Class 10th

Mathematics

Quadratic Equation

MOST IMPORTANT QUESTIONS

Section –A (1 Mark)

1. Equation of $(x + 1)^2 - x^2 = 0$ has number of real roots equal to:

(A) 1 (B) 2

(C) 3 (D) 4

Ans. (A)

Sol. $(x + 1)^2 - x^2 = 0$

$x^2 + 2x + 1 - x^2 = 0$

$2x + 1 = 0$

$x = -\frac{1}{2}$

Hence, there is one real root.

2. Which of the following equations has 2 as a root?

(A) $x^2 - 4x + 5 = 0$

(B) $x^2 + 3x - 12 = 0$

(C) $2x^2 - 7x + 6 = 0$

(D) $3x^2 - 6x - 2 = 0$

Ans. (C)

Sol. We have, $2x^2 - 7x + 6 = 0$

$2(2)^2 - 7(2) + 6 = 0$

$2 \times 4 - 14 + 6 = 0$

$8 + 6 - 14 = 0$

$14 - 14 = 0$ satisfied the equation.

3. If the product of roots of the equation $x^2 - 3x + k = 10$ is –2, then the value of k is:

(A) –2

(B) –8

(C) 8

(D) 12

Ans. (C)

Sol. Given equation is $x^2 - 3x + (k - 10) = 0$

$\therefore$ Product of roots $= \frac{c}{a} = k - 10$

So, $k - 10 = -2$

or, $k = -2 + 10$

or, $k = 8$

4. If one root of equation $4x^2 - 2x + k - 4 = 0$ is reciprocal of the other. The value of k is:

(A) –8 (B) 8

(C) –4 (D) 4

Ans. (B)

Sol. Let one root of equation be α then other will be $\frac{1}{\alpha}$

Then $\alpha \times \frac{1}{\alpha} = \frac{k-4}{4}$

$\Rightarrow 1 = \frac{(k-4)}{4}$

$\Rightarrow 4 = k - 4$

$\Rightarrow 4 + 4 = k$

$\boxed{k = 8}$

5. A quadratic equation $ax^2 + bx + c = 0$ has no real roots, if

(A) $b^2 - 4ac > 0$

(B) $b^2 - 4ac = 0$

(C) $b^2 - 4ac < 0$

(D) $b^2 - ac < 0$

Ans. (C)

Sol. A quadratic equation $ax^2 + bx + c = 0$ has no real roots, if $b^2 - 4ac < 0$. That means, the quadratic equation contains imaginary roots.

6. If the equation

$(\ell + m^2) x^2 + (2mc) x + (c^2 - a^2) = 0$ has equal roots, Then:

(A) $lc^2 - a^2 = \ell + m^2$ (B) $c^2 = a^2 (\ell + m^2)$

(C) $lc^2a^2 = (\ell + m^2)$ (D) $c^2 + a^2 = \ell + m^2$

Ans. (B)

Sol. Since the equation has two equal roots, D = 0

$\Rightarrow (2mc)^2 - 4(\ell + m^2)(c^2 - a^2) = 0$

$\Rightarrow 4m^2c^2 - 4c^2 \ell + 4a^2\ell - 4m^2c^2 + 4m^2a^2 = 0$

$\Rightarrow -4c^2\ell + 4a^2\ell + 4m^2a^2 = 0 \Rightarrow 4c^2\ell = 4a^2\ell + 4m^2a^2$

$\Rightarrow 4c^2\ell = 4a^2 (\ell + m^2)$

$\Rightarrow c^2 l = a^2 (l + m^2)$

$\Rightarrow lc^2 = a^2 (l + m^2)$

7. The quadratic equation $2x^2 - \sqrt{5x} + 1 = 0$ has

(A) two distinct real roots

(B) two equal real roots

(C) no real roots

(D) more than 2 real roots

Ans. (C)

Sol. Given,

$2x^2 - \sqrt{5x} + 1 = 0$

Comparing with the standard form of a quadratic equation,

a = 2, b = $-\sqrt{5}$, c = 1

Now,

$b^2 - 4ac = (-\sqrt{5})^2 - 4(2)(1)$

$= 5 - 8$

$= -3 < 0$

Therefore, the given equation has no real roots.

8. The roots of $100x^2 - 20x + 1 = 0$ is:

(A) $\frac{1}{20}$ and $\frac{1}{20}$ (B) $\frac{1}{10}$ and $\frac{1}{20}$

(C) $\frac{1}{10}$ and $\frac{1}{10}$ (D) None of the above

Ans. (C)

Sol. Given, $100x^2 - 20x + 1=0$

$100x^2 - 10x - 10x + 1 = 0$

$10x(10x - 1) -1(10x - 1) = 0$

$(10x - 1)^2 = 0$

$\therefore (10x - 1) = 0$ or $(10x - 1) = 0$

$\Rightarrow x = \frac{1}{10}$ or $x = \frac{1}{10}$

9. The roots of the quadratic equation $6x^2 - x - 2 = 0$ are

(A) $\frac{2}{3}, \frac{1}{2}$ (B) $-\frac{2}{3}, \frac{1}{2}$

(C) $\frac{2}{3}, -\frac{1}{2}$ (D) $-\frac{2}{3}, -\frac{1}{2}$

Ans. (C)

Sol. $6x^2 + 3x - 4x - 2 = 0$

$\Rightarrow 3x(2x + 1) -2(2x + 1) = 0$

$\Rightarrow (2x + 1)(3x - 2) = 0$

$\Rightarrow 2x + 1 = 0$ or $3x - 2 = 0$

$\therefore x = -\frac{1}{2}, x = \frac{2}{3}$

10. The quadratic equation whose one rational root is $3 + \sqrt{2}$ is

(A) $x^2 - 7x + 5 = 0$

(B) $x^2 + 7x + 6 = 0$

(C) $x^2 - 7x + 6 = 0$

(D) $x^2 - 6x + 7 = 0$

Ans. (D)

Sol. $\because$ We have one root is $3 + \sqrt{2}$

$\therefore$ other possible root is $3 - \sqrt{2}$

$\Rightarrow$ SoR = $\left(3+\sqrt{2}+3-\sqrt{2}\right) = 6$

PoR = $\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right) = 9 - = 7$

x^2 – (SoR) x + PoR = 0

11. Which of the following is a quadratic equation?

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$

(B) $-2x^2 = (5 - x)2x - \frac{2}{5}$

(C) $(k+1)x^2 + \frac{3}{2}x = 7$, where $k = -1$

(D) $x^3 - x^2 = (x-1)^3$

Ans. (D)

Sol. An equation is of the form $ax^2 + bx + c = 0$, $a \neq 0$ is called a quadratic equation

(A) Given that

$x^2 + 2x + 1 = (4 - x)^2 + 3$

$\Rightarrow x^2 + 2x + 1 = 16 + x^2 - 8x + 3$

$\Rightarrow 10x - 2 = 0$, which is not a quadratic equation

(B) Given that,

$-2x^2 = (5-x)2x - \frac{2}{5}$

$\Rightarrow -2x^2 = 10x - 2x^2 - \frac{2}{5}$

$\Rightarrow 50x - 2 = 0$

$\Rightarrow 50x - 2 = 0$, which is not a quadratic equation

(C) Given that,

$x^2(k+1) + \frac{3}{2}x = 7$, $\therefore$ where $k = -1$

$\Rightarrow x^2(-1+1) + \frac{3}{2}x = 7$

$\Rightarrow 3x - 14 = 0$, which is not a quadratic equation

(D) Given that

$x^3 - x^2 = x^3 - 1 - 3x(x - 1)$

$\Rightarrow x^3 - x^2 = x^3 - 1 - 3x^2 + 3x$

$\Rightarrow x^3 - x^2 - x^3 + 1 + 3x^2 - 3x = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

which is a quadratic equation hence the correct answer is options (D).

12. A quadratic equation can have

(A) at least two roots

(B) at most two roots

(C) exactly two roots

(D) any number of roots

Sol.(B) A quadratic equation can have atmost two roots, zero, one and two.

Hence, the correct answer is option (B).

Section –B (2 Mark)

13. Show quadratic equation $x^2 + 7x - 60$ has two real and unequal roots.

Sol. Given,

$x^2 + 7x - 60 = 0$

Comparing with the standard form,

$a = 1, b = 7, c = -60$

$b^2 - 4ac = (7)^2 - 4(1)(-60) = 49 + 240 = 289 > 0$

Hence $x^2+7x-60$ has two read and unequal roots

14. The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. Find the quadratic Equation:

Sol. The breadth of the rectangular plot = x m

Thus, the length of the plot = (2x + 1) m

As we know,

Area of rectangle = length × breadth = 528 m^2

Putting the value of the length and breadth of the plot in the formula, we get,

$(2x + 1) \times x = 528$

$\Rightarrow 2x^2 + x = 528$

$\Rightarrow 2x^2 + x - 528 = 0$

15. The sum of two numbers is 27 and the product is 182. Find the numbers:

Sol. Let x is one number

Another number = 27 – x

Product of two numbers = 182

$x(27 - x) = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x - 13) - 14(x - 13) = 0$

$\Rightarrow (x - 13)(x - 14) = 0$

$\Rightarrow x = 13$ or $x = 14$

Hence the number are 13, 14

16. Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

Sol. Here $a = 3$, $b = -2$ and $c = \frac{1}{3}$

Therefore, discriminant = $b^2 - 4ac$

$= (-2)^2 - 4 \times 3 \times \frac{1}{3}$

$= 4 - 4 = 0$

Hence, the given quadratic equation has two equal real roots.

The roots are $\frac{-b}{2a}, \frac{-b}{2a}$, i.e., $\frac{2}{6}, \frac{2}{6}$, i.e., $\frac{1}{3}, \frac{1}{3}$.

Section –C (3 Mark)

17. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Sol. Let us say the shorter side of the rectangle is x m.

Then, larger side of the rectangle = (x + 30) m

Diagonal of the rectangle $= \sqrt{x^2 + (x+30)^2}$

As given, the length of the diagonal is = x + 60 m

Therefore,

$\sqrt{x^2 + (x+30)^2} = x + 60$

$\Rightarrow x^2 + (x + 30)^2 = (x + 60)^2$

$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$

$\Rightarrow x^2 - 60x - 2700 = 0$

$\Rightarrow x^2 - 90x + 30x - 2700 = 0$

$\Rightarrow x(x - 90) + 30(x - 90) = 0$

$\Rightarrow (x - 90)(x + 30) = 0$

$\Rightarrow x = 90, -30$

However, the side of the field cannot be negative. Therefore, the length of the shorter side will be 90 m.

And the length of the larger side will be (90 + 30) m = 120 m.

18. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Sol. Let us say the base of the right triangle is x cm.

Given, the altitude of right triangle = (x – 7) cm

From Pythagoras' theorem, we know,

$Base^2 + Altitude^2 = Hypotenuse^2$

$\therefore x^2 + (x - 7)^2 = 132$

$\Rightarrow x^2 + x^2 + 49 - 14x = 169$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x - 12) + 5(x - 12) = 0$

$\Rightarrow (x - 12)(x + 5) = 0$

Thus, either x – 12 = 0 or x + 5 = 0,

$\Rightarrow x = 12$ or $x = -5$

Since sides cannot be negative, x can only be 12.

Therefore, the base of the given triangle is 12 cm, and the altitude of this triangle will be (12 – 7) cm = 5 cm.

19. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hrs more, Then find the Quadratic equation

Sol. Let us consider,

The speed of the train = x km/h

And

Time taken to travel 480 km = $\frac{480}{x}$ hr

As per second condition, the speed of train = (x – 8) km/h

Also given, the train will take 3 hours more to cover the same distance.

Therefore, time taken to travel 480 km = $\left(\frac{480}{x}\right) + 3$ km/h

As we know,

Speed × Time = Distance

Therefore,

$(x - 8)\left(\frac{480}{x} + 3\right) = 480$

$(x - 8)(480 + 3x) = 480x$

$480x + 3x^2 - 480x\ 8 - 24x$

$= 480x$

$3x^2 - 24x - 480x\ 8 = 0$

$x^2 - 8x4 - 1280 = 0$

20. If $\frac{1}{2}$ is a root of the quadratic equation $x^2 - mx - \frac{5}{4} = 0$, then find the value of m.

Sol. Given x= $\frac{1}{2}$ is root of equation $x^2 - mx - \frac{5}{4}$=0.

$$\left(\frac{1}{2}\right)^2 - m\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$

$$\frac{1}{4} - \frac{m}{2} - \frac{5}{4} = 0$$

$$-\frac{m}{2} = \frac{5}{4} - \frac{1}{4}$$

$$-\frac{m}{2} = 1$$

m= –2

Section –E (5 Mark)

21. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

Sol. Let original speed of train = x km/h

We know,

$$\text{Time} = \frac{\text{distance}}{\text{Time}}$$

According to the question, we have,

Time taken by train = $\frac{360}{x}$ hour

And, time taken by train if its speed increased by 5 km/h = $\frac{360}{(x+5)}$

It is given that,

Time taken by train in first case time taken by train in 2nd case = 48 min = $\frac{48}{60}$ hour

$$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$$

$$360\left(\frac{1}{x} - \frac{1}{x+5}\right) = \frac{4}{5}$$

$$\Rightarrow 360\left(\frac{x+5-x}{x(x+5)}\right) = \frac{4}{5}$$

$$\Rightarrow \frac{5}{x(x+5)} = \frac{4}{5\times 360}$$

$$\Rightarrow \frac{5\times 5\times 360}{4} = x(x+5)$$

$\Rightarrow 5 \times 5 \times 90 = x^2 + 5x$

$\Rightarrow 2250 = x^2 + 5x$

$\Rightarrow x^2 + 5x - 2250 = 0$

$$\Rightarrow \therefore x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{(-5)^2 - 4.(-2250)}}{2\times 1}$$

$$= \frac{-5 \pm 95}{2}$$

x= –50, 45

But x ≠ -50 because speed cannot be negative

So, x = 45 km/h

Hence, original speed of train = 45 km/h

22. John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

Sol. Let us say the number of marbles John has = x

Therefore, the number of marble Jivanti has = 45 –x

After losing 5 marbles each,

Number of marbles John has = x – 5

Number of marble Jivanti has = 45 – x – 5 = 40 – x

Given that the product of their marbles is 124.

$\therefore$ (x – 5)(40 – x) = 124

$\Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0$

$\Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 36)(x - 9) = 0$

Thus, we can say,

x – 36 = 0 or x – 9 = 0

$\Rightarrow$ x = 36 or x = 9

Therefore,

If John's marbles = 36

Then, Jivanti's marbles = 45 – 36 = 9

And if John's marbles = 9

Then, Jivanti's marbles = 45 – 9 = 36

23. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of a larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time at which each tap can separately fill the tank

Sol. Let the time taken by the smaller pipe to fill the tank = x hr.

Time taken by the larger pipe = (x – 10) hr

Part of the tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of the tank filled by larger pipe in 1 hour = $\frac{1}{(x-10)}$

As given, the tank can be filled in

$9\frac{3}{8}=\frac{75}{8}$ hours by both the pipes together.

Therefore,

$\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$

$\Rightarrow \frac{x-10+x}{x(x-10)}=\frac{8}{75}$

$\Rightarrow \frac{2x-10}{x^2-10}=\frac{8}{75}$

$\Rightarrow 75(2x-10)=8(x^2-10x)$

$\Rightarrow 150x-750=8x^2-80x$

$\Rightarrow 8x^2-80x-150x+750=0$

$\Rightarrow 8x^2-230x+750=0$

$\Rightarrow 8x^2-200x-30x+750=0$

$\Rightarrow 8x(x-25)-30(x-25)=0$

$\Rightarrow (x-25)(8x-30)=0$

$\Rightarrow x=25, \frac{30}{8}$

Time taken by the smaller pipe cannot be $\frac{30}{8}$ = 3.75 hours, as the time taken by the larger pipe will become negative, which is logically not possible.

Therefore, the time taken individually by the smaller pipe and the larger pipe will be 25 and 25 – 10 =15 hours, respectively.