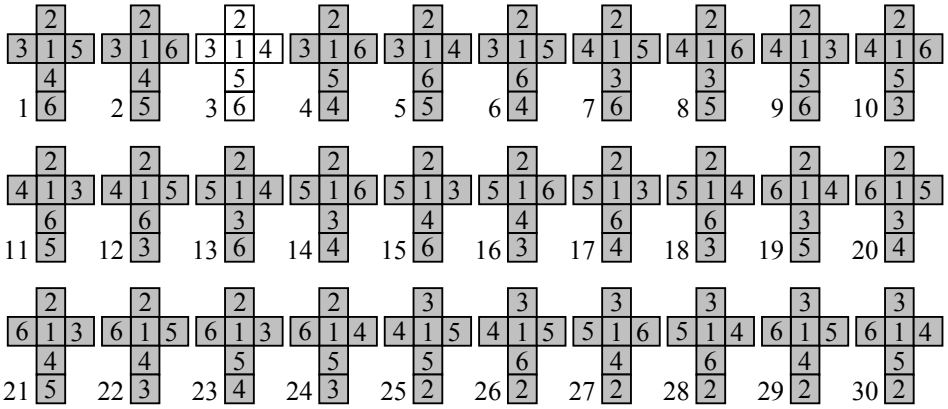
12 Dice and Cube

Dice: The dice has six sides for six numbers. If we try all possible ways to spread them out, we get 30 dice. Every dice has its reflection. We can find the possibilities if we reverse the numbers beside the 1. We can mark every dice by a number with six digits: Start with 1, go upwards, go around 1 counter clockwise and add the number that appears opposite the number 1 on the dice. The red dice has 123546 for instance. The dice on the left are ordered by these numbers.



There are certain rules with the help of these rules question on dice can easily determine.

- **Rule No. 1:** Two opposite faces cannot be adjacent to one another.
- **Rule No. 2:** If two different positions of a dice are shown and one of the two common faces is in the same position then of the remaining faces will be opposite to each other.
- **Rule No. 3:** If in two different positions of dice, the position of a common face be the same, then each of the opposite faces of the remaining faces will be in the same position.
- **Rule No. 4:** If in two different positions of a dice, the position of the common face be not the same, then opposite face of the common face will be that which is not shown on any face in these two positions. Besides, the opposite faces of the remaining faces will not be the same.

A "Mathematics trick" with two dice: Take 2 dice. Dice are made up so that opposite faces add up to 7. Make sure that 6 is opposite 1, the 5 is opposite 2, and the 4 is opposite 3. Roll the dice and then

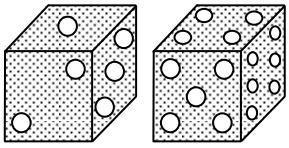
- multiply the top two numbers.
- multiply the bottom two numbers.
- multiply the top of one die by the bottom of the other.
- and now multiply the other top and bottom.
- Now add up all of your (four) answers, and it always adds up to 49.

Question on dice have been classified under three different categories. In the following paragraphs different types of questions have been discussed with the help of examples under different categories.

Note

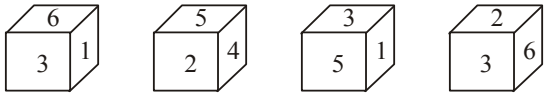
The dice have 21 points (1+2+3+4+5+6 = 21). The numbers 1 to 6 appear on the six sides of a cube. You throw the dice by hand or by a leather dice cup. You turn the cup upside down and you let the dice roll to a standstill.

It is by chance which number appears on top. This is the attraction of throwing the dice. The dice are used in many board games. They often determine the playing. They give the game unexpected turns and add excitement to the games.



Category I:

Example 1. A dice has been thrown four times and produces following results.



(i) (ii) (iii) (iv)

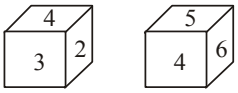
Which number will appear opposite to the number 3?

a. 4 **b.** 5 **c.** 6 **d.** 1

Solution: (a) From the figures (i), (ii) and (iv) we find that numbers 6, 1, 5 and 2 appear on the adjacent surfaces to the number 3. Therefore, number 4 will be opposite to number 3. Hence option (a) is the answer.

Category II:

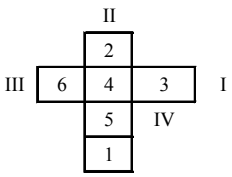
Example 2. The figures given below show the two different positions of a dice. Which number will appear to number 2.



(i) (ii)

a. 3 **b.** 4 **c.** 5 **d.** 6

Solution: (c) The above question, where only two positions of a dice are given, can easily be solved with the following method.



Step I: The dice, when unfolded, will appear as shown in the figure given on the right side.

Step II: Write the common number to both the dice in the middle block. Since common number is 4, hence number 4 will appear in the central block.

Step III: Consider the figure (i) and write the first number in the anti-clockwise direction of number 4, (common number) in block I and second number in block II. Therefore, numbers 3 and 2 being the first and second number to 4 in anticlockwise directions respectively, will appear in block I and II respectively.

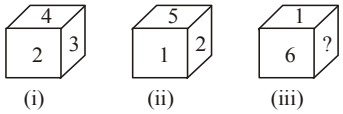
Step IV: Consider fig (ii) and write first and second number in the anticlockwise direction to number 4, (common number) in block (iii) and (iv) Hence number 6 and 5 will appear in the blocks III and IV respectively.

Step V: Write remaining number in the remaining block. Therefore, number 1 will come in the remaining block.

Now, from the unfolded figures we find that number opposite to 6 is 3, number opposite to 2 is 5 and number opposite to 4 is 1. Therefore, option (c) is our answer.

Category III

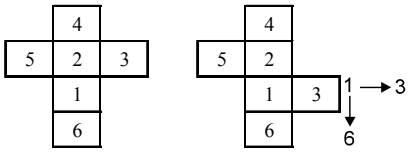
Example 3. From the following figures of dice. Find which number will come in place of?



(i) (ii) (iii)

a. 4 **b.** 5 **c.** 2 **d.** 3

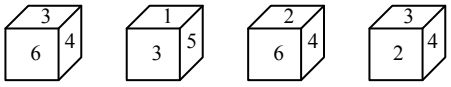
Solution: (d) If the above dice is unfolded, it will look like as the fig given below. (Student should follow the methods as explained in the previous example to find the appropriate place of the numbers appearing on the different surfaces of the dice in the figure).



Now the number in place of '?' can be obtained by making a slight change in the figure as given here. Now comparing fig (iii) as above, we get that number in place of ? is 3

Category IV:

Example 4. Which of the following dices is identical to the unfolded figure as shows here?

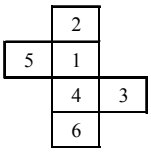


(i) (ii) (iii) (iv)

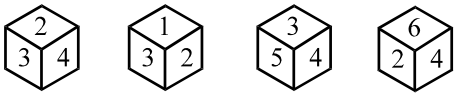
a. (i) **b.** (ii) **c.** (iii) **d.** (iv)

Solution: (a) From the unfolded figure of dice, we find that number opposite to 2 is 4, for 5 it is 3 and for 1 it is 6. From this result we can definitely say that figure (ii), (iii) and (iv) cannot be the answer figure as number lying on the opposite pair of surface are present on the adjacent surfaces.

Hence fig (i) is our answer.



Example 5. A die is thrown four times and its four different positions are given below. Find the number on the face opposite the face showing 2.



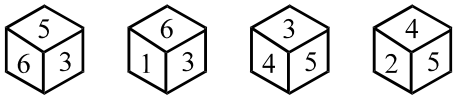
(i) (ii) (iii) (iv)

a. 3 **b.** 4 **c.** 5 **d.** 6

Solution: (b) Here, the number 2 appears in three dice, namely (i), (ii) and (iv). In these dice, we ovserve that the numbers 2, 4, 1 and 6 appear adjacent to 3. So, none of these numbers can be present opposite 2. The only number left is 5.

Hence, 5 is present on the face opposite (b).

Example 6. Shown below are, four different positions of the same dice. Find the number on the face opposite the face showing 6.



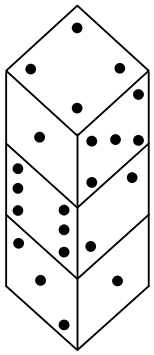
(i) (ii) (iii) (iv)

a. 1 **b.** 2 **c.** 4 **d.** 5

Solution: (c) In this case, the number 6 appears in only two dice from which we observe that the numbers 1, 3 and 5 appear adjacent to 6, so that 2 or 4 can appear opposite 6. So, we beign finding a number which appears at least in three of the given dice. 3 is such a number, which appears in (i), (ii) and (iii). We observe in these dice that, the numbers 1, 4, 5 and 6 appear adjacent to 3. So, they cannot appear opposite 3. The only number that can appear opposite 3 is 2.
So, 2 cannot appear opposite 6.
Hence, 4 appears opposite 6, so that (c) is the answer.
We are now in a position to solve the following exercise.

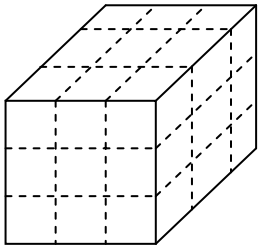
Example 7. What the sum of the does on the hidden faces of these dice?
Solution: (41) Opposite sides of a dice add up to 7, giving 21 dots per dice. Multiply this by three and take away the dots you can see.

Cube: The questions from this topic are of the type wherein, a cube with side measuring unit 'x' is painted on all faces and is cut into smaller cubes with sides measuring unit 'y'. You are then required to find the number of cubes having 'n' faces painted. The first thing that you need to figure out is the number of smaller cubes. For this, you look at one particular edge of the big cube and figure out how many smaller cubes can fit into this. It will be x/y. So, the number of smaller cubes will be $(x/y)^3$.



Since all the smaller cubes will have at least one face facing inside, i.e. not on the surface of the original cube, hence, none of the smaller cubes will have all faces painted. Further, since the maximum number of faces of the larger cube that intersect at a point are 3(at the corners), hence, the smaller cubes can have a maximum of 3 faces painted. So, number of smaller cubes with 3 faces painted = No of corners of larger cube = 8 (always), provided none of the faces of the larger cube is left unpainted.

Example 8. A wooden cube is painted black on four adjoining faces and green on two opposite faces, that is, top and bottom. It is then cut into 27 smaller, identical cubes.



1. How many smaller cubes have only one of their faces painted black?
a. 1 **b.** 2 **c.** 3 **d.** 4

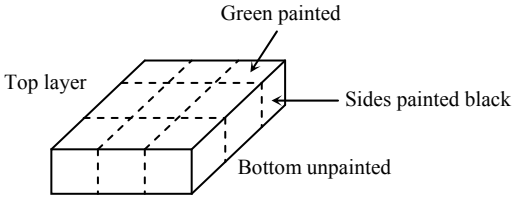
2. How many smaller cubes have only one of their faces painted green?
a. 1 **b.** 2 **c.** 3 **d.** 4

3. How many smaller cubes have only two of their faces painted black?
a. 2 **b.** 4 **c.** 6 **d.** 8

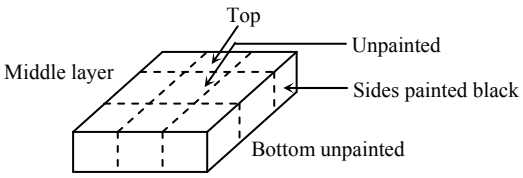
4. How many smaller cubes have at least three of their faces painted?
a. 8 **b.** 6 **c.** 3 **d.** 2

5. How many smaller cubes have none of their faces painted at all?
a. 1 **b.** 2 **c.** 3 **d.** 4

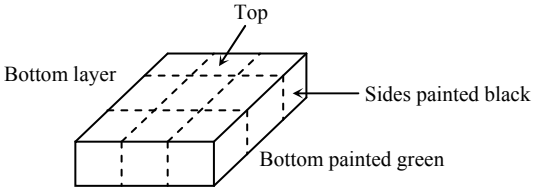
Solution: 1. (d) 2. (b) 3. (b) 4. (a) 5. (a)
This problem can be analysed by considering the three horizontal layers separately. In the top layer, the central cube has only one of its faces painted green; the four cubes at the corner have three of their faces painted one face green and the other two faces black. The remaining four cubes have two of their faces painted – one green and one black.



In the middle layer, the central cube has none of the faces painted. Four cubes at the corners have two of their faces painted black. The remaining four cubes have only one of their faces painted black.



In the bottom layer, the central cube has one of its faces painted green, and four cubes at the corners have three of their faces painted – two black and one green. The remaining four cubes have two of their faces painted – one black and one green.



Example 9. A solid cube is painted on only three adjacent faces and then cut into 64 smaller cubes of equal size.

1. How many smaller cubes have three of their faces coloured?
a. 0 **b.** 1 **c.** 2 **d.** 4

2. How many smaller cubes have only two of their faces coloured?
a. 6 **b.** 8 **c.** 9 **d.** 16

3. How many smaller cubes have only one of their faces coloured?
a. 36 **b.** 27 **c.** 16 **d.** 9

Solution: 1. (b) 2. (c) 3. (b)

The cube is painted on three adjacent faces and cut into 64 smaller, identical cubes. In this case, only one corner cube is painted on three of its faces.

Each pair, out of three coloured adjacent faces, has only one common edge. Thus, there are three edges, along which two coloured faces meet. There are three smaller cubes along each such edge which are painted only on two of their faces. Hence, there are totally $3 \times 3 = 9$ smaller cbes which are coloured on two of their faces.

There are nine smaller cubes on each coloured face which are coloured only on one of their faces. Thus, there are totally $9 \times 3 = 27$ smaller cubes which are coloured only on one of their faces.

Example 10. Each of the faces of a cube is painted with different colours. The face painted red is opposite to the one painted green. The face painted blue is between the red and green coloured faces. The face painted yellow is adjacent to the one painted pink.

The face painted white is adjacent to the yellow coloured one, and the green face is facing down.

1. What is the colour of the face at the top?
a. Blue **b.** White **c.** Red **d.** Yellow

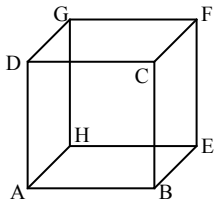
2. What is the colour of the face which is opposite to the one coloured pink?
a. White **b.** Green **c.** Blue **d.** Yellow

3. What is the colour of the face which is opposite to the one coloured blue?
a. Red **b.** Yellow **c.** White **d.** Pink

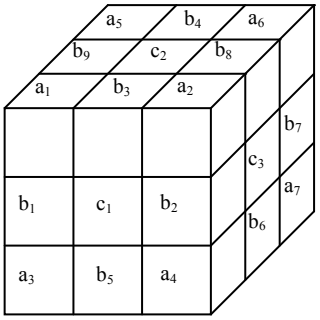
4. Which are the four colours on the faces adjacent to yellow coloured face?
a. Red, white, blue, pink
b. Green, white, blue, pink
c. White, pink, red, green
d. Blue, pink, red, green

Solution: 1. (c) 2. (a) 3. (b) 4. (d)

Face ABEH – green
CDGF – red
CBEF – blue
ABCD – white or pink
AHGD – yellow
EFGH – pink or white



Example. 11 A cube of 3 cm edge is painted red on all its faces. It is then cut at equal distances, at right angles, four times vertically (top to bottom) and twice horizontally (along the sides), as show in figure, where the lines represent the cuts made. Study the diagram and answer the following questions:



1. How many smaller cubes have three of their faces painted red?
a. 64 **b.** 4 **c.** 12 **d.** 8

2. How many cubes have two of their faces painted?
a. 4 **b.** 8 **c.** 12 **d.** 6

3. How many cubes have only one of their sides painted red?
a. 9 **b.** 6 **c.** 1 **d.** 4

4. How many cubes have none of their sides painted?
a. 1 **b.** 4 **c.** 0 **d.** 4

Solution: 1. (d) 2. (c) 3. (b) 4. (a)

Here n = 3.

Therefore, number of cubes with three sides painted red = number of corners – cubes = 8

Number of cubes with two faces painted

$= (n - 2) \times 12 = (3 - 2) \times 12 = 12.$

Number of cubes with one face painted

$= (n - 2)^2 \times 6 = (3 - 2)^2 \times 6 = 6.$

Number of cubes with no face painted

$= (n - 2)^3 = (3 - 2)^3 = 1.$

Multiple Choice Questions

Directions (1 to 5): A cube of 10″ dimensions is coloured yellow, brown, and orange on the three sets of opposite faces. The cube is then cut into smaller cubes of 2″ dimensions.

1. How many smaller cubes are painted yellow and brown on two adjacent faces?
a. 12 **b.** 24 **c.** 20 **d.** 8

2. How many smaller cubes are painted only on one of their faces?
a. 18 **b.** 54 **c.** 36 **d.** 42

3. How many smaller cubes are painted yellow on one face and brown on the opposite face?
a. 0 **b.** 4
c. 8 **d.** None of these

4. How many smaller cubes are painted brown at least on one of their faces?
a. 18 **b.** 20 **c.** 30 **d.** 50

5. How many smaller cubes are painted only on two of their faces, one with brown and the other with orange?
a. 12 **b.** 16 **c.** 18 **d.** 30

Directions (6 to 9): A solid cube is painted red on two adjacent faces, green on the faces opposite to the faces painted red, and black on the remaining faces. The cube is then cut into two parts through the centre without touching the black face. One part is cut into 32 smaller identical cubes and the other part is cut into four smaller cubes of equal size. Answer the following questions by considering the smaller cubes of both the sizes.

6. How many smaller cubes have black colour face(s)?
a. 10 **b.** 12 **c.** 20 **d.** 32

7. How many smaller cubes have only red colour face(s)?
a. 6 **b.** 12 **c.** 8 **d.** 5

8. How many smaller cubes have none of their faces painted?
a. 2 **b.** 8 **c.** 6 **d.** 4

9. How many smaller cubes have only two of their faces painted?
a. 8 **b.** 12 **c.** 24 **d.** 16

Directions (10 to 14): A cube of dimensions 10″ is painted red, blue, and green, respectively, on the three sets of opposite faces. The cube is then cut into 104 smaller cubes, some with 4″ dimensions and some with 2″ dimensions. Two of the cubes of dimensions 4″ have only one of their faces painted red. The remaining cubes of dimensions 4″ have only one of their faces painted green. The following questions are based on the smaller cubes of dimensions 4″ and 2″.

10. How many smaller cubes have three of their faces painted red, blue, and green?
a. 4 **b.** 6 **c.** 10 **d.** 8

11. How many smaller cubes of 4″ dimensions have two of their faces coloured red and blue?
a. 1 **b.** 0 **c.** 4 **d.** 3

12. How many smaller cubes have at least two of their faces coloured?
a. 24 **b.** 36 **c.** 44 **d.** 32

13. How many smaller cubes have at least one of their faces painted blue?
a. 36 **b.** 48 **c.** 42 **d.** 50

14. How many smaller cubes have none of their faces coloured?
a. 27 **b.** 15 **c.** 18 **d.** 12

Directions (15 to 18): A solid cube of size 4″ is cut into smaller cubes of ½″, 1″, and 2″ dimensions. The following questions are based on the smaller cubes.

15. If 1″ and 2″ cubes are one each in number, what will be the total number of ½″ cubes?
a. 510 **b.** 440 **c.** 424 **d.** 460

16. If the numbers of 1″ and 2″ cubes are multiples of 7, what will be the total number of smaller cubes (of all sizes together)?
a. 21 **b.** 8 **c.** 20 **d.** 22

17. If we get maximum possible number of 2″ cubes, then how many ½″ cubes will be obtained (along with the maximum number of 2″ cubes)?
a. 0 **b.** 2 **c.** 4 **d.** 8

18. If the numbers of 1″ and 2″ cubes are multiplies of 5, how many ½″ cubes can be obtained?
a. 36 **b.** 80 **c.** 112 **d.** 120

Directions (19 to 22): The six faces of a cube are painted in such a manner that no two adjacent faces have the same colour. The three colours used are red, white, and black. The cube is then cut into 36 smaller cubes in such a manner that 32 cubes are of one size and the rest are of bigger size, and each of the bigger cubes has no red-coloured face. Answer the following questions by considering the smaller cubes of both the sizes.

19. How many smaller cubes have two or more of their faces painted?

a. 32 **b.** 36 **c.** 28 **d.** 16

20. How many smaller cues have a red-coloured face?

a. 20 **b.** 32 **c.** 16 **d.** 0

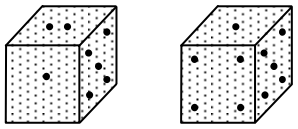
21. How many smaller cubes have three of their faces painted?

a. 16 **b.** 32 **c.** 20 **d.** 8

22. How many smaller cubes have only two of their faces painted?

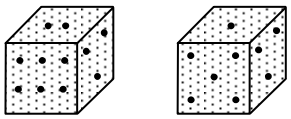
a. 36 **b.** 28 **c.** 32 **d.** 16

23. Observe the dots on the dice (one to six dots) in the following figures. How many dots are contained on the face opposite to the face containing four dots?



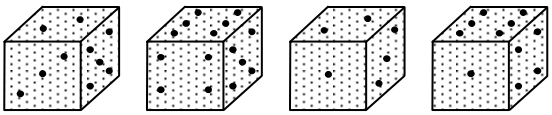
a. 2 **b.** 3 **c.** 5 **d.** 6

24. Two positions of a dice are shown below. How many points will be on the top when 2 points are at the bottom?



a. 6 **b.** 5 **c.** 4 **d.** 1

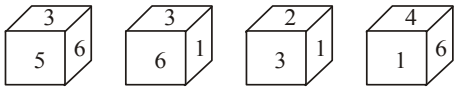
25. How many points will be on the face opposite to the face which contains 2 points?



a. 1 **b.** 5 **c.** 4 **d.** 6

Directions (26 to 30): In each of the following questions four positions of the same dice have been shown. You have to see these figures and select the number opposite to the number as asked in each question.

26.

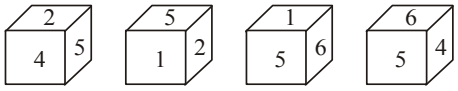


(i) (ii) (iii) (iv)

Which number is on the opposite surface of number 3?

a. 4 **b.** 6 **c.** 5 **d.** 1

27.

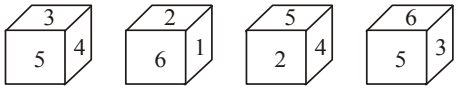


(i) (ii) (iii) (iv)

Which number is opposite to number 1?

a. 4 **b.** 6 **c.** 2 **d.** 3

28.

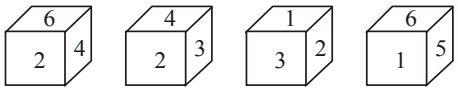


(i) (ii) (iii) (iv)

Which number is opposite to number 5?

a. 6 **b.** 5 **c.** 1 **d.** 3

29.



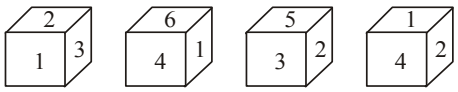
(i) (ii) (iii) (iv)

Which number is opposite to number 2?

a. 4 **b.** 6

c. 5 **d.** 3

30.



(i) (ii) (iii) (iv)

Which number is opposite to number 5?

a. 4 **b.** 6 **c.** 1 **d.** 3

ANSWERS

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
c	b	5	d	a	c	d	d	b	d
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
b	b	d	b	b	d	a	c	c	b
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
d	d	a	d	d	a	a	c	c	d

□ □ □