propagation of mechanical waves elasticity and density of the medium play an important role, therefore mechanical waves are also known as **elastic waves**.

Example : Sound waves in water, seismic waves in earth's crust.

Note : Apart from mechanical (elastic) and non-mechanical (electromagnetic) waves there is also another kind of waves called 'matter waves'. They represent wavelike properties of particles and are governed by the laws of quantum physics.

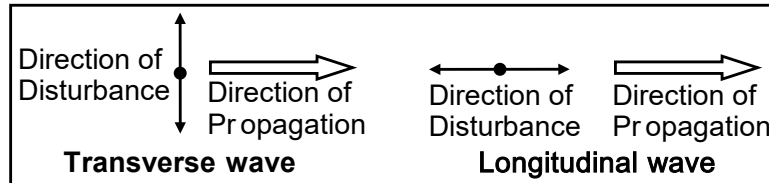
1.2 Based on Direction of Energy Propagation

Waves can be one, two or three dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one-dimensional.

Surface waves or ripples on water are two dimensional, while sound or light waves from a point source are three dimensional.

1.3 Based on Vibrational Direction

Waves are of two types on the basis of motion of particles of the medium.



(i) Transverse waves

(ii) Longitudinal waves

In the transverse wave the direction associated with the disturbance (i.e. motion of particles of the medium) is at right angle to the direction of propagation of wave while in the longitudinal wave the direction of disturbance is along the direction of propagation.

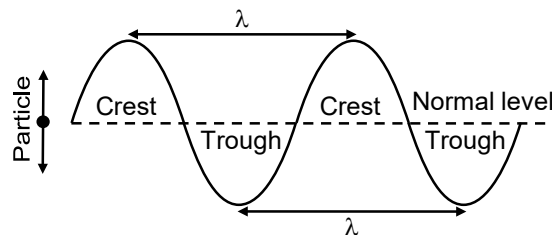
(i) Transverse Wave Motion

Mechanical transverse waves produced in such type of medium which have shearing property, so they are known as shear wave or S-wave.

Note:

Shearing is the property of a body by which it changes its shape on application of force.

⇒ Mechanical transverse waves are generated only in solids & surface of liquid.



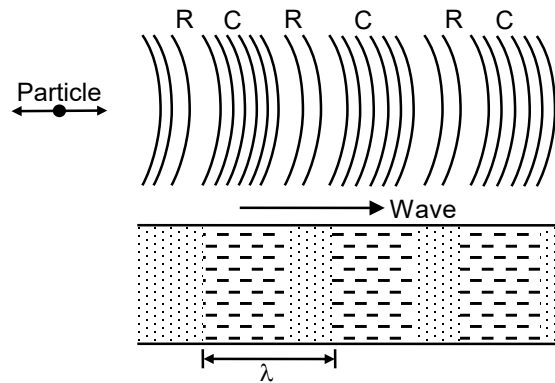
In this, individual particles of the medium execute SHM about their mean position in direction $\perp$ to the direction of propagation of wave motion.

A **crest** is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium, when a transverse wave passes.

A **trough** is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium, when a transverse wave passes.

(ii) Longitudinal Wave Motion

In this type of waves, oscillatory motion of the medium particles produces regions of compression (high pressure) and rarefaction (low pressure) which propagated in space with time (see figure).



Note:

The regions of high particle density are called compressions and regions of low particle density are called rarefactions.

The propagation of sound waves in air is visualized as the propagation of pressure or density fluctuations. The pressure fluctuations are of the order of 1 Pa, whereas atmospheric pressure is 10^5 Pa.

1.4 Based on Energy Propagation

Waves can be divided into two parts on the basis of energy propagation

- (i) Progressive wave
- (ii) Stationary waves

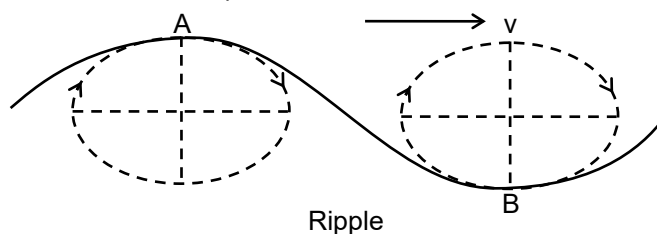
The progressive wave propagates with constant velocity in a medium. In stationary waves particles of the medium vibrate with different amplitude but energy does not propagate.

2. Mechanical Waves in Different Media

- A mechanical wave will be transverse or longitudinal depends on the nature of medium and mode of excitation.
- In strings mechanical waves are always transverse when string is under a tension. In gases and liquids mechanical waves are always longitudinal e.g. sound waves in air.
- In solids, mechanical waves can be either transverse or longitudinal depending on the mode of excitation. The speed of the two waves in the same solid are different. (Longitudinal waves travel faster than transverse waves).
- Furthermore, in case of seismic waves produced by Earthquakes both S (shear) and P (pressure) waves are produced simultaneously which travel through the rock in the crust at different speeds

$[v_s \cong 5 \text{ km/s while } v_p \cong 9 \text{ km/s}]$ S-waves are transverse while P-waves longitudinal.

- Some waves in nature are neither transverse nor longitudinal but a combination of the two. These waves are called 'ripple' and waves on the surface of a liquid are of this type. In these waves particles of the medium vibrate up and down and back and forth simultaneously describing ellipses in a vertical plane.



3. Characteristics of Wave Motion

- (a) Amplitude (A) :** Maximum displacement of vibrating particle from its equilibrium position.
- (b) Time Period (T) :** Time taken by wave to travel a distance equal to one wavelength.
- (c) Frequency (n) :** Number of vibrations (Number of complete wavelengths) complete by a particle in one second.

(d) Angular Frequency (ω) : It is defined as $\omega = \frac{2\pi}{T} = 2\pi n$

(e) Wavelength (λ) [Length of One Wave] : Distance travelled by the wave during the time, any one particle of the medium completes one vibration about its mean position. We may also define wavelength as the distance between any two nearest particles of the medium, vibrating in the same phase.

(f) Wave Number ($\bar{\nu}$) : It is defined as $\bar{\nu} = \frac{1}{\lambda}$ = number of waves in a unit length of the wave pattern.

(g) Angular Wave Number (k) : It is defined as $k = \frac{2\pi}{\lambda}$

4. Mathematical Representation of Progressive Wave

To describe a wave of any type mathematically functions which depend on two variables space and time i.e., $f(x, t)$ are required. It has been shown analytically that any function of space and time which satisfies the equation,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad \text{.....(i)}$$

Represents a wave, e.g., functions $y = A \sin \omega t$ or $y = A \sin kx$ don't satisfy the above equation, so don't represent waves.

The functions represent a travelling wave, i.e., the equation of a travelling wave is of the form $y = F(at \pm bx)$ (ii)

Negative sign between t and x implies that the wave is travelling along positive x -axis and vice-versa.

If a travelling wave is a sin or cos function of $F(at - bx)$ or $(at + bx)$, the wave is said to be harmonic or plane progressive wave. Here we shall limit ourselves to 1-D plane progressive wave which is its most general form is given by

$$y = A \sin (\omega t \pm kx + \phi) \quad \text{.....(iii)}$$

Example 1:

The wave function, $y = A\sqrt{(x - vt)}$, $y = (x + vt)^3$, $y = A(x - vt)$ or $y = A \log (x + vt)$ though are of the form $y = f(x \pm vt)$, but not useful in wave motion. Explain why ?

Solution:

In dealing with wave motion we usually assume that the wave function is continuous, single valued and finite. No doubt the given wave functions are continuous and single valued, are not useful in wave motion as they are not finite for all values of x and t [as $x \rightarrow \infty$ or $t \rightarrow \infty$, $f(x, t) \rightarrow \infty$].

Example 2:

Which of the following functions represent a travelling wave ?

(a) $(x - vt)^2$

(b) $\ln(x + vt)$

(c) $e^{-(x-vt)^2}$

(d) $\frac{1}{x + vt}$

Solution:

Although all the four functions are written in the form $f(ax \pm bt)$, only (c) among the four functions is finite everywhere at all times. Hence only (c) represents a travelling wave.

Example 3:

An observer standing at sea coast observes 54 waves reaching the coast per minute. If the wavelength of the waves is 10 m, find the velocity. What type of waves did he observe ?

Solution:

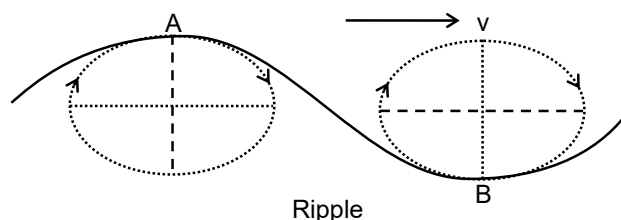
As 54 waves reach the shore per minute,

$$f = \frac{54}{60} = \frac{9}{10} \text{ Hz}$$

And as wavelength of waves is 10 m

$$v = f\lambda = \frac{9}{10} \times 10 = 9 \text{ m/s}$$

The waves on the surface of water are combined transverse and longitudinal called 'ripples'. In case of surface waves the particles of the medium move in elliptical paths in a vertical plane so that the vibrations are simultaneously back and forth and up and down.



Example 4:

A man generates a symmetrical pulse in a string by moving his hand up and down. At $t = 0$ the point in his hand moves downward. The pulse travels with speed of 3 m/s on the string & his hands passes 6 times in each second from the mean position. Then the point on the string at a distance 3m will reach its upper extreme first time at time $t =$

- (1) 1.25 sec. (2) 1 sec (3) $\frac{13}{12}$ sec (4) none

Solution:

$$\text{Frequency of wave} = \frac{6}{2} = 3 \Rightarrow T = \frac{1}{3} \text{ s};$$

$$\lambda = vT = (3) \left(\frac{1}{3} \right) = 1 \text{ m}$$

$$\text{Total time taken} = \frac{3}{3} + \frac{3T}{4} = 1.25 \text{ sec}$$

Concept Builder-1

- Q.1** Which of the following functions represent a travelling wave ?
 (1) $(x - vt)^2$ (2) $(x - vt)^3$ (3) $2^{-(x-vt)^2}$ (4) $e^{(x - vt)}$
- Q.2** Distinguish between sound and radio waves of the same frequency (say 15 kHz).
- Q.3** Explain why (a) transverse mechanical waves cannot be propagated in gases while (b) waves on strings are always transverse ?

5. Kinematic Parameters of a Progressive Wave**5.1 Phase, Phase Difference, Path Difference**

For any harmonic plane progressive wave represented by $y = A \sin (\omega t - kx)$

The argument of harmonic function $(\omega t - kx)$ is called phase of the wave and is constant if the shape of the wave remains unchanged.

Further if we consider two points at positions x_1 and x_2 on a wave at a given instant, then

$$\phi_1 = \omega t - kx_1 \quad \text{and} \quad \phi_2 = \omega t - kx_2$$

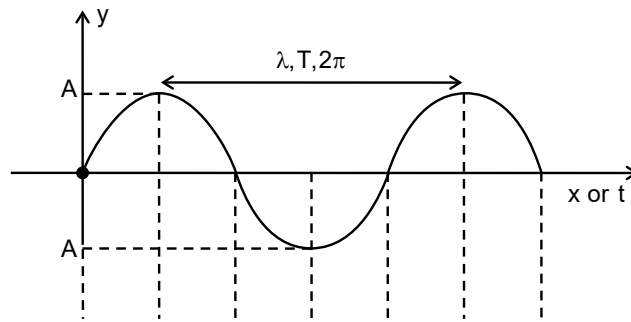
$$\text{So,} \quad \phi_2 - \phi_1 = k(x_2 - x_1)$$

$$\text{i.e.,} \quad \Delta\phi = \frac{2\pi}{\lambda} \Delta x \quad \left[\text{as } k = \frac{2\pi}{\lambda} \right]$$

From this it is clear that if $\Delta x = \lambda$, $\Delta\phi = 2\pi$, i.e., a path difference λ corresponds to a phase change of (2π) rad.

Phase is a quantity which contains all information related to any vibrating particle in a wave.

- **Relation between Phase difference, Path difference & Time difference**



Phase (ϕ)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
Wave length ($\Delta\lambda$)	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ	$\frac{5\lambda}{4}$	$\frac{3\lambda}{2}$
Time Period (ΔT)	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T	$\frac{5T}{4}$	$\frac{3T}{2}$

$$\Rightarrow \frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T}$$

$$\Rightarrow \text{Path difference} = \left(\frac{\lambda}{2\pi} \right) \text{Phase difference}$$

5.2 Wave Velocity & Particle Velocity

Wave Velocity

The velocity with which the disturbance, or planes of equal phase (wave front), travel through the medium is called wave (or phase) velocity.

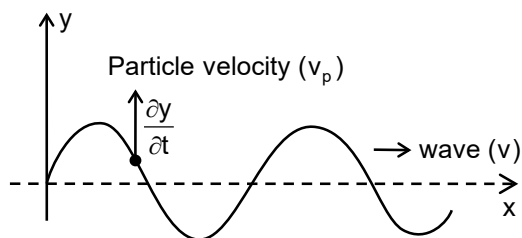
The so called wave or phase velocity will be given by

$$v = \frac{dx}{dt} = \frac{\omega}{k} = \frac{2\pi f}{(2\pi/\lambda)} = f\lambda$$

And depends only on the nature of the medium in which the wave propagates and is independent of the source producing the waves.

Particle Velocity in Wave Motion

The individual particles which make up the medium do not travel through the medium with the waves. They simply oscillate about their equilibrium positions. The instantaneous velocity of an oscillating particle of the medium, through which a wave is travelling, is known as "Particle velocity".



Relation Between Particle Velocity and Wave Velocity

Wave equation :- $y = A \sin (\omega t - kx)$,

$$\text{Particle velocity } v_p = \frac{\partial y}{\partial t} = A\omega \cos (\omega t - kx).$$

$$\text{Wave velocity } = v = \frac{\lambda}{T} = \lambda \frac{\omega}{2\pi} = \frac{\omega}{k}$$

$$\frac{\partial y}{\partial x} = -Ak \cos (\omega t - kx)$$

$$= -\frac{A}{\omega} \omega k \cos (\omega t - kx) = -\frac{1}{v} \frac{\partial y}{\partial t}$$

$$\Rightarrow \frac{\partial y}{\partial x} = -\frac{1}{v} \frac{\partial y}{\partial t} \quad \Rightarrow \quad \text{Slope} = -\frac{1}{v}(v_p) \quad \text{or} \quad v_p = -v (\text{Slope})$$

Note : $\frac{\partial y}{\partial x}$ represent the slope of the string (wave) at the point x.

5.3 Acceleration of Particle

For Any Harmonic Plane Progressive Wave

$$y = A \sin (\omega t - kx)$$

$$v_{pa} = \frac{\partial y}{\partial t} = A\omega \cos (\omega t - kx)$$

$$\& \quad a_{pa} = \frac{\partial v_{pa}}{\partial t} = -\omega^2 A \sin (\omega t - kx)$$

i.e., the acceleration of the particle equals $-\omega^2$ times its displacement, which is the result we obtained for SHM. Thus, $a_p = -\omega^2$ (displacement)

Example 5:

The equation for the displacement of a stretched string is given by

$$y = 4 \sin 2\pi \left[\frac{t}{0.02} - \frac{x}{100} \right]$$

where y and x are in cm and t in sec. Determine the (a) direction in which wave is propagating (b) amplitude (c) time period (d) frequency (e) angular frequency (f) wavelength (g) propagation constant (h) velocity of wave (i) phase constant and (j) the maximum particle velocity.

Solution:

Comparing the given equation with the general wave equation –

$$y = A \sin (\omega t - kx + \phi), \text{ i.e., } y = A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda} + \phi \right]$$

we find that :

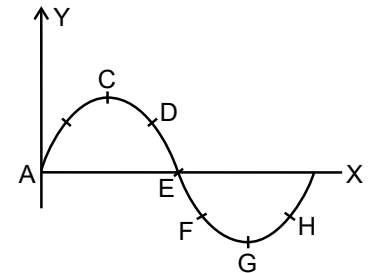
- (a) As there is negative sign between t and x terms, the wave is propagating along positive x-axis.
- (b) The amplitude of the wave $A = 4 \text{ cm} = 0.04 \text{ m}$.
- (c) The time period of the wave $T = 0.02 \text{ s} = (1/50\text{s})$.
- (d) The frequency of the wave $f = (1/T) = 50 \text{ Hz}$.

- (e) Angular frequency of the wave $\omega = 2\pi f = 100\pi \text{ rad/s}$.
- (f) The wavelength of the wave $\lambda = 100 \text{ cm} = 1 \text{ m}$.
- (g) The propagation constant. [= angular wave number = $(2\pi/\lambda) = 2\pi \text{ rad/m}$.]
- (h) The velocity of wave $v = f\lambda = 50 \times 1 = 50 \text{ m/s}$.
- (i) the phase constant, i.e., initial phase $\phi = 0$.
- (j) The max particle velocity $(v_{pa})_{\max} = A\omega = 0.04 \times 100\pi = 4\pi \text{ m/s}$.

Example 6:

A transverse wave is travelling along a string from left to right. Figure below represents the shape of the string (snap shot) at a given instant. At this instant

- (a) which points has an upward velocity ?
- (b) which points will have downward velocity ?
- (c) which points have zero velocity ?
- (d) which points have maximum magnitude of velocity ?

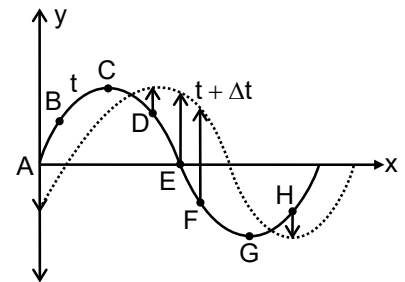


Solution:

For a wave $v_{pa} = -v \times (\text{slope})$

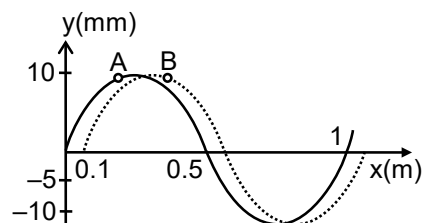
i.e., particle velocity is proportional to the negative of the slope of y/x curve; so

- (a) For upward velocity $v_{pa} = \text{positive}$, so slope must be negative which is at points D, E and F.
- (b) For downward velocity $v_{pa} = \text{negative}$, so slope must be positive which is at points A, B and H.
- (c) For zero velocity slope must be zero which is at C and G.
- (d) For maximum magnitude of velocity $|\text{slope}| = \text{max}$ which is at A and E.



Example 7:

Two identical waves A and B are produced from the origin at different instants t_A and t_B along the positive x-axis, as shown in the figure. If the speed of wave is 5m/s then



- (1) the wavelength of the waves is 1m
- (2) the amplitude of the waves is 10 mm
- (3) the wave A leads B by 0.0167 s
- (4) the wave B leads A by 1.67 s

Solution:

Wavelength of the waves = 1m; Amplitude of the waves = 10 mm

Example 8:

A wave is governed by the equation: $y = (4 \text{ cm}) \sin \left(2\pi t - \frac{\pi}{2} x \right)$

Here x is in cm. What is the phase difference between 2 points 20 mm part.

Solution:

$$\lambda = \frac{2\pi}{K} = 4 \text{ cm} \quad \text{So } \Delta\phi = \frac{2\pi}{4} \left(\frac{20}{10} \right) = \pi$$

Concept Builder-2

Q.1 The equation of a wave is,

$$y(x, t) = 0.05 \sin \left[\frac{\pi}{2} (10x - 40t) - \frac{\pi}{4} \right] \text{ m}$$

Find :

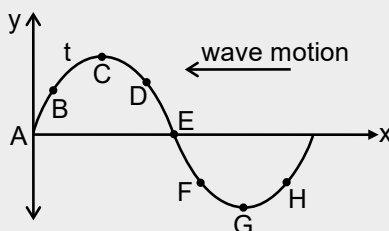
- (i) The wavelength, the frequency and the wave velocity
- (ii) The particle velocity and acceleration at $x=0.5 \text{ m}$ and $t=0.05 \text{ s}$.

Q.2 The equation of travelling wave is :

$$y = 0.07 \sin (12\pi x - 500\pi t)$$

where the distances are in m and time t in s respectively. Calculate the wavelength and velocity of the wave.

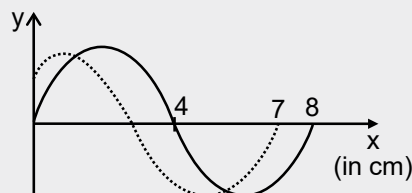
Q.3 A transverse wave is travelling along a stretched string from right to left. The figure shown represents the shape of the string (snap shot) at a given instant. At this instant :



- (1) the particles at A, B and H have upward velocity
- (2) the particles at D, E and F have downward velocity
- (3) the particles at C, E and G have zero velocity
- (4) the particles at A and E have maximum velocity

Q.4 Transverse wave on a string have wave speed 12.0 m/s , amplitude 0.05 m and wavelength 0.4 m . The waves travel in the $+x$ direction and at $t = 0$, the $x = 0$ end of the string has zero displacement and is moving upwards. Write a wave function describing the wave.

- Q.5** The snapshot of two identical waves at an instant is shown. Find out the phase difference between them.



- Q.6** The equation of a wave is $y = 6 \sin\left(\pi t + \frac{\pi}{8}x\right)$,
Here y is in cm & x is in mm. Find out the path difference between two points having phase difference of $\frac{\pi}{3}$ at any instant.

6. Energy in Wave Motion

Every wave motion has energy associated with it. In wave motion, energy and momentum are transferred or propagated.

To produce any of the wave motions, we have to apply a force to a portion of the wave medium. The point where the force is applied moves, so we do work on the system. As the wave propagates, each portion of the medium exerts a force and does work on the adjoining portion. In this way a wave can transport energy from one region of space to other.

6.1 Kinetic Energy Density

We can calculate the kinetic energy of unit volume of the string from the wave function. Mass of unit volume is density ρ . The kinetic energy of unit volume ΔK is then

$$\Delta K = \frac{1}{2} (\Delta m) v_y^2 = \frac{1}{2} \rho \left(\frac{\delta y}{\delta t} \right)^2$$

So the kinetic energy per unit volume is

$$\Delta K = \frac{1}{2} \rho \omega^2 A^2 \cos^2(kx - \omega t)$$

6.2 Potential Energy Density

The Potential energy of a segment is the work done in stretching the string and depends on the slope $\frac{\delta y}{\delta x}$.

$$\Delta U = \frac{1}{2} \rho v^2 \left(\frac{\delta y}{\delta x} \right)^2$$

$$\Delta U = \frac{1}{2} \rho \omega^2 A^2 \cos^2(kx - \omega t)$$

6.3 Energy Density

By the energy density of a plane progressive wave mean the total mechanical energy (kinetic + potential) per unit volume of the medium through which the wave is passing. Let us proceed to obtain an expression for it.

The total energy per unit volume is

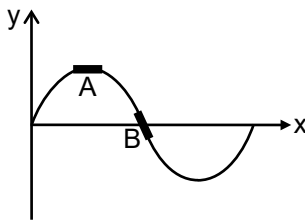
$$\Delta E = \Delta K + \Delta U = \rho \omega^2 A^2 \cos^2(kx - \omega t)$$

We see that ΔE varies with time. Since the average value of $\cos^2(kx - \omega t)$ at any point is $\frac{1}{2}$, the average energy per unit volume (called the energy density u) is

$$u = \frac{1}{2} \rho \omega^2 A^2$$

Important Facts

For a string segment, the potential energy depends on the slope of the string and is maximum when the slope is maximum, which is at the equilibrium position of the segment, the same position for which the kinetic energy is maximum.



At A : Kinetic energy and potential energy both are zero.

At B : Kinetic energy and potential energy both are maximum.

6.4 Power

Power is the instantaneous rate at which energy is transferred along the string (if we consider a transverse wave on a string).

In unit time, the wave will travel a distance v . If s be the area of cross-section of the string, then volume of this length would be sv and energy transmitted per unit time (called power) would be,

$$P = (\text{energy density}) (\text{volume})$$

$$\therefore P = \frac{1}{2} \rho \omega^2 A^2 sv$$

Note : This power is really the average power. The instantaneous power is given by,

$$P(x, t) = F_y(x, t) \cdot v_y(x, t)$$

6.5 Intensity

Flow of energy per unit area of cross-section of the string in unit time is known as the intensity of the wave. Thus,

$$I = \frac{\text{power}}{\text{area of cross-section}} = \frac{P}{s}$$

$$\text{or } I = \frac{1}{2} \rho \omega^2 A^2 v$$

Again this is the average intensity transmitted through the string. The instantaneous intensity $\rho \omega^2 A^2 v \sin^2(kx - \omega t)$ or $\rho \omega^2 A^2 v \cos^2(kx - \omega t)$ depends on x and t .

If P is the power of an isotropic point source, then intensity at a distance r is given by,

$$I = \frac{P}{4\pi r^2} \text{ or } I \propto \frac{1}{r^2} \quad (\text{for a point source})$$

If P is the power of a line source, then intensity at a distance r is given by,

$$I = \frac{P}{2\pi r \ell} \text{ or } I \propto \frac{1}{r} \quad (\text{for a line source}) \text{ As, } I \propto A^2$$

$$\text{Therefore, } A \propto \frac{1}{r} \quad (\text{for a point source})$$

$$\text{and } A \propto \frac{1}{\sqrt{r}} \quad (\text{for a line source})$$

Example 9:

Two mechanical waves, $y_1 = 2 \sin 2\pi (50t - 2x)$ & $y_2 = 4 \sin 2\pi (ax + 100t)$ propagate in a medium with same speed.

(1) The ratio of their intensities is 1 : 16

(2) The ratio of their intensities is 1 : 4

(3) The value of 'a' is 4 units

(4) The value of 'a' is 2 units

Solution:

(1, 3)

$$v = \frac{\omega}{k} \Rightarrow \frac{50}{2} = \frac{100}{a} \Rightarrow a = 4$$

$$\frac{I_1}{I_2} = \left(\frac{\omega_1}{\omega_2} \right)^2 \left(\frac{A_1}{A_2} \right)^2 = \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) = \frac{1}{16}$$

$$I = \frac{1}{2} \rho v \omega^2 A^2 \text{ and velocity} = \frac{\omega}{k}$$

Concept Builder-3



Q.1 Find out the ratio of intensities of two waves, travelling into same medium & represented

$$\text{as } y_1 = 4 \sin \pi(10t - 4x)$$

$$y_2 = 2 \sin 2\pi(4x + 10t)$$

Q.2 If the intensity ratio of two same frequency waves and of same speed, travelling in same medium is 4 : 1, the ratio of their amplitude is :

(1) 2 : 1

(2) 1 : 1

(3) 4 : 1

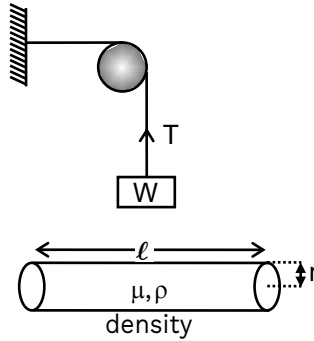
(4) 1 : 4

7. Velocity of Transverse Wave in a String

Mass of per unit length $\mu = \frac{\pi r^2 \ell \times \rho}{\ell} = \pi r^2 \rho$, where ρ = Density of matter.

Velocity of transverse wave in any wire

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\pi r^2 \rho}} = \sqrt{\frac{T}{A \rho}} \quad \because \pi r^2 = A$$



- If m is constant then, $v \propto \sqrt{T}$
 $\leftarrow$ It is called tension law.
- If tension is constant then $v \propto \sqrt{\frac{1}{\mu}}$
 $\leftarrow$ It is called law of mass
- If T is constant & take wire of different radius for same material then $v \propto \frac{1}{r}$
 $\leftarrow$ It is called law of radius
- If T is constant & taking wire of same radius for different material. Then
 $\leftarrow$ It is called law of density

Example 10:

A string of length 4 m & mass 20gm is clamped between two ends. If the tension in the string is 50N then find

- Speed of wave in string
- time taken by the wave to travel between two ends
- By what amount tension must be increased so that speed of the wave becomes 120 m/s

Solution:

$$T = 50 \text{ N}, \mu = \frac{20 \times 10^{-3}}{4}$$

$$\text{or } \mu = 5 \times 10^{-3} \text{ kg/m}$$

$$(a) \quad v = \sqrt{\frac{T}{\mu}} = 100 \text{ m/s}$$

$$(b) \quad t = \frac{d}{v} = \frac{4}{100} = 0.04 \text{ s}$$

$$(c) \quad T = \mu v^2 = 5 \times 10^{-3} (120)^2$$

$$\Rightarrow T = 72 \text{ N so tension must be increased by 22 N}$$

Example 11:

A wave moves with speed 300 m/s on a wire which is under a tension of 500 N. Find how much the tension be changed to increase the speed to 312 m/s ?

Solution:

Speed of a transverse wave on a wire is,

$$v \propto \sqrt{T}$$

$$\text{So, } \left(\frac{v_1}{v_2} \right)^2 = \frac{T_1}{T_2}$$

$$T_2 = \frac{v_2^2}{v_1^2} \times T_1$$

$$T_2 = \left(\frac{312}{300} \right)^2 \times 500 = 540.8 \text{ N}$$

$$\Delta T = T_2 - T_1 = 540.8 - 500$$

$$\Delta T = 40.8 \text{ N}$$

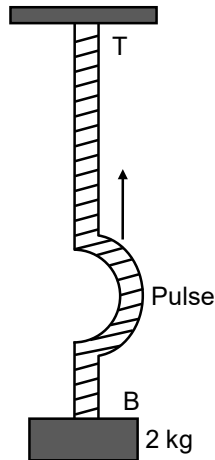
i.e., tension should be increased by 40.8 N

Example 12:

A uniform rope of length 12m and mass 6kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope ?

Solution:

As the rope has a mass and m mass is also suspended from the lower end, the tension in the rope will be different at different points.



$$\text{Now as } v = \sqrt{(T / m)}$$

$$\frac{v_T}{v_B} = \sqrt{\frac{T_T}{T_B}} = \sqrt{\frac{(6+2)g}{2g}} = 2$$

$$\text{or } \left[\frac{f_T \lambda_T}{f_B \lambda_B} \right] = 2 \quad [\text{as } v = f\lambda]$$

Here $f_T = f_B$ as frequency is the characteristic of the source producing the waves.

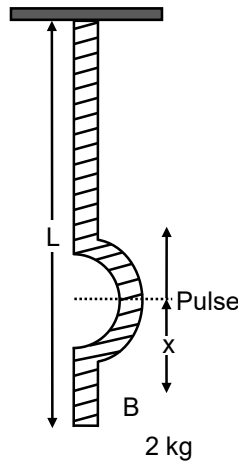
$$\text{So, } \lambda_T = 2\lambda_B = 2 \times 0.06 = 0.12 \text{ m}$$

Example 13:

A uniform rope of mass 0.1 kg and length 2.45 m hangs from a ceiling. (a) Find the speed of transverse wave in the rope at a point 0.5 m distant from the lower end, (b) Calculate the time taken by a transverse wave to travel the full length of the rope. ($g = 9.8 \text{ m/s}^2$)

Solution:

(a) As the string has mass and it is suspended vertically, tension in it will be different at different points. For a point at a distance x from the free end, tension will be due to the weight of the string below it. So if M is the mass of string of length L , the mass of length x of the string will be $(M/L)x$.



$$\therefore T = \left[\frac{M}{L} x \right] g$$

$$\text{So, } v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mgx}{L(M/L)}}$$

$$\sqrt{gx} = \dots(i)$$

Here $x = 0.5 \text{ m}$

$$\text{So, } v = \sqrt{0.5 \times 9.8} = 2.21 \text{ m/s}$$

(b) From part (a) it is clear that the tension and so the velocity of the wave is different at different points. So if at point x the wave travels a distance dx in time dt ,

$$v = \frac{dx}{dt} \quad \text{or} \quad \sqrt{gx} = \frac{dx}{dt} \quad [\text{from eqn. (i)}]$$

$$\text{or } \int dt = \int \frac{dx}{\sqrt{gx}}, \quad \text{i.e.,} \quad t = \frac{1}{\sqrt{g}} \int_0^L x^{-1/2} dx$$

$$\text{i.e.,} \quad t = 2\sqrt{L/g}$$

Hence $L = 2.45 \text{ m}$

$$\text{So} \quad t = 2\sqrt{(2.45/9.8)} = 1 \text{ sec}$$



Q.1 A transverse wave described by equation $y = 0.02 \sin (x + 30t)$ (where x and t are in meters and sec. respectively) is travelling along a wire of area of cross-section 1 mm^2 and density 8000 kg/m^3 . What is the tension in the string ?

- (1) 20 N (2) 7.2 N (3) 30 N (4) 14.4N

Paragraph For (2 & 3)

A mass of 2 kg is hanging from a rigid support by a string of mass 10 gm and length 2m. Find the

Q.2 Speed of the transverse wave ($g = 10 \text{ m/s}^2$)

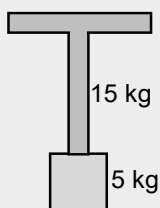
- (1) 60.25 m/s (2) 63.25 m/s (3) 66.25 m/s (4) 69.25 m/s

Q.3 Time taken by a transverse wave pulse to move from bottom to top of the wire.

- (1) 0.03 s (2) 0.04 s (3) 0.05 s (4) 0.06 s

Q.4 The wave moves in a wire with a speed of 100 m/s. The tension in the wire is 60 N. What will be the speed of wave if the tension is decreased by 3N

Q.5 A uniform rope of length 10 cm and mass 15 kg hangs vertically from a rigid support. A block of mass 5kg is attached to the free end of the rope. A transverse pulse of wavelength 0.08 m is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope will be :



- (1) 0.08 m (2) 0.04 m (3) 0.16 m (4) 0 m

8. Sound Wave

8.1 Sound Wave

- According to their frequencies longitudinal mechanical waves are divided into the following three categories.

Infrasonic Waves

- Longitudinal waves having frequencies below 20 Hz are called infrasonic waves.
- They cannot be heard by human beings.
- They are produced during earthquakes.
- Infrasonic waves can be heard by snakes.

Audible Waves

- Longitudinal waves having frequencies lying between 20-20,000 Hz are called audible waves.

Ultrasonic Waves

- Longitudinal waves having frequencies above 20,000 Hz are called ultrasonic waves.
- They are produced and heard by bats.
- They have a large energy content.

Newton Formula

$$v_{\text{medium}} = \sqrt{\frac{E}{\rho}} \quad (\text{Use for every medium})$$

Where E = Elasticity coefficient of medium

& ρ = Density of medium

- **For Solid Medium**

$$v_{\text{solid}} = \sqrt{\frac{Y}{\rho}} \quad \text{Where } E = Y = \text{Young's modulus}$$

- **For Liquid Medium**

$$v_{\text{liquid}} = \sqrt{\frac{B}{\rho}} \quad \text{Where } E = B,$$

where B = volume elasticity coefficient of liquid

- **For Gas Medium**

The formula for velocity of sound in air was first obtained by Newton. He assumed that sound propagates through air and temperature remains constant. (i.e. the process is isothermal) so Isothermal Elasticity

$$B = P \therefore v_{\text{air}} = \sqrt{P / \rho}$$

At NTP for air

$$P = 1.01 \times 10^5 \text{ N/m}^2 \text{ and}$$

$$\rho = 1.3 \text{ kg/m}^3$$

$$\text{so } v_{\text{air}} = \sqrt{\frac{1.01 \times 10^5}{1.3}} = 279 \text{ m/s}$$

However, the experimental value of sound in air is 332 m/s which is much higher than given by Newton's formula.

- **Laplace Correction**

In order to remove the discrepancy between theoretical and experimental values of velocity of sound, Laplace modified Newton's formula assuming that propagation of sound in air is adiabatic process,

i.e. Adiabatic Elasticity = γP

$$\text{so that } v = \sqrt{\frac{\gamma P}{\rho}}$$

$$\text{i.e. } v = \sqrt{1.41} \times 279 = 331.3 \text{ m/s [as } \gamma_{\text{air}} = 1.41]$$

Which is in good agreement with the experimental value (332 m/s).

- In a cloud-lightening, though light and sound are produced simultaneously but as $c > v$, light proceeds thunder.

$$v_s = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M_w}}$$

- from kinetic-theory of gases

$$v_{rms} = \sqrt{(3RT / M_w)} \text{ So } \frac{v_s}{v_{rms}} = \sqrt{\frac{\gamma}{3}}$$

- **Effect of Various Quantities**

(a) Effect of Temperature

For a gas γ & M_w is constant $v \propto \sqrt{T}$

$$\Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{t + 273}{273}}$$

$$\Rightarrow v_t = v_0 \left[1 + \frac{t}{273} \right]^{\frac{1}{2}}$$

By applying Binomial theorem.

(i) For any gas medium $v_t = v_0 \left[1 + \frac{t}{546} \right]$

(ii) For air : $v_t = v_0 + 0.61 t \text{ m/sec. } (v_0 = 332 \text{ m/sec.})$

(b) Effect of Relative Humidity

With increase in humidity, density decreases since $v = \sqrt{\gamma P / \rho}$. We conclude that with rise in humidity velocity of sound increases. This is why sound travels faster in humid air (rainy season) than in dry air (summer) at same temperature. Due to this in rainy season the sound of factories siren and whistle of train can be heard more than summer.

(c) Effect of Pressure

As velocity of sound $v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

So pressure has no effect on velocity of sound in a gas as long as temperature remain constant. This is why in going up in the atmosphere, though both pressure and density decreases, velocity of sound remains constant as long as temperature remains constant. Furthermore it has also been established that all other factors such as amplitude, frequency, phase, loudness pitch, quality etc. has partially no effect on velocity of sound.

(d) Effect of Motion of Air

If air is blowing then the speed of sound changes. If the actual speed of sound is v and the speed of air is w , then the speed of sound in the direction in which air is blowing will be $(v + w)$, and in the opposite direction it will be $(v - w)$.

(e) Effect of Frequency

There is no effect of frequency on the speed of sound. Sound waves of different frequencies travel with the same speed in air although their wavelength in air are different. If the speed of sound were dependent on the frequency, then we could not have enjoyed orchestra.

Example 14:

- (i) Speed of sound in air is 332 m/s at NTP. What will the speed of sound in hydrogen at NTP if the density of hydrogen at NTP is (1/16) that of air.
- (ii) Calculate the ratio of the speed of sound in neon to that in water vapour at any temperature. [Molecular weight of neon = 2.02×10^{-2} kg/mol and for water vapours = 1.8×10^{-2} kg/mol]

Solution:

The velocity of sound in air is given by

$$v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

- (i) In terms of density and pressure

$$\frac{v_H}{v_{air}} = \sqrt{\frac{P_H}{\rho_H} \times \frac{\rho_{air}}{P_{air}}} = \sqrt{\frac{\rho_{air}}{P_H}} \quad [\text{as } P_{air} = P_H]$$

$$\Rightarrow v_H = v_{air} \times \sqrt{\frac{\rho_{air}}{P_H}} = 332 \times \sqrt{\frac{16}{1}} = 1328 \text{ m/s}$$

- (ii) In terms of temperature and molecular weight

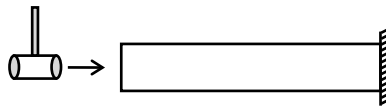
$$\frac{v_{Ne}}{v_W} = \sqrt{\frac{\gamma_{Ne}}{M_{Ne}} \times \frac{M_W}{\gamma_W}} \quad [\text{as } T_N = T_W]$$

Now as neon is mono atomic ($\gamma = 5/3$) while water vapours poly atomic ($\gamma = 4/3$) so

$$\begin{aligned} \frac{v_{Ne}}{v_W} &= \sqrt{\frac{(5/3) \times 1.8 \times 10^{-2}}{(4/3) \times 2.02 \times 10^{-2}}} \\ &= \sqrt{\frac{5}{4} \times \frac{1.8}{2.02}} = 1.055 \end{aligned}$$

Example 15:

A 1000 m long rod of density $10.0 \times 10^4 \text{ kg/m}^3$ and having young's modulus $Y = 10^{11} \text{ Pa}$, is clamped at one end. It is hammered at the other free end as shown in the figure. The longitudinal pulse goes to right end, gets reflected and again returns to the left end. How much time (in sec) the pulse takes to go back to initial point?

**Solution:**

(2)

Velocity of longitudinal

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{10^{11}}{10 \times 10^4}} = 10^3 \text{ ms}^{-1}$$

$$\text{Required time } \frac{2l}{v} = \frac{2 \times 1000}{10^3} = 2 \text{ s}$$

8.3 Intensity of Sound

The physiological sensation of loudness is closely related to the intensity of wave producing the sound. At a frequency of 1 kHz people are able to detect sounds with intensities as low as 10^{-12} W/m^2 . On the other hand, an intensity of 1 W/m^2 can cause pain, and prolonged exposure to sound at this level will damage a person's ears. Because the range in intensities over which people hear is so large, it is convenient to use a logarithmic scale to specify intensities. If the intensity of sound in watts per meter square is I , then the intensity level β in decibels (dB) is given by

$$\beta = 10 \log \frac{I}{I_0}$$

where the base of the logarithm is 10 and $I_0 = 10^{-12} \text{ W/m}^2$ (roughly the minimum intensity that can be heard).

On the decibel scale, the pain threshold of 1 W/m^2 is then

$$\beta = 10 \log \frac{1}{10^{-12}} = 120 \text{ dB}$$

8.4 Echo of Sound

Multiple reflection of sound is called an echo. If the distance of reflector from the source is d then, $2d = vt$

$$\therefore d = \frac{vt}{2}$$

Since, the effect of ordinary sound remains on our ear for $1/10$ second, therefore, if the sound returns to the starting point before $1/10$ second, then it will not be distinguished from the original sound and no echo will be heard. Therefore, the minimum distance of the reflector is,

$$d_{\min} = \frac{v \times t}{2} = \left(\frac{330}{2} \right) \left(\frac{1}{10} \right) = 16.5 \text{ m}$$

Example 16:

Calculate the change in intensity level when the intensity of sound increases by 10^6 times its original intensity.

Solution:

$$\text{Here, } \frac{I}{I_0} = \frac{\text{Final intensity}}{\text{Initial intensity}} = 10^6$$

Increase in intensity level,

$$L = 10 \log_{10} \left(\frac{I}{I_0} \right) = 10 \log_{10} (10^6) = 10 \times \log_{10} 10 = 10 \times 6 \times 1 = 60 \text{ decibels.}$$



- Q.1** Calculate the temperature at which the speed of sound in air becomes double of its value at 0°C .
- Q.2** Velocity of sound in a tube containing air at 27°C and at a pressure of 76 cm of Hg is 300 m/s. What will its velocity be when the pressure is increased to 100 cm of Hg and the temperature is kept constant ?

9. Superposition of Waves

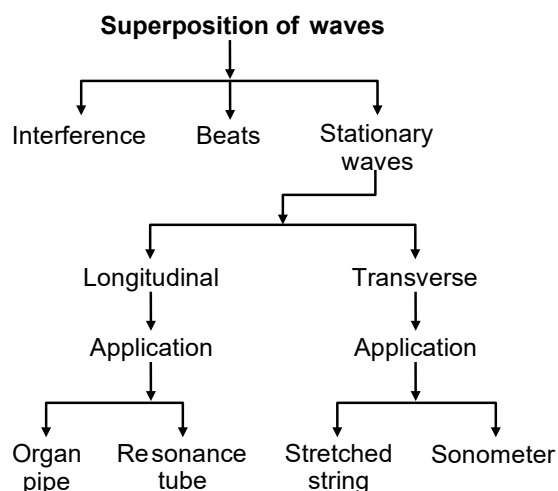
Two or more waves can propagate in the same medium without affecting the motion of one another. If several waves propagate in a medium simultaneously, then the resultant displacement of any particle of the medium at any instant is equal to the vector sum of the displacements produced by individual wave. The phenomenon of intermixing of two or more waves to produce a new wave is called Superposition of waves. Therefore, according to superposition principle.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, \dots$ are the displacement of particle at a particular time due to individual waves, then the resultant displacement is given by $\vec{y}_R = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \dots$

Principle of superposition holds for all types of waves, i.e., mechanical as well as electromagnetic waves. But this principle is not applicable to the waves of very large amplitude.

Due to superposition of waves the following phenomenon can be seen

- **Interference** : Superposition of two waves having equal frequency and nearly equal amplitude.
- **Beats** : Superposition of two waves of nearly equal frequency in same direction.
- **Stationary waves** : Superposition of equal wave in opposite direction.



Example 17:

Three simple harmonic waves, identical in frequency n and amplitude A moving in the same direction are superimposed in air in such a way, that the first, second and the third wave have the phase angles $\phi, \phi + \frac{\pi}{2}$ and $(\phi + \pi)$ respectively at a given point P in the superposition. Then as

the waves progress, the superposition will result in

- (1) a periodic, non-simple harmonic wave of amplitude $3A$
- (2) a stationary simple harmonic wave of amplitude $3A$
- (3) a simple harmonic progressive wave of amplitude A
- (4) the velocity of the superposed resultant wave will be the same as the velocity of each wave

Solution:

(3, 4)

Since the first wave and the third wave moving in the same direction have the phase angles ϕ and $(\phi + \pi)$, they superpose with opposite phase at every point of the vibrating medium and thus cancel out each other, in displacement, velocity and acceleration. They, in effect, destroy each other out. Hence we are left with only the second wave which progresses as a simple harmonic wave of amplitude A . The velocity of this wave is the same as if it were moving alone.

Concept Builder-6

Q.1 Two waves having equations

$$x_1 = a \sin(\omega t + \phi_1), \quad x_2 = a \sin(\omega t + \phi_2)$$

If in the resultant wave the frequency and amplitude remain equal to those of superimposing waves. Then phase difference between them is

- (1) $\frac{\pi}{6}$ (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

Q.2 Write the equation of resultant wave formed after superposition of waves given by

$$y_1 = A \sin(\omega t - kx),$$

$$y_2 = -A \cos(\omega t - kx),$$

$$y_3 = 2A \sin(\omega t - kx + \pi),$$

$$y_4 = -A \cos(\omega t - kx + \frac{\pi}{2}).$$

Q.3 Two identical travelling waves moving in the same direction are out of phase by ϕ radians. What is the amplitude of the combined wave in term of the common amplitude A of the combining waves?

10. Interference of Waves

When two waves of equal frequency and nearly equal amplitude travelling in same direction having same state of polarisation in medium superimpose, then intensity is different at different points. At some points intensity is large, whereas at other points it is nearly zero.

Consider two waves

$$y_1 = A_1 \sin(\omega t - kx) \text{ and } y_2 = A_2 \sin(\omega t - kx + \phi)$$

By principle of superposition

$$y = y_1 + y_2 = A \sin(\omega t - kx + \delta)$$

$$\text{where } A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

and $\tan \delta = \frac{A_2 \sin \phi}{A_1 + A_2 \cos \phi}$

As intensity $I \propto A^2$

so $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$

- **Constructive Interference (Maximum Intensity)**

Phase difference $\phi = 2n\pi$ or

path difference $= n\lambda$

where $n = 0, 1, 2, 3, \dots$

$\Rightarrow A_{\max} = A_1 + A_2$

and $I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

- **Destructive Interference (Minimum Intensity)**

Phase difference $\phi = (2n-1)\pi$, or

path difference $= (2n-1) \frac{\lambda}{2}$

where $n = 1, 2, 3, \dots$

$\Rightarrow A_{\min} = A_1 - A_2$

and $I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = (\sqrt{I_1} - \sqrt{I_2})^2$

Key Points

- Maximum and minimum intensities in any interference wave form.

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} \right)^2 = \left(\frac{A_1 + A_2}{A_1 - A_2} \right)^2$$

- Average intensity of interference wave form %&

$$\langle I \rangle \text{ or } I_{\text{av}} = \frac{I_{\max} + I_{\min}}{2} = I_1 + I_2$$

if $A = A_1 = A_2$

and $I_1 = I_2 = I$

then $I_{\max} = 4I$, $I_{\min} = 0$

and $I_{\text{AV}} = 2I$

- Degree of interference Pattern (f) : Degree of hearing (Sound Wave) or Degree of visibility (Light Wave)

$$f = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \times 100$$

In condition of perfect interference degree of interference pattern is maximum

$f_{\max} = 1$ or 100%

- Condition of maximum contrast in interference wave form $a_1 = a_2$ and $I_1 = I_2$ then $I_{\max} = 4I$ and $I_{\min} = 0$

For perfect destructive interference we have a maximum contrast in interference wave form.

Example 18:

Two sources of intensities I and $4I$ are used in an interference experiment. Find the intensity at points where the waves from the two sources superimpose with a phase difference of (a) zero, (b) $(\pi/2)$ and (c) π .

Solution:

In case of interference of two waves of intensities I_1 and I_2 with phase difference ϕ ,

$$I = I_1 + I_2 + 2(\sqrt{I_1 I_2}) \cos \phi$$

Here, $I_1 = I$ and $I_2 = 4I$,

$$\text{So, } I = I + 4I + 2\sqrt{4I \times I} \cos \phi = 5I + 4I \cos \phi$$

So

$$(a) \quad \text{For } \phi = 0 \quad I = 5I + 4I \times 1 = 9I \quad [\text{as } \cos 0 = 1]$$

$$(b) \quad \text{For } \phi = (\pi/2) \quad I = 5I + 4I \times 0 = 5I \quad [\text{as } \cos(\pi/2) = 0] \quad \text{and}$$

$$(c) \quad \text{For } \phi = \pi \quad I = 5I + 4I(-1) = I \quad [\text{as } \cos(\pi) = -1]$$

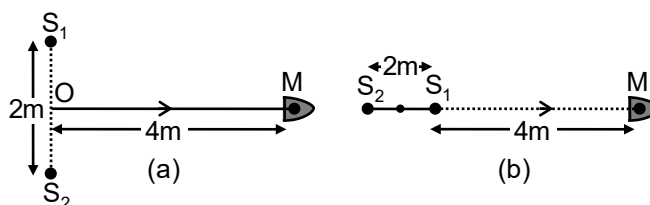
Example 19:

Two speakers connected to the same source of fixed frequency are placed 2.0 m apart in a box. A sensitive microphone placed at a distance of 4.0 m from their mid-point along the perpendicular bisector shows maximum response. The box is slowly rotated till the speakers are in line with the microphone. The distance between the mid-point of the speakers and microphone remains unchanged. Exactly 5 maximum responses are observed in the microphone in doing this. Calculate the wavelength of the sound wave.

Solution:

As shown in figure (a) initially $S_1M = S_2M$,

$\Delta x = 0$, i.e., at M there is zero order maximum.



According to the given problem on rotation to speakers about O when S_1 and S_2 are in line with microphone M is shown in figure (b), 5 maximum responses are observed,

$$\text{i.e., } S_2M - S_1M = 5\lambda$$

$$\text{or } S_2S_1 = 5\lambda$$

$$\text{i.e., } \lambda = 2/5 = 0.4 \text{ m} \quad [\text{as } S_1S_2 = 2\text{m}]$$

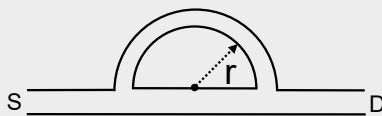
Concept Builder-7

Q.1 Two sources of intensity I interfere. What is the intensity at the point where the phase difference is

$$(A) \quad \frac{\pi}{3}$$

$$(B) \quad \frac{\pi}{2}$$

- Q.2** A sound wave of 40 cm wavelength enters the tube as shown in figure. What must be the smallest radius r such that a minimum will be heard at the detector ?



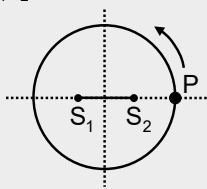
- Q.3** A sound source of frequency 170 Hz is placed near a wall. A man walking from the source normally towards the wall finds that there is a periodic rise and fall of sound intensity. If speed of sound in air is 340 m/s, the distance in meter between two successive position of minimum intensity is :

- (1) $\frac{1}{2}$ m (2) 1 m (3) $\frac{3}{2}$ m (4) 2 m

- Q.4** The two waves having intensities in the ratio 1 : 9 produce interference. The ratio of the maximum to the minimum intensities is equal to :

- (1) 10 : 8 (2) 9 : 1 (3) 4 : 1 (4) 2 : 1

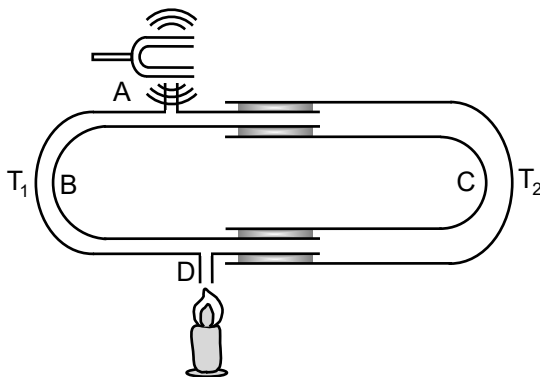
- Q.5** Find out the total maxima observed by a person moving along a circular track along the two coherent sources shown in figure. $s_1s_2 = 2\lambda$ (Assume S_1 & S_2 are at equal distance from centre)



Apparatus for Determining Speed of Sound

Quinck's Tube

It consists of two U shaped metal tubes. Sound waves with the help of tuning fork are produced at A which travel through B & C and comes out at D where a sensitive flame is present. Now the two waves coming through different path interfere and flame flares up. But if they are not in phase, destructive interference occurs and flame remains undisturbed.



Suppose destructive interference occurs at D for some position of C.

If now the tube C is moved so that interference condition is disturbed and again by moving a distance x , destructive interference occurs so that $2x = \lambda$.

Similarly, if the distance moved between successive constructive and destructive interference is x then $2x = \frac{\lambda}{2}$

Now by having value of x , speed of sound is given by $v = n\lambda$.

11. Beats

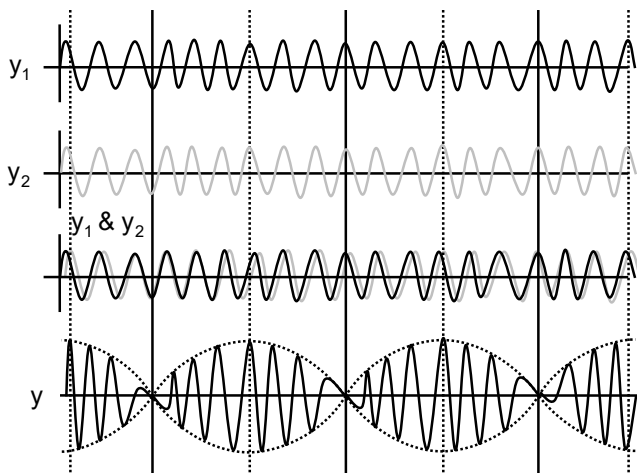
When two sound waves of same amplitude travelling in same direction with slightly different frequency superimpose, then intensity varies periodically with time. This effect is called Beats. Suppose two waves of frequencies f_1 and f_2 ($< f_1$) are meeting at some point in space.

The corresponding periods are T_1 and T_2 ($> T_1$).

If the two waves are in phase at $t=0$, they will again be in phase when the first wave has gone through exactly one more cycle than the second. This will happen at a time $t=T$ (the period of the beat).

Let n be the number of cycles of the first wave in time T , then the number of cycles of the second wave in the same time is $(n-1)$.

Hence, $T = nT_1 = (n-1)T_2$.



Eliminating n we have

$$T = \frac{T_1 T_2}{T_2 - T_1} = \frac{1}{\frac{1}{T_1} - \frac{1}{T_2}} = \frac{1}{f_1 - f_2}$$

The reciprocal of the beat period is the beat frequency $f = \frac{1}{T} = f_1 - f_2$

Waves Interference on The Basis of Beats

Two equal frequency waves travel in same direction. If displacement of first wave

$$y_1 = a \sin \omega_1 t \longrightarrow (f_1, a)$$

Displacement of second wave

$$y_2 = a \sin \omega_2 t \longrightarrow (f_2, a)$$

By superposition $y = y_1 + y_2$

Equation of resulting wave

$$y = a \{ \sin 2\pi f_1 t + \sin 2\pi f_2 t \}$$

$$y = a \left\{ 2 \sin 2\pi t \frac{(f_1 + f_2)}{2} \cos 2\pi t \frac{(f_1 - f_2)}{2} \right\}$$

$$= \left\{ 2a \cos 2\pi t \frac{(f_1 - f_2)}{2} \right\} \sin 2\pi t \frac{(f_1 + f_2)}{2} = A \sin 2\pi f' t$$

Amplitude

$$A = 2a \cos 2\pi t \left(\frac{f_1 - f_2}{2} \right) = 2a \cos \pi t (f_1 - f_2)$$

Frequency : $f' = \frac{f_1 + f_2}{2}$

Key Points



- When we added wax on tuning fork then the frequency of fork decreases.
- When we file the tuning fork then the frequency of fork increases.

Example 20:

A tuning fork having $n = 300$ Hz produces 5 beats/s with another tuning fork. If impurity (wax) is added on the arm of known tuning fork, the number of beats decreases then calculate the frequency of unknown tuning fork.

Solution:

The frequency of unknown tuning fork should be $300 \pm 5 = 295$ Hz or 305 Hz.

When wax is added, if it would be 305 Hz, beats would have increases but with 295 Hz beats is decreases so, frequency of unknown tuning fork is 295 Hz.

Example 21:

If two sound waves, $y_1 = 0.3 \sin 596\pi [t - x/330]$ and $y_2 = 0.5 \sin 604\pi [t - x/330]$ are superimposed, what will be the (a) frequency of resultant wave, (b) frequency at which the amplitude of resultant waves varies, (c) frequency at which beats are produced ? Find also the ratio of maximum and minimum intensities of beats.

Solution:

$A_1 = 0.3$ and $\omega_1 = 2\pi f_1 = 596\pi$, i.e., $f_1 = 298$ Hz

and $A_2 = 0.5$ and $\omega_2 = 2\pi f_2 = 604\pi$, i.e., $f_2 = 302$ Hz

So,

(a) The frequency of the resultant wave :

$$f_{av} = \frac{f_1 + f_2}{2} = \frac{(298 + 302)}{2} = 300 \text{ Hz}$$

(b) The frequency at which amplitude of resultant wave varies :

$$f_A = \frac{f_1 - f_2}{2} = \frac{(298 - 302)}{2} = 2 \text{ Hz}$$

(c) The frequency at which beats are produced

$$f_b = 2f_A = f_1 - f_2 = 4 \text{ Hz}$$

(d) The ratio of maximum to minimum intensities of beat

$$\frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{(0.3 + 0.5)^2}{(0.3 - 0.5)^2} = \frac{64}{4} = 16$$



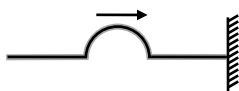
- Q.1** A tuning fork having $n = 158$ Hz, produce 3 beats/s with another. As we file the arm of unknown, beats become 7 then calculate the frequency of unknown.
- Q.2** Two vibrating tuning forks produce progressive waves given by $y_1 = 4 \sin(500\pi t)$ and $y_2 = 2 \sin(506\pi t)$. These tuning forks are held near the ear of a person. The person will hear α beats/s with intensity ratio between maxima and minima equal to β . Find the value of β & α .
- Q.3** Two tuning fork having frequency 320 Hz & 324 Hz produce beat phenomena. Determine-
- Beat time period
 - Minimum time interval in which maximum intensity becomes minimum.
 - Number of beats per three seconds.
- Q.4** 51 tuning fork are arranged in series in such a way that each fork produce five beats/sec with neighbouring tuning fork. If frequency of last is six time of first then determine frequency of first, 11th, 17th, 27th, 33th and last tuning fork.

12. Reflection of Wave in a String

12.1 From Rigid End

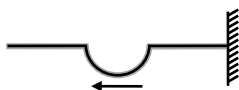
- When the pulse reaches the right end which is clamped at the wall, the element at the right end exerts a force on the clamp and the clamp exerts equal and opposite force on the element.
- Thus, the wave is reflected from the fixed end and the reflected wave is inverted with respect to the original wave.
- The shape of the string at any time, while the pulse is being reflected, can be found by adding an inverted image pulse to the incident pulse.
- Equation of wave propagating in +ve x-axis

Incident wave



$$y_1 = a \sin(\omega t - kx)$$

Reflected wave



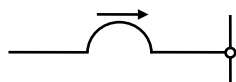
$$y_2 = a \sin(\omega t + kx + \pi)$$

or $y_2 = -a \sin(\omega t + kx)$

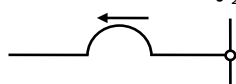
12.2 From Free End

- The right end of the string is attached to a light frictionless ring which can freely move on a vertical rod.
- A wave pulse is sent on the string from left. When the wave reaches the right end, the element at this end is acted on by the force from the left to go up.
- The element at the end is acted upon by both the incident and the reflected wave and the displacements add.
- Thus, a wave is reflected by the free end without inversion.

Incident wave $y_1 = a \sin (\omega t - kx)$

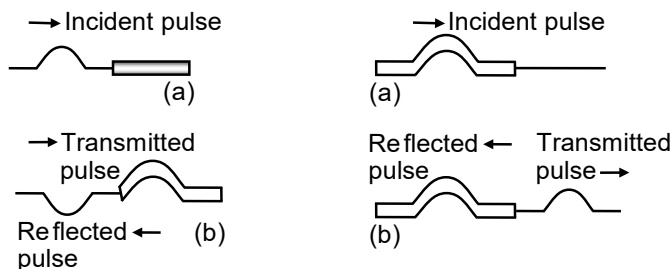


Reflected wave $y_2 = a \sin (\omega t + kx)$



13. Transmission of Wave

We may have a situation in which the boundary is intermediate between these two extreme cases, that is, one in which the boundary is neither rigid nor free. In this case, part of the incident energy is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string as in (figure). When a pulse travelling on the light reaches the knot, some part of it is reflected and inverted and some part of it is transmitted to the heavier string.



As one would expect, the reflected pulse has a smaller amplitude than the incident pulse, since part of the incident energy is transferred to the pulse in the heavier string. The inversion in the reflected wave is similar to the behaviour of a pulse meeting a rigid boundary, when it is totally reflected. When a pulse travelling on a heavy string strikes the boundary of a lighter string, as in (figure), again part is reflected and part is transmitted. However, in this case the reflected pulse is not inverted. In either case, the relative height of the reflected and transmitted pulses +depends on the relative densities of the two string. In the previous section, we found that the speed of a wave on a string increases as the density of the string decreases. That is, a pulse travels more slowly on a heavy string than on a light string, if both are under the same tension. The following general rules apply to reflected waves. When a wave pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is denser than A), the pulse will be inverted upon reflection. When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is denser than B), it will not be inverted upon reflection.

Key Points



Phenomenon of reflection and transmission of waves obeys the laws of reflection and refraction. The frequency of these wave remains constant i.e. does not change.

$$\omega_i = \omega_r = \omega_t = \omega$$

Example 22:

A wave on a string is represented as

$$y = 6 \sin 2\pi(t - 4x)$$

When this wave is made to fall on denser medium, what will be the equation of reflected wave ? (Assume 64% reflection).

Solution:

As we known $I \propto A^2$

$$\& \quad I_R = 0.64 I \quad \text{So, } A_R = 0.8 A = 4.8$$

So equation of reflected wave will be

$$y_R = 4.8 \sin \{2\pi (t + 4x) + \pi\}$$

14. Stationary Waves

Definition

- The superposition of two waves, travelling in same medium, with same speed but in opposite direction, generates stationary wave.
- When any travelling wave superimposes with it's reflected wave, stationary waves are formed.
- The wave propagating in such a medium will be reflected at the boundary and produce a wave of the same kind travelling in the opposite direction. The superposition of the two waves will give rise to a stationary wave.
- Formation of stationary wave is possible only and only in bounded medium.

Analytical Method for Stationary Waves

(i) **From Rigid End** : We know equation for progressive wave in positive x-direction.

$$y_1 = a \sin (\omega t - kx)$$

After reflection from rigid end

$$y_2 = a \sin (\omega t + kx + \pi) = -a \sin (\omega t + kx)$$

By principle of super position.

$$\begin{aligned} y &= y_1 + y_2 = a \sin (\omega t - kx) - a \sin (\omega t + kx) \\ &= -2a \sin kx \cos \omega t \end{aligned}$$

This is equation of stationary wave reflected from rigid end

$$\text{Amplitude} = 2a \sin kx$$

Velocity of particle

$$v_{pa} = \frac{dy}{dt} = 2a \omega \sin kx \sin \omega t$$

- **Node** $x = 0, \frac{\lambda}{2}, \lambda, \dots$

$$A = 0, v_{pa} = 0, \text{strain} \rightarrow \text{max.}$$

Change in pressure $\rightarrow \text{max}$

- **Antinode** $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \dots$

$$A \rightarrow \text{max}, v_{pa} \rightarrow \text{max. strain} = 0$$

Change in pressure = 0

(ii) **From Free End** : we know equation for progressive wave in positive x-direction

$$y_1 = a \sin (\omega t - kx)$$

After reflection from free end

$$y_2 = a \sin (\omega t + kx)$$

By Principle of superposition

$$\begin{aligned} y &= y_1 + y_2 = a \sin (\omega t - kx) + a \sin (\omega t + kx) \\ &= 2a \sin \omega t \cos kx \end{aligned}$$

$$\text{Amplitude} = 2a \cos kx,$$

Velocity of particle

$$v_{pa} = \frac{dy}{dt} = 2a \omega \cos \omega t \cos kx$$

- **Antinode** : $x = 0, \frac{\lambda}{2}, \lambda, \dots$ $A \rightarrow \text{Max},$

$$v_{pa} = \frac{dy}{dt} \rightarrow \text{max. Strain} = 0, \text{ change in pressure} = 0$$

- **Node** : $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$

$$A = 0, v_{pa} = \frac{dy}{dt} = 0, \text{strain} \rightarrow \text{max}, \text{ change in pressure} \rightarrow \text{max}$$

Important Characteristics of Stationary Waves are

- Stationary waves are produced in the bounded medium and the boundaries of bounded medium may be rigid or free.
- In stationary waves nodes and antinodes are formed alternately. Nodes are the points which are always in rest having maximum strain. Antinodes are the points where the particles vibrate with maximum amplitude having minimum strain.
- All the particles except at the nodes vibrate simple harmonically with the same period.
- The distance between any two successive nodes or antinodes is $\lambda/2$.
- The amplitude of vibration gradually increases from zero to maximum value from node to

antinode.

- All the particles in one particular segment vibrate in the same phase, but the particle of two adjacent segments differ in phase by 180°
- All points of the medium pass through their mean position simultaneously twice in each period.
- In longitudinal stationary waves, condensation (compression) and rarefaction do not travel forward as in progressive waves but they appear and disappear alternately at the same place.
- These waves do not transfer energy in the medium. Transmission of energy is not possible in a stationary wave.

Example 23:

A transverse wave, travelling along the positive x-axis, given by $y = A \sin (kx - \omega t)$ is superposed with another wave travelling along the negative x-axis given by $y = -A \sin (kx + \omega t)$. The point $x = 0$ is

- | | |
|------------------------------------|--|
| (1) a node | (2) an antinode |
| (3) neither a node nor an antinode | (4) a node or antinode depending on t. |

Solution:

(2)

At $x = 0$, $y_1 = A \sin (-\omega t)$

and $y_2 = -A \sin \omega t$;

$y_1 + y_2 = -2A \sin \omega t$ (antinode)

Example 24:

The vibrations of a string of length 60 cm fixed at both ends are represented by the equation

$$y = 4 \sin \left(\frac{\pi x}{15} \right) \cos(96\pi t) \text{ where } x \text{ and } y \text{ are in cm and } t \text{ in seconds.}$$

- What is the maximum displacement of a point at $x = 5\text{ cm}$?
- Where are the nodes located along the string?
- What is the velocity of the particle at $x = 7.5\text{ cm}$ and at $t = 0.25\text{ s}$?
- Write down the equation of the component wave whose superposition give the above wave.

Solution:

- (a) At $x = 5\text{ cm}$ the standing wave equation gives

$$\begin{aligned} y &= 4 \sin \left(\frac{5\pi}{15} \right) \cos(96\pi t) \\ &= 4 \sin \frac{\pi}{3} \cos(96\pi t) = 4 \times \frac{\sqrt{3}}{2} \cos(96\pi t) \end{aligned}$$

$\therefore$ Maximum displacement $= 2\sqrt{3}\text{ cm}$

- (b) The nodes are the point of permanent rest. Thus, they are those points for which

$$\sin\left(\frac{\pi x}{15}\right) = 0$$

$$\text{i.e., } \frac{\pi x}{15} = n\pi, \quad n = 0, 1, 2, 3, 4 \dots$$

$$x = 15n, \text{ i.e.,}$$

$$\text{at } x = 0, 15, 30, 45 \text{ and } 60 \text{ cm}$$

(c) The particle velocity is equal to

$$\left(\frac{\partial y}{\partial t}\right) = 4 \sin\left(\frac{\pi x}{15}\right) (96\pi) (-\sin 96\pi t)$$

$$= -384\pi \sin\left(\frac{\pi x}{15}\right) \sin(96\pi t)$$

at $x = 7.5$ and $t = 0.25$ s, we get

$$\left(\frac{\partial y}{\partial t}\right) = -384\pi \sin\left(\frac{\pi x}{15}\right) \sin(96\pi t)$$

$$= -384\pi \sin\left(\frac{\pi x}{15}\right) \sin(24\pi) = 0$$

(d) The equations of the component waves are :

$$y_1 = 2 \sin\left(\frac{\pi x}{15} + 96\pi t\right)$$

$$\text{and } y_2 = 2 \sin\left(\frac{\pi x}{15} - 96\pi t\right)$$

as we can see that $y = y_1 + y_2$.

Concept Builders-9



Q.1 A wave $y = 10 \sin(\omega t + kx)$ is incident on a rarer medium, If 36% of the wave is reflected back to the same medium write the equations of reflected wave.

Q.2 Two transverse waves A and B superimpose to produce a node at $x = 0$. If the equation of wave A is $y = a \cos(kx + \omega t)$, then the equation of wave B is

(1) $+a \cos(kx - \omega t)$ (2) $-a \cos(kx + \omega t)$ (3) $-a \cos(kx - \omega t)$ (4) $+a \cos(\omega t - kx)$

Q.3 The displacement of a standing wave on a string is given by

$$y(x, t) = 0.4 \sin(0.5x) \cos(30t)$$

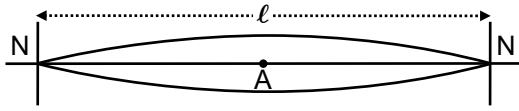
Where x and y are in centimeters.

(a) Find the frequency, amplitude and wave speed & wavelength of the component waves.

(b) What is the particle velocity at $x = \pi/2$ cm at $t = \pi/20$ s ?

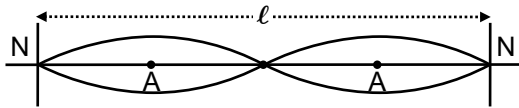
15. Transverse Stationary Waves & Their Behaviour

Fundamental Harmonic



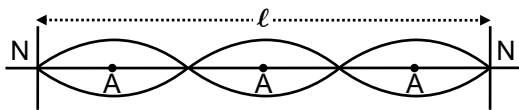
$$\frac{\lambda}{2} = l \Rightarrow \lambda = 2l \quad \Rightarrow f = \frac{v}{2l}$$

Second Harmonic



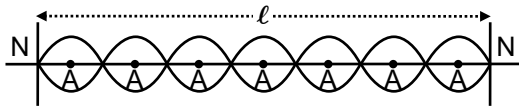
$$\frac{2\lambda}{2} = l \Rightarrow \lambda = l \quad \Rightarrow f = \frac{2v}{2l}$$

Third Harmonic



$$\frac{3\lambda}{2} = l \Rightarrow \lambda = \frac{2l}{3} \Rightarrow f = \frac{3v}{2l}$$

p^{th} Harmonic



$$\frac{p\lambda}{2} = l \Rightarrow \lambda = \frac{2l}{p} \quad \Rightarrow f = \frac{pv}{2l}$$

- **Law of length :** For a given string, under a given tension, the fundamental frequency of vibration is inversely proportional to the length of the string, i.e., $n \propto \frac{1}{l}$ (T and μ are constant)
- **Law of tension :** The fundamental frequency of vibration of stretched string is directly proportional to the square root of the tension in the string, provided that length and mass per unit length of the string are kept constant. $n \propto \sqrt{T}$ (l and μ are constant)
- **Law of mass :** The fundamental frequency of vibration of a stretched string is inversely proportional to the square root of its mass per unit length provided that length of the string and tension in the string are kept constant, i.e., $n \propto \frac{1}{\sqrt{\mu}}$ (l and T are constant)

16. Comparison of Progressive & Stationary Waves

Progressive Waves

1. These waves travel in a medium with finite velocity.
2. These waves transmit energy in the medium.
3. The phase of vibration varies continuously from particle to particle.
4. No particle of medium is permanently at rest.
5. All particle of the medium vibrate and amplitude of vibration is same.
6. All the particles do not attain the maximum displacement position simultaneously.

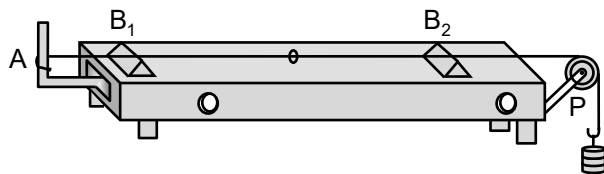
Stationary Waves

1. These waves do not travel and remain confined between two boundaries in the medium.
2. These waves do not transmit energy in the medium.
3. The phase of all the particles in between two nodes is always same. But particles of two Adjacent nodes differ in phase by 180° .
4. Particles at nodes are permanently at rest.
5. The amplitude of vibration changes from particles to particle. The amplitude is zero for all at nodes and maximum at antinodes.
6. All the particles attain the maximum displacement.

17. Experimental Analysis of Stationary Wave

Sonometer

Sonometer consists of a hollow rectangular box of light wood. One end of the experimental wire is fastened to one end of the box. The wire passes over a frictionless pulley P at the other end of the box. The wire is stretched by a tension T.



The box serves the purpose of increasing the loudness of the sound produced by the vibrating wire. If the length of the wire between the two bridges is ℓ , then the frequency of vibration is

$$n = \frac{1}{2\ell} \sqrt{\frac{T}{\mu}}$$

A small paper rider is placed on the string. When a vibrating tuning fork is placed on the box, and if the length between the bridges is properly adjusted, then when the two frequencies are exactly equal, the string quickly picks up the vibrations of the fork and the rider is thrown off the wire.

Example 25:

A sonometer wire resonates with a given tuning fork forming a standing wave with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass 'M' kg, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. Find the value of M.

(1) 25

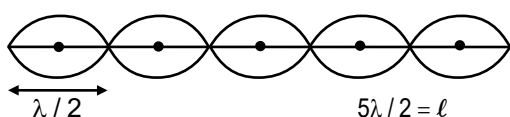
(2) 20

(3) 15

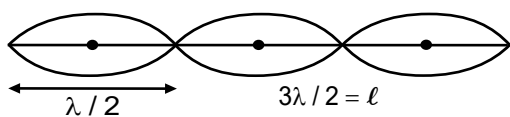
(4) 10

Solution:

(1)



$$f = \frac{\sqrt{\frac{9g}{\mu}}}{\frac{2\ell}{5}}$$



$$f = \frac{\sqrt{\frac{Mg}{\mu}}}{\frac{2\ell}{3}} \Rightarrow f = \frac{\sqrt{\frac{9g}{\mu}}}{\frac{2\ell}{5}} = \frac{\sqrt{\frac{Mg}{\mu}}}{\frac{2\ell}{3}} \Rightarrow M = 25$$

Example 26:

A standing wave is created on a string of length 120 m and it is vibrating in 6th harmonic. Maximum possible amplitudes of any particle is 10 cm and maximum possible velocity will be 10 cm/s. Choose the correct statement.

(1) Angular wave number of two waves will be $\frac{\pi}{20}$.

(2) Time period of any particle's SHM will be 4π sec.

(3) Any particle will have same kinetic energy as potential energy.

(4) Amplitude of interfering waves are 10 cm each.

Solution:

(1)

$$6\left(\frac{\lambda}{2}\right) = 120 \Rightarrow \lambda = 40\text{ m} \Rightarrow k = \frac{\pi}{20} \Rightarrow A\omega = v_{\max} \Rightarrow \omega = 1 \Rightarrow T = 2\pi\text{ sec.}$$

Example 27:

A sonometer wire has a length of 114 cm between two fixed ends. Where should two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1 : 3 : 4 ?

Solution:

In case of a given wire under specific tension, fundamental frequency of vibration $f \propto (1/L)$. So for having fundamental frequency in the ratio of 1 : 3 : 4, the vibrating length should be in the ratio $1 : (1/3) : (1/4)$, i.e., $L_1 : L_2 : L_3 :: 12 : 4 : 3$

If common factor is x,

$$12x + 4x + 3x = 114 = \text{length of the string}$$

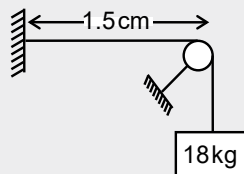
$$\text{i.e., } 19x = 114 \quad \text{or} \quad x = 6$$

$$\text{So, } L_1 = 12 \times 6 = 72\text{ m, } L_2 = 4 \times 6 = 24\text{ cm}$$

$$\text{and } L_3 = 3 \times 6 = 18\text{ cm}$$



- Q.1** The length of the wire shown in Fig. between the pulley and fixed support is 1.5 cm and mass is 12.0g. The frequency of vibration with which the wire vibrates in two loops leaving the middle point of the wire between the pulleys at rest is :



- (1) 100 Hz (2) 300 Hz (3) 1000 Hz (4) 700 Hz

- Q.2** Two wires are kept tight between the same pair of supports. The tension in the wire are in the ratio 2 : 1, the radii are in the ratio 3 : 1 and the densities are in the ratio 1 : 2. The ratio of their fundamental frequencies is :

- (1) 2 : 3 (2) 2 : 4 (3) 2 : 5 (4) 2 : 6

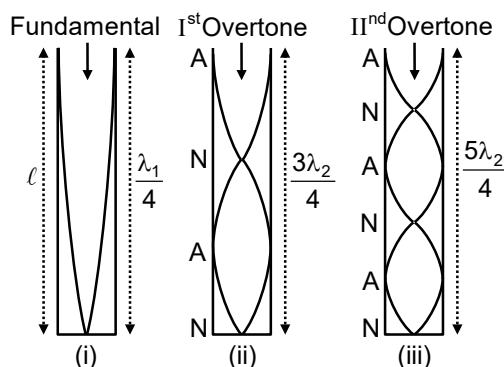
18. Longitudinal Stationary Waves (Organ Pipes)

Vibration in Organ Pipes

When two longitudinal waves of same frequency and amplitude travel in a medium in opposite directions then by superposition, standing waves are produced. These waves are produced in air columns in cylindrical tube of uniform diameter. These sound producing tubes are called organ pipes.

(i) Vibration of Air Column in Closed Organ Pipe

The tube which is closed at one end and open at the other end is called close organ pipe. On blowing air at the open end, a wave travels towards closed end from where it is reflected towards open end. As the wave reaches open end, it is reflected again. So two longitudinal waves travel in opposite directions to superpose and produce stationary waves. At the closed end there is a node since particles do not have freedom to vibrate whereas at open end there is an antinode because particles have greatest freedom to vibrate.



Hence on blowing air at the open end, the column vibrates forming antinode at free end and node at closed end. If l is length of pipe and λ be the wavelength and v be the velocity of sound in organ pipe then

$$\text{Case (i) } \ell = \frac{\lambda}{4} \Rightarrow \lambda = 4\ell$$

$$\Rightarrow n_1 = \frac{v}{\lambda} = \frac{v}{4\ell} \text{ fundamental frequency.}$$

$$\text{Case (ii) } \ell = \frac{3\lambda}{4} \Rightarrow \lambda = \frac{4\ell}{3}$$

$$\Rightarrow n_2 = \frac{v}{\lambda} = \frac{3v}{4\ell} \text{ First overtone/III}^{\text{rd}} \text{ Harmonic}$$

$$\text{Case (iii) } \ell = \frac{5\lambda}{4} \Rightarrow \lambda = \frac{4\ell}{5}$$

$$\Rightarrow n_3 = \frac{v}{\lambda} = \frac{5v}{4\ell} \text{ Second overtone/V}^{\text{th}} \text{ Harmonic}$$

When closed organ pipe vibrates in m^{th} overtone then $\ell = (2m+1)\frac{\lambda}{4}$

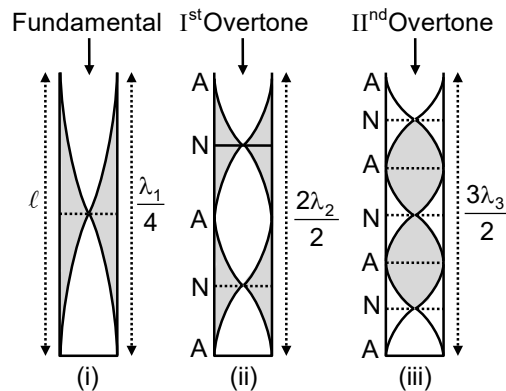
$$\text{So } \lambda = \frac{4\ell}{(2m+1)} \Rightarrow \boxed{n = (2m+1) \frac{v}{4\ell}}$$

Hence frequency of overtones is given by

$$n_1 : n_2 : n_3 : \dots = 1 : 3 : 5 : \dots$$

(ii) Vibration of Air Columns in Open Organ Pipe

The tube which is open at both ends is called an open organ pipe. On blowing air at the open end, a wave travels towards the other end after reflection from the open end waves travel in opposite direction to superpose and produce a stationary wave.



Now the pipe is open at both ends by which an antinode is formed at open end. Hence on blowing air at the open end antinodes are formed at each end and nodes in the middle. If ℓ is length of the pipe and λ be the wavelength and v is velocity of sound in organ pipe.

$$\text{Case (i) } \ell = \frac{\lambda}{2} \Rightarrow \lambda = 2\ell$$

$$\Rightarrow n_1 = \frac{v}{\lambda} = \frac{v}{2\ell} \text{ Fundamental frequency.}$$

$$\text{Case (ii) } \ell = \frac{2\lambda}{2} \Rightarrow \lambda = \frac{2\ell}{2}$$

$$\Rightarrow n_2 = \frac{v}{\lambda} = \frac{2v}{2\ell} \text{ First overtone frequency.}$$

$$\text{Case (iii) } \ell = \frac{3\lambda}{2} \Rightarrow \lambda = \frac{2\ell}{3}$$

$$\Rightarrow n_3 = \frac{v}{\lambda} = \frac{3v}{2\ell} \text{ Second overtone frequency.}$$

Hence frequency of overtones are given by the relation $n_1 : n_2 : n_3 : \dots = 1 : 2 : 3 : \dots$

When open organ pipe vibrates in m^{th} overtone then

$$\ell = (m+1) \frac{\lambda}{2} \Rightarrow \lambda = \frac{2\ell}{(m+1)} \Rightarrow n = \frac{(m+1)v}{2\ell}$$

Key Points

- A rod clamped at one end or a string fixed at one end is similar to vibration of closed end organ pipe.
- A rod clamped in the middle is similar to the vibration of open end organ pipe.
- If an open pipe is half submerged in water, it becomes a closed organ pipe of length half that of open pipe i.e. frequency remains same.
- Due to finite motion of air molecules in organ pipes reflection takes place not exactly at open end but some what above it so in an organ pipe antinode is not formed exactly at free-end but above it at a distance $e = 0.6r$ (called end correction or Rayleigh's correction) with r being the radius of pipe. So for closed organ pipe $L \rightarrow L + 0.6r$ while for open $L \rightarrow L + 2 \times 0.6r$ (as both ends are open)

$$\text{so that } f_c = \frac{v}{4(L+0.6r)} \text{ while } f_o = \frac{v}{2(L+1.2r)}$$

This is why for a given v and L narrower the pipe higher will be the frequency or pitch and shriller will be the sound.

- For an organ pipe (closed or open) if $v = \text{constant}$.

$$f \propto (1/L)$$

So with decrease in length of vibrating air column, frequency or pitch will increase and vice-versa.

This is why the pitch increases gradually as an empty vessel fills slowly.

Example 28:

For a certain organ pipe, three successive resonant frequencies are observed at 425, 595 and 765 Hz respectively. Taking the speed for sound in air to be 340 m/s (a) Explain whether the pipe is closed at one end or open at both ends (b) determine the fundamental frequency and length of the pipe.

Solution:

- (a) The given frequencies are in the ratio

$$425 : 595 : 765, \text{ i.e., } 5 : 7 : 9$$

And as clearly these are odd integers so the given pipe is closed pipe.

- (b) From part (a) it is clear that the frequency of 5th harmonic (which is third overtone) is 425 Hz

$$\text{so } 425 = 5f_c \Rightarrow f_c = 85 \text{ Hz}$$

$$\text{Further as } f_c = \frac{v}{4L}, L = \frac{v}{4f_c} = \frac{340}{4 \times 85} = 1 \text{ m}$$



Q.1 Column-I represents the standing waves in air columns and string. Column-II represents frequency of the note. Match the column-I with column-II. [v = velocity of the sound in the medium]

Column -I	Column-II
(A) Second harmonic for the tube open at both ends	(P) $\frac{v}{4\ell}$
(B) Fundamental frequency for the tube closed at one end	(Q) $\frac{v}{2\ell}$
(C) First overtone for the tube closed at one end	(R) $\frac{3v}{4\ell}$
(D) Fundamental frequency for the string fixed at both ends	(S) $\frac{v}{\ell}$
	(T) $\frac{5v}{4\ell}$

Q.2 Where will a person hear maximum sound at (displacement) node or antinode ?

Q.3 For a certain organ pipe, three successive resonance frequencies are observed at 214, 321 & 428 Hz respectively. Taking the speed of sound in air to be 321 m/s. (a) Explain whether the pipe is closed at one end or open at both ends. (b) Determine the fundamental frequency and length of the pipe.

Q.4 Third overtone of a closed organ pipe is in unison with fourth harmonic of an open organ pipe. Find the ratio of lengths of the pipes.

Q.5 Two organ pipes of same length open at both ends produce sounds of different pitch if their radii are different. Why ?

Q.6 A tube of certain diameter and length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320 m/s. Estimate the diameter of the tube. One end of the tube is now closed. Calculate the frequency of resonance for the tube.

Q.7 In resonance tube experiment if $V = 300 \text{ m/s}$, $n = 500 \text{ Hz}$, $L = 125 \text{ m}$

- (i) Find out maximum order of resonance that can be established?
- (ii) Maximum number of resonance?
- (iii) Maximum & minimum water level kept at resonance condition?

Q.8 Fill in the blanks for COP and OOP.

- (i) In COP if the frequency of 7th overtone is 600 Hz then fundamental frequency of COP is....
- (ii) In COP if the frequency of 3rd overtone is 1400 Hz then fundamental frequency of same length OOP is.....
- (iii) In OOP if the frequency of 7th overtone is 800 Hz then fundamental frequency of OOP is....
- (iv) In COP if the frequency of 7th overtone is 600 Hz then frequency of 3rd overtone is....
- (v) In OOP if the frequency of 7th overtone is 1600 Hz then frequency of 3rd overtone is....
- (vi) In OOP if the frequency of 7th overtone is 1600 Hz then fundamental frequency of same length COP is....
- (vii) In OOP if the frequency of 7th overtone is 800 Hz then frequency of OOP corresponding to first overtone is....
- (viii) Length of OOP is 44 cm speed of sound 340 m/s fundamental frequency is 340 Hz then value of end correction is....
- (ix) Length of OOP is 38 cm speed of sound 340 m/s fundamental frequency is 340 Hz then value of pipe is....

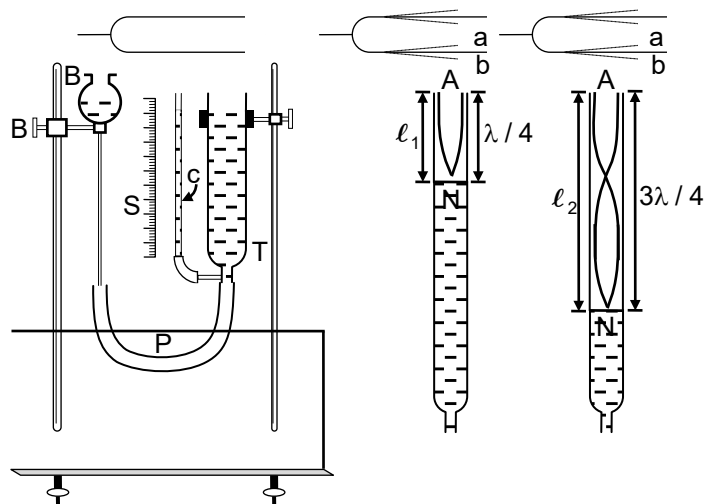
Q.9 Fill in the blanks for Resonance tube.

- (i) In resonance tube first resonating length is 25 cm then its second resonating length will be (if $e = 0$).....
- (ii) In resonance tube first resonating length is 25 cm then its second resonating length will be (if $e \neq 0$).....
- (iii) Length of resonance tube is 140 cm. How many resonances are possible for wave having wave length 40 cm....
- (iv) Length of resonance tube is 150 cm. How many resonance are possible for wave having frequency 400 Hz and speed of sound 320 m/s....
- (v) Length of resonance tube is 150 cm. Maximum level of liquid, in resonance condition for wave having wavelength 80 cm. is....
- (vi) Length of resonance tube is 150 cm. Minimum level of liquid, in resonance condition for wave having wavelength 80 cm. is....
- (vii) In resonance tube having first resonating length is 17 cm, second resonating length is 55 cm then wavelength of wave is.....
- (viii) In resonance tube having first resonating length is 17 cm, second resonating length is 55 cm then radius of tube is.....

Experimental Techniques

Resonance Tube

Construction : The resonance tube is a tube T (in figure) made of brass or glass, about 1 meter long and 5 cm in diameter and fixed on a vertical stand. Its lower end is connected to a water reservoir B by means of a flexible rubber tube. The rubber tube carries a pinch-cock P. The level of water in T can be raised or lowered by water adjusting the height of the reservoir B and controlling the flow of water from B to T or from T to B by means of the pinch-cock P. Thus the length of the air-column in T can be changed. The position of the water level in T can be read by means of a side tube C and a scale S.



Determination of the Speed of Sound in Air by Resonance Tube

First of all the water reservoir B is raised until the water level in the tube T rises almost to the top of the tube. Then the pinch-cock P is tightened and the reservoir B is lowered. The water level in T stays at the top. Now a tuning fork is sounded and held over the mouth of tube. The pinch-cock P is opened slowly so that the water level in T falls and the length of the air-column increases. At a particular length of air-column in T, a loud sound is heard. This is the first state of resonance. In this position the following phenomenon takes place inside the tube.

(i) For first resonance $\ell_1 = \lambda/4$

(ii) For second resonance $\ell_2 = 3\lambda/4$

$$\ell_2 - \ell_1 = \lambda/2 \quad \lambda = 2(\ell_2 - \ell_1)$$

If the frequency of the fork be n and the temperature of the air-column be $t^\circ\text{C}$, then the speed of sound at $t^\circ\text{C}$ is given by

$$v_t = n\lambda = 2n(\ell_2 - \ell_1)$$

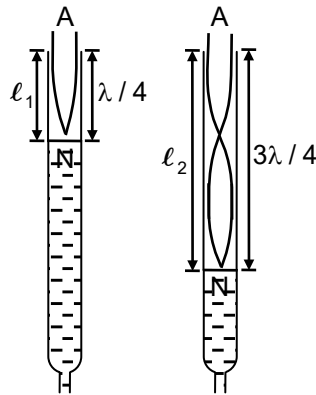
The speed of sound wave at 0°C

$$v_0 = (v_t - 0.61 t) \text{ m/s.}$$

End Correction : In the resonance tube, the antinode is not formed exactly at the open but slightly outside at a distance x .

Hence the length of the air-column in the first and second states of resonance are $(\ell_1 + x)$ and $(\ell_2 + x)$ then

(i) For first resonance $\ell_1 + x = \lambda/4$ (i)



(ii) For second resonance

$$\ell_2 + x = 3\lambda/4 \quad \dots(ii)$$

Subtract Equation (ii) from Equation (i)

$$\ell_2 - \ell_1 = \lambda/2$$

$$\lambda = 2 (\ell_2 - \ell_1)$$

Put the value of λ in Equation (i)

$$\ell_1 + x = \frac{2(\ell_2 - \ell_1)}{4}$$

$$\ell_1 + x = \frac{\ell_2 - \ell_1}{2} \Rightarrow \boxed{x = \frac{\ell_2 - 3\ell_1}{2}}$$

19. Doppler's Effect

The apparent change in the frequency of sound when the source of sound, the observer and the medium are in relative motion is called Doppler effect. While deriving these expressions, we make the following assumptions.

- The velocity of the source, the observer and the medium are along the line joining the positions of the source and the observer.
- The velocity of the source and the observer is less than velocity of sound.

Doppler effect takes place both in sound and light. In sound it depends on whether the source or observer or both are in motion while in light it depends on whether the distance between source and observer is increasing or decreasing.

Notations

$n \rightarrow$ actual frequency

$n' \rightarrow$ observed (apparent) frequency

$\lambda \rightarrow$ actual wavelength

$\lambda' \rightarrow$ observed (apparent) wavelength

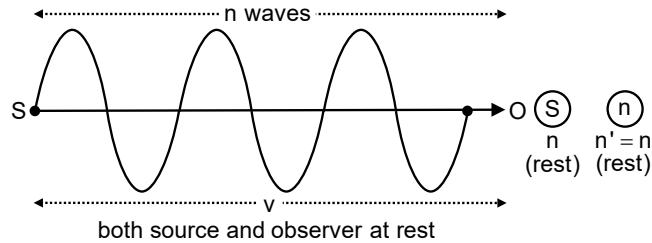
$v \rightarrow$ velocity of sound

$v_s \rightarrow$ velocity of source

$v_o \rightarrow$ velocity of observer

$v_m \rightarrow$ medium (wind) velocity

Case I : Source in Motion, Observer at Rest, Medium at Rest

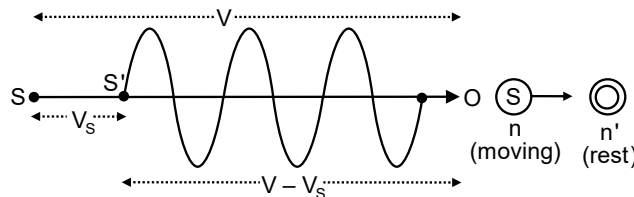


Suppose the source S and observer O are separated by distance v . Where v is the velocity of sound. Let n be the frequency of sound emitted by the source. Then n waves will be emitted by the source in one second. These n waves will be accommodated in distance v .

So, wavelength $\lambda = \frac{\text{total distance}}{\text{total number of waves}} = \frac{v}{n}$

(i) Source Moving Towards Stationary Observer

Let the sources start moving towards the observer with velocity v_s . After one second, the n waves will be crowded in distance $(v - v_s)$. Now the observer shall feel that he is listening to sound of wavelength λ' and frequency n'



Now apparent wavelength

$$\lambda' = \frac{\text{total distance}}{\text{total number of waves}} = \frac{v - v_s}{n}$$

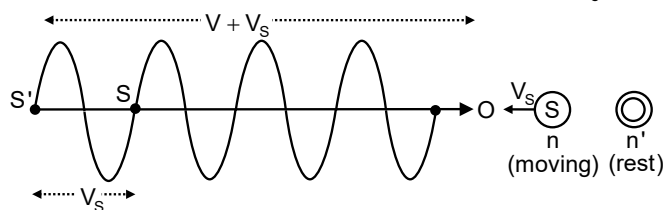
and changed frequency,

$$n' = \left(\frac{v}{v - v_s} \right) n \quad \text{i.e. } n' > n$$

So, as the source of sound approaches the observer the apparent frequency n' becomes greater than the true frequency n .

(ii) Source Move Away from Stationary Observer

For this situation n waves will be crowded in distance $v + v_s$.



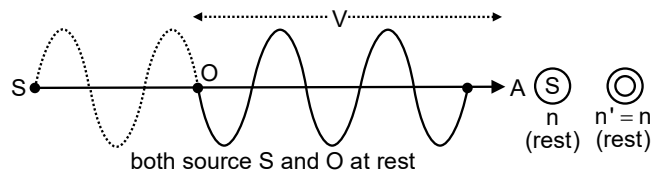
So, apparent wavelength $\lambda' = \frac{v + v_s}{n}$

and Apparent frequency

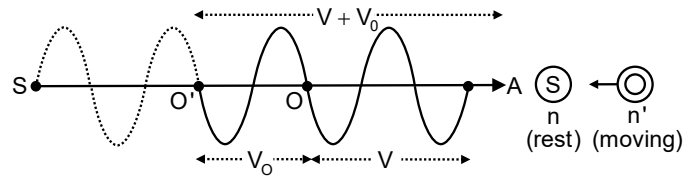
$$n' = \frac{v}{\lambda'} = \frac{v}{\left(\frac{v + v_s}{n} \right)} = n \left(\frac{v}{v + v_s} \right) \quad \text{So } n' < n$$

Case II : Observer in Motion, Source at Rest, Medium at Rest

Let the source (S) and observer (O) are in rest at their respective places. Then n waves given by source 'S' would be crossing observer 'O' in one second and fill the space OA ($=v$)



(i) Observer Moves Towards Stationary Source



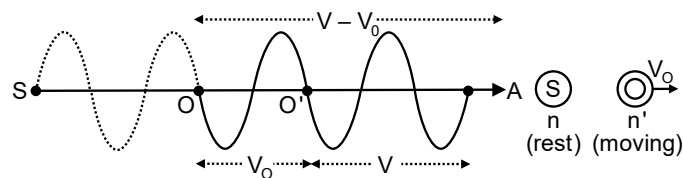
When observer 'O' moves towards 'S' with velocity v_o , it will cover v_o distance in one second. So the observer has received not only the n waves occupying OA but also received additional number of Δn waves occupying the distance OO' ($=v_o$).

So, total waves received by observer in one second i.e.,
apparent frequency (n') = Actual waves (n) + Additional waves (Δn)

$$n' = \frac{v}{\lambda} + \frac{v_o}{\lambda} = \frac{v + v_o}{(v/n)} = n \left(\frac{v + v_o}{v} \right) \text{ (so, } n' > n \text{)}$$

(ii) Observer Moves Away from Stationary Source

For this situation n waves will be crowded in distance $v - v_o$.



When observer move away from source with v_o velocity then he will get Δn waves less than real number of waves. So, total number of waves received by observer i.e.

Apparent frequency (n') = Actual waves (n) – reduction in number of waves (Δn)

$$n' = \frac{v}{\lambda} - \frac{v_o}{\lambda} = \frac{v - v_o}{(v/n)} = \left(\frac{v - v_o}{v} \right) n \text{ (so } n' < n \text{)}$$

Key Points

- If medium (air) is also moving with v_m velocity along the line connecting source and observer. Then velocity of sound relative to observer will be $v \pm v_m$ (–ve sign, if v_m is opposite to sound velocity). So,

$$n' = n \left(\frac{v \pm v_m \pm v_o}{v \pm v_m \mp v_s} \right)$$

- Source in motion towards the observer. Both medium and observer are at rest.

$$n' = \left(\frac{v}{v - v_s} \right) n$$

So, when a source of sound approaches a stationary observer, the apparent frequency is more than the actual frequency.

- Source in motion away from the observer. Both medium and observer are at rest.

$$n' = \left(\frac{v}{v + v_s} \right) n.$$

So, when a source of sound moves away from a stationary observer, the apparent frequency is less than actual frequency.

- Observer in motion towards the source. Both medium and source are at rest. $n' = \left(\frac{v + v_o}{v} \right) n.$

So, when observer is in motion towards the source, the apparent frequency is more than the actual frequency.

- Observer in motion away from the source. Both medium and source are at rest.

$$n' = \left(\frac{v - v_o}{v} \right) n.$$

So, when observer is in motion away from the source, the apparent frequency is less than the actual frequency.

- Both source and observer are moving away from each other. Medium at rest. $n' = \left(\frac{v - v_o}{v + v_s} \right) n$

Conditions When Doppler's Effect is Not Observed for Sound Waves

- When the source of sound and observer both are at rest.
- When the source and observer both are moving with same velocity in same direction.
- When the source and observer are moving mutually in perpendicular directions.
- When the medium only is moving.
- When the distance between the source and observer is constant.

Doppler Effect for Light

Doppler effect holds not only for sound (mechanical waves) but also for electromagnetic waves (non-mechanical waves) including microwaves, radio waves and visible light. However, as electromagnetic waves do not require a medium for their propagation and the motion of source relative to detector or of detector relative to source represents same physical situation (as speed of light is independent of relative motion between source and observer), the formulae are different from that of sound. Here when either source or detector or both are in motion, only two cases are possible, viz., of approach and recession and for these apparent frequencies is given by,

$$f_A = f \sqrt{\frac{1+v/c}{1-v/c}} \quad \text{and} \quad f_R = f \sqrt{\frac{1-v/c}{1+v/c}}$$

where c is the velocity of light and v is relative speed of approach or recession.

For $v \ll c$, the above formulae with the help of binomial theorem reduces to :

$$f_A = f \left[1 + \frac{v}{c} \right] \quad \text{and} \quad f_R = f \left[1 - \frac{v}{c} \right]$$

which are same as in case of sound (for $u \ll v$).

So at low speed Doppler effect in light and sound is governed by the same formulae.

The change in wavelength is given as

$$\frac{|\Delta\lambda|}{\lambda} = \left(\frac{v}{c} \right) \quad |\Delta\lambda| = \frac{v}{c} \lambda$$

In case of approach frequency increases while wavelength decreases, i.e., shift $\Delta\lambda$ is towards blue end of the spectrum ($\lambda_A = \lambda - \Delta\lambda$)

while In case of recession frequency decreases and wavelength increases, i.e., shift is towards red end ($\lambda_R = \lambda + \Delta\lambda$).

The effect allowed astronomers to determine the speeds of stars and galaxies relative to the earth by studying the wavelength (or frequency) of radiations (light) coming from them.

Example 29:

Two trains travelling in opposite directions at 126 km/hr each, cross each other while one of them is whistling. If the frequency of the horn is 2.22 kHz find the apparent frequency as heard by an observer in the other train :

(a) Before the trains cross each other

(b) After the trains have crossed each other. ($v_{\text{sound}} = 335 \text{ m/sec}$)

Solution:

Here $v_1 = 126 \times \frac{5}{18} = 35 \text{ m/s}$

(a) In this situation $\circ \rightarrow v_1$ $v_1 \leftarrow \circ$

Observed frequency

$$n' = \left(\frac{v + v_1}{v - v_1} \right) \times n = \left(\frac{335 + 35}{335 - 35} \right) \times 2220 = 2738 \text{ Hz}$$

(b) In this situation $v_1 \leftarrow \circ$ $\circ \rightarrow v_1$

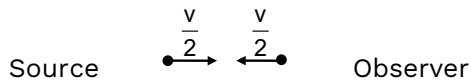
Observed frequency

$$n' = \left(\frac{v - v_1}{v + v_1} \right) \times n = \left(\frac{335 - 35}{335 + 35} \right) \times 2220 = 1800 \text{ Hz}$$

Example 30:

When both source and observer approach each other with a speed equal to the half the speed of sound, then determine the percentage change in frequency of sound as detected by the listener.

Solution:



$$n' = \left(\frac{v + \frac{v}{2}}{v - \frac{v}{2}} \right) n = \left(\frac{\frac{3}{2}v}{\frac{1}{2}v} \right) n = 3n$$

Percentage change

$$= \frac{n' - n}{n} \times 100 = \frac{3n - n}{n} \times 100 = \frac{2n}{n} \times 100 = 200\%$$

Example 31:

A stationary source emits sound of frequency 1200 Hz. If wind blows at the speed of $0.1v$, deduce

- The change in the frequency for a stationary observer on the wind side of the source.
- Report the calculations for the case when there is no wind but the observer moves at $0.1v$ speed towards the source.

(Given : velocity of sound = v)

Solution:

- Medium moves in the direction of sound propagation i.e. from source to observer

so effective velocity of sound $v_{\text{eff}} = v + v_m$

since both source and observer are at rest

$$n' = \left(\frac{v + v_m + 0}{v + v_m + 0} \right) n = \left(\frac{v + 0.1v}{v + 0.1v} \right) n = n$$

so there is no change in frequency

- When observer move towards source

$$n' = \left(\frac{v + v_o}{v} \right) n = \left(\frac{v + 0.1v}{v} \right) n$$

$$= 1.1n = 1.1 \times 1200 \text{ Hz} = 1320 \text{ Hz}$$

Example 32:

A whistle of frequency 540 Hz rotates in a circle of radius 2 m at an angular speed of 15 rad/sec.

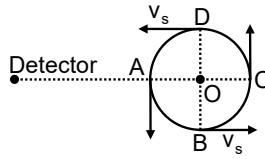
What is the lowest and highest frequency heard by a listener, a long distance away at rest with respect to the centre of the circle ($v = 330 \text{ m/s}$)? Can the apparent frequency be ever equal to actual ?

Solution:

In case of circular motion $v = r\omega$, so here

$$v_s = 2 \times 15 = 30 \text{ m/s}$$

and as detector is at rest,



$$f_{Ap} = f \left[\frac{v}{v \pm v_s} \right]$$

So, frequency will be minimum when the source is at B and moving away from the listener

$$f_{\min} = f \left[\frac{v}{v + v_s} \right]$$

$$= 540 \left[\frac{330}{330 + 30} \right] = 495 \text{ Hz}$$

and frequency will be maximum when source is at D and approaching the listener.

$$f_{\max} = f \left[\frac{v}{v - v_s} \right]$$

$$= 540 \left[\frac{330}{330 - 30} \right] = 594 \text{ Hz}$$

Further when source is at A or C, speed of source along line of sight $v_s \cos 90^\circ = 0$

$$\text{i.e., } f_{Ap} = f \frac{v}{v \pm 0} = f = 540 \text{ Hz}$$

i.e., apparent frequency will be equal to actual frequency when the source is moving perpendicular to the line joining the listener to the centre of circle, i.e., A and C.

Example 33:

An astronaut is approaching the moon. He sends out a radio signal of frequency 5×10^9 Hz and finds out that the frequency shift in echo received is 10^3 Hz. Find his speed of approach.

Solution:

If the astronaut (source) at speed v is approaching the moon, the frequency 'heard' by the moon will be,

$$f_1 = f \left[1 + \frac{2v}{c} \right] \text{ [as } v \ll c]$$

$$\text{or } \frac{f_1 - f}{f} = \frac{2v}{c},$$

$$\text{i.e., } v = \frac{c}{2} \left[\frac{\Delta f}{f} \right]$$

So substituting the given data

$$v = \frac{3 \times 10^8}{2} \left[\frac{10^3}{5 \times 10^9} \right] = 30 \text{ m/s}$$

Concept Builder-12

- Q.1** A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall ?
- Q.2** An engine moving towards a wall with a velocity 50 ms^{-1} emits a note 1.2 kHz. Speed of sound in air is 350 ms^{-1} . The frequency of the note after reflection from the wall as heard by the driver of the engine is :
(1) 2.4 kHz (2) 0.24 kHz (3) 1.6 kHz (4) 1.2 kHz
- Q.3** A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1 . If the train's speed is reduced to 17 m/s, the frequency registered is f_2 . If the speed of sound is 340 m/s, then the ratio $\frac{f_1}{f_2}$ is :
(1) $\frac{18}{19}$ (2) $\frac{1}{2}$ (3) 2 (4) $\frac{19}{18}$
- Q.4** The wavelength of light coming from a distant galaxy is found to be 0.05% more than that coming from a sources on the earth. Calculate the velocity of the galaxy.
- Q.5** A star, which emits a line of wavelength $5000\text{\AA}$ is receding from you in the line of sight with a velocity (1/100) of speed of light. The wavelength of this line as observed by you will be :
(1) $4900 \text{\AA}$ (2) $4950 \text{\AA}$ (3) $5050 \text{\AA}$ (4) $5100 \text{\AA}$

ANSWER KEY FOR CONCEPT BUILDERS

CONCEPT BUILDER-1

1. (3) 2. Based on theory
3. Based on theory

CONCEPT BUILDER-2

1. (i) 0.4 m, 10 Hz, 4 m/s
(ii) $\frac{\pi}{\sqrt{2}}$ m/s, $10\sqrt{2}\pi^2$ m/s²
2. $\lambda = \frac{1}{6}$ m, $v = 41.7$ m/s
3. (1), (2), (4)
4. $y = 0.05 \sin \{60\pi t - 5\pi x\}$
5. $\frac{\pi}{4}$ 6. $\frac{8}{3}$ mm

CONCEPT BUILDER-3

1. 1 : 1 2. (1)

CONCEPT BUILDER-4

1. (2) 2. (2)
3. (1) 4. 97.5 m/s
5. (3)

CONCEPT BUILDER-5

1. 819°C 2. 300 m/s

CONCEPT BUILDER-6

1. (2) 2. $y = -A \cos(\omega t - kx)$
3. $2A \cos(\phi/2)$

CONCEPT BUILDER-7

1. (A) 3 I (B) 2 I
2. 17.5 cm
3. (2) 4. (3)
5. 8 Maxima

CONCEPT BUILDER-8

1. 155 & 161 Hz. 2. 3 beats/sec,
3. (a) $\frac{1}{4}$ s (b) $\frac{1}{8}$ s (c) 12 beats
4. $n_1 = 50$ Hz $n_{11} = 100$ Hz $n_{17} = 130$ Hz
 $n_{27} = 180$ Hz $n_{33} = 210$ Hz $n_{51} = 300$ Hz

CONCEPT BUILDER-9

1. $y = 6 \sin(\omega t - kx)$
2. (3)
3. (a) $\frac{15}{\pi}$ Hz, 0.2 cm, 60 cm/s, 4π cm
(b) $6\sqrt{2}$ cm/s

CONCEPT BUILDER-10

1. (3) 2. (1)

CONCEPT BUILDER-11

1. (A) $\rightarrow$ S; (B) $\rightarrow$ P; (C) $\rightarrow$ R; (D) $\rightarrow$ Q
2. Displacement node & Pressure-antinode.
3. (a) Pipe is open at both ends
(b) 107 Hz, 1.5 m
4. 7/8
5. Due to end correction
6. 3.33 cm, 163.3 Hz
7. (i) fourth order (ii) 4
(iii) 110 cm, 20 cm
8. (i) 40 Hz (ii) 400 Hz (iii) 100 Hz
(iv) 280 Hz (v) 800 Hz (vi) 100 Hz
(vii) 200 Hz (viii) 3 cm (ix) 10 cm
9. (i) 75 cm (ii) Slightly more than 75 cm
(iii) 7 (iv) 4 (v) 130 cm
(vi) 10 cm (vii) 76 cm (viii) 3.33 cm

CONCEPT BUILDER-12

1. 42.47 kHz 2. (3)
3. (4) 4. 1.5×10^5 m/s
5. (3)

Exercise - I

Introduction, Characteristic Equation of Wave

1. A boat at anchor is rocked by waves whose crests are 100m apart and velocity is 25m/s. The boat bounces up once in every :-
 (1) 2500 s
 (2) 75 s
 (3) 4 s
 (4) 0.25 s

2. A wave of frequency 500 Hz travels between X and Y, a distance of 600 m in 2 sec. How many wavelength are there in distance XY :-
 (1) 1000
 (2) 300
 (3) 180
 (4) 2000

3. The distance between two consecutive crests in a wave train produced in string is 5 m. If two complete waves pass through any point per second, the velocity of wave is:-
 (1) 2.5 m/s
 (2) 5 m/s
 (3) 10 m/s
 (4) 15 m/s

4. Two wave are represented by equation $y_1 = a \sin \omega t$ and $y_2 = a \cos \omega t$ the first wave:
 (1) leads the second by π
 (2) lags the second by π
 (3) leads the second by $\pi / 2$
 (4) lags the second by $\pi / 2$

5. Two waves traveling in a medium in the x-direction are represented by $y_1 = A \sin(\alpha t - \beta x)$ and $y_2 = A \cos \left(\beta x + \alpha t - \frac{\pi}{4} \right)$, where y_1 and y_2 are the displacements of the particles of the medium, t is time, and α and β are constants. The two waves have different:-
 (1) speeds
 (2) directions of propagation
 (3) wavelengths
 (4) frequencies

6. The equation of displacement of two waves are given as $y_1 = 10 \sin (3\pi t + \pi/3)$ and $y_2 = 5(\sin 3\pi t + \sqrt{3} \cos 3\pi t)$, then what is the ratio of their amplitude:-
 (1) 1 : 2
 (2) 2 : 1
 (3) 1 : 1
 (4) None of these

7. A plane progressive wave is represented by the equation $y = 0.25 \cos (2\pi t - 2\pi x)$. The equation of a wave is with double the amplitude and half frequency but travelling in the opposite direction will be:-
 (1) $y = 0.5 \cos (\pi t - \pi x)$
 (2) $y = 0.5 \cos (2\pi t + 2\pi x)$
 (3) $y = 0.25 \cos (\pi t + 2\pi x)$
 (4) $y = 0.5 \cos (\pi t + \pi x)$

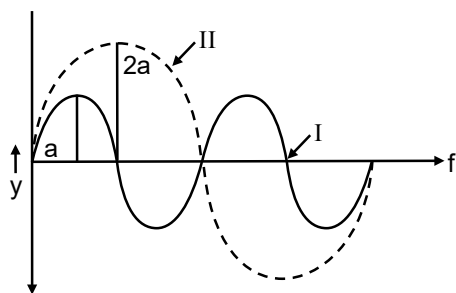
8. The equation of a progressive wave is $y = a \sin \left(\frac{\pi}{2} x - 200\pi t \right)$. The frequency of the wave will be -
 (1) 0.1 Hz
 (2) 25 Hz
 (3) 100 Hz
 (4) 200 Hz

9. Of the following properties of a wave, the one that is independent of the other is its-
 (1) Amplitude (2) Velocity
 (3) Wavelength (4) Frequency
10. If the equation of progressive wave given by $y = 4 \sin \pi \left[\frac{t}{5} - \frac{x}{9} + \frac{\pi}{6} \right]$ m Which of the following is correct?
 (1) $v = 5$ cm/s (2) $\lambda = 18$ m
 (3) $A = 0.04$ cm (4) $f = 50$ Hz
11. The displacement y of a wave travelling in the x -direction is given by $y = 10^{-4} \sin \left(600t - 2x + \frac{\pi}{3} \right)$ metre, where, x is expressed in metres and t in seconds. The speed of the wave-motion, in ms^{-1} is-
 (1) 300 (2) 600
 (3) 1200 (4) 200
12. A wave travelling along the x -axis is described by the equation $y(x, t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave in 0.08m and 2.0 s respectively then α and β in appropriate units are
 (1) $\alpha = 25.00\pi$, $\beta = \pi$
 (2) $\alpha = \frac{0.08}{\pi}$; $\beta = \frac{2.0}{\pi}$
 (3) $\alpha = \frac{0.04}{\pi}$, $\beta = \frac{1.0}{\pi}$
 (4) $\alpha = 12.50 \pi$, $\beta = \frac{\pi}{2.0}$
13. The waves in which the particles of the medium vibrate in a direction perpendicular to the direction of wave motion is known as-
 (1) transverse waves
 (2) propagated waves
 (3) longitudinal waves
 (4) stationary waves
14. Energy is not carried by which of the following waves ?
 (1) stationary (2) transverse
 (3) progressive (4) electromagnetic
15. If equation of a sound wave is $y = 0.0015 \sin(62.8x + 314t)$ then its wavelength will be-
 (1) 2 unit (2) 0.3 unit
 (3) 0.1 unit (4) 0.2 unit
16. The equation $y = 4 + 2 \sin(6t - 3x)$ represents a wave motion with
 (1) amplitude 6 units
 (2) amplitude 4 units
 (3) wave speed 2 units
 (4) wave speed 1/2 units
17. Due to propagation of Longitudinal wave in a medium, the following quantities also propagate in the same direction:
 (1) Energy, Momentum and Mass
 (2) Energy
 (3) Energy and Mass
 (4) Energy and Linear Momentum
18. The waves produced by motorboat sailing on water are
 (1) Transverse
 (2) Longitudinal
 (3) Longitudinal and Transverse
 (4) Stationary

Kinematic Parameters of progressive Wave, Energy Intensity of Wave

19. A transverse wave is described by the equation $y = y_0 \sin 2\pi \left(ft - \frac{x}{\lambda} \right)$. The maximum particle velocity is equal to four times the wave velocity if:-
 (1) $\lambda = \frac{\pi y_0}{4}$ (2) $\lambda = \frac{\pi y_0}{2}$
 (3) $\lambda = \pi y_0$ (4) $\lambda = 2\pi y_0$

20. Disturbances of two waves are shown as a function of time in the following I_1/I_2 figure. The ratio of their intensities will be -



- (1) 1 : 4 (2) 4 : 1
(3) 1 : 2 (4) 1 : 1
21. Two sound waves are respectively $y = a \sin(\omega t - kx)$ and $y = b \cos(\omega t - kx)$ the phase difference between the two waves is -
- (1) $\pi/2$ (2) $\pi/4$
(3) π (4) $3\pi/4$
22. The two waves having intensities in the ratio 25 : 9 produce interference. The ratio of the maximum to the minimum intensities is equal to-
- (1) 10 : 8 (2) 9 : 1
(3) 16 : 1 (4) 2 : 1
23. A transverse wave along a string is given by $y = 2 \sin\left(2\pi(3t - x) + \frac{\pi}{4}\right)$ where, x and y are in cm and t is second. The acceleration of a particle located at $x = 4$ cm at $t = 1$ s is -
- (1) $36\sqrt{2} \pi^2 \text{ cm/s}^2$ (2) $36\pi^2 \text{ cm/s}^2$
(3) $-36\sqrt{2} \pi^2 \text{ cm/s}^2$ (4) $-36\pi^2 \text{ cm/s}^2$
24. The ratio of intensities of two waves is 9 : 4. When they superimpose, the ratio of maximum to minimum intensity will become-
- (1) 25 : 1 (2) 13 : 5
(3) 4 : 1 (4) 5 : 1

25. A sings with a frequency (n) and B sings with a frequency $1/8$ that of A. If the energy remains the same and the amplitude of A is a, then amplitude of B will be-

- (1) 2a (2) 8a
(3) 4a (4) a

26. In a sinusoidal wave, the time required for a particular particle to move from mean position to maximum displacement is 0.17 sec then the frequency of wave is-

- (1) 1.47 Hz (2) 0.36 Hz
(3) 2.94 Hz (4) 2.48 Hz

27. In a string the speed of wave is 10 m/s and its frequency at a distance 2.5 cm will be:

- (1) $\pi/2$ (2) $\pi/8$
(3) $3\pi/2$ (4) 2π

28. Two sound waves have phase difference of 60° , then they will have the path difference of-

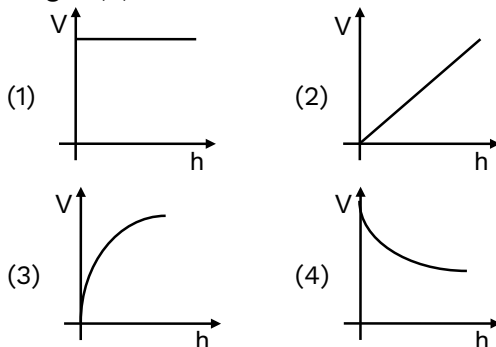
- (1) 3λ
(2) $\frac{\lambda}{3}$
(3) $\frac{\lambda}{6}$
(4) λ

Velocity of Wave in String

29. The linear density of a vibrating string is $1.3 \times 10^{-4} \text{ kg/m}$. A transverse wave is propagating on the string and is described by the equation $y = 0.021 \sin(x + 30t)$ where x and y are measured in meter and t in second the tension in the string is :-

- (1) 0.12 N (2) 0.48 N
(3) 1.20 N (4) 4.80 N

30. A uniform rope having some mass hinges vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed (v) of the wave pulse varies with height (h) from the lower end as:-



31. The time taken by a transverse wave to travel the full length of a uniform rope of mass 0.1 kg and length 2.45 m hanging from the ceiling, is-
- (1) 1 s (2) 0.5 s
(3) 2 s (4) 1.5 s
32. A wave is represented by the equation : $y = a \sin (0.01x - 2t)$ where a and x are in cm. Velocity of propagation of wave is-
- (1) 20 cm/s (2) 50 cm/s
(3) 100 cm/s (4) 200 cm/s
33. A transverse wave passes through a string with the equation $y = 10 \sin \pi(0.02x - 2.00t)$, where x is in metre and t in second. The maximum velocity of the particle in wave motion is-
- (1) 100 m/s (2) 63 m/s
(3) 120 m/s (4) 161 m/s

Sound Wave : Loudness, ECHO, Velocity of Sound

34. If at some point the amplitude of the sound becomes double and the frequency becomes on fourth then at that point the intensity of sound will be:
- (1) Become double
(2) be half
(3) Become one fourth
(4) Remain unchanged

35. When sound wave travels from air to water, which are of the following remain constant:

(1) wavelength (2) velocity
(3) frequency (4) intensity

36. Intensity level of a sound of intensity I is 30 dB. The ratio I/I_0 is (I_0 is the threshold of hearing)

(1) 1000 (2) 3000
(3) 300 (4) 30

37. The velocities of sound at the same pressure in two monoatomic gases of densities ρ_1 and ρ_2 are v_1 and v_2 respectively. If $\frac{\rho_1}{\rho_2} = 4$, then the value of

$\frac{v_1}{v_2}$ is:

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$
(3) 2 (4) 4

38. A thunder tap is heard 5.5 s after the lightening flash. The distance of the flash is (velocity of sound in air is 330 m/s) :-

(1) 3560 m (2) 300 m
(3) 1780 m (4) 1815 m

39. At the room temperature the velocity of sound in O_2 gas is v . Then in mixture of H_2 and O_2 gas the speed of sound at same temperature:-

(1) will be less than v
(2) will be more than v
(3) will be equal to v
(4) nothing can be said

40. An underwater sonar source operating at a frequency of 60 kHz directs its beam towards the surface. If velocity of sound in air is 330 m/s, wavelength and frequency of the waves in air are:-

(1) 5.5 mm, 60 kHz (2) 3.30 m, 60kHz
(3) 5.5 mm, 30 kHz (4) 5.5 mm, 80 kHz

41. 'SONAR' emits which of the following waves ?

- (1) radio waves (2) ultrasonic waves
(3) magnetic waves (4) light waves

42. At what temperature the speed of sound in air will become double of its value at 27°C ?

- (1) 54°C (2) 627°C
(3) 927°C (4) 327°C

43. A man standing on a cliff claps his hand and hears its echo after one second. If the sound is reflected from another mountain then the distance between the man & reflection points is $V_{\text{sound}} = 340 \text{ m/sec}$.

- (1) 680 m (2) 340 m
(3) 170 m (4) 85 m

Superposition and Interference of Wave

44. The resultant amplitude, when two waves of same frequency but with amplitudes a_1 and a_2 superimpose at phase difference of $\pi/2$ will be:-

- (1) $a_1 + a_2$ (2) $a_1 - a_2$
(3) $\sqrt{a_1^2 + a_2^2}$ (4) $a_1^2 + a_2^2$

45. If the amplitudes of two sources, having equal frequency are not equal, then-

- (1) There will be no interference
(2) The intensity of sound will be the same at all the points
(3) The intensity of sound at any point will decrease and increase
(4) The interference will be there, but the minimum intensity will not be zero.

46. Two vertical antennas situated at a distance $3\lambda/2$ emit radio signals of same wavelength. The intensity of each is I_0 . The intensity at a point equidistant from two antennas will be -

- (1) $4 I_0$ (2) $2 I_0$
(3) zero (4) I_0

47. Two sound waves, originating from the same sound source travel along different paths in air and then meet at a point. The speed of the sound is 330m/s. If the source vibrates at a frequency of 500 Hz and one path is 33 cm longer than the other, then the nature of interference is -

- (1) Destructive
(2) Constructive
(3) Neither destructive nor constructive
(4) Nothing can be predicted

48. What is the path difference for destructive interference ?

- (1) $n\lambda$ (2) $n(\lambda + 1)$
(3) $\frac{(n+1)\lambda}{2}$ (4) $\frac{(2n+1)\lambda}{2}$

49. The ratio of intensities of two waves is 16 : 9. When they superimpose, the ratio of maximum to minimum intensity will become:-

- (1) 49 : 1 (2) 3 : 1
(3) 2 : 1 (4) 1 : 1

50. In case of interference of two waves each of intensity I_0 , the intensity at a point of constructive interference will be :

- (i) $4I_0$ for coherent sources
(ii) $2I_0$ for coherent sources
(iii) $4I_0$ for incoherent sources
(iv) $2I_0$ for incoherent sources
(1) (i, iv) (2) (ii, iv)
(3) (ii, iii) (4) (i, iii)

51. Two coherent sources of different intensities send waves which interfere. The ratio of the maximum intensity to the minimum intensity is 25. The intensities of the sources are in the ratio :

- (1) 25 : 1 (2) 5 : 1
(3) 9 : 4 (4) 625 : 1

52. Waves from two sources superimpose on each other at a particular point, Amplitude and frequency of both the waves are equal. The ratio of intensities when both waves reach in the same phase and they reach with the phase difference of 90° will be

- (1) 1 : 1 (2) $\sqrt{2} : 1$
(3) 2 : 1 (4) 4 : 1

Beats

53. Two vibrating tuning forks produce progressive waves given by $y_1 = 4 \sin 500\pi t$ and $y_2 = 2 \sin 506 \pi t$.

Number of beats produced per minute is:-

- (1) 3 (2) 360
(3) 180 (4) 60

54. Frequency of tuning fork A is 256 Hz. It produces 4 beats/second with tuning fork B. When wax is applied at tuning fork B then 6 beats/second are heard. Frequency of B is:-

- (1) 250 Hz
(2) 260 Hz
(3) 252 Hz
(4) (2) & (3) both may possible

55. 16 tuning forks are arranged in increasing order of frequency. Any two consecutive tuning forks when sounded together produce 8 beats per second. If the frequency of last tuning fork is twice that of first, the frequency of first tuning fork is:-

- (1) 60 (2) 80
(3) 100 (4) 120

56. The beats are produced by two sound sources of same amplitude and of nearly equal frequencies. The maximum intensity of beats will be that of one source -

- (1) Same (2) Double
(3) Four times (4) Eight times

57. The keys of two pianos are simultaneously pressed. The frequencies of notes produced by them are n_1 and n_2 . The number of beats produced per second is -

- (1) $(n_1 - n_2)$ (2) $\frac{(n_1 - n_2)}{2}$
(3) $\frac{(n_1 + n_2)}{2}$ (4) $2(n_1 - n_2)$

58. On sounding two tuning forks A and B together 5 beats per second are produced. On filling A slightly, the number of beats per second becomes 2. If the frequency of the A is 384 Hz. Then the frequency of B will be -

- (1) 319 Hz (2) 314 Hz
(3) 389 Hz (4) 334 Hz

59. When a unknown fork is sounded with a known fork of frequency 288Hz then 5 beats per second are produced. The unknown fork is again sounded after loading it with wax and again 5 beats are produced. The frequency of unknown fork will be-

- (1) 283 Hz (2) 293 Hz
(3) 288 Hz (4) 292 Hz

60. Two source have frequency 256 Hz and 258 Hz, then time difference between two consecutive maxima is-

- (1) 1 s (2) 0.5 s
(3) 2 ms (4) None

Reflection & Transmission of Wave, Transverse Stationary Wave

- 61.** A wave is represented by the equation $y = a \sin(kx - \omega t)$ is superimposed with another wave to form a stationary wave such that the point $x = 0$ is a node. Then the equation of other wave is:-
 (1) $y = a \cos(kx - \omega t)$
 (2) $y = a \cos(kx + \omega t)$
 (3) $y = -a \sin(kx + \omega t)$
 (4) $y = a \sin(kx + \omega t)$
- 62.** A wave $y = a \sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is-
 (1) $y = a \sin(\omega t + kx)$ (2) $y = -a \sin(\omega t + kx)$
 (3) $y = a \sin(\omega t - kx)$ (4) $y = -a \sin(\omega t - kx)$
- 63.** If a wave is represented by the following equation $y = A \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda}$ then it is a:
 (1) Progressive wave
 (2) Stationary wave
 (3) Longitudinal progressive wave
 (4) Transverse progressive wave
- 64.** Stationary wave is represented by :
 $y = A \sin(100t) \cos(0.01x)$
 where y and A are in mm, t in sec, and x in m. The velocity of the wave -
 (1) 1 m/s (2) 10^2 m/s
 (3) 10^4 m/s (4) zero
- 65.** A standing wave having 3 nodes and 2 antinodes is formed between 1.21 Å distance then the wavelength is:-
 (1) 1.21 Å (2) 2.42 Å
 (3) 0.605 Å (4) 4.84 Å
- 66.** Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it, is-
 (1) 20 (2) 80
 (3) 40 (4) 120
- 67.** If vibrations of a string are to be increased to a factor of two, then tension in the string must be made-
 (1) half (2) thrice
 (3) four times (4) eight times
- 68.** Stationary waves are so called because in them-
 (1) The particles of the medium are not disturbed at all
 (2) The particles of the medium do not execute SHM
 (3) There occur no flow of energy along the wave
 (4) The interference effect can't be observed
- 69.** Fundamental frequency of sonometer wire is n . If the length, tension and diameter of wire are tripled, the new fundamental frequency is:
 (1) $n/\sqrt{3}$ (2) $n/3$
 (3) $n\sqrt{3}$ (4) $n/3\sqrt{3}$
- 70.** A sonometer wire of density d and radius r is held between two bridges at a distance L apart. The wire has a tension T . The fundamental frequency of the wire will be-
 (1) $f = \frac{1}{2Lr} \sqrt{\frac{T}{\pi d}}$ (2) $f = \frac{r}{2L} \sqrt{\frac{\pi d}{T}}$
 (3) $f = \frac{1}{2Lr} \sqrt{\frac{d}{\pi T}}$ (4) $f = \frac{1}{2L} \sqrt{\frac{d}{T}}$

- 71.** The tension in a piano wire is 10N. What should be the tension in the wire to produce a note of double the frequency ?
 (1) 10N (2) 20N
 (3) 40N (4) 80N
- 72.** Stationary waves are produced in 10m long stretched string. If the string vibrates in 5 segments and wave velocity 20m/s then the frequency is:-
 (1) 10 Hz (2) 5 Hz
 (3) 4 Hz (4) 2Hz
- 73.** A string in a musical instrument is 50 cm long and its fundamental frequency is 800 Hz. If a frequency of 1000 Hz is to be produced, then required length of string is-
 (1) 62.5 cm (2) 50 cm
 (3) 40 cm (4) 37.5 cm
- 74.** A stretched string is vibrating according to the equation $y = 5 \sin \left(\frac{\pi x}{2} \right) \cos 4\pi t$, where y and a are in cm and t is in sec. The distance between two consecutive nodes on the string is:
 (1) 2 cm (2) 4 cm
 (3) 8 cm (4) 16 cm
- 75.** A stretched string is 1 m long. Its mass per unit length is 3.5 g/m,. It is stretched with a force of 20 N. It plucked at a distance of 25 cm from one end. The frequency of note emitted by it will be:
 (1) 400 Hz (2) 300 Hz
 (3) 200 Hz (4) 100 Hz
- 76.** A stretched wire of length 114 cm is divided into three segments whose frequencies are in the ratio 1 : 3 : 4, the length of the segments must be in the ratio -
 (1) 18 : 24 : 72 (2) 24 : 72 : 18
 (3) 24 : 18 : 72 (4) 72 : 24 : 18
- 77.** A sonometer wire with a suspended mass of $M = 1$ kg is in resonance with a given tuning fork. The apparatus is taken to the moon where the acceleration due to gravity is $1/6$ that on the earth. To obtain resonance on the moon, the value of M should be -
 (1) 1 kg (2) $\sqrt{6}$ kg
 (3) 6 kg (4) 24 kg
- 78.** In a sonometer wire, the tension is maintained by suspending a mass M from free end of wire. The fundamental frequency of the wire is N Hz. If the suspended mass is completely immersed in water the fundamental frequency will
 (1) increases (2) constant
 (3) decrease (4) can't say
- 79.** Length of a sonometer wire is either 95 cm or 100 cm. In both the cases a tuning fork produces 4 beats then the frequency of tuning fork is:-
 (1) 152 (2) 156
 (3) 160 (4) 164
- 80.** If a vibrating tuning fork is put in contact with the sonometer box then the rider on the wire falls down. The frequency of tuning fork, as the compared to that of the sonometer wire will be -
 (1) More
 (2) Less
 (3) Equal
 (4) No relation between the two

Longitudinal Stationary Waves, Organ Pipes

- 81.** Two open pipes of length 25 cm and 25.5 cm produced 0.1 beat/second in fundamental mode. The velocity of sound will be:-
 (1) 255 cm/s (2) 250 cm/s
 (3) 350 cm/s (4) none of these

- 82.** A tube, closed at one end and containing air, produces, when excited, the fundamental note of frequency 512 Hz. If the tube is opened at both ends the fundamental frequency that can be excited is (in Hz.):—
 (1) 1024 (2) 512
 (3) 256 (4) 128
- 83.** A cylindrical tube, open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now:—
 (1) $\frac{f}{2}$ (2) $\frac{3f}{4}$
 (3) f (4) $2f$
- 84.** An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 and P_2 is:—
 (1) $\frac{8}{3}$ (2) $\frac{3}{8}$
 (3) $\frac{1}{6}$ (4) $\frac{1}{3}$
- 85.** The velocity of sound in air is 333 m/s. If the frequency of the fundamental tone is 333 Hz, the length of the open pipe to generate second harmonic is:—
 (1) 0.5m (2) 1.0m
 (3) 2.0m (4) 4.0 m
- 86.** The maximum length of a closed pipe that would produce a just audible sound is ($v_{\text{sound}} = 336 \text{ m/s}$):—
 (1) 4.2 cm (2) 4.2 m
 (3) 4.2 mm (4) 1.0 cm
- 87.** If oil of density higher than that of water be used (in place of water) in a resonance tube, then its frequency will -
 (1) Increase
 (2) Decrease
 (3) Remain the same
 (4) Depend upon the density of the material of the tube
- 88.** The frequency of an open pipe is 300Hz. The first overtone of this pipe is the same as the first overtone of a closed pipe. The length of the closed organ pipe is ($v=340 \text{ m/s}$):—
 (1) 21 cm (2) 42 cm
 (3) 11 cm (4) 84 cm
- 89.** In a resonance tube, using a tuning fork of frequency 325 Hz, two successive resonance length are observed at 25.4 cm and 77.4 cm respectively. The velocity of sound in air is -
 (1) 338 ms^{-1} (2) 328 ms^{-1}
 (3) 330 ms^{-1} (4) 320 ms^{-1}
- 90.** Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tubes A and B is—
 (1) 1 : 2 (2) 1 : 4
 (3) 2 : 1 (4) 4 : 1
- 91.** An air column in pipe, which is closed at one end will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is: [$v = 330 \text{ m/s}$]
 (1) 31.25 (2) 62.50
 (3) 110 (4) 125

92. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m neglecting end corrections, the air column in the pipe can resonant for sound of frequency.

- (1) 80 Hz (2) 240 Hz
(3) 500 Hz (4) 400 Hz

93. An air column having one end closed contains minimum resonance length 50 cm. If it is vibrated by same tuning fork then its next resonance length will be-

- (1) 250 cm (2) 200 cm
(3) 150 cm (4) 100 cm

94. An open organ pipe of length 33 cm, vibrates with frequency 1000 Hz. Its velocity of sound is 330 m/s, then its frequency is:

- (1) Fundamental frequency
(2) First overtone of pipe
(3) Second overtone
(4) Fourth overtone

95. The length of two closed organ pipes are 0.750 m and 0.770 m. If they are sounded together, 3 beats per second are produced. The velocity of sound will be:

- (1) 350.5 m/sec
(2) 335.5 m/sec
(3) 346.5 m/sec
(4) None of these

96. An organ pipe closed at one end has fundamental frequency of 1500 Hz. The maximum number of overtones generated by this pipe which a normal person can hear is

- (1) 14 (2) 13
(3) 6 (4) 9

97. The end correction of a resonance column is 1 cm. If the shortest length resonating with the tuning fork is 10 cm, the next resonating length should be

- (1) 32 cm (2) 40 cm
(3) 28 cm (4) 36 cm

Doppler's Effect

98. A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer (in Hz) is : (speed of sound 333 m/s)

- (1) 409 (2) 429
(3) 517 (4) 500

99. A whistle revolves in a circle with angular speed $\omega = 20$ rad/s using a string of length 50 cm. If the frequency of sound from the whistle is 385 Hz, then what is the minimum frequency heard by an observer which is far away from the centre:-

($v_{\text{sound}} = 340$ m/s)

- (1) 385 Hz (2) 374 Hz
(3) 394 Hz (4) 333 Hz

100. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. what is the percentage increase in the apparent frequency ?

- (1) zero (2) 0.5%
(3) 5% (4) 20%

101. A siren emitting sound of frequency 500 Hz is going away from a static listener with a speed of 50 m/sec. The frequency of sound to be heard directly from the siren is-

($v_{\text{sound}} = 330$ m/s)

- (1) 434.2 Hz (2) 589.3 Hz
(3) 484.2 Hz (4) 256.5 Hz

- 102.** A vehicle, with a horn of frequency n is moving with a velocity of 30 m/s in a direction perpendicular to the straight line joining the observer and the vehicle. The observer perceives the sound to have a frequency $n + n_1$. Then –
(Take velocity of sound in air 330 m/s)
(1) $n_1 = 10n$ (2) $n_1 = -n$
(3) $n_1 = 0$ (4) $n_1 = 2n$
- 103.** A source of sound of frequency 1000 Hz is moving with a uniform velocity 20 m/s. The ratio of apparent frequency heard by the observer before and after the source crosses him would be : [$v = 340$ m/s]
(1) 9 : 8 (2) 8 : 9
(3) 1 : 1 (4) 9 : 10
- 104.** A railway engine moving with a speed of 60 m/s passes in front of a stationary listener. The real frequency of whistle is 400 Hz. Calculate the apparent frequency heard by listener (a) when the engine is approaching the listener, (b) when the engine moving away from the listener ($v = 340$ m/s)
(1) 485.7 Hz, 340 Hz (2) 220 Hz, 180 Hz
(3) 320 Hz, 155 Hz (4) 400 Hz, 330 Hz
- 105.** A source of sound of frequency 90 vibrations/s is approaching a stationary observer with a speed equal to $1/10$ the speed of sound. What will be the frequency heard by the observer ?
(1) 80 vibrations/s (2) 90 vibrations/s
(3) 100 vibrations/s (4) 120 vibrations/s
- 106.** A source of sound of frequency 500 Hz is moving towards an observer with velocity 30 m/s. The speed of sound is 330 m/s. The frequency heard by the observer will be
(1) 550 Hz (2) 458.3 Hz
(3) 530 Hz (4) 545.5 Hz
- 107.** A source of sound having frequency 300 Hz emitted waves of wavelength 1 m. The listener is at rest the source is moving away from it with a velocity of 30 m/s. Find the frequency heard by observer.
(1) 270 Hz (2) 273 Hz
(3) 375 Hz (4) 300 Hz
- 108.** A locomotive engine is approaching a hill with speed of 30 m/s. It blows whistle of frequency 600 Hz, the frequency of the echo of the whistle as heard by the driver of the engine is
($v = 330$ m/s)
(1) 600 Hz (2) 660 Hz
(3) 720 Hz (4) 550 Hz
- 109.** A bus is moving with a velocity of 5 m/s towards a huge wall. The driver sounds a horn of frequency of 165 Hz. If the speed of sound in air is 335 m/s, number of beats heard by a passenger on bus will be :
(1) 6 (2) 5
(3) 3 (4) 4
- 110.** Two trains A and B are moving in the same direction with velocities 30 m/s and 10 m/s respectively. B is behind from A and A blows a horn of frequency 450 Hz. Then the apparent frequency heard by observer on train B is (speed of sound is 330 m/s):
(1) 425 Hz (2) 300 Hz
(3) 450 Hz (4) 350 Hz
- 111.** If a star emitting light of wave length $5000\text{\AA}$ is moving towards earth with a velocity of 1.5×10^6 m/s then the shift in the wavelength due to Doppler's effect will be:
(1) $2.5\text{\AA}$ (2) $250\text{\AA}$
(3) $25\text{\AA}$ (4) Zero

- 112.** Velocity of star is 10^6 m/s and frequency of emitted light is 4.5×10^{14} Hz. If star is moving away, then apparent frequency will be:

 - 4.5 Hz
 - 4.5×10^{16} Hz
 - 4.485×10^{14} Hz
 - 4.5×10^8 Hz

113. What happens in the red shift which confirm that the universe is expanding ?

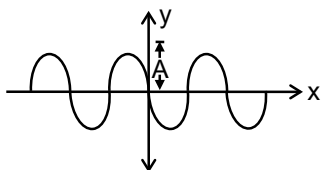
 - wavelength of light emitted by galaxies appears to decrease
 - wavelength of light emitted by galaxies appears to be the same
 - wavelength of light emitted by galaxies appears to increase
 - none of these

ANSWER KEY																											
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25		
Ans.	3	1	3	4	2	3	4	3	1	2	1	1	1	1	3	3	4	3	2	4	1	3	3	1	2		
Que.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50		
Ans.	1	1	3	1	3	1	4	2	3	3	1	2	4	2	1	2	3	3	3	4	1	1	4	1	1		
Que.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75		
Ans.	3	3	3	4	4	3	1	3	2	2	4	2	2	4	1	2	3	3	4	1	3	2	3	1	3		
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100		
Ans.	4	3	3	2	3	1	1	3	3	2	2	3	2	1	3	1	3	3	2	3	3	1	4	2	4		
Que.	101	102	103	104	105	106	107	108	109	110	111	112	113														
Ans.	1	3	1	1	3	1	2	3	2	1	3	3	3														

Exercise - II

1. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin(kx - \omega t + \phi)$.

Its snapshot at $t = 0$ is given in the figure



For this wave, the phase ϕ is:

- (1) 0 (2) $-\frac{\pi}{2}$
- (3) π (4) $\frac{\pi}{2}$
2. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound ? [Given reference intensity of sound as 10^{-12} W/m^2]
- (1) 10 cm (2) 30 cm
- (3) 40 cm (4) 20 cm
3. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3 \sin(0.157x) \cos(200\pi t)$. The length of the string is : (All quantities are in SI units.)
- (1) 20 m (2) 80 m
- (3) 60 m (4) 40 m
4. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is :
- (1) 320 m/s, 120 Hz (2) 180 m/s, 80 Hz
- (3) 180 m/s, 120 Hz (4) 320 m/s, 80 Hz

5. Two cars A and B are moving away from each other in opposite direction. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B ?

(speed of sound in air = 340 ms^{-1}):

- (1) 2250 Hz (2) 2060 Hz
- (3) 2150 Hz (4) 2300 Hz
6. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are, in ms^{-1} , (Given speed of sound = 300 m/s)
- (1) 16, 14 (2) 12, 18
- (3) 12, 16 (4) 8, 18
7. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him ? (Take velocity of sound in air is 350 m/s)
- (1) 857 Hz (2) 807 Hz
- (3) 750 Hz (4) 1143 Hz
8. A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν . The value of ν is close to : (Speed of sound in water = 1500 ms^{-1})
- (1) 499 Hz (2) 502 Hz
- (3) 507 Hz (4) 504 Hz

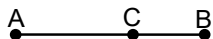
9. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals:
 (1) 2.5 m/s (2) 15.0 m/s
 (3) 5.5 m/s (4) 10.0 m/s
10. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m ($m \ll M$). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1} . When the car has acceleration a , the wave-speed increases to 61.5 ms^{-1} . The value of a , in terms of gravitational acceleration g , is closed to :
 (1) $\frac{g}{5}$ (2) $\frac{g}{20}$
 (3) $\frac{g}{10}$ (4) $\frac{g}{30}$
11. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to :
 (1) 753 Hz (2) 500 Hz
 (3) 333 Hz (4) 666 Hz
12. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to:
 (1) 16.6 cm (2) 20.0 cm
 (3) 10.0 cm (4) 33.3 cm
13. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1 . If the speed of the train is reduced to 17 m/s, the frequency registered is f_2 . If speed of sound is 340 m/s, then the ratio f_1/f_2 is-
 (1) 18 / 17 (2) 19 / 18
 (3) 20 / 19 (4) 21 / 20
14. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be: (Assume that the highest frequency a person can hear is 20,000 Hz)
 (1) 7 (2) 5
 (3) 6 (4) 4
15. Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03 \sin (450 t - 9x)$ where distance and time are measured in SI units. The tension in the string is-
 (1) 10 N (2) 12.5 N
 (3) 7.5 N (4) 5 N
16. A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3} \sin (50 t + 2x)$, where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave ?
 The wave is propagating along the
 (1) negative x -axis with speed 25 ms^{-1}
 (2) positive x -axis with speed 25 ms^{-1}
 (3) positive x -axis with speed 100 ms^{-1}
 (4) negative x -axis with speed 100 ms^{-1}

- 17.** Two sitar strings A and B playing the note 'Dha' are slightly out of true and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease by 3 Hz. If the frequency of A is 425 Hz, the original frequency of B is
 (1) 428 Hz (2) 430 Hz
 (3) 422 Hz (4) 420 Hz
- 18.** A standing wave is formed by the superposition of two waves travelling in opposite directions. The transverse displacement is given by
 $y(x, t) = 0.5 \sin(5\pi x/4) \cos(200\pi t)$
 x and t are in meter and second respectively. What is the speed of the travelling wave moving in the positive x direction?
 (1) 160 m/s (2) 90 m/s
 (3) 180 m/s (4) 120 m/s
- 19.** Two engines pass each other moving in opposite directions with uniform speed of 30 m/s. One of them is blowing a whistle of frequency 540 Hz. Calculate the frequency heard by driver of second engine before they pass each other. Speed of sound is 330 m/sec.
 (1) 450 Hz (2) 540 Hz
 (3) 648 Hz (4) 270 Hz
- 20.** A toy-car, blowing its horn, is moving with a steady speed of 5 m/s, away from a wall. An observer, towards whom the toy car is moving, is able to hear 5 beats per second. If the velocity of sound in air is 340 m/s, the frequency of the horn of the toy car is close to:
 (1) 680 Hz (2) 510 Hz
 (3) 340 Hz (4) 170 Hz
- 21.** A bat moving at 10 ms^{-1} towards a wall sends a sound signal of 8000 Hz towards it. On reflection it hears a sound of frequency. The value of f in Hz is close to (speed of sound = 320 ms^{-1})
 (1) 8258 (2) 8516
 (3) 8000 (4) 8424
- 22.** A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillation of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s.
 (1) 4 (2) 12
 (3) 8 (4) 6
- 23.** A transverse wave is represented by :
 $y = \frac{10}{\pi} \sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$
 For what value of the wavelength the wave velocity is twice the maximum particle velocity ?
 (1) 40 cm (2) 20 cm
 (3) 10 cm (4) 60 cm
- 24.** Two factories are sounding their sirens at 800 Hz. A man goes from one factory to other at a speed of 2 m/s. The velocity of sound is 320 m/s. The number of beats heard by the person in one second will be:
 (1) 2 (2) 4
 (3) 8 (4) 10
- 25.** When two sound waves travel in the same direction in a medium, the displacement of a particle is located at 'x' at time 't' is given by :
 $y_1 = 0.05 \cos(0.50\pi x - 100\pi t)$
 $y_2 = 0.05 \cos(0.46\pi x - 92\pi t)$
 Where y_1 , y_2 and x are in metre and t in seconds. The speed of the sound in the medium is :
 (1) 332 m/s (2) 200 m/s
 (3) 92 m/s (4) 100 m/s

26. An engine approaches a hill with a constant speed. When it is at a distance of 0.9 km, it blows a whistle whose echo is heard by the driver after 5 seconds. If the speed of sound in air is 330 m/s, then the speed of the engine is :

(1) 60 m/s (2) 32 m/s
(3) 30 m/s (4) 27.5 m/s

27. 'A' and 'B' are two sources generating sound waves. A listener is situated at 'C'. The frequency of the source at 'A' is 500 Hz. 'A' now, moves towards 'C' with speed 4 m/s. The number of beats heard at 'C' is 6. When 'A' moves away from 'C' with speed 4 m/s, the number of beats heard at 'C' is 18. The speed of sound is 340 m/s. The frequency of the source at 'B' is :



(1) 506 Hz (2) 512 Hz
(3) 500 Hz (4) 494 Hz

28. A sonometer wire of length 114 cm is fixed at both the ends. Where should the two bridge be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1 : 3 : 4 ?

(1) At 72 cm and 96 cm from one end
(2) At 48 cm and 96 cm from one end
(3) At 24 cm and 72 cm from one end
(4) At 36 cm and 84 cm from one end

29. The disturbance $y(x, t)$ of a wave propagating in the positive x-direction is given by $y = \frac{1}{1+x^2}$ at time $t = 0$ and by

$$y = \frac{1}{[1+(x-1)^2]} \text{ at } t = 2\text{s, where } x \text{ and } y$$

are in meters. The shape of the wave disturbance does not change during the propagation.

The velocity of wave in m/s is-

(1) 2.0 (2) 4.0
(3) 0.5 (4) 1.0

30. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is (velocity of sound - 330 m/s)

(1) 125.00 (2) 93.75
(3) 62.50 (4) 187.50

31. A wave represented by the $y_1 = a \cos(kx - \omega t)$ is superimposed with another wave to form a stationary wave such that the point $x = 0$ is node. The equation for the other wave is

(1) $a \cos(kx - \omega t + \pi)$
(2) $a \cos(kx + \omega t + \pi)$
(3) $a \cos(kx + \omega t + \frac{\pi}{2})$
(4) $a \cos(kx - \omega t + \frac{\pi}{2})$

32. This question has Statement 1 and Statement 2. Of the four choice given after the statements, choose the one that best describes the two statements.

Statement 1 : Bats emitting ultrasonic waves can detect the location of a prey by hearing the waves reflected from it.

Statement 2 : When the source and the detector are moving, the frequency of reflected wave is changed.

(1) Statement 1 is false, Statement 2 is true
(2) Statement 1 is true, Statement 2 is false
(3) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1
(4) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1

- 33.** A plane wave $y = a \sin(bx + ct)$ is incident on a surface. Equation of the reflected wave is $y' = a' \sin(ct - bx)$. Which of the following statements is not correct?

 - The wave is incident on the surface normally.
 - Reflecting surface is $y-z$ plane.
 - Medium, in which incident wave is travelling, is denser than the other medium.
 - a' cannot be greater than a .

(1) (i), (ii) (2) (iii), (iv)
(3) (i), (iv) (4) All correct

34. A travelling wave represented by $y = A \sin(\omega t - kx)$ is superimposed on another wave represented by $y = A \sin(\omega t + kx)$. The resultant is :-

 - A standing wave having nodes at $x = \left(n + \frac{1}{2}\right) \frac{\lambda}{2}, n = 0, 1, 2$
 - A wave travelling along $+x$ direction
 - A wave travelling along $-x$ direction
 - A standing wave having nodes at $x = \frac{n\lambda}{2}; n = 0, 1, 2$

35. The power of sound from the speaker of a radio is 20 MW by turning the knob of the volume control the power of the sound is increased to 400 MW. The power increase in describe as compared to the original power is :-

(1) 13 dB (2) 10 dB
(3) 20 dB (4) 800 dB

36. Two tuning forks having frequency 256 Hz (A) and 262 Hz (B) tuning fork. A produces some beats per second with unknown tuning fork, same unknown tuning fork produce double beats per second from B tuning fork then the frequency of unknown tuning fork is:-

(1) 262 (2) 260
(3) 250 (4) 300

37. A person observes a change of 2.5% in frequency of sound of horn of a car. If the car is approaching forward the person & sound velocity is 320 m/s, then velocity of car in m/s will be approximately:-

(1) 8 (2) 800
(3) 7 (4) 6

38. Two cars A and B approach a stationary observer from opposite sides as shown in figure. Observer hears no beats. If the frequency of the horn of the car B is 504 Hz, the frequency of horn of car A will be- [$v = 330$ m/s]

(1) 529.2 Hz (2) 295.2 Hz
(3) 440.5 Hz (4) none of these

39. The displacement y of a particle executing periodic motion is given by :

$$y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$$

This expression may be considered to be a result of the superposition of independent, simple harmonic motions.

(1) two (2) three
(3) four (4) five

ANSWER KEY																									
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	3	3	2	4	1	2	3	2	1	1	4	2	2	1	2	1	4	1	3	4	2	4	1	4	2
Que.	26	27	28	29	30	31	32	33	34	35	36	37	38	39											
Ans.	3	2	1	3	2	2	3	4	1	1	3	1	1	2											

Exercise – III (Previous Year Question)

1. Which one of the following statements is true- **[AIPMT-2006]**
 - (1) Both light and sound waves in air are transverse
 - (2) The sound waves in air are longitudinal while the light waves are transverse
 - (3) Both light and sound waves in air are longitudinal
 - (4) Both light and sound waves can travel in vacuum

2. The driver of a car travelling with speed 30m/sec towards a hill sounds a horn of frequency 600 Hz. If the velocity of sound in air is 330 m/s, the frequency of reflected sound as heard by driver is- **[AIPMT-2009]**
 - (1) 500 Hz
 - (2) 550 Hz
 - (3) 555.5 Hz
 - (4) 720 Hz

3. A wave in a string has an amplitude of 2cm. The wave travels in the +ve direction of x-axis with a speed of 128 m/sec and it is noted that 5 complete waves fit in 4m length of the string. The equation describing the wave is- **[AIPMT-2009]**
 - (1) $y = (0.02)\text{m} \sin(7.85x - 1005t)$
 - (2) $y = (0.02)\text{m} \sin(7.85x + 1005t)$
 - (3) $y = (0.02)\text{m} \sin(15.7x - 2010t)$
 - (4) $y = (0.02)\text{m} \sin(15.7x + 2010t)$

4. A transverse wave is represented by $y = A \sin(\omega t - kx)$. For what value of the wavelength is the wave velocity equal to the maximum particle velocity? **[AIPMT Pre-2010]**
 - (1) $\frac{\pi A}{2}$
 - (2) πA
 - (3) $2\pi A$
 - (4) A

5. A tuning fork of frequency 512 Hz makes 4 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per sec when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was- **[AIPMT Pre-2010]**
 - (1) 510 Hz
 - (2) 514 Hz
 - (3) 516 Hz
 - (4) 508 Hz

6. Two waves are represented by the equations
 $y_1 = a \sin(\omega t + kx + 0.57) \text{ m}$ and
 $y_2 = a \cos(\omega t + kx) \text{ m}$,
 where x is in meter and t in sec. The phase difference between them is: **[AIPMT Pre-2011]**
 - (1) 0.57 radian
 - (2) 1 radian
 - (3) 1.25 radian
 - (4) 1.57 radian

7. Sound waves travel at 350 m/s through a warm air and at 3500 m/s through brass. The wavelength of a 700 Hz acoustic wave as it enters brass from warm air : **[AIPMT Pre-2011]**
 - (1) decreases by a factor 10
 - (2) increases by a factor 20
 - (3) increase by a factor 10
 - (4) decreases by a factor 20

8. Two identical piano wires, kept under the same tension T have a fundamental frequency of 600Hz. The fractional increase in the tension of one of the wires which will lead to occurrence of 6 beats/s when both the wires oscillate together would be: **[AIPMT Mains-2011]**
 - (1) 0.01
 - (2) 0.02
 - (3) 0.03
 - (4) 0.04

9. Two sources of sound placed close to each other, are emitting progressive waves given by

$$y_1 = 4 \sin 600 \pi t \text{ and } y_2 = 5 \sin 608 \pi t:$$

An observer located near these two sources of sound will hear-

[AIPMT (Pre)-2012]

- (1) 8 beats per second with intensity ratio 81 : 1 between waxing and waning
- (2) 4 beats per second with intensity ratio 81 : 1 between waxing and waning
- (3) 4 beats per second with intensity ratio 25 : 16 between waxing and waning
- (4) 8 beats per second with intensity ratio 25 : 16 between waxing and waning

10. The equation of a simple harmonic wave is given by

$$y = 3 \sin \frac{\pi}{2} (50t - x)$$

where x and y are in meters and t is in seconds. The ratio of maximum particle velocity to the wave velocity is -

[AIPMT Mains-2012]

- (1) $\frac{3}{2} \pi$
- (2) 3π
- (3) $\frac{2}{3} \pi$
- (4) 2π

11. A wave travelling in the +ve x -direction having displacement along y -direction as 1m, wavelength 2π m and frequency of $1/\pi$ Hz is represented by: **[NEET-2013]**

- (1) $y = \sin (2\pi x + 2\pi t)$
- (2) $y = \sin (x - 2t)$
- (3) $y = \sin (2\pi x - 2\pi t)$
- (4) $y = \sin (10\pi x - 20\pi t)$

12. A source of unknown frequency gives 4 beats/ s, when sounded with a source of known frequency 250 Hz, the second harmonic of the source of unknown frequency gives five beats per second, when sounded with a source of frequency 513 Hz. The unknown frequency is :

[NEET-2013]

- (1) 260 Hz
- (2) 254 Hz
- (3) 246 Hz
- (4) 240 Hz

13. if we study the vibration of a pipe open at both ends, then the following statement is not true : **[NEET - 2013]**

- (1) Pressure change will be maximum at both ends
- (2) Open end will be antinode
- (3) Odd harmonics of the fundamental frequency will be generated
- (4) All harmonics of the fundamental frequency will be generated

14. If n_1 , n_2 and n_3 are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency n of the string is given by : **[AIPMT - 2014]**

- (1) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$
- (2) $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}} + \frac{1}{\sqrt{n_3}}$
- (3) $\sqrt{n} = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3}$
- (4) $n = n_1 + n_2 + n_3$

15. The number of possible natural oscillations of air column in a pipe closed at one end of length 85 cm whose frequencies lie below 1250 Hz are: (velocity of sound = 340 ms^{-1})

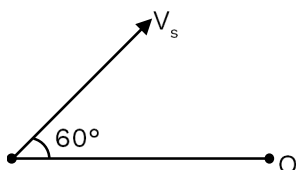
[AIPMT - 2014]

- (1) 4
- (2) 5
- (3) 7
- (4) 6

16. The fundamental frequency of a closed organ pipe of length 20 cm is equal to the second overtone of an organ pipe open at both the ends. The length of organ pipe open at both the ends is : **[AIPMT-2015]**

(1) 100 cm (2) 120 cm
(3) 140 cm (4) 80 cm

17. A source of sound S emitting waves of frequency 100 Hz and an observer O are located at some distance from each other. The source is moving with a speed of 19.4 ms^{-1} at an angle of 60° with the source observer line as shown in the figure. The observer is at rest. The apparent frequency observed by the observer (velocity of sound in air 300 ms^{-1}) is : **[Re-AIPMT-2015]**



(1) 97 Hz (2) 100 Hz
(3) 103 Hz (4) 106 Hz

18. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. The lowest resonant frequencies for this string is : **[Re-AIPMT-2015]**

(1) 105 Hz (2) 155 Hz
(3) 205 Hz (4) 10.5 Hz

19. A siren emitting a sound of frequency 800 Hz moves away from an observe towards a cliff at a speed of 15 ms^{-1} . Then, the frequency of sound that the observer hears in the echo reflected from the cliff is : (Take velocity of sound in air = 300 ms^{-1}) **[NEET -I - 2016]**

(1) 765 Hz (2) 800 Hz
(3) 838 Hz (4) 885 Hz

20. A uniform rope of length L and mass m_1 hangs vertically from a rigid support. A block of mass m_2 is attached to the free end of the rope. A transverse pulse of wavelength λ_1 is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is λ_2 . The ratio λ_2/λ_1 is : **[NEET - I - 2016]**

(1) $\sqrt{\frac{m_1}{m_2}}$ (2) $\sqrt{\frac{m_1 + m_2}{m_2}}$
(3) $\sqrt{\frac{m_2}{m_1}}$ (4) $\sqrt{\frac{m_1 + m_2}{m_1}}$

21. An air column, closed at one end and open at the other, resonates with a tuning fork when the smallest length of the column is 50 cm. The next larger length of the column resonating with the same tuning fork is : **[NEET -I - 2016]**

(1) 66.7 cm (2) 100 cm
(3) 150 cm (4) 200 cm

22. The second overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe L metre long. The length of the open pipe will be : **[NEET - II - 2016]**

(1) $\frac{L}{2}$ (2) 4 L
(3) L (4) 2 L

23. Three sound waves of equal amplitude have frequencies $(n-1)$, n , $(n+1)$. They superimpose to give beats. The number of beats produced per second will be : **[NEET - II - 2016]**

(1) 3 (2) 2
(3) 1 (4) 4

24. The two nearest harmonics of a tube closed at one end and open at other end are 220 Hz and 260 Hz. What is the fundamental frequency of the system? **[NEET - 2017]**

(1) 10 Hz (2) 20 Hz
(3) 30 Hz (4) 40 Hz

