Speed Test-3

1. **(b)** $\vec{u} = \hat{i} + 2\hat{j} = u_x\hat{i} + u_y\hat{j} \Rightarrow u\cos\theta = 1,\ u\sin\theta = 2$

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u_x^2}$$

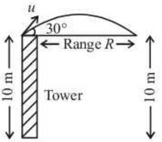
$$\therefore\ y = 2x - \frac{1}{2}gx^2 = 2x - 5x^2$$

2. **(c)** $500\cos\theta = 250 \Rightarrow \cos\theta = \frac{1}{2}$

or $\theta = 60°$.

3. **(c)** As time periods are equal therefore ratio of angular speeds will be 1 : 1. $\left(\omega = \frac{2\pi}{T}\right)$.

4. **(d)**

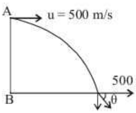


From the figure it is clear that range is required

$$R = \frac{u^2\sin 2\theta}{g} = \frac{(10)^2\sin(2\times 30°)}{10} = 5\sqrt{3} = 8.66 \text{ m}$$

5. **(a)** Horizontal component of velocity $v_x = 500$ m/s and vertical component of velocity while striking the ground.

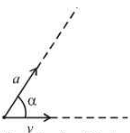
$u_v = 0 + 10 \times 10 = 100$ m/s



$\therefore$ Angle with which it strikes the ground

$$\theta = \tan^{-1}\left(\frac{u_v}{u_x}\right) = \tan^{-1}\left(\frac{100}{500}\right) = \tan^{-1}\left(\frac{1}{5}\right)$$

6. **(b)**



The velocity of first particle, $v_1 = v$

The velocity of second particle, $v_2 = at$

Relative velocity, $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$

or $v_{12}^2 = v^2 + (at)^2 - 2v(at\cos\alpha)$

For least value of relative velocity, $\frac{dv_{12}}{dt} = 0$

or $\frac{d}{dt}\left[v^2 + a^2t^2 - 2vat\cos\alpha\right] = 0$

or $0 + a^2 \times 2t - 2va\cos\alpha = 0$

or $t = \frac{v\cos\alpha}{a}$

7. **(d)** $t = \frac{2u\sin 30°}{g\cos 30°} = \frac{2(10)(1/2)}{10(\sqrt{3}/2)} = \frac{2}{\sqrt{3}}\text{sec}$

$$R = 10\cos 30°\, t - \frac{1}{2}\, g\sin 30°\, t^2$$

$$= \frac{10\sqrt{3}}{2}\left(\frac{2}{\sqrt{3}}\right) - \frac{1}{2}(10)\left(\frac{1}{2}\right)\frac{4}{3} = 10 - \frac{10}{3} = \frac{20}{3}\text{ m}$$

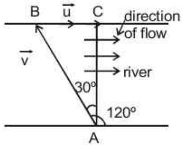
8. **(b)** $\overline{AB} = (4\hat{i} + 5\hat{j} + 6\hat{k}) - (3\hat{i} + 4\hat{j} + 5\hat{k}) = \hat{i} + \hat{j} + \hat{k}$

$\overline{CD} = (4\hat{i} + 6\hat{j}) - (7\hat{i} + 9\hat{j} + 3\hat{k}) = 3\hat{i} - 3\hat{j} + 3\hat{k}$

$\overline{AB}$ and $\overline{CD}$ are parallel, because its cross-product is 0.

9. **(c)** Here v = 0.5 m/sec. u = ?

so $\sin\theta = \frac{u}{v} \Rightarrow \frac{u}{.5} = \frac{1}{2}$ or $u = 0.25\ \text{ms}^{-1}$

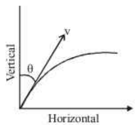


10. **(d)** Max. height $= H = \frac{v^2\sin^2(90-\theta)}{2g}$ .....(i)

Time of flight, $T = \frac{2v\sin(90-\theta)}{g}$ ...(ii)

From (i), $\frac{v\cos\theta}{g} = \sqrt{\frac{2H}{g}}$

From (ii), $T = 2\sqrt{\frac{2H}{g}} = \sqrt{\frac{8H}{g}}$



11. (c) Yes, the person can catch the ball when horizontal velocity is equal to the horizontal component of ball's velocity, the motion of ball will be only in vertical direction with respect to person for that,

$$\frac{v_o}{2} = v_o \cos\theta \text{ or } \theta = 60°$$

12. (b) Two vectors are

$\vec{A} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$

$\vec{B} = \cos\frac{\omega t}{2}\hat{i} + \sin\frac{\omega t}{2}\hat{j}$

For two vectors $\vec{A}$ and $\vec{B}$ to be orthogonal $A.B = 0$

$\vec{A}.\vec{B} = 0 = \cos\omega t.\cos\frac{\omega t}{2} + \sin\omega t.\sin\frac{\omega t}{2}$

$= \cos\left(\omega t - \frac{\omega t}{2}\right) = \cos\left(\frac{\omega t}{2}\right)$

So, $\frac{\omega t}{2} = \frac{\pi}{2}$ $\therefore$ $t = \frac{\pi}{\omega}$

13. (a) $\vec{v_1} = 50\,\text{km h}^{-1}$ due North;

$\vec{v_2} = 50\,\text{km h}^{-1}$ due West. Angle between $\vec{v_1}$ and $\vec{v_2} = 90°$

$-\vec{v_1} = 50\,\text{km h}^{-1}$ due South

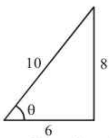
$\therefore$ Change in velocity

$= |\vec{v_2} - \vec{v_1}| = |\vec{v_2} + (-\vec{v_1})|$

$= \sqrt{v_2^2 + v_1^2} = \sqrt{50^2 + 50^2} = 70.7$ km/h

The direction of this change in velocity is in South-West.

14. (b) $\vec{v} = 6\hat{i} + 8\hat{j}$



Comparing with $\vec{v} = v_x\hat{i} + v_y\hat{j}$, we get

$v_x = 6\text{ms}^{-1}$ and $v_y = 8\text{ ms}^{-1}$

Also, $v^2 = v_x^2 + v_y^2 = 36 + 64 = 100$

or $v = 10\text{ ms}^{-1}$

$\sin\theta = \frac{8}{10}$ and $\cos\theta = \frac{6}{10}$

$R = \frac{v^2\sin 2\theta}{g} = \frac{2v^2\sin\theta\cos\theta}{g}$

$R = 2\times10\times10\times\frac{8}{10}\times\frac{6}{10}\times\frac{1}{10} = 9.6\text{m}$

15. (d) $s = t^3 + 5$

$\Rightarrow$ velocity, $v = \frac{ds}{dt} = 3t^2$

Tangential acceleration $a_t = \frac{dv}{dt} = 6t$

Radial acceleration $a_c = \frac{v^2}{R} = \frac{9t^4}{R}$

At $t = 2s$, $a_t = 6\times2 = 12\text{ m/s}^2$

$a_c = \frac{9\times16}{20} = 7.2\text{ m/s}^2$

$\therefore$ Resultant acceleration

$= \sqrt{a_t^2 + a_c^2} = \sqrt{(12)^2 + (7.2)^2} = \sqrt{144 + 51.84}$

$= \sqrt{195.84} = 14\text{ m/s}^2$

16. (b) $\frac{B}{2} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ ...... (i)

$\therefore$ $\tan 90° = \frac{B\sin\theta}{A + B\cos\theta}$ $\Rightarrow A + B\cos\theta = 0$

$\therefore$ $\cos\theta = -\frac{A}{B}$

Hence, from (i) $\frac{B^2}{A} = A^2 + B^2 - 2A^2 \Rightarrow A = \sqrt{3}\frac{B}{2}$

$\Rightarrow$ $\cos\theta = -\frac{A}{B} = -\frac{\sqrt{3}}{2}$ $\therefore \theta = 150°$

17. (b) Suppose velocity of rain

$\vec{v}_R = v_x\hat{i} - v_y\hat{j}$

and the velocity of the man

$\vec{v}_m = u\hat{i}$

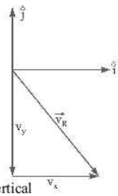
$\therefore$ Velocity of rain relative to man

$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m = (v_x - u)\hat{i} - v_y\hat{j}$

According to given condition that rain appears to fall vertically, so $(v_x - u)$ must be zero.

$\therefore$ $v_x - u = 0$ or $v_x = u$

When he doubles his speed,



$\vec{v}'_m = 2u\hat{i}$

Now $\vec{v}_{Rm} = \vec{v}_R - \vec{v}'_m$

$= (v_x\hat{i} - v_y\hat{j}) - (2u\hat{i})$

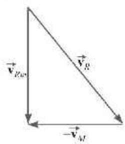
$= (v_x - 2u)\hat{i} - v_y\hat{j}$

The $\vec{v}_{Rm}$ makes an angle θ with the vertical

$\tan\theta = \frac{x - \text{componend of } \vec{v}_{Rm}}{y - \text{componend of } \vec{v}_{Rm}}$

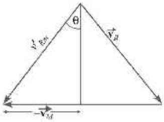
$= \frac{(v_x - 2u)}{-v_y}$

$= \frac{u - 2u}{-v_y}$



which gives

$$v_y = \frac{u}{\tan\theta}$$



Thus the velocity of rain

$$\vec{\mathbf{v}}_R = v_x\hat{\mathbf{i}} - v_y\hat{\mathbf{i}}$$

$$= u\,\hat{\mathbf{i}} - \frac{u}{\tan\theta}\hat{\mathbf{j}}.$$

18. (c) For projectile A

Maximum height, $H_A = \frac{u_A^2 \sin^2 45°}{2g}$

For projectile B

Maximum height, $H_B = \frac{u_B^2 \sin^2 \theta}{2g}$

As we know, $H_A = H_B$

$$\frac{u_A^2 \sin^2 45°}{2g} = \frac{u_B^2 \sin^2 \theta}{2g}$$

$$\frac{\sin^2\theta}{\sin^2 45°} = \frac{u_A^2}{u_B^2}$$

$$\sin^2\theta = \left(\frac{u_A}{u_B}\right)^2 \sin^2 45°$$

$$\sin^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{4}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

19. (a) The angle for which the ranges are same is complementary.

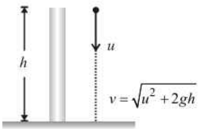
Let one angle be θ, then other is $90° - \theta$

$$T_1 = \frac{2u\sin\theta}{g}, T_2 = \frac{2u\cos\theta}{g}$$

$$T_1T_2 = \frac{4u^2\sin\theta\cos\theta}{g} = 2R \quad \left(\because R = \frac{u^2\sin^2\theta}{g}\right)$$

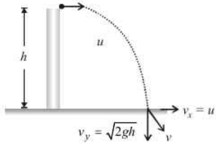
Hence it is proportional to R.

20. (c) When particle thrown in vertically downward direction with velocity u then final velocity at the ground level



$$v^2 = u^2 + 2gh \;\therefore\; v = \sqrt{u^2 + 2gh}$$

Another particle is thrown horizontally with same velocity then velocity of particle at the surface of earth.



Horizontal component of velocity $v_x = u$

$\therefore$ Resultant velocity, $v = \sqrt{u^2 + 2gh}$

For both the particles, final velocities when they reach the earth's surface are equal.

21. (b) $\hat{r} = 0.5\hat{i} + 0.8\hat{j} + c\hat{k}$

$$|\hat{r}| = 1 = \sqrt{(0.5)^2 + (0.8)^2 + c^2}$$

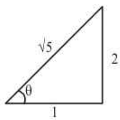
$$(0.5)^2 + (0.8)^2 + c^2 = 1$$

$$c^2 = 0.11 \Rightarrow c = \sqrt{0.11}$$

22. (c) The pilot will see the ball falling in straight line because the reference frame is moving with the same horizontal velocity but the observer at rest will see the ball falling in parabolic path.

23. (a) R = 2H (given)

We know, $R = 4H\cot\theta \Rightarrow \cot\theta = \frac{1}{2}$



From triangle we can say that $\sin\theta = \frac{2}{\sqrt{5}}, \cos\theta = \frac{1}{\sqrt{5}}$

$\therefore$ Range of projectile $R = \frac{2v^2\sin\theta\cos\theta}{g}$

$$= \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$$

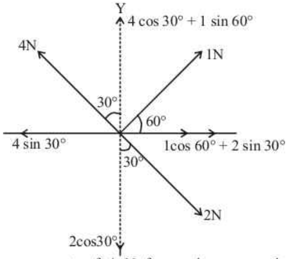
24. (a) Note that the given angles of projection add upto 90°. For complementary angles of projection $(45° + \alpha)$ and $(45° - \alpha)$ with same initial velocity u, range R is same.

$$\theta_1 + \theta_2 = (45° + \alpha) + (45° - \alpha) = 90°$$

So, the ratio of horizontal ranges is 1 : 1.

25. (a) The components of 1 N and 2N forces along + x axis = 1 cos 60° + 2 sin 30°

$$= 1 \times \frac{1}{2} + 2 \times \frac{1}{2} = \frac{1}{2} + 1 = \frac{3}{2} = 1.5\text{N}$$



The component of 4 N force along –x-axis $= 4\sin 30° = 4\times\frac{1}{2} = 2\text{N}$.

Therefore, if a force of 0.5N is applied along + x-axis, the resultant force along x-axis will become zero and the resultant force will be obtained only along y-axis.

26. (d) $F_x = \frac{dp_x}{dt} = -2\sin\theta.$

Similarly, $F_y = \frac{dp_y}{dx} = 2\cos\theta.$

Angle θ between two vectors

$$\cos\theta = \frac{F_xp_x + F_yp_y}{|\vec{F}||\vec{p}|}$$

$$= \frac{(-2\sin\theta)(2\cos\theta)+(2\cos\theta)(2\sin\theta)}{|\vec{F}||\vec{p}|}$$

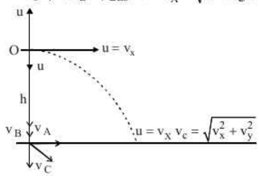
$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°$

27. (a) The motion of the train will affect only the horizontal component of the velocity of the ball. Since, vertical component is same for both observers, the y_m will be same, but R will be different.

28. (d) As body covers equal angle in equal time intervals. Its angular velocity and hence magnitude of linear velocity is constant.

29. (a) **For A:** It goes up with velocity *u* will it reaches its maximum height (i.e. velocity becomes zero) and comes back to O and attains velocity *u*.

Using $v^2 = u^2 + 2as \Rightarrow v_A = \sqrt{u^2+2gh}$



For B, going down with velocity u

$\Rightarrow v_B = \sqrt{u^2+2gh}$

For C, horizontal velocity remains same, i.e. u. Vertical velocity $= \sqrt{0+2gh} = \sqrt{2gh}$

The resultant $v_C = \sqrt{v_x^2+v_y^2} = \sqrt{u^2+2gh}$.

Hence $v_A = v_B = v_C$

30. (d) $\vec{v}_{av} = \frac{\Delta\vec{r}\text{ (displacement)}}{\Delta t\text{ (time taken)}}$

$$= \frac{(13-2)\hat{i}+(14-3)\hat{j}}{5-0} = \frac{11}{5}(\hat{i}+\hat{j})$$

31. (c) Position vector

$\vec{r} = \cos wt\ \hat{x} + \sin\omega t\ \hat{y}$

$\therefore$ Velocity, $\vec{v} = -\omega\sin\omega t\ \hat{x} + \omega\cos\omega t\ \hat{y}$

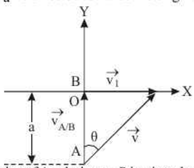
and acceleration,

$\vec{a} = -\omega^2\cos\omega t\ \hat{x} + \omega\sin\omega t\ \hat{y} = -\omega 2\ \vec{r}$

$\vec{r}\cdot\vec{v} = 0$ hence $\vec{r}\perp\vec{v}$ and

$\vec{a}$ is directed towards the origin.

32. (d)



Velocity of A relative to *B* is given by

$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B = \vec{v} - \vec{v}_1$ ....(1)

By taking x-components of equation (1), we get

$0 = v\sin\theta - v_1 \Rightarrow \sin\theta = \frac{v_1}{v}$ ....(2)

By taking *Y*-components of equation (1), we get

$v_y = v\cos\theta$ .....(3)

Time taken by boy at *A* to catch the boy at *B* is given by

$$t = \frac{\text{Relative displacement along Y - axis}}{\text{Relative velocity along Y -axis}}$$

$$= \frac{a}{v\cos\theta} = \frac{a}{v.\sqrt{1-\sin^2\theta}} = \frac{a}{v.\sqrt{1-\left(\frac{v_1}{v}\right)^2}}$$

[From equation (1)]

$$= \frac{a}{v.\sqrt{\frac{v^2-v_1^2}{v^2}}} = \frac{a}{\sqrt{v^2-v_1^2}} = \sqrt{\frac{a^2}{v^2-v_1^2}}$$

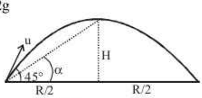
33. (b) $H = \frac{u^2\sin^2 45°}{2g} = \frac{u^2}{4g}$ ...(1)

$$R = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g}$$

$$\therefore \frac{R}{2} = \frac{u^2}{2g} \quad ...(2)$$

$$\therefore \tan\alpha = \frac{H}{R/2}$$

$$= \frac{\frac{u^2}{4g}}{\frac{u^2}{2g}} = \frac{1}{2} \qquad \therefore \alpha = \tan^{-1}\left(\frac{1}{2}\right)$$



34. (b) Here, $x = 4\sin(2\pi t)$ ...(*i*)

$y = 4\cos(2\pi t)$ ...(*ii*)

Squaring and adding equation (*i*) and (*ii*)

$x^2 + y^2 = 4^2 \Rightarrow R = 4$

Motion of the particle is circular motion, acceleration vector is along $-\vec{R}$ and its magnitude $= \frac{v^2}{R}$

Velocity of particle, $v = \omega R = (2\pi)(4) = 8\pi$

35. (d) $|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$

$|\vec{A} + \vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2\vec{A}.\vec{B} = A^2 + B^2 + 2AB\cos\theta$

$|\vec{A} - \vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 - 2\vec{A}.\vec{B}$

$= A^2 + B^2 - 2AB\cos\theta$

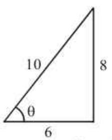
So, $A^2 + B^2 + 2AB\cos\theta$
$= A^2 + B^2 - 2AB\cos\theta$

$4AB\cos\theta = 0 \Rightarrow \cos\theta = 0$

$\therefore \theta = 90°$

So, angle between A & B is 90°.

36. (b) $\vec{v} = 6\hat{i} + 8\hat{j}$



Comparing with $\vec{v} = v_x\hat{i} + v_y\hat{j}$, we get

$v_x = 6\,ms^{-1}$ and $v_y = 8\,ms^{-1}$

Also, $v^2 = v_x^2 + v_y^2 = 36 + 64 = 100$

or $v = 10\ ms^{-1}$

$\sin\theta = \frac{8}{10}$ and $\cos\theta = \frac{6}{10}$

$$R = \frac{v^2 \sin 2\theta}{g} = \frac{2v^2 \sin\theta\cos\theta}{g}$$

$$R = 2 \times 10 \times 10 \times \frac{8}{10} \times \frac{6}{10} \times \frac{1}{10} = 9.6\,m$$

37. (a) Range of a projectile is maximum when it is projected at an angle of 45° and is given by

$R_{max} = \frac{u^2}{g}$, where u is the velocity of projection

$\Rightarrow R = \frac{u^2}{g} \qquad \therefore u^2 = Rg \quad ...(i)$

Now, to hit a target at a distance (R/2) from the gun, we must have

$\frac{R}{2} = \frac{u^2 \sin 2\theta}{g}$, where θ is the angle of projection.

$\Rightarrow \frac{R}{2} = \frac{Rg\sin 2\theta}{g}$; from (i)

$\Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow \sin 2\theta = \sin 30°$

$\Rightarrow 2\theta = 30° \quad \therefore \theta = 15°$

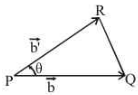
38. (a) Distance covered in one circular loop $= 2\pi r$
$= 2 \times 3.14 \times 100 = 628$ m

Speed $= \frac{628}{62.8} = 10$ m/sec

Displacement in one circular loop = 0

Velocity $= \frac{0}{\text{time}} = 0$

39. (a) $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$



$\therefore \overrightarrow{QR} = \vec{b'} - \vec{b}$

Now $|\vec{b'} - \vec{b}|^2 = (\vec{b'} - \vec{b}).(\vec{b'} - \vec{b})$

$= b'^2 - 2bb'\cos\theta + b^2$

$= 2b^2(1 - \cos\theta) \qquad [\because b' = b]$

$\vec{b'} - \vec{b} = \sqrt{2}b\sqrt{1 - \cos\theta}$

$= \sqrt{2}b\left(\sqrt{2}\sin\frac{\theta}{2}\right) = 2b\sin\frac{\theta}{2}$

40. (a) $H_1 = \frac{u^2 \sin^2\theta}{2g}$

and $H_2 = \frac{u^2 \sin^2(90° - \theta)}{2g} = \frac{u^2 \cos^2\theta}{2g}$

$H_1H_2 = \frac{u^2\sin^2\theta}{2g} \times \frac{u^2\cos^2\theta}{2g} = \frac{(u^2\sin 2\theta)^2}{16g^2} = \frac{R^2}{16}$

$\therefore R = 4\sqrt{H_1H_2}$

41. (c) $\vec{P}$ = vector sum $= \vec{A} + \vec{B}$

$\vec{Q}$ = vector differences $= \vec{A} - \vec{B}$

Since $\vec{P}$ and $\vec{Q}$ are perpendicular

$\therefore\ \vec{P}\,.\,\vec{Q} = 0$

$\Rightarrow\ (\vec{A}+\vec{B}).(\vec{A}-\vec{B}) = 0\ \Rightarrow A^2 = B^2 \Rightarrow |A| = |B|$

42. **(b)** $y = bx^2$

Differentiating w.r.t to t an both sides, we get

$$\frac{dy}{dx} = b2x\frac{dx}{dt}$$

$v_y = 2bxv_x$

Again differentiating w.r.t to t on both sides we get

$$\frac{dv_y}{dt} = 2bv_x\frac{dx}{dt} + 2bx\frac{dv_x}{dt} = 2bv_x^2 + 0$$

[$\frac{dv_x}{dt}$ = 0, because the particle has constant acceleration along y-direction]

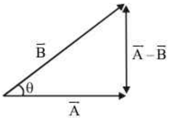
Now, $\frac{dv_y}{dt} = a = 2bv_x^2$;

$$v_x^2 = \frac{a}{2b}$$

$$v_x = \sqrt{\frac{a}{2b}}$$

43. **(a)** Arc length = radius × angle

So, $|\overline{B}-\overline{A}| = |\overline{A}|\,\Delta\theta$



44. **(c)** Speed, V = constant (from question)

Centripetal acceleration,

$$a = \frac{V^2}{r}$$

ra = constant

Hence graph (c) correctly describes relation between acceleration and radius.

45. **(c)** From question,

Horizontal velocity (initial),

$$u_x = \frac{40}{2} = 20\,\text{m/s}$$

Vertical velocity (initial), $50 = u_y t + \frac{1}{2}gt^2$

$\Rightarrow\ u_y \times 2 + \frac{1}{2}(-10)\times 4$

or, $50 = 2u_y - 20$

or, $u_y = \frac{70}{2} = 35\,\text{m/s}$

$\therefore\ \tan\theta = \frac{u_y}{u_x} = \frac{35}{20} = \frac{7}{4}$

$\Rightarrow$ Angle $\theta = \tan^{-1}\frac{7}{4}$