Session 3

Geometric Sequence or Geometric Progression (GP)

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A geometric progression is a sequence, if the ratio of any term and its just preceding term is constant throughout. This constant quantity is called the common ratio and is generally denoted by 'r'.

Or

A geometric progression (GP) is a sequence of numbers, whose first term is non-zero and each of the term is obtained by multiplying its just preceding term by a constant quantity. This constant quantity is called common ratio of the GP.

Let $t_1, t_2, t_3, \ldots, t_n; t_1, t_2, t_3, \ldots$ be respectively a finite or an infinite sequence. Assume that none of t_n''s is 0 and that $\frac{t_k}{t_{k-1}} = r$, a constant (i.e., independent of k).

For $k = 2, 3, 4, \ldots, n$ or $k = 2, 3, 4, \ldots$ as the case may be. We then call $\{t_k\}_{k=1}^{n}$ or $\{t_k\}_{k=1}^{\infty}$ as the case may be a geometric progression (GP). The constant ratio r is called the common ratio (CR) of the GP.

i.e., $$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \ldots = \frac{t_n}{t_{n-1}}$$

If $t_1 = a$ is the first term of a GP, then

$$t_2 = ar,\ t_3 = t_2 r = ar^2,\ t_4 = t_3 r = ar^3, \ldots,$$
$$t_n = t_{n-1} r = ar^{n-1}$$

It follows that, given that first term a and the common ratio r, the GP can be rewritten as

$a, ar, ar^2, \ldots, ar^{n-1}$ (standard GP) or $a, ar, ar^2, \ldots, ar^{n-1}, \ldots$ (standard GP)
according as it is finite or infinite.

Important Results

1. In a GP, neither $a = 0$ nor $r = 0$.
2. In a GP, no term can be equal to '0'.
3. If in a GP, the terms are alternatively positive and negative, then its common ratio is always negative.
4. If we multiply the common ratio with any term of GP, we get the next following term and if we divide any term by the common ratio, we get the preceding term.
5. The common ratio of GP may be positive, negative or imaginary.
6. If common ratio of GP is equal to unity, then GP is known as **Constant GP**.
7. If common ratio of GP is imaginary or real, then GP is known as **Imaginary GP.**
8. **Increasing and Decreasing GP**
   For a GP to be increasing or decreasing $r > 0$. If $r < 0$, terms of GP are alternatively positive and negative and so neither increasing nor decreasing.

a	$a > 0$	$a > 0$	$a < 0$	$a < 0$
r	$0 < r < 1$	$r > 1$	$r > 1$	$0 < r < 1$
Result	Decreasing	Increasing	Decreasing	Increasing

Example 26.

(i) 1, 2, 4, 8, 16, ... (ii) 9, 3, 1, $\frac{1}{3}, \frac{1}{9}$, ...

(iii) $-2, -6, -18, \ldots$ (iv) $-8, -4, -2, -1, -\frac{1}{2}, \ldots$

(v) $5, -10, 20, \ldots$ (vi) 5, 5, 5, 5,...

(vii) $1, 1+i, 2i, -2+2i, \ldots; i = \sqrt{-1}$

Sol. (i) Here, $a = 1$

and $r = \frac{2}{1} = \frac{4}{2} = \frac{8}{4} = \frac{16}{8} = \ldots = 2$ i.e. $a = 1$ and $r = 2$

Increasing GP $(a > 0, r > 1)$

(ii) Here, $a = 9$

and $r = \frac{3}{9} = \frac{1}{3} = \frac{\frac{1}{3}}{1} = \frac{\frac{1}{9}}{\frac{1}{3}} = \ldots = \frac{1}{3}$ i.e. $a = 9, r = \frac{1}{3}$

Decreasing GP $(a > 0, 0 < r < 1)$

(iii) Here, $a = -2$

and $r = \frac{-6}{-2} = \frac{-18}{-6} = \ldots = 3$

i.e. $a = -2, r = 3$

Decreasing GP $(a < 0, r > 1)$

(iv) Here, $a = -8$

and $r = \frac{-4}{-8} = \frac{-2}{-4} = \frac{-1}{-2} = \frac{-\frac{1}{2}}{-1} = \ldots. = \frac{1}{2}$

i.e. $a = -8, r = \frac{1}{2}$

Increasing GP $(a < 0, 0 < r < 1)$

(v) Here, $a = 5$

and $r = \frac{-10}{5} = \frac{20}{-10} = \ldots = -2$ i.e., $a = 5, r = -2$

Neither increasing nor decreasing $(r < 0)$

(vi) Here, $a = 5$

and $r = \frac{5}{5} = \frac{5}{5} = \frac{5}{5} = \ldots = 1$ i.e., $a = 5, r = 1$

Constant GP $(r = 1)$

(vii) Here, $a = 1$

and $r = \frac{1+i}{1} = \frac{2i}{1+i} = \frac{-2+2i}{2i} = \ldots$

$$= (1+i) = \frac{2i(1-i)}{(1+i)(1-i)} = \frac{(-1+i)\,i}{i^2} = \ldots$$

$$= (1+i) = (i+1) = (1+i) = \ldots$$

i.e., $a = 1, r = 1 + i$

Imaginary GP (r = imaginary)

Example 27. Show that the sequence $< t_n >$ defined by $t_n = \frac{2^{2n-1}}{3}$ for all values of $n \in N$ is a GP. Also, find its common ratio.

Sol. We have, $t_n = \frac{2^{2n-1}}{3}$

On replacing n by $n - 1$, we get

$$t_{n-1} = \frac{2^{2n-3}}{3} \Rightarrow \frac{t_n}{t_{n-1}} = \frac{\frac{2^{2n-1}}{3}}{\frac{2^{2n-3}}{3}} = 2^2 = 4$$

Clearly, $\frac{t_n}{t_{n-1}}$ is independent of n and is equal to 4. So, the given sequence is a GP with common ratio 4.

Example 28. ***Show that the sequence*** $< t_n >$ ***defined by*** $t_n = 2 \cdot 3^n + 1$ ***is not a GP.***

Sol. We have, $t_n = 2 \cdot 3^n + 1$

On replacing n by $(n - 1)$ in t_n, we get

$$t_{n-1} = 2 \cdot 3^{n-1} + 1$$

$$\Rightarrow \quad t_{n-1} = \frac{(2 \cdot 3^n + 3)}{3}$$

$$\therefore \quad \frac{t_n}{t_{n-1}} = \frac{(2 \cdot 3^n + 1)}{\frac{(2 \cdot 3^n + 3)}{3}} = \frac{3(2 \cdot 3^n + 1)}{(2 \cdot 3^n + 3)}$$

Clearly, $\frac{t_n}{t_{n-1}}$ is not independent of n and is therefore not constant. So, the given sequence is not a GP.

General Term of a GP

Let 'a' be the first term, 'r' be the common ratio and 'l' be the last term of a GP having 'n' terms. Then, GP can be written as $a, ar, ar^2, \ldots, \frac{l}{r^2}, \frac{l}{r}, l$

(i) nth Term of a GP from Beginning

1st term from beginning $= t_1 = a = ar^{1-1}$

2nd term from beginning $= t_2 = ar = ar^{2-1}$

3rd term from beginning $= t_3 = ar^2 = ar^{3-1}$

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

nth term from beginning $= t_n = ar^{n-1}, \forall\, n \in N$

Hence, n th term of a GP from beginning

$$t_n = ar^{n-1} = l \qquad \text{[last term]}$$

(ii) nth Term of a GP from End

1st term from end $= t'_1 = l = \frac{l}{r^{1-1}}$

2nd term from end $= t'_2 = \frac{l}{r} = \frac{l}{r^{2-1}}$

3rd term from end $= t'_3 = \frac{l}{r^2} = \frac{l}{r^{3-1}}$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

nth term from end $= t'_n = \frac{l}{r^{n-1}}, \forall\, n \in N$

Hence, nth term of a GP from end

$$= t'_n = \frac{l}{r^{n-1}} = a \qquad \text{[first term]}$$

Now, it is clear that $t_k \times t'_k = ar^{k-1} \times \frac{l}{r^{k-1}} = a \times l$

or $\quad t_k \times t'_k = a \times l, \forall\, 1 \le k \le n$

i.e. in a finite GP of n terms, the product of the k th term from the beginning and the k th term form the end is independent of k and equals the product of the first and last terms.

Remark

1. nth term is also called the general term.
2. If last term of GP be t_n and CR is r, then terms of GP from end are $t_n, \frac{t_n}{r}, \frac{t_n}{r^2}, \ldots$
3. If in a GP, the terms are alternatively positive and negative, then its common ratio is always negative.

4. If a and l represent first and last term of a GP respectively, then common ratio of GP $= r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}}$

5. If t_n, t_{n+1}, t_{n+2} are three consecutive terms of a GP, then
$$\frac{t_{n+1}}{t_n} = \frac{t_{n+2}}{t_{n+1}} \Rightarrow t_{n+1}^2 = t_n\, t_{n+2}$$
In particular, if a, b, c are in GP, then $\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$

On squaring, $\frac{b^2}{a^2} = \frac{c^2}{b^2}$

Hence, a^2, b^2, c^2 are also in GP.

Example 29. If first term of a GP is a, third term is b and $(n+1)$th term is c. The $(2n+1)$th term of a GP is

(a) $c\sqrt{\frac{b}{a}}$ (b) $\frac{bc}{a}$ (c) abc (d) $\frac{c^2}{a}$

Sol. Let common ratio $= r$

$$\therefore \quad b = ar^2 \Rightarrow r = \sqrt{\frac{b}{a}}$$

$$\text{Also,} \quad c = ar^n \Rightarrow r^n = \frac{c}{a}$$

$$\therefore \quad t_{2n+1} = ar^{2n} = a(r^n)^2 = a\left(\frac{c}{a}\right)^2 = \frac{c^2}{a}$$

Hence, (d) is the correct answer.

Example 30. The $(m+n)$th and $(m-n)$th terms of a GP are p and q, respectively. Then, the mth term of the GP is

(a) $p\left(\frac{q}{p}\right)^{\frac{m}{2n}}$ (b) $\sqrt{pq}$

(c) $\sqrt{\frac{p}{q}}$ (d) None of these

Sol. Let a be the first term and r be the common ratio, then

$$t_{m+n} = p \Rightarrow ar^{m+n-1} = p \quad \text{...(i)}$$

$$t_{m-n} = q \Rightarrow ar^{m-n-1} = q \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$ar^{m+n-1} \times ar^{m-n-1} = p \times q$$

$$\Rightarrow \quad a^2 r^{2m-2} = pq \Rightarrow ar^{m-1} = \sqrt{pq}$$

$$\Rightarrow \quad t_m = \sqrt{pq}$$

Hence, (b) is the correct answer.

Example 31. If $\sin\theta, \sqrt{2}(\sin\theta + 1), 6\sin\theta + 6$ are in GP, then the fifth term is

(a) 81 (b) $81\sqrt{2}$ (c) 162 (d) $162\sqrt{2}$

Sol. $[\sqrt{2}(\sin\theta + 1)]^2 = \sin\theta(6\sin\theta + 6)$

$$\Rightarrow [(\sin\theta + 1)\, 2(\sin\theta + 1) - 6\sin\theta] = 0$$

We get, $\quad \sin\theta = -1, \frac{1}{2}$

$$\therefore \quad \sin\theta = \frac{1}{2} \quad [\sin\theta = -1 \text{ is not possible}]$$

then first term $= a = \sin\theta = \frac{1}{2}$ and common ratio

$$= r = \frac{\sqrt{2}\left(\frac{1}{2} + 1\right)}{\left(\frac{1}{2}\right)} = 3\sqrt{2}$$

$$\therefore \quad t_5 = ar^4 = \frac{1}{2}(3\sqrt{2})^4 = 162$$

Hence, (c) is the correct answer.

Example 32. The 1025th term in the sequence 1, 22, 4444, 88888888, ... is

(a) 2^9 (b) 2^{10}

(c) 2^{11} (d) 2^{12}

Sol. The number of digits in each term of the sequence are 1, 2, 4, 8, ... which are in GP. Let 1025th term is 2^n.

Then,

$$1 + 2 + 4 + 8 + \ldots + 2^{n-1} < 1025 \le 1 + 2 + 4 + 8 + \ldots + 2^n$$

$$\Rightarrow \quad \frac{(2-1)(1 + 2 + 2^2 + 2^3 + \ldots + 2^{n-1})}{(2-1)} < 1025$$

$$\le \frac{(2-1)(1 + 2 + 2^2 + 2^3 + \ldots + 2^n)}{(2-1)}$$

$$\Rightarrow \quad 2^n - 1 < 1025 \le 2^{n+1} - 1 \Rightarrow 2^n < 1026 \le 2^{n+1} \quad \text{...(i)}$$

$$\text{or} \quad 2^{n+1} \ge 1026 > 1024$$

$$\Rightarrow \quad 2^{n+1} > 2^{10} \Rightarrow n + 1 > 10$$

$$\therefore \quad n > 9 \quad \therefore n = 10$$

[which is always satisfy Eq. (i)]

Hence, (b) is the correct answer.

Example 33. ***If a, b, c are real numbers such that $3(a^2 + b^2 + c^2 + 1) = 2(a + b + c + ab + bc + ca)$, then a, b, c are in***

(a) AP only (b) GP only

(c) GP and AP (d) None of these

Sol. Given, $3(a^2 + b^2 + c^2 + 1) = 2(a + b + c + ab + bc + ca)$

$$\Rightarrow \quad 2(a^2 + b^2 + c^2 - ab - bc - ca) + (a^2 + b^2 + c^2 - 2a - 2b - 2c + 3)$$

$$\Rightarrow \quad \{(a-b)^2 + (b-c)^2 + (c-a)^2\} + \{(a-1)^2 + (b-1)^2 + (c-1)^2\} = 0$$

$$\Rightarrow a - b = b - c = c - a = 0 \text{ and } a - 1 = b - 1 = c - 1 = 0$$

$$\Rightarrow \quad a = b = c = 1$$

$\Rightarrow$ a, b, c are in GP and AP.

Hence, (c) is the correct answer.

Sum of a Stated Number of Terms of a Geometric Series

The game of chess was invented by **Grand Vizier Sissa Ben Dhair** for the Indian king **Shirham.** Pleased with the game, the king asked the **Vizier** what he would like as reward. The **Vizier** asked for one grain of wheat to be placed on the first square of the chess, two grains on the second, four grains on the third and so on (each time doubling the number of grains). The king was surprised of the request and told the **vizier** that he was fool to ask for so little.

The inventor of chess was no fool. He told the king ''What I have asked for is more wheat that you have in the entire kingdom, in fact it is more than there is in the whole world'' He was right. There are 64 squares on a chess board and on the nth square he was asking for 2^{n-1} grains, if you add the numbers

i.e., $$S = 1 + 2 + 2^2 + 2^3 + \dots + 2^{62} + 2^{63} \quad \dots\text{(i)}$$

On multiplying both sides by 2, then

$$2S = 2 + 2^2 + 2^3 + 2^4 + \dots + 2^{63} + 2^{64} \quad \dots\text{(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$S = 2^{64} - 1 = 18, 446, 744, 073, 709, 551, 615$ grains i.e., represent more wheat that has been produced on the Earth.

Sum of n Terms of a GP

Let a be the first term, r be the common ratio, l be the last term of a GP having n terms and S_n the sum of n terms, then

$$S_n = a + ar + ar^2 + \dots + \frac{l}{r^2} + \frac{l}{r} + l \quad \dots\text{(i)}$$

On multiplying both sides by r (the common ratio)

$$r\,S_n = ar + ar^2 + ar^3 + \dots + \frac{l}{r} + l + lr \quad \dots\text{(ii)}$$

On subtracting Eq. (ii) from Eq. (i), we have

$$S_n - r\,S_n = a - lr \text{ or } S_n(1-r) = a - lr$$

$$\therefore \quad \boldsymbol{S_n = \frac{a - lr}{1 - r}}, \text{ when } r < 1$$

$$\boldsymbol{S_n = \frac{lr - a}{r - 1}}, \text{ when } r > 1$$

Now, $\quad l = t_n = ar^{n-1}$

Then, above formula can be written as

$$\boldsymbol{S_n = \frac{a(1 - r^n)}{(1 - r)}} \text{ when } \boldsymbol{r < 1, S_n = \frac{a(r^n - 1)}{(r - 1)}},$$

when $\boldsymbol{r > 1}$

If $r = 1$, the above formulae cannot be used. But, then the GP reduces to $a, a, a, \dots$

$\therefore \quad S_n = a + a + a + \dots n \text{ times} = na$

Sum to Infinity of a GP, when the Numerical Value of the Common Ratio is Less than Unity, i.e. It is a Proper Fraction

If a be the first term, r be the common ratio of a GP, then

$$S_n = \frac{a(1 - r^n)}{(1 - r)} = \frac{a}{(1 - r)} - \frac{ar^n}{(1 - r)}$$

Let $-1 < r < 1$ i.e. $|r| < 1$, then $\lim\limits_{n \to \infty} r^n \to 0$

Let S_∞ denote the sum to infinity of the GP, then

$$S_\infty = \frac{a}{(1 - r)},$$

where $\quad -1 < r < 1$

Recurring Decimal

Recurring decimal is a very good example of an infinite geometric series and its value can be obtained by means of infinite geometric series as follows

$0.3\dot{2}\dot{7} = 0.327272727\dots$ to infinity

$$= 0.3 + 0.027 + 0.00027 + 0.0000027 + \dots \text{ upto infinity}$$

$$= \frac{3}{10} + \frac{27}{10^3} + \frac{27}{10^5} + \frac{27}{10^7} + \dots \text{ upto infinity}$$

$$= \frac{3}{10} + \frac{27}{10^3}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + \dots \text{ upto infinity}\right)$$

$$= \frac{3}{10} + \frac{27}{10^3}\left(\frac{1}{1 - \frac{1}{10^2}}\right)$$

$$= \frac{3}{10} + \frac{27}{990} = \frac{297 + 27}{990}$$

$$= \frac{324}{990} \qquad \text{[rational number]}$$

Aliter (Best method)

Let P denotes the figure which do not recur and suppose them p in number, Q denotes the recurring period consisting of q figures. Let R denotes the value of the recurring decimal.

Then, $\quad R = 0 \cdot PQQQ \dots$

$\therefore \quad 10^p \times R = P \cdot QQQ \dots$

and $\quad 10^{p+q} \times R = PQ \cdot QQQ \dots$

$\therefore$ Therefore, by subtraction $R = \dfrac{PQ - P}{(10^{p+q} - 10^p)}$.

***Corollary* I** If $R = 0 \cdot QQQ \ldots$

Then, $R = \dfrac{Q}{10^q - 1}$ (when Q denote the recurring period consisting of q figures)

For example, If $R = 0.\dot{3}$, then $R = \dfrac{3}{10^1 - 1} = \dfrac{1}{3}$

***Corollary* II** The value of recurring decimal is always rational number.

Example 34. Find the value of $0.32\dot{5}\dot{8}$.

Sol. Let $\quad R = 0.32\dot{5}\dot{8}$

$\Rightarrow \quad R = 0.3258585858 \ldots \quad \ldots$(i)

Here, number of figures which are not recurring is 2 and number of figures which are recurring is also 2.

Then, $\quad 100R = 32.58585858 \ldots \quad \ldots$(ii)

and $\quad 10000R = 3258.58585858 \ldots \quad \ldots$(iii)

On subtracting Eq. (ii) from Eq. (iii), we get

$$9900R = 3226$$

$$\therefore \quad R = \frac{3226}{9900}$$

Hence, $\quad R = \dfrac{1613}{4950}$

Shortcut Methods for Recurring Decimals

1. The numerator of the vulgar fraction is obtained by subtracting the non-recurring figure from the given figure.
2. The denominator consists of as many 9's as there are recurring figure and as many zero as there are non-recurring figure.

   For example,

   (i) $0.36\dot{5}\dot{4} = \dfrac{3654 - 36}{9900} = \dfrac{3618}{9900}$

   (ii) $1.3\dot{2}\dot{7} = 1 + 0.3\dot{2}\dot{7} = 1 + \dfrac{327 - 3}{990} = \dfrac{1314}{990}$

   (iii) $0.\dot{3} = \dfrac{3 - 0}{9} = \dfrac{1}{3}$

Example 35. ***Find the sum upto n terms of the series $a + aa + aaa + aaaa + \ldots, \ \forall\, a \in N$ and $1 \le a \le 9$.***

Sol. Let $S = a + aa + aaa + aaaa + \ldots$ upto n terms

$$= a(1 + 11 + 111 + 1111 + \ldots \text{ upto } n \text{ terms})$$

$$= \frac{a}{9}(9 + 99 + 999 + 9999 + \ldots \text{ upto } n \text{ terms})$$

$$= \frac{a}{9}\{(10^1 - 1) + (10^2 - 1) + (10^3 - 1) + (10^4 - 1) + \ldots \text{ upto } n \text{ terms}\}$$

$$= \frac{a}{9}\{(10 + 10^2 + 10^3 + \ldots \text{ upto } n \text{ terms}) - (1 + 1 + 1 + \ldots n \text{ times})\}$$

$$= \frac{a}{9}\left\{\frac{10(10^n - 1)}{10 - 1} - n\right\} = \frac{a}{9}\left\{\frac{10}{9}(10^n - 1) - n\right\}$$

[Remember]

In Particular

(i) For $a = 1$, $1 + 11 + 111 + \ldots = \dfrac{1}{9}\left\{\dfrac{10}{9}(10^n - 1) - n\right\}$

(ii) For $a = 2$, $2 + 22 + 222 + \ldots = \dfrac{2}{9}\left\{\dfrac{10}{9}(10^n - 1) - n\right\}$

(iii) For $a = 3$, $3 + 33 + 333 + \ldots = \dfrac{3}{9}\left\{\dfrac{10}{9}(10^n - 1) - n\right\}$

(iv) For $a = 4$, $4 + 44 + 444 + \ldots = \dfrac{4}{9}\left\{\dfrac{10}{9}(10^n - 1) - n\right\}$

(v) For $a = 5$, $5 + 55 + 555 + \ldots = \dfrac{5}{9}\left\{\dfrac{10}{9}(10^n - 1) - n\right\}$

(vi) For $a = 6$, $6 + 66 + 666 + \ldots = \dfrac{6}{9}\left\{\dfrac{10}{9}(10^n - 1) - n\right\}$

(vii) For $a = 7$, $7 + 77 + 777 + \ldots = \dfrac{7}{9}\left\{\dfrac{10}{9}(10^n - 1) - n\right\}$

(viii) For $a = 8$, $8 + 88 + 888 + \ldots = \dfrac{8}{9}\left\{\dfrac{10}{9}(10^n - 1) - n\right\}$

(ix) For $a = 9$, $9 + 99 + 999 + \ldots = \dfrac{9}{9}\left\{\dfrac{10}{9}(10^n - 1) - n\right\}$

Example 36. Find the sum upto n terms of the series $0.b + 0.bb + 0.bbb + 0.bbbb + \ldots, \forall\, b \in N$ and $1 \le b \le 9$.

Sol. Let $S = 0.b + 0.bb + 0.bbb + 0.bbbb + \ldots$ upto n terms

$$= b(0.1 + 0.11 + 0.111 + 0.1111 + \ldots \text{ upto } n \text{ terms})$$

$$= \frac{b}{9}(0.9 + 0.99 + 0.999 + 0.9999 + \ldots \text{ upto } n \text{ terms})$$

$$= \frac{b}{9}\{(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + (1 - 0.0001) + \ldots \text{ upto } n \text{ terms}\}$$

$$= \frac{b}{9}\{(1 + 1 + 1 + 1 + \ldots \text{ upto } n \text{ times}) - (0.1 + 0.01 + 0.001 + 0.0001 + \ldots \text{ upto } n \text{ terms})\}$$

$$= \frac{b}{9}\left\{n - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \frac{1}{10^4} + \ldots \text{ upto } n \text{ terms}\right)\right\}$$

$$= \frac{b}{9}\left\{n - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^n\right)}{1 - \frac{1}{10}}\right\} = \frac{b}{9}\left\{n - \frac{1}{9}\left[1 - \left(\frac{1}{10}\right)^n\right]\right\}$$

[Remember]

In Particular

(i) For $b=1$,

$$0.1+0.11+0.111+\ldots=\frac{1}{9}\left\{n-\frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]\right\}$$

(ii) For $b=2$,

$$0.2+0.22+0.222+\ldots=\frac{2}{9}\left\{n-\frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]\right\}$$

(iii) For $b=3$,

$$0.3+0.33+0.333+\ldots=\frac{3}{9}\left\{n-\frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]\right\}$$

(iv) For $b=4$,

$$0.4+0.44+0.444+\ldots=\frac{4}{9}\left\{n-\frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]\right\}$$

(v) For $b=5$,

$$0.5+0.55+0.555+\ldots=\frac{5}{9}\left\{n-\frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]\right\}$$

(vi) For $b=6$,

$$0.6+0.66+0.666+\ldots=\frac{6}{9}\left\{n-\frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]\right\}$$

(vii) For $b=7$,

$$0.7+0.77+0.777+\ldots=\frac{7}{9}\left\{n-\frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]\right\}$$

(viii) For $b=8$,

$$0.8+0.88+0.888+\ldots=\frac{8}{9}\left\{n-\frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]\right\}$$

(ix) For $b=9$,

$$0.9+0.99+0.999+\ldots=\frac{9}{9}\left\{n-\frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]\right\}$$

Example 37. If N, the set of natural numbers is partitioned into groups $S_1=\{1\}, S_2=\{2,3\}$, $S_3=\{4,5,6,7\}$, $S_4=\{8,9,10,11,12,13,14,15\},\ldots$, then find the sum of the numbers in S_{50}.

Sol. The number of terms in the groups are $1, 2, 2^2, 2^3, \ldots$

$\therefore$ The number of terms in the 50th group $=2^{50-1}=2^{49}$

$\because$ The first term of 1st group $=1=2^0=2^{1-1}$

The first term of 2nd group $=2=2^1=2^{2-1}$

The first term of 3rd group $=4=2^2=2^{3-1}$

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

The first term of 50th group $=2^{50-1}=2^{49}$

$$\therefore \quad S_{50}=\frac{2^{49}}{2}\{2\times 2^{49}+(2^{49}-1)\times 1\}$$

$$=2^{48}(2^{50}+2^{49}-1)$$

$$=2^{48}[2^{49}(2+1)-1]=2^{48}(3\cdot 2^{49}-1)$$

Example 38. ***If*** $S_n=1+\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^{n-1}}$, ***then calculate the least value of*** n ***such that*** $2-S_n<\frac{1}{100}$.

Sol. Given, $S_n=1+\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^{n-1}}=\dfrac{1\cdot\left[1-\left(\frac{1}{2}\right)^n\right]}{\left(1-\frac{1}{2}\right)}$

$$\Rightarrow \quad S_n=2-\frac{1}{2^{n-1}}$$

$$\Rightarrow \quad 2-S_n=\frac{1}{2^{n-1}}<\frac{1}{100} \qquad \left[\because 2-S_n<\frac{1}{100}\right]$$

$$\Rightarrow \quad 2^{n-1}>100>2^6$$

$$\Rightarrow \quad 2^{n-1}>2^6$$

$$\therefore \quad n-1>6 \Rightarrow n>7$$

Hence, the least value of n is 8.

Example 39. If $x=1+a+a^2+a^3+\ldots+\infty$ and $y=1+b+b^2+b^3+\ldots+\infty$ show that $1+ab+a^2b^2+a^3b^3+\ldots+\infty=\frac{xy}{x+y-1}$, where $0<a<1$ and $0<b<1$.

Sol. Given, $x=1+a+a^2+a^3+\ldots+\infty=\frac{1}{1-a}$

$$\Rightarrow \quad x-ax=1$$

$$\therefore \quad a=\left(\frac{x-1}{x}\right) \qquad \ldots\text{(i)}$$

and $\quad y=1+b+b^2+b^3+\ldots+\infty$

Similarly, $\quad b=\left(\frac{y-1}{y}\right) \qquad \ldots\text{(ii)}$

Since, $\quad 0<a<1, 0<b<1$

$\therefore \quad 0<ab<1$

Now, $1+ab+a^2b^2+a^3b^3+\ldots+\infty=\frac{1}{1-ab}$

$$=\frac{1}{1-\left(\frac{x-1}{x}\right)\left(\frac{y-1}{y}\right)} \qquad \text{[from Eqs. (i) and (ii)]}$$

$$=\frac{xy}{xy-xy+x+y-1}$$

Hence, $1+ab+a^2b^2+a^3b^3+\ldots+\infty=\frac{xy}{x+y-1}$

Properties of Geometric Progression

1. If $a_1, a_2, a_3, \ldots$ are in GP with common ratio r, then $a_1 k, a_2 k, a_3 k, \ldots$ and $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \ldots$ are also in GP $(k \neq 0)$ with common ratio r.
2. If $a_1, a_2, a_3, \ldots$ are in GP with common ratio r, then $a_1 \pm k, a_2 \pm k, a_3 \pm k, \ldots$. are not in GP $(k \neq 0)$.
3. If $a_1, a_2, a_3, \ldots$ are in GP with common ratio r, then
   (i) $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots$ are also in GP with common ratio $\frac{1}{r}$.
   (ii) $a_1^n, a_2^n, a_3^n, \ldots$ are also in GP with common ratio r^n and $n \in Q$.
   (iii) $\log a_1, \log a_2, \log a_3, \ldots$ are in AP $(a_i > 0, \forall\, i)$
   In this case, the converse also holds good.
4. If $a_1, a_2, a_3, \ldots$ and $b_1, b_2, b_3, \ldots$ are two GP's with common ratios r_1 and r_2, respectively. Then,
   (i) $a_1 b_1, a_2 b_2, a_3 b_3, \ldots$ and $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \ldots$ are also in GP with common ratios $r_1 r_2$ and $\frac{r_1}{r_2}$, respectively.
   (ii) $a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3, \ldots$ are not in GP.
5. If $a_1, a_2, a_3, \ldots, a_{n-2}, a_{n-1}, a_n$ are in GP. Then,
   (i) $a_1 a_n = a_2 a_{n-1} = a_3 a_{n-2} = \ldots$
   (ii) $a_r = \sqrt{a_{r-k}\, a_{r+k}}, \forall\, k, 0 \leq k \leq n-r$
   (iii) $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \ldots = \frac{a_n}{a_{n-1}}$
   $\Rightarrow \quad a_2^2 = a_3 a_1, a_3^2 = a_2 a_4, \ldots$
   Also, $\quad a_2 = a_1 r, a_3 = a_1 r^2,$
   $a_4 = a_1 r^3, \ldots, a_n = a_1 r^{n-1}$
   where, r is the common ratio of GP.
6. If three numbers in GP whose product is given are to be taken as $\frac{a}{r}, a, ar$ and if five numbers in GP whose product is given are to be taken as $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$, etc.

   In general If $(2m+1)$ numbers in GP whose product is given are to be taken as $(m \in N)$
   $$\frac{a}{r^m}, \frac{a}{r^{m-1}}, \ldots, \frac{a}{r}, a, ar, \ldots, ar^{m-1}, ar^m$$

Remark

1. Product of three numbers $= a^3$
   Product of five numbers $= a^5$
   $\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$
   Product of $(2m + 1)$ numbers $= a^{2m+1}$
2. From given conditions, find two equations in a and r and then solve them. Now, the numbers in GP can be obtained.

7. If four numbers in GP whose product is given are to be taken as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$ and if six numbers in GP whose product is given are to be taken as $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$, etc.

   In general If $(2m)$ numbers in GP whose product is given are to be taken as $(m \in N)$
   $$\frac{a}{r^{2m-1}}, \frac{a}{r^{2m-3}}, \ldots, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, \ldots, ar^{2m-3}, ar^{2m-1}$$

Remark

1. Product of four numbers $= a^4$
   Product of six numbers $= a^6$
   $\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$
   Product of $(2m)$ numbers $= a^{2m}$
2. From given conditions, find two equations in a and r and then solve them. Now, the numbers in GP can be obtained.

Example 40. If $S_1, S_2, S_3, \ldots, S_p$ are the sum of infinite geometric series whose first terms are 1, 2, 3, ..., p and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{p+1}$ respectively, prove that
$$S_1 + S_2 + S_3 + \ldots + S_p = \frac{p\,(p+3)}{2}.$$

Sol. $\because \quad S_p = \frac{p}{1 - \frac{1}{p+1}} = (p+1)$

$\therefore \quad S_1 = 2, S_2 = 3, S_3 = 4, \ldots$

$\therefore \quad \text{LHS} = S_1 + S_2 + S_3 + \ldots + S_p$

$= 2 + 3 + 4 + \ldots + (p+1) = \frac{p}{2}(2 + p + 1)$

$= \frac{p\,(p+3)}{2} = \text{RHS}$

Example 41. ***Let x_1 and x_2 be the roots of the equation $x^2 - 3x + A = 0$ and let x_3 and x_4 be the roots of the equation $x^2 - 12x + B = 0$. It is known that the numbers x_1, x_2, x_3, x_4 (in that order) form an increasing GP. Find A and B.***

Sol. $\because x_1, x_2, x_3, x_4$ are in GP.

Let $x_2 = x_1 r, x_3 = x_1 r^2, x_4 = x_1 r^3$

[here, product of x_1, x_2, x_3, x_4 are not given]

Given, $x_1 + x_2 = 3, x_1 x_2 = A$

$\Rightarrow \quad x_1(1 + r) = 3, x_1^2 r = A$ ...(i)

and $x_3 + x_4 = 12, x_3 x_4 = B$

$\Rightarrow \quad x_1 r^2 (1 + r) = 12, x_1^2 r^5 = B$ ...(ii)

From Eqs. (i) and (ii),

$r^2 = 4 \Rightarrow r = 2$ [for increasing GP]

From Eq. (i), $x_1 = 1$

Now, $A = x_1^2 r = 1^2 \cdot 2 = 2$ [from Eq. (i)]

and $B = x_1^2 r^5 = 1^2 \cdot 2^5 = 32$ [from Eq. (ii)]

Example 42. ***Suppose*** a, b, c ***are in AP and*** a^2, b^2, c^2 ***are in GP, if*** $a > b > c$ ***and*** $a + b + c = \frac{3}{2}$, ***then find the values of*** a ***and*** c.

Sol. Since, a, b, c are in AP and sum of a, b, c is given.

Let $a = b - D, c = b + D$ $[D < 0][\because a > b > c]$

and given $a + b + c = \frac{3}{2}$

$\Rightarrow \quad b - D + b + b + D = \frac{3}{2}$

$\therefore \quad b = \frac{1}{2}$

Then, $a = \frac{1}{2} - D$ and $c = \frac{1}{2} + D$

Also, given a^2, b^2, c^2 are in GP, then $(b^2)^2 = a^2 c^2$

$\Rightarrow \quad \pm b^2 = ac \Rightarrow \pm \frac{1}{4} = \frac{1}{4} - D^2$

$\Rightarrow \quad D^2 = \frac{1}{4} \pm \frac{1}{4} = \frac{1}{2}$ $[\because D \neq 0]$

$\therefore \quad D = \pm \frac{1}{\sqrt{2}} \Rightarrow D = -\frac{1}{\sqrt{2}}$ $[\because D < 0]$

Hence, $a = \frac{1}{2} + \frac{1}{\sqrt{2}}$ and $c = \frac{1}{2} - \frac{1}{\sqrt{2}}$

Example 43. If the continued product of three numbers in GP is 216 and the sum of their products in pairs is 156, then find the sum of three numbers.

Sol. Here, product of numbers in GP is given.

$\therefore$ Let the three numbers be $\frac{a}{r}, a, ar$.

Then, $\frac{a}{r} \cdot a \cdot ar = 216$

$\Rightarrow \quad a^3 = 216$

$\therefore \quad a = 6$

Sum of the products in pairs = 156

$\Rightarrow \quad \frac{a}{r} \cdot a + a \cdot ar + ar \cdot \frac{a}{r} = 156$

$\Rightarrow \quad a^2 \left(\frac{1}{r} + r + 1\right) = 156 \Rightarrow 36 \left(\frac{1 + r^2 + r}{r}\right) = 156$

$\Rightarrow \quad 3\left(\frac{1 + r + r^2}{r}\right) = 13 \Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow \quad (3r - 1)(r - 3) = 0 \Rightarrow r = \frac{1}{3}$ or $r = 3$

Putting the values of a and r, the required numbers are 18, 6, 2 or 2, 6, 18. Hence, the sum of numbers is 26.

Example 44. Find a three-digit number whose consecutive digits form a GP. If we subtract 792 from this number, we get a number consisting of the same digits written in the reverse order. Now, if we increase the second digit of the required number by 2, then the resulting digits will form an AP.

Sol. Let the three digits be a, ar, ar^2, then according to hypothesis

$$100a + 10ar + ar^2 - 792 = 100ar^2 + 10ar + a$$

$\Rightarrow \quad 99a(1 - r^2) = 792$

$\Rightarrow \quad a(1 + r)(1 - r) = 8$ ...(i)

and $a, ar + 2, ar^2$ are in AP.

Then, $2(ar + 2) = a + ar^2$

$\Rightarrow \quad a(r^2 - 2r + 1) = 4 \Rightarrow a(r - 1)^2 = 4$ ...(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{r + 1}{r - 1} = -2 \Rightarrow r = \frac{1}{3}$$

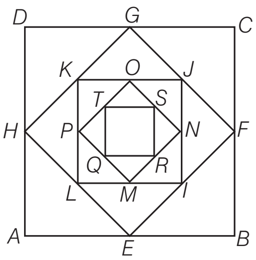
From Eq. (ii), $a = 9$

Thus, digits are 9, 3, 1 and so the required number is 931.

Examples on Application of Progression in Geometrical Figures

Example 45. ***A square is drawn by joining the mid-points of the sides of a given square. A third square is drawn inside the second square in the same way and this process continues indefinitely. If a side of the first square is 16 cm, then determine the sum of the areas of all the squares.***

Sol. Let a be the side length of square, then



$AB = BC = CD = DA = a$

$\because$ E, F, G, H are the mid-points of AB, BC, CD and DA, respectively.

$\therefore \quad EF = FG = GH = HE = \dfrac{a}{\sqrt{2}}$

and I, J, K, L are the mid-points of EF, FG, GH and HE, respectively.

$\therefore \quad IJ = JK = KL = LI = \dfrac{a}{2}$

Similarly, $\quad MN = NO = OP = PM = \dfrac{a}{2\sqrt{2}}$ and

$$QR = RS = ST = TQ = \frac{a}{4}, \dots$$

S = Sum of areas

$$= ABCD + EFGH + IJKL + MNOP + QRST + \dots$$

$$= a^2 + \left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2\sqrt{2}}\right)^2 + \dots$$

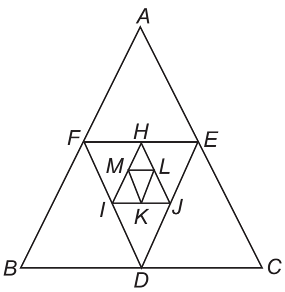
$$= a^2\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\right)$$

$$= a^2\left(\frac{1}{1 - \frac{1}{2}}\right) = 2a^2 = 2(16)^2 \qquad [\because a = 16 \text{ cm}]$$

$$= 512 \text{ sq cm}$$

Example 46. One side of an equilateral triangle is 24 cm. The mid-points of its sides are joined to form another triangle whose mid-points, in turn, are joined to form still another triangle. This process continues, indefinitely. Find the sum of the perimeters of all the triangles.

Sol. Let a be the side length of equilateral triangle, then $AB = BC = CA = a$



$\because$ D, E, F are the mid-points of BC, CA and AB, respectively.

$\therefore \quad EF = FD = DE = \dfrac{a}{2}$

and H, I, J are the mid-points of EF, FD and DE, respectively.

$\therefore \quad IJ = JH = HI = \dfrac{a}{4}$

Similarly, $\quad KL = ML = KM = \dfrac{a}{8}, \dots$

$$P = \text{Sum of perimeters} = 3\left(a + \frac{a}{2} + \frac{a}{4} + \frac{a}{8} + \dots\right)$$

$$= 3\left(\frac{a}{1 - \frac{1}{2}}\right) = 6a = 6 \times 24 = 144 \text{ cm} \qquad [\because a = 24 \text{ cm}]$$

Example 47. ***Let $S_1, S_2, \dots$ be squares such that for each $n \ge 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm and the area of S_n less than 1 sq cm. Then, find the value of n.***

Sol. We have, length of a side of

S_n = length of diagonal of S_{n+1}

$\Rightarrow$ Length of a side of $S_n = \sqrt{2}$ (length of a side of S_{n+1})

$$\Rightarrow \quad \frac{\text{Length of a side of } S_{n+1}}{\text{Length of a side of } S_n} = \frac{1}{\sqrt{2}}, \text{ for all } n \ge 1$$

$\Rightarrow$ Sides of $S_1, S_2, S_3, \dots$ form a GP with common ratio $\dfrac{1}{\sqrt{2}}$ and first term 10.

$$\therefore \quad \text{Side of } S_n = 10\left(\frac{1}{\sqrt{2}}\right)^{n-1} = \frac{10}{2^{\frac{(n-1)}{2}}}$$

$$\Rightarrow \text{Area of } S_n = (\text{Side})^2 = \frac{100}{2^{n-1}}$$

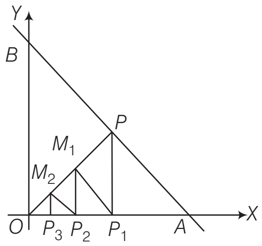
Now, given area of $S_n < 1$

$$\Rightarrow \quad \frac{100}{2^{n-1}} < 1 \Rightarrow 2^{n-1} > 100 > 2^6$$

$$\Rightarrow \quad 2^{n-1} > 2^6 \Rightarrow n - 1 > 6$$

$$\therefore \quad n > 7 \text{ or } n \ge 8$$

Example 48. The line $x + y = 1$ meets X-axis at A and Y-axis at B, P is the mid-point of AB, P_1 is the foot of perpendicular from P to OA, M_1 is that of P_1 from OP; P_2 is that of M_1 from OA, M_2 is that of P_2 from OP; P_3 is that of M_2 from OA and so on. If P_n denotes the nth foot of the perpendicular on OA, then find OP_n.



Sol. We have,

$$(OM_{n-1})^2 = (OP_n)^2 + (P_n M_{n-1})^2$$

$$= (OP_n)^2 + (OP_n)^2 = 2(OP_n)^2 = 2\alpha_n^2 \quad [\text{say}]$$

Also, $(OP_{n-1})^2 = (OM_{n-1})^2 + (P_{n-1}M_{n-1})^2$

$$\Rightarrow \quad \alpha_{n-1}^2 = 2\alpha_n^2 + \frac{1}{2}\alpha_{n-1}^2 \Rightarrow \alpha_n^2 = \frac{1}{4}\alpha_{n-1}^2$$

$$\Rightarrow \quad \alpha_n = \frac{1}{2}\alpha_{n-1}$$

$$\Rightarrow \quad OP_n = \alpha_n = \frac{1}{2}\alpha_{n-1} = \frac{1}{2^2}\alpha_{n-2} = \dots = \frac{1}{2^n}$$

$$\therefore \quad OP_n = \left(\frac{1}{2}\right)^n$$

Use of GP in Solving Practical Problems

In this part, we will see how the formulae relating to GP can be made use of in solving practical problems.

Example 49. Dipesh writes letters to four of his friends. He asks each of them to copy the letter and mail to four different persons with the request that they continue the chain similarly. Assuming that the chain is not broken and that it costs 25 paise to mail one letter, find the total money spent on postage till the 8th set of letters is mailed.

Sol. Number of letters in the 1st set = 4 (These are letters sent by Dipesh)

Number of letters in the 2nd set = 4 + 4 + 4 + 4 = 16

Number of letters in the 3rd set

$$= 4 + 4 + 4 + \dots + 16 \text{ terms} = 64$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

The number of letters sent in the 1st set, 2nd set, 3rd set, ... are respectively 4, 16, 64, ... which is a GP with $a = 4$,

$$r = \frac{16}{4} = \frac{64}{16} = 4$$

$\therefore$ Total number of letters in all the first 8 sets

$$= \frac{4(4^8 - 1)}{4 - 1} = 87380$$

$\therefore$ Total money spent on letters $= 87380 \times \frac{25}{100} =$ ₹21845

Example 50. An insect starts from a point and travels in a straight path 1 mm in the first second and half of the distance covered in the previous second in the succeeding second. In how much time would it reach a point 3 mm away from its starting point.

Sol. Distance covered by the insect in the 1st second = 1 mm

Distance covered by it in the 2nd second $= 1 \times \frac{1}{2} = \frac{1}{2}$ mm

Distance covered by it in the 3rd second $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ mm

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

The distance covered by the insect in 1st second, 2nd second, 3rd second, ... are respectively $1, \frac{1}{2}, \frac{1}{4}, \dots$, which are in GP with $a = 1$, $r = \frac{1}{2}$. Let time taken by the insect in covering 3 mm be n seconds.

$\therefore 1 + \frac{1}{2} + \frac{1}{4} + \dots + n$ terms $= 3$

$$\Rightarrow \quad \frac{1 \cdot \left[1 - \left(\frac{1}{2}\right)^n\right]}{1 - \frac{1}{2}} = 3$$

$$\Rightarrow \quad 1 - \left(\frac{1}{2}\right)^n = \frac{3}{2}$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^n = -\frac{1}{2}$$

$$\Rightarrow \quad 2^n = -2$$

which is impossible because $2^n > 0$

$\therefore$ Our supposition is wrong.

$\therefore$ There is no $n \in N$, for which the insect could never 3 mm in n seconds.

Hence, it will never to able to cover 3 mm.

Remark

The maximum distance that the insect could cover is 2 mm.

i.e., $\quad 1 + \frac{1}{2} + \frac{1}{4} + \dots = \frac{1}{1 - \frac{1}{2}} = 2$

Example 51. The pollution in a normal atmosphere is less than 0.01%. Due to leakage of a gas from a factory, the pollution is increased to 20%. If every day 80% of the pollution is neutralised, in how many days the atmosphere will be normal?

Sol. Let the pollution on 1st day = 20

The pollution on 2nd day = 20 × 20% = 20 (0.20)

The pollution on 3rd day = $20(0.20)^2$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

Let in n days the atmosphere will be normal

$$\therefore \quad 20(0.20)^{n-1} < 0.01$$

$$\Rightarrow \quad \left(\frac{2}{10}\right)^{n-1} < \frac{1}{2000}$$

Taking logarithm on base 10, we get

$$(n-1)(\log 2 - \log 10) < \log 1 - \log 2000$$

$$\Rightarrow \quad (n-1)(0.3010 - 1) < 0 - (0.3010 + 3)$$

$$\Rightarrow \quad n - 1 > \frac{3.3010}{0.6990}$$

$$\Rightarrow \quad n > 5.722$$

Hence, the atmosphere will be normal in 6 days.

Exercise for Session 3

1. The fourth, seventh and the last term of a GP are 10, 80 and 2560, respectively. The first term and number of terms in GP are

(a) $\frac{4}{5}$, 12 (b) $\frac{4}{5}$, 10 (c) $\frac{5}{4}$, 12 (d) $\frac{5}{4}$, 10

2. If the first and the nth terms of a GP are a and b respectively and if P is the product of the first n terms, then P^2 is equal to

(a) ab (b) $(ab)^{n/2}$ (c) $(ab)^n$ (d) None of these

3. If $a_1, a_2, a_3, (a_1 > 0)$ are three successive terms of a GP with common ratio r, the value of r for which $a_3 > 4a_2 - 3a_1$ holds is given by

(a) $1 < r < 3$ (b) $-3 < r < -1$ (c) $r < 1$ or $r > 3$ (d) None of these

4. If $x, 2x + 2, 3x + 3$ are in GP, the fourth term is

(a) 27 (b) – 27 (c) 13.5 (d) – 13.5

5. In a sequence of 21 terms the first 11 terms are in AP with common difference 2 and the last 11 terms are in GP with common ratio 2, if the middle term of the AP is equal to the middle term of GP, the middle term of the entire sequence is

(a) $-\frac{10}{31}$ (b) $\frac{10}{31}$ (c) $-\frac{32}{31}$ (d) $\frac{32}{31}$

6. Three distinct numbers x, y, z form a GP in that order and the numbers $7x + 5y, 7y + 5z, 7z + 5x$ form an AP in that order. The common ratio of GP is

(a) – 4 (b) –2 (c) 10 (d) 18

7. The sum to n terms of the series $11 + 103 + 1005 + \dots$ is

(a) $\frac{1}{9}(10^n - 1) + n^2$ (b) $\frac{1}{9}(10^n - 1) + 2n$ (c) $\frac{10}{9}(10^n - 1) + n^2$ (d) $\frac{10}{9}(10^n - 1) + 2n$

8. In an increasing GP, the sum of the first and last term is 66, the product of the second and the last but one is 128 and the sum of the sum of the terms is 126, then the number of terms in the series is

(a) 6 (b) 8 (c) 10 (d) 12

9. If S_1, S_2, S_3 be respectively the sum of $n, 2n$ and $3n$ terms of a GP, then $\frac{S_1(S_3 - S_2)}{(S_2 - S_1)^2}$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

10. If $|a| < 1$ and $|b| < 1$, then the sum of the series $1 + (1 + a)b + (1 + a + a^2)b^2 + (1 + a + a^2 + a^3)b^3 + \dots$ is

(a) $\frac{1}{(1-a)(1-b)}$ (b) $\frac{1}{(1-a)(1-ab)}$ (c) $\frac{1}{(1-b)(1-ab)}$ (d) $\frac{1}{(1-a)(1-b)(1-ab)}$

11. If the sides of a triangle are in GP and its larger angle is twice the smallest, then the common ratio r satisfies the inequality

(a) $0 < r < \sqrt{2}$ (b) $1 < r < \sqrt{2}$ (c) $1 < r < 2$ (d) $r > \sqrt{2}$

12. If $ax^3 + bx^2 + cx + d$ is divisible by $ax^2 + c$, then a, b, c, d are in

(a) AP (b) GP (c) HP (d) None of these

13. If $(r)_n$ denotes the number $r\,r\,r \dots$ (n digits), where $r = 1, 2, 3, \dots, 9$ and $a = (6)_n, b = (8)_n, c = (4)_{2n}$, then

(a) $a^2 + b + c = 0$ (b) $a^2 + b - c = 0$ (c) $a^2 + b - 2c = 0$ (d) $a^2 + b - 9c = 0$

14. $0.4\dot{2}\dot{7}$ represents the rational number

(a) $\frac{47}{99}$ (b) $\frac{47}{110}$ (c) $\frac{47}{999}$ (d) $\frac{49}{99}$

15. If the product of three numbers in GP be 216 and their sum is 19, then the numbers are

(a) 4, 6, 9 (b) 4, 7, 8 (c) 3, 7, 9 (d) None of these