

CHAPTER-5

CONTINUITY AND DIFFERENTIABILITY

CONTINUITY

TWO MARK QUESTIONS

1. Check the continuity of the function f given by $f(x) = 2x + 3$ at $x = 1$. (U)
2. Examine whether the function f given by $f(x) = x^2$ is continuous at $x = 0$. (U)
3. Discuss the continuity of the function f given by $f(x) = |x|$ at $x = 0$. (U)
4. Show that the function f given by $f(x) = \begin{cases} x^3 + 3, & \text{for } x \neq 0 \\ 1, & \text{for } x = 0 \end{cases}$ is not continuous at $x = 0$. (U)
5. Check the points where the constant function $f(x) = k$ is continuous. (U)
6. Prove that the identity function on real numbers given by $f(x) = x$ is continuous at every real number. (U)
7. Is the function defined by $f(x) = |x|$, a continuous function? (U)
8. Discuss the continuity of the function f given by $f(x) = x^3 + x^2 - 1$. (U)
9. Discuss the continuity of the function f defined by $f(x) = \frac{1}{x}$, $x \neq 0$. (U)
10. Show that every polynomial function is continuous. (U)
11. Show that every rational function is continuous. (U)
12. Prove that the function $f(x) = 5x - 3$ is, continuous at $x = 0$. (U)
13. Prove that the function $f(x) = 5x - 3$ is, continuous at $x = -3$. (U)
14. Prove that the function $f(x) = 5x - 3$ is, continuous at $x = 5$. (U)
15. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$. (U)
16. Examine the following functions for continuity: **(Each question of 2 Marks)**
 - a) $f(x) = x - 5$
 - b) $f(x) = |x - 5|$
 - c) $f(x) = \frac{x^2 - 25}{x + 5}$, $x \neq -5$
 - d) $f(x) = \frac{1}{x - 5}$, $x \neq 5$. (U)
17. Prove that the function $f(x) = x^n$ is continuous at $x = n$, where n is a positive integer. (U)
18. Discuss the continuity of the following functions: **(Each question is of 2 Marks)**
 - a) $f(x) = \sin x + \cos x$
 - b) $f(x) = \sin x - \cos x$
 - c) $f(x) = \sin x \cdot \cos x$. (U)
19. Discuss the continuity of the cosine, cosecant, secant and cotangent functions. (U)

THREE MARK QUESTIONS

1. Discuss the continuity of the function f defined by $f(x) = \begin{cases} x+2, & \text{if } x \leq 1 \\ x-2, & \text{if } x > 1 \end{cases}$. (U)
2. Find all the points of discontinuity of the function f defined by $f(x) = \begin{cases} x+2, & \text{if } x < 1 \\ 0, & \text{if } x = 1 \\ x-2, & \text{if } x > 1 \end{cases}$. (U)
3. Discuss the continuity of the function f defined by $f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ -x+2, & \text{if } x > 0 \end{cases}$. (U)
4. Discuss the continuity of the function f defined by $f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ x^2, & \text{if } x < 0 \end{cases}$. (U)
5. Discuss the continuity of the sine function. (U)
6. Prove that the function defined by $f(x) = \tan x$ is a continuous function. (U)
7. Show that the function defined by $f(x) = \sin(x^2)$ is a continuous function. (U)
8. Is the function f defined by $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$ continuous at $x=0$? At $x=1$? At $x=2$? (U)

FOUR MARK QUESTIONS

1. Find all points of discontinuity of the greatest integer function defined by $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x . (U)
2. Show that the function f defined by $f(x) = |1-x+|x||$, where x is any real number, is a continuous function. (U)
3. Find all points of discontinuity of f , where f is defined by: $f(x) = \begin{cases} 2x+3, & \text{if } x \leq 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$. (U)
4. Find all points of discontinuity of f , where f is defined by: $f(x) = \begin{cases} |x|+3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2, & \text{if } x \geq 3 \end{cases}$. (U)
5. Find all points of discontinuity of f , where f is defined by: $f(x) = \begin{cases} x+1, & \text{if } x \geq 1 \\ x^2+1, & \text{if } x < 1 \end{cases}$. (U)
6. Find all points of discontinuity of f , where f is defined by: $f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$. (U)

7. Find all points of discontinuity of f , where f is defined by: $f(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$. (U)

8. Find all points of discontinuity of f , where f is defined by: $f(x) = \begin{cases} x^3 - 3, & \text{if } x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$. (U)

9. Find all points of discontinuity of f , where f is defined by: $f(x) = \begin{cases} x^{10} - 1, & \text{if } x \leq 1 \\ x^2, & \text{if } x > 1 \end{cases}$. (U)

10. Is the function defined by $f(x) = \begin{cases} x+5, & \text{if } x \leq 1 \\ x-5, & \text{if } x > 1 \end{cases}$ a continuous function? (U)

11. Discuss the continuity of the function f , where f is defined by:

$$f(x) = \begin{cases} 3, & \text{if } 0 \leq x \leq 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \leq x \leq 10 \end{cases} \text{ . (U)}$$

12. Discuss the continuity of the function f , where f is defined by:

$$f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \leq x \leq 1 \\ 4x, & \text{if } x > 1 \end{cases} \text{ . (U)}$$

13. Discuss the continuity of the function f , where f is defined by:

$$f(x) = \begin{cases} -2, & \text{if } x \leq -1 \\ 2x, & \text{if } -1 < x \leq 1 \\ 2, & \text{if } x > 1 \end{cases} \text{ . (U)}$$

14. Find the relationship between 'a' and 'b' so that the function 'f' defined by

$$f(x) = \begin{cases} ax+1, & \text{if } x \leq 3 \\ bx+3, & \text{if } x > 3 \end{cases} \text{ is continuous at } x=3. \text{ (U)}$$

15. For what value of λ is the function defined by $f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x+1, & \text{if } x > 0 \end{cases}$ is continuous at $x=0$? What about continuity at $x=1$? (U)

16. Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points. Here $[x]$ denotes the greatest integer less than or equal to x . (U)

17. Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at $x = \pi$? (U)

18. Find all the points of discontinuity of f , where $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x+1, & \text{if } x \geq 0 \end{cases}$. (U)

19. Determine if f defined by $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ is continuous function? (U)

20. Examine the continuity of f , where f is defined by $f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$. (U)

21. Determine the value of k , if $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$. (U)

22. Find the value of k if $f(x) = \begin{cases} kx^2, & \text{if } x \leq 2 \\ 3, & \text{if } x > 2 \end{cases}$ is continuous at $x = 2$. (U)

23. Find the value of k so that the function $f(x) = \begin{cases} kx+1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$, is continuous at $x = \pi$. (U)

24. Find the value of k so that the function $f(x) = \begin{cases} kx+1, & \text{if } x \leq 5 \\ 3x-5, & \text{if } x > 5 \end{cases}$, at $x = 5$ is a continuous function. (U)

25. Find the values of a and b such that $f(x) = \begin{cases} 5, & \text{if } x \leq 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \geq 10 \end{cases}$ is a continuous functions. (U)

26. Show that the function defined by $f(x) = \cos(x^2)$ is a continuous function. (U)

27. Show that the function defined by $f(x) = |\cos x|$ is a continuous function. (U)

28. Examine that $\sin|x|$ is a continuous function. (U)

29. Find all the points of discontinuity of f defined by $f(x) = |x| - |x+1|$. (U)

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DIFFERENTIABILITY

ONE MARK QUESTIONS

1. Find the derivative of $y = \tan(2x+3)$. (U)

2. If $y = \sin(x^2 + 5)$, find $\frac{dy}{dx}$. (U)

3. If $y = \cos(\sin x)$, find $\frac{dy}{dx}$. (U)

4. If $y = \sin(ax+b)$, find $\frac{dy}{dx}$. (U)

5. If $y = \cos(\sqrt{x})$, find $\frac{dy}{dx}$. (U)

6. Find $\frac{dy}{dx}$, if $y = \cos(1-x)$. (U)

7. If $y = \log(\sin x)$, find $\frac{dy}{dx}$. (U)

8. Find $\frac{dy}{dx}$, if $x-y = \pi$. (U)

9. Find $\frac{dy}{dx}$, if $y = e^{-x}$. (U)

10. Find $\frac{dy}{dx}$, if $y = \sin(\log x)$, $x > 0$. (U)

11. Find $\frac{dy}{dx}$, if $y = \cos^{-1}(e^x)$. (K)

12. If $y = e^{\cos x}$, find $\frac{dy}{dx}$. (U)

13. Find $\frac{dy}{dx}$, if $y = e^{\sin^{-1} x}$. (A)

14. Find $\frac{dy}{dx}$, if $y = e^{x^3}$. (U)

15. Find $\frac{dy}{dx}$, if $y = \log(\log x)$, $x > 0$. (U)

16. Find $\frac{dy}{dx}$, if $y = x^3 + \tan x$. (K)

17. Find $\frac{dy}{dx}$, if $y = x^2 + 3x + 2$. (K)

18. Find $\frac{dy}{dx}$, if $y = x^{20}$. (K)

19. Find $\frac{dy}{dx}$, if $y = x \cos x$. (U)

20. Find $\frac{dy}{dx}$, if $y = \log x$. (U)

- 21.** Find $\frac{dy}{dx}$, if $y = \tan^{-1} x$. (U)
- 22.** Find $\frac{dy}{dx}$, if $y = \sin(\log x)$. (U)
- 23.** If $y = e^{\log x}$, prove that $\frac{dy}{dx} = 1$. (A)
- 24.** Find $\frac{dy}{dx}$, if $y = 5^x$. (U)

TWO MARK QUESTIONS

- 1.** If $y = (2x+1)^3$, find $\frac{dy}{dx}$. (K)
- 2.** Find the derivative of the function given by $f(x) = \sin(x^2)$. (U)
- 3.** Find $\frac{dy}{dx}$, if $y + \sin y = \cos x$. (U)
- 4.** Find $\frac{dy}{dx}$, if $2x + 3y = \sin x$. (U)
- 5.** Find $\frac{dy}{dx}$, if $2x + 3y = \sin y$. (U)
- 6.** Find $\frac{dy}{dx}$, if $ax + by^2 = \cos y$. (U)
- 7.** Find $\frac{dy}{dx}$, if $x^2 + xy + y^2 = 100$. (U)
- 8.** Find $\frac{dy}{dx}$, if $\sin^2 x + \cos^2 y = 1$. (U)
- 9.** If $\sqrt{x} + \sqrt{y} = 10$, show that $\frac{dy}{dx} + \sqrt{\frac{y}{x}} = 0$. (U)
- 10.** Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$. (U)
- 11.** Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$. (U)
- 12.** If $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$, find $\frac{dy}{dx}$. (U)
- 13.** Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$. (U)
- 14.** Find $\frac{dy}{dx}$, if $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, $-1 < x < 1$. (U)
- 15.** Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, $\frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$. (U)

- 16.** Find $\frac{dy}{dx}$, if $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$, $0 < x < \frac{1}{\sqrt{2}}$. (U)
- 17.** Find $\frac{dy}{dx}$, if $y = \log_a x$. (A)
- 18.** Find $\frac{dy}{dx}$, if $y = \frac{e^x}{\sin x}$. (K)
- 19.** Find $\frac{dy}{dx}$, if $y = \sin(\tan^{-1} e^{-x})$. (A)
- 20.** Find $\frac{dy}{dx}$, if $y = \log(\cos e^x)$. (A)
- 21.** Find $\frac{dy}{dx}$, if $y = e^x + e^{x^2} + e^{x^3} + \dots + e^{x^5}$. (U)
- 22.** Find $\frac{dy}{dx}$, if $y = \sqrt{e^{\sqrt{x}}}$, $x > 0$. (U)
- 23.** Find $\frac{dy}{dx}$, if $y = \frac{\cos x}{\log x}$, $x > 0$. (K)
- 24.** Find $\frac{dy}{dx}$, if $y = \cos(\log x + e^x)$, $x > 0$. (U)
- 25.** Differentiate a^x with respect to x , where a is a positive constant. (K)
- 26.** Differentiate $x^{\sin x}$, $x > 0$ with respect to x . (U)
- 27.** Differentiate $(\log x)^{\cos x}$ with respect to x . (U)
- 28.** If $y = x^x$, find $\frac{dy}{dx}$. (U)
- 29.** Differentiate $\left(x + \frac{1}{x}\right)^x$ w. r. to x . (U)
- 30.** Find $\frac{dy}{dx}$, if $y = x^{\left(x + \frac{1}{x}\right)}$. (U)
- 31.** Find $\frac{dy}{dx}$, if $y = (\log x)^x$ OR $y = x^{(\log x)}$. (U)
- 32.** Find $\frac{dy}{dx}$, if $y = (\sin x)^x$ OR $y = \sin^{-1} \sqrt{x}$. (U)
- 33.** Find $\frac{dy}{dx}$, if $y = x^{\sin x}$ OR $y = (\sin x)^{(\cos x)}$. (U)
- 34.** Find $\frac{dy}{dx}$, if $y = \log_7(\log x)$. (A)
- 35.** Find $\frac{dy}{dx}$, if $y = \cos^{-1}(\sin x)$. (U)
- 36.** Find $\frac{dy}{dx}$, if $y = (3x^2 - 9x + 5)^9$. (U)

37. Find $\frac{dy}{dx}$, if $y = \sin^3 x + \cos^6 x$. (U)
38. Find $\frac{dy}{dx}$, if $y = (5x)^{3\cos 2x}$. (U)
39. Find $\frac{dy}{dx}$, if $y = \sin^{-1}(x\sqrt{x})$, $0 \leq x \leq 1$. (K)
40. Find $\frac{dy}{dx}$, if $y = \frac{\cos^{-1} \frac{x}{2}}{\sqrt{2x+7}}$, $-2 < x < 2$. (K)
41. Find $\frac{dy}{dx}$, if $y = (\log x)^{\log x}$, $x > 1$. (U)
42. Find $\frac{dy}{dx}$, if $y = \cos(a \cos x + b \sin x)$, for some constant 'a' and 'b'. (U)
43. Find $\frac{dy}{dx}$, if $y = x^3 \log x$. (U)
44. Find $\frac{dy}{dx}$, if $y = e^x \sin 3x$. (U)
45. Find $\frac{dy}{dx}$, if $y = e^{6x} \cos 3x$. (U)

THREE MARK QUESTIONS

1. Differentiate $\sin(\cos(x^2))$ w. respect to x . (U)
2. If $y = \sec(\tan(\sqrt{x}))$, find $\frac{dy}{dx}$. (U)
3. If $y = \frac{\sin(ax+b)}{\cos(cx+d)}$, find $\frac{dy}{dx}$. (U)
4. If $y = \cos x^3 \cdot \sin^2(x^5)$, find $\frac{dy}{dx}$. (U)
5. Prove that the function f given by $f(x) = |x-1|$, $x \in R$ is not differentiable at $x=1$. (K)
6. Prove that the greatest integer function defined by $f(x) = [x]$, $0 < x < 3$ is not differentiable $x=1$ and $x=2$. (A)
7. If $y = 2\sqrt{\cot(x^2)}$, find $\frac{dy}{dx}$. (U)
8. Find $\frac{dy}{dx}$, if $x + \sin xy - y = 0$. (K)
9. Find $\frac{dy}{dx}$, if $xy + y^2 = \tan x + y$. (K)
10. Find $\frac{dy}{dx}$, if $x^3 + x^2y + xy^2 + y^3 = 81$. (K)

- 11.** Find $\frac{dy}{dx}$, if $\sin^2 x + \cos xy = k$. (K)
- 12.** Find the derivative of f given by $f(x) = \sin^{-1} x$ assuming it exists. (K)
- 13.** Find the derivative of f given by $f(x) = \tan^{-1} x$ assuming it exists. (K)
- 14.** Differentiate e^x w. r. to x from first principle method. (K)
- 15.** Differentiate $\log_e x$ w. r. to x from first principle method. (K)
- 16.** Differentiate $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ with respect to x . (K)
- 17.** Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$. (K)
- 18.** Find $\frac{dy}{dx}$, if $y = \cos x \cdot \cos 2x \cdot \cos 3x$. (K)
- 19.** Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect to x . (K)
- 20.** Find $\frac{dy}{dx}$, if $x^x - 2^{\sin x}$. (K)
- 21.** Find $\frac{dy}{dx}$, if $y = (x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$. (K)
- 22.** Find $\frac{dy}{dx}$, if $x^y + y^x = 1$. (U)
- 23.** Find $\frac{dy}{dx}$, if $x^y = y^x$. (U)
- 24.** Find $\frac{dy}{dx}$, if $xy = e^{x-y}$. (U)
- 26.** Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$. (K)
- 27.** Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ by using product rule. (K)
- 28.** Find $\frac{dy}{dx}$, if $y = x^{x \cos x}$ OR $y = \frac{x^2 + 1}{x^2 - 1}$. (U)
- 29.** Find $\frac{dy}{dx}$, if $y = (x \cos x)^x$ OR $y = (x \sin x)^{\frac{1}{x}}$. (U)
- 30.** If u, v and w are functions of x , then show that
- $$\frac{d}{dx}(uvw) = uv \frac{d}{dx} w + vw \frac{d}{dx} u + wu \frac{d}{dx} v$$
- 31.** Find $\frac{dy}{dx}$, if $x = a \cos \theta$, $y = a \sin \theta$. (U)
- 32.** Find $\frac{dy}{dx}$, if $x = at^2$, $y = 2at$. (U)

33. Find $\frac{dy}{dx}$, if $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$. (U)
34. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. (U)
35. If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, prove that $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$. (A)
36. Find $\frac{dy}{dx}$, if $x = 2at^2$, $y = at^4$. (U)
37. Find $\frac{dy}{dx}$, if $x = a \cos \theta$, $y = b \cos \theta$. (U)
38. Find $\frac{dy}{dx}$, if $x = \sin t$, $y = \cos 2t$. (U)
39. Find $\frac{dy}{dx}$, if $x = 4t$, $y = \frac{4}{t}$. (U)
40. Find $\frac{dy}{dx}$, if $x = \cos \theta - \cos 2\theta$, $y = \sin \theta - \sin 2\theta$. (U)
41. If $x = a(\theta - \sin \theta)$ and $y = a(1 + \cos \theta)$ then prove that $\frac{dy}{dx} = -\cot\left(\frac{\theta}{2}\right)$. (A)
42. Find $\frac{dy}{dx}$, if $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$, $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$. (U)
43. Find $\frac{dy}{dx}$, if $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$. (A)
44. Find $\frac{dy}{dx}$, if $x = a \sec \theta$, $y = b \tan \theta$. (U)
45. Find $\frac{dy}{dx}$, if $x = a(\cos \theta + \theta \sin \theta)$, $y = a(\sin \theta - \theta \cos \theta)$. (U)
46. If $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$, then prove that $\frac{dy}{dx} = -\frac{y}{x}$. (A)
48. If $x = a(\theta + \sin \theta)$ and $y = a(1 - \cos \theta)$. Prove that $\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$. (A)

49. Verify Rolle's Theorem for the function $y = x^2 + 2$, $x \in [-2, 2]$. (K) **"OR"**

Verify Rolle's Theorem for the function $y = x^2 + 2$, $a = -2$ and $b = 2$. (K)

50. Verify Mean Value Theorem for the function $y = x^2$ in the interval $[2, 4]$. (K)

51. Verify Rolle's Theorem for the function $y = x^2 + 2x - 8$, $x \in [-4, 2]$. (K)

52. Verify Mean Value theorem, if $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$, where $a = 1$ and $b = 4$. (K)

53. Verify Mean Value Theorem if $f(x) = x^3 - 5x^2 - 3x$ in the interval $[1, 3]$. (K)

54. Find $\frac{dy}{dx}$, if $y = \sqrt{3x+2} + \frac{1}{\sqrt{2x^2+4}}$. (U)
55. Find $\frac{dy}{dx}$, if $y = e^{\sec^2 x} + 3\cos^{-1} x$. (K)
56. Find $f'(x)$, if $f(x) = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$. (A)
57. Find $f'(x)$, if $f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$. (A)
58. Find $f'(x)$ if $f(x) = (\sin x)^{\sin x}$ for all $0 < x < \pi$. (U)
59. For a positive constant 'a' find $\frac{dy}{dx}$, where $y = a^{t+\frac{1}{t}}$ and $y = \left(t + \frac{1}{t}\right)^a$. (U)
60. Differentiate $\sin^2 x$ with respect to $e^{\cos x}$. (K)
61. Find $\frac{dy}{dx}$, if $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$, $0 < x < \frac{\pi}{2}$. (A)
62. Find $\frac{dy}{dx}$, if $y = (\sin x - \cos x)^{(\sin x - \cos x)}$, $\frac{\pi}{4} < x < \frac{3\pi}{4}$. (K)
63. Find $\frac{dy}{dx}$, if $x^x + x^a + a^x + a^a$, for some fixed $a > 0$ and $x > 0$. (K)
64. Find $\frac{dy}{dx}$, if $y = (x)^{x^2-3} + (x-3)^{x^2}$, for $x > 3$. (U)
65. Find $\frac{dy}{dx}$, if $y = 12(1 - \cos t)$, $x = 10(t - \sin t)$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$. (U)
66. Find $\frac{dy}{dx}$, if $y = \sin^{-1} x + \sin^{-1} \sqrt{1-x^2}$, $0 < x < 1$. (A)
67. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1 < x < 1$. Prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$. (K)
68. If $\cos y = x \cos(a+y)$, with $\cos a \neq \pm 1$ prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. (K)
69. If $f(x) = |x|^3$, $f''(x)$ exists for all real x and find it. (U)
70. Using mathematical induction prove that $\frac{d}{dx} x^n = nx^{n-1}$ for all positive integer n . (U)
71. Using the fact that $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and the differentiation, obtain the sum formula for cosines. (U)
72. Does there exists a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer. (A)

73. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, prove that, $\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$. (S)

FIVE MARKS QUESTIONS

- 1.** If $y = A \sin x + B \cos x$, then prove that $\frac{d^2 y}{dx^2} + y = 0$. (K)
- 2.** If $y = 3e^{2x} + 2e^{3x}$, then prove that $\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$. (K)
- 3.** If $y = \sin^{-1} x$, then prove that $(1-x^2)\frac{d^2 y}{dx^2} - x\frac{dy}{dx} = 0$. (U)
- 4.** If $y = 5 \cos x - 3 \sin x$, then prove that $\frac{d^2 y}{dx^2} + y = 0$. (U)
- 5.** If $y = \cos^{-1} x$, find $\frac{d^2 y}{dx^2}$ in terms of y alone. (S)
- 6.** If $y = 3 \cos(\log x) + 4 \sin(\log x)$, show that $x^2 y_2 + xy_1 + y = 0$. (U)
- 7.** If $y = Ae^{mx} + Be^{nx}$, prove that $\frac{d^2 y}{dx^2} - (m+n)\frac{dy}{dx} + (mn)y = 0$. (K)
- 8.** If $y = 500e^{7x} + 600e^{-7x}$, show that $y_2 = 49y$. (K)
- 9.** If $e^y (x+1) = 1$, show that $y_2 = y_1^2$. (K)
- 10.** If $e^y (x+1) = 1$, Prove that $\frac{dy}{dx} = -e^y$ hence prove that $\frac{d^2 y}{dx^2} = \left(\frac{dy}{dx}\right)^2$. (K)
- 11.** If $y = (\tan^{-1} x)^2$, show that $(1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$. (U)
- 12.** If $y = e^{a \cos^{-1} x}$, $-1 \leq x \leq 1$, show that $(1-x^2)\frac{d^2 y}{dx^2} - x\frac{dy}{dx} - a^2 y = 0$. (A)
- 13.** If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2 y}{dx^2}$. (A)
- 14.** If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$ is a constant independent of a and b . (A)