Chapter-26 : Atoms

1. **(c)** Kinetic energy of α nucleus is equall to electrostatic potential energy of the system of the α particle and the heavy nucleus. That is,

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha Ze}{r_0}$$

where r_0 is the distance of closest approach

$$r_0 = \frac{2}{4\pi\varepsilon_0}\frac{q_\alpha Ze}{mv^2}$$

$$\Rightarrow r_0 \propto Ze \propto q_\alpha \propto \frac{1}{m} \propto \frac{1}{v^2}$$

2. **(b)** In Paschen series $\frac{1}{\lambda_{max}} = R\left[\frac{1}{(3)^2} - \frac{1}{(4)^2}\right]$

$$\Rightarrow \lambda_{max} = \frac{144}{7R} = \frac{144}{7 \times 1.1 \times 10^7} = 1.89 \times 10^{-6} m$$

$= 1.89\ \mu m$

Similarly $\lambda_{min} = \frac{9}{R} = \frac{9}{1.1 \times 10^7} = 0.818\ \mu m$

3. **(c)** $U = -K\frac{ze^2}{r}$; $T.E = -\frac{k}{2}\frac{ze^2}{r}$

$K.E = \frac{k}{2}\frac{ze^2}{r}$. Here r decreases

4. **(d)** $h\nu_L = E_\infty - E_1$ ...(i)

$h\nu_f = E_\infty - E_5$ ...(ii)

$$E \propto \frac{z^2}{n^2} \Rightarrow \frac{E_5}{E_1} = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$$

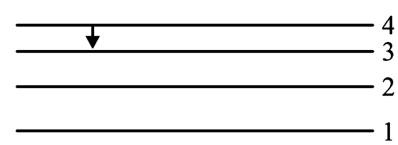
Eqn (i) / (ii) $\Rightarrow \frac{h\nu_L}{h\nu_f} = \frac{E_1}{E_5}$

$$\Rightarrow \frac{\nu_L}{\nu_f} = \frac{25}{1} \Rightarrow \nu_f = \frac{\nu_L}{25}$$

5. **(d)**

6. **(c)** Transition A (n = ∞ to 1) : Series lime of Lyman series.
Transition B (n = 5 to n = 2) : Third spectral line of Balmer series.
Transition C (n = 5 to n = 3) : Second spectral line of Paschen series.

7. **(c)** $\frac{n(n-1)}{2} = 6$

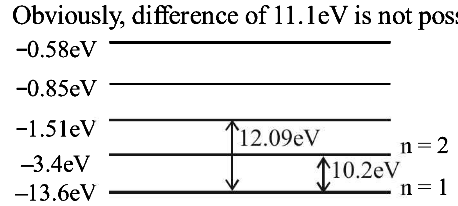


$n^2 - n - 12 = 0$

$(n-4)(n+3) = 0$ or $n = 4$

8. **(d)**

9. **(b)** Obviously, difference of 11.1eV is not possible.



10. **(c)** $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$\Rightarrow \frac{1}{970.6 \times 10^{-10}} = 1.097 \times 10^7\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right] \Rightarrow n_2 = 4$$

$\therefore$ Number of emission line $N = \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$

11. **(c)**

12. **(c)** According to Bohr's theory, the wave number of the last line of the Balmer series in hydrogen spectrum,
For hydrogen atom z = 1
$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) = 10^7 \times 1^2\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$
$\Rightarrow$ wave number $\frac{1}{\lambda} = 0.25 \times 10^7\ m^{-1}$.

13. **(a)** **14.** **(d)**

15. **(a)** Excitation energy $\Delta E = E_2 - E_1 = 13.6\,Z^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$
$$\Rightarrow 40.8 = 13.6 \times \frac{3}{4} \times Z^2 \Rightarrow Z = 2.$$
Now required energy to remove the electron from ground state $= \frac{+13.6Z^2}{(1)^2} = 13.6(Z)^2 = 54.4$ eV.

16. **(d)** No. of particles scattered through an angle
$$\theta = N(\theta) = \frac{kZ^2}{\sin^4\left(\frac{\theta}{2}\right)(K.E.)^2}$$
$$\therefore 28 = \frac{4kcz^2}{(K.E.)^2} \text{ for } \theta = 90°$$
$$\therefore \frac{kz^2}{(K.E.)^2} = \frac{28}{4} = 7$$
$$\therefore N(60°) = \frac{7}{\sin^4\left(\frac{60°}{2}\right)} = 16 \times 7 = 112/\text{min}.$$
$$N(120°) = \frac{7}{\sin^4\left(\frac{120°}{2}\right)} = 12.4 / \text{min}$$

17. **(b)** Transition from higher states to n = 2 lead to emission of radiation with wavelengths 656.3 nm and 365.0 nm. These wavelengths fall in the visible region and constitute the Balmer series.

18. **(b)** $E = -3.4$ eV and $r = \frac{kze^2}{2E}$
angular momentum = mvr
$$\Rightarrow \frac{1}{2}mv^2 = E = 3.4 \times (10^{-19} \times 1.6)$$
$$\Rightarrow m^2v^2 = (9.1 \times 10^{-31})2 \times 3.4 \times 1.6 \times 10^{-19}$$
$$= 99.008 \times 10^{-50}$$
$$mv = 9.95028 \times 10^{-25}$$
$$\therefore L = (9.95028 \times 10^{-25})\left(\frac{9 \times 10^9 \times 1 \times (1.6 \times 10^{-19})^2}{2 \times (3.4)}\right)$$
$$= 2.10 \times 10^{-34} \text{ Js.}$$

19. **(b)**

20. **(c)** For third line of Balmer series $n_1 = 2, n_2 = 5$
$$\therefore \frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \text{gives } Z^2 = \frac{n_1^2 n_2^2}{(n_2^2 - n_1^2)\lambda R}$$
On putting values Z = 2
$$\text{From } E = -\frac{13.6Z^2}{n^2} = \frac{-13.6(2)^2}{(1)^2} = -54.4 \text{ eV}$$

21. **(a)** The kinetic energy of the projectile is given by
$$\frac{1}{2}mv^2 = \frac{Ze(2e)}{4\pi\varepsilon_0 r_0}$$
$$= \frac{Z_1 Z_2}{4\pi\varepsilon_0 r_0}$$
Thus energy of the projectile is directly proportional to Z_1, Z_2

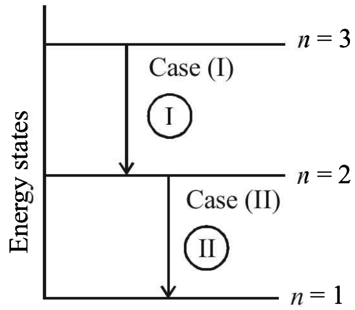
22. **(d)** $\lambda_{IR} > \lambda_{UV}$ also wavelength of emitted radiation $\lambda \propto \frac{1}{\Delta E}$.

23. **(a)** Energy of a H-like atom in it's n^{th} state is given by
$$E_n = -Z^2 \times \frac{13.6}{n^2} eV$$
For, first excited state of He^+, n = 2, Z = 2
$$\therefore \quad E_{He^+} = -\frac{4}{2^2} \times 13.6 = -13.6eV$$

24. **(d)** For first line of Lyman series of hydrogen
$$\frac{hc}{\lambda_1} = Rhc\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$
For second line of Balmer series of hydrogen like ion
$$\frac{hc}{\lambda_2} = Z^2Rhc\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$
By question, $\lambda_1 = \lambda_2$
$$\Rightarrow \left(\frac{1}{1} - \frac{1}{2}\right) = Z^2\left(\frac{1}{4} - \frac{1}{16}\right) \text{ or } Z = 2$$

25. **(b)** $KE_{max} = 10$eV
$\phi = 2.75$ eV
Total incident energy
$E = \phi + KE_{max} = 12.75$ eV
$\therefore$ Energy is released when electron jumps from the excited state n to the ground state.
$\because E_4 - E_1 = \{-0.85 - (-13.6)\text{ ev}\}$
$= 12.75$eV
$\therefore$ value of n = 4

26. (c)



The wave number $(\bar{v})$ of the radiation $= \frac{1}{\lambda}$

$$= R_{\infty}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Now for case (I) $n_1 = 3, n_2 = 2$

$$\frac{1}{\lambda_1} = R_{\infty}\left[\frac{1}{9} \frac{-1}{4}\right], R_{\infty} = \text{ Rydberg constant}$$

$$\frac{1}{\lambda_1} = R_{\infty}\left[\frac{4-9}{36}\right] = \frac{-5R_{\infty}}{36} \Rightarrow \lambda_1 = \frac{-36}{5R_{\infty}}$$

$$\frac{1}{\lambda_2} = R_{\infty}\left[\frac{1}{4} - \frac{1}{1}\right] = \frac{-3R_{\infty}}{4}$$

$$\lambda_2 = \frac{-4}{3R_{\infty}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{-36}{5R_{\infty}} \times \frac{3R_{\infty}}{-4} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{27}{5}$$

27. (a) For emission, the wave number of the radiation is given as

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

R = Rydberg constant, Z = atomic number

$$= R\left(\frac{1}{1^2} - \frac{1}{5^2}\right) = R\left(1 - \frac{1}{25}\right) \Rightarrow \frac{1}{\lambda} = R\frac{24}{25}$$

linear momentum

$$P = \frac{h}{\lambda} = h \times R \times \frac{24}{25} \text{ (de-Broglie hypothesis)}$$

$$\Rightarrow mv = \frac{24hR}{25} \Rightarrow V = \frac{24hR}{25m}$$

28. (d) For ground state, the principal quantum no $(n) = 1$. There is a 3rd excited state for principal quantum number.
The possible number of the spectral lines is given

$$= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

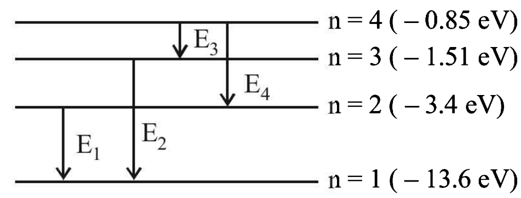
29. (c) For the $\lambda = 975$ Å; $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

where R is the Rydberg constant

Solving we get $n_2 = n = 4$ ($\because n_1 = 1$ ground state)
Therefore number of spectral lines

$$= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

30. (b) From diagram



$E_1 = -13.6 - (-3.4) = -10.2$ eV
$E_2 = -13.6 - (-1.51) = -12.09$ eV
$E_3 = -1.51 - (-0.85) = -0.66$ eV
$E_4 = -3.4 - (-0.85) = (-2.55)$ eV

E_3 is least i.e., frequency is lowest.

31. (b) $E = Rhc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

E will be maximum for the transition for which $\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ is maximum. Here n_2 is the higher energy level.

Clearly, $\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ is maximum for the third transition, i.e. $2 \rightarrow 1$.
I transition represents the absorption of energy.

32. (d) $E = -Z^2 \times 13.6$ eV $= -9 \times 13.6$ eV $= -122.4$ eV
So ionisation energy = + 122.4 eV.

33. (a) $T \propto n^3$. Given $T_{n_1} = 8T_{n_2}$, hence $n_1 = 2n_2$

34. (a) We know that $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

The wave length of first spectral line in the Balmer series of hydrogen atom is 6561Å. Here $n_2 = 3$ and $n_1 = 2$

$$\therefore \frac{1}{6561} = R(1)^2\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36} \quad ...(i)$$

For the second spectral line in the Balmer series of singly ionised helium ion $n_2 = 4$ and $n_1 = 2$; Z = 2

$$\therefore \frac{1}{\lambda} = R(2)^2\left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3R}{4} \quad ...(ii)$$

Dividing equation (i) and equation (ii) we get

$$\frac{\lambda}{6561}=\frac{5R}{36}\times\frac{4}{3R}=\frac{5}{27}$$

$\therefore\ \lambda = 1215\ \text{Å}$

35. (d) Radius of n^{th} orbit $r_n \propto n^2$, graph between r_n and n is a parabola. Also, $\frac{r_n}{r_1}=\left(\frac{n}{1}\right)^2 \Rightarrow \log_e\left(\frac{r_n}{r_1}\right)=2\log_e(n)$

Comparing this equation with $y = mx + c$,

Graph between $\log_e\left(\frac{r_n}{r_1}\right)$ and $\log_e(n)$ will be a straight line, passing from origin.

Similarly it can be proved that graph between $\log_e\left(\frac{f_n}{f_1}\right)$ and $\log_e n$ is a straight line. But with negative slops.

36. (d) $\frac{1}{\lambda}=RZ^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$

37. (a) $\frac{1}{\lambda_{max}}=R\left[\frac{1}{(1)^2}-\frac{1}{(2)^2}\right]\Rightarrow \lambda_{max}=\frac{4}{3R}\approx 1213\text{Å}$

and $\frac{1}{\lambda_{min}}=R\left[\frac{1}{(1)^2}-\frac{1}{\infty}\right]\Rightarrow \lambda_{min}=\frac{1}{R}\approx 910\text{Å}.$

38. (d) $\because\ B=\frac{\mu_0 I}{2r}$ and $I=\frac{e}{T}$

$B=\frac{\mu_0 e}{2rT}\ [r\propto n^2, T\propto n^5]$; $B\propto\frac{1}{n^5}$

39. (b) For 2^{nd} line of Balmer series in hydrogen spectrum

$$\frac{1}{\lambda}=R\,(1)\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=\frac{3}{16}R$$

For Li^{2+} $\left[\frac{1}{\lambda}=R\times 9\left(\frac{1}{x^2}-\frac{1}{12^2}\right)=\frac{3R}{16}\right]$

which is satisfied by $n = 12 \rightarrow n = 6$.

40. (b) $\frac{1}{\lambda}=R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

$$\frac{1}{\lambda_0}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=R\left(\frac{1}{4}-\frac{1}{9}\right)=\frac{5R}{36}$$

$$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=R\left(\frac{1}{4}-\frac{1}{16}\right)=\frac{3R}{16}$$

$$\frac{\lambda}{\lambda_0}=\frac{5}{36}\times\frac{16}{3}=\frac{20}{27}$$

41. (b) $\frac{1}{\lambda'}=\frac{1}{\lambda}\sqrt{\frac{c-v}{c+v}}$

Here, $\lambda' = 706\,nm, \lambda = 656\,nm$

$\therefore\ \frac{c-v}{c+v}=\left(\frac{\lambda}{\lambda'}\right)^2=\left(\frac{656}{706}\right)^2=0.86$

$\Rightarrow \frac{v}{c}=\frac{0.14}{1.86}$

$\Rightarrow v = 0.075\times 3\times 10^8 = 2.25\times 10^7 m/s$

42. (d) $\frac{1}{\lambda}=RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

For Balmer series $n = 2$

$\frac{1}{\lambda_{max}}=RZ^2\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right)$ and

$\frac{1}{\lambda_{min}}=RZ^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$

or $\frac{1/\lambda_{min}}{1/\lambda_{max}}=\frac{(1/2^2-1/3^2)}{(1/2^2)}$

$=\frac{\frac{1}{4}-\frac{1}{9}}{1/4}=1-\frac{4}{9}=5/9 \Rightarrow \frac{\lambda_{max}}{\lambda_{min}}=\frac{5}{9}$

43. (a) For Bracket series $\frac{1}{\lambda_{max}}=R\left[\frac{1}{4^2}-\frac{1}{5^2}\right]=\frac{9}{25\times 16}R$

and $\frac{1}{\lambda_{min}}=R\left[\frac{1}{4^2}-\frac{1}{\infty^2}\right]=\frac{R}{16}\Rightarrow\frac{\lambda_{max}}{\lambda_{min}}=\frac{25}{9}$

44. (c) $N\propto\frac{1}{\sin^4\theta/2}$; $\frac{N_2}{N_1}=\frac{\sin^4(\theta_1/2)}{\sin^4(\theta_2/2)}$

or $\frac{N_2}{5\times 10^6}=\frac{\sin^4(60°/2)}{\sin^4(120°/2)}$

or $\frac{N_2}{5\times 10^6}=\frac{\sin^4 30°}{\sin^4 60°}$

or $N_2 = 5\times 10^6\times\left(\frac{1}{2}\right)^4\left(\frac{2}{\sqrt{3}}\right)^4=\frac{5}{9}\times 10^6$

45. (a) Distance of closest approach

$$r_0=\frac{Ze(2e)}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$$

Energy, $E = 5\times 10^6\times 1.6\times 10^{-19}$ J

$$\therefore r_0=\frac{9\times 10^9\times(92\times 1.6\times 10^{-19})(2\times 1.6\times 10^{-19})}{5\times 10^6\times 1.6\times 10^{-19}}$$

$\Rightarrow r = 5.2\times 10^{-14} m = 5.3\times 10^{-12}$ cm

46. (b) When $F=\frac{k}{r}$ = centripetal force, then $\frac{k}{r}=\frac{mv^2}{r}$

$\Rightarrow mv^2$ = constat $\Rightarrow$ kinetic energy is constant

$\Rightarrow T$ is independent of n.

47. (b) The smallest frequency and largest wavelength in ultraviolet region will be for transition of electron from orbit 2 to orbit 1.

$$\therefore \quad \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{122 \times 10^{-9} m} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R\left[1 - \frac{1}{4}\right] = \frac{3R}{4}$$

$$\Rightarrow R = \frac{4}{3 \times 122 \times 10^{-9}} m^{-1}$$

The highest frequency and smallest wavelength for infrared region will be for transition of electron from ∞ to 3rd orbit.

$$\therefore \quad \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \frac{1}{\lambda} = \frac{4}{3 \times 122 \times 10^{-9}}\left(\frac{1}{3^2} - \frac{1}{\infty}\right)$$

$$\therefore \quad \lambda = \frac{3 \times 122 \times 9 \times 10^{-9}}{4} = 823.5\text{nm}$$

48. (d) For an atom following Bohr's model, the radius is given by

$r_m = \frac{r_0 m^2}{Z}$ where r_0 = Bohr's radius and m = orbit number.

For *Fm*, $m = 5$ (Fifth orbit in which the outermost electron is present)

$$\therefore \quad r_m = \frac{r_0 5^2}{100} = n r_0 \text{ (given)} \Rightarrow n = \frac{1}{4}$$

49. (c) Number of possible spectral lines emitted when an electron jumps back to ground state from n^{th} orbit $= \frac{n(n-1)}{2}$

Here, $\frac{n(n-1)}{2} = 6 \Rightarrow n = 4$

Wavelength λ from transition from n = 1 to n = 4 is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{1} - \frac{1}{4^2}\right) \Rightarrow \lambda = \frac{16}{15R} = 975 \text{ Å}$$

50. (d) From, $\Delta E = h\nu$

$$\nu = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{k(2n-1)}{n^2(n-1)^2}$$

$$\approx \frac{2k}{n^3} \text{ or, } \nu \propto \frac{1}{n^3}.$$