16

Oscillations

IN THIS CHAPTER ....

Periodic Motion

A motion that repeats itself over and over again after a regular interval of time is called a periodic motion. The regular interval of time after which the periodic motion is repeated is called its *time period*. Revolution of the Earth around the Sun, rotation of the Earth about its axis are the common examples of periodic motion.

Oscillatory Motion

A special type of periodic motion in which a particle moves to and fro (back and forth or up and down) about a fixed point after regular interval of time is termed as oscillatory or vibratory motion. The fixed point about which the body oscillates is called *mean position* or *equilibrium position*. Thus, a periodic or bounded motion of a body about a fixed point is called an oscillatory or vibratory motion. Examples of oscillatory motion are simple pendulum, spring pendulum, etc.

A body that undergoes oscillatory motion always have a stable equilibrium position (where net force on the body is zero). As the body is displaced from its mean/equilibrium position, a force (torque) comes into existence, which tends to bring the body back to the equilibrium position, this force or torque is termed as the *restoring force* (*torque*).

Note There is no significant difference between oscillations and vibrations. When to and fro motion of the body about a fixed position has small frequency, we call it oscillation such as the oscillation of a simple pendulum.
When to and fro motion of the body about a fixed position has high frequency, we call it as vibrations such as oscillation of a musical instrument.

Periodic Functions

Those functions which are used to represent periodic motion are known as periodic functions.

A function $f(t)$ is said to be periodic, if $f(t) = f(t+T) = f(t+2T)$ ...(i)

$\because$ sine and cosine functions are example of periodic functions.

When T is the period of this periodic motion, then for periodic motion,

$$y = A\sin\omega t = A\sin\omega(t+T) \quad \text{...(ii)}$$

and $$x = A\cos\omega t = A\cos\omega(t+T) \quad \text{...(iii)}$$

But the value of sine or cosine functions repeat after a period of 2π radian.

$\therefore \quad \omega(t + T) = \omega t + 2\pi$

or $\quad \omega T = 2\pi \quad$ ...(iv)

or $\quad \omega = 2\pi\nu \quad$ ...(v)

Consider a linear combination of sine and cosine functions are given as

$$x = f(t) = A \sin \omega t + B \cos \omega t$$

Taking, $\quad A = R\cos\phi$ and $B = R\sin\phi$

Then, $x = R\cos\phi \sin \omega t + R\sin\phi\cos\omega t = R\sin(\omega t + \phi) \quad$ ...(vi)

It represents a period function of time period T and amplitude R,

where $\quad R = \sqrt{A^2 + B^2}$ and $\tan\phi = B/A$.

The combination of any number of periodic functions will also be periodic one, whose time period will be minimum of the periodic functions used in the combinations.

Example 1. *A function of time is represented as follows* $\sin \omega t + \cos 2\omega t + \sin 4\omega t$.
The motion represented by it is

(a) *non-periodic*
(b) *periodic*
(c) *both non-periodic and periodic*
(d) *data insufficient*

Sol. (*b*) This is an example of a periodic motion. It can be noted that each term represents a periodic function with different angular frequency. Since period is the least interval of time after which a function repeats its value, $\sin \omega t$ has a period $T_0 = \frac{2\pi}{\omega}$,

$$\cos 2\omega t \text{ has a period } \frac{\pi}{\omega} = \frac{T_0}{2}$$

and $$\sin 4\omega t \text{ has a period } \frac{2\pi}{4\omega} = \frac{T_0}{4}$$

The period of the first term is a multiple of the periods of the last two terms. Therefore, the smallest interval of time after which the sum of the three terms repeats is T_0 and thus, the sum is a periodic function with a period $\frac{2\pi}{\omega}$.

Example 2. *Among the following, what is the time period of* $\sin^2 \omega t$?

(a) $\frac{\omega}{\pi}$ (b) $\frac{2\omega}{\pi}$ (c) $\frac{\pi}{\omega}$ (d) $\frac{\pi}{2\omega}$

Sol. (*c*) Given, $\sin^2 \omega t$

Using the trigonometric identity,

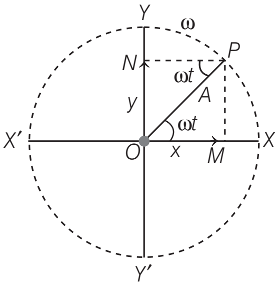
$$\cos 2\theta = 1 - 2\sin^2\theta$$

We have, $$\sin^2 \omega t = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$$

The function is periodic having a period $T = \frac{\pi}{\omega}$. It also represents a harmonic motion with the point of equilibrium occuring at $\frac{1}{2}$ instead of zero.

Simple Harmonic Motion

The type of oscillatory motion in which the particle moves to and fro in a straight line about a fixed point under a restoring force (or torque) whose magnitude is directly proportional to its displacement (or angular displacement), is known as simple harmonic motion.



A simple harmonic motion may also be considered as the projection of a uniform circular motion on any diameter of the circle. As shown in figure, let us consider uniform circular motion in a plane with constant angular velocity ω and let at an instant t the particle executing circular motion be at point P making an angle $\theta = \omega t$ from X-axis, then the projection of this circular motion along a diameter YY' is given by,

$$y = A \sin \omega t$$

and projection along diameter XX' is given by

$$x = A \cos \omega t$$

Such a motion is a simple harmonic motion. The radius A of the reference circle is the maximum value of displacement (to and fro motion about the mean position O) and is known as the *amplitude of SHM.*

Terms Related to SHM

- **Displacement** Displacement of a particle in the case of SHM is always measured from the mean position.
  - (a) If the particle is at the mean position at $t = 0$, then displacement is given by $x = A \sin \omega t$.
  - (b) If the particle is at the extreme position at $t = 0$, then displacement is given by $x = A\cos\omega t$.
  - (c) In general displacement is given by $x = A\sin(\omega t + \phi)$, where ϕ is the *initial phase* or *phase constant*.
- **Time period** The time taken by the particle to complete one oscillation is called *time period of oscillation*. It is denoted by T and given by $T = \frac{2\pi}{\omega}$.
- **Frequency** The number of oscillations made by the particle in one second is called *frequency of oscillations*. It is denoted by ν or n. Thus, $\nu = \frac{1}{T} = \frac{\omega}{2\pi}$. ω being the angular frequency of the oscillating particle.
- **Velocity** As, $x = A\sin(\omega t + \phi)$

$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$$

$$= A\omega\,[1 - \sin^2(\omega t + \phi)]^{1/2} \quad \text{(In terms of time)}$$

$$= A\omega\left(1 - \frac{x^2}{A^2}\right)^{1/2}$$

So, $$v = \omega\sqrt{A^2 - x^2}$$

(In terms of displacement from mean position)

This is the required expression of velocity of the particle executing SHM.

(a) At mean position ($x = 0$) velocity is maximum, *i.e.* $v_{\max} = A\omega$.

(b) At extreme positions ($x = \pm A$), velocity is zero.

- **Acceleration** Acceleration is

$$a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \phi) = -\omega^2 x$$

(a) Acceleration is zero at the mean position, *i.e.* $a = 0$, at $x = 0$.

(b) Acceleration is maximum at extreme position, *i.e.* $a_{\max} = -\omega^2 A$ at $x = A$.

In above expressions negative sign indicate that in SHM, the acceleration is proportional to the displacement but is in opposite.

Also in linear SHM, the force and displacement are related by

$$F = -(\text{ a positive constant })\, x$$

which says that the force is proportional to the displacement but is in opposite direction.

- **Phase relationship between displacement, velocity and acceleration of SHM**

The term $(\omega t + \phi)$ is called phase and ϕ is called phase constant.

As, we have seen that

$$x = A\sin(\omega t + \phi)$$

$$v = A\omega\cos(\omega t + \phi) \quad \text{...(i)}$$

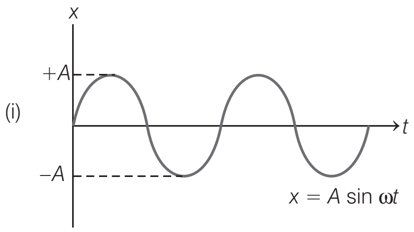
$$= A\omega\sin\left(\omega t + \phi + \frac{\pi}{2}\right)$$

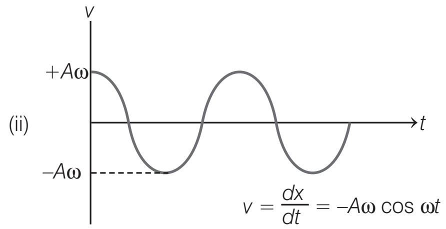
and $$a = -A\omega^2 \sin(\omega t + \phi)$$

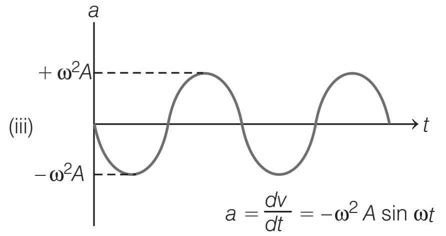
$$= A\omega^2 \sin(\omega t + \phi + \pi) \quad \text{...(ii)}$$

Thus from Eqs. (i) and (ii), we conclude that in SHM, particle velocity is ahead in phase to the displacement by $\pi/2$ and acceleration is further ahead in phase by $\pi/2$.

In figure, x, v and a as functions of time are illustrated.







Example 3. *The displacement of a particle executing periodic motion is given by*

$$y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$$

Find independent constituents of SHMs.

(a) $y_1 = 2\sin 1000\,t,\ y_2 = \sin 1001t,\ y_3 = \sin 999\,t$

(b) $y_1 = 3\sin 1000\,t,\ y_2 = \sin 1000\,t,\ y_3 = \sin 899\,t$

(c) $y_1 = \sin 1001t,\ y_2 = \sin 999\,t,\ y_3 = 2\sin 1000\,t$

(d) None of the above

Sol. (a) $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$

$$= 2(1 + \cos t)\sin(1000t)$$

$$= 2\sin(1000t) + 2\cos t\sin(1000t)$$

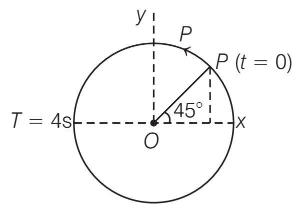
$$= 2\sin(1000t) + \sin(1001t) + \sin(999t)$$

Thus, the given periodic motion is a combination of three independent SHMs, which are given by

$$y_1 = 2\sin 1000t,\ y_2 = \sin 10001t$$

and $$y_3 = 999\,t$$

Example 4. *The following figure depicts circular motion. The radius of the circle, the period of revolution, the initial position and sense of revolution are indicated in the figure.*



The simple harmonic motion of the x-projection of the radius vector of the rotating particle P is as follows

(a) $x(t) = A\cos\left(\frac{2\pi}{4}t + \frac{\pi}{4}\right)$ (b) $x(t) = A\cos\left(\frac{\pi}{4}t - \frac{2\pi}{4}\right)$

(c) $x(t) = A\sin\left(\frac{2\pi}{4}t + \frac{\pi}{2}\right)$ (d) $x(t) = A\sin\left(\frac{\pi}{4}t - \frac{\pi}{2}\right)$

Sol. (a) At $t = 0$, OP makes an angle of $45° = \frac{\pi}{4}$ rad with the positive direction of X-axis. After time t, it covers an angle $\frac{2\pi}{T}t$ in the anti-clockwise sense, and makes an angle of $\frac{2\pi}{T}t + \frac{\pi}{4}$ with the X-axis.

The projection of OP on the X-axis at time t is given by

$$x(t) = A\cos\left(\frac{2\pi}{T}t + \frac{\pi}{4}\right)$$

For, $T = 4$ s $\quad x(t) = A\cos\left(\frac{2\pi}{4}t + \frac{\pi}{4}\right)$

which is a SHM of amplitude A, period 4 s and an initial phase $= \frac{\pi}{4}$.

Example 5. *The periodic time of a body executing SHM is 2 s. After what interval from t = 0, will its displacement be half of its amplitude?*

(a) (1/5) *s* (b) (1/6) *s* (c) (1/10) *s* (d) (1/8) *s*

Sol. (b) Given, $T = 2$ s, $t = ?$; $x = \frac{A}{2}$

Now, $\quad x = A\sin\omega t = A\sin\frac{2\pi}{T}t$

$\therefore \quad \frac{A}{2} = A\sin\frac{2\pi}{2}t = A\sin\pi t$

$\Rightarrow \quad \sin\pi t = \sin 30° = \sin\frac{\pi}{6}$

or $\quad \pi t = \frac{\pi}{6}$ or $t = \frac{1}{6}$ s

Example 6. *A particle in SHM is described by the displacement function.*

$$x = A\cos(\omega t + \phi), \omega = 2\pi/T$$

If the initial (t = 0) *position of the particle is* 1 *cm and its initial velocity is* π *cm* s^{-1}, *what is the initial phase angle?* (*The angular frequency of the particle is* π s^{-1}.)

(a) $3\pi/4$ (b) $2\pi/4$ (c) $5\pi/4$ (d) $7\pi/4$

Sol. (a) Here, at $t = 0$, $x = 1$ cm and $v = \pi$ cms^{-1}; $\phi = ?$; $\omega = \pi$ s^{-1}

Given, $\quad x = A\cos(\omega t + \phi)$

$1 = A\cos(\pi \times 0 + \phi)$

$1 = A\cos\phi \quad$ ...(i)

Velocity, $v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$

$\therefore \quad \pi = -A\pi\sin(\pi \times 0 + \phi) = -A\pi\sin\phi$

or $\quad 1 = -A\sin\phi$

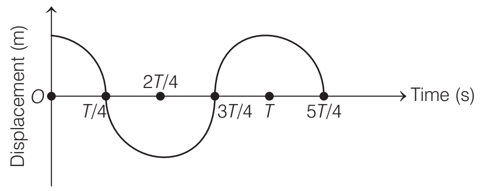
or $\quad A\sin\phi = -1 \quad$ ...(ii)

Dividing Eq. (ii) by (i), we get

$$\tan\phi = -1 = -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$$

or $\quad \phi = \frac{3\pi}{4}$

Example 7. *The displacement-time graph of a particle executing SHM is given in figure (sketch is schematic and not to scale).* **[JEE Main 2020]**



Which of the following statement(s) is/are true for this motion?

A. *The force is zero at* $t = \frac{3T}{4}$.

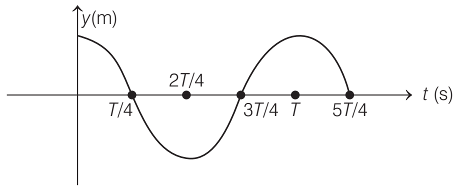
B. *The acceleration is maximum at* $t = T$.

C. *The speed is maximum at* $t = \frac{T}{4}$.

D. *The potential energy is equal to kinetic energy of the oscillation at* $t = \frac{T}{2}$.

(a) *A, B and D* (b) *B, C and D*
(c) *A, B and C* (d) *A and D*

Sol. (c) The displacement-time graph is as shown below,



At $t = \frac{3T}{4}$, particle is at mean position. At mean position of SHM, acceleration of particle is zero. *i.e.* force is zero.

Statement (A) is correct.

At $t = T$, particle is at extreme position. As direction is changing, so its acceleration is maximum. Statement (B) is correct.

At $t = \frac{T}{4}$, particle is at mean position, so whole of energy is kinetic. *i.e.* its speed is maximum. Statement (C) is correct.

PE = KE at $t = \frac{T}{8}$, so statement (D) is incorrect.

Hence, option (c) is correct.

Example 8. *If two SHMs are represented by equations* $y_1 = 10\sin\left(3\pi t + \frac{\pi}{4}\right)$ *and* $y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$, *then the ratio of their amplitudes is*

(a) 1 : 2 (b) 2 : 1
(c) 1 : 1 (d) 1 : 3

Sol. (c) Here, $y_1 = 10\sin\left(3\pi t + \frac{\pi}{4}\right)$

But $y_2 = 5(\sin 3\pi + \sqrt{3}\cos 3\pi t)$ does not appear to be a single compact function of time t. We therefore, assume that

$$5 = A\cos\phi \quad \text{and} \quad 5\sqrt{3} = A\sin\phi$$

Thus, $A = \sqrt{(5)^2 + (5\sqrt{3})^2} = 10$

and $\tan \phi = \sqrt{3}$

or $\phi = \frac{\pi}{3}$

then, $y_2 = A \cos \phi \sin 3\pi t + A \sin \phi \cos 3\pi t$

$= A \sin (3\pi t + \phi)$

$= 10 \sin \left(3\pi t + \frac{\pi}{3} \right)$

Now, we find that $A_1 = 10$ and $A_2 = 10$

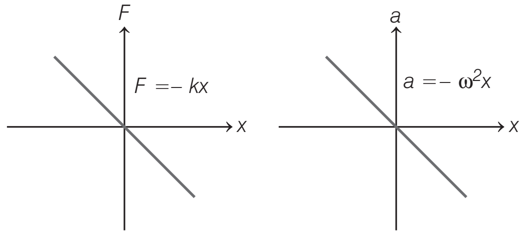
or $A_1 : A_2 = 10 : 10 = 1 : 1$

Differential Equations of SHM

For linear SHM, $\frac{d^2x}{dt^2} + \omega^2 x = 0$

or $a = -\omega^2 x$

In SHM, $F = -kx$ or $a = -\omega^2 x$, *i.e.* F-x graph or a-x graph is a straight line passing through the origin with negative slope. The corresponding graphs are shown below.



(a) Slope = – k (b) Slope = – ω^2

Energy in SHM

A particle executing SHM possesses two types of energy. If a particle executes SHM, its kinetic energy changes into potential energy and *vice-versa* keeping total energy constant (if friction of air is neglected).

Kinetic energy $K = \frac{1}{2} mv^2$

$$= \frac{1}{2} mA^2\omega^2 \cos^2 (\omega t + \phi) = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

(because $x = A \sin(\omega t + \phi)$)

Potential energy $U = \frac{1}{2} m\omega^2 x^2$

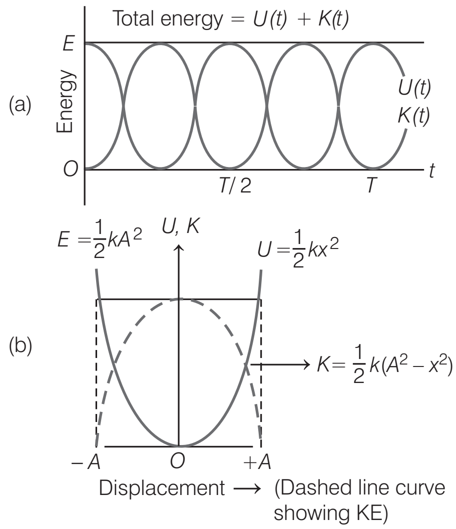
$$= \frac{1}{2} mA^2\omega^2 \sin^2 (\omega t + \phi)$$

Thus, total energy = kinetic energy + potential energy

or $E = \frac{1}{2} m\omega^2 A^2$ (constant)

The total energy is constant and is proportional to the square of amplitude (A) of motion.

Figures show the variations of total energy (E), potential energy (U) and kinetic energy (K) with displacement (x).



Example 9. *A particle executes SHM of amplitude A. At what distance from the mean position is its KE equal to its PE?*

(a) 0.71 A (b) 0.61 A (c) 0.65 A (d) 0.8 A

Sol. (a) As, $\text{KE} = \frac{1}{2} m\omega^2 (A^2 - x^2)$

and $\text{PE} = \frac{1}{2} m\omega^2 x^2$

As, KE = PE

$\therefore$ $\frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} m\omega^2 x^2$

or $A^2 - x^2 = x^2$

or $x^2 = A^2/2$

or $x = A/\sqrt{2} = 0.71A$

Example 10. *A block of mass m attached to a massless spring is performing oscillatory motion of amplitude A on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system becomes fA. The value of f is* **[JEE Main 2020]**

(a) $\sqrt{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

Sol. (c) In the equilibrium position, the velocity is maximum and it is equal to $v_{\text{max}} = \omega A$

where, $\omega = \sqrt{\frac{k}{m}}$ = angular frequency

and k = spring constant.

Now, kinetic energy in equilibrium position,

$$E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} m\left(\frac{k}{m}\right) A^2 \quad \text{...(i)}$$

When half of the mass of the block breaks off, also the kinetic energy of the system will become half, *i.e.* $E/2$.

The angular frequency will become,

$$\omega' = \sqrt{\frac{k}{(m/2)}}$$

Now, the amplitude will become fA.

$\therefore$ The new kinetic energy,

$$E' = \frac{E}{2} = \frac{1}{2}\left(\frac{m}{2}\right)\omega'^2 (fA)^2$$

$$\Rightarrow \quad \frac{E}{2} = \frac{1}{2}\left(\frac{m}{2}\right)\left(\frac{2k}{m}\right)(fA)^2$$

$$\Rightarrow \quad \frac{1}{2}\left[\frac{1}{2}m\left(\frac{k}{m}\right)A^2\right] = \frac{1}{2}\cdot\frac{m}{2}\cdot\frac{2k}{m}\cdot f^2A^2 \qquad \text{[Using Eq. (i)]}$$

$$\Rightarrow \quad 1 = 2f^2 \Rightarrow f = \frac{1}{\sqrt{2}}$$

Example 11. *A particle undergoing simple harmonic motion has time dependent displacement given by* $x(t) = A\sin\frac{\pi t}{90}$. *The ratio of kinetic to potential energy of this particle at* $t = 210$ *s will be* ***[JEE Main 2019]***

(a) 2 (b) $\frac{1}{3}$ (c) $\frac{1}{9}$ (d) 3

Sol. *(b)* Here given, displacement, $x(t) = A\sin\frac{\pi t}{90}$

where A is amplitude of SHM, t is time taken by particle to reach a point where its potential energy $U = \frac{1}{2}kx^2$ and kinetic energy $= \frac{1}{2}k(A^2 - x^2)$ here k is force constant and x is position of the particle.

Potential energy (U) at $t = 210$ s is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2\left(\frac{210}{90}\pi\right)$$

$$= \frac{1}{2}kA^2\sin^2\left(2\pi + \frac{3}{9}\pi\right) = \frac{1}{2}kA^2\sin^2\left(\frac{\pi}{3}\right)$$

Kinetic energy at $t = 210$ s, is

$$K = \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kA^2\left[1 - \sin^2\left(\frac{210\pi}{90}\right)\right]$$

$$= \frac{1}{2}kA^2\cos^2(210\,\pi/90)$$

$$\Rightarrow \quad K = \frac{1}{2}kA^2\cos^2(\pi/3)$$

So, ratio of kinetic energy to potential energy is

$$\frac{K}{U} = \frac{\frac{1}{2}kA^2\cos^2(\pi/3)}{\frac{1}{2}kA^2\sin^2(\pi/3)} = \cot^2(\pi/3) = \frac{1}{3}$$

Angular Simple Harmonic Motion

If the angular displacement of the body at an instant is θ, the resultant torque acting on the body in angular SHM should be

$$\tau = -k\theta$$

If the moment of inertia is I, the angular acceleration is

$$\alpha = \frac{\tau}{I} = -\frac{k}{I}\theta$$

or

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta \qquad \text{...(i)}$$

where

$$\omega = \sqrt{\frac{k}{I}}$$

The Eq. (i) may be integrated in the similar manner and we shall get

$$\theta = \theta_0\sin(\omega t + \delta) \qquad \text{...(ii)}$$

where, θ_0 is the maximum angular displacement on either side.

The angular velocity at time t is

$$\omega = \frac{d\theta}{dt} = \theta_0\omega\cos(\omega t + \delta) \qquad \text{...(iii)}$$

The time period of oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{k}} \qquad \text{...(iv)}$$

and the frequency of oscillation is

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{I}} \qquad \text{...(v)}$$

The quantity $\omega = \sqrt{\frac{k}{I}}$ is the angular frequency.

Energy in Angular SHM

The potential energy is, $U = \frac{1}{2}k\theta^2 = \frac{1}{2}I\omega^2\theta^2$

The kinetic energy is, $K = \frac{1}{2}I\omega^2$

Total energy, $E = U + K$

$$= \frac{1}{2}I\omega^2\theta^2 + \frac{1}{2}I\omega^2$$

From $\theta = \theta_0\sin(\omega t + \delta)$ we have,

$$E = \frac{1}{2}I\omega^2\theta_0^2\sin^2(\omega t + \delta) + \frac{1}{2}I\theta_0^2\omega^2\cos^2(\omega t + \delta)$$

$$= \frac{1}{2}I\omega^2\theta_0^2 \text{ (Constant)}$$

Identifying Angular SHM

When a system undergoes simple harmonic motion, its angular acceleration α and angular displacement θ are related by

$$\alpha = -\text{(a positive constant)}\,\theta$$

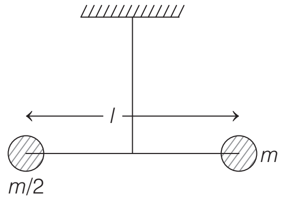
which says that the angular acceleration α is proportional to the angular displacement θ from the equilibrium position but tends to rotate the system in the direction opposite to the displacement.

Also in angular SHM, the torque τ in terms of the angular displacement θ is given by

$$\tau = -\text{(a positive constant)}\,\theta$$

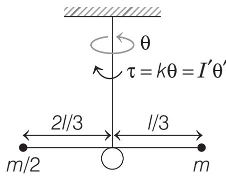
It says that the torque τ is proportional to the angular displacement θ from the equilibrium position but tends to rotate the system in an opposite direction.

Example 12. *Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be* ***[JEE Main 2019]***



(a) $\frac{2k\theta_0^2}{l}$ (b) $\frac{k\theta_0^2}{l}$ (c) $\frac{3k\theta_0^2}{l}$ (d) $\frac{k\theta_0^2}{2l}$

Sol. (*b*) Since in the given question, rotational torque, $\tau \propto$ angular displacement.

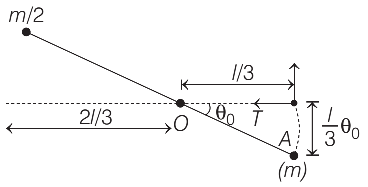


Thus, when it will be released, the system will execute SHM with a time period, $T = 2\pi\sqrt{\frac{I}{k}}$

(Where I is moment of inertia and k is torsional constant)

and the angular frequency is given as, $\omega = \sqrt{\frac{k}{I}}$.

If we now look at the top view of the above figure, we have



At some angular displacement 'θ_0', at point 'A' the maximum velocity will be

$$v_{max} = \frac{l}{3}\theta_0\omega = \frac{l}{3}\theta_0\sqrt{\frac{k}{I}} \quad \text{...(i)}$$

Then, tension in the rod when it passes through mean position will be

$$T = \frac{m \times v_{max}^2}{\frac{l}{3}} = \frac{ml^2\theta_0^2 k \times 3}{9 \times I \times l} \quad \text{[using Eq. (i)]}$$

$$= \frac{ml\,\theta_0^2 k}{3I}$$

The moment of inertia I at point O,

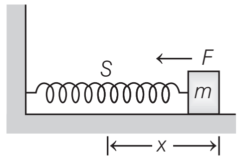
$$= \frac{m}{2}\left(\frac{2l}{3}\right)^2 + m\left(\frac{l}{3}\right)^2$$

$$= \frac{2l^2m}{9} + \frac{ml^2}{9} = \frac{3ml^2}{9} = \frac{ml^2}{3}$$

$$\Rightarrow \quad T = \frac{ml\,\theta_0^2\,k \times 3}{3 \times ml^2} = \frac{\theta_0^2 k}{l} = \frac{k\theta_0^2}{l}$$

Spring Block System

Let a mass m be attached to the free end of a massless spring of spring constant (also known as force constant or spring factor or stiffness) k, with its other end fixed to a rigid support. If the mass be displaced through a distance x and then released, a linear restoring force $F = -kx$ acts on the mass due to elastic nature of the spring.



Force Constant for Spring-Block system

From Newton's second law of motion and the expression for acceleration of a particle undergoing SHM, the force acting on a particle of mass m in SHM is

$$F(t) = ma = -m\omega^2 x(t)$$

i.e. $$F(t) = -kx(t)$$

where, $$k = m\omega^2$$

or $$\omega = \sqrt{\frac{k}{m}}$$

Force is always directed towards mean position and is also called *restoring force* and k is called the *spring constant*, its value is governed by the elastic properties of the spring. A stiff spring has large k and a soft spring has small k.

Example 13. *A 5 kg collar is attached to a spring of spring constant 500 Nm^{-1}. It slides without friction over a horizontal rod. The collar is displaced from its equilibrium position by 10 cm and released. The maximum speed is*

(a) 1 ms^{-1} (b) 5 ms^{-1} (c) 10 ms^{-1} (d) 20 ms^{-1}

Sol. (*a*) The velocity of the collar executing SHM is given by

$$v(t) = -A\omega\sin(\omega t + \phi)$$

The maximum speed is given by

$$v_m = A\omega$$

Given, $$A = 10 \text{ cm} = 0.1 \text{ m},$$

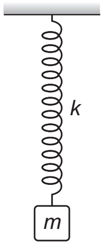
$$\omega = \sqrt{\frac{k}{m}}$$

$$v_m = 0.1 \times \sqrt{\frac{500}{5}}$$

$$v_m = 1 \text{ ms}^{-1}$$

Spring Pendulum

A point mass suspended from a massless (or light) spring constitutes a spring pendulum. If the mass is once pulled downward so as to stretch the spring and then released, the system oscillates up and down about its mean position simple harmonically. Time period and frequency of oscillations are given by



$$T = 2\pi\sqrt{\frac{m}{k}}$$

or
$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

If the spring is not light but has a definite mass m_s, then it can be easily shown that period of oscillation will be

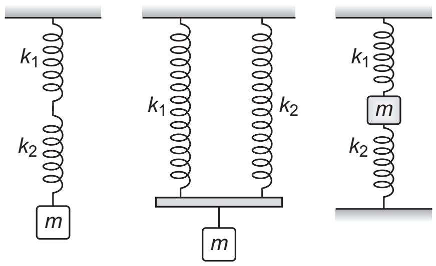
$$T = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$$

Oscillations of Spring Combination

For different combinations of spring block system, following cases occurs

- If a spring pendulum is constructed by using two springs in series and a mass m as shown in Fig. (a), the resultant spring constant of the combination is given by

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1 + k_2}{k_1 k_2}, \text{ and hence}$$
$$T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{(k_1 + k_2)}{k_1 k_2}}$$



(a) (b) (c)

- If in a spring pendulum, two springs are joined in parallel arrangement as shown in Fig. (b) and (c), then

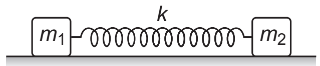
$$k_p = k_1 + k_2, \text{ and hence, } T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

These rules are applicable for horizontal motion of spring mass systems too.

- The force constant of a spring is inversely proportional to its length, *i.e.* $k \propto \frac{1}{l}$. Thus, if we change the length of a spring, the time of oscillation of spring pendulum will change. As an example, if a spring is cut into two parts of equal length and a pendulum is prepared by using one part of spring and some mass m, then the new time period will be

$$T' = 2\pi\sqrt{\frac{m}{2k}} = \frac{T}{\sqrt{2}}$$

- If two masses m_1 and m_2 are connected by a spring as shown in figure and the arrangement is made to oscillate on a horizontal surface, then time period is given by



$$T' = 2\pi\sqrt{\frac{\mu}{k}}$$

where, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass.

Example 14. *A block with a mass of 2 kg hangs without vibrating at the end of a spring of spring constant 800 Nm^{-1}, which is attached to the ceiling of an elevator. The elevator is going upwards with an acceleration g/3. At a certain instant, the acceleration suddenly ceases and elevator starts moving with constant velocity. What is the angular frequency of oscillation of block when the acceleration ceases?*

(a) 12 *rad* s^{-1} (b) 20 *rad* s^{-1} (c) 21 *rad* s^{-1} (d) 19 *rad* s^{-1}

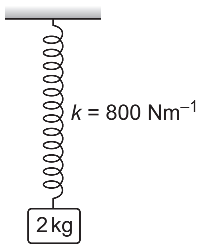
Sol. (*b*) Angular frequency, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{2}} = 20 \text{ rads}^{-1}$

Example 15. *In the above example, by what amount is the spring stretched during the time when the elevator is accelerating?*

(a) 2.3 *cm* (b) 3.3 *cm*
(c) 5.3 *cm* (d) 4.4 *cm*

Sol. (*b*) When the elevator is accelerating upwards with an acceleration $a = g/3$, the equation of motion of the block is

$$\left(ky - mg = ma = m\frac{g}{3}\right)$$



$$\Rightarrow \quad ky = mg + \frac{mg}{3} = \frac{4}{3}mg$$

$$\Rightarrow \quad y = \frac{4mg}{3k} = \frac{4 \times 2 \times 10}{3 \times 800} = 0.033 \text{ m or } 3.3 \text{ cm}$$

Example 16. *In the above example, what is the amplitude of oscillation? (Take, $g = 10\ ms^{-2}$)*

(a) 0.1 cm (b) 0.7 cm (c) 0.8 cm (d) 0.5 cm

Sol. (c) In equilibrium, when the elevator has no acceleration, the equation of motion is

$$ky_0 = mg$$

$$\Rightarrow \quad y_0 = \frac{mg}{k} = \frac{2 \times 10}{800} = 0.025 \text{ m} \quad \text{or } 2.5 \text{ cm}$$

$\therefore$ Amplitude of oscillation,

$$A = y - y_0 = 3.3 - 2.5 = 0.8 \text{ cm}$$

Example 17. *A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1 / k_2 of the corresponding force constants k_1 and k_2 will be* **[JEE Main 2019]**

(a) n (b) $\frac{1}{n^2}$ (c) $\frac{1}{n}$ (d) n^2

Sol. (c) If parameters like material, number of loops per unit length, area of cross-section, etc., are kept same, then force constant of spring is inversely proportional to its length.

In given case, all other parameters are same for both parts of spring.

So, $$k_1 \propto \frac{1}{l_1} \text{ and } k_2 \propto \frac{1}{l_2}$$

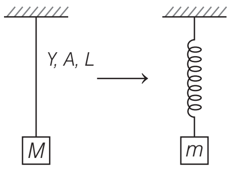
$$\therefore \quad \frac{k_1}{k_2} = \frac{l_2}{l_1} = \frac{l_2}{nl_2} = \frac{1}{n} \qquad [\because l_1 = nl_2]$$

Example 18. *An object of mass m is suspended at the end of a massless wire of length L and area of cross-section A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is* **[JEE Main 2020]**

(a) $f = \frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$ (b) $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

(c) $f = \frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$ (d) $f = \frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$

Sol. (b)



As we know that an elastic wire behaves as an elastic spring with spring constant K given by,

$$K = \frac{YA}{L}$$

This block-wire system can be taken as spring block system with time period of small oscillation given by

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{mL}{YA}}$$

$\therefore$ Frequency of oscillation, $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

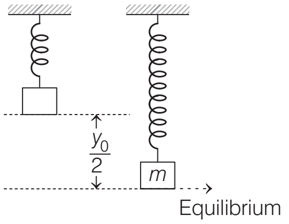
Example 19. *When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t) = y_0 \sin^2 \omega t$, where y is measured from the lower end of unstretched spring. Then ω is* **[JEE Main 2020]**

(a) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$ (b) $\sqrt{\frac{g}{y_0}}$ (c) $\sqrt{\frac{g}{2y_0}}$ (d) $\sqrt{\frac{2g}{y_0}}$

Sol. (c) Given that, displacement of particle at any instant

$$y(t) = y_0 \sin^2 \omega t, \; y = y_0\left(\frac{1 - \cos 2\omega t}{2}\right)$$

$$y - \frac{y_0}{2} = -\frac{y_0}{2}\cos 2\omega t \quad \text{or} \quad Y = A\cos 2\omega t$$



where, amplitude (displacement), $A = \frac{y_0}{2}$

and $$2\omega = \sqrt{\frac{k}{m}} \qquad \text{...(i)}$$

Maximum displacement, $\frac{y_0}{2} = \frac{mg}{k}$ (see figure)

or $$k = \frac{2mg}{y_0} \qquad \text{...(ii)}$$

Putting the value of k from Eq. (ii) in Eq. (i), we get

$$2\omega = \sqrt{\frac{2mg}{y_0} \times \frac{1}{m}} \quad \text{or} \quad \omega = \sqrt{\frac{g}{2y_0}}$$

Example 20. *A spring mass system (mass m, spring constant k and natural length l) rests in equilibrium on horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, ($k >> m\omega^2$), the relative change in the length of the spring is best given by the option* **[JEE Main 2020]**

(a) $\frac{m\omega^2}{3k}$ (b) $\frac{2m\omega^2}{k}$ (c) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$ (d) $\frac{m\omega^2}{k}$

Sol. (d) When disc and spring mass system rotates about central axis, spring force provides necessary centripetal pull for rotation of spring mass system.

Let extension in spring = x.

Then, total length of rotating spring = $l_0 + x$.

Equating spring force and centripetal pull, we have

$$kx = m\omega^2(l_0 + x)$$

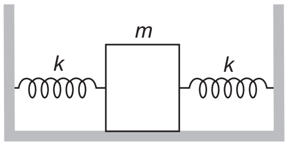
$$\Rightarrow \quad \frac{x}{l_0} = \frac{m\omega^2}{k - m\omega^2}$$

$$\Rightarrow \quad \frac{x}{l_0} = \frac{m\omega^2}{k\left(1 - \frac{m\omega^2}{k}\right)}$$

As $k >> m\omega^2$, so $1 - \frac{m\omega^2}{k} \approx 1$

Hence, $$\frac{x}{l_0} = \frac{m\omega^2}{k}$$

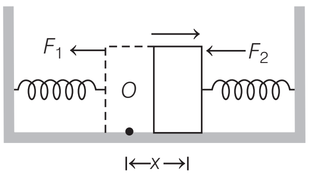
Example 21. *Two identical springs of spring constant k are attached to a block of mass m and to fixed supports as shown, when the mass is displaced from its equilibrium position on either side, it executes SHM, the period of oscillation is*



[NCERT]

(a) $2\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{2m}{k}}$ (c) $2\pi\sqrt{\frac{m}{3k}}$ (d) $2\pi\sqrt{\frac{m}{2k}}$

Sol. (*d*) Let the mass be displaced by a small distance x to the right side of the equilibrium position. Under this situation, the spring on the left side elongated by a length equal to x and that on the right side gets compressed by the same length. The forces acting on the mass are then



(force exerted by the spring on the left side, trying to pull the mass towards the mean position)

$$F_l = -kx$$

(force exerted by the spring on the right side, trying to push the mass towards the mean position)

$$F_2 = -kx$$

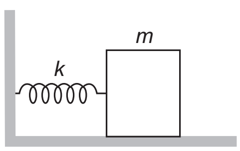
The net force, F acting on the mass is then

$$F = -2kx$$

Hence, the force acting on the mass is proportional to the displacement and is directed towards the mean position, therefore the motion executed by the mass is simple harmonics. The time period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

Example 22. *A block whose mass is 1 kg is fastened to a spring. The spring has a spring constant of 50 Nm^{-1}. The block is pulled to a distance $x = 10$ cm from its equilibrium position at $x = 0$ on a frictionless surface from rest at $t = 0$. The total energy of the block when it is 5 cm away from the mean position is*



(a) 0.19 *J* (b) 0.0625 *J*
(c) 0.09 *J* (d) *0.25 J*

Sol. (*d*) The block executes SHM to angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{1}} = 7.07 \text{ rad s}^{-1}$$

Its displacement at any time t is given by

$$x(t) = 0.1\cos(7.07t)$$

Therefore, when the particle is 5 cm away from the mean position, we have

$$0.05 = 0.1\cos(7.07t)$$

or $$\cos(7.07t) = 0.5$$

$$\sin(7.07t) = \frac{\sqrt{3}}{2} = 0.866$$

Then, the velocity of the block at $x = 5$ cm is

$$= 0.1 \times 7.07 \times 0.866 \text{ ms}^{-1} = 0.61 \text{ ms}^{-1}$$

Hence, the kinetic energy of the block

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}[1\text{ kg} \times (0.6123)^2] = 0.19 \text{ J}$$

and the potential energy,

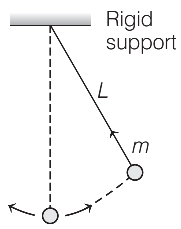
$$\text{PE} = \frac{1}{2}kx^2 = \frac{1}{2}(50 \times 0.05 \times 0.05) = 0.0625 \text{ J}$$

$\therefore$ Total energy = KE + PE = 0.25 J

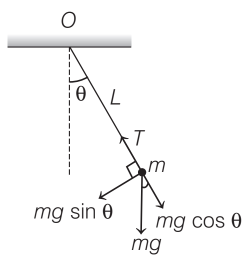
Note At maximum displacement kinetic energy is zero and hence the total energy of the system is equal to potential energy. The result is also in conformity with the principle of conservation of energy.

Simple Pendulum

Given figure shows simple pendulum in which a small bob of mass m tied to an in extensible massless string of length L. The other end of the string is fixed to a support in the ceiling. The bob oscillates in a plane about the vertical line through the support.



The various force acting on the system are as shown in the diagram. If θ is the angle made by the string with the vertical. When the bob is at the mean position $\theta = 0$, there are only two forces acting on the bob, the tension T along the string and the vertical force due to gravity (= mg). Resolving force (= mg) into the component $mg\cos\theta$ along the string and $mg\sin\theta$ perpendicular to it.



Taking torque about point O.

$$\tau = -mgL\sin\theta$$

$$\tau = -mgL\theta \text{ (for small angular displacement } \sin\theta \simeq \theta)$$

$$I\alpha = -mgL\theta$$

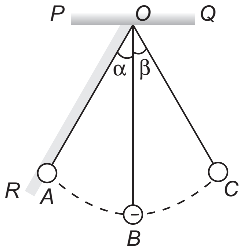
$$\alpha = \frac{-mgL\theta}{I}$$

$$\alpha = \frac{-mgL}{mL^2}\theta = \left(\frac{-g}{L}\right)\theta \Rightarrow \omega = \sqrt{\frac{g}{L}}$$

So, the time period of the simple pendulum can be given as

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Example 23. *A ball is suspended by a thread of length l at the point O on the wall PQ. Another wall OR is inclined to the vertical by α. The thread with the ball is displaced by a small angle β away from the vertical and also away from the wall OR. If the ball is released, assuming the collision to be perfectly elastic the period of oscillation of the pendulum for $\beta > \alpha$ is*



(a) $\sqrt{\frac{L}{g}}\left[\pi + 2\cos^{-1}\frac{\beta}{\alpha}\right]$ (b) $\sqrt{\frac{L}{g}}\left[\pi + 2\sin^{-1}\frac{\alpha}{\beta}\right]$

(c) $\sqrt{\frac{g}{L}}\left[\pi + 2\tan^{-1}\frac{\alpha}{\beta}\right]$ (d) $\sqrt{\frac{g}{L}}\left[2\pi + 2\tan^{-1}\frac{\beta}{\alpha}\right]$

Sol. (b) When $\beta > \alpha$ times taken by pendulum from B to C and C to B is

$$t_1 = \frac{T}{2} = \frac{1}{2}\times 2\pi\sqrt{\frac{L}{g}} = \pi\sqrt{\frac{L}{g}}$$

and $$t_2 = 2t = \frac{2}{\omega}\sin^{-1}\left(\frac{\alpha}{\beta}\right)$$

Using, $$\theta = \theta_0 \sin\omega t$$
$$\alpha = \beta\sin\omega t$$

or $$t = \frac{1}{\omega}\sin^{-1}\left(\frac{\alpha}{\beta}\right)$$

Time period of motion, $T = t_1 + t_2 = \sqrt{\frac{L}{g}}\left[\pi + 2\sin^{-1}\frac{\alpha}{\beta}\right]$

Example 24. *Length of a simple pendulum which ticks seconds is*

(a) 1 *m* (b) 2 *m* (c) 3 *m* (d) 4 *m*

Sol. (a) The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow L = \frac{gT^2}{4\pi^2}$$

The time period of a simple pendulum which ticks seconds is 2 s. Therefore, for $g = 9.8 \text{ ms}^{-2}$ and $T = 2$ s, L is

$$L = \frac{9.8\times 4}{4\pi^2} \approx 1\text{m}$$

Example 25. *The bob of a simple pendulum executes SHM in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $\left(\frac{4}{3}\right)\times 1000\ kgm^{-3}$. What relationship between t and t_0 is true?*

(a) $t = t_0$ (b) $t = 4t_0$

(c) $t = 2t_0$ (d) $t = \frac{t_0}{2}$

Sol. (c) Here density of bob, $\rho = \frac{4}{3}\times 1000 \text{ kgm}^{-3}$

and density of water, $\sigma = 1000 \text{ kgm}^{-3}$

$\therefore$ In air $t_0 = 2\pi\sqrt{\frac{L}{g}}$ and in water

$$t = 2\pi\sqrt{\frac{L}{g\left(1-\frac{\sigma}{\rho}\right)}}$$

$$= 2\pi\sqrt{\frac{L}{g\left(1-\frac{3}{4}\right)}}$$

$$= 2\times 2\pi\sqrt{\frac{L}{g}} = 2t_0$$

Important Points Related to Simple Pendulum

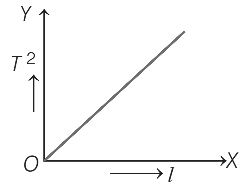
- The time period of a simple pendulum is

$$T = 2\pi\sqrt{l/g}$$

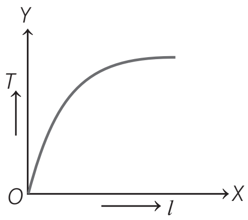
$$\Rightarrow \quad T \propto \sqrt{l} \quad \text{or} \quad T \propto \frac{1}{\sqrt{g}} \Rightarrow T \propto \sqrt{\frac{l}{g}}$$

Using these relations. We may conclude

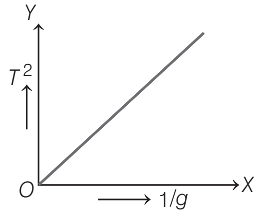
(a) The graph between T^2 and l is a straight line.



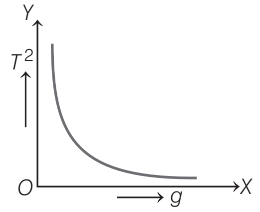
(b) The graph between T and l is a parabola.



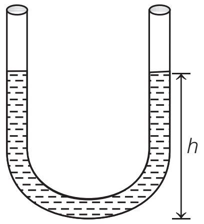
(c) The graph between T^2 and $1/g$ is a straight line.



(d) The graph between T^2 and g is a rectangular hyperbola.



- In the case of water oscillating in a U-tube



$$T = 2p\sqrt{\left(\frac{h}{g}\right)}$$

where, h is the height of liquid column in each limb.

- When a pendulum is kept in a car which is sliding down, then

$$T = 2\pi\sqrt{\left(\frac{l}{g\cos\theta}\right)}$$

where, θ is the angle of inclination.

- If a simple pendulum oscillates in a non-viscous liquid of density ρ, then its time period is

$$T = 2p\sqrt{\left[\frac{l}{\left(1-\frac{s}{r}\right)g}\right]}$$

where, ρ = density of suspended mass.

- If the mass m attached to a spring oscillates in a non-viscous liquid of density σ, then its time period is

$$T = 2\pi\left[\frac{m}{k}\left(1-\frac{\sigma}{\rho}\right)\right]^{1/2}$$

where, k = force constant.

- For a body executing SHM in a tunnel dug along any chord of earth.

Time period, $T = 2\pi\sqrt{\frac{R_e}{g}} = 84.6$ min

where, R_e is the radius of Earth.

- If the time period of simple pendulum is 2s, then it is called as second's pendulum.
- If the simple pendulum is placed in some non-inertial frame of reference like an accelerated lift, g is replaced by g_{eff} whose value can be computed by considering the inertial force. In these cases, the equilibrium position may also change.
- If the length of simple pendulum is very large, then g can't be taken along vertical direction.

In this case, $T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}}$

where, R = Radius of length of the pendulum.

- If temperature of system changes, then time period of simple pendulum changes due to change in length of the simple pendulum.
- If a simple pendulum is in a carriage which is accelerating with an acceleration **a**, then

$$\mathbf{g}_{eff} = \mathbf{g} - \mathbf{a}$$

e.g., if the acceleration **a** is upwards, then

$$|\mathbf{g}_{eff}| = g + a \text{ and } T = 2\pi\sqrt{\frac{l}{g+a}}$$

If the acceleration **a** is downwards, then $(g > a)$

$$|\mathbf{g}_{eff}| = g - a \text{ and } T = 2\pi\sqrt{\frac{l}{g-a}}$$

If the acceleration a is in the horizontal direction, then

$$|\mathbf{g}_{eff}| = \sqrt{a^2 + g^2}$$

In a freely falling lift, $g_{eff} = 0$ and $T = \infty$, *i.e.* the pendulum will not oscillate.

- If in addition to gravity one additional force **F** (*e.g.* electrostatic force $\mathbf{F}_e$) is also acting on the bob, then in that case

$$\mathbf{g}_{eff} = \mathbf{g} + \frac{\mathbf{F}}{m}$$

Here, m is the mass of the bob.

Example 26. *The acceleration due to gravity on the surface of the moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the moon, if its time period on the earth is 3.5 s? (g on the Earth 9.8 ms^{-2}).*

(a) 8.4 *s* (b) 8.2 *s* (c) 7.4 *s* (d) 6.4 *s*

Sol. (a) Given, $g_m = 1.7 \text{ ms}^{-2}$,

$$g_e = 9.8 \text{ ms}^{-2}, T_m = ?; T = 3.5 \text{ s}$$

As, $$T_e = 2\pi\sqrt{\frac{l}{g_e}}$$

and $$T_m = 2\pi\sqrt{\frac{l}{g_m}}$$

$\therefore$ $$\frac{T_m}{T_e} = \sqrt{\frac{g_e}{g_m}}$$

or $$T_m = T_e\sqrt{\frac{g_e}{g_m}} = 3.5\sqrt{\frac{9.8}{1.7}} = 8.4 \text{ s}$$

Example 27. *A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is* **[JEE Main 2019]**

(a) $2T\sqrt{\frac{1}{10}}$ (b) $2T\sqrt{\frac{1}{14}}$ (c) $4T\sqrt{\frac{1}{14}}$ (d) $4T\sqrt{\frac{1}{15}}$

Sol. (d) We know that,
Time period of a pendulum is given by

$$T = 2\pi\sqrt{L/g_{eff}} \quad \text{...(i)}$$

Here, L is the length of the pendulum and g_{eff} is the effective acceleration due to gravity in the respective medium in which bob is oscillating.
Initially, when bob is oscillating in air, $g_{eff} = g$.

So, initial time period, $T = 2\pi\sqrt{\frac{L}{g}}$ ...(ii)

Let ρ_{bob} be the density of the bob.
When this bob is dipped into a liquid whose density is given as

$$\rho_{liquid} = \frac{\rho_{bob}}{16} = \frac{\rho}{16} \quad \text{(given)}$$

$\therefore$ Net force on the bob is $F_{net} = V\rho g - V \cdot \frac{\rho}{16} \cdot g$ ...(iii)

(where, V = volume of the bob = volume of displaced liquid by the bob when immersed in it). If effective value of gravitational acceleration on the bob in this liquid is g_{eff}, then net force on the bob can also be written as

$$F_{net} = V\rho g_{eff} \quad \text{...(iv)}$$

Equating Eqs. (iii) and (iv), we have

$$V\rho g_{eff} = V\rho g - V\rho g/16$$

$$\Rightarrow \quad g_{eff} = g - g/16 = \frac{15}{16}g \quad \text{...(v)}$$

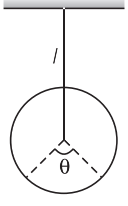
Substituting the value of g_{eff} from Eq. (v) in Eq. (i), the new time period of the bob will be

$$T' = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{16}{15}\frac{L}{g}}$$

$$\Rightarrow \quad T' = \sqrt{\frac{16}{15}} \times 2\pi\sqrt{\frac{L}{g}} = \frac{4}{\sqrt{15}} \times T \quad \text{[using Eq. (ii)]}$$

Torsional pendulum

In a torsional pendulum, an object is suspended from a wire. If such a wire is twisted due to elasticity, it exerts a restoring torque $\tau = C\theta$



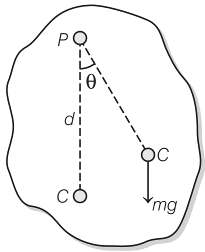
In this case, time period is given by

$$T = 2\pi\sqrt{\frac{I}{C}}$$

where, I = Moment of inertia of the object,
C = Torsional constant of wire = $\frac{\pi\eta r^n}{2l}$,
η = Modulus of elasticity of wire,
r = Radius of wire
and l = Length of wire.

Physical Pendulum

When a rigid body of any shape is capable of oscillating about an axis (may or may not be passing through it), it constitutes a physical pendulum.



So, the time period of physical-pendulum can be given as

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{mgd}}$$

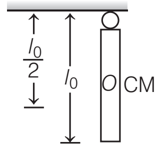
where, I = moment of inertia of body about axis passing through point of suspension.
d = distance of COM of body from point of suspension.
The simple pendulum whose time period is same as that of a physical pendulum is termed as an *equivalent simple pendulum*.

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{l}{g}}$$

The length of an equivalent simple pendulum is given by

$$l = \frac{I}{md}$$

Example 28. *A uniform rod of mass m and length l_0 is pivoted at one end and is hanging in the vertical direction. The period of small angular oscillations of the rod is*



(a) $T = 3\pi\sqrt{\frac{2l_0}{3g}}$ (b) $T = 4\pi\sqrt{\frac{l_0}{3g}}$

(c) $T = 4\pi\sqrt{\frac{2l_0}{3g}}$ (d) $T = 2\pi\sqrt{\frac{2l_0}{3g}}$

Sol. (d) Here the rod is oscillating about an end point O. Hence, moment of inertia of rod about the point of oscillating is

$$I = \frac{1}{3}ml_0^2$$

Moreover, length l of the pendulum = distance from the oscillation axis to centre of mass of rod = $l_0/2$

$\therefore$ Time period of oscillation,

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{\frac{1}{3}ml_0^2}{mg\left(\frac{l_0}{2}\right)}} \Rightarrow T = 2\pi\sqrt{\frac{2l_0}{3g}}$$

Example 29. *Two physical pendulums perform small oscillations about the same horizontal axis with frequencies ω_1 and ω_2. Their moments of inertia relative to the given axes are I_1 and I_2 respectively. In the equilibrium positions, they are joined rigidly. The frequency of small oscillations of the combined pendulum is*

(a) $\sqrt{\frac{I_1\omega_1^2 + I_2\omega_2^2}{I_1 + I_2}}$ (b) $\sqrt{\frac{I_1 + I_2}{I_1^2\omega_1 + I_2^2\omega_2}}$

(c) $\sqrt{\frac{I_1\omega_1^2 - I_2\omega_2^2}{I_1 - I_2}}$ (d) $\sqrt{\frac{I_1 - I_2}{I_1^2\omega_1 - I_2^2\omega_2}}$

Sol. (a) When the pendulums are rigidly joined and set to oscillate, each exerts a torque on the other. These torques are equal and opposite, thus

$$I_1\alpha = -\omega_1^2 I_1\theta + G \quad \text{...(i)}$$

$$I_2\alpha = -\omega_2^2 I_2\theta - G \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we get

$$\alpha = -\frac{(I_1\omega_1^2 + I_2\omega_2^2)\,\theta}{I_1 + I_2}$$

Comparing with $\alpha = -\omega^2\theta$, we get

Frequency, $$\omega = \sqrt{\frac{I_1\omega_1^2 + I_2\omega_2^2}{I_1 + I_2}}$$

Free, Forced, Damped and Resonant Vibrations

Free Vibrations If a given body is once set into vibrations and then let free to vibrate with its own natural frequency, the vibrations are said to be free vibrations.

Forced Vibrations The vibrations in which a body oscillates under the effect of an external periodic force, whose frequency is different from the natural frequency of oscillating body are called forced vibrations.

Damped Vibrations When a body is set in free vibrations, there is a dissipation of energy due to dissipative causes like viscous drag of a fluid, frictional force, hysteresis, electromagnetic damping force, etc. and as a result, amplitude of vibration regularly decreases with time. Such vibrations of continuously falling amplitudes are called damped vibrations.

If the velocity of an oscillator is v, the damping force

$$F_d = -bv$$

where, b = damping constant.

Resultant force on a damped oscillator is given by

$$F = F_R + F_D = -kx - bv$$

or $$\frac{md^2x}{dt^2} + \frac{bdx}{dt} + kx = 0$$

Displacement of a damped oscillator is given by

$$x = x_m e^{-bt/2m}\sin(\omega' t + \phi)$$

$$\omega' = \sqrt{\omega_0^2 - (b/2m)^2}$$

where, ω' = angular frequency of the damped oscillator.

Mechanical energy of a damped oscillator which decreases exponentially with time can be given as

$$E = \frac{1}{2}kx_m^2 e^{-bt/m}$$

Resonant Vibrations It is a special case of forced vibrations in which frequency of external force is exactly same as the natural frequency of oscillator. As a result, the oscillating body begins to vibrate with a large amplitude leading to the resonance phenomenon to occur. Resonant vibrations play a very important role in music and tuning of station/channel in a radio/TV.

Example 30. *A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to* **[JEE Main 2019]**

(a) 20 *s* *(b)* 50 *s*

(c) 100 *s* *(d)* 10 *s*

Sol. (a) Given, frequency of oscillations is

$$f = 5 \text{ osc s}^{-1}$$

$\Rightarrow$ Time period of oscillations is $T = \frac{1}{f} = \frac{1}{5}$ s

So, time for 10 oscillations is $= \frac{10}{5} = 2$s

Now, if A_0 = initial amplitude at $t = 0$ and γ = damping factor, then for damped oscillations, amplitude after t second is given as

$$A = A_0 e^{-\gamma t}$$

$\therefore$ After 2 s,

$$\frac{A_0}{2} = A_0 e^{-\gamma(2)}$$

$$\Rightarrow \quad 2 = e^{2\gamma}$$

$$\Rightarrow \quad \gamma = \frac{\log 2}{2} \quad \text{...(i)}$$

Now, when amplitude is $\frac{1}{1000}$ of initial amplitude, *i.e.*

$$\frac{A_0}{1000} = A_0 e^{-\gamma t}$$

$$\Rightarrow \quad \log(1000) = \gamma t$$

$$\Rightarrow \quad \log(10^3) = \gamma t$$

$$3\log 10 = \gamma t$$

$$\Rightarrow \quad t = \frac{2 \times 3\log 10}{\log 2} \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad t = 19.93 \text{ s}$$

or $$t \approx 20 \text{ s}$$

Practice Exercise

ROUND I Topically Divided Problems

Oscillatory Motion and Simple Harmonic Motion

1. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t}\cos(10\pi t + \phi)$. Here, t is in seconds.
The time taken for its amplitude of vibration to drop to half of its initial value is close to **[JEE Main 2019]**
(a) 27 s
(b) 13 s
(c) 4 s
(d) 7 s

2. The displacement of the particle varies with time according to the relation.
$$y = a\sin\omega t + b\cos\omega t, \text{ then}$$
[NCERT Exemplar]
(a) The motion is oscillating but not SHM
(b) The motion is SHM with amplitude $a + b$
(c) The motion is SHM with amplitude $a^2 + b^2$
(d) The motion is SHM with amplitude $\sqrt{a^2 + b^2}$

3. The displacement of a particle is represented by the equation $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$. The motion of the particle is **[NCERT Exemplar]**
(a) simple harmonic with period 2p/ω
(b) simple harmonic with period π/ω
(c) periodic but not simple harmonic
(d) non-periodic

4. The displacement of a particle is represented by the equation $y = \sin^3\omega t$. The motion is **[NCERT Exemplar]**
(a) non-periodic
(b) periodic but not simple harmonic
(c) simple harmonic with period $2\pi/\omega$
(d) simple harmonic with period π/ω

5. A simple harmonic oscillator has amplitude a and time period T. The time required by it to travel from $x = a$ to $x = \frac{a}{2}$ is
(a) $\frac{T}{6}$ (b) $\frac{T}{4}$
(c) $\frac{T}{3}$ (d) $\frac{T}{2}$

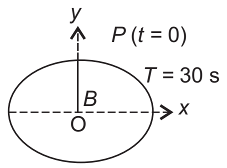
6. Motion of an oscillating liquid column in a U-tube is **[NCERT Exemplar]**
(a) periodic but not simple harmonic
(b) non-periodic
(c) simple harmonic and time period is independent of the density of the liquid
(d) simple harmonic and time-period is directly proportional to the density of the liquid

7. A particle is acted simultaneously by mutually perpendicular simple hormonic motions $x = a\cos\omega t$ and $y = a\sin\omega t$. The trajectory of motion of the particle will be **[NCERT Exemplar]**
(a) an ellipse (b) a parabola
(c) a circle (d) a straight line

8. The acceleration d^2x/dt^2 of a particle varies with displacement x as $\frac{d^2x}{dt^2} = -kx$ where k is a constant of the motion. The time period T of the motion is equal to
(a) $2\pi k$ (b) $2\pi\sqrt{k}$
(c) $2\pi/\sqrt{k}$ (d) $2\pi/k$

9. A block is resting on a piston which is moving vertically with SHM of period 1.0 s. At what amplitude of motion will the block and piston separate?
(a) 0.2 m (b) 0.25 m
(c) 0.3 m (d) 0.35 m

10. Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution and the initial position are indicated on the figure. The simple harmonic motion of the x-projection of the radius vector of the rotating particle P is



[NCERT Exemplar]
(a) $x(t) = B\sin\left(\frac{2\pi t}{30}\right)$ (b) $x(t) = B\cos\left(\frac{\pi t}{15}\right)$
(c) $x(t) = B\sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$ (d) $x(t) = B\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$

11. Two pendulums have time period T and $5T/4$. They start SHM at the same time from the mean position. What will be the phase difference between them after the bigger pendulum completed one oscillation?
(a) 45° (b) 90° (c) 60° (d) 30°

12. The displacement of two particles executing SHM are represented by equations

$$y_1 = 2\sin(10t + \theta),\ y_2 = 3\cos 10t.$$

The phase difference between the velocity of these particles is
(a) θ (b) $-\theta$ (c) $\theta + \pi/2$ (d) $\theta - \pi/2$

13. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time (in seconds) is **[JEE Main 2019]**
(a) $\frac{4\pi}{3}$ (b) $\frac{8\pi}{3}$ (c) $\frac{7}{3}\pi$ (d) $\frac{3}{8}\pi$

14. Two pendulums of length 121 cm and 100 cm start vibrating. At some instant both are in the mean position in the same phase. After how many vibrations of the shorter pendulum, both will be again in phase at the mean position?
(a) 10 (b) 11 (c) 20 (d) 21

15. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what is its maximum speed? **[NCERT]**
(a) 100 m/min (b) 200 m/min
(c) 300 m/min (d) 50 m/min

16. Two points are located at a distance of 10 m and 15 m from the source of oscillation. The period of oscillation is 0.05 s and the velocity of the wave is 300 m/s. What is the phase difference between the oscillations of two points?
(a) π (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

17. A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of 40 cm. The block just loses contact with the plank when the later is momentarily at rest, then
(a) the block becomes weighless
(b) the block weighs double its weight when the plank is at one of the positions of momentary at rest
(c) the block weighs 1.5 times its weight on the plank at extreme position
(d) the block weighs its true weight on the plank, at one of the positions of momentary at rest

18. A body has a time period T_1 under the action of one force and T_2 under the action of another force, the square of the time period when both the forces are acting in the same direction is
(a) $T_1^2 T_2^2$ (b) $T_1^2 T_2^2$
(c) $T_1^2 + T_2^2$ (d) $T_1^2 T_2^2 / (T_1^2 + T_2^2)$

19. Two linear SHMs of equal amplitude A and angular frequencies ω and 2ω are impressed on a particle along the axes x and y respectively. If the initial phase difference between them is $\pi/2$, the resultant path followed by the particle is
(a) $y^2 = x^2(1 - x^2/A^2)$ (b) $y^2 = 2x^2(1 - x^2/A^2)$
(c) $y^2 = 4x^2(1 - x^2/A^2)$ (d) $y^2 = 8x^2(1 - x^2/A^2)$

20. A coin is placed on a horizontal platform, which undergoes horizontal SHM about a mean position O. The coin placed on platform does not slip, coefficient of friction between the coin and the platform is μ. The amplitude of oscillation is gradually increased. The coil will begin to slip on the platform for the first time
(a) at the mean position
(b) at the extreme position of oscillations
(c) for an amplitude of $\mu g/\omega^2$
(d) for an amplitude of $g/\mu\omega^2$

21. A particle in SHM is described by the displacement function $x(t) = A\cos(\omega t + \phi)$, $\omega = 2\pi/T$. If the initial ($t = 0$) position of the particle is 1 cm, its initial velocity is π cm s^{-1} and its angular frequency is πs^{-1}, then the amplitude of its motion is
(a) π cm (b) 2 cm (c) $\sqrt{2}$ cm (d) 1 cm

22. A cylindrical plastic bottle of negligible mass is filled with 310 mL of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm, then ω is close to
(Take, density of water = 10^3 kg/m^3) **[JEE Main 2019]**
(a) 2.50 rad s^{-1} (b) 8.00 rad s^{-1}
(c) 1.25 rad s^{-1} (d) 3.75 rad s^{-1}

23. A large horizontal surface moves up and down in SHM with an amplitude of 1 cm. If a mass of 10 kg (which is placed on the surface) is to remain continuously in contact with it, then the maximum frequency of SHM will be
(a) 5 Hz (b) 0.5 Hz
(c) 1.5 Hz (d) 10 Hz

24. A horizontal platform vibrates with simple harmonic motion in the horizontal direction with a period 2 s. A body of mass 0.5 kg is placed on the platform. The coefficient of static friction between the body and platform is 0.3. What is the maximum frictional force on the body when the platform is oscillating with an amplitude 0.2 m? Assume $\pi^2 = 10 = g$.

(a) 0.5 N (b) 1 N

(c) 1.5 N (d) 2 N

25. The composition of two simple harmonic motions of equal periods at right angle to each other and with a phase difference of π results in the displacement of the particle along

(a) circle (b) figure of eight

(c) straight line (d) ellipse

26. The bob of a simple pendulum of length L is released at time $t = 0$ from a position of small angular displacement. Its linear displacement at time t is given by

(a) $x = a \sin 2\pi \sqrt{\frac{L}{g}} \times t$ (b) $x = a \cos 2\pi \sqrt{\frac{g}{L}} \times t$

(c) $x = a \sin \sqrt{\frac{g}{L}} \times t$ (d) $x = a \cos \sqrt{\frac{g}{L}} \times t$

27. Displacement-time equation of a particle executing SHM is , $x = 3 \sin \omega t + 4 \sin (\omega t + \pi/3)$. Here x is in centimetre and t in second. The amplitude of oscillation of the particle is approximately

(a) 5 cm (b) 6 cm

(c) 7 cm (d) 9 cm

28. Which one of the following equations does not represent SHM, x = displacement and t = time? (parameters a, b and c are the constants of motion)

(a) $x = a \sin bt$ (b) $x = a \cos bt + c$

(c) $x = a \sin bt + c \cos bt$ (d) $x = a \sec bt + c \operatorname{cosec} bt$

29. A particle is performing simple harmonic motion along X-axis with amplitude 4 cm and time period 1.2 s. The minimum time taken by the particle to move from $x = +2$ to $x = 4$ cm and back again is given by

(a) 0.4 s (b) 0.3 s (c) 0.2 s (d) 0.6 s

Energy in Simple Harmonic Motion

30. The angular velocity and the amplitude of a simple pendulum is ω and a respectively. At a displacement x from the mean position, if its kinetic energy is T and potential energy is V, then the ratio of T to V is

(a) $(a^2 - x^2\omega^2)/x^2\omega^2$ (b) $x^2\omega^2/(a^2 - x^2\omega^2)$

(c) $(a^2 - x^2)/x^2$ (d) $x^2/(a^2 - x^2)$

31. A particle is vibrating in a simple harmonic motion with an amplitude of 4 cm. At what displacement from the equilibrium position is its energy half potential and half kinetic?

(a) 1 cm (b) $\sqrt{2}$ cm

(c) 3 cm (d) $2\sqrt{2}$ cm

32. When the potential energy of a particle executing simple harmonic motion is one-fourth of its maximum value during the oscillation, the displacement of the particle from the equilibrium position in terms of its amplitude a is

(a) $\frac{a}{4}$ (b) $\frac{a}{3}$ (c) $\frac{a}{2}$ (d) $\frac{2a}{3}$

33. The potential energy of a particle (U_x) executing SHM is given by

(a) $U_x = \frac{k}{2}(x-a)^2$ (b) $U_x = k_1x + k_2x^2 + k_3x^3$

(c) $U_x = Ae^{-bx}$ (d) U_x = constant

34. A particle of mass m is executing oscillations about the origin on the X-axis with amplitude A. Its potential energy is given as $U(x) = \alpha x^4$, where α is positive constant. The x-coordinate of mass where potential energy is one-third of the kinetic energy of particle, is

(a) $\pm \frac{A}{\sqrt{3}}$ (b) $\pm \frac{A}{\sqrt{2}}$ (c) $\pm \frac{A}{3}$ (d) $\pm \frac{A}{2}$

35. A particle starts SHM from the mean position. Its amplitude is a and total energy E. At one instant, its kinetic energy is $3E/4$. Its displacement at this instant is

(a) $y = a/\sqrt{2}$ (b) $y = \frac{a}{2}$

(c) $y = \frac{a}{\sqrt{3/2}}$ (d) $y = a$

36. A point particle of mass 0.1 kg is executing SHM of amplitude 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. The equation of motion of this particle, if its initial phase of oscillation is 45°, is

(a) $y = 0.1 \sin\left(\frac{r}{4} + \frac{\pi}{4}\right)$ (b) $y = 0.1 \sin\left(\frac{t}{2} + \frac{\pi}{4}\right)$

(c) $y = 0.1 \sin\left(4t - \frac{\pi}{4}\right)$ (d) $y = 0.1 \sin\left(4t + \frac{\pi}{4}\right)$

37. If a simple pendulum of length l has maximum angular displacement θ, then the maximum kinetic energy of bob of mass m is

(a) $\frac{1}{2} \times \left(\frac{l}{g}\right)$ (b) $\frac{1}{2} \times \frac{mg}{l}$

(c) $mgl \times (1 - \cos\theta)$ (d) $\frac{1}{2} \times mgl \sin\theta$

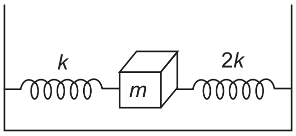
38. For a particle executing SHM, the kinetic energy K is given by $K = K_0 \cos^2 \omega t$. The equation of its displacement can be

(a) $\left(\frac{K_0}{m\omega^2}\right)^{1/2} \sin \omega t$ (b) $\left(\frac{2K_0}{m\omega^2}\right)^{1/2} \sin \omega t$

(c) $\left(\frac{2\omega^2}{mK_0}\right)^{1/2} \sin \omega t$ (d) $\left(\frac{2K_0}{m\omega}\right)^{1/2} \sin \omega t$

Springs and their Oscillations

39. Two springs of force constants k and $2k$ are connected to a mass as shown below. The frequency of oscillation of the mass is



(a) $\frac{1}{2\pi}\sqrt{k/m}$ (b) $\frac{1}{2\pi}\sqrt{2k/m}$
(c) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$

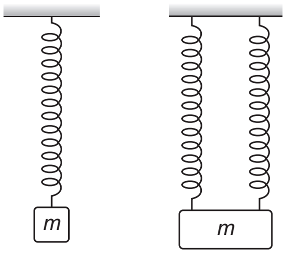
40. A simple spring has length l and force constant k. It is cut into two springs of length l_1 and l_2 such that $l_1 = nl_2$ (n = an integer). The force constant of the spring of length l_2 is

(a) $k(1+n)$ (b) $\frac{(n+1)k}{n}$ (c) k (d) $k/(n+1)$

41. A weightless spring which has a force constant k oscillates with frequency n when a mass m is suspended from it. The spring is cut into two equal halves and a mass 2 m is suspended from one part of spring. The frequency of oscillation will now become

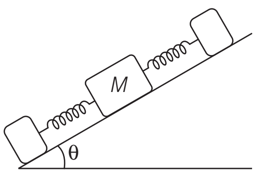
(a) n (b) $2n$ (c) $\frac{n}{\sqrt{2}}$ (d) $n(2)^{1/2}$

42. An object suspended from a spring exhibits oscillations of period T. Now, the spring is cut in two halves and the same object is suspended with two halves as shown in figure. The new time period of oscillation will become



(a) $\frac{T}{2\sqrt{2}}$ (b) $\frac{T}{2}$ (c) $\frac{T}{\sqrt{2}}$ (d) $2T$

43. On a smooth inclined plane, a body of mass M is attached between two springs. The other ends of the springs are fixed to firm support. If each spring has force constant k, then the period of oscillation of the body (assuming the springs as massless) is

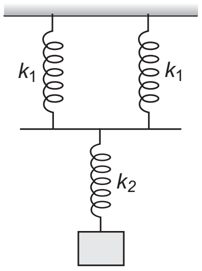


(a) $2\pi[M/2k]^{1/2}$ (b) $2\pi[2M/k]^{1/2}$
(c) $2\pi[Mg\sin\theta/2k]^{1/2}$ (d) $2\pi[2Mg/k]^{1/2}$

44. Time period of mass m suspended by a spring is T. If the spring is cut to one-half and made to oscillate by suspending double mass, the time period of the mass will be

(a) $8T$ (b) $4T$
(c) $\frac{T}{2}$ (d) T

45. What will be the force constant of the spring system shown in figure?



(a) $\frac{k_1}{2}+k_2$ (b) $\left(\frac{1}{2k_1}+\frac{1}{k_2}\right)^{-1}$
(c) $\frac{1}{2k_1}+\frac{1}{k_2}$ (d) $\left(\frac{2}{k_1}+\frac{1}{k_2}\right)^{-1}$

46. A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length l. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then, the stretch in the spring is **[JEE Main 2020]**

(a) $\frac{ml\omega^2}{k+m\omega}$ (b) $\frac{ml\omega^2}{k+m\omega^2}$
(c) $\frac{ml\omega^2}{k-m\omega^2}$ (d) $\frac{ml\omega^2}{k-m\omega}$

47. A massless spring (k = 800 N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released, so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K) **[JEE Main 2019]**

(a) 10^{-4} K (b) 10^{-3} K
(c) 10^{-1} K (d) 10^{-5} K

48. A mass M, attached to a spring, oscillates with a period of 2 s. If the mass is increased by 4 kg, the time period increases by 1 s, the initial mass M was

(a) 3.2 kg (b) 1 kg
(c) 2 kg (d) 8 kg

49. A mass M is suspended from a light spring. An additional mass m is added and displaces the spring further by a distance X. Now the combined mass will oscillate on the spring with period

(a) $T = 2\pi\sqrt{\frac{mg}{X(M+m)}}$ (b) $T = 2\pi\sqrt{\frac{(M+m)X}{mg}}$

(c) $T = \pi/2\sqrt{\frac{mg}{X(M+m)}}$ (d) $T = 2\pi\sqrt{\frac{(M+m)}{mg}}$

50. Two blocks with masses $m_1 = 1$ kg and $m_2 = 2$ kg are connected by a spring of spring constant $k = 24$ Nm^{-1} and placed on a frictionless horizontal surface. The block m_1 is imparted an initial velocity $v_0 = 12$ cms^{-1} to the right, the amplitude of oscillation is

(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm

51. A mass 1 kg suspended from a spring whose force constant is 400 Nm^{-1}, executes simple harmonic oscillation. When the total energy of the oscillator is 2 J, the maximum acceleration experienced by the mass will be

(a) 2 ms^{-2} (b) 4 ms^{-2}

(c) 40 ms^{-2} (d) 400 ms^{-2}

Simple Pendulum and Other Oscillating Systems

52. A simple pendulum of length l and mass (bob) m is suspended vertically. The string makes an angle θ with the vertical. The restoring force acting on the pendulum is

(a) $mg\tan\theta$ (b) $-mg\sin\theta$

(c) $mg\sin\theta$ (d) $-mg\cos\theta$

53. A man measures the period of a simple pendulum inside a stationary lift and finds it to be T second. If the lift accelerates upwards with an acceleration $g/4$, then the period of pendulum will be

(a) $2T\sqrt{5}$ (b) T (c) $\frac{2T}{\sqrt{5}}$ (d) $\frac{T}{4}$

54. A simple pendulum of length l has been set up inside a railway wagon sliding down a frictionless inclined plane having an angle of inclination $\theta = 30°$ with the horizontal. What will be its period of oscillation as recorded by an observer inside the wagon?

(a) $2\pi\sqrt{\frac{2l}{\sqrt{3}g}}$ (b) $2\pi\sqrt{2l/g}$

(c) $2\pi\sqrt{l/g}$ (d) $2\pi\sqrt{\frac{\sqrt{3}l}{2g}}$

55. If a simple pendulum is taken to a place where g decreases by 2%, then the time period

(a) increases by 0.5% (b) increases by 1%

(c) increases by 2.0% (d) decreases by 0.5%

56. A heavy sphere of mass m is suspended by string of length l. The sphere is made to revolve about a vertical line passing through the point of suspension in a horizontal circle such that the string always remains inclined to the vertical at an angle θ . What is its period of revolution?

(a) $T = 2\pi\sqrt{\frac{l}{g}}$ (b) $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

(c) $T = 2\pi\sqrt{\frac{l\sin\theta}{g}}$ (d) $T = 2\pi\sqrt{\frac{l\tan\theta}{g}}$

57. A ring is hung on a nail. It can oscillate without slipping or sliding

(i) in its plane with a time period T_1 and

(ii) back and forth in a direction perpendicular to its plane, with a period T_2.

The ratio $\frac{T_1}{T_2}$ will be **[JEE Main 2020]**

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{2}{3}$ (c) $\frac{3}{\sqrt{2}}$ (d) $\frac{\sqrt{2}}{3}$

58. The mass and the diameter of a planet are three times the respective values for the earth. The period of oscillation of a simple pendulum on the earth is 2 s. The period of oscillation of the same pendulum on the planet would be **[JEE Main 2019]**

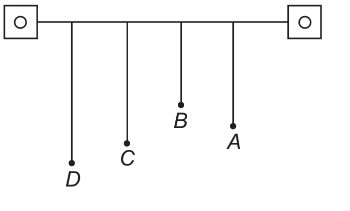
(a) $\frac{2}{\sqrt{3}}$ s (b) $\frac{3}{2}$ s (c) $2\sqrt{3}$ s (d) $\frac{\sqrt{3}}{2}$ s

59. A simple pendulum has a length l. The inertial and gravitational masses of the bob are m_1 and m_g respectively. Then the time period T is given by

(a) $T = 2\pi\sqrt{\frac{m_g l}{m_i g}}$ (b) $T = 2\pi\sqrt{\frac{m_i l}{m_g g}}$

(c) $T = 2\pi\sqrt{\frac{m_i \times m_g \times l}{g}}$ (d) $T = 2\pi\sqrt{\frac{l}{m_i \times m_g \times g}}$

60. Four pendulums A, B, C and D are suspended from the same elastic support as shown in figure. A and C are of the same length, while B is smaller than A and D is larger than A. **[NCERT Exemplar]**



(a) D will vibrate with maximum amplitude

(b) C will vibrate with maximum amplitude

(c) B will vibrate with maximum amplitude

(d) All the four will oscillate with equal amplitude

61. If the length of second's pendulum is increased by 2%. How many seconds it will lose per day?

(a) 3927 s (b) 3427 s

(c) 3737 s (d) 864 s

62. A pendulum bob of mass m is hanging from a fixed point by a light thread of length l. A horizontal speed v_0 is imparted to the bob, so that it takes up horizontal position. If g is the acceleration due to gravity, then v_0 is

(a) mgl (b) $\sqrt{2gl}$

(c) $\sqrt{gl}$ (d) gl

63. The bob of a pendulum of length l is pulled a side from its equilibrium position through an angle θ and then released. The bob will then pass through its equilibrium position with a speed v, where v equals

(a) $\sqrt{2gl(1-\cos\theta)}$

(b) $\sqrt{2gl(1+\sin\theta)}$

(c) $\sqrt{2gl(1-\sin\theta)}$

(d) $\sqrt{2gl(1+\cos\theta)}$

64. A tunnel is made across the earth of radius R, passing through its centre. A ball is dropped from a height h in the tunnel. The motion will be periodic with time period

(a) $2\pi\sqrt{\frac{R}{g}}+4\sqrt{\frac{h}{g}}$

(b) $2\pi\sqrt{\frac{R}{g}}+4\sqrt{\frac{2h}{g}}$

(c) $2\pi\sqrt{\frac{R}{g}}+\sqrt{\frac{h}{g}}$

(d) $2\pi\sqrt{\frac{R}{g}}+\sqrt{\frac{2h}{g}}$

65. Two simple pendulums of length 0.5 m and 20 m respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed ...... oscillations.

(a) 5 (b) 1

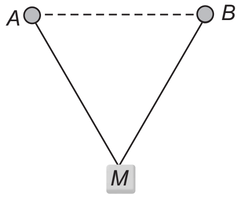
(c) 2 (d) 3

66. In damped oscillation the amplitude of oscillations is reduced to one-third of its initial value a_0 at the end of 100 oscillation. When the oscillation completes 200 oscillations, its amplitude must be

(a) $\frac{a_0}{2}$ (b) $\frac{a_0}{4}$

(c) $\frac{a_0}{6}$ (d) $\frac{a_0}{9}$

67. A and B are fixed points and the mass M is tied by strings at A and B. If the mass M is displaced slightly out of this plane and released, it will execute oscillations with period
(Given, $AM = BM = L$, $AB = 2d$)



(a) $2\pi\sqrt{\frac{L}{g}}$ (b) $2\pi\sqrt{\frac{(L^2-d^2)^{1/2}}{g}}$

(c) $2\pi\sqrt{\frac{(L^2+d^2)^{1/2}}{g}}$ (d) $2\pi\sqrt{\frac{(2d^2)^{3/2}}{g}}$

68. A piece of wood has dimensions a, b and c. Its relative density is d. It is floating in water such that the side c is vertical. It is now pushed down gently and released. The time period is

(a) $T=2\pi\sqrt{\left(\frac{abc}{g}\right)}$ (b) $T=2\pi\sqrt{\left(\frac{ba}{dg}\right)}$

(c) $T=2\pi\sqrt{\left(\frac{g}{dc}\right)}$ (d) $T=2\pi\sqrt{\left(\frac{dc}{g}\right)}$

69. A pendulum clock is placed on the Moon, where object weighs only one-sixth as much as on the Earth. How many seconds the clock tick out in an actual time of 1 min the clock keeps good time on the Earth?

(a) 12.25 (b) 24.5 (c) 2.45 (d) 0.245

70. A uniform cylinder of length L and mass M having cross-sectional area A is suspended with its vertical length, from a fixed point by a massless spring, such that it is half submerged in a liquid of density d at equilibrium position. When released, it starts oscillating vertically with a small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is

(a) $\frac{1}{2\pi}\left(\frac{k-Adg}{M}\right)^{1/2}$ (b) $\frac{1}{2\pi}\left(\frac{k+Adg}{M}\right)^{1/2}$

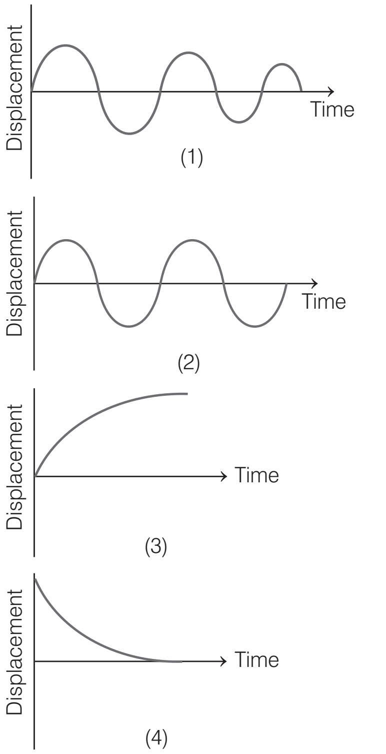
(c) $\frac{1}{2\pi}\left(\frac{k-dgL}{M}\right)^{1/2}$ (d) $\frac{1}{2\pi}\left(\frac{k+AgL}{Adg}\right)^{1/2}$

71. A particle, with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force, $F = F_0 \sin \omega t$.
If the amplitude of the particle is maximum for $\omega=\omega_1$ and the energy of the particle is maximum for $\omega=\omega_2$, then

(a) $\omega_1=\omega_0$ and $\omega_2\neq\omega_0$ (b) $\omega_1=\omega_0$ and $\omega_2=\omega_0$

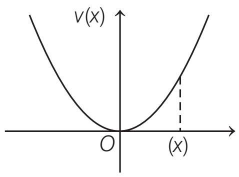
(c) $\omega_1\neq\omega_0$ and $\omega_2=\omega_0$ (d) $\omega_1\neq\omega_0$ and $\omega_2\neq\omega_0$

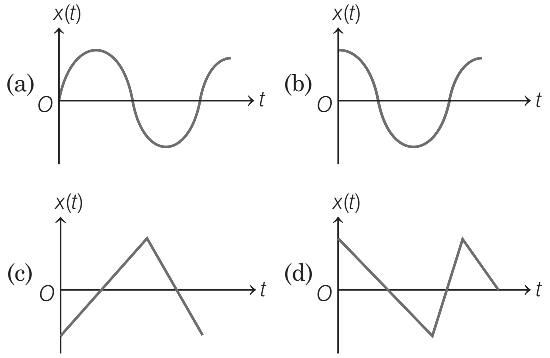
72. Which of the following figure represent(s) damped simple harmonic motions?



(a) Fig. 1 (b) Fig. 2
(c) Fig. 4 (d) Figs. 3 and 4

73. A particle of mass m is released from rest and follows a parabolic path as shown. Assuming that the displacement of the mass from the origin is small. Which graph correctly depicts the position of the particle as a function of time?





74. The amplitude of damped oscillator becomes (1/3)rd in 2 s. Its amplitude after 6 s is $1/n$ times the original. Then n is equal to
(a) 2^3 (b) 3^2 (c) $3^{1/3}$ (d) 3^3

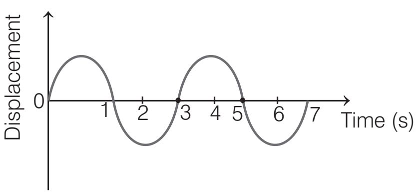
ROUND II Mixed Bag

Only One Correct Option

1. Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower point is **[NCERT Exemplar]**
(a) simple harmonic motion (b) non-periodic motion
(c) periodic but not SHM (d) None of the above

2. The time period of a mass suspended from a spring is 5 s. The spring is cut into four equal parts and the same mass is now suspended from one of its parts. The period is now
(a) 5 s (b) 2.5 s (c) 1.25 s (d) $\frac{1}{16}$ s

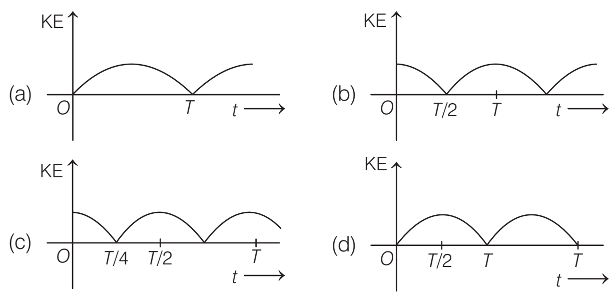
3. Displacement vs. time curve for a particle executing SHM is shown in figure. Choose the correct statements. **[NCERT Exemplar]**



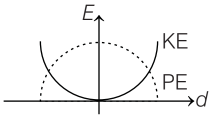
(a) Phase of the oscillator is same at $t = 0$ s and $t = 2$ s
(b) Phase of the oscillator is different at $t = 2$ s and $t = 6$ s
(c) Phase of the oscillator is same at $t = 1$ s and $t = 7$ s
(d) Phase of the oscillator is same at $t = 1$ s and $t = 5$ s

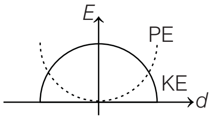
4. The total energy of a particle executing SHM is 80 J. What is the potential energy when the particle is at a distance of 3/4 of amplitude from the mean position?
(a) 60 J (b) 10 J (c) 40 J (d) 45 J

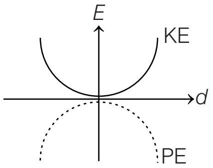
5. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like **[JEE Main 2017]**

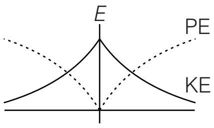


6. For a simple pendulum, a graph is plotted between its Kinetic Energy (KE) and Potential Energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale) **[JEE Main 2015]**

(a) 

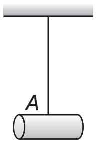
(b) 

(c) 

(d) 

7. Two simple harmonic motions act on a particle. These harmonic motions are
$x = A\sin(\omega t + \alpha)$; $y = A\cos(\omega t + \alpha)$, then path of particle is
(a) an ellipse and the actual motion is counter clockwise
(b) an ellipse and the actual motion is clockwise
(c) a circle and the actual motion is counter clockwise
(d) a circle and the actual motion is clockwise

8. A uniform cylindrical metal rod A of length L and radius R is suspended at its mid-point from a rigid support through a strong metal wire of length l. The rod is given a small angular twist and released so that it oscillates to and fro about its mean position with a time period T_1. The rotational inertia of metal rod about the wire as an axis is



(a) $\dfrac{ML^2}{12}$ (b) $\dfrac{MR^2}{2}$
(c) $M\left[\dfrac{L^2}{12} + \dfrac{R^2}{2}\right]$ (d) $M\left[\dfrac{L^2}{12} + \dfrac{R^2}{4}\right]$

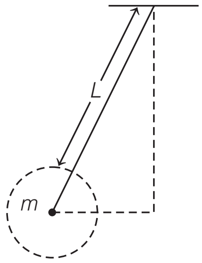
9. A particle is having kinetic energy 1/3 of the maximum value at a distance of 4 cm from the mean position. Find the amplitude of motion.
(a) $2\sqrt{6}$ cm (b) $2/\sqrt{6}$ cm
(c) $\sqrt{2}$ cm (d) $6/\sqrt{2}$

10. If a spring extends by x on loading, then the energy stored in the spring is (if T is the tension and k is the force constant of the spring)
(a) $\dfrac{T^2}{2x}$ (b) $\dfrac{T^2}{2k}$ (c) $\dfrac{2k}{T^2}$ (d) $\dfrac{2T^2}{k}$

11. The period of oscillation of a mass m suspended from a spring is 2 s. If along with it another mass 2 kg is also suspended, the period of oscillation increases by 1 s. The mass m will be
(a) 2 kg (b) 1 kg (c) 1.6 kg (d) 2.6 kg

12. Two pendulums begin to swing simultaneously. The first pendulum makes 9 full oscillations when the other makes 7. The ratio of lengths of the two pendulums is
(a) 9/7 (b) 7/9 (c) 49/81 (d) 81/49

13. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that string can bear is 324 N.The maximum possible value of angular velocity of ball (in rad/s) is



(a) 9 (b) 18 (c) 27 (d) 36

14. A point mass is subjected to two simultaneous sinusoidal displacement in X-direction
$X_1(t) = A\sin\omega t$ and $X_2(t) = A\sin\left(\omega t + \dfrac{2\pi}{3}\right)$.
Adding a third sinusoidal displacement $X_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are
(a) $\sqrt{2}A, \dfrac{3\pi}{4}$ (b) $A, \dfrac{4\pi}{3}$ (c) $\sqrt{3}A, \dfrac{5\pi}{6}$ (d) $A, \dfrac{\pi}{3}$

15. Average value of kinetic energy and potential energy over entire time period is
(a) $0, \dfrac{1}{2}m\omega^2A^2$ (b) $\dfrac{1}{2}m\omega^2A^2, 0$
(c) $\dfrac{1}{2}m\omega^2A^2, \dfrac{1}{2}m\omega^2A^2$ (d) $\dfrac{1}{4}m\omega^2A^2, \dfrac{1}{4}m\omega^2A^2$

16. A particle in SHM is described by the displacement function $x(t) = A\cos(\omega t + \theta)$. If the initial ($t = 0$) position of the particle is 1 cm and its initial velocity is π cms^{-1}, what is its amplitude? The angular frequency of the particle is π s^{-1}.
(a) 1 cm (b) $\sqrt{2}$ cm (c) 2 cm (d) 2.5 cm

17. The period of particle in SHM is 8 s. At $t = 0$, particle is at the mean position. The ratio of the distances travelled by it in Ist second and 2nd second is
(a) 1.6 : 1 (b) 2.4 : 1 (c) 3.2 : 1 (d) 4.2 : 1

18. A simple pendulum of length l has a bob of mass m, with a charge q on it. A vertical sheet of charge, with surface charge density σ passes through the point of suspension. At equilibrium, the spring makes an angle θ with the vertical. Its time period of oscillations is T in this position. Then
(a) $\tan\theta = \dfrac{\sigma q}{2\varepsilon_0 mg}$ (b) $\tan\theta = \dfrac{\sigma q}{\varepsilon_0 mg}$
(c) $T > 2\pi\sqrt{\dfrac{1}{g}}$ (d) $T = 2\pi\sqrt{\dfrac{1}{g}}$

19. The time period of a particle in simple harmonic motion is 8 s. At $t = 0$, it is at the mean position. The ratio of the distances travelled by it in the first and second, seconds is

(a) $1:2$ (b) $1:\sqrt{2}$ (c) $1:(\sqrt{2}-1)$ (d) $1:\sqrt{3}$

20. The bob of a simple pendulum is of mass 10 g. It is suspended with a thread of 1 m. If we hold the bob so as to stretch the string horizontally and release it, what will be the tension at the lowest position? (Take, $g = 10\ \text{ms}^{-2}$)

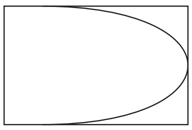
(a) Zero (b) 0.1 N (c) 0.3 N (d) 1.0 N

21. A block of mass M is suspended from a light spring of force constant k. Another mass m moving upwards with velocity v hits the mass M and gets embedded in it. What will be the amplitude of oscillation of the combined mass?

(a) $\frac{mv}{k\sqrt{(M-m)}}$ (b) $\frac{mv}{\sqrt{(M-m)k}}$

(c) $\frac{mv}{k\sqrt{(M+m)}}$ (d) $\frac{mv}{\sqrt{(M+m)k}}$

22. Lissajous figure shown below corresponds to which one of the following?



(a) Phase difference $\pi/2$ and period 1 : 2
(b) Phase difference $3\pi/4$ and period 1 : 2
(c) Phase difference $\pi/4$ and period 2 : 1
(d) Phase difference $2\pi/3$ and period 2 : 1

23. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s, it will decrease to α times its original magnitude, where α equals **[JEE Main 2013]**

(a) 0.7 (b) 0.81 (c) 0.729 (d) 0.6

24. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} per second. What is the force constant of the bonds connecting one atom with the other? (Take, molecular weight of silver = 108 and Avogadro number = 6.02×10^{23} g mol^{-1})

(a) 6.4 N/m (b) 7.1 N/m **[JEE Main 2018]**
(c) 2.2 N/m (d) 5.5 N/m

25. A particle moves with simple harmonic motion in a straight line. In first t sec, after starting from rest, it travels a distance a and in next t sec, it travels $2a$ in same direction, then **[JEE Main 2014]**

(a) amplitude of motion is $3a$
(b) time period of oscillations is $5t$
(c) amplitude of motion is $4a$
(d) time period of oscillations is $6t$

26. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then **[JEE Main 2019]**

(a) $K_2 = 2K_1$ (b) $K_2 = \frac{K_1}{2}$

(c) $K_2 = \frac{K_1}{4}$ (d) $K_2 = K_1$

27. A simple harmonic motion is represented by $y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm. The amplitude and time period of the motion are **[JEE Main 2019]**

(a) 10 cm, $\frac{3}{2}$ s (b) 5 cm, $\frac{2}{3}$ s

(c) 5 cm, $\frac{3}{2}$ s (d) 10 cm, $\frac{2}{3}$ s

28. A highly rigid cubical block A of small mass M and side L is fixed rigidly on the another cubical block of same dimensions and low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of Z. After the force is withdrawn, block A executes small oscillations, the time period of which is given by

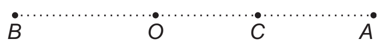
(a) $2\pi\sqrt{ML\eta}$ (b) $2\pi\sqrt{M\eta/L}$

(c) $2\pi\sqrt{ML/\eta}$ (d) $2\pi\sqrt{M/\eta L}$

29. Two pendulums of length 1 m and 16 m start vibrating one behind the other from the same stand. At some instant, the two are in the mean position in the same phase. The time period of shorter pendulum is T. The minimum time after which the two threads of the pendulum will be one behind the other is

(a) $T/4$ (b) $T/3$ (c) $4T/3$ (d) $4T$

30. A particle is in linear simple harmonic motion between two points A and B, 10 cm apart (Fig). Take the direction from A to B as the +ve direction and choose the correct statements. **[NCERT Exemplar]**



$AO = OB = 5$ cm, $BC = 8$ cm

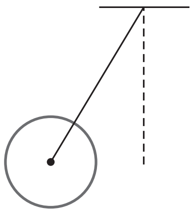
(a) The sign of velocity, acceleration and force on the particle when it is 3 cm away from A going towards B are positive
(b) The sign of velocity of the particle at C going towards O is negative
(c) The sign of velocity, acceleration and force on the particle when it is 5 cm away from B going towards A are positive
(d) The sign of acceleration and force on the particle when it is at point B is positive

31. A particle performs harmonic oscillation along the X-axis about the equilibrium position $x = 0$. The oscillation frequency is $\omega = 4.00\ s^{-1}$. At a certain moment of time the particle has a coordinate $x_0 = 25.0$ cm and its velocity is equal to $v_{x_0} = 100\ cms^{-1}$. Find the equation of motion of the particle.

(a) $y = 13\sqrt{3}\sin\left(4t + \frac{\pi}{4}\right)$ (b) $y = 25\sqrt{2}\sin\left(4t + \frac{\pi}{4}\right)$

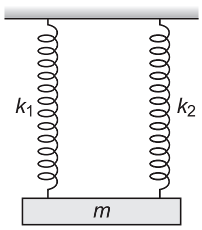
(c) $y = 27\sqrt{2}\sin\left(4t + \frac{\pi}{4}\right)$ (d) $y = 27\sqrt{5}\sin\left(t + \frac{\pi}{2}\right)$

32. A metal rod of length L and mass m is pivoted at one end. A thin disc of mass M and radius $R (< L)$ is attached at its centre to the free end of the rod. Consider two ways, the disc is attached (case A), the disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre. The rod-disc system perform SHM in vertical plane after being released from the same displaced position in each case. Which of the following statements is (are) true?



(a) Restoring torque in case A = Restoring torque in case B.
(b) Restoring torque in case A < Restoring torque in case B .
(c) Angular frequency for case A > Angular frequency for case B.
(d) None of the above

33. A mass m is suspended separately by two different springs in successive order, then time periods is t_1 and t_2 respectively. If m is connected by both springs as shown in figure, then time period is t_0, the correct relation is



(a) $t_0^2 = t_1^2 + t_2^2$
(b) $t_0^{-2} = t_1^{-2} + t_2^{-2}$
(c) $t_0^{-1} = t_1^{-1} + t_2^{-1}$
(d) $t_0 = t_1 + t_2$

34. A body of mass 4.9 kg hangs from a spring and oscillates with a period 0.5 s on the removal of the body, the spring is shortened by
(Take g =10 ms^{-2}, $\pi^2 = 10$)

(a) 6.3 m (b) 0.63 m
(c) 6.25 cm (d) 63.5 cm

35. A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

(a) $2\pi\sqrt{\dfrac{3l}{g + \dfrac{v^2}{R}}}$ (b) $2\pi\sqrt{l\Big/\left(g^2 + \dfrac{v^4}{R^2}\right)}$

(c) $2\pi\sqrt{\dfrac{2l}{\left(g^2 + \dfrac{v^2}{R^2}\right)}}$ (d) $2\pi\sqrt{\dfrac{2l}{(g^2 + v^2/R)}}$

36. When a body of mass 1.0 kg is suspended from a certain light spring hanging vertically, its length increases by 5 cm. By suspending 2.0 kg block to the spring and if the block is pulled through 10 cm and released, the maximum velocity of it, (in ms^{-1}) is (Take, $g = 10\ ms^{-2}$)

(a) 0.5 (b) 1
(c) 2 (d) 4

37. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by

[JEE Main 2019]

(a) 1 rad/s (b) 10^{-5} rad /s
(c) 10^{-3} rad /s (d) 10^{-1} rad /s

38. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant when it is at a distance $\frac{2}{3}A$ from equilibrium position. The new amplitude of the motion is

[JEE Main 2016]

(a) $\frac{A}{3}\sqrt{41}$ (b) $3A$ (c) $A\sqrt{3}$ (d) $\frac{7}{3}A$

39. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is p_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency **[JEE Main 2013]**

(a) $\frac{1}{2\pi}\frac{A_{\gamma}\,p_0}{V_0 M}$

(b) $\frac{1}{2\pi}\frac{V_0 M p_0}{A^2\gamma}$

(c) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma\,p_0}{M\,V_0}}$

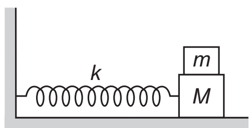
(d) $\frac{1}{2\pi}\sqrt{\frac{M\,V_0}{A_{\gamma}\,p_0}}$

40. A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 min, the amplitude decrease to 6 cm. Determine the value of the damping constant for this motion. **[JEE Main 2021]**

(a) $0.69\times10^2\text{kg s}^{-1}$ (b) $3.3\times10^2\text{kg s}^{-1}$

(c) $1.16\times10^2\text{kg s}^{-1}$ (d) $5.7\times10^{-3}\text{ kg s}^{-1}$

41. A mass M is attached to a horizontal spring of force constant k fixed on one side to a rigid support as shown in figure. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, another mass m is gently placed on it. What will be the new amplitude of oscillations?



(a) $A\sqrt{\left(\frac{M}{M-m}\right)}$ (b) $A\sqrt{\left(\frac{M-m}{M}\right)}$

(c) $A\sqrt{\left(\frac{M}{M+m}\right)}$ (d) $A\sqrt{\left(\frac{M+m}{M}\right)}$

42. A bottle weighing 200 g and area of cross-section 50 cm^2 and height 4 cm oscillates on the surface of water in vertical position. Its frequency of oscillation is

(a) 1.5 Hz
(b) 2.5 Hz
(c) 3.5 Hz
(d) 4.5 Hz

43. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. The suction pump is removed, the column of mercury in the U-tube will show **[NCERT]**

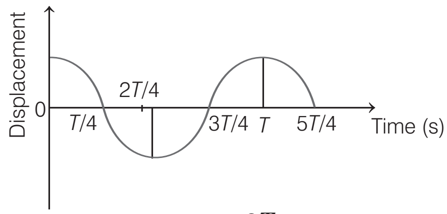
(a) periodic motion
(b) oscillation
(c) simple harmonic motion
(d) None of the above

44. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body? **[NCERT]**

(a) 222.13 N (b) 200.13 N

(c) 193.13 N (d) 219.13 N

45. The displacement time graph of a particle executing SHM is shown in figure. Which of the following statement is/are true? **[NCERT Exemplar]**



(a) The force is zero at $t=\frac{3T}{4}$.

(b) The acceleration is maximum at $t=\frac{3T}{4}$.

(c) The velocity is maximum at $t=\frac{T}{2}$.

(d) The PE is equal to KE of oscillation at $t=\frac{T}{2}$.

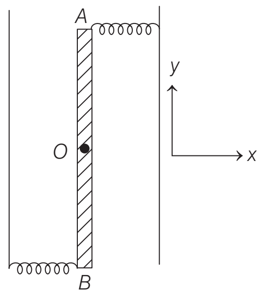
46. A pendulum is made to hang from a ceilling of an elevator.It has period of T_{sec}. (for small angles). The elevator is made to accelerate upwards with 10 m/s^2.The period of the pendulum now will be (assume g =10 m/s^2)

(a) $T\sqrt{2}$ (b) infinite

(c) $T/\sqrt{2}$ (d) zero

47. Two light identical springs of spring constant k are attached horizontally at the two ends of an uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure.

The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is



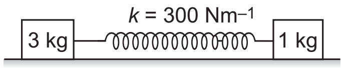
[JEE Main 2019]

(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Numerical Value Questions

48. Two point masses of 3.0 kg and 1.0 kg are attached to opposite ends of a horizontal spring whose spring constant is 300 Nm^{-1} as shown in adjacent figure. The natural frequency (in Hz) of vibration of the system is ....

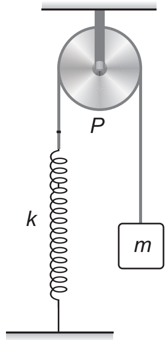


49. A particle performs simple harmonic motion with a period of 2 s. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $(1/a)$ s . The value of a to the nearest integer is ....
[JEE Main 2021]

50. Two springs of force constants 20 N/m and 10 N/m, have equal highest velocities when executing SHM, then the ratio of their amplitudes (given their masses are equal) will be $\frac{1}{\sqrt{x}}$. Find the value of x.

51. A particle of mass 4 cm is executing oscillations about the origin on the X-axis. Its potential energy is $U(x) = k|x|^3$, where k is a positive constant. If the amplitude of oscillation is 4 cm, then its time period T will be proportional to ....

52. As shown in figure, system consisting of massless pulley, a light spring of force constant $k = 16$ N/m and a block of mass 0.01 kg. If the block is slightly displaced vertically downwards from its equilibrium position and released, then the frequency of vertical oscillations will be $\frac{n}{\pi}$ Hz. Find the value of n.



53. Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown in figure. Fig. 1 shows one of them and Fig. 2 shows their series combination. The ratios of time period of oscillation of the two SHM is $\frac{T_b}{T_a} = \sqrt{x}$, where value of x is .... (Rounded off to the nearest integer)
[JEE Main 2021]

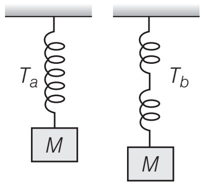


Fig. 1 Fig. 2

54. A particle is executing SHM of amplitude 25 cm and time period 3 s. The minimum time (in s) required for the particle to move between two points 12.5 cm on either side of the mean position will be .... s.

Answers

Round I

1. (d)	2. (d)	3. (b)	4. (b)	5. (a)	6. (c)	7. (c)	8. (c)	9. (b)	10. (a)
11. (b)	12. (d)	13. (b)	14. (b)	15. (a)	16. (d)	17. (b)	18. (d)	19. (c)	20. (c)
21. (c)	22. (b)	23. (a)	24. (b)	25. (c)	26. (d)	27. (b)	28. (d)	29. (a)	30. (c)
31. (d)	32. (c)	33. (a)	34. (b)	35. (b)	36. (d)	37. (c)	38. (d)	39. (c)	40. (b)
41. (a)	42. (b)	43. (a)	44. (d)	45. (b)	46. (c)	47. (d)	48. (a)	49. (b)	50. (b)
51. (c)	52. (b)	53. (c)	54. (a)	55. (b)	56. (b)	57. (a)	58. (c)	59. (b)	60. (b)
61. (d)	62. (b)	63. (a)	64. (b)	65. (a)	66. (d)	67. (b)	68. (d)	69. (b)	70. (b)
71. (c)	72. (a)	73. (a)	74. (d)						

Round II

1 (a)	2. (b)	3. (d)	4. (d)	5. (c)	6. (b)	7. (c)	8. (d)	9. (a)	10. (d)
11. (c)	12. (c)	13. (d)	14. (b)	15. (d)	16. (b)	17. (b)	18. (a)	19. (c)	20. (c)
21. (d)	22. (a)	23. (c)	24. (b)	25. (d)	26. (b)	27. (d)	28. (d)	29. (c)	30. (a)
31. (b)	32. (a)	33. (b)	34. (c)	35. (b)	36. (b)	37. (c)	38. (d)	39. (c)	40. (*)
41. (c)	42. (b)	43. (c)	44. (d)	45. (a)	46. (c)	47. (c)	48. 3	49. 6	50. 2
51. 0.5	52. 20	53. 2	54. 0.5						

Solutions

Round I

1. Given, displacement is

$$x(t) = e^{-0.1t} \cos(10\pi t + \phi)$$

Here, amplitude of the oscillator is

$$A = e^{-0.1t} \quad \dots \text{(i)}$$

Let it takes t seconds for amplitude to be dropped by half.

At $t = 0 \Rightarrow A = 1$ [from Eq. (i)]

At $t = t \Rightarrow A' = \frac{A}{2} = \frac{1}{2}$

So, Eq. (i) can be written as

$$e^{-0.1t} = \frac{1}{2}$$

or $e^{0.1t} = 2$

or $0.1\,t = \ln(2)$

or $t = \frac{1}{0.1} \ln(2) = 10 \ln(2)$

Now, $\ln(2) = 0.693$

$\therefore$ $t = 10 \times 0.693 = 6.93$ s

or $t \approx 7$ s

2. Given, $y = a\sin\omega t + b\cos\omega t$

Let $a = A\cos\theta$ and $b = A\sin\theta$ ...(i)

then $y = A\cos\theta\sin\omega t + A\sin\theta\cos\omega t$

$y = A\sin(\omega t + \theta)$

which is in the form of SHM.

From Eq. (i)

$$a^2 + b^2 = A^2\cos^2\theta + A^2\sin^2\theta$$

$\Rightarrow$ $A = \sqrt{a^2 + b^2}$

3. Given, $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$ ... (i)

Velocity, $v = \frac{dy}{dt} = 3 \times 2\,\omega \sin\left(\frac{\pi}{4} - 2\,\omega t\right)$

Acceleration,

$$A = \frac{dv}{dt} = -4\,\omega^2 \times 3\cos\left(\frac{\pi}{4} - 2\omega t\right) = -4\,\omega^2 y$$

As $A \propto y$ and – ve sign shows that it is directed towards equilibrium (or mean position), hence particle will execute SHM. Comparing Eq. (i) with equation

$$y = r\cos(\phi - \omega' t)$$

we have, $\omega' = 2\,\omega$

or $\frac{2\pi}{T'} = 2\,\omega$

or $T' = \frac{\pi}{\omega}$

4. Given, $y = \sin^3 \omega t = \frac{1}{4}(3\sin\omega t - \sin 3\,\omega t)$

As this motion is not represented by single harmonic function, hence it is not an SHM. As this motion involves sine and cosine functions, hence it is periodic motion.

5. It is required to calculate the time for extreme position.

Hence in this case, equation of displacement of particle can be written as $x = a\sin\left(\omega t + \frac{\pi}{2}\right) = a\cos\omega t$

$\Rightarrow$ $\frac{a}{2} = a\cos\omega t$ (as per question)

$\frac{1}{2} = \cos\omega t$

$\Rightarrow$ $\cos\frac{\pi}{3} = \cos\omega t$ $\left\{\because \cos\frac{\pi}{3} = \frac{1}{2}\right\}$

$\Rightarrow$ $\omega t = \frac{\pi}{3}$

$\Rightarrow$ $\frac{2\pi}{T} \cdot t = \frac{\pi}{3}$

$\Rightarrow$ $t = \frac{T}{6}$

6. Motion of an oscillating liquid column in a U-tube is SHM with period, $T = 2\pi\sqrt{\frac{l}{g}}$, where l is the height of liquid column in one arm of U-tube in equilibrium position of liquid. Therefore, T is independent of density of liquid.

7. $x = a\cos\omega t$ and $y = a\sin\omega t$

$\therefore$ $x^2 + y^2 = a^2(\cos^2\omega t + \sin^2\omega t) = a^2$

It is an equation of a circle. Thus, trajectory of motion of the particle will be a circle.

8. As, $d^2x/dt^2 = -kx$

and $T = 2\pi\sqrt{\frac{\text{displacemant}}{\text{acceleration}}}$

So, $T = 2\pi\sqrt{\frac{x}{kx}} = 2\pi\sqrt{\frac{1}{k}}$

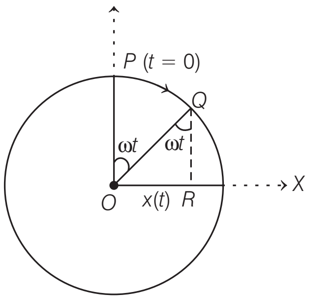
9. Weight kept on the system will separate from the piston, when the maximum force just exceeds the weight of the body. Hence,

$$m\omega^2 y = mg$$

or $y = g/\omega^2 = 9.8/(2\pi)^2 = 0.25$ m

10. Given, $T = 30\,\text{s}, OQ = B$. The projection of the radius vector on the diameter of the circle when a particle is moving with uniform angular velocity (ω) on a circle of reference is SHM. Let the particle move from P to Q in time t, then $\angle POQ = \omega t = \angle OQP$.

The projection of radius OQ on X-axis will be $OR = x(t)$ say.



In ΔOQR, $\sin \omega t = \frac{x(t)}{B}$

or $x(t) = B\sin \omega t = B\sin \frac{2\pi}{T} t = B\sin \frac{2\pi}{30} t$

11. When bigger pendulum of time period $(5T/4)$ completes one vibrations, the smaller pendulum will complete $(5/4)$ vibrations. It means the smaller pendulum will be leading the bigger pendulum by phase $T/4$ sec $= \frac{\pi}{2}$ rad $= 90°$.

12. We can find the velocities, $v_1 = \frac{dy_1}{dt} = 2 \times 10 \cos(10t + \theta)$

and $v_2 = -3 \times 10 \sin 10t = 30 \cos(10t + \pi/2)$

$\therefore$ Phase difference $= (10t + \theta) - (10t + \pi/2) = \theta - \pi/2$

13. In simple harmonic motion, position (x), velocity (v) and acceleration (a) of the particle are given by

$$x = A\sin \omega t$$
$$v = \omega\sqrt{A^2 - x^2}$$

or $v = A\omega \cos \omega t$

and $a = -\omega^2 x$

or $a = -\omega^2 A \sin \omega t$

Given, amplitude $A = 5$ cm and displacement $x = 4$ cm.

At this time (when $x = 4$ cm), velocity and acceleration have same magnitude.

$\Rightarrow$ $|v_{x=4}| = |a_{x=4}|$

or $|\omega\sqrt{5^2 - 4^2}| = |-4\omega^2|$

$\Rightarrow$ $3\omega = +4\omega^2$

$\Rightarrow$ $\omega = (3/4)$ rad/s

So, time period, $T = \frac{2\pi}{\omega}$

$\Rightarrow$ $T = \frac{2\pi}{3} \times 4 = \frac{8\pi}{3}$ s

14. The time periods, $T_1 = 2\pi\sqrt{\frac{121}{g}}$ and $T_2 = 2\pi\sqrt{\frac{100}{g}}$

So, $T_1 > T_2$, Let the shorter pendulum makes n vibrations, then the longer pendulum will make less than n vibrations to come in phase again.

So, $nT_2 = (n-1)T_1$

or $n \times 2\pi\sqrt{\frac{100}{g}} = (n-1) \times 2\pi\sqrt{\frac{121}{g}}$

or $10n = (n-1)11$

or $n = 11$

15. Given, angular frequency of the piston, $\omega = 200$ rad/min

and stroke length = 1 m

$\therefore$ Amplitude of SHM, $A = \frac{\text{Stroke length}}{2} = \frac{1}{2} = 0.5$ m

Now, $v_{max} = \omega A$

$= 200 \times 0.5 = 100$ m/min

16. As, Wavelength = Velocity of wave × Time period

i.e. $\lambda = 300 \times 0.05 = 15$ m

According to the problem path difference between two points

$= 15 - 10 = 5$ m

$\therefore$ Phase difference $= \frac{2\pi}{\lambda} \times$ path difference

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{15} \times 5 = \frac{2\pi}{3}$$

17. At one of the extreme position, weight of block = restoring force. At the other extreme position where it is momentary at rest, weight of block and restoring force both act in downward direction. So the block weighs double than its weight.

18. As, $F_1 = \frac{m4\pi^2 a}{T_1^2}$ and $F_2 = \frac{m4\pi^2 a}{T_2^2}$

Net force, $F = F_1 + F_2$

$$= \frac{4\pi^2 ma}{T_1^2} + \frac{4\pi^2 ma}{T_2^2}$$

$$= 4\pi^2 ma\left(\frac{1}{T_1^2} + \frac{1}{T_2^2}\right)$$

or $\frac{4\pi^2 ma}{T^2} = 4\pi^2 ma\left(\frac{1}{T_1^2} + \frac{1}{T_2^2}\right)$

or $\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$

or $\frac{1}{T^2} = \frac{T_1^2 + T_2^2}{T_1^2 T_2^2}$

or $T^2 = \frac{T_1^2 T_2^2}{T_1^2 + T_2^2}$

19. As, $x = A\sin(\omega + \pi/2) = A\cos \omega t$

$\therefore$ $\cos \omega t = x/A$

and $\sin \omega t = \sqrt{1 - (x^2/A^2)}$

$$y = A\sin 2\omega t = \sqrt{1 - (x^2/A^2)}$$

$$y = A\sin 2\omega t = 2A\sin \omega t \cos \omega t$$

or $y^2 = 4A^2 \sin^2 \omega t \cos^2 \omega t$

$$= 4A^2 \times \frac{x^2}{A^2} \times \left(\frac{A^2 - x^2}{A^2}\right)$$

$$= 4x^2 \left(1 - \frac{x^2}{A^2}\right)$$

20. Let, O be the position and x be the distance of coin from O. The coin will slip if pseudo force on coin just becomes equal to force of friction, *i.e.* $mx\omega^2 = \mu mg$.

The coin will slip if, x = maximum = amplitude A

$$m A\omega^2 = \mu mg$$

or $A = \mu g / \omega^2$

21. As, $x(t) = A\cos(\omega t + \phi)$ ...(i)

$\therefore$ $1 = A\cos(\pi \times 0 + \phi) = A\cos\phi$ ...(ii)

$$\text{velocity} = \frac{d[x(t)]}{dt} = -A\omega \sin(\omega t + \phi)$$

$$\pi = -A \times \pi \sin(0 + \phi) = -\pi A \sin\phi - 1$$

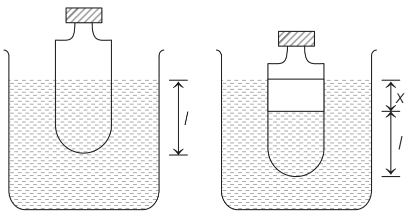
$= A\sin\phi$ ...(ii)

Squaring and adding Eqs. (ii) and (iii), we have

$$1 + 1 = A^2(\cos^2\omega + \sin^2\omega) = A^2$$

or $A = \sqrt{2}$ cm

22. In equilibrium condition bottle floats in water and its length l inside water is same as the height of water upto which bottle is filled.



So, l = Volume of water in bottle/Area

$$= \frac{310}{\pi \times (2.5)^2} = 15.8 \text{ cm} = 0.158 \text{ m}$$

When bottle is slightly pushed inside by an amount x then, restoring force acting on the bottle is the upthrust of fluid displaced when bottle goes into liquid by amount x.

So, restoring force,

$F = -(\rho A x) g$ ...(i)

where ρ = density of water,

A = area of cross-section of bottle

and x = displacement from equilibrium position.

But $F = ma$ ...(ii)

where, m = mass of water and bottle system

$= Al\rho$

From Eqs. (i) and (ii), we have

‘ $$Al\rho\alpha = -\rho Axg$$

or $a = -\frac{g}{l}x$

As for SHM, $a = -\omega^2 x$

We have $\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{10}{0.158}} = \sqrt{63.29} \approx 8 \text{ rad s}^{-1}$

23. Here, $a = 1$ cm $= 0.01$ m. The mass will remain in contact with surface, if

$$mg = m\omega^2 a$$

or $\omega = \sqrt{g/a}$

or $2\pi\nu = \sqrt{g/a}$

or $\nu = \frac{1}{2\pi}\sqrt{\frac{g}{a}}$

$$= \frac{7}{2 \times 22}\sqrt{\frac{980}{1}}$$

$= 4.9$ Hz ≈ 5 Hz

24. Maximum force on body while in SHM

$$= m\omega^2 a = 0.5\,(2\pi/2)^2 \times 0.2 = 1 \text{ N}$$

Maximum force of friction $= \mu mg = 0.3 \times 0.5 \times 10 = 1.5$ N

Since, the maximum force on the body due to SHM of the platform is less than the maximum possible frictional force, so the maximum force of friction will be equal to the maximum force acting on body due to SHM of platform, *i.e.* 1 N.

25. As, $x = a\sin\omega t$

and $y = b\sin(\omega t + \pi) = -b\sin\omega t$

$\Rightarrow \frac{x}{a} = -\frac{y}{b} \Rightarrow y = -\frac{b}{a}x$

It is an equation of a straight line.

26. Time period, $T = 2\pi\sqrt{\frac{L}{g}}$ and $\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$

$\therefore$ Displacement, $x = a\cos\omega t = a\cos\sqrt{\frac{g}{L}}\,t$

27. Given, $x = 3\sin\omega t + 4\sin(\omega t + \pi/3)$

Comparing it with the equation

$$x = r_1 \sin\omega t + r_2 \sin(\omega t + \phi)$$

We have, $r_1 = 3$ cm, $r_2 = 4$ cm and $\phi = \pi/3$

The amplitude of combination is

$$r = \sqrt{r_1^2 + r_2^2 + 2r_1 r_2 \cos\phi}$$

$$= \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \times \cos\pi/3}$$

$= \sqrt{37} \approx 6$ cm

28. As, $\sec bt$ is not define for $bt = \pi/2$

and $x = a\sec bt + c\,\text{cosec}\, bt = \frac{a\sin bt + c\cos bt}{\sin bt \cos bt}$

This equation cannot be modified in the form of simple equation of SHM containing sine or cosine function.

So, it cannot represent SHM.

29. When particle is at $x = 2$, the displacement is $y = 4 - 2 = 2$ cm. If t is the time taken by the particle to go from $x = 4$ cm to $x = 2$ cm, then

$$y = a\cos\omega t = a\cos\frac{2\pi t}{T} = a\cos\frac{2\pi t}{1.2}$$

or $\cos\frac{2\pi t}{1.2} = \frac{y}{a} = \frac{2}{4}$

$\cos\frac{2\pi t}{1.2} = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow \quad \frac{2\pi t}{1.2} = \frac{\pi}{3}$

or $\frac{2t}{1.2} = \frac{1}{3}$

or $t = \frac{1.2}{6} = 0.2$ s

Time taken to move from $x = +2$ cm to $x = +4$ cm and back again $= 2t = 2 \times 0.2$ s $= 0.4$ s.

30. Potential energy, $V = \frac{1}{2}m\omega^2 x^2$

and kinetic energy $E, T = \frac{1}{2}m\omega^2(a^2 - x^2)$

$\therefore \quad \frac{T}{V} = \frac{a^2 - x^2}{x^2}$

31. Let x be point, where KE = PE

Hence, $\frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2}m\omega^2 x^2$

$\Rightarrow \quad 2x^2 = a^2$

$\Rightarrow \quad x = \frac{a}{\sqrt{2}} = \frac{y}{\sqrt{2}} = 2\sqrt{2}$ cm

32. As, $\frac{U}{U_{\max}} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} = \frac{1}{4}$

$\frac{y^2}{a^2} = \frac{1}{4} \Rightarrow y = \frac{a}{2}$

33. Potential energy of body in SHM at an instant,

$$U_x = \frac{1}{2}ky^2$$

If the displacement, $y = (a - x)$, then

$$U_x = \frac{1}{2}k(a - x)^2 = \frac{1}{2}k(x - a)^2$$

34. Total energy of oscillation, $E = \alpha A^4$

(at maximum displacement)

Kinetic energy of mass at $x = x$ is

$$K = E - U = \alpha(A^4 - x^4)$$

As $K = 3U$

$\alpha(A^4 - x^4) = 3\alpha x^4$

or $x = \pm\frac{A}{\sqrt{2}}$

35. Total energy, $E = \frac{1}{2}m\omega^2 a^2$ ...(i)

$\text{KE} = \frac{3E}{4} = \frac{1}{2}m\omega^2(a^2 - y^2)$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{3}{4} = \frac{a^2 - y^2}{a^2}$$

or $y^2 = a^2/4$

or $y = a/2$

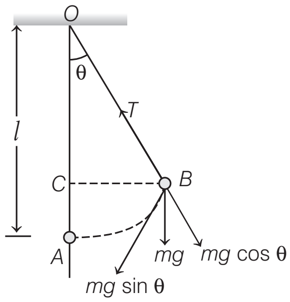
36. Kinetic energy at mean position $= \frac{1}{2}m\omega^2 a^2 = 8 \times 10^{-3}$

or $\omega = \left(\frac{2 \times 8 \times 10^{-3}}{ma^2}\right)^{1/2} = \left[\frac{2 \times 8 \times 10^{-3}}{0.1 \times (0.1)^2}\right]^{1/2} = 4$ rad/s

Equation of SHM is,

$$y = a\sin(\omega t + \theta) = 0.1\sin\left(4t + \frac{\pi}{4}\right)$$

37. From the figure, $OC = l\cos\theta$



$\therefore \quad AC = OA - OC$

$= l - l\cos\theta$

$= l(1 - \cos\theta)$

Maximum KE of bob at O or A = Maximum PE of bob at B

$$= ma \times AC = mgl(1 - \cos\theta)$$

38. If m is the mass and r is the amplitude of oscillation, then maximum kinetic energy,

$$K_0 = \frac{1}{2}m\omega^2 r^2$$

or $r = \left(\frac{2K_0}{m\omega^2}\right)^{1/2}$

The displacement equation can be

$$y = r\sin\omega t = \left(\frac{2K_0}{m\omega}\right)^{1/2}\sin\omega t$$

39. As, $n = \frac{1}{2\pi}\sqrt{\frac{k_{\text{effective}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k + 2k}{m}} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

(for parallel combination of spring, $k_{eq} = k_1 + k_2$)

40. Let k be the force constant of spring of length l_2. Since, $l_1 = nl_2$, where n is an integer, so the spring is made of $(n + 1)$ equal parts in length, each of length l_2.

$\therefore \quad \frac{1}{k} = \frac{(n+1)}{k}$

or $k = (n + 1)k$

The spring of length $l_1 = n\,l_2$ will be equivalent to n spring connected in series where spring constant $k' = \frac{k}{n} = \frac{(n+1)k}{n}$

41. We have, $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$;

$$n' = \frac{1}{2\pi}\sqrt{\frac{k'}{2m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{2m}} = n \qquad (\because k' = 2k)$$

42. Let k be the spring constant of each half part of the spring. For a complete spring, the spring constant $k' = k/2$ (spring in series). When two splitted parts of a spring are connected to the body, then spring are in parallel. Their effective spring constant, $k' = k + k = 2k$.

As $$T = 2\pi\sqrt{\frac{m}{k}}$$

or $$T \propto \frac{1}{\sqrt{k}} \qquad \text{(for a fixed value of } m)$$

$\therefore$ $$\frac{T'}{T} = \sqrt{\frac{k/2}{2k}} = \frac{1}{2}$$

or $$T' = \frac{T}{2}$$

43. It is a system of two springs in parallel. The restoring force on the body is due to springs and not due to gravity.

Therefore slope is irrelevant.

Here, the effective spring constant $= k + k = 2k$

Thus time period, $T = 2\pi\sqrt{M/2k}$

44. As, $T = 2\pi\sqrt{\frac{m}{k}}$

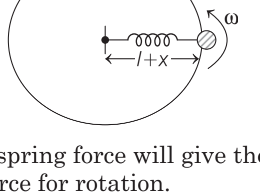
and $$T' = 2\pi\sqrt{\frac{2m}{2k}} = 2\pi\sqrt{\frac{m}{k}} = T$$

45. Two spring, each of spring constant k_1 in parallel, give equivalent spring constant of $2k_1$ and this is in series with another spring of spring constant k_2, so equivalent spring constant

$$k = \left(\frac{1}{k_2} + \frac{1}{2k_1}\right)^{-1}$$

46. Initially, it is given that the unstretched length of the spring is l. When it is given an angular speed ω, then let x = stretched length of spring.

Then, total length of the spring system while rotating will be $(l + x)$ as shown in the figure.



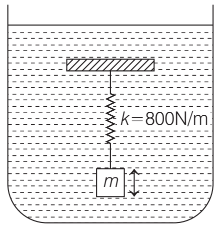
As we know, spring force will give the necessary centripetal force for rotation.

So, $$kx = m(l + x)\omega^2$$

$\Rightarrow$ $$(k - m\omega^2)x = ml\omega^2$$

$\Rightarrow$ $$x = \frac{ml\omega^2}{(k - m\omega^2)}$$

47. The given situation is shown in the figure given below.



When vibrations of mass are suddenly stopped, oscillation energy (or stored energy of spring) is dissipated as heat, causing rise of temperature.

So, conversation of energy gives

$$\frac{1}{2}kx_m^2 = (m_1 s_1 + m_2 s_2)\Delta T$$

where, x_m = amplitude of oscillation,

s_1 = specific heat of mass,

s_2 = specific heat of water

and ΔT = rise in temperature.

Substituting values given in question, we have

$$\frac{1}{2} \times 800 \times (2 \times 10^{-2})^2 = \left[\left(\frac{500}{1000}\right) \times 400 + 1 \times 4184\right]\Delta T$$

$\Rightarrow$ $$\Delta T = \frac{16 \times 10^{-2}}{4384}$$

$$= 3.65 \times 10^{-5} \text{ K}$$

So, the order of magnitude of change in temperature is approx 10^{-5}.

48. As, $T = 2 = 2\pi\sqrt{\frac{M}{k}}$

and $$2 + 1 = 2\pi\sqrt{\frac{M+4}{k}} \qquad \text{(from question)}$$

or $$3 = 2\pi\sqrt{\frac{k+4}{k}}$$

So, $$\frac{4}{9} = \frac{M}{M+4}$$

or $$4M + 16 = 9M$$

or $$M = \frac{16}{5} = 3.2 \text{ kg}$$

49. A total restoring force, $F = kX = mg$

or $$k = mg/X$$

$\therefore$ $$T = 2\pi\sqrt{\frac{(M+m)}{mg/X}} = 2\pi\sqrt{\frac{(M+m)X}{mg}}$$

50. The amplitude of oscillations will be the maximum when compression in the spring is maximum. At the time of maximum compression, velocities of both the blocks are equal say v, then using law of conservation of momentum,

$$m_1 v_0 = (m_1 + m_2)v$$

or $$1 \times 12 = (1 + 2)v$$

or $$v = 4 \text{ cms}^{-1}$$

Using law of conservation of energy, we have

$$\frac{1}{2}m_1v_0^2=\frac{1}{2}kx^2+\frac{1}{2}(m_1+m_2)v^2$$

Putting the value of $v=4\ \text{cms}^{-1}$ and solving, we get $x=2$ cm.

51. As, energy stored = work done

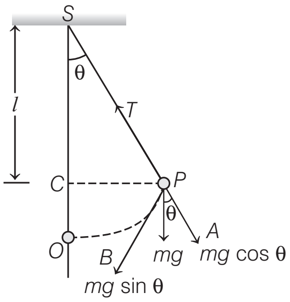
$\Rightarrow \quad E=\frac{1}{2}kr^2$ (where, r = displacement)

or $\quad r=\sqrt{\frac{2E}{k}}=\sqrt{\frac{2\times2}{400}}=\frac{1}{10}\text{m}$

Now, $\quad a=\omega^2r=\left(\sqrt{\frac{k}{m}}\right)^2\times\frac{1}{10}$

$$=\left(\frac{400}{1}\right)\times\frac{1}{10}=40\ \text{ms}^{-2}$$

52.



When the bob is displaced to position P, through a small angle θ from the vertical, the various forces acting on the bob at P are

(i) the weight mg of the bob acting vertically downwards

(ii) the tension T in the string acting along PS

Resolving mg into two rectangular components, we get

(a) $mg\cos\theta$ acts along PA, opposite to tension,

(b) $mg\sin\theta$ acts along PB, tangent to the arc OP and directed towards O.

If the string neither slackens nor breaks but remains taut, then

$$T=mg\cos\theta$$

The force $mg\sin\theta$ tends to bring the bob back to its mean position O.

$\therefore$ Restoring force acting on the bob is

$$F=-mg\sin\theta$$

53. As, $T=2\pi\sqrt{\frac{l}{g}}$. When lift is accelerated upwards with acceleration a (= $g/4$), the effective acceleration due to gravity inside the lift,

$$g_1=g+a=g+\frac{g}{4}=\frac{5g}{4}$$

$\therefore \quad T_1=2\pi\sqrt{\frac{l}{5g/4}}$

$$=2\pi\sqrt{\frac{l}{g}}\times\frac{2}{\sqrt5}=\frac{2T}{\sqrt5}$$

54. On the inclined plane, the effective acceleration due to gravity

$$g'=g\cos30^\circ=g\times\sqrt3/2$$

$$T=2\pi\sqrt{\frac{l}{g'}}=2\pi\sqrt{\frac{2l}{\sqrt3\,g}}$$

55. As, $T=2\pi\sqrt{l/g}$

$\Rightarrow \quad \log T=\log2+\log\pi+\frac{1}{2}\log l-\frac{1}{2}\log g$

Differentiating it, we get

$$\frac{dT}{T}=\frac{1}{2}\frac{dl}{l}-\frac{1}{2}\frac{dg}{g}=-\frac{1}{2}\frac{dg}{g} \quad (\because l\text{ is constant})$$

% change in time period

$$=\frac{dT}{T}\times100=\frac{1}{2}\frac{dg}{g}\times100$$

$$=-\frac{1}{2}\left(\frac{-2}{100}\right)\times100=1\% \quad \text{(increase)}$$

56. Resolving tension T in string into two rectangular components, we get

$$T\cos\theta=mg$$

and $\quad T\sin\theta=mr\omega^2$

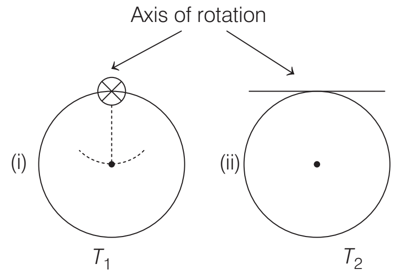
So, $\quad \frac{T\sin\theta}{T\cos\theta}=\tan\theta=\frac{r\omega^2}{g}$

or $\quad g\tan\theta=r\omega^2=r\times4\pi^2/T^2$

or $\quad T=2\pi\sqrt{\frac{r}{g\tan\theta}}=2\pi\sqrt{\frac{l\sin\theta}{g\tan\theta}}$

$$=2\pi\sqrt{\frac{l\cos\theta}{g}}$$

57. Let I_1 be the moment of inertia in case (i) and I_2 be the moment of inertia in case (ii).



The time period of physical pendulum is given by

$$T=2\pi\sqrt{\frac{I}{mgd}} \quad \text{(where, } d=R)$$

Here, $\quad I_1=2\,mR^2 \Rightarrow I_2=\frac{3}{2}mR^2$

$\therefore \quad T_1=2\pi\sqrt{\frac{I_1}{mgd}}$

and $\quad T_2=2\pi\sqrt{\frac{I_2}{mgd}}$

$\Rightarrow \quad \frac{T_1}{T_2}=\sqrt{\frac{I_1}{I_2}}=\sqrt{\frac{2\,mR^2}{\frac{3}{2}mR^2}}=\frac{2}{\sqrt3}$

58. Period of motion of a pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \ldots(i)$$

On the surface of Earth, let period of motion is T_e and acceleration due to gravity is g_e

$$\therefore \quad T_e = 2\pi\sqrt{\frac{l}{g_e}} \quad \ldots(ii)$$

On the another planet, let period of motion is T_p and gravitational acceleration is g_p

$$\therefore \quad T_p = 2\pi\sqrt{\frac{l}{g_p}} \quad \ldots(iii)$$

($\because$ Pendulum is same, so l will be same)

From Eqs. (ii) and (iii), we get

$$\frac{T_e}{T_p} = \frac{2\pi\sqrt{\frac{l}{g_e}}}{2\pi\sqrt{\frac{l}{g_p}}} = \sqrt{\frac{g_p}{g_e}} \quad \ldots(iv)$$

Now, $g_e = \frac{GM_e}{R_e^2}$ and $g_p = \frac{GM_p}{R_p^2}$

Given, $M_p = 3M_e$ and $R_p = 3R_e$

$$\therefore \quad g_p = \frac{G \times 3M_e}{9R_e^2} = \frac{1}{3} \cdot \frac{GM_e}{R_e^2} = \frac{1}{3} g_e$$

$$\Rightarrow \quad \frac{g_p}{g_e} = \frac{1}{3} \text{ or } \sqrt{\frac{g_p}{g_e}} = \frac{1}{\sqrt{3}} \quad \ldots(v)$$

From Eqs. (iv) and (v), $T_p = \sqrt{3}\ T_e$

or $\quad T_p = 2\sqrt{3}$ s $\quad$ ($\because T_e = 2$ s)

59. Torque acting on the bob $= I\alpha = -(mg)l \sin\theta$

or $\quad (m_i l^2)\,\alpha = -(m_g g) l\,\theta$

or $\quad \alpha = -\left(\frac{m_g g}{m_i l}\right)\theta = -\omega^2\theta$

where, $\quad \omega^2 = \frac{m_g g}{m_i l}$

$$\therefore \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m_i l}{m_g g}}$$

60. Since length of pendulums A and C is same and $T = 2\pi\sqrt{l/g}$, hence their time period is same and they will have same frequency of vibration. Due to it, a resonance will take place and the pendulum C will vibrate with maximum amplitude.

61. According to question,

$$T' = T\sqrt{\frac{l + 2\,l/100}{l}}$$

$$= T\left(1 + \frac{2}{100}\right)^{1/2} = 2\left(1 + \frac{1}{100}\right)$$

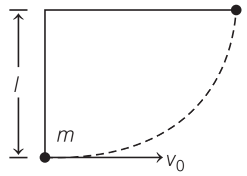
$\left[\because \text{ using binomial theorem, } (1 + x)^n = 1 + nx \Rightarrow \left(1 + \frac{2}{100}\right)^{1/2} = \left(1 + \frac{1}{100}\right)\right]$

$$\therefore \quad T' - T = \frac{2}{100} = \frac{1}{50}\text{ s}$$

Therefore, loss in seconds per day

$$= \frac{1/50}{2} \times 24 \times 60 \times 60 = 864\text{ s}$$

62. According to the law of conservation of mechanical energy, we get



$$\frac{1}{2}mv_0^2 = mgl \Rightarrow v_0 = \sqrt{2gl}$$

63. When the bob of pendulum is brought to a position making an angle θ with the equilibrium position, the height of bob of pendulum will be,

$h = l - l\cos\theta = l(1 - \cos\theta)$.

Taking free fall of the bob,

$$u = 0, a = g, g = h = l(1 - \cos\theta), v = ?$$

Now, $\quad v^2 = u^2 + 2\,gh = 0 + 2\,gl(1 - \cos\theta)\,f$

or $\quad v = \sqrt{2\,gl(1 - \cos\theta)}$

64. When the ball of mass m falls from a height h, it reaches the surface of Earth in time, $t = \sqrt{2h/g}$ with velocity $v = \sqrt{2gh}$. It then moves into the tunnel and reaches upto a height h from other side of Earth. The ball then returns back and thus executes periodic motion. Outside the Earth, ball crosses distance h four times. When the ball is in the tunnel at distance x from the centre of the Earth, then gravitational force acting on ball is

$$F = \frac{Gm}{x^2} \times \left(\frac{4}{3}\pi x^3 \rho\right) = G \times \left(\frac{4}{3}\pi\rho\right) mx$$

Mass of the Earth, $M = \frac{4}{3}\pi R^3 \rho$

or $\quad \frac{4}{3}\pi\rho = \frac{M}{R^3}$

$$\therefore \quad F = \frac{GMmx}{R^3}$$

i.e. $\quad F \propto x$

As, this force, F is directed towards the centre of Earth, *i.e.* the mean position. So, the ball will execute periodic motion about the centre of Earth.

Here, Inertia factor = mass of ball = m

Spring factor $= \frac{GMm}{R^3} = \frac{gm}{R}$

$\therefore$ Time period of oscillation of ball in the tunnel is

$$T' = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

$$= 2\pi\sqrt{\frac{m}{gm/R}} = 2\pi\sqrt{\frac{R}{g}}$$

Time spent by ball outside the tunnel on both the sides will be $= 4\sqrt{2h/g}$.

Therefore, total time period of oscillation of ball is

$$= 2\pi\sqrt{\frac{R}{g}} + 4\sqrt{\frac{2h}{g}}$$

65. Let T_1 and T_2 be the time period of shorter length and longer length pendulums respectively. As per question, $nT_1 = (n-1)T_2$

So $\quad n\,2\pi\sqrt{\frac{0.5}{g}} = (n-1)\,2\pi\sqrt{\frac{20}{g}}$

or $\quad n = (n-1)\sqrt{40} \approx (n-1)6$

Hence, $\quad 5n = 6$

Hence, after 5 oscillations they will be in same phase.

66. In damped oscillation, amplitude goes on decaying exponentially,

$$a = a_0\, e^{-bt}$$

where, b = damping coefficient initially

$$\frac{a_0}{3} = a_0 e^{-b\times 100T},$$

T = time of one oscillation

or $\quad \frac{1}{3} = e^{-100bT} \quad \ldots(\text{i})$

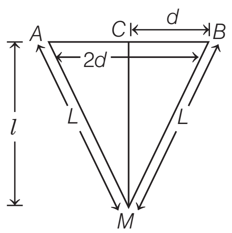
Finally, $\quad a = a_0 e^{-b\times 200T}$

or $\quad a = a_0 (e^{-100bt})^2$

or $\quad a = a_0 \times \left(\frac{1}{3}\right)^2 \quad [\because \text{from Eq. (i)}]$

$$a = \frac{a_0}{9}$$

67. The motion of M is SHM, with length, $l = \sqrt{L^2 - d^2}$



$\therefore$ Time period, $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{(L^2 - d^2)^{1/2}}{g}}$

68. Let the distance x of vertical side c of block be pushed in liquid, when block is floating, the buoyancy force

$$= abxd\omega\, g = abxg \qquad (\because d\omega = 1)$$

The mass of piece of wood $= abcd$

So, $\quad$ acceleration $= -abxg/abcd = -\left(\frac{g}{cd}\right)x$

Hence, time period, $\quad T = 2\pi\sqrt{\frac{dc}{g}}$

69. We have, $T' = 2\pi\sqrt{l/(g/6)} = \sqrt{6}T$

Hence, the clock will tick in one minute,

$$n = 60/\sqrt{6} = 24.5 \text{ times}$$

70. When the cylinder is given a small downward displacement, say y, the additional restoring force is due to (i) additional extension y, which is, $F_1 = ky$ (ii) additional buoyancy which is $F_2 = AYd\ g$.

Total restoring force,

$$-F = F_1 + F_2 = (k + Adg)Y$$
$$= \text{new force constant}$$

$\therefore \quad n = \frac{1}{2\pi}\sqrt{\frac{k'}{M}}$

$$= \frac{1}{2\pi}\sqrt{\frac{k + Adg}{M}}$$

71. Amplitude resonance takes place at a frequency of external force which is less than the frequency of undamped maximum vibration, i.e. $\omega_1 \neq \omega_0$. Velocity-resonance takes place (*i.e.* maximum energy), when frequency of external periodic force is equal to natural frequency of undamped vibrations, i.e. $\omega_2 = \omega_0$.

72. Fig. (1) alone represents damped SHM as displacement is decreasing regularly with time.

73. Motion given here is SHM starting from rest. Therefore, the graph shown in option (a) is correct.

74. For damped motion, $a = a_0 e^{-bt}$

For first case,

$$\frac{a_0}{3} = a_0 e^{-b\times 2} \quad \text{or} \quad \frac{1}{3} = e^{-2b}$$

For second case, $\frac{a_0}{n} = a_0 e^{-(b\times 6)}$

or $\quad \frac{1}{n} = e^{-6b} = (e^{-2b})^3 = \left(\frac{1}{3}\right)^3 \Rightarrow n = 3^3$

Round II

1. A ball bearing when released a little above the lower limit inside a smooth curved bowl, will execute SHM with a definite period.

2. As, $\quad T = 2\pi\sqrt{\frac{m}{k}}$

and $\quad T' = 2\pi\sqrt{\frac{m}{4k}} = \frac{T}{2} = \frac{5}{2}\text{ s} = 2.5\text{ s}$

3. Phase is the state of a particle as regards with its position and direction of motion w.r.t. mean position. In the given curve phase is same when $t = 1$ s and $t = 5$ s. Also, phase is same when $t = 2$ s and $t = 6$ s.

4. Given, $\frac{1}{2}m\omega^2 r^2 = 80$ J;

$\therefore \quad \text{PE} = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 \times \left(\frac{3}{4}r\right)^2$

$$= \frac{9}{16}\left(\frac{1}{2}m\omega^2 r^2\right) = \frac{9}{16}\times 80 = 45\text{ J}$$

5. KE is maximum at mean position and minimum at extreme position $\left(\text{at } t = \frac{T}{4}\right)$, so correct graph is (c).

6. During oscillation, (motion of a simple pendulum) KE is maximum at mean position where PE is minimum. At extreme position, KE is minimum and PE is maximum. Thus, correct graph is depicted in option (b).

7. Given, $x = A\sin(\omega t + \alpha)$ ...(i)

and $y = A\cos(\omega t + \alpha)$ ...(ii)

Squaring and adding Eqs. (i) and (ii), we get

$$x^2 + y^2 = A^2[\sin^2(\omega t + \alpha) + \cos^2(\omega t + \alpha)] = A^2$$

It is an equation of a circle. The given motion is counter-clockwise.

8. The moment of inertia of a cylindrical rod about axis of wire(*i.e.* an axis passing through the centre of rod and perpendicular to its length) is

$$I = M\left(\frac{L^2}{12} + \frac{R^2}{4}\right)$$

9. As, $\frac{1}{2}m\omega^2(r^2 - y^2) = \frac{1}{3}\times\frac{1}{2}m\omega^2 r^2$

or $r^2 - y^2 = \frac{1}{3}r^2$

or $3r^2 - 3y^2 = r^2$

or $2r^2 - 3y^2 = 0$

or $r = \sqrt{\frac{3}{2}}\times y = \sqrt{\frac{3}{2}}\times 4 = 2\sqrt{6}$ cm

10. In equilibrium, $T = mg$

Work done $= mg = mgx = \frac{1}{2}kx^2$

or $x = \frac{2mg}{k} = \frac{2T}{k}$ ...(i)

Energy stored $= mgx = T\,x$

$= T\times\frac{2T}{k} = \frac{2T^2}{k}$ [from Eq. (i)]

11. Here, $2 = 2\pi\sqrt{\frac{m}{k}}$

and $3 = 2\pi\sqrt{\frac{m+2}{k}}$

So, $\frac{3}{2} = \sqrt{\frac{m+2}{m}}$

or $9m = 4m + 8$

or $m = 1.6$ kg

12. Since the time period of simple pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{t_0}{9} = 2\pi\sqrt{\frac{l_1}{g}} \text{ and } \frac{t_0}{7} = 2\pi\sqrt{\frac{l_2}{g}}$$

$\therefore$ $\frac{l_1}{l_2} = \left(\frac{7}{9}\right)^2 = \left(\frac{49}{81}\right)$

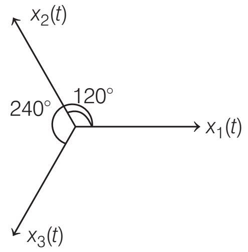
13. From the figure, $T\sin\theta = mL\sin\theta\omega^2$

$$324 = 0.5\times0.5\times\omega^2$$

$$\omega^2 = \frac{324}{0.5\times0.5}$$

$\Rightarrow$ $\omega = \sqrt{\frac{324}{0.5\times0.5}} = \frac{18}{0.5} = 36$ rad/s

14.



It is clear from figure that, magnitude of $x_3(t)$, must be the resultant of $x_1(t)$ and $x_2(t)$, to bring the mass at rest.

So, $B = A$ and $\phi = 240° = \frac{4\pi}{3}$.

15. Maximum KE $= \frac{1}{2}m\omega^2A^2$; minimum KE $= 0$

Average KE $= \frac{0 + \frac{1}{2}m\omega^2A^2}{2} = \frac{1}{4}m\omega^2A^2$

Similarly, average PE $= \left(\frac{0 + \frac{1}{2}m\omega^2A^2}{2}\right) = \frac{1}{4}m\omega^2A^2$

16. Given, $x(t) = A\cos(\omega t + \theta)$

Velocity, $v = \frac{dx}{dt} = -A\omega\sin(\omega t + \theta)$

$= -A\omega\sqrt{1 - \cos^2(\omega t + \theta)}$

$\Rightarrow$ $v = -A\,\omega\sqrt{1 - x^2/A^2} = -\omega\sqrt{A^2 - x^2}$

Here, $v = \pi$ cms^{-1}, $x = 1$ cm, $\omega = \pi$ s^{-1}

So, $\pi = -\pi\sqrt{A^2 - 1^2}$

or $(-1)^2 = A^2 - 1$

or $A^2 = 2$

or $A = \sqrt{2}$ cm

17. As, $x_1 = a\sin(\omega\times1) = a\sin\omega$

and $x_2 = a\sin(\omega\times2) - a\sin\omega$

Now $\frac{x_2}{x_1} = \frac{\sin(2\omega) - \sin\omega}{\sin\omega}$

$= \sin 2\times(2\pi/8) - \sin 2\pi/8$ (from question)

$= \frac{1 - (1/\sqrt{2})}{(1/\sqrt{2})} = \frac{\sqrt{2} - 1}{1}$

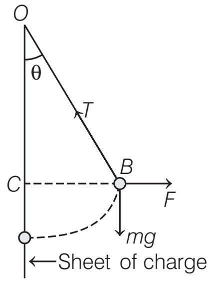
or $\frac{x_1}{x_2} = \frac{1}{\sqrt{2} - 1} = \frac{\sqrt{2} + 1}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$

$= \frac{\sqrt{2} + 1}{2 - 1}$

$= 2.414 = 2.4$

18. Electric intensity at B due to sheet of charge,

$$E = \frac{1}{2}\frac{\sigma q}{\varepsilon_0}$$



Force on the bob due to sheet of charge,

$$F = qE = \frac{1}{2}\frac{\sigma q}{\varepsilon_0}$$

As, the bob is in equilibrium, so

$$\frac{mg}{OC} = \frac{F}{CB} = \frac{T}{BO}$$

$\therefore$ $$\tan\theta = \frac{CB}{OC} = \frac{F}{mg} = \frac{\frac{1}{2}\sigma q/\varepsilon_0}{mg} = \frac{\sigma q}{2\varepsilon_0 mg}$$

19. When $t = 1\text{ s},\ y_1 = r\sin\omega\times 1 = r\sin\omega$

when $t = 2\text{ s},\ y_2 = r\sin\omega\times 2 = r\sin 2\omega$

$\therefore$ $$\frac{y_1}{y_2} = \frac{r\sin\omega}{r\sin 2\omega}$$

$$= \frac{1}{2\cos\omega} = \frac{1}{2\cos 2\pi/T}$$

$$= \frac{1}{2\cos 2\pi/8}$$

$$= \frac{1}{2\cos\pi/4} = \frac{1}{2(1/\sqrt{2})} = \frac{1}{\sqrt{2}}$$

$\therefore$ $y_2 = \sqrt{2}y_1$

Distance covered in 2nd second $= y_2 - y_1 = (\sqrt{2}-1)y_1$

$\therefore$ Ratio $= 1 : (\sqrt{2}-1)$

20. When the bob falls through a vertical height h, the velocity acquired at the lowest point,

$$v = \sqrt{2gh} = \sqrt{2\times 10\times 1} = \sqrt{20}\ \text{ms}^{-1}$$

Centrifugal force $= \frac{mv^2}{r} = \frac{0.01\times 20}{1} = 0.20$ N

Net tension = Weight + Centrifugal force

$= (0.01\times 10 + 0.20) = 0.30$ N

21. v and v' are the velocities of the block of mass M and $(M+m)$ respectively while passing from the mean position when executing SHM.

Using law of conservation of linear momentum, we have

$$mv = (M+m)\,v'$$

or $$v' = mv/(M+m)$$

Also, maximum PE = maximum KE

$\therefore$ $$\frac{1}{2}kA'^2 = \frac{1}{2}(M+m)\,v'^2$$

or $$A' = \left(\frac{M+m}{k}\right)^{1/2}\times\frac{mv}{(M+m)}$$

$$= \frac{mv}{\sqrt{(M+m)k}}$$

22. The Lissajous figure will be parabola, if period ratio is 1 : 2 and phase difference is $\pi/2$.

Let $x = a\sin(2\omega t + \pi/2)$ and $y = b\sin\omega t$

$\therefore$ $\sin\omega t = y/b$

Now, $$\frac{x}{a} = \sin(2\omega t + \pi/2) = \cos 2\omega/t$$

$\Rightarrow$ $$\frac{x}{a} = 1 - 2\sin^2\omega t = 1 - \frac{2y^2}{b^2}$$

or $$\frac{2y^2}{b^2} = 1 - \frac{x}{a} = -\left(\frac{x-a}{a}\right)$$

or $$y^2 = -\frac{b^2}{2a}(x-a)$$

It is an equation of a parabola as given in figure.
Hence, period ratio is 1 : 2 and phase difference is $\pi/2$.

23. Amplitude of damped oscillator is given by

$$A = A_0\, e^{-\frac{bt}{2m}}$$

After 5 s, $0.9\,A_0 = A_0\,e^{-\frac{b(5)}{2m}}$

$\Rightarrow$ $$0.9 = e^{-\frac{b(5)}{2m}} \quad \text{...(i)}$$

After, 10 s, $A = A_0 e^{-b\frac{(15)}{2m}}$

$\Rightarrow$ $$A = A_0\,(e^{-\frac{5b}{2m}})^3 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$A = 0.729\,A_0$$

Hence, $\alpha = 0.729$

24. For a harmonic oscillator,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where, k = force constant and $T = \frac{1}{f}$.

$\therefore$ $$k = 4\pi^2 f^2 m$$

$$= 4\times\left(\frac{22}{7}\right)^2\times(10^{12})^2\times\frac{108\times 10^{-3}}{6.02\times 10^{23}}$$

$\Rightarrow$ $k = 7.1$ N/m

25. In SHM, a particle starts from rest, we have

$x = A\cos\omega t$, at $t = 0$, $x = A$

When $t = t$, then $x = A - a$

When $t = 2t$, then $x = A - 3a$

So, $$A - a = A\cos\omega t$$

$$A - 3a = A\cos 2\omega t$$

As, $$\cos 2\omega\tau = 2\cos^2\omega t - 1$$

$\Rightarrow$ $$\frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

$$\Rightarrow \quad \frac{A-3a}{A} = \frac{2A^2+2a^2-4Aa-A^2}{A^2}$$

$$A^2 - 3aA = A^2 + 2a^2 - 4Aa$$

$$a^2 = 2aA \Rightarrow A = 2a$$

Now, $A - a = A\cos\omega t \Rightarrow \cos\omega t = 1/2$

$$\frac{2\pi t}{T} = \frac{\pi}{3}$$

$$T = 6t$$

26. Kinetic energy of a pendulum is maximum at its mean position. Also, maximum kinetic energy of pendulum

$$K_{\max} = \frac{1}{2}m\omega^2 a^2$$

where, angular frequency

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{\frac{l}{g}}}$$

or $$\omega = \sqrt{\frac{g}{l}} \text{ or } \omega^2 = \frac{g}{l}$$

and a = amplitude.

As amplitude is same in both cases, so

$$K_{\max} \propto \omega^2$$

or $$K_{\max} \propto \frac{1}{l} \quad [\because g \text{ is constant}]$$

According to given data, $K_1 \propto \frac{1}{l}$ and $K_2 \propto \frac{1}{2l}$

$\therefore$ $$\frac{K_1}{K_2} = \left(\frac{1/l}{1/2l}\right) = 2$$

or $$K_1 = 2K_2$$

$\Rightarrow$ $$K_2 = \frac{K_1}{2}$$

27. Equation for SHM is given as

$$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$$

$$= 5\times 2\left(\frac{1}{2}\times\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right)$$

$$= 5\times 2\left(\cos\frac{\pi}{3}.\sin 3\pi t + \sin\frac{\pi}{3}.3\pi t\right)$$

$$= 5\times 2\sin\left(3\pi t + \frac{\pi}{3}\right)$$

[using, $\sin(a+b) = \sin a\cos b + \cos a\sin b$]

or $$y = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

Comparing this equation with the general equation of SHM, *i.e.*

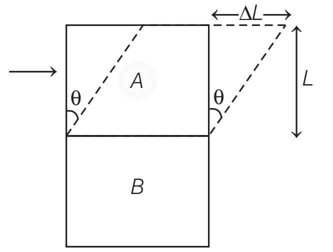
$$y = A\sin\left(\frac{2\pi t}{T} + \phi\right)$$

We get, amplitude, $A = 10$ cm

and $$3\pi = \frac{2\pi}{T}$$

or Time period, $T = \frac{2}{3}$ s

28. When the force F is applied to one side of block A, let the upper face of A be displaced through distance ΔL. Then



$$\eta = \frac{F/L^2}{\Delta L/L} \text{ or } F = \eta L\Delta L \quad \text{...(i)}$$

So, $F \propto \Delta L$ and this force is restoring one. So, if the force is removed, the block will execute SHM.

From Eq. (i) spring factor $= \eta L$

Here, inertia factor $= M$

$\therefore$ Time period, $$T = 2\pi\sqrt{\frac{M}{\eta L}}$$

29. As, $$\frac{T_1}{T} = \sqrt{\frac{l_1}{l}} = \sqrt{\frac{16}{1}} = 4 \text{ or } T_1 = 4T$$

Let after time t, the pendulum be in the same phase. It will be so, then

$$\frac{t}{T_1} = \frac{t}{T} - 1 = \frac{t-T}{T}$$

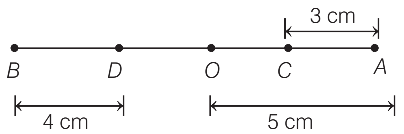
or $$\frac{t}{4T} = \frac{t-T}{T}$$

or $$t = 4t - 4T$$

or $$3t = 4T$$

or $$t = 4T/3$$

30. As per question, the direction from A to B, *i.e.* from A towards mean position O is positive, therefore if a particle starting from A reaches at D, where $AD = 3$ cm, then its direction of motion is towards the mean position O. Hence, its velocity is positive, acceleration is positive and force is positive.



When a particle from B reaches point O, where $BO = 4$ cm, then its direction of motion is towards BA, *i.e.* along BO, then velocity, acceleration and force are negative.

When particle reaches at B, its velocity becomes zero but its acceleration and force are towards BA, *i.e.* negative.

31. Let $x(t) = A\sin(\omega t + \phi_0)$

and $v(t) = A\omega\cos(\omega t + \phi_0)$

We have,

$$x(0) = A\sin\phi_0 = 25$$

$$\Rightarrow \quad \sin\phi_0 = \frac{25}{A} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad A = 25\sqrt{2}$$

Also, $\quad v(0) = A\omega\cos\phi_0$

$$\Rightarrow \quad \cos\phi_0 = \frac{100}{A\omega} = \frac{100}{25\sqrt{2}\times 4} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad \phi_0 = \frac{\pi}{4}$$

The equation of motion,

$$x(t) = 25\sqrt{2}\sin\left(4t + \frac{\pi}{4}\right)$$

or

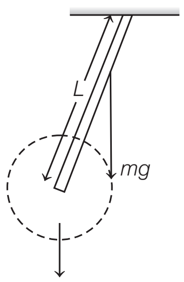
$$y = 25\sqrt{2}\;\sin\left(4t + \frac{\pi}{4}\right)$$

32. Torque is same for both the cases, *i.e.* restoring torque is same for both cases.

$$\therefore \quad T = 2\pi\sqrt{\frac{I}{mgd}}$$

Since, $\quad I_A > I_B$

therefore, $\omega_a < \omega_b$



33. As, $t_1 = 2\pi\sqrt{\dfrac{m}{k_1}}$

or $\quad t_1^2 = \dfrac{2\pi^2 m}{k_1}$

or $\quad k_1 = \dfrac{4\pi^2 m}{t_1^2}$ ...(i)

Similarly, $\quad k_2 = \dfrac{4\pi^2 m}{t_2^2}$ ...(ii)

and $\quad (k_1 + k_2) = \dfrac{4\pi^2 m}{t_0^2}$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$\therefore \quad \frac{4\pi^2 m}{t_0^2} = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$$

or

$$\frac{1}{t_0^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$$

34. Time period of oscillation, $T = 2\pi\sqrt{\dfrac{m}{k}}$

where m is the mass of body suspended from a spring and K is spring constant of the spring and $K = \dfrac{4\pi^2 m}{T^2}$.

Substituting the given values, we get

$$K = \frac{4\times 10\times 4.9}{(0.5)^2}\ \text{Nm}^{-1} \quad \text{...(i)}$$

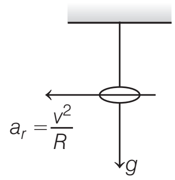
On the removal of the body the spring is shortened by x

$$\therefore \quad mg = kx$$

$$\Rightarrow \quad x = \frac{mg}{k} = \frac{4.9\times 10\times(0.5)^2}{4\times 10\times 4.9} \quad \text{[from Eq. (i)]}$$

$$= \frac{0.25}{4} = 0.0625\ \text{m} = 6.25\ \text{cm}$$

35. The bob is subjected to two simultaneous, accelerations perpendicular to each other *viz* acceleration due to gravity g and radial acceleration $a_R = \dfrac{v^2}{R}$ towards the centre of the circular path.



$\therefore$ Effective acceleration, $a_{\text{eff}} = \sqrt{g^2 + \left(\dfrac{v^2}{R}\right)^2}$

$\therefore$ Time period of the simple pendulum, $T = 2\pi\sqrt{\dfrac{l}{a_{\text{eff}}}}$

$$= 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$$

36. As, $F = mg = kx$

For first case, $k = \dfrac{mg}{x} = \dfrac{1\times 10\ \text{N}}{0.05} = 200\ \text{Nm}^{-1}$

For second case, $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{200}{2.0}} = \sqrt{100} = 10\text{Hz}$

$$r = \frac{m'g}{k} = \frac{2\times 10}{200} = 0.1\ \text{m}$$

$$\therefore \quad v_{\max} = r\omega = 0.1\ \times 10 = 1\ \text{ms}^{-1}$$

37. We know that, time period of a pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

So, angular frequency, $\omega = \dfrac{2\pi}{T} = \sqrt{\dfrac{g}{l}}$ ..(i)

Now, differentiate both side w.r.t g

$$\therefore \quad \frac{d\omega}{dg} = \frac{1}{2\sqrt{g}\sqrt{l}}$$

$$d\omega = \frac{dg}{2\sqrt{g}\sqrt{l}} \quad \text{...(ii)}$$

By dividing Eq. (ii) by Eq. (i), we get

$$\frac{d\omega}{\omega} = \frac{dg}{2g}$$

Or we can write

$$\frac{\Delta\omega}{\omega} = \frac{\Delta g}{2g} \quad \text{...(iii)}$$

As Δg is due to oscillation of support.

$$\therefore \quad \Delta g = 2\omega^2 A \quad (\omega_1 = 1\ \text{rad/s, support})$$

Putting value of Δg in Eq. (iii) we get

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\cdot\frac{2\,\omega_1^2 A}{g} = \frac{\omega_1^2 A}{g}; \qquad (A = 10^{-2}\text{m})$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = \frac{1\times10^{-2}}{10} = 10^{-3} \text{ rad/s}$$

38. The velocity of a particle executing SHM at any instant, is defined as the time rate of change of its displacement at that instant.

$$v = \omega\sqrt{A^2 - x^2}$$

where, ω is angular frequency, A is amplitude and x is displacement of a particle.

Suppose that the new amplitude of the motion be A'.

Initial velocity of a particle performing SHM,

$$v^2 = \omega^2\left[A^2 - \left(\frac{2A}{3}\right)^2\right] \qquad \text{... (i)}$$

where, A is initial amplitude and ω is angular frequency.

Final velocity,

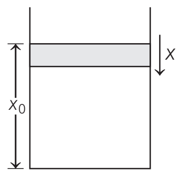
$$(3v)^2 = \omega^2\left[A'^2 - \left(\frac{2A}{3}\right)^2\right] \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{9} = \frac{A^2 - \frac{4A^2}{9}}{A'^2 - \frac{4A^2}{9}} \Rightarrow A' = \frac{7A}{3}$$

39. Pressure applied by piston,

$$\frac{Mg}{A} = p_0$$



$$Mg = p_0 A \qquad \text{...(i)}$$

As no exchange of heat, so process is adiabatic.

$$p_0 V_0^\gamma = pV^\gamma$$

$$p_0 A x_0^\gamma = pA(x_0 - x)^\gamma$$

$$p = \frac{p_0 x_0^\gamma}{(x_0 - x)^\gamma}$$

Let piston be displaced by x.

$$Mg - \left(\frac{p_0 x_0^\gamma}{(x_0 - x)^\gamma}\right)A = f_{\text{restoring}}$$

$$p_0 A\left(1 - \frac{x_0^\gamma}{(x_0 - x)^\gamma}\right) = F_{\text{restoring}} \qquad (\because x_0 - x \approx x_0)$$

$$F = -\frac{\gamma\, p_0\, Ax}{x_0}$$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{\gamma p_0 A}{x_0 M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma p_0 A^2}{MV_0}}$$

40. For damped motion, $A = A_0 e^{-rt}$

$$12 = 6e^{-\frac{bt}{2m}}$$

or
$$\ln 2 = \frac{b}{2m}\times 120$$

$$b = \frac{0.693\times2\times1}{120}$$

$$= 1.16\times10^{-2} \text{ kg s}^{-1}$$

41. When a mass m is placed on mass M, the new system is of mass $= (M+m)$, attached to the spring. New time period of oscillation,

$$T' = 2\pi\sqrt{\frac{M+m}{k}}$$

$$T = 2\pi\sqrt{\frac{M}{k}}$$

Let v = velocity of mass M while passing through the mean position.

v' = velocity of mass $(M+m)$, while passing through the mean position.

According to law of conservation of linear momentum,

$$Mv = (M+m)v'$$

At mean position, $\quad v = A\,\omega =$ and $v' = A'\omega'$

$$\therefore \quad MA\,\omega = (M+m)A'\omega'$$

or
$$A' = \left(\frac{M}{M+m}\right)\frac{\omega}{\omega'}A$$

$$= \frac{M}{M+m}\times\frac{T'}{T}\times A$$

$$= \left(\frac{M}{M+m}\right)\times\sqrt{\frac{M+m}{M}}\times A$$

$$= A\sqrt{\left(\frac{M}{M+m}\right)}$$

42. Let h be the depth of in water, then

$$A\,h\,\rho\,g = mg$$

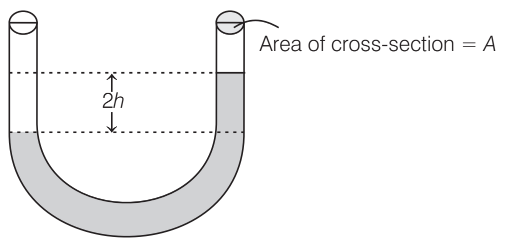
or
$$h = \frac{m}{A\rho} = \frac{200}{50\times1} = 4 \text{ cm} \qquad (\because \rho = 1 \text{ gcm}^{-3})$$

$$T = 2\pi\sqrt{\frac{h}{g}}$$

Now,
$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{h}} = \frac{7}{2\times22}\sqrt{\frac{980}{4}} = 2.5 \text{ Hz}$$

43. Density of mercury column $= \rho$

Acceleration due to gravity $= g$



Restoring force,

$F = -$ Weight of mercury column in excess of one arm

$= -(\text{Volume} \times \text{density} \times g)$

$= -(A \times 2h \times \rho \times g)$

$= -2A\rho g h = -k \times$ Displacement in one arm (h)

Clearly, $2A\rho g = \text{constant} = k$ (say)

As, $F = -kx \Rightarrow F \propto -h$

Hence, motion is SHM.

$\therefore$ Time period, $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{m}{2A\rho g}}$

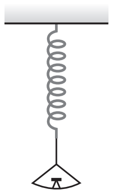
where, m = mass of the mercury column of length l.

If ρ_{Hg} is density of mercury, then

$$m = Al\rho$$

$$\therefore \quad T = 2\pi\sqrt{\frac{Al\rho}{2A\rho g}} = 2\pi\sqrt{\frac{l}{2g}}$$

44. As, the length of the scale is 20 cm and it can read upto 50 kg. The maximum extension of 20 cm will correspond to maximum weight of 50 kg $\times$ 9.8 m/s^2.



Using, $F = -kx$

$|\mathbf{F}| = F = kx$

Here, $x = 20 \times 10^{-2}$ m

$$k = \frac{50 \times 9.8}{20 \times 10^{-2}} = 2450 \text{ N/m}$$

We have for loaded oscillation

$$T = 2\pi\sqrt{\frac{m}{k}}$$

or $$T^2 = 4\pi^2\frac{m}{k}$$

$$m = \frac{T^2 k}{4\pi^2} = \frac{(0.60)^2 \times 2450}{4 \times (3.14)^2}$$

$$= 22.36 \text{ kg}$$

$\therefore$ Weight $= mg = 22.36 \times 9.8 = 219.13$ N

45. For the given SHM, the displacement is given by $y = a\cos\omega t$

Velocity, $v = \dfrac{dy}{dt} = -\omega\sin\omega t = a\omega\sin(\omega t + \pi)$

Acceleration, $A = \dfrac{dV}{dt} = a\omega^2\cos\omega t$

Force = mass $\times$ acceleration $= -m\,a\,\omega^2\cos\omega t$

Force is zero, when $\cos\omega t = 0$ or $\omega t = \dfrac{\pi}{2}$ or $\dfrac{3\pi}{2}$

i.e. $\dfrac{2\pi}{T}t = \dfrac{\pi}{2}$ or $\dfrac{3\pi}{2}$

If $\dfrac{2\pi}{T}t = \dfrac{\pi}{2}$, then, $t = \dfrac{T}{4}$

If $\dfrac{2\pi}{T}t = \dfrac{3\pi}{2}$, then $t = \dfrac{3T}{4}$ (given)

So for $t = \dfrac{3T}{4}$, force is zero.

Acceleration is maximum, if $\cos\omega t = 1$ or 2π

or $\dfrac{2\pi}{T}t = 2\pi$ or $t = T = \dfrac{4T}{4}$ s (given)

Velocity is maximum, if $\sin(\omega t + \pi) = 1$ or $\omega t + \pi = \pi/2$

or $\omega t = \dfrac{\pi}{2} - \pi = -\pi/2$

or $\dfrac{2\pi}{T}t = -\dfrac{\pi}{2}$ or $t = -\dfrac{T}{4}s$

$$\text{PE} = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 a^2\cos^2\omega t$$

$$\text{KE} = \frac{1}{2}m\omega^2 a^2\sin^2\omega t$$

If PE = KE, then $\cos^2\omega t = \sin^2\omega t$ or $\cos\omega t = \sin\omega t$ or $\tan\omega t = 1$

or $\omega t = \dfrac{\pi}{4}$ or $\dfrac{2\pi}{T}t = \dfrac{\pi}{4}$ or $t = \dfrac{T}{8}$ s

46. $T' = 2\pi\sqrt{\dfrac{l}{g_{net}}}$

$g_{net} = g + a = 10 + 10 = 20 \text{ m/s}^2$

$T' = \dfrac{T}{\sqrt{2}}$

47. When a system oscillates, the magnitude of restoring torque of system is given by

$$\tau = C\theta \quad \ldots\text{(i)}$$

where, C = constant that depends on system.

Also, $\tau = I\alpha$ ...(ii)

where, I = moment of inertia

and α = angular acceleration

From Eqs. (i) and (ii), we get

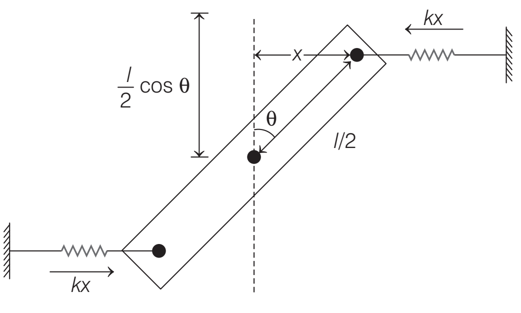
$$\alpha = \frac{C}{I}\cdot\theta \quad \ldots\text{(iii)}$$

and time period of oscillation of system will be

$$T = 2\pi\sqrt{\frac{I}{C}}$$

In given case, magnitude of torque is

τ = Force $\times$ Perpendicular distance



$$\tau = 2kx \times \frac{l}{2}\cos\theta$$

For small deflection, $\tau = \left(\frac{kl^2}{2}\right)\theta$ ...(iv)

$\because$ For small deflections, $\sin\theta = \frac{x}{(l/2)} \approx \theta$

$\Rightarrow \quad x = \frac{l\theta}{2}$

Also, $\cos\theta \approx 1$

Comparing Eqs. (iv) and (i), we get

$$C = \frac{kl^2}{2} \Rightarrow \alpha = \frac{(kl^2/2)}{\left(\frac{1}{12}ml^2\right)} \cdot \theta \Rightarrow \alpha = \frac{6k}{m} \cdot \theta$$

Hence, time period of oscillation is $T = 2\pi\sqrt{\frac{m}{6k}}$

Frequency of oscillation is given by

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

48. Here reduced mass of the system, $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{3\times 1}{3+1}$

$= 0.75\text{kg}$

$\therefore$ Vibrational frequency,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{300}{0.75}} = \frac{20}{2\pi} = \frac{10}{\pi} \cong 3\text{ Hz}$$

49. For half displacement, $t = \frac{T}{12} = \frac{2}{12} = \frac{1}{6}$ s

$\therefore \quad a = 6$

50. At highest velocities, $A_1\omega_1 = A_2\omega_2$

$\therefore \quad \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \frac{\sqrt{k_2/m_2}}{\sqrt{k_1/m_1}} = \sqrt{\frac{k_2}{k_1}} = \frac{1}{\sqrt{2}}$ (since, $m_1 = m_2$)

$\therefore \quad x = 2$

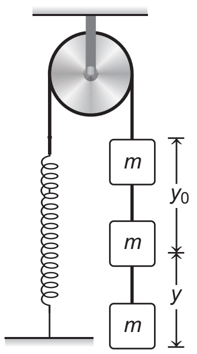
51. As potential energy $U(x) = k|x|^3$, hence maximum value of potential energy,

$$U_{\max} = U(a) = \frac{1}{2}m\omega^2 a^2 = ka^2$$

$\Rightarrow \quad \omega \propto \sqrt{a}$

As, $T = \frac{2\pi}{\omega}$, hence $T \propto \frac{1}{\sqrt{4}} = \frac{1}{2} = 0.5$

52. In equilibrium, due to weight mg spring is stretched by y_0 as shown in figure. When further depressed by a small distance y, the restoring force will be



$$F = -[k(y + y_0) - mg]$$
$$= [k(y + y_0) - ky_0] = -ky$$

$\therefore$ Acceleration, $a = \frac{F}{m} = \frac{k}{m}y$

As acceleration is proportional to the displacement y and opposite to y, the motion will be SHM. Frequency of oscillations will be

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
$$= \frac{1}{2\pi}\sqrt{\frac{16}{0.01}}$$
$$= \frac{20}{\pi}\text{ Hz}$$

$\therefore \quad n = 20$

53. $T_a = 2\pi\sqrt{\frac{M}{k}}$

In series combination, $k' = \frac{k}{2}$

$$T_b = 2\pi\sqrt{\frac{M}{k'}}$$
$$= 2\pi\sqrt{\frac{M}{k/2}} = 2\pi\sqrt{\frac{2M}{k}}$$
$$\frac{T_b}{T_a} = \sqrt{2}$$

$\therefore \quad x = 2$

54. Here, $A = 25$ cm; $T = 3$ s;

Let the particle be at the locating -12.5 cm at time t_1 and $+12.5$ cm at time t_2.

Using the relation, $x = A\cos\left(\frac{2\pi t}{T} + \phi\right)$

First condition $-12.5 = 25\cos\left(\frac{2\pi t_1}{3} + \phi\right)$ ...(i)

Second condition $12.5 = 25\cos\left(\frac{2\pi t_2}{3} + \phi\right)$ ...(ii)

From Eq. (i), $\cos\left(\frac{2\pi t_1}{3} + \phi\right) = \frac{-12.5}{25} = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\therefore \quad \frac{2\pi t_1}{3} + \phi = \frac{2\pi}{3}$

or $\quad 2\pi t_1 + 3\phi = 2\pi$ ...(iii)

From Eq. (ii), $\cos\left(\frac{2\pi t_2}{2} + \phi\right) = \frac{12.5}{25} = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore \quad \frac{2\pi t_2}{3} + \phi = \frac{\pi}{3}$

or $\quad 2\pi t_2 + 3\phi = \pi$ ...(iv)

Subtracting Eq. (iv) from Eq. (iii), we get

$$2\pi(t_1 - t_2) = \pi$$

or $\quad (t_1 - t_2) = \pi/2\pi = 1/2 = 0.5$ s