23 Indefinite Integrals

Consider the following for the next two (02) items that follow:

Consider $\int x\tan^{-1}x\,dx = A(x^2+1)\tan^{-1}x + Bx + C$

where C is the constant of integration. **[2014-II]**

1. What is the value of A?

(*a*) 1 (*b*) 1/2 (*c*) −1/2 (*d*) 1/4

2. What is the value of B?

(*a*) 1 (*b*) 1/2 (*c*) −1/2 (*d*) 1/4

Consider the following for the next two (02) items that follow:

Consider the function $f''(x) = \sec^4 x + 4$ with $f(0) = 0$ and $f'(0) = 0$.

[2014-II]

3. What is $f'(x)$ equal to?

(*a*) $\tan x - \frac{\tan^3 x}{3} + 4x$ (*b*) $\tan x + \frac{\tan^3 x}{3} + 4x$

(*c*) $\tan x - \frac{\sec^3 x}{3} + 4x$ (*d*) $-\tan x - \frac{\tan^3 x}{3} - 4x$

4. What is $f(x)$ equal to?

(*a*) $\frac{2\ln\sec x}{3} + \frac{\tan^2 x}{6} + 2x^2$ (*b*) $\frac{3\ln\sec x}{2} + \frac{\cot^2 x}{6} + 2x^2$

(*c*) $\frac{4\ln\sec x}{3} + \frac{\sec^2 x}{6} + 2x^2$ (*d*) $\ln\sec x + \frac{\tan^4 x}{12} + 2x^2$

5. What is $\int \frac{xe^x dx}{(x+1)^2}$ equal to? **[2015-I]**

(*a*) $(x+1)^2 e^x + c$ (*b*) $(x+1)e^x + c$

(*c*) $\frac{e^x}{x+1} + c$ (*d*) $\frac{e^x}{(x+1)^2} + c$

where c is the constant integration.

Consider the following for the next two (02) items that follow:

The integral $\int \frac{dx}{a\cos x + b\sin x}$ is of the form $\frac{1}{r}\ln\left[\tan\left(\frac{x+\alpha}{2}\right)\right]$.

6. What is r equal to? **[2015-I]**

(*a*) $a^2 + b^2$ (*b*) $\sqrt{a^2+b^2}$

(*c*) $a + b$ (*d*) $\sqrt{a^2-b^2}$

7. What is α equal to?

(*a*) $\tan^{-1}\left(\frac{a}{b}\right)$ (*b*) $\tan^{-1}\left(\frac{b}{a}\right)$

(*c*) $\tan^{-1}\left(\frac{a+b}{a-b}\right)$ (*d*) $\tan^{-1}\left(\frac{a-b}{a+b}\right)$

8. What is $\int \frac{dx}{\sqrt{x^2+a^2}}$ equal to? **[2015-I]**

(*a*) $\ln\left|\frac{x+\sqrt{x^2+a^2}}{a}\right| + c$ (*b*) $\ln\left|\frac{x-\sqrt{x^2+a^2}}{a}\right| + c$

(*c*) $\ln\left|\frac{x^2+\sqrt{x^2+a^2}}{a}\right| + c$ (*d*) None of these

where c is the constant of integration.

9. $\int \frac{dx}{1+e^{-x}}$ is equal to **[2015-II]**

(*a*) $1 + e^x + c$ (*b*) $\ln(1 + e^{-x}) + c$

(*c*) $\ln(1 + e^x) + c$ (*d*) $2\ln(1 + e^{-x}) + c$

where c is the constant of integration

10. What is $\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}dx$ equal to? **[2016-II]**

(*a*) $\sqrt{\frac{x^4+x^2+1}{x}} + c$ (*b*) $\sqrt{x^4+2-\frac{1}{x^2}} + c$

(*c*) $\sqrt{x^2+\frac{1}{x^2}+1} + c$ (*d*) $\sqrt{\frac{x^4-x^2+1}{x}} + c$

11. What is $\int e^{\sin x}\frac{x\cos^3 x - \sin x}{\cos^2 x}dx$ equal to? **[2016-II]**

(*a*) $(x + \sec x)e^{\sin x} + c$ (*b*) $(x - \sec x)e^{\sin x} + c$

(*c*) $(x + \tan x)e^{\sin x} + c$ (*d*) $(x - \tan x)e^{\sin x} + c$

12. What is $\int \frac{dx}{x(x^7+1)}$ equal to? **[2017-I]**

(a) $\frac{1}{2}\ln\left|\frac{x^7-1}{x^7+1}\right|+c$ (b) $\frac{1}{7}\ln\left|\frac{x^7+1}{x^7}\right|+c$

(c) $\ln\left|\frac{x^7-1}{7x}\right|+c$ (d) $\frac{1}{7}\ln\left|\frac{x^7}{x^7+1}\right|+c$

13. What is $\int \frac{(x^{e-1}+e^{x-1})dx}{x^e+e^x}$ equal to? **[2017-I]**

(a) $\frac{x^2}{2}+c$ (b) $\ln(x+e)+c$

(c) $\ln(x^e+e^x)+c$ (d) $\frac{1}{e}\ln(x^e+e^x)+c$

14. Let $f(x)$ be an indefinite integral of $\sin^2 x$. Consider the following statements: **[2017-I]**

Statement 1: The function $f(x)$ satisfies $f(x+\pi)=f(x)$ for all real x.

Statement 2: $\sin^2(x+\pi)=\sin^2 x$ for all real x.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and Statement 2 is the correct explanation of Statement 1

(b) Both the statements are true but Statement 2 is not the correct explanation of Statement 1

(c) Statement 1 is true but Statement 2 is false

(d) Statement 1 is false but Statement 2 is true

15. If $l_1=\frac{d}{dx}\left(e^{\sin x}\right)$ **[2017-II]**

$l_2=\lim_{h\to 0}\frac{e^{\sin(x+h)}-e^{\sin x}}{h}$

$l_3=\int e^{\sin x}\cos x\,dx$

then which one of the following is correct?

(a) $l_1\neq l_2$ (b) $\frac{d}{dx}(l_3)=l_2$

(c) $\int l_3 dx=l_2$ (d) $l_2=l_3$

16. What is $\int \tan^{-1}(\sec x+\tan x)dx$ equal to? **[2017-II]**

(a) $\frac{\pi x}{4}+\frac{x^2}{4}+c$ (b) $\frac{\pi x}{2}+\frac{x^2}{4}+c$

(c) $\frac{\pi x}{4}+\frac{\pi x^2}{4}+c$ (d) $\frac{\pi x}{4}-\frac{x^2}{4}+c$

17. What is $\int(\ln x)^{-1}dx-\int(\ln x)^{-2}dx$ is equal to **[2017-II]**

(a) $x(\ln x)^{-1}+c$ (b) $x(\ln x)^{-2}+c$

(c) $x(\ln x)+c$ (d) $x(\ln x)^2+c$

18. What is $\int \frac{dx}{2^x-1}$ equal to? **[2018-I]**

(a) $\ln(2^x-1)+c$ (b) $\frac{\ln(1-2^{-x})}{\ln 2}+c$

(c) $\frac{\ln(2^{-x}-1)}{2\ln 2}+c$ (d) $\frac{\ln(1+2^{-x})}{\ln 2}+c$

19. What is $\int \sin^3 x\cos x\,dx$ equal to? **[2018-II]**

(a) $\cos^4 x+c$ (b) $\sin^4 x+c$

(c) $\frac{(1-\sin^2 x)^2}{4}+c$ (d) $\frac{(1-\cos^2 x)^2}{4}+c$

where c is the constant of integration.

20. What is $\int e^{\ln(\tan x)}dx$ equal to? **[2018-II]**

(a) $\ln|\tan x|+c$ (b) $\ln|\sec x|+c$

(c) $\tan x+c$ (d) $e^{\tan x}+c$

Where c is the constant of integration.

21. What is $\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$ equal to? **[2018-II]**

(a) $c+\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)$

(b) $c-\frac{1}{ab}\tan^{-1}\left(\frac{b\tan x}{a}\right)$

(c) $c+\frac{1}{ab}\tan^{-1}\left(\frac{b\tan x}{a}\right)$

(d) None of these

22. What is $\int ln(x)^2\,dx$ equal to ? **[2019-I]**

(a) $2xln(x)-2x+c$ (b) $\frac{2}{x}+c$

(c) $2x\,ln(x)+c$ (d) $\frac{2ln(x)}{x}-2x+c$

23. What is $\int e^{x\,ln(a)}\,dx$ equal to? **[2019-I]**

(a) $\frac{a^x}{ln(a)}+c$ (b) $\frac{a^x}{ln(a)}+c$ (c) $\frac{e^x}{ln(ae)}+c$ (d) $\frac{ae^x}{ln(a)}+c$

24. What is $\int \frac{dx}{2x^2-2x+1}$ equal to? **[2019-II]**

(a) $\frac{\tan^{-1}(2x-1)}{2}+c$ (b) $2\tan^{-1}(2x-1)+c$

(c) $\frac{\tan^{-1}(2x+1)}{2}+c$ (d) $\tan^{-1}(2x-1)+c$

25. What is $\int \frac{dx}{x(1+\ln x)^n}$ equal to $(n\neq 1)$? **[2019-II]**

(a) $\frac{1}{(n-1)(1+\ln x)^{n-1}}+C$ (b) $\frac{1-n}{(1+\ln x)^{1-n}}+C$

(c) $\frac{n+1}{(1+\ln x)^{n+1}}+C$ (d) $\frac{-1}{(n-1)(1+\ln x)^{n-1}}+C$

26. If $p(x)=(4e)^{2x}$, then what is $\int p(x)dx$ equal to ? **[2020-I&II]**

(a) $\frac{p(x)}{1+2\ln 2}+c$ (b) $\frac{p(x)}{2(1+2\ln 2)}+c$

(c) $\frac{2p(x)}{1+\ln 4}+c$ (d) $\frac{p(x)}{1+\ln 2}+c$

27. What is $\int \left(e^{\log x} + \sin x\right) \cos x \, dx$ equal to? **[2020-I&II]**

(a) $\sin x + x\cos x + \frac{\sin^2 x}{2} + c$ (b) $\sin x - x\cos x + \frac{\sin^2 x}{2} + c$

(c) $x\sin x + \cos x + \frac{\sin^2 x}{2} + c$ (d) $x\sin x - x\cos x + \frac{\sin^2 x}{2} + c$

28. What is $\int \frac{dx}{x(x^n+1)}$ equal to? **[2020-I&II]**

(a) $\frac{1}{n}\ln\left(\frac{x^n}{x^n+1}\right) + c$ (b) $\ln\left(\frac{x^n+1}{x^n}\right) + c$

(c) $\ln\left(\frac{x^n}{x^n+1}\right) + c$ (d) $\frac{1}{n}\ln\left(\frac{x^n+1}{x^n}\right) + c$

29. What is the value of k such that integration of $\frac{3x^2+8-4k}{x}$ with respect to x, may be a rational function? **[2020-I&II]**

(a) 0 (b) 1

(c) 2 (d) −2

30. What is $\int \frac{dx}{\sec x + \tan x}$ equal to? **[2021-I]**

(a) $\ln(\sec x) + \ln|\sec x + \tan x| + c$

(b) $\ln(\sec x) - \ln|\sec x + \tan x| + c$

(c) $\sec x \tan x - \ln|\sec x - \tan x| + c$

(d) $\ln|\sec x + \tan x| - \ln|\sec x| + c$

31. What is $\int \frac{dx}{\sec^2(\tan^{-1} x)}$ equal to? **[2021-I]**

(a) $\sin^{-1} x + c$ (b) $\tan^{-1} x + c$

(c) $\sec^{-1} x + c$ (d) $\cos^{-1} x + c$

32. What is $\int e^{(2\ln x + \ln x^2)} dx$ equal to? **[2021-I]**

(a) $\frac{x^4}{4} + C$ (b) $\frac{x^3}{3} + C$

(c) $\frac{2x^5}{5} + C$ (d) $\frac{x^5}{5} + C$

33. What is the integral of $f(x) = 1 + x^2 + x^4$ with respect to x^2? **[2021-II]**

(a) $x + \frac{x^3}{3} + \frac{x^5}{5} + C$ (b) $\frac{x^3}{3} + \frac{x^5}{5} + C$

(c) $x^2 + \frac{x^4}{4} + \frac{x^6}{6} + C$ (d) $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$

34. If $\int \sqrt{1 - \sin 2x}\, dx = A\sin x + B\cos x + C$, where $0 < x < \frac{\pi}{4}$, then which one of the following is correct? **[2021-II]**

(a) $A + B = 0$ (b) $A + B - 2 = 0$

(c) $A + B + 2 = 0$ (d) $A + B - 1 = 0$

35. What is $\int \frac{dx}{x(x^2+1)}$ equal to? **[2021-II]**

(a) $\frac{1}{2}\ln\left(\frac{x^2}{x^2+1}\right) + C$ (b) $\ln\left(\frac{x^2}{x^2+1}\right) + C$

(c) $\frac{3}{2}\ln\left(\frac{x^2+1}{x^2}\right) + C$ (d) $\frac{1}{2}\ln\left(\frac{x^2+1}{x^2}\right) + C$

36. What is $\int (\sin x)^{-1/2}(\cos x)^{-3/2}\, dx$ equal to? **[2022-I]**

(a) $\sqrt{\tan x} + c$ (b) $2\sqrt{\tan x} + c$

(c) $\sqrt{\cot x} + c$ (d) $\sqrt{2\tan x} + c$

37. If $I_1 = \int \frac{e^x dx}{e^x + e^{-x}}$ and $I_2 = \int \frac{dx}{e^{2x}+1}$, then what is $I_1 + I_2$ equal to? **[2022-I]**

(a) $\frac{x}{2} + c$ (b) $x + c$

(c) $\ln(e^x + e^{-x}) + c$ (d) $\ln(e^x - e^{-x}) + c$

38. What is $\int (x^x)^2 (1 + \ln x) dx$ equal to? **[2022-II]**

(a) $x^{2x} + c$ (b) $\frac{1}{2}x^{2x} + c$

(c) $2x^{2x} + c$ (d) $\frac{1}{2}x^x + c$

39. What is $\int e^x \{1 + \ln x + x\ln x\} dx$ equal to? **[2022-II]**

(a) $xe^x \ln x + c$ (b) $x^2 e^x \ln x + c$

(c) $x + e^x \ln x + c$ (d) $xe^x + \ln x + c$

40. What is $\int \frac{(\cos x)^{1.5} - (\sin x)^{1.5}}{\sqrt{\sin x \cdot \cos x}} dx$ equal to? **[2022-II]**

(a) $\sqrt{\sin x} - \sqrt{\cos x} + c$ (b) $\sqrt{\sin x} + \sqrt{\cos x} + c$

(c) $2\sqrt{\sin x} + 2\sqrt{\cos x} + c$ (d) $\frac{1}{2}\sqrt{\sin x} + \frac{1}{2}\sqrt{\cos x} + c$

Consider the following for the next two (02) items that follow:

Given that $\int \frac{3\cos x + 4\sin x}{2\cos x + 5\sin x} dx = \frac{\alpha x}{29} + \frac{\beta}{29} \ln|2\cos x + 5\sin x| + c$

41. What is the value of α? **[2024-I]**

(a) 7 (b) 13 (c) 17 (d) 26

42. What is the value of β? **[2024-I]**

(a) 7 (b) 13 (c) 17 (d) 26

Consider the following for the next two (02) items that follow:

Let $\int \frac{dx}{\sqrt{x+1} - \sqrt{x-1}} = \alpha(x+1)^{\frac{3}{2}} + \beta(x-1)^{\frac{3}{2}} + c$

43. What is the value of α? **[2024-I]**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

44. What is the value of β? **[2024-I]**

(a) $-\frac{2}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

ANSWER KEY

1. (*b*)	**2.** (*c*)	**3.** (*b*)	**4.** (*a*)	**5.** (*c*)	**6.** (*b*)	**7.** (*a*)	**8.** (*a*)	**9.** (*c*)	**10.** (*c*)
11. (*b*)	**12.** (*d*)	**13.** (*d*)	**14.** (*d*)	**15.** (*b*)	**16.** (*a*)	**17.** (*a*)	**18.** (*b*)	**19.** (*d*)	**20.** (*b*)
21. (*a*)	**22.** (*a*)	**23.** (*a*)	**24.** (*d*)	**25.** (*d*)	**26.** (*b*)	**27.** (*c*)	**28.** (*a*)	**29.** (*c*)	**30.** (*d*)
31. (*b*)	**32.** (*d*)	**33.** (*d*)	**34.** (*b*)	**35.** (*a*)	**36.** (*b*)	**37.** (*b*)	**38.** (*b*)	**39.** (*a*)	**40.** (*c*)
41. (*d*)	**42.** (*a*)	**43.** (*a*)	**44.** (*c*)						

EXPLANATIONS

1. (*b*) Given,

$$\int x\tan^{-1}x\,dx = A(x^2+1)\tan^{-1}x + Bx + C$$

where, C is the constant of integration

Consider, $\int x\tan^{-1}x\,dx$

$$= \tan^{-1}x.\frac{x^2}{2} - \int\frac{d}{dx}(\tan^{-1}x).\frac{x^2}{2}dx$$

(using integration by parts)

$$= \frac{x^2.\tan^{-1}x}{2} - \frac{1}{2}\int\frac{x^2}{1+x^2}dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\left(\int\left(\frac{1+x^2-1}{1+x^2}\right)dx\right)$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\left(\int dx - \int\frac{dx}{1+x^2}\right)$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}(x - \tan^{-1}x) + C$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{x}{2} + \frac{\tan^{-1}x}{2} + C$$

$$= \frac{1}{2}(x^2+1)\tan^{-1}x - \frac{x}{2} + C$$

$$\therefore\ A = \frac{1}{2}$$

2. (*c*) $B = -\frac{1}{2}$, from above.

3. (*b*) $f'(x) = \int f''(x)dx + C$

$$= \int\sec^2 x\sec^2 x\,dx + \int 4dx + C$$

$$= \int(1+\tan^2 x)\sec^2 x\,dx + 4x + C$$

$$= I_1 + 4x + C$$

Put $\tan x = t$ in the integral I_1, then $\sec^2 x\,dx = dt$

$$\therefore I_1 = \int(1+t^2)dt = t + \frac{t^3}{3} + C'$$

$$= \tan x + \frac{\tan^3 x}{3} + C'$$

$$\therefore f'(x) = \tan x + \frac{\tan^3 x}{3} + 4x + C_1$$

where, $C_1 = C + C'$

Given, $f'(0) = 0 \Rightarrow C = 0$

Thus, $f'(x) = \tan x + \frac{\tan^3 x}{3} + 4x$

4. (*a*) $f(x) = \int f'(x)dx + C_2$

$$= \int\tan x\,dx + \frac{1}{3}\int\tan x(\sec^2 x - 1)dx + 4.\frac{x^2}{2} + C_2$$

$$= \frac{2}{3}\int\tan x\,dx + \frac{1}{3}\int\tan x.\sec^2 x\,dx + 2x^2 + C_2$$

$$= \frac{2}{3}\ln(\sec x) + \frac{1}{3}I_2 + 2x^2 + C_2$$

Consider $I_2 = \int\tan x\sec^2 x\,dx$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$\Rightarrow I_2 = \int t\,dt = \frac{t^2}{2} + C_3 = \frac{\tan^2 x}{2} + C_3$$

$$\therefore f(x) = \frac{2}{3}\ln(\sec x) + \frac{1}{6}\tan^2 x + 2x^2 + C_4$$

Here, $C_4 = C_2 + \left(\frac{C_3}{3}\right)$

Given, $f(0) = 0 \quad \Rightarrow C_4 = 0$

$$\therefore f(x) = \frac{2}{3}\ln(\sec x) + \frac{1}{6}\tan^2 x + 2x^2$$

5. (*c*) $\int\frac{xe^x}{(1+x)^2}dx$

$$= \int e^x\left(\frac{x+1-1}{(x+1)^2}\right)dx$$

$$= \int e^x\left[\frac{1}{1+x} - \frac{1}{(1+x)^2}\right]dx$$

$$= \frac{e^x}{1+x} + C$$

$$\left\{\because \int e(f(x) + f'(x))dx = e^x f(x) + C\right\}$$

6. (*b*) Given that,

$$\int\frac{dx}{a\cos x + b\sin x} = \frac{1}{r}\ln\left[\tan\left(\frac{x+\alpha}{2}\right)\right]$$

Put $a = r\sin\alpha$, $b = r\cos\alpha$

$$\int\frac{dx}{r\sin\alpha\cos x + r\cos\alpha\sin x} = \frac{1}{r}\int\frac{1}{\sin(x+\alpha)}dx$$

$$= \frac{1}{r}\int\text{cosec}\,(x+\alpha)dx = \frac{1}{r}\ln\left[\tan\left(\frac{x+\alpha}{2}\right)\right]$$

$a = r\sin\alpha \Rightarrow a^2 = r^2\sin^2\alpha$ ...(i)

$b = r\cos\alpha \Rightarrow b^2 = r^2\cos^2\alpha$ ...(ii)

Adding (i) and (ii), we get

$r^2 = a^2 + b^2$

$\Rightarrow r = \sqrt{a^2 + b^2}$

7. (*a*) $a = r\sin\alpha$ ...(i)

$b = r\cos\alpha$ ...(ii)

Dividing (i) from (ii),

$$\frac{a}{b} = \tan\alpha$$

$$\alpha = \tan^{-1}\left(\frac{a}{b}\right)$$

8. (*a*) $\int \frac{dx}{\sqrt{x^2+a^2}}$

$= \log\left|x+\sqrt{a^2+x^2}\right| - \log|a| + C$

$= \log\left|\frac{x+\sqrt{a^2+x^2}}{a}\right| + C$

9. (*c*) Let $I = \int \frac{dx}{1+e^{-x}} \Rightarrow I = \int \frac{e^x}{e^x+1}dx$

Let $e^x + 1 = t$

$e^x dx = dt$

$\therefore I = \int \frac{dt}{t}$

$\Rightarrow \log t + c \Rightarrow \log(e^x+1)+c$

10. (*c*) $I = \int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}dx$

$= \int \frac{x-\frac{1}{x^3}}{\sqrt{\frac{x^4+x^2+1}{x^2}}}dx = \int \frac{x-\frac{1}{x^3}}{\sqrt{x^2+1+\frac{1}{x^2}}}dx$

Let $x^2+1+\frac{1}{x^2} = t \Rightarrow \left(2x-\frac{2}{x^3}\right)dx = dt$

$\Rightarrow 2\left(x-\frac{1}{x^3}\right)dx = dt$

$\therefore I = \int \frac{\frac{dt}{2}}{\sqrt{t}} = \frac{1}{2}\int t^{-1/2}dt$

$= \frac{1}{2}\frac{t^{1/2}}{\frac{1}{2}} + C = t^{1/2} + C$

$= \left(x^2+1+\frac{1}{x^2}\right)^{1/2} + C$

$= \sqrt{x^2+\frac{1}{x^2}+1} + C$

11. (*b*) $I = \int e^{\sin x}\left(\frac{x\cos^3 x - \sin x}{\cos^2 x}\right)dx$

$= \int e^{\sin x}(x\cos x - \tan x \sec x)dx$

$= \int xe^{\sin x}\cos x dx - \int e^{\sin x}\tan x \sec x\, dx$

Integrate by parts.

$\left[xe^{\sin x} - \int e^{\sin x}\cdot 1\cdot dx\right] - \left[e^{\sin x}\sec x - \int e^{\sin x}\cos x \sec x\, dx\right] + C$

$= xe^{\sin x} - e^{\sin x}\cdot \sec x + C$

12. (*d*) $\int \frac{dx}{x(x^7+1)} = \frac{x^6}{x^7(x^7+1)}.dx$

Let $x^7 = t \quad \Rightarrow 7x^6.dx = dt$

$\therefore \int \frac{x^6 dx}{x^7(x^7+1)} = \frac{1}{7}\int \frac{dt}{t(t+1)}$

$= \frac{1}{7}\left[\int \frac{1}{t}dt - \int \frac{1}{t+1}dt\right]$

$= \frac{1}{7}[\ln|t| - \ln|t+1|] + c = \frac{1}{7}\ln\left|\frac{t}{t+1}\right| + c$

$= \frac{1}{7}\ln\left|\frac{x^7}{x^7+1}\right| + c$

13. (*d*) $\int \frac{(x^{e-1}+e^{x-1})dx}{x^e+e^x}$

Put $x^e + e^x = t$

$\Rightarrow (ex^{e-1} + e^x)\, dx = dt$

$\therefore \int \frac{(x^{e-1}+e^{x-1})dx}{x^e+e^x} = \frac{1}{e}\int \frac{e.x^{e-1}+e^x}{x^e+e^x}.dx$

$= \frac{1}{e}\int \frac{dt}{t} = \frac{1}{e}.\ln t + c = \frac{1}{e}.\ln(x^e+e^x)+c$

14. (*d*) $f(x) = \int \sin^2 x.dx = \int \frac{1-\cos 2x}{2}.dx$

$= \frac{1}{2}x - \frac{\sin 2x}{2}.\frac{1}{2} + c = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$

(1) $f(\pi+x) = \frac{1}{2}(\pi+x) - \frac{1}{4}\sin 2(\pi+x)$

$= \frac{1}{2}\pi + \frac{1}{2}x - \frac{\sin 2x}{4} + c$

So, $f(x+\pi) \neq f(x)$

So, statement 1 is false.

(2) We know that

$\sin^2(x+\pi) = \sin^2 x$ is true for all real x.

15. (*b*)

$I_1 = \frac{d}{dx}(e^{\sin x})$

$I_2 = \lim_{h\to 0}\frac{e^{\sin(x+h)} - e^{\sin x}}{h}$

$I_3 = \int e^{\sin x}\cdot \cos x\, dx$

Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$I_3 = \int e^t \cdot dt = e^t + c = e^{\sin x} + c$

$\frac{d}{dx}(I_3) = \frac{d}{dx}(e^{\sin x}+c)$

$= \frac{d}{dx}(e^{\sin x}) = I_1 = I_2$

16. (*a*) $\int \tan^{-1}(\sec x + \tan x)dx$

$= \int \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)dx$

$= \int \tan^{-1}\left(\frac{1+\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}\right)dx$

$= \int \tan^{-1}\left(\frac{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)dx$

$= \int \tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)dx$

$= \int \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)dx$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right) = \int\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$

$= \frac{\pi x}{4} + \frac{x^2}{4} + c$

17. (*a*) $\int (\ln x)^{-1}dx - \int (\ln x)^{-2}dx$

$\int\left[\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right]dx$

Put $\ln x = t \Rightarrow x = e^t$

$dx = e^t.dt.$

$\therefore \int\left[\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right]dx = \int\left(\frac{1}{t}-\frac{1}{t^2}\right).e^t.dt$

$= \int e^t.\left(\frac{1}{t}-\frac{1}{t^2}\right)dt$

$= \frac{e^t}{t} + c = \frac{x}{\ln x} + c$

$\left[\because \int e^x(f(x)+f'(x)dx = e^x f(x) + C\right]$

$= x(\ln x)^{-1} + c$

18. (*b*) $\int \frac{dx}{2^x-1} = \int \frac{2^{-x}}{1-2^{-x}}dx$

Let $1 - 2^{-x} = t$

$\Rightarrow 2^{-x}.dx = \frac{dt}{\log 2} \therefore \int \frac{2^{-x}}{1-2^{-x}}dx = \frac{1}{\log 2}\int \frac{dt}{t}$

$= \frac{1}{\log 2}(\log t) + c$

$= \frac{1}{\log 2}(\log(1-2^{-x})) + c$

19. (*d*) Let $t = \sin x \Rightarrow dt = \cos x\, dx$

$\int \sin^3 x \cos x\, dx = \int t^3 dt = \frac{t^4}{4} + c = \frac{\sin^4 x}{4} + c$

$= \frac{(1-\cos^2 x)^2}{4} + c$

20. (b) $\int e^{\ln(\tan x)}dx = \int \tan x\, dx$

$= \ln|\sec x| + c$

21. (a) $I = \int \frac{dx}{a^2\sin^2 x + b^2\cos^2 x}$

$I = \frac{1}{a^2}\int \frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2}$

$= \frac{1}{a^2} \times \frac{a}{b}\tan^{-1}\left(\frac{a\tan x}{b}\right) + c$

$\left[\because \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c\right]$

$= \frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right) + c$

22. (a) $\int \ln(x^2)dx = \ln(x^2).x - \int \frac{1}{x^2}.2x.xdx$

$= x\ln(x^2) - 2x + C$

23. (a) $\int e^{x\ln(a)}dx = \int e^{\ln(a)^x}dx$

$= \int a^x dx = \frac{a^x}{\ln a} + c$

24. (d) $\int \frac{dx}{2x^2-2x+1} = 2\int \frac{dx}{(4x^2-4x+1)+1}$

$= \int \frac{2dx}{(2x-1)^2+1}$

Let $2x - 1 = t$

$\Rightarrow 2dx = dt$

$\therefore \int \frac{dx}{2x^2-2x+1} = \int \frac{d}{1+t^2}$

$\Rightarrow \int \frac{dx}{2x^2-2x+1} = \tan^{-1}t + c$

$\Rightarrow \int \frac{dx}{2x^2-2x+1} = \tan^{-1}(2x-1) + c$

25. (d) Consider, $I = \int \frac{dx}{x(1+\ln x)^n}$

Let $1 + \ln x = t$

$\frac{1}{x}dx = dt$

$\therefore I = \int \frac{dt}{t^n} = \int t^{-n}dt$

$= \frac{t^{-n+1}}{-n+1} + C$

$= \frac{-1}{(n-1)t^{n-1}} + C$

$= \frac{-1}{(n-1)(1+\log x)^{n-1}} + C$

26. (b) Given, $p(x) = (4e)^{2x}$

$\Rightarrow \int p(x)dx = \int (4e)^{2x}dx$

put $2x = t \Rightarrow dx = \frac{dt}{2}$

$\int p(x)dx = \frac{1}{2}\int (4e)^t dt = \frac{1}{2}\cdot\frac{(4e)^t}{\ln 4e} + c$

$= \frac{p(x)}{2(1+2\ln 2)} + c$

27. (c) Given -

Let $I = \int (e^{\log x} + \sin x)\cos x\,dx$.

$= \int (x\cos x + \sin x\cos x)dx$

$= \int x\cos x\,dx + \int \sin x\cos x\,dx$

Let $I = I_1 + I_2$

Here, $I_1 = \int x\cos x\,dx$

$\int x\cos x\,dx - \int\left(\frac{dx}{dx}\int \cos x\,dx\right)dx$

$= x\sin x + \cos x + c$

Now, $I_2 = \int \sin x\cos x\,dx$

Let $\sin x = t \Rightarrow \cos x\,dx = dt$

Differentiating with respect to x -

$I_2 = \int t\,dt = \frac{t^2}{2} + c = \frac{\sin^2 x}{2} + c$

$\therefore I = x\sin x + \cos x + \frac{\sin^2 x}{2} + c$

28. (a) Given, $\int \frac{dx}{x(x^n+1)}$.

Multiplying and dividing the integral by (x^{n-1})-

$I = \int \frac{x^{n-1}dx}{x^n(x^n+1)}$ ...(i)

Let $x^n = t \Rightarrow x^{n-1}dx = \frac{dt}{n}$

Equation (i) becomes -

$I = \frac{1}{n}\int \frac{dt}{t(t+1)}$ ...(ii)

Now, consider

$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$ ...(iii)

$\therefore \frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{(t+1)}$

So, equation (ii) is -

$I = \frac{1}{n}\int\left(\frac{1}{t} - \frac{1}{(t+1)}\right)dt$

$I = \frac{1}{n}\left[\log t - \log(t+1)\right] + c$

Substitute the value of t -

$I = \frac{1}{n}\left[\log\frac{x^n}{(x^n+1)}\right] + c$

29. (c) Given, The expression is $\frac{3x^2+8-4k}{x}$.

The first value is $k = 0$.

Integrate the expression with respect to x -

$\Rightarrow \int f(x)dx = \int \frac{3x^2+8}{x}dx$

$\Rightarrow \int f(x)dx = \int\left(3x + \frac{8}{x}\right)dx$

$\int f(x)dx = \left(\frac{3x^2+16\log x}{2}\right)dx$

Here, the numerator is not a polynomial. Therefore, the function is not a rational function at $k = 0$.

The second value is $k = 1$.

Integrate the expression with respect to x–

$\int f(x)dx = \int \frac{3x^2+4}{x}dx$

$\int f(x)dx = \left(\frac{3x^2+8\log x}{2}\right)dx$

Here, the numerator is not a polynomial. Therefore, the function is not a rational function at $k = 1$.

The third value is $k = 2$.

$f(x) = \frac{3x^2+8-4(2)}{x}$

Integrate the expression with respect to x -

$\int f(x)dx = \int \frac{3x^2}{x}dx$

$\int f(x)dx = \int 3x\,dx$

$\int f(x)dx = \frac{3x^2}{2} = \frac{P(x)}{Q(x)}$

Here, $p(x)$ and $Q(x)$ are polynomials with $Q(x) \neq 0$

Therefore, the function is a rational function at $k = 2$.

The fourth value is $k = -2$.

$f(x) = \frac{3x^2+8-4(-2)}{x}$

Integrate the expression with respect to x –

$\int f(x)dx = \int \frac{3x^2+16}{x}dx$

$\int f(x)dx = \left(\frac{3x^2}{2} + 8\log x\right)dx$

$\int f(x)dx = \left(\frac{3x^2+16\log x}{2}\right)dx$

Here, the numerator is not a polynomial. Therefore, the function is not a rational function at $k = -2$.

30. (*d*) Let,

$$I=\int\frac{1}{(\sec x+\tan x)}\times\frac{(\sec x-\tan x)}{(\sec x-\tan x)}dx$$

$$I=\int\frac{(\sec x-\tan x)}{\sec^2 x-\tan^2 x}dx$$

$$=\int\frac{(\sec x-\tan x)}{1}dx$$

$$I=\int\sec x\,dx-\int\tan x\,dx$$

$$I=ln|\sec x+\tan x|-ln|\sec x|+C$$

31. (*b*) Let $I=\int\frac{dx}{\sec^2(\tan^{-1}x)}$

$$\Rightarrow I=\int\frac{dx}{1+\tan^2(\tan^{-1}x)}=\int\frac{dx}{1+x^2}$$

$=\tan^{-1}x+C$

32. (*d*) Suppose $I=\int e^{(2\ln x+\ln x^2)}dx$

$$=\int e^{(\ln x^2+\ln x^2)}dx=\int x^4dx$$

$$I=\frac{x^5}{5}+C$$

33. (*d*) Given function, $f(x)=1+x^2+x^4$

$\therefore$ Integral of $f(x)$ w.r.t x^2.

$$\therefore\int f(x)dx^2=\int(1+x^2+x^4)\cdot 2x\,dx$$

$$=\int(2x+2x^3+2x^5)dx=\frac{2x^2}{2}+\frac{2x^4}{4}+\frac{2x^6}{6}+C$$

$$=x^2+\frac{x^4}{2}+\frac{x^6}{3}+C$$

34. (*b*) Given that,

$\int\sqrt{1-\sin 2x}\;dx=A\sin x+B\cos x+C$, where $0\le x\le\frac{\pi}{4}$.

Let

$$I=\int\sqrt{1-\sin 2x}\;dx$$

$$=\int\sqrt{\cos^2 x+\sin^2 x-2\cos x.\sin x}dx$$

$$I=\int\sqrt{(\cos x-\sin x)^2}\;dx$$

$$\left\{\because\ \cos x>\sin x\text{ when }0<x<\frac{\pi}{4}\right\}$$

$$I=\int(\cos x-\sin x)dx$$

$I=\sin x+\cos x+C$

$\therefore$ $\sin x+\cos x+C=A\sin x+B\cos x+C$

Equating the coefficient of $\sin x$ and $\cos x$

$\therefore\quad A=1, B=1$

$\therefore\quad A+B-2=1+1-2=0$

35. (*a*) Let us suppose, $I=\int\frac{dx}{x(x^2+1)}$

$$\because\ \frac{1}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$$

(by using partial fraction)

$$\frac{1}{x(x^2+1)}=\frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)}$$

$1=(A+B)x^2+Cx+A$

Equating the coefficient of similar power of x.

$\Rightarrow\quad A+B=0, C=0, A=1$

$\therefore\quad B=-A=-1$

$$\therefore\frac{1}{x(x^2+1)}=\frac{1}{x}-\frac{x}{x^2+1}$$

Now,

$$\therefore I=\int\frac{dx}{x(x^2+1)}=\int\left(\frac{1}{x}-\frac{x}{x^2+1}\right)dx$$

$$=\int\frac{1}{x}dx-\frac{1}{2}\int\frac{2x}{x^2+1}dx$$

$$=\frac{1}{2}ln\left(\frac{x^2}{x^2+1}\right)+C$$

36. (*b*) $I=\int(\sin x)^{-1/2}(\cos x)^{-3/2}dx$

$$=\int\frac{\text{cosec}^2 x dx}{\cot x\cdot\sqrt{\cot x}}$$

Let, $\cot x=\text{t}\Rightarrow-\text{cosec}^2x\,dx=dt$

$$\therefore I=\int\frac{-dt}{t^{3/2}}=-\frac{-t^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)}$$

$$=\frac{2}{\sqrt{t}}+c=\frac{2}{\sqrt{\cot x}}+c$$

$$=2\sqrt{\tan x}+c$$

37. (*b*) $I_1+I_2=\int\frac{e^x dx}{e^x+e^{-x}}+\int\frac{dx}{e^{2x}+1}$

$$=\int\frac{e^{2x}dx}{e^{2x}+1}+\int\frac{dx}{e^{2x}+1}$$

$$=\int\left(\frac{1+e^{2x}}{1+e^{2x}}\right)dx=x+c$$

38. (*b*) Given $\text{I}=\int(x^x)^2(1+\ln x)\,dx$ ...(i)

Consider $x^{2x}=u\Rightarrow 2x\ln x=\ln u$

On differentiating , we get

$$2\ln x+2=\frac{1}{u}\frac{du}{dx}$$

$\Rightarrow(1+\ln x)\,dx=\frac{1}{2u}du$ ...(ii)

On substituting (ii) in (i)

$$\int u\frac{1}{2u}\;du=\frac{1}{2}u+C=\frac{1}{2}x^{2x}+C$$

39. (*a*) $I=\int e^x dx+\int e^x\ln x dx+\int e^x x\ln x\,dx$

Compute the last integral by taking $f(x)=x\ln x$ and $g(x)=e^x$.

40. (*c*) $I=\int\frac{(\cos x)^{1.5}-(\sin x)^{1.5}}{\sqrt{\sin x\cos x}}dx$

$$I=\int\frac{\cos x}{\sqrt{\sin x}}dx-\int\frac{\sin x}{\sqrt{\cos x}}dx=I_1-I_2$$

Put $\sin x=t$ in I_1 and $\cos x=t$ in I_2

On solving, we get

$I=2\sqrt{\sin x}+2\sqrt{\cos x}+C$

41. (*d*) Let $u=2\cos x+5\sin x$

$\Rightarrow du=-2\sin x\,dx+5\cos x\,dx=(5\cos x-2\sin x)dx$

Let us take k and m such that:

$3\cos x+4\sin x=k(5\cos x-2\sin x)+m(2\cos x+5\sin x)$

Equating coefficients:

$3=5k+2m$ and $4=-2k+5m$

$\Rightarrow 15=25\text{ K}+10\text{ M}$ ...(*i*)

$\Rightarrow 8=-4k+10\text{ m}$ ... (*ii*)

Subtracting (*ii*) from (*i*)

$15-8=25k+10m-(-9k+10m)$

$7=29k=k=\frac{7}{29}$ and $m=\frac{26}{29}$

Now we have:

$3\cos x+4\sin x=\frac{7}{29}[5\cos x-2\sin x]+\frac{26}{29}(2\cos x+5\sin x)$

$=\int\frac{3\cos x+4\sin x}{2\cos x+5\sin x}dx=\frac{7}{29}|2\cos x+5\sin x|+\frac{26}{29}x+\text{C}$

Given form $\frac{dx}{29}+\frac{B}{29}\;ln|2\cos x+5\sin x|+C$

$\therefore\alpha=26,\beta=7$

42. (*a*)

43. (*a*)

Let $I=\int\frac{dx}{\sqrt{x+1}-\sqrt{x-1}}$

$$=\int\left(\frac{\sqrt{x+1}+\sqrt{x-1}}{(x+1)-(x-1)}\right)dx$$

$$=\int\left(\frac{\sqrt{x+1}+\sqrt{x-1}}{2}\right)dx$$

$$=\frac{1}{2}\left[\frac{(x+1)^{3/2}}{\frac{3}{2}}+\frac{(x-1)^{3/2}}{\frac{3}{2}}\right]$$

$$=\frac{1}{3}(x+1)^{\frac{3}{2}}+\frac{1}{3}(x+1)^{\frac{3}{2}}$$

Comparing the given

$\alpha=\frac{1}{3}$ and $\beta=\frac{1}{3}$

44. (*c*)