

LINES AND ANGLES

CONTENTS

- Line segment, Ray, Collinear points, Angle
- Measure of an angle, Types of angles
- Bisector of an angle
- Complementary, Supplementary, Adjacent angles
- Linear pair of angles
- Vertically opposite angles

► SOME BASIC TERMS

◆ Line segment:

A part (or portion) of a line with two end points

◆ Ray:

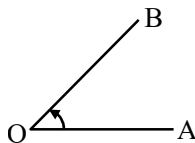
A part of a line with one end point.

◆ Collinear points:

If three or more points lie on the same line otherwise these are called non collinear points.

◆ Angle:

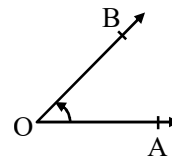
Two rays with a common end point form an angle.



OA, OB are rays & O is end point.

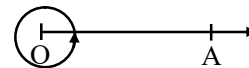
◆ Measure of an angle:

The amount of turning from OA to OB is called the measure of $\angle AOB$, written as $m\angle AOB$. An angle is measured in degrees denoted by '°'.



◆ An angle of 360°:

If a ray OA starting from its original position OA, rotates about O, in the anticlockwise direction and after making a complete revolution it comes back to its original position, we say that it has rotated through 360 degrees, written as 360°.



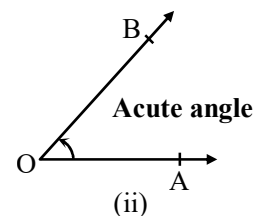
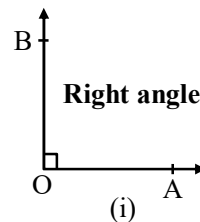
This complete rotation is divided into 360 equal parts. Each part measures 1°.

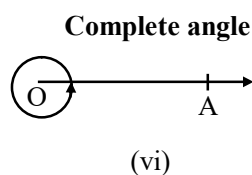
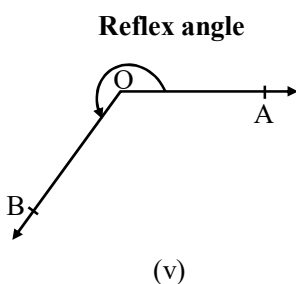
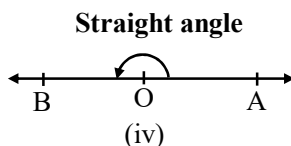
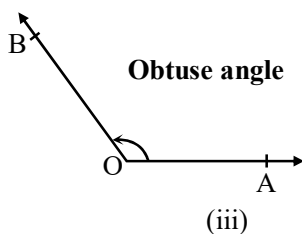
1° = 60 minutes, written as 60'.

1' = 60 seconds, written as 60".

We use a protractor to measure an angle.

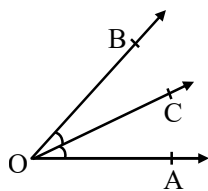
◆ Types of angles:





◆ Bisector of an angle:

A ray OC is called the bisector of $\angle AOB$, if $m\angle AOC = m\angle BOC$.



In this case, $\angle AOC = \angle BOC = \frac{1}{2} \angle AOB$.

◆ Complementary angles:

Two angles are said to be complementary, if the sum of their measures is 90° .

Two complementary angles are called the complement of each other.

Ex. : Angles measuring 20° and 70° are complementary angles.

◆ Supplementary angles:

Two angles are said to be supplementary, if the sum of their measures is 180° .

Two supplementary angles are called the supplement of each other.

Ex. : Angles measuring 60° and 120° are supplementary angles.

◆ EXAMPLES ◆

Ex.1 Find the measure of an angle which is 20° more than its complement.

Sol. Let the measure of the required angle be x° .
Then, measure of its complement $= (90 - x)^\circ$.
 $\therefore x - (90 - x) = 20 \Leftrightarrow 2x = 110 \Leftrightarrow x = 55$
Hence, the measure of the required angle is 55° .

Ex.2 Find the measure of an angle which is 40° less than its supplement.

Sol. Let the measure of the required angle be x° .
Then, measure of its supplement $= (180 - x)^\circ$.
 $\therefore (180 - x) - x = 40 \Leftrightarrow 2x = 140 \Leftrightarrow x = 70$
Hence, the measure of the required angle is 70° .

Ex.3 Find the measure of an angle, if six times its complement is 12° less than twice its supplement.

Sol. Let the measure of the required angle be x° .
Then, measure of its complement $= (90 - x)^\circ$.
Measure of its supplement $= (180 - x)^\circ$.
 $\therefore 6(90 - x) = 2(180 - x) - 12$
 $\Leftrightarrow 540 - 6x = 360 - 2x - 12$
 $\Leftrightarrow 4x = 192 \Leftrightarrow x = 48$.

Hence the measure of the required angle is 48° .

Ex.4 Convert 180° in degree, minute & second.

Sol. $180^\circ = 179^\circ 59' 60''$.

Ex.5 Find the measure of the supplement of an angle of $87^\circ 28' 43''$.

Sol. We may write, $180^\circ = 179^\circ 59' 60''$.
 \therefore supplement of an angle of $(87^\circ 28' 43'')$
 $=$ an angle of $[180^\circ - (87^\circ 28' 43'')]$
 $=$ an angle of $[179^\circ 59' 60'' - 87^\circ 28' 43'']$
 $=$ an angle of $(92^\circ 31' 17'')$.
Hence, the measure of the required angle $= (92^\circ 31' 17'')$.

Ex.6 If $\angle A = 36^\circ 27' 46''$ and $\angle B = 28^\circ 43' 39''$, find $\angle A + \angle B$.

Sol. $\angle A + \angle B = 36^\circ 27' 46'' + 28^\circ 43' 39''$
 $= 64^\circ 70' 85'' = 64^\circ 71' 25'' = 65^\circ 11' 25''$

Ex.7 Find the complement of each of the following angles :

- (i) 58° (ii) 16°
(iii) $\frac{2}{3}$ of a right angle (iv) $46^\circ 30'$

Sol. (i) $90^\circ - 58^\circ = 32^\circ$
(ii) $90^\circ - 16^\circ = 74^\circ$
(iii) $90^\circ - \frac{2}{3}(90^\circ) = 90^\circ - 60^\circ = 30^\circ$
(iv) $90^\circ - 46^\circ 30'$
 $= 89^\circ 60' - 46^\circ 30'$
 $= 43^\circ 30'$

Ex.8 Find the measure of an angle which is complement of itself.

Sol. Let the measure of the angle be x° . Then,
Then, the measure of its complement is given to be x° .
Since, the sum of the measures of an angle and its complement is 90°
 $\therefore x^\circ + x^\circ = 90^\circ \Rightarrow 2x^\circ = 90^\circ \Rightarrow x^\circ = 45^\circ$

Ex.9 Find the measure of an angle which forms a pair of supplementary angles with itself.

Sol. Let the measure of the angle be x° . Then,
 $x^\circ + x^\circ = 180^\circ \Rightarrow 2x^\circ = 180^\circ \Rightarrow x^\circ = 90^\circ$

Ex.10 An angle is equal to five times its complement. Determine its measure.

Sol. Let the measure of the given angle be x degrees. Then, its complement is $(90 - x)^\circ$.
It is given that :
Angle = $5 \times$ Its complement
 $\Rightarrow x = 5(90 - x) \Rightarrow x = 450 - 5x$
 $\Rightarrow 6x = 450 \Rightarrow x = 75$
Thus, the measure of the given angles is 75° .

Ex.11 An angle is equal to one-third of its supplement. Find its measure.

Sol. Let the measure of the required angle be x degrees. Then,

Its supplement = $180^\circ - x$. It is given that :

$$\text{Angle} = \frac{1}{3} (\text{Its supplement})$$

$$\Rightarrow x = \frac{1}{3} (180^\circ - x) \Rightarrow 3x = 180^\circ - x$$

$$\Rightarrow 4x = 180^\circ \Rightarrow x = 45^\circ$$

Thus, the measure of the given angle is 45° .

Ex.12 Two supplementary angles are in the ratio 2 : 3. Find the angles.

Sol. Let the two angles be $2x$ and $3x$ in degrees. Then,

$$2x + 3x = 180^\circ$$

$$\Rightarrow 5x = 180^\circ \Rightarrow x = 36^\circ$$

Thus, the two angles are $2x = 2 \times 36^\circ = 72^\circ$

and $3x = 3 \times 36^\circ = 108^\circ$

Ex.13 Write the complement of the following angles:
 $30^\circ 20'$

Sol. Complement of
 $30^\circ 20' = 90^\circ - 30^\circ 20'$
 $= 90^\circ - (30^\circ + 20')$
 $= (89^\circ - 30^\circ) + (1^\circ - 20')$
 $= 59^\circ + (60' - 20') \quad [\ominus 1^\circ = 60']$
 $= 59^\circ + 40' = 59^\circ 40'$

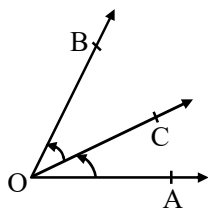
Ex.14 Find the supplement of the following angles :
 $134^\circ 30' 26''$

Sol. Supplement of an angle of $134^\circ 30' 26''$
 $= 180^\circ - (134^\circ 30' 26'')$
 $= (179^\circ - 134^\circ) + (1^\circ - 30' 26'')$
 $= 45^\circ + (60' - (30' + 26'')) \quad [\ominus 1^\circ = 60']$
 $= 45^\circ + (59' - 30') + (1' - 26'')$
 $= 45^\circ + 29' + 34'' = 45^\circ 29' 34''$

◆ Adjacent angles:

Two angles are called adjacent angles, if

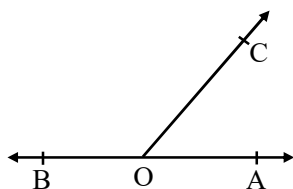
- (i) they have the same vertex,
- (ii) they have a common arm and
- (iii) their non-common arms are on either side of the common arm.



In the given figure, $\angle AOC$ and $\angle BOC$ are adjacent angles having the same vertex O, a common arm OC and their non-common arms OA and OB on either side of OC.

◆ Linear pair of angles:

Two adjacent angles are said to form a linear pair of angles, if their non-common arms are two opposite rays.



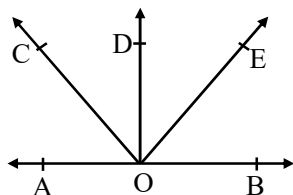
In the adjoining figure, $\angle AOC$ and $\angle BOC$ are two adjacent angles whose non-common arms OA and OB are two opposite rays, i.e., BOA is a line

$\therefore \angle AOC$ and $\angle BOC$ form a linear pair of angles.

Theorem 1 :

Prove that the sum of all the angles formed on the same side of a line at a given point on the line is 180° .

Given : AOB is a straight line and rays OC, OD and OE stand on it, forming $\angle AOC$, $\angle COD$, $\angle DOE$ and $\angle EOB$.



To prove : $\angle AOC + \angle COD + \angle DOE + \angle EOB = 180^\circ$.

Proof : Ray OC stands on line AB.

$$\therefore \angle AOC + \angle COB = 180^\circ$$

$$\Rightarrow \angle AOC + (\angle COD + \angle DOE + \angle EOB) = 180^\circ$$

$$[\ominus \angle COB = \angle COD + \angle DOE + \angle EOB]$$

$$\Rightarrow \angle AOC + \angle COD + \angle DOE + \angle EOB = 180^\circ.$$

Hence, the sum of all the angles formed on the same side of line AB at a point O on it is 180° .

Theorem 2 :

Prove that the sum of all the angles around a point is 360° .

Given : A point O and the rays OA, OB, OC, OD and OE make angles around O.

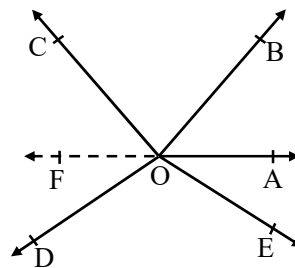
To prove : $\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360^\circ$

Construction : Draw a ray OF opposite to ray OA.

Proof : Since ray OB stands on line FA, we have : $\angle AOB + \angle BOF = 180^\circ$ [linear pair]

$$\therefore \angle AOB + \angle BOC + \angle COF = 180^\circ \quad \dots(i)$$

$$[\ominus \angle BOF = \angle BOC + \angle COF]$$



Again, ray OD stands on line FA.

$$\therefore \angle FOD + \angle DOA = 180^\circ \quad [\text{linear pair}]$$

$$\text{or } \angle FOD + \angle DOE + \angle EOA = 180^\circ \quad \dots(ii)$$

$$[\ominus \angle DOA = \angle DOE + \angle EOA]$$

Adding (i) and (ii), we get :

$$\angle AOB + \angle BOC + \angle COF + \angle FOD + \angle DOE + \angle EOA = 360^\circ$$

$$\therefore \angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360^\circ$$

$$[\ominus \angle COF + \angle FOD = \angle COD]$$

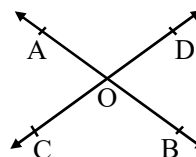
Hence, the sum of all the angles around a point O is 360° .

◆ Vertically opposite angles:

Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays.

Let two lines AB and CD intersect at a point O. Then, two pairs of vertically opposite angles are formed :

$$(i) \angle AOC \text{ and } \angle BOD \quad (ii) \angle AOD \text{ and } \angle BOC$$



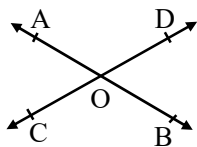
Theorem 3 :

If two lines intersect then the vertically opposite angles are equal.

Given : Two lines AB and CD intersect at a point O.

To prove : (i) $\angle AOC = \angle BOD$, (ii) $\angle AOD = \angle BOC$

Proof : Since ray OA stands on line CD, we have:



$$\angle AOC + \angle AOD = 180^\circ \text{ [linear pair].}$$

Again, ray OD stands on line AB.

$$\therefore \angle AOD + \angle BOD = 180^\circ \text{ [linear pair]}$$

$$\begin{aligned} \therefore \angle AOC + \angle AOD &= \angle AOD + \angle BOD \\ &\text{[each equal to } 180^\circ] \end{aligned}$$

$$\therefore \angle AOC = \angle BOD$$

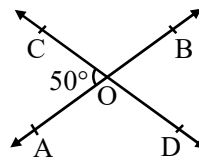
Similarly, $\angle AOD = \angle BOC$

▶ IMPORTANT POINTS

- ◆ Two angles are called adjacent angles if –
 - (i) they have the same vertex,
 - (ii) they have a common arm, and
 - (iii) uncommon arms are on either side of the common arm.
- ◆ Two adjacent angles are said to form a linear pair of angles, if the non-common arms are two opposite rays.
- ◆ If a ray stands on a line, then the sum of the adjacent angles so formed is 180° .
- ◆ If the sum of two adjacent angles is 180° , then their non-common arms are two opposite rays.
- ◆ The sum of all the angles round a point is equal to 360° .
- ◆ Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays.
- ◆ If two lines intersect, then the vertically opposite angles are equal.

❖ EXAMPLES ❖

Ex.15 Two lines AB and CD intersect at O. If $\angle AOC = 50^\circ$, find $\angle AOD$, $\angle BOD$ and $\angle BOC$.



Sol. $\angle AOD + \angle AOC = 180^\circ$ (linear pair)

$$\angle AOD + 50^\circ = 180^\circ$$

$$\angle AOD = 130^\circ$$

$$\text{Also } \angle BOD = \angle AOC$$

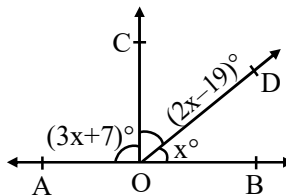
(vertically opposite angles)

$$\& \angle BOC = \angle AOD = 130^\circ$$

(vertically opposite angles)

$$\therefore 130^\circ, 50^\circ, 130^\circ.$$

Ex.16 In the adjoining figure, AOB is a straight line. Find the value of x . Hence, find $\angle AOC$, $\angle COD$ and $\angle BOD$.



Sol. $(3x + 7)^\circ + (2x - 19)^\circ + x^\circ = 180^\circ$ (linear pair)

$$\Rightarrow (6x - 12) = 180^\circ$$

$$\Rightarrow 6x = 192^\circ$$

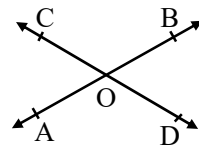
$$\Rightarrow x = 32^\circ$$

$$\therefore \angle AOC = 3x + 7 = 3(32) + 7 = 96 + 7 = 103^\circ$$

$$\angle COD = 2x - 19 = 2(32) - 19 = 64 - 19 = 45^\circ$$

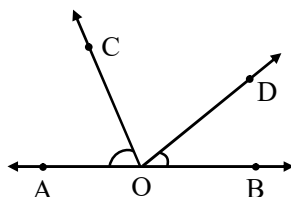
$$\angle BOD = x^\circ = 32^\circ.$$

Ex.17 Two lines AB and CD intersect at a point O such that $\angle BOC + \angle AOD = 280^\circ$, as shown in the figure. Find all the four angles.



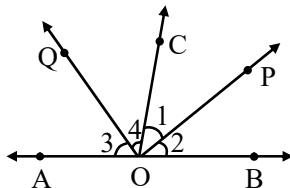
Sol. $\angle AOC = \angle BOD = x$ (Let)
 (vertically opposite angles)
 $\therefore \angle AOC + (\angle AOD + \angle BOC) + \angle BOD = 360^\circ$
 $\Rightarrow x + 280^\circ + x = 360^\circ$
 $\Rightarrow 2x = 80^\circ$
 $\Rightarrow x = 40^\circ$
 $\therefore \angle AOC = \angle BOD = x^\circ = 40^\circ$.
 and $\angle BOC = \angle AOD = \frac{280}{2} = 140^\circ$.

Ex.18 In figure, OA, OB are opposite rays and $\angle AOC + \angle BOD = 90^\circ$. Find $\angle COD$.



Sol. Since OA and OB are opposite rays. Therefore, AB is a line. Since ray OC stands on line AB.
 $\therefore \angle AOC + \angle COB = 180^\circ$
 $\Rightarrow \angle AOC + \angle COD + \angle BOD = 180^\circ$
 $[\angle COB = \angle COD + \angle BOD]$
 $\Rightarrow (\angle AOC + \angle BOD) + \angle COD = 180^\circ$
 $\Rightarrow 90^\circ + \angle COD = 180^\circ$
 $[\angle AOC + \angle BOD = 90^\circ \text{ (Given)}]$
 $\Rightarrow \angle COD = 180^\circ - 90^\circ = 90^\circ$

Ex.19 In figure, OP bisects $\angle BOC$ and OQ, $\angle AOC$. Show that $\angle POQ = 90^\circ$.



Sol.

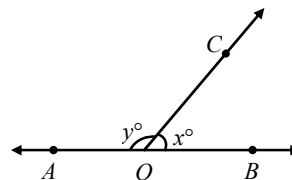
According to question, OP is bisector of $\angle BOC$
 $\therefore \angle 1 = \angle 2$
 or $\angle 1 = \frac{\angle BOC}{2}$
 and OQ is bisector of $\angle AOC$
 $\therefore \angle 3 = \angle 4$

$$\text{or } \angle 4 = \frac{\angle AOC}{2}$$

$$\therefore \angle 1 + \angle 4 = \frac{\angle BOC}{2} + \frac{\angle AOC}{2}$$

$$= \frac{\angle BOC + \angle AOC}{2} = \frac{180}{2} = 90^\circ$$

Ex.20 In figure OA and OB are opposite rays :



- (i) If $x = 75$, what is the value of y ?
- (ii) If $y = 110$, what is the value of x ?

Sol. Since $\angle AOC$ and $\angle BOC$ form a linear pair.

Therefore, $\angle AOC + \angle BOC = 180^\circ$
 $\Rightarrow x + y = 180^\circ \quad \dots(1)$

- (i) If $x = 75$, then from (i)

$$75 + y = 180^\circ$$

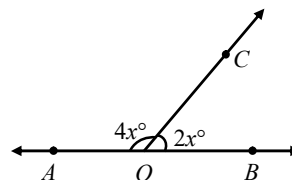
$$y = 105^\circ$$

- (ii) If $y = 110$ then from (i)

$$x + 110 = 180$$

$$\Rightarrow x = 180 - 110 = 70$$

Ex.21 In figure $\angle AOC$ and $\angle BOC$ form a linear pair. Determine the value of x .



Sol. Since $\angle AOC$ and $\angle BOC$ form a linear pair.

$$\therefore \angle AOC + \angle BOC = 180^\circ$$

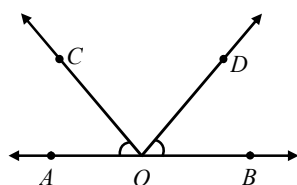
$$\Rightarrow 4x + 2x = 180^\circ$$

$$\Rightarrow 6x = 180^\circ$$

$$\Rightarrow x = \frac{180^\circ}{6} = 30^\circ$$

Thus, $x = 30^\circ$

Ex.22 In figure OA, OB are opposite rays and $\angle AOC + \angle BOD = 90^\circ$. Find $\angle COD$.



Sol. Since OA and OB are opposite rays. Therefore, AB is a line. Since ray OC stands on line AB. Therefore,

$$\angle AOC + \angle COB = 180^\circ \text{ [Linear Pairs]}$$

$$\Rightarrow \angle AOC + \angle COD + \angle BOD = 180^\circ$$

$$[\ominus \angle COB = \angle COD + \angle BOD]$$

$$\Rightarrow (\angle AOC + \angle BOD) + \angle COD = 180^\circ$$

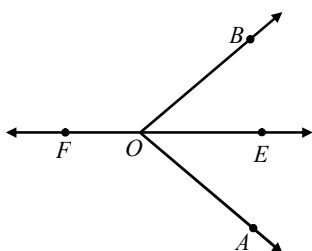
$$\Rightarrow 90^\circ + \angle COD = 180^\circ$$

$$[\ominus \angle AOC + \angle BOD = 90^\circ \text{ (Given)}]$$

$$\Rightarrow \angle COD = 180^\circ - 90^\circ$$

$$\Rightarrow \angle COD = 90^\circ$$

Ex.23 In figure ray OE bisects angle $\angle AOB$ and OF is a ray opposite to OE. Show that $\angle FOB = \angle FOA$.



Sol. Since ray OE bisects angle AOB. Therefore,
 $\angle EOB = \angle EOA$ (i)

Now, ray OB stands on the line EF.

$$\therefore \angle EOB + \angle FOB = 180^\circ \text{ ... (ii) [linear pair]}$$

Again, ray OA stands on the line EF.

$$\therefore \angle EOA + \angle FOA = 180^\circ \text{(iii)}$$

Form (ii) and (iii), we get

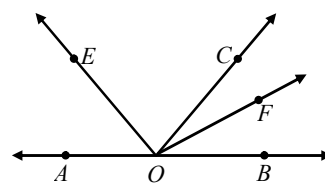
$$\angle EOB + \angle FOB = \angle EOA + \angle FOA$$

$$\Rightarrow \angle EOA + \angle FOB = \angle EOA + \angle FOA$$

$$[\ominus \angle EOB = \angle EOA \text{ (from (i))}]$$

$$\Rightarrow \angle FOB = \angle FOA.$$

Ex.24 In figure OE bisects $\angle AOC$, OF bisects $\angle COB$ and $OE \perp OF$. Show that A, O, B are collinear.



Sol. Since OE and OF bisect angles AOC and COB respectively. Therefore,

$$\angle AOC = 2\angle EOC \text{(i)}$$

$$\text{and } \angle COB = 2\angle COF \text{(ii)}$$

Adding (i) and (ii), we get

$$\angle AOC + \angle COB = 2\angle EOC + 2\angle COF$$

$$\Rightarrow \angle AOC + \angle COB = 2(\angle EOC + \angle COF)$$

$$\Rightarrow \angle AOC + \angle COB = 2(\angle EOF)$$

$$\Rightarrow \angle AOC + \angle COB = 2 \times 90^\circ$$

$$[\ominus OE \perp OF \therefore \angle EOF = 90^\circ]$$

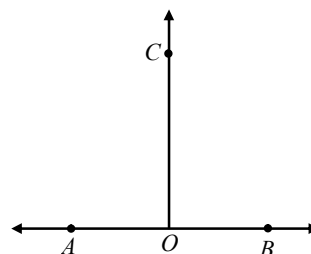
$$\Rightarrow \angle AOC + \angle COB = 180^\circ$$

But $\angle AOC$ and $\angle COB$ are adjacent angles.

Thus, $\angle AOC$ and $\angle COB$ are adjacent supplementary angles. So, $\angle AOC$ and $\angle COB$ form a linear pair. Consequently OA and OB are two opposite rays. Hence, A, O, B are collinear.

Ex.25 If ray OC stands on line AB such that $\angle AOC = \angle COB$, then show that $\angle AOC = 90^\circ$.

Sol. Since ray OC stands on line AB. Therefore,
 $\angle AOC + \angle COB = 180^\circ$ [Linear pair] ... (i)

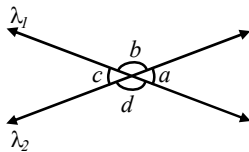


$$\text{But } \angle AOC = \angle COB \text{ (Given)}$$

$$\therefore \angle AOC + \angle AOC = 180^\circ$$

$$\Rightarrow 2\angle AOC = 180^\circ \Rightarrow \angle AOC = 90^\circ$$

Ex.26 In Fig., lines λ_1 and λ_2 intersect at O, forming angles as shown in the figure. If $a = 35^\circ$, find the values of b, c, and d.



Sol. Since lines λ_1 and λ_2 intersect at O. Therefore,

$$\angle a = \angle c \text{ [Vertically opposite angles]}$$

$$\Rightarrow \angle c = 35^\circ \quad [\because \angle a = 35^\circ]$$

$$\text{Clearly, } \angle a + \angle b = 180^\circ$$

[Since $\angle a$ and $\angle b$ are angles of a linear pair]

$$\Rightarrow 35^\circ + \angle b = 180^\circ$$

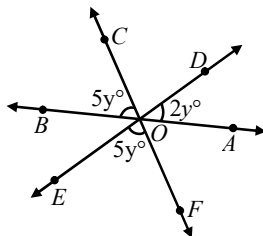
$$\Rightarrow \angle b = 180^\circ - 35^\circ$$

$$\Rightarrow \angle b = 145^\circ$$

Since $\angle b$ and $\angle d$ are vertically opposite angles. Therefore,

$$\angle d = \angle b \Rightarrow \angle d = 145^\circ \quad [\because \angle b = 145^\circ]$$

Ex.27 In Fig., determine the value of y.



Sol. Since $\angle COD$ and $\angle EOF$ are vertically opposite angles. Therefore,

$$\angle COD = \angle EOF \Rightarrow \angle COD = 5y^\circ$$

$$[\because \angle EOF = 5y^\circ \text{ (Given)}]$$

Now, OA and OB are opposite rays.

$$\therefore \angle AOD + \angle DOC + \angle COB = 180^\circ$$

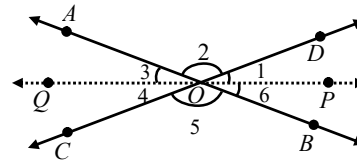
$$\Rightarrow 2y^\circ + 5y^\circ + 5y^\circ = 180^\circ$$

$$\Rightarrow 12y^\circ = 180^\circ$$

$$\Rightarrow y^\circ = \frac{180^\circ}{12} = 15.$$

Thus, $y^\circ = 15$.

Ex.28 In Fig., AB and CD are straight lines and OP and OQ are respectively the bisectors of angles BOD and AOC. Show that the rays OP and OQ are in the same line.



Sol. In order to prove that OP and OQ are in the same line, it is sufficient to prove that $\angle POQ = 180^\circ$.

Now, OP is the bisector of $\angle AOC$

$$\Rightarrow \angle 1 = \angle 6 \quad \dots(i)$$

and, OQ is the bisector of $\angle AOC$

$$\Rightarrow \angle 3 = \angle 4 \quad \dots(ii)$$

Clearly, $\angle 2$ and $\angle 5$ are vertically opposite angles.

$$\therefore \angle 2 = \angle 5 \quad \dots(iii)$$

We know that the sum of the angles formed at a point is 360° .

Therefore,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360^\circ$$

$$\Rightarrow (\angle 1 + \angle 6) + (\angle 3 + \angle 4) + (\angle 2 + \angle 5) = 360^\circ$$

$$\Rightarrow 2\angle 1 + 2\angle 3 + 2\angle 2 = 360^\circ$$

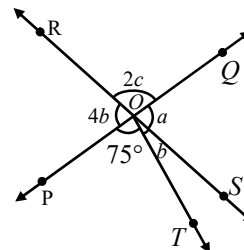
[Using (i), (ii) and (iii)]

$$\Rightarrow 2(\angle 1 + \angle 3 + \angle 2) = 360^\circ$$

$$\Rightarrow \angle 1 + \angle 2 + \angle 3 = 180^\circ \Rightarrow \angle POQ = 180^\circ$$

Hence, OP and OQ are in the same straight line.

Ex.29 In Fig., two straight lines PQ and RS intersect each other at O. If $\angle POT = 75^\circ$, find the values of a, b and c.



Sol. Since OR and OS are in the same line. Therefore,

$$\begin{aligned}\angle ROP + \angle POT + \angle TOS &= 180^\circ \\ \Rightarrow 4b^\circ + 75^\circ + b^\circ &= 180^\circ \Rightarrow 5b^\circ + 75^\circ = 180^\circ \\ \Rightarrow 5b^\circ &= 105^\circ \Rightarrow b^\circ = 21\end{aligned}$$

Since PQ and RS intersect at O. Therefore,
 $\angle QOS = \angle POR$

[Vertically opposite angles]

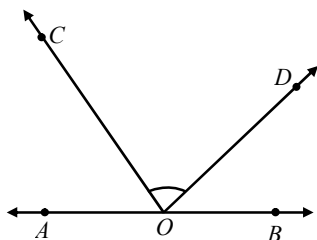
$$\begin{aligned}\Rightarrow a &= 4b \\ \Rightarrow a &= 4 \times 21 = 84 \quad [\because b = 21]\end{aligned}$$

Now, OR and OS are in the same line. Therefore,

$$\begin{aligned}\angle ROQ + \angle QOS &= 180^\circ \quad [\text{Linear pair}] \\ \Rightarrow 2c + a &= 180 \\ \Rightarrow 2c + 84 &= 180 \quad [\because b = 84] \\ \Rightarrow 2c &= 96 \Rightarrow c = 48\end{aligned}$$

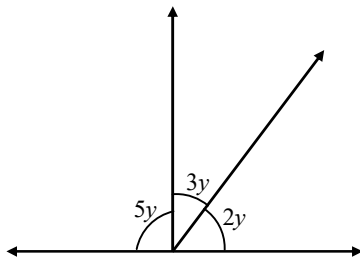
Hence, $a = 84$, $b = 21$ and $c = 48$

Ex.30 In fig if $\angle AOC + \angle BOD = 70^\circ$, find $\angle COD$.



Sol. $\angle AOC + \angle COD + \angle BOD = 180^\circ$
or $(\angle AOC + \angle BOD) + \angle COD = 180^\circ$
or $70^\circ + \angle COD = 180^\circ$
or $\angle COD = 180^\circ - 70^\circ$
or $\angle COD = 110^\circ$

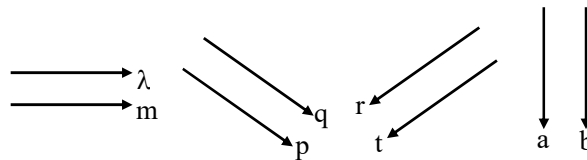
Ex.31 In fig. find the value of y .



Sol. $2y + 3y + 5y = 180^\circ$
 $\Rightarrow 10y = 180^\circ \Rightarrow y = \frac{180^\circ}{10} = 18^\circ$

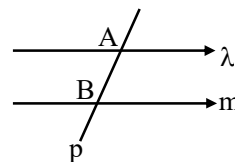
◆ Parallel lines:

The lines which are in same plane and do not intersect each other anywhere, i.e. distance between parallel lines is same anywhere



◆ Transversal line :

A line which intersects two or more given lines at distinct points, is called a transversal of the given lines.



Here $\lambda \parallel m$ and p is transversal line.

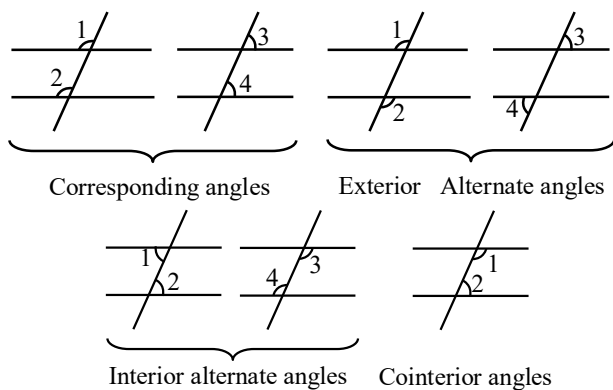
* Part of transversal which is between the two lines is called intercept (AB).

➤ IMPORTANT POINTS

- ◆ Two angles on the same side of a transversal are known as the corresponding angles if both lie either above the two lines or below the two lines.
- ◆ The pairs of interior angles on the same side of the transversal are called pairs of consecutive interior angles.
- ◆ If a transversal intersects two parallel lines, then each pair of corresponding angles are equal.
- ◆ If a transversal intersects two parallel lines, then each pair of alternate interior angles are equal.
- ◆ If a transversal intersects two lines in such a way that a pair of alternate interior angles are equal, then the two lines are parallel.
- ◆ If a transversal intersects two parallel lines, then each pair of consecutive interior angles are supplementary.

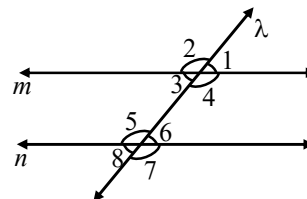
- ◆ If a transversal intersects two lines in such a way that a pair of consecutive interior angles are supplementary, then the two lines are parallel.
- ◆ If two parallel lines are intersected by a transversal, the bisectors of any pair of alternate interior angles are parallel.
- ◆ If two parallel lines are intersected by a transversal, then bisectors of any two corresponding angles are parallel.
- ◆ If the bisectors of a pair of corresponding angles formed by a transversal with two given lines are parallel, prove that the given lines are parallel.
- ◆ If a line is perpendicular to one of two given parallel lines, then it is also perpendicular to the other line.
- ◆ If a side of a triangle is produced, the exterior angle so formed is equal to the sum of two interior opposite angles.
- ◆ If all sides of a polygon are equal it is called a regular polygon.
- ◆ **Sum of all the interior angles of a polygon of n-sides** $= (n - 2) \times 180^\circ$ ($n \geq 3$)
- ◆ **Each interior angle of a regular polygon of n-sides** $= \frac{(n - 2) \times 180^\circ}{n}$
- ◆ **Sum of all the exterior angles formed by producing the sides of polygon** $= 360^\circ$.
- ◆ **No. of sides of polygon**

$$= \frac{360^\circ}{180^\circ - \text{each interior angle}}$$



❖ EXAMPLES ❖

Ex.32 In figure $m \parallel n$ and $\angle 1 = 65^\circ$. Find $\angle 5$ and $\angle 8$.



Sol. We have,

$$\angle 1 = \angle 3 \text{ [Vertically opposite angles]}$$

$$\text{and, } \angle 3 = \angle 8 \text{ [Corresponding angles]}$$

$$\therefore \angle 1 = \angle 8$$

$$\Rightarrow \angle 8 = 65^\circ \text{ } [\angle 1 = 65^\circ \text{ (Given)}]$$

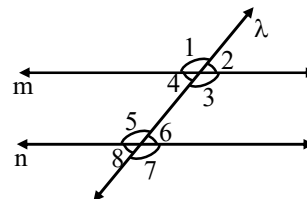
$$\text{Now, } \angle 5 + \angle 8 = 180^\circ$$

$$\Rightarrow \angle 5 + 65^\circ = 180^\circ$$

$$\Rightarrow \angle 5 = 180^\circ - 65^\circ = 115^\circ$$

$$\text{Thus, } \angle 5 = 115^\circ \text{ and } \angle 8 = 65^\circ.$$

Ex.33 In figure $m \parallel n$ and angles 1 and 2 are in the ratio 3 : 2. Determine all the angles from 1 to 8.



Sol. It is given that $\angle 1 : \angle 2 = 3 : 2$. So, let

$$\angle 1 = 3x^\circ \text{ and } \angle 2 = 2x^\circ$$

But $\angle 1$ and $\angle 2$ form a linear pair.

$$\therefore \angle 1 + \angle 2 = 180^\circ$$

$$\Rightarrow 3x^\circ + 2x^\circ = 180^\circ \Rightarrow 5x^\circ = 180^\circ$$

$$\Rightarrow x = \frac{180^\circ}{5} = 36^\circ$$

$$\therefore \angle 1 = 3x^\circ = (3 \times 36)^\circ = 108^\circ$$

$$\text{and, } \angle 2 = 2x^\circ = (2 \times 36)^\circ = 72^\circ$$

$$\text{Now, } \angle 1 = \angle 4 \text{ and } \angle 2 = \angle 3$$

[Vertically opposite angles]

$$\therefore \angle 4 = 72^\circ \text{ and } \angle 3 = 108^\circ$$

Now, $\angle 6 = \angle 2^\circ$ and $\angle 3 = \angle 7$

[Corresponding angles]

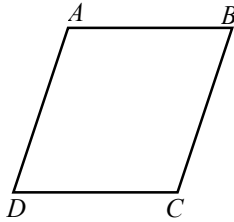
$$\Rightarrow \angle 6 = 72^\circ \text{ and } \angle 7 = 108^\circ \quad [\ominus \angle 2 = 72^\circ]$$

Again, $\angle 5 = \angle 7$ and $\angle 8 = \angle 6$

$$\therefore \angle 5 = 108^\circ \text{ and } \angle 8 = 72^\circ$$

Hence, $\angle 1 = 108^\circ$, $\angle 2 = 72^\circ$, $\angle 3 = 108^\circ$, $\angle 4 = 72^\circ$, $\angle 5 = 108^\circ$, $\angle 6 = 72^\circ$, $\angle 7 = 108^\circ$ and $\angle 8 = 72^\circ$.

Ex.34 In figure $AB \parallel DC$ and $AD \parallel BC$. Prove that $\angle DAB = \angle DCB$.



Sol. Since $AD \parallel BC$ and AB is a transversal intersecting them at A and B respectively. Therefore

$$\angle DAB + \angle ABC = 180^\circ$$

[Consecutive interior angles] ... (i)

Again, $AB \parallel CD$ and BC is a transversal intersecting them at B and C respectively. Therefore,

$$\angle ABC + \angle DCB = 180^\circ$$

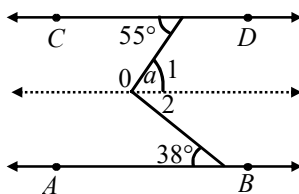
[Consecutive interior angles] (ii)

From (i) and (ii), we get

$$\angle DAB + \angle ABC = \angle ABC + \angle DCB$$

$$\Rightarrow \angle DAB = \angle DCB$$

Ex.35 In figure $AB \parallel CD$. Determine $\angle a$



Sol. Through O draw a line λ parallel to both AB and CD .

$$\text{Clearly, } \angle a = \angle 1 + \angle 2 \quad \dots (ii)$$

$$\text{Now, } \angle 1 = 55^\circ \quad [\text{Alternate } \angle s]$$

$$\text{and } \angle 2 = 38^\circ \quad [\text{Alternate } \angle s]$$

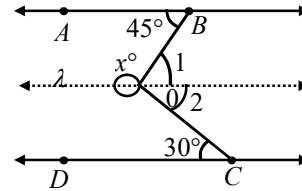
$$\therefore \angle a = 55^\circ + 38^\circ \quad [\text{Using (i)}]$$

$$\Rightarrow \angle a = 93^\circ.$$

$$\text{Thus, } \angle a = 93^\circ$$

Ex.36 In figure $AB \parallel CD$. Determine X .

Sol. Through O , draw a line λ parallel to both AB and CD . Then,



$$\angle 1 = 45^\circ \quad [\text{Alternate } \angle s]$$

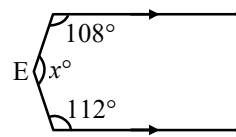
$$\text{and } \angle 2 = 30^\circ \quad [\text{Alternate } \angle s]$$

$$\therefore \angle BOC = \angle 1 + \angle 2 = 45^\circ + 30^\circ = 75^\circ$$

$$\text{So, } x = 360 - \angle BOC = 360 - 75 = 285^\circ$$

$$\text{Hence, } x = 285^\circ$$

Ex.37 In figure $AB \parallel CD$. Find the value of x .



Sol. Draw EF parallel to both AB and CD .

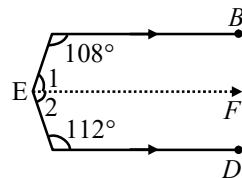
Now, $AB \parallel EF$ and transversal AE cuts them at A and E respectively.

$$\angle BAE + \angle FEA = 180^\circ$$

$$\Rightarrow 108^\circ + \angle 1 = 180^\circ \Rightarrow \angle 1 = 180^\circ - 108^\circ = 72^\circ$$

Again, $EF \parallel CD$ and transversal CE cuts them at E and F respectively.

$$\therefore \angle FEC + \angle ECD = 180^\circ$$



$$\Rightarrow \angle 2 + 112^\circ = 180^\circ$$

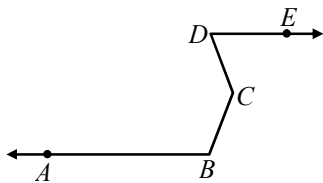
$$\Rightarrow \angle 2 = 180^\circ - 112^\circ$$

$$\Rightarrow \angle 2 = 68^\circ$$

$$\text{Now, } x = \angle 1 + \angle 2$$

$$\Rightarrow x = 72^\circ + 68^\circ = 140^\circ$$

Ex.38 In Figure $AB \parallel DE$. Prove that $\angle ABC + \angle BCD = 180^\circ + \angle CDE$.



Sol. Through C, draw CF parallel to both AB and DE. Since $AB \parallel CF$ and the transversal BC cuts them at B and C respectively. Therefore,

$$\angle ABC + \angle 1 = 180^\circ \quad \dots(i)$$

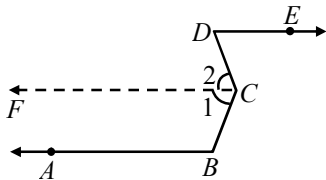
[Θ consecu. interior angles are supplementary]

Similarly, $DE \parallel CF$ and transversal CD intersects them at C and D respectively. Therefore,

$$\angle CDE = \angle 2 \quad [\text{Alternate angles}] \quad \dots(ii)$$

Adding (i) and (ii), we get

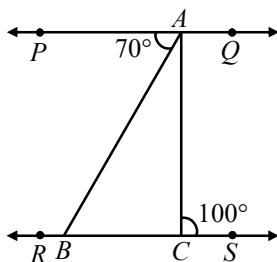
$$\angle ABC + \angle 1 + \angle 2 = 180^\circ + \angle CDE$$



$$\Rightarrow \angle ABC + \angle BCD = 180^\circ + \angle CDE$$

$$[\Theta \angle 1 + \angle 2 = \angle BCD]$$

Ex.39 In Figure $PQ \parallel RS$, $\angle PAB = 70^\circ$ and $\angle ACS = 100^\circ$. Determine $\angle ABC$, $\angle BAC$ and $\angle CAQ$.



Sol. Since $PQ \parallel RS$ and transversal AB cuts them at A and B respectively.

$$\therefore \angle ABC = \angle PAB \quad [\text{Alternate angles}]$$

$$\Rightarrow \angle ABC = 70^\circ \quad [\Theta \angle PAB = 70^\circ (\text{Given})]$$

Now, $PQ \parallel RS$ and transversal AC cuts them at A and C respectively.

$$\therefore \angle PAC = \angle ACS \quad [\text{Alternate angles}]$$

$$\Rightarrow \angle PAC = 100^\circ \quad [\Theta \angle ACS = 100^\circ]$$

$$\Rightarrow \angle PAB + \angle BAC = 100^\circ$$

$$[\Theta \angle PAC = \angle PAB + \angle BAC]$$

$$\Rightarrow 70^\circ + \angle BAC = 100^\circ$$

$$\Rightarrow \angle BAC = 30^\circ$$

Now, ray AB stands at A on PQ.

$$\therefore \angle PAC + \angle CAQ = 180^\circ$$

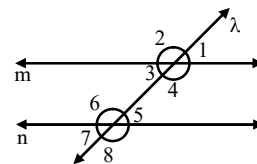
$$\Rightarrow 100^\circ + \angle CAQ = 180^\circ$$

$$\Rightarrow \angle CAQ = 80^\circ$$

Hence, $\angle ABC = 70^\circ$,

$\angle BAC = 30^\circ$ and $\angle CAQ = 80^\circ$.

Ex.40 In Figure if $\angle 2 = 120^\circ$ and $\angle 5 = 60^\circ$, show that $m \parallel n$.



Sol. We have

$$\angle 2 = 120^\circ \text{ and } \angle 5 = 60^\circ$$

But $\angle 2 = \angle 4$ [Vertically opposite angles]

$$\therefore \angle 4 = 120^\circ, \angle 5 = 60^\circ$$

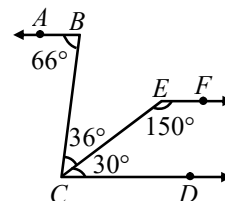
$$\Rightarrow \angle 4 + \angle 5 = 120^\circ + 60^\circ = 180^\circ$$

$\Rightarrow \angle 4$ and $\angle 5$ are supplementary angles.

\Rightarrow Consecutive interior angles are supplementary.

$$\Rightarrow m \parallel n.$$

Ex.41 In figure show that $AB \parallel EF$.



Sol. We have,

$$\angle BCD = \angle BCE + \angle ECD$$

$$= 36^\circ + 30^\circ = 66^\circ$$

$$\therefore \angle ABC = \angle BCD$$

Thus, lines AB and CD are intersected by the line BC such that $\angle ABC = \angle BCD$ i.e. the alternate angles are equal. Therefore,

$$AB \parallel CD \quad \dots(i)$$

$$\text{Now, } \angle ECD + \angle CEF = 30^\circ + 150^\circ = 180^\circ$$

This shows that the sum of the interior angles on the same side of the transversal CE is 180° i.e. they are supplementary.

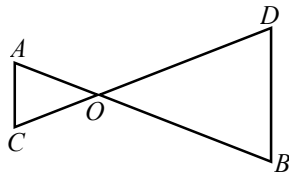
$$\therefore EF \parallel CD \quad \dots(ii)$$

From (i) and (ii), we have

$$AB \parallel CD \text{ and } CD \parallel EF \Rightarrow AB \parallel EF.$$

Hence, $AB \parallel EF$

Ex.42 In figure given that $\angle AOC = \angle ACO$ and $\angle BOD = \angle BDO$. Prove that $AC \parallel DB$.



Sol. We have,

$$\angle AOC = \angle ACO \text{ and } \angle BOD = \angle BDO$$

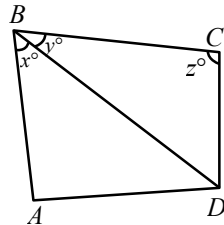
But $\angle AOC = \angle BOD$ [Vertically opp. \angle s]

$$\therefore \angle ACO = \angle BOD \text{ and } \angle BOD = \angle BDO$$

$$\Rightarrow \angle ACO = \angle BDO$$

Thus, AC and BD are two lines intersected by transversal CD such that $\angle ACO = \angle BDO$ i.e. alternate angles are equal. Therefore, $AC \parallel DB$.

Ex.43 In figure $AB \parallel DC$ if $x = \frac{4}{3}y$ and $y = \frac{3}{8}z$, find the values of x, y and z.



Sol. Since $AB \parallel DC$ and transversal BD intersects them at B and D respectively. Therefore,

$$\angle ABD = \angle CDB \Rightarrow \angle CDB = x^\circ$$

In $\triangle BCD$, we have

$$y^\circ + z^\circ + x^\circ = 180^\circ$$

$$\Rightarrow \frac{3}{8}z^\circ + z^\circ + \frac{4}{3} \times \frac{3}{8}z^\circ = 180$$

$$\Rightarrow \frac{3}{8}z^\circ + z^\circ + \frac{1}{2}z^\circ = 180$$

$$\Rightarrow \frac{15}{8}z^\circ = 180^\circ$$

$$[\Theta x = \frac{4}{3}y \text{ and } y = \frac{3}{8}z]$$

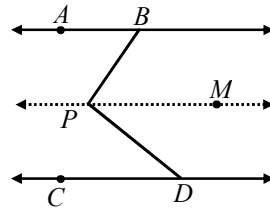
$$\therefore x = \frac{4}{3} \times \frac{3}{8}z = \frac{z}{2}]$$

$$\Rightarrow z^\circ = 180^\circ \times \frac{8}{15} = 96^\circ$$

$$\text{Now, } y = \frac{3}{8}z \Rightarrow y = \frac{3}{8} \times 96^\circ = 36^\circ$$

$$\text{and } x = \frac{4}{3}y \Rightarrow x = \frac{4}{3} \times 36^\circ = 48$$

Ex.44 In figure lines AB and CD are parallel and P is any point between the two lines. Prove that $\angle ABP + \angle CDP = \angle DPB$.



Sol. Through point P draw a line PM parallel to AB or CD.

Now,

$$PM \parallel AB \quad [\text{By construction}]$$

$$\Rightarrow \angle ABP = \angle MPB \quad [\text{Alternate angles}] \quad \dots(i)$$

It is given that $CD \parallel AB$ and $PM \parallel AB$ by construction. Therefore,

$$PM \parallel CD$$

[Θ Lines parallel to the same line are parallel to each other]

$$\Rightarrow \angle CDP = \angle MPD \quad [\text{Alternate angles}] \quad \dots(ii)$$

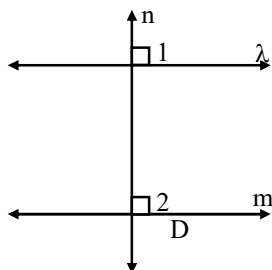
Adding (i) and (ii), we get

$$\angle ABP + \angle CDP = \angle MPB + \angle MPD = \angle DPB$$

Ex.45 Prove that two lines perpendicular to the same line are parallel to each other.

Sol. Let lines λ , m , n be such that $\lambda \perp n$ and $m \perp n$ as shown in figure.

We have to prove that $\lambda \parallel m$



Now,

$\lambda \perp n$ and $m \perp n$

$$\Rightarrow \angle 1 = 90^\circ \text{ and } \angle 2 = 90^\circ$$

$$\Rightarrow \angle 1 = \angle 2$$

Thus, the corresponding angles made by the transversal n with lines λ and m are equal.

Hence, $\lambda \parallel m$.

Ex.46 Prove that two angles which have their arms parallel are either equal or supplementary.

Sol. **Given :** Two angles $\angle ABC$ and $\angle DEF$ such that $BA \parallel ED$ and $BC \parallel EF$.

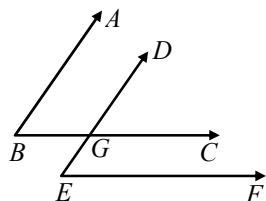
To prove : $\angle ABC = \angle DEF$

$$\text{or } \angle ABC + \angle DEF = 180^\circ$$

Proof : We have the following three cases:

Case I : When both pairs of arms are parallel in the same sense fig. in this case,

$AB \parallel DE$ and transversal BC cuts them at B and G respectively



$$\therefore \angle ABC = \angle DGC \quad \dots(i)$$

[Corresponding angles]

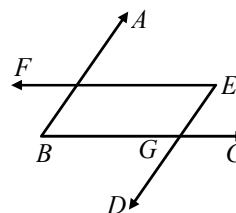
Again, $BC \parallel EF$ and transversal DE cuts them at G and E respectively.

$$\therefore \angle DGC = \angle DEF \quad \dots(ii)$$

From (i) and (ii), we get

$$\angle ABC = \angle DEF$$

Case II : When both pairs of arms are parallel in opposite sense in this case,



$AB \parallel DE$ and transversal BC cuts them at B and G respectively.

$$\therefore \angle ABC = \angle EGC \quad \dots(iii)$$

[Corresponding angles]

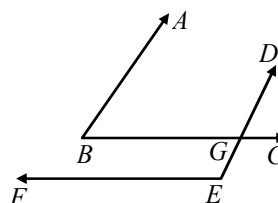
Again, $BC \parallel EF$ and transversal DE cuts them at G and E respectively.

$$\therefore \angle DEF = \angle EGC \text{ [Alternate angles]} \quad \dots(iv)$$

From (iii) and (iv), we get

$$\angle ABC = \angle DEF.$$

Case III : When one pair of arms is parallel in the same sense and the other in opposite sense. In this case,



$AB \parallel DE$ and transversal BC cuts them

$$\therefore \angle ABC = \angle BGE \text{ [Alternate angles]} \quad \dots(v)$$

Again, $BC \parallel FE$ and transversal DE cuts them

$$\therefore \angle DEF + \angle BGE = 180^\circ \quad \dots(vi)$$

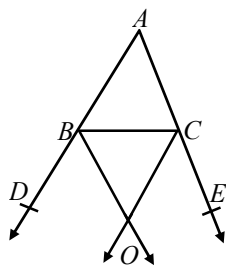
[\odot Consecutive interior angles are supplementary]

From (v) and (vi), we get

$$\angle ABC + \angle DEF = 180^\circ$$

Ex.47 In figure bisectors of the exterior angles B and C formed by producing sides AB and AC of $\triangle ABC$ intersect each other at the point O .

Prove that $\angle BOC = 90^\circ - \frac{1}{2} \angle A$.



Sol. $\angle DBC = 180^\circ - \angle B$

$$\therefore \frac{1}{2} \angle DBC = \angle OBC = 90^\circ - \angle B/2$$

Similarly, $\angle OCB = 90^\circ - \angle C/2$

In $\angle OBC$, we have,

$$\angle OBC + \angle OCB + \angle BOC = 180^\circ$$

$$\text{or } (90^\circ - \angle B/2) + (90^\circ - \angle C/2) + \angle BOC = 180^\circ$$

$$\angle BOC = 180^\circ - (180^\circ - \angle B/2 + \angle C/2)$$

$$= \angle B/2 + \angle C/2 \quad \dots(i)$$

But $\angle A + \angle B + \angle C = 180^\circ$

$$\text{or } \angle B + \angle C = 180^\circ - \angle A$$

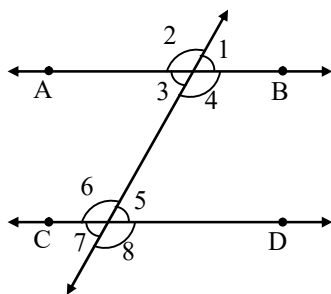
$$\therefore \frac{1}{2} (\angle B + \angle C) = \frac{1}{2} (180^\circ - \angle A)$$

$$\therefore \frac{1}{2} \angle B + \frac{1}{2} \angle C = 90^\circ - \angle A/2$$

Hence, from (i), we have,

$$\angle BOC = 90^\circ - \angle A/2$$

Ex.48 In fig, given that $AB \parallel CD$.



(i) If $\angle 1 = (120 - x)^\circ$ and $\angle 5 = 5x^\circ$, find the measures of $\angle 1$ and $\angle 5$.

(ii) If $\angle 4 = (x + 20)^\circ$ and $\angle 5 = (x + 8)^\circ$, find the measure of $\angle 4$ and $\angle 5$.

(iii) If $\angle 2 = (3x - 10)^\circ$ and $\angle 8 = (5x - 30)^\circ$, determine the measures of $\angle 2$ and $\angle 8$.

(iv) If $\angle 1 = (2x + y)^\circ$ and $\angle 6 = (3x - y)^\circ$, determine the measures of $\angle 2$ in terms of y .

(v) If $\angle 2 = (2x + 30)^\circ$, $\angle 4 = (x + 2y)^\circ$ and $\angle 6 = (3y + 10)^\circ$, find the measure of $\angle 5$.

(vi) If $\angle 2 = 2(\angle 1)$, determine $\angle 7$.

(vii) If the ratio of the measures of $\angle 3$ and $\angle 8$ is 4 : 5, find the measure $\angle 3$ and $\angle 8$

(viii) If the complement of $\angle 5$ equals the supplement of $\angle 4$, find the measures of $\angle 4$ and $\angle 5$.

Sol. (i) since $\angle 1$ and $\angle 5$ are the corresponding angles and corresponding angles are equal.

$$\therefore \angle 1 = \angle 5 \Rightarrow (120 - x)^\circ = 5x^\circ$$

$$\Rightarrow 120^\circ = 6x \Rightarrow x = \frac{120}{6} = 20^\circ$$

$$\therefore \angle 1 = (120 - x)^\circ = (120 - 20)^\circ = 100^\circ$$

$$\text{and, } \angle 5 = 5x^\circ = (5 \times 20)^\circ = 100^\circ$$

(ii) Since $\angle 4$ and $\angle 5$ are consecutive interior angles. Therefore,

$$\angle 4 + \angle 5 = 180^\circ$$

[\therefore Consecutive interior angles are supplementary]

$$\Rightarrow (x + 20)^\circ + (x + 8)^\circ = 180^\circ$$

$$\Rightarrow 2x^\circ + 28^\circ = 180^\circ \Rightarrow 2x^\circ = 180^\circ - 28^\circ$$

$$\Rightarrow 2x = 152^\circ \Rightarrow x = 76^\circ$$

$$\therefore \angle 4 = (x + 20)^\circ = (76 + 20)^\circ = 96^\circ$$

$$\text{and, } \angle 5 = (x + 8)^\circ = (76 + 8)^\circ = 84^\circ$$

(iii) We have,

$$\angle 2 = \angle 4 \quad [\text{Vertically opposite angles}]$$

$$\text{and } \angle 4 = \angle 8 \quad [\text{Corresponding angles}]$$

$$\therefore \angle 2 = \angle 8$$

$$\Rightarrow (3x - 10)^\circ = (5x - 30)^\circ \Rightarrow 3x - 10 = 5x - 30$$

$$\Rightarrow 3x - 5x = -30 + 10 \Rightarrow -2x = -20$$

$$\Rightarrow x = 10$$

$$\therefore \angle 2 = (3x - 10)^\circ = (3 \times 10 - 10)^\circ = 20^\circ$$

$$\text{and } \angle 8 = (5x - 30)^\circ = (5 \times 10 - 30)^\circ = 20^\circ$$

(iv) Since $\angle 3$ and $\angle 6$ are consecutive interior angles. Therefore

$$\angle 3 + \angle 6 = 180^\circ$$

$$\text{But } \angle 1 = \angle 3 \quad \therefore \angle 1 + \angle 6 = 180^\circ$$

$$\Rightarrow (2x + y)^\circ + (3x - y)^\circ = 180^\circ$$

$$\Rightarrow 5x = 180^\circ \quad \Rightarrow x = 36.$$

$$\therefore \angle 1 = (2x + y)^\circ = (72 + y)^\circ \quad [\because x = 36]$$

$$\text{But } \angle 1 + \angle 2 = 180^\circ$$

$$\therefore (72 + y)^\circ + \angle 2 = 180^\circ$$

$$\Rightarrow \angle 2 = (180 - (72 + y))^\circ \Rightarrow \angle 2 = (108 - y)^\circ.$$

(v) We have,

$$\angle 2 = \angle 4 \quad [\text{Vertically opposite angles}]$$

$$\text{and } \angle 4 = \angle 6 \quad [\text{Alternate angles}]$$

$$\therefore \angle 2 = \angle 4 = \angle 6$$

$$\text{Now, } \angle 2 = \angle 4$$

$$\Rightarrow 2x + 30 = x + 2y \Rightarrow 2x - x - 2y + 30 = 0$$

$$\Rightarrow x - 2y + 30 = 0 \quad \dots(1)$$

$$\text{And, } \angle 4 = \angle 6 \Rightarrow (x + 2y) = (3y + 10)$$

$$\Rightarrow x - y - 10 = 0 \quad \dots(2)$$

Subtracting (2) from (1), we get

$$(x - 2y + 30) - (x - y - 10) = 0$$

$$\Rightarrow -y + 40 = 0 \Rightarrow y = 40.$$

Putting $y = 40$ in (2), we get $x = 50$.

$$\therefore \angle 4 = (x + 2y)^\circ = (50 + 2 \times 40)^\circ = 130^\circ$$

$$\text{But } \angle 4 + \angle 5 = 180^\circ$$

$$\therefore 130^\circ + \angle 5 = 180^\circ \Rightarrow \angle 5 = 50^\circ$$

(vi) We have,

$$\angle 1 + \angle 2 = 180^\circ \quad [\text{Linear pairs}]$$

$$\therefore \angle 2 = 2\angle 1 \Rightarrow \angle 1 + 2\angle 1 = 180^\circ$$

$$\Rightarrow 3\angle 1 = 180^\circ \Rightarrow \angle 1 = 60^\circ$$

$$\text{But } \angle 1 = \angle 3 \quad [\text{Vertically opposite angles}]$$

$$\therefore \angle 3 = 60^\circ$$

$$\text{But } \angle 3 = \angle 5 \quad [\text{Alternate angles}]$$

$$\text{and } \angle 5 = \angle 7 \quad [\text{Vertically opposite angles}]$$

$$\therefore \angle 3 = \angle 7 \Rightarrow \angle 7 = 60^\circ \quad [\because \angle 3 = 60^\circ]$$

(vii) We have, $\angle 3 : \angle 8 = 4 : 5$. So, let

$$\angle 3 = 4x \text{ and } \angle 8 = 5x.$$

$$\Rightarrow \angle 5 = 4x \text{ and } \angle 8 = 5x$$

$$[\because \angle 3 = \angle 5 \text{ (Alternate angles)}]$$

$$\Rightarrow \angle 5 + \angle 8 = 4x + 5x$$

$$\Rightarrow 180^\circ = 9x \Rightarrow x = 20^\circ$$

$$\therefore \angle 3 = 4x = 4 \times 20^\circ = 80^\circ$$

$$\text{and } \angle 8 = 5x = 5 \times 20^\circ = 100^\circ$$

(viii) We have, Complement of $\angle 5 =$ Supplement of $\angle 4$

$$\Rightarrow 90^\circ - \angle 5 = 180^\circ - \angle 4$$

$$\Rightarrow 90^\circ - \angle 5 = 180^\circ - (180^\circ - \angle 5)$$

$$\begin{cases} \ominus \angle 4 + \angle 5 = 180^\circ \\ \ominus \angle 4 = 180^\circ - \angle 5 \end{cases}$$

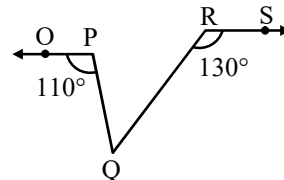
$$\Rightarrow 90^\circ - \angle 5 = \angle 5$$

$$\Rightarrow 2\angle 5 = 90^\circ \quad \Rightarrow \angle 5 = 45^\circ$$

$$\therefore \angle 4 + \angle 5 = 180^\circ$$

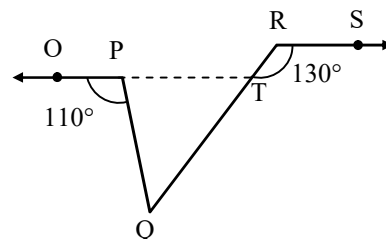
$$\Rightarrow \angle 4 + 45^\circ = 180^\circ \quad \Rightarrow \angle 4 = 135^\circ$$

Ex.49 In fig, $OP \parallel RS$. Determine $\angle PQR$.



Sol. Produce OP to intersect RQ in a point T.

Now, $OT \parallel RS$ and transversal RT intersect them at T and R respectively.



$$\therefore \angle RTP = \angle SRT \quad [\text{Alternate angles}]$$

$$\Rightarrow \angle RTP = 130^\circ$$

$$\Rightarrow \angle PTQ = 180^\circ - 130^\circ = 50^\circ$$

$$\begin{cases} \ominus \angle RTP + \angle PTQ = 180^\circ \\ \text{Linear Pairs} \end{cases}$$

Since, ray QP stands at P on OT.

$$\therefore \angle OPQ + \angle QPT = 180^\circ$$

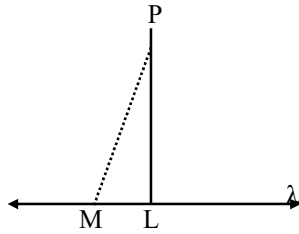
$$\Rightarrow 110^\circ + \angle QPT = 180^\circ \Rightarrow \angle QPT = 70^\circ$$

$$\therefore \angle PQR = 180^\circ - (70^\circ + 50^\circ) = 60^\circ$$

$[\because \text{Sum of the angles of a triangle is } 180^\circ]$

Ex.50 Prove that through a given point we can draw only one perpendicular to a given line.

Sol. If possible, let PL and PM be two perpendicular from a point P on a line λ as shown in fig.



We know that two lines perpendicular to the same line are parallel to each other. Therefore,

$$PL \parallel PM$$

But there cannot be two parallel lines passing through the same point. Therefore, through a given point we can draw only one line perpendicular to a given line.

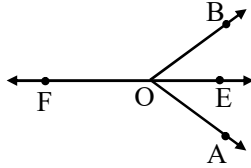
IMPORTANT POINTS TO BE REMEMBERED

1. An angle is the union of two non-collinear rays with a common initial point.
2. An angle whose measure is 90° is called a right angle.
3. An angle whose measure is less than 90° is called an acute angle.
4. An angle whose measure is more than 90° but less than 180° is called an obtuse angle.
5. An angle whose measure is 180° is called a straight angle.
6. An angle whose measure is more than 180° is called a reflex angle.
7. Two angles are complementary if their sum is 90° .
8. Two angles are supplementary if their sum is 180° .
9. Two angles having a common vertex and a common arm are called adjacent angles if their uncommon arms are on either side of the common arm.
10. Two adjacent angles are said to form a linear pair of angles, if their non-common arms are two opposite rays.
11. Two angles are pair of vertically opposite angles if their arms form two pairs of opposite rays.
12. If two lines intersect, then vertically opposite angles are equal.
13. If a transversal intersects two parallel lines, then each pair of -
 - (i) corresponding angles are equal
 - (ii) alternate interior angles are equal
 - (iii) interior angles on the same side of the transversal are supplementary.
14. If a transversal intersects two lines such that, either -
 - (i) any one pair of corresponding angles are equal, or
 - (ii) any one pair of alternate interior angles are equal, or
 - (iii) any one pair of interior angles on the same side of the transversal are supplementary then the lines are parallel.
15. Lines which are parallel to a given line are parallel to each other.

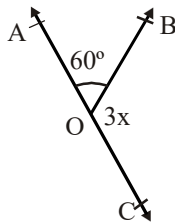
EXERCISE # 1

A. Very Short Answer Type Questions

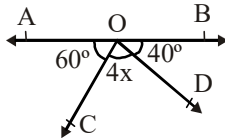
- Q.1** In given figure, Ray OE bisects $\angle AOB$ and OF is a ray opposite to OE. Show that $\angle FOB = \angle FOA$.



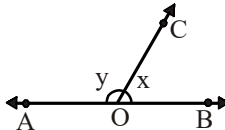
- Q.2** In figure, AOC is a line, find x.



- Q.3** In figure, AOB is a line, determine x.

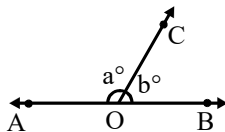


- Q.4** In figure, OA and OB are the opposite rays:

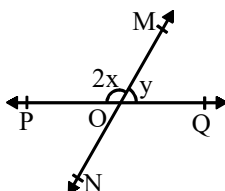


- (i) If $y = 110^\circ$, what is the value of x ?
(ii) If $x = 75^\circ$, what is the value of y ?

- Q.5** In figure, $\angle AOC$ and $\angle BOC$ form a linear pair. If $a - b = 80^\circ$, find the values of a and b.

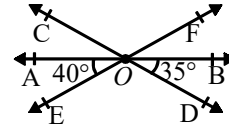


- Q.6** In figure, \overleftrightarrow{PQ} and \overleftrightarrow{MN} intersect at O.

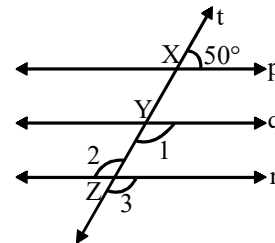


- (i) Determine y when $x = 60^\circ$
(ii) Determine x when $y = 40^\circ$

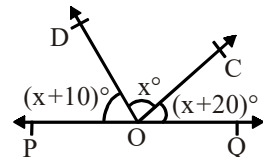
- Q.7** In figure, lines AB, CD and EF intersect at O. Find the measures of $\angle AOC$, $\angle COF$.



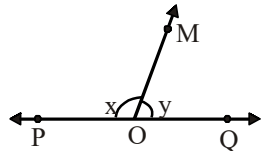
- Q.8** In figure, p, q and r are parallel lines intersected by transversal t at X, Y and Z respectively. Find $\angle 1$, $\angle 2$ and $\angle 3$.



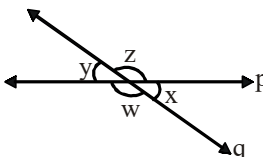
- Q.9** In figure, OP and OQ are opposite rays. Find x.



- Q.10** In figure, $\angle POM$ and $\angle QOM$ form a linear pair. If $x - 2y = 30^\circ$, find x and y.

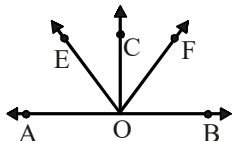


- Q.11** In figure, lines p and q intersect at O. If $x = 35^\circ$, find the values of y, z, w.

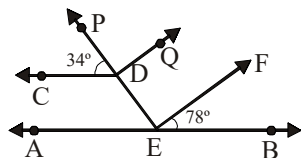


B. Short Answer Type Questions

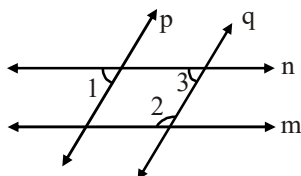
- Q.12** In figure, OE bisects $\angle AOC$, OF bisects $\angle COB$ and $OE \perp OF$. Show that A, O, B are collinear.



- Q.13** In figure, $AB \parallel CD$ and $EF \parallel DQ$. Determine $\angle PDQ$, $\angle AED$ and $\angle DEF$.

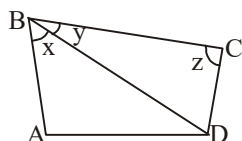


- Q.14** In figure, $m \parallel n$ and $p \parallel q$. If $\angle 1 = 75^\circ$, prove that $\angle 2 = \angle 1 + \frac{1}{3}$ of a right angle.



- Q.15** Prove that two angles which have their arms parallel are either equal or supplementary.

- Q.16** In figure, $AB \parallel DC$. If $x = \frac{4}{3}y$ and $y = \frac{3}{8}z$, find the values of x, y, z .



- Q.17** Two lines AB and CD intersect at O. If $\angle AOC + \angle COB + \angle BOD = 270^\circ$, find the measures of $\angle AOC, \angle COB, \angle BOD, \angle DOA$.

- Q.18** If the bisectors of angles $\angle ABC$ and $\angle ACB$ of a triangle ABC meet at a point O, then prove that $\angle BOC = 90^\circ + \frac{1}{2} \angle A$.

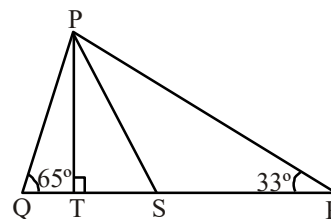
- Q.19** If two parallel lines are intersected by a transversal, prove that the bisectors of the two pairs of interior angles enclose a rectangle.

- Q.20** The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 10° , find all the three angles.

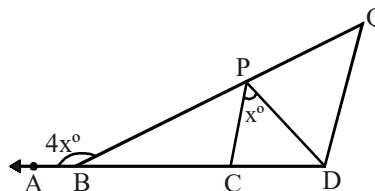
- Q.21** In a $\triangle ABC$, $\angle ABC = \angle ACB$ and the two bisectors of $\angle ABC$ and $\angle ACB$ intersect at O such that $\angle BOC = 120^\circ$. Show that $\angle A = \angle B = \angle C = 60^\circ$.

C. Long Answer Type Questions

- Q.22** In figure, $PT \perp QR$ and PS bisects $\angle QPR$. If $\angle Q = 65^\circ$ and $\angle R = 33^\circ$, find $\angle TPS$.

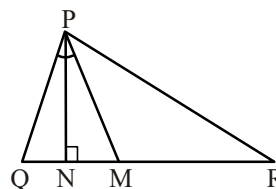


- Q.23** In figure, ABCD and BPQ are lines. $BP = BC$ and $DQ \parallel CP$. Prove that
(i) $CP = CD$ (ii) DP bisects $\angle CDQ$.

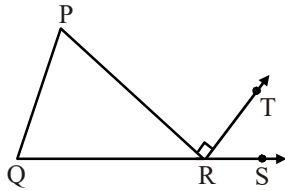


- Q.24** ABCDE is a regular pentagon. Find each angle of $\triangle BDE$.

- Q.25** In figure $\angle Q > \angle R$ and M is a point on QR such that PM is the bisector of $\angle QPR$. If the perpendicular from P on QR meets QR at N, then prove that $\angle MPN = \frac{1}{2}(\angle Q - \angle R)$.



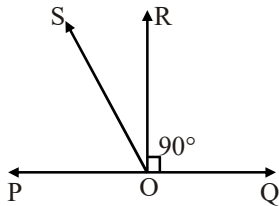
- Q.26** In figure side QR of $\angle PQR$ has been produced to S, if $\angle P : \angle Q : \angle R = 3 : 2 : 1$ and $RT \perp PR$, find $\angle TRS$.



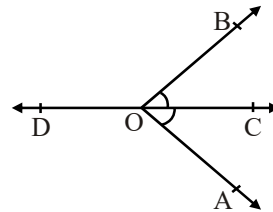
- Q.27** If the angles of a triangle are in the ratio $2 : 3 : 4$, find the three angles.

- Q.28** In figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

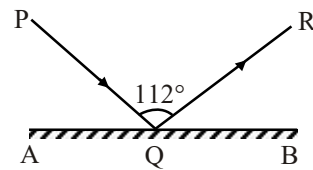
$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS).$$



- Q.29** In the given figure, ray OC is the bisector of $\angle AOB$ and OD is the ray opposite to OC. Show that $\angle AOD = \angle BOD$.



- Q.30** In the given figure, AB is a mirror, PQ is the incident ray and QR, the reflected ray. If $\angle PQR = 112^\circ$, find $\angle PQA$.



ANSWER KEY

A. VERY SHORT ANSWER TYPE QUESTIONS:

- | | |
|---|---|
| 2. 40° | 3. 20° |
| 4. (i) 70° , (ii) 105° | 5. $a = 130^\circ$, $b = 50^\circ$ |
| 6. (i) $y = 60^\circ$, (ii) $x = 70^\circ$ | 7. $\angle AOC = 35^\circ$, $\angle COF = 105^\circ$ |
| 8. $\angle 130^\circ$, $\angle 130^\circ$, $\angle 130^\circ$ | 9. 50° |
| 10. $x = 130^\circ$, $y = 50^\circ$ | 11. $y = 35^\circ$, $z = w = 145^\circ$ |

B. SHORT ANSWER TYPE QUESTIONS :

- | | |
|---|--|
| 13. $\angle AED = 34^\circ$, $\angle PDQ = 68^\circ$, $\angle DEF = 68^\circ$ | 16. $x = 48^\circ$, $y = 36^\circ$, $z = 96^\circ$ |
| 17. 90° | 20. 50° , 60° , 70° |

C. LONG ANSWER TYPE QUESTIONS :

- | | |
|-----------------------------|--|
| 22. 16° | 24. $\angle EBD = 36^\circ$, $\angle BED = \angle BDE = 72^\circ$ |
| 26. $\angle TRS = 60^\circ$ | 27. 40° , 60° , 80° |
| 30. $\angle PQA = 34^\circ$ | |

EXERCISE # 2

Q.1 At 4.24 pm, how many degrees has the hour hand of a clock moved from its position at noon?

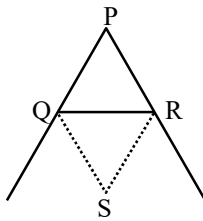
Q.2 Define adjacent angles.

Q.3 Find the sum of all interior angles of hexagon.

Q.4 Find the sum of all interior angles of pentagon.

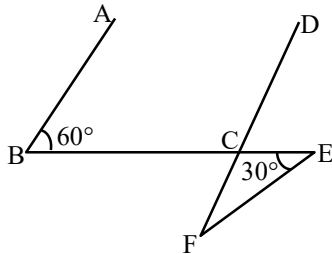
Q.5 If an angle is three times as large as its complement then find it.

Q.6 In given figure, QS and RS are bisectors of exterior angles Q and R. Then find $\angle QSR + \angle P/2$.

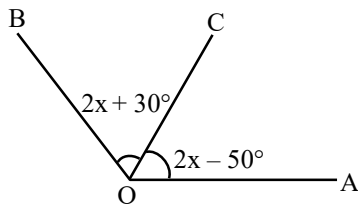


Q.7 Find the angle which exceeds its complement by 20° .

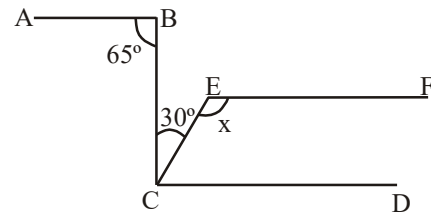
Q.8 In the figure $AB \parallel CD$, then find $\angle EFD$.



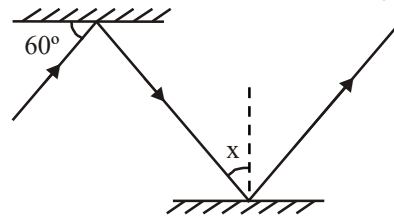
Q.9 What value of x will make AOB a straight line?



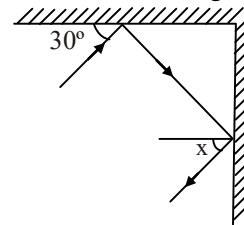
Q.10 What value of x will make $CD \parallel EF$, if $AB \parallel CD$?



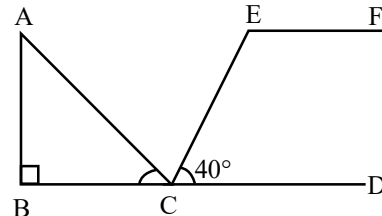
Q.11 Find the value of x in the following figure?



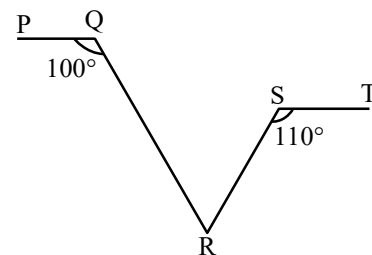
Q.12 Find the value of x in the given figure.



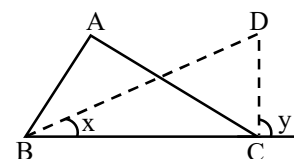
Q.13 In the figure if $BD \parallel EF$, then find $\angle CEF$.



Q.14 In the figure $PQ \parallel ST$, then find $\angle QRS$.



Q.15 In the adjoining figure, BD and CD are angle bisectors. Then, find the relation between $\angle D$ & $\angle A$.



ANSWER KEY

1. 132°
2. They lie in the same plane and have a common vertex, they have a ray in common, the intersection of their interiors is empty.
3. 720°
4. 540°
5. 67.5°
6. 90°
7. 55°
8. 30°
9. 50°
10. 145°
11. 30°
12. 30°
13. 140°
14. 30°
15. $\angle D = \frac{1}{2} \angle A$