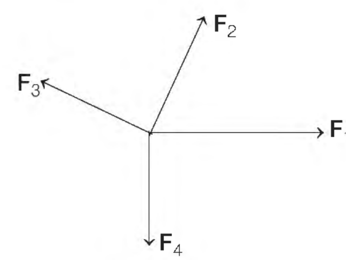
CHAPTER 04

Laws of Motion

Equilibrium of Forces

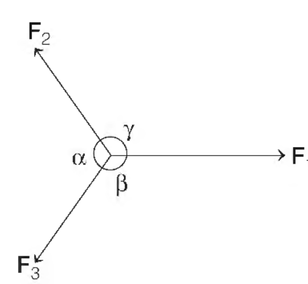
- **Concurrent coplanar forces** If all forces are in equilibrium or $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 = 0$, then we can write

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$



where, x and y are any two mutually perpendicular directions.

- **Lami's theorem** If a body is in equilibrium under three concurrent forces as shown in figure.



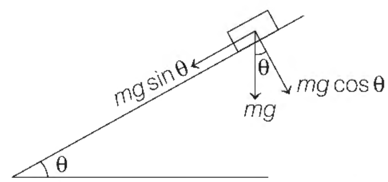
Then, we can write

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

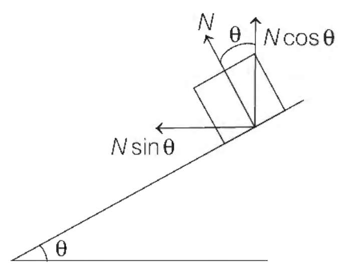
- **Non concurrent coplanar forces** If a body is in equilibrium under non concurrent coplanar forces, then we can write $\Sigma F_x = 0, \Sigma F_y = 0$ and Σ(moment about any point) $= 0$

Important Components

- Component of weight mg along the plane is $mg\sin\theta$ and component perpendicular to plane is $mg\cos\theta$.



- If an inclined plane makes an angle θ with horizontal, then normal reaction N acting on a block kept over this surface makes θ with vertical. So, this normal reaction has vertical component $N\cos\theta$ and the horizontal component $N\sin\theta$.



- Net contact force (F) can have two components, **friction** (f) which is tangential and **normal reaction** (N). Thus, N acts towards the body. Further N and f are in mutually perpendicular directions, hence

$$F = \sqrt{N^2 + f^2}$$

Resolution of Forces

Two situations normally occur :

(i) **Permanent rest, equilibrium, net force equal to zero, net acceleration equal to zero or moving with constant velocity** From forces point of view all situations are similar. In this case, we can resolve the forces in any direction. In all directions, net force should be zero.

(ii) **Momentary rest, accelerated or rotating in a circle** In all these cases, body has an acceleration **a**. If a particle is projected vertically upwards, then its highest position is the momentary rest position, where $\mathbf{a} \neq 0$ but $\mathbf{v} = 0$.

Extreme positions of a pendulum are also the momentary rest positions, where $\mathbf{v} = 0$ but $\mathbf{a} \neq 0$. The direction of **a** in the momentary rest condition is the direction where the body will move just after few seconds. In a uniform circular motion, direction of **a** is towards centre.

In the above situations, we normally resolve the forces in the direction of acceleration and perpendicular to it. In the direction of acceleration, net force should be $m\mathbf{a}$ and perpendicular to acceleration, net force should be zero.

Note *In the above situations, we can also resolve the forces in any third direction (say x-direction). In that case, net force along x-direction = m (component of **a** along x-direction)*

- If **a** is the acceleration of a body, then $m\mathbf{a}$ force does not act on the body but this much force is required to provide **a** acceleration to the body. The different available forces acting on the body provide this $m\mathbf{a}$ force or we can say that vector sum of all forces acting on the body is equal to $m\mathbf{a}$.

  The available forces may be weight, tension, normal reaction, friction or any externally applied force, etc.

- If all bodies of a system have a common acceleration, then that common acceleration can be given by

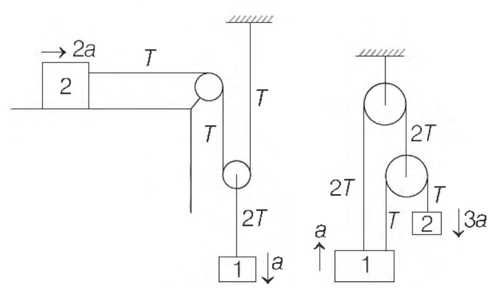
$$a = \frac{\text{net pulling/ pusing force}}{\text{total mass}} = \frac{\text{NPF}}{\text{TM}}$$

  Net pulling/pushing force (NPF) is actually the net force.

  After finding that common acceleration, we will have to draw free body diagrams of different blocks to find normal reaction or tension, etc.

- In some cases, acceleration of a block is inversely proportional to tension force acting on the block (or its component in the direction of motion or acceleration).

  If tension is double (as compared to other block), then acceleration will be half.



(a) (b)

In Fig. (a), tension force on block-1 is double ($=2T$) the tension force on block-2 ($=T$). Therefore, acceleration of block-1 will be half.

If block-1 has an acceleration a in downward direction, then block-2 will have an acceleration $2a$ towards right.

In Fig. (b), tension force on block-1 is three times ($2T + T = 3T$), the tension force on block-2 ($=T$). Therefore, acceleration of block-2 will be three times. If block-1 has an acceleration a in upward direction, then acceleration of block-2 will be $3a$ downwards.

Pseudo Force

If we observe an object from a non-inertial frame, then the motion conditions are changed from this frame. To justify these changed motion conditions from equation point of view, we will have to apply a pseudo force ma in the opposite direction of acceleration of frame.

In the pseudo force, mass m is the mass of object which is being observed and a is the acceleration of frame from where this object is observed.

Inertial Frame of Reference

A non-accelerating frame of reference is called an inertial frame of reference. A frame of reference moving with a constant velocity is an inertial frame of reference.

Non-inertial Frame of Reference

An accelerating frame of reference is called a non-inertial frame of reference.

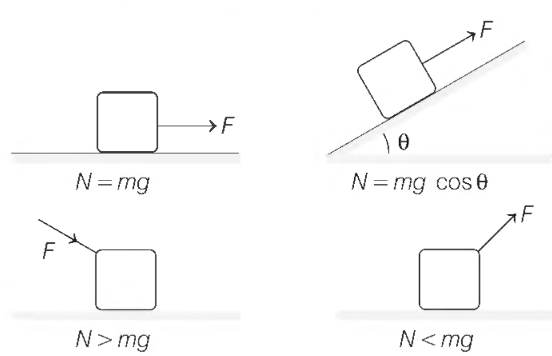
Note *(i) A rotating frame of reference is a non-inertial frame of reference, because it is also an accelerating reference.*

(ii) Earth is rotating about its axis of rotation and it is revolving around the centre of sun also. So, it is a non-inertial frame of reference. But for most of the cases, we consider it as an inertial frame of reference.

Friction

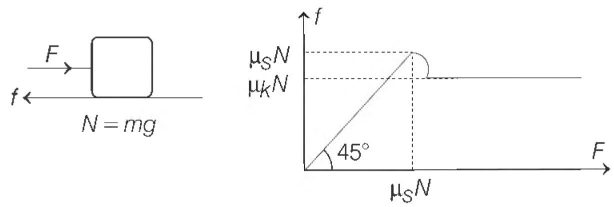
- Friction is a tangential component of net contact force between two solid bodies in contact.
- This force starts acting between them when there is relative motion (or have the tendency of relative motion).
- Like other forces, this force also makes a pair of equal and opposite forces acting on two different bodies.
- Direction of frictional force on a body is opposite to the direction of relative motion (or its tendency) of this body with respect to the other body.
- Normal reaction N (component of net contact force in perpendicular direction) plays a very important role while deciding limiting value of static friction $\mu_s N$ or constant value of kinetic friction $\mu_k N$.
- As long as forces are acting on a body parallel to the plane over which body is kept, normal reaction will be $mg \cos\theta$ (if plane is inclined).

  If forces are acting at some angle with plane, normal reaction is greater than mg (or $mg \cos\theta$) or less than this, depending upon whether the external forces are of pushing nature or pulling nature.



- Static friction is of self adjusting nature with its value varying between 0 and $\mu_s N$. This force acts when there is only tendency of relative motion.

  On the other hand, kinetic friction is constant of value $\mu_k N$. This force acts when relative motion actually takes place.

- Coefficient of kinetic friction (μ_k) is always less than the coefficient of static friction (μ_s).
- If μ_s and μ_k are not separately given, but only one value of μ is given, then in this case, limiting value of static friction and constant value of kinetic friction are same and equal to μN.
- 

  (a) (b)

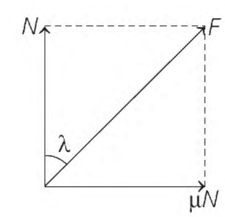
  In Fig. (a), a force F is applied on a block of mass m. Force of friction f starts acting on the block in opposite direction to stop its relative motion with ground. Following cases may arise depending on the value of F.

  If $F \leq \mu_s N, f = F, F_{net} = 0 \therefore \quad a = 0$

  If $F > \mu_s N, f = \mu_k N, F_{net} = F - f$ and $a = \dfrac{F_{net}}{m}$

  Corresponding graph is shown in Fig. (b).

Angle of Friction (λ)



In critical condition when slipping is about to occur, the two forces acting are the normal reaction N and frictional force μN.

The resultant of these two forces is F and it makes an angle λ with the normal reaction, where

$$\tan \lambda = \frac{\mu N}{N} = \mu$$

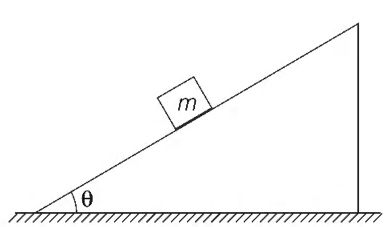
or

$$\lambda = \tan^{-1} (\mu)$$

This angle λ is called the angle of friction.

Angle of Repose (α)

Suppose a block of mass m is placed on an inclined plane whose inclination θ can be increased or decreased. Let μ be the coefficient of friction between the block and the plane.



At a general angle θ,

$N = mg\cos\theta,\ f_L = \mu N = \mu mg\cos\theta$ and downward force $F = mg\sin\theta$.

As θ increases, F increases and f_L decreases. At angle $\theta = \alpha$, called angle of repose, $F = f_L$ and the block starts sliding.

$$\therefore \quad mg \sin\alpha = \mu mg \cos\alpha$$

$$\text{or} \quad \tan\alpha = \mu \quad \text{or } \alpha = \tan^{-1}(\mu)$$

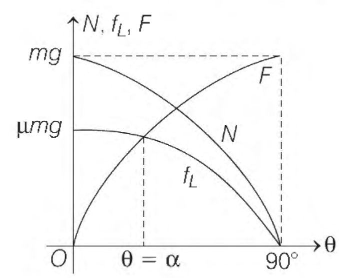
If $\theta < \alpha$, $F < f_L$, the block is stationary.

If $\theta = \alpha$, $F = f_L$, the block is on the verge of sliding.

and if $\theta > \alpha, F > f_L$, the block slides down with an acceleration

$$a = \frac{F - f_L}{m} = g(\sin\theta - \mu \cos\theta)$$

Variation of N, f_L and F with θ, is shown graphically in figure.



$$N = mg\cos\theta \quad \text{or} \quad N \propto \cos\theta$$
$$f_L = \mu mg\cos\theta \quad \text{or} \quad f_L \propto \cos\theta$$
$$F = mg\sin\theta \quad \text{or} \quad F \propto \sin\theta$$

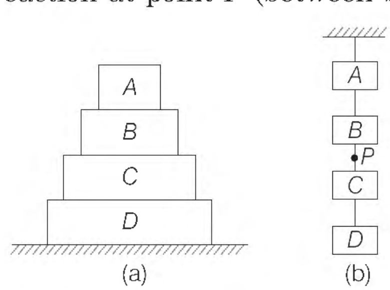
Normally $\mu < 1$, so $f_L < N$.

Method of Finding Tension at Some Point (say P), if it is Non-uniform

Find a common acceleration a of all the blocks attached with the given string. In some cases, a will be given in the question. Cut the string at P and make free body diagram of any one part.

In addition to other forces which are actually acting on this part, apply one tension at P. Direction of this tension should be such that string looks as if it is stretched. Now apply, $F_{net} = ma$ for this part to find the tension at P.

- In Fig. (a), normal reaction at point P (between blocks C and D) is given by



$$N = [\Sigma(\text{mass above } P)] \times g_{eff} = (m_A + m_B + m_C)\, g_{eff}$$

In Fig. (b), tension at point P is given by

$$T = \Sigma\, [(\text{mass above } P)] \times g_{eff} = (m_A + m_B) g_{eff}$$

If the strings are massless, then $T = (m_C + m_D) g_{eff}$

Here, $g_{eff} = g$ if acceleration of system is zero
$= (g + a)$ if acceleration a is upwards
$= (g - a)$ if acceleration a is downwards

Few Importants Points Related to Newton's Laws of Motion

- **Feeling of weight to a person is due to the normal reaction**. Under normal conditions, $N = mg$, feeling of weight is the actual weight mg. If we are standing on a lift and the lift has an acceleration a upwards then $N = m(g + a)$. Therefore, feeling of weight is more than the actual weight mg.
  Similarly, if a is downwards, then $N = m(g - a)$ and feeling of weight is less than the actual weight mg.
- If a car (or any other vehicle) accelerates and decelerates by friction, then the maximum acceleration or deceleration of a car on horizontal ground can be $\mu g \left(= \frac{f_L}{m} = \frac{\mu N}{m} = \frac{\mu mg}{m} \right)$, unless some other force is applied.
- Kilogram weight (kg-wt) or kilogram force (kg-f), gram weight (g-wt) or gram-force (g-f) are also the units of force. The CGS unit of force is dyne.

$$1 \text{ N} = 10^5 \text{ dyne}$$
$$1 \text{ kg-wt} = 1 \text{ kg-f} = 9.81 \text{ N}$$
$$1 \text{ g-wt} = 1 \text{ g-f} = 981 \text{ dyne}$$

- Newton's second law of motion is called real law of motion because first and third laws of motion can be obtained by it.
- Newton's third law of motion is a consequence of law of conservation of momentum. Rocket propulsion is based on law of conservation of momentum or Newton's third law of motion.
- Kinetic friction is less than the limiting value of static friction. This is because, to start motion of a body molecular bonds formed between two bodies in contact are broken. But once the motion starts, new bonds are formed and broken and this process continues.

Examples of Newton's First Law

(i) When a carpet or a blanket is beaten with a stick, then the dust particles separate out from it.

(ii) If a moving vehicle suddenly stops, then the passengers inside the vehicle bend in forward direction.

Examples of Newton's Third Law

(i) Swimming becomes possible because of third law of motion.

(ii) Jumping of a man from a boat onto the bank of a river.

(iii) Jerk is produced in a gun when bullet is fired from it.

(iv) Pulling of cart by a horse.