

CBSE Board Class 10 Chapter 5- Arithmetic Progression Objective Questions

General Term of AP

1. Find the number of terms in each of the following APs:

(i) 7, 13, 19, ..., 205

(ii) 18, $31\frac{1}{2}$, 13, ..., -47

(A) 26, 35

(B) 27, 34

(C) 35, 26

(D) 34, 27

Answer: (D) 34, 27

Solution: (i) 7, 13, 19, ..., 205

First term, $a=7$

Common difference, $d=13-7=6$

$a_n=205$

Using formula $a_n=a+(n-1)d$ to find n^{th} term of arithmetic progression, we get

$$205=7+(n-1)6$$

$$\Rightarrow 205=6n+1$$

$$\Rightarrow 204=6n$$

$$\Rightarrow n=204/6=34$$

Therefore, there are 34 terms in the given arithmetic progression.

(ii) 18, $31\frac{1}{2}$, 13, ..., -47

First term, $a=18$

Common difference, $d = (31/2) - 18 = -5/2$

$$a_n = -47$$

Using formula $a_n = a + (n-1)d$ to find n^{th} term of arithmetic progression, we get

$$-47 = 18 + (n-1)(-5/2)$$

$$\Rightarrow -94 = 36 - 5n + 5$$

$$\Rightarrow 5n = 135$$

$$\Rightarrow n = 135/5 = 27$$

Therefore, there are 27 terms in the given arithmetic progression.

2. Find the 31st term of an AP whose 11th term is 38 and 16th term is 73.

- (A) 185
- (B) 210
- (C) 178
- (D) 150

Answer: (C) 178

Solution: We are given that $a_{11} = 38$ and $a_{16} = 73$ where, a_{11} is the 11th term and a_{16} is the 16th term of an AP.

Using formula $a_n = a + (n-1)d$ to find n^{th} term of arithmetic progression, we get

$$38 = a + 10d \dots (i)$$

$$73 = a + 15d \dots (ii)$$

equation (ii) - equation (i) gives,

$$35 = 5d$$

$$d = 7 \dots (iii)$$

Substituting (iii) in (i) we get, $a = -32$

$$a_{31} = -32 + (31-1)(7)$$

$$\Rightarrow -32 + 210 = 178$$

Therefore, 31st term of AP is 178.

3. If the third and the ninth terms of an AP are 4 and -8 respectively, which term of this AP is zero?

- (A) 5th
- (B) 4th

- (C) 3rd
(D) 6th

Answer: (B) 4th

Solution: It is given that 3rd and 9th term of AP are 4 and -8 respectively.

It means $a_3=4$ and $a_9=-8$

Where, a_3 and a_9 are third and ninth terms respectively.

Using formula $a_n=a+(n-1)d$ to find n^{th} term of arithmetic progression, we get

$$4=a+(3-1)d$$

$$\Rightarrow 4 = a+2d \dots (i)$$

$$-8 = a+(9-1)d$$

$$\Rightarrow -8=a+8d \dots (ii)$$

From equation (i) we have $a=4-2d$

Substituting in equation (ii), we have

$$-8=4-2d+8d$$

$$\Rightarrow -12 = 6d$$

$$\Rightarrow d = -12/6 = -2$$

Solving for (a), we get

$$\Rightarrow -8 = a-16$$

$$\Rightarrow a = 8$$

Therefore, first term $a = 8$ and Common Difference $d = -2$

We know $a_n = a + (n-1)d$ (where a_n is the n^{th} term)

Finding value of n where $a_n=0$

$$0=8+(n-1)(-2)$$

$$\Rightarrow 0 = 8-2n+2$$

$$\Rightarrow 0 = 10-2n$$

$$\Rightarrow 2n = 10$$

$$\Rightarrow n = 10/2=5$$

Therefore, 5th term is equal to 0.

4. Which term of the AP: 3, 15, 27, 39, ... will be 132 more than its 54th term?

- (A) 70
(B) 65
(C) 80
(D) 55

Answer: (B) 65

Solution: Let's first calculate 54th of the given AP.

First term = $a = 3$

Common difference = $d = 15 - 3 = 12$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression, we get

$$a_{54} = a + (54-1)d$$

$$a_{54} = 3 + 53(12) = 3 + 636 = 639$$

We want to find which term is 132 more than its 54th term. Let's suppose it is n^{th} term which is 132 more than 54th term.

Therefore, we can say that

$$a_n = a_{54} + 132$$

$$a_n = a + (n-1)d = 3 + (n-1)(12)$$

$$\Rightarrow 3 + (n-1)12 = 639 + 132$$

$$\Rightarrow 3 + 12n - 12 = 771$$

$$\Rightarrow 12n - 9 = 771$$

$$\Rightarrow 12n = 780$$

$$\Rightarrow n = 780/12 = 65$$

Therefore, 65th term is 132 more than its 54th term.

5. Two AP's have the same common difference. The difference between their 100th terms 100, what is the difference between their 1000th terms.

- (A) 200
- (B) 150
- (C) 100
- (D) 55

Answer: (C) 100

Solution: Let first term of first AP = a

Let first term of 2nd AP = a'

It is given that their common difference is same. Let their common difference be d.

It is given that difference between their 100th terms is 100. Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we can say that

$$a + (100-1)d - (a' + (100-1)d) = a + 99d - a' - 99d = 100$$

$$\Rightarrow a - a' = 100 \quad \dots\dots (1)$$

We want to find difference between their 1000th terms which means we want to calculate:

$$a + (1000-1)d - (a' + (1000-1)d) = a + 999d - a' - 999d = a - a'$$

Putting equation (1) in the above equation we get,

$$a + (1000-1)d - (a' + (1000-1)d) = a + 999d - a' - 999d = a - a' = 100$$

Therefore, difference between their 1000th terms would be equal to 100

6. How many three-digit numbers are divisible by 7?

- (A) 112
- (B) 114
- (C) 128
- (D) 110

Answer: (C) 128

Solution: We have an AP starting at 105 because it is the first three digit number divisible by 7.

AP will end at 994 because it is the last three digit number divisible by 7.

Therefore, we have an AP 105, 112, 119...994

First term, a = 105

Common difference, $d = 112 - 105 = 7$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression, we can say that

$$994 = 105 + (n-1)(7)$$

$$\Rightarrow 994 = 105 + (n-1)(7)$$

$$\Rightarrow 889 = 7(n-1)$$

$$\Rightarrow n-1 = 889/7 \Rightarrow n = 127 + 1 = 128$$

994 is the 128^{th} term of AP. Therefore, there are 128 terms in AP. In other words, we can also say that there are 128 three digit numbers divisible by 7.

7. For what value of n , are the n^{th} terms of two AP's: 63, 65, 67 and 3, 10, 17,.. equal?

(A) 11

(B) 14

(C) 12

(D) 13

Answer: (D) 13

Solution: Let's first consider AP: 63, 65, 67.....

First term $= a = 63$

Common difference $= d = 65 - 63 = 2$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression, we can say that

$$a_n = 63 + (n-1)(2) \quad (1)$$

Now, consider second AP 3, 10, 17...

First term $= a = 3$

Common difference $= d = 10 - 3 = 7$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression, we can say that

$$a_n = 3 + (n-1)(7) \quad (2)$$

According to the given condition, we can write

$$(1) = (2)$$

$$\Rightarrow 63 + (n-1)(2) = 3 + (n-1)(7)$$

$$\Rightarrow 63 + 2n - 2 = 3 + 7n - 7$$

$$\Rightarrow 65 = 5n$$

$$\Rightarrow n = 65/5 = 13$$

Therefore, 13th terms of both the AP's are equal.

Introduction to AP

8. If $(x+1)$, $3x$ and $(4x+2)$ are first three terms of an AP, then its 5th term is :

(A) 14

(B) 19

(C) 24

(D) 28

Answer: (D) 28

Solution: Given, $(x+1)$, $3x$, $(4x+2)$ are in AP.

Hence, the difference of two consecutive terms will be same.

$$\text{Hence, } 3x - (x+1) = (4x+2) - 3x$$

$$\Rightarrow 2x - 1 = x + 2$$

$$\Rightarrow x = 3$$

So, the first term,

$$a = (x+1) = 4.$$

The common difference,

$$d = 9 - 4 = 5.$$

The n^{th} term of an AP is given by

$$t_n = a + (n-1)d$$

$$\Rightarrow t_5 = 4 + 4(5) = 24.$$

9. The sum of first ten terms of an A.P. is four times the sum of its first five terms, then ratio of the first term and common difference is

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) 4
- (D) 1

Answer: (A) $\frac{1}{2}$

Solution: Let S_{10} be the sum of first 10 terms and S_5 be the sum of first 5 terms.

We know,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Given,

$$S_{10} = 4S_5$$

$$\Rightarrow \frac{10}{2} [2a + (10-1)d] = 4 \times \frac{5}{2} [2a + (5-1)d]$$

$$\Rightarrow \frac{10}{2} [2a + 9d] = 4 \times \frac{5}{2} [2a + 4d]$$

$$\Rightarrow 2a + 9d = 4a + 8d$$

$$\Rightarrow a/d = \frac{1}{2}$$

10. Find the common difference (d) in the following APs respectively.

(i) 20, 40, 60, 80, 100,...

(ii) 5, 0, -5, -10, -15, ...

(iii) 2, 2, 2, 2, 2, ..

- (A) 20, 5, 0
- (B) -20, -5, 0
- (C) -20, 5, 0
- (D) 20, -5, 0

Answer: (D) 20, -5, 0

Solution: Common difference is

(i) 20, 40, 60, 80, 100,

$$d = 40 - 20 = 20$$

(ii) 5, 0, -5, -10, -15, ...

$$d = 0 - 5 = -5$$

(iii) 2, 2, 2, 2, 2....

$$d = 2 - 2 = 0$$

11. Which of the following sequences form an AP?

(i) 2, 4, 8, 16.....

(ii) 2, 3, 5, 7, 11.....

(iii) -1, -1.25, -1.5, -1.75.....

(iv) 1, -1, -3, -5, -7.....

Answer: (iii) -1, -1.25, -1.5, -1.75..... and

(iv) 1, -1, -3, -5, -7.....

Solution: Consider each list of numbers:

(i) 2, 4, 8, 16.....

$$\text{Difference between the first two terms} = 4 - 2 = 2$$

$$\text{Difference between the third and second term} = 8 - 4 = 4$$

Since $a_2 - a_1 \neq a_3 - a_2$, this sequence is not an AP

(ii) 2, 3, 5, 7, 11.....

$$\text{Difference between the first two terms} = 3 - 2 = 1$$

$$\text{Difference between the third and second term} = 5 - 3 = 2$$

Since $a_2 - a_1 \neq a_3 - a_2$, this sequence is not an AP

(iii) -1, -1.25, -1.5, -1.75.....

Difference between the first two terms = $-1.25 - (-1) = -0.25$

Difference between the third and second term = $-1.5 - (-1.25) = -0.25$

Difference between the fourth and third term = $-1.75 - (-1.5) = -0.25$

Since $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$, this sequence is an AP

(iv) 1, -1, -3, -5, -7.....

Difference between the first two terms = $-1 - 1 = -2$

Difference between the third and second term = $-3 - (-1) = -2$

Difference between the fourth and third term = $-5 - (-3) = -2$

Since $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$, this sequence is an AP

12. What are the conditions for a sequence to be an AP?

- (A) The sum of two consecutive terms should be constant.
- (B) The product of two consecutive numbers should be constant.
- (C) The difference between two consecutive terms should be constant.
- (D) The ratio of two consecutive terms should be constant.

Answer: (C) The difference between two consecutive terms should be constant.

Solution: Let $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \dots$ be a sequence.

For this sequence to be an AP, the difference between any two consecutive terms should be constant.

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = d$$

This difference is called the common difference of the AP and is denoted by d .

So an AP can also be represented in this form as well $a, a+d, a+2d \dots$

13. Which of the following list of numbers forms an AP?

(i) $4, 4 + \sqrt{3}, 4 + 2\sqrt{3}, 4 + 3\sqrt{3}, 4 + 4\sqrt{3}$

(ii) $0.3, 0.33, 0.333, 0.3333, 0.33333$

(iii) $3/5, 6/5, 9/5, 12/5, 3$

(iv) $-1/5, -1/5, -1/5, -1/5, -1/5$

Answer: (i), (iii) and (iv)

Solution: Consider each series (i) $4, 4 + \sqrt{3}, 4 + 2\sqrt{3}, 4 + 3\sqrt{3}, 4 + 4\sqrt{3}$

Difference between first two consecutive terms $= 4 + \sqrt{3} - 4 = \sqrt{3}$

Difference between third and second consecutive terms $= 4 + 2\sqrt{3} - 4 + \sqrt{3} = \sqrt{3}$

Difference between fourth and third consecutive terms $= 4 + 3\sqrt{3} - 4 + 2\sqrt{3} = \sqrt{3}$

Since $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

This series is an AP

(ii) $0.3, 0.33, 0.333, 0.3333, 0.33333$

Difference between first two consecutive terms $= 0.33 - 0.3 = 0.03$

Difference between third and second consecutive terms $= 0.333 - 0.33 = 0.003$

Since $a_2 - a_1 \neq a_3 - a_2$

This series is not an AP

(iii) $3/5, 6/5, 9/5, 12/5, 3$

Difference between first two consecutive terms $= (6/5) - (3/5) = 3/5$

Difference between third and second consecutive terms $= (9/5) - (6/5) = 3/5$

Difference between fourth and third consecutive terms $= (12/5) - (9/5) = 3/5$

Since $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

This series is an AP

(iv) $-1/5, -1/5, -1/5, -1/5, -1/5$

Difference between first two consecutive terms = $-1/5 - (-1/5) = 0$

Difference between third and second consecutive terms = $-1/5 - (-1/5) = 0$

Difference between fourth and third consecutive terms = $-1/5 - (-1/5) = 0$

Since $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

This series is an AP

Sum of Terms in AP

14. The first term of an AP is 5, the last term is 50 and the sum is 440. Find the number of terms and the common difference

(A) 17; 3

(B) 16; 2

(C) 17; 2

(D) 16; 3

Answer: (D) 16; 3

Solution: First term, $a=5$

Last term, $l=50$

$S_n=440$

Applying formula, $S_n = n/2 (a+l)$ to find sum of n terms of AP, we get

$$440 = n/2 (5+50)$$

$$\Rightarrow 440/55 = n/2$$

$$\Rightarrow 8 = n/2$$

$$\Rightarrow n = 16$$

Applying formula, $S_n = n/2 (2a + (n-1)d)$ to find sum of n terms of AP and putting value of n , we get

$$440 = 16/2(2(5) + (16-1) d)$$

$$\Rightarrow 440 = 8(10 + 15d)$$

$$\Rightarrow 10 + 15d = 55$$

$$\Rightarrow 15d = 45$$

$$\Rightarrow d = 45/15 = 3$$

- 15.** The first and the last terms of an AP are 17 and 350 respectively. If, the common difference is 9, how many terms are there and what is their sum?

(A) 38; 7114

(B) 32; 7114

(C) 38; 6973

(D) 32; 6973

Answer: (C) 38; 6973

Solution: First term = $a = 17$

Last term = $l = 350$

Common difference = $d = 9$

Using formula $a_n = a + (n-1) d$, to find n th term of arithmetic progression, we can say that

$$350 = 17 + (n-1) (9)$$

$$\Rightarrow 350 = 17 + 9n - 9$$

$$\Rightarrow 342 = 9n$$

$$\Rightarrow n = 342/9 = 38$$

Applying formula, $S_n = n/2(2a + (n-1) d)$ to find sum of n terms of AP and putting value of n , we get

$$S_{38} = 38/2(34 + (38-1) (9))$$

$$\Rightarrow S_{38} = 19(34 + 333) = 6973$$

Therefore, there are 38 terms and their sum is equal to 6973.

16. Find the sum of first 22 terms of an AP in which $d = 7$ and the 22nd term is 149.

- (A) 1623
- (B) 1712
- (C) 1542
- (D) 1661

Answer: (D) 1661

Solution: It is given that 22nd term is equal to 149.

It means $a_{22}=149$

Using formula $a_n=a+(n-1)d$, to find n^{th} term of AP, we can say that

$$149=a+(22-1)(7)$$

$$\Rightarrow 149=a+147$$

$$\Rightarrow a=2$$

Applying formula, $S_n = n/2(2a + (n-1)d)$ to find Sum of n terms of AP and putting value of a , we get

$$S_{22} = 22/2(4 + (22-1)(7))$$

$$\Rightarrow S_{22}=11(4+147)$$

$$\Rightarrow S_{22}=1661$$

Therefore, sum of first 22 terms of AP is equal to 1661.

17. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

- (A) 5610
- (B) 5840
- (C) 5320
- (D) 5000

Answer: (A) 5610

Solution: It is given that second and third terms of AP are 14 and 18 respectively.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression, we can say that

$$14 = a + (2-1)d$$

$$\Rightarrow 14 = a + d \quad (1)$$

$$\text{And, } 18 = a + (3-1)d$$

$$\Rightarrow 18 = a + 2d \quad (2)$$

These are equations consisting of two variables. We can solve them by the method of substitution.

Using equation (1), we can say that $a = 14 - d$

Putting value of a in equation (2), we can say that

$$18 = 14 - d + 2d$$

$$\Rightarrow d = 4$$

Therefore, common difference $d = 4$

Putting value of d in equation number (1), we can say that

$$18 = a + 2(4)$$

$$\Rightarrow a = 10$$

Applying formula, $S_n = \frac{n}{2}(2a + (n-1)d)$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}(20 + (15-1)4) = \frac{15}{2}(20 + 200) = 15 \times 110 = 5610$$

Therefore, sum of first 15 terms of an AP is equal to 5610.

18. The sum of four consecutive numbers in an A.P. with $d > 0$ is 20. Sum of their square is 120, then the middle terms are

(A) 8, 10

(B) 6, 8

(C) 4, 6

(D) 2,4

Answer: (C) 4, 6

Solution: Let the numbers are $a-3d, a-d, a+d, a+3d$

Given: $a-3d+a-d+a+d+a+3d=20$

$$4a=20$$

$$a=5 \text{ and}$$

$$(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 120$$

$$(4)(a)^2 + 20d^2 = 120$$

$$(4)(5)^2 + 20d^2 = 120$$

$$d^2 = 1$$

$$d = +1 \text{ or } -1$$

Hence numbers are 2,4,6, and 8

19. Find the sum of all the non-negative terms of the following sequence: 100, 97, 94,

(A) 1717

(B) 1719

(C) 1721

(D) 1723

Answer: (A) 1717

Solution: Here, $t_1 = 100$, common difference, $d = t_2 - t_1 = 97 - 100 = -3$

$$t_n = t_1 + (n-1)d \Rightarrow 100 + (n-1)(-3) = 100 - 3(n-1) = 103 - 3n.$$

Let t_n be the first negative term. i.e,

$$t_n < 0 \Rightarrow 103 - 3n < 0 \Rightarrow n > 103/3 > 34$$

That is, the 35th term will be negative.

Sum of the first 34 terms = $(34/2)$ (first term + last term)

$$= 17(100 + 100 + (34-1)(-3)) = 1717$$

20. What is the sum of all 3 digit numbers that leave a remainder of '2' when divided by 3?

- (A) 149700
- (B) 164749
- (C) 164850
- (D) 897

Answer: (C) 164850

Solution: The smallest 3 digit number that will leave a remainder of 2 when divided by 3 is 101.
The next number that will leave a remainder of 2 when divided by 3 is 104, 107,
The largest 3 digit number that will leave a remainder of 2 when divided by 3 is 998.

So, it is an AP with the first term being 101 and the last term being 998 and common difference being 3.

Sum of an AP = $n/2(a+l)$

we know that in an A.P., the nth term $a_n = a_1 + (n-1) \times d$

In this case, therefore,

$$998 = 101 + (n - 1) \times 3$$

$$\Rightarrow 897 = (n-1) \times 3$$

$$\Rightarrow 299 = (n-1)$$

$$\Rightarrow n = 300$$

Sum of the AP will therefore, be $(101+998)/ 2 \times 300 = 164, 850$