CHAPTER

25

DIFFERENTIAL EQUATIONS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The differential equation representing the family of parabolas having vertex at origin and axis along positive direction of x-axis is

(a) $y^2y'' - 2xy' = 0$ (b) $y^2 - 2xyy'' = 0$

(c) $y^2 - 2xyy' = 0$ (d) None of these

2. The differential equation of all non-horizontal lines in a plane is

(a) $\frac{d^2y}{dx^2}$ (b) $\frac{d^2x}{dy^2} = 0$ (c) $\frac{dy}{dx} = 0$ (d) $\frac{dx}{dy} = 0$

3. The differential equation which represent the family of curves $y = ae^{bx}$, where a and b are arbitrary constants.

(a) $y' = y^2$ (b) $y'' = y\,y'$

(c) $y\,y'' = y'$ (d) $y\,y'' = (y')^2$

4. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circle with

(a) variable radii and fixed centre (0, 1)

(b) variable radii and fixed centre (0, –1)

(c) fixed radius 1 and variable centre on x-axis

(d) fixed radius 1 and variable centre on y-axis

5. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is

(a) $e^x = \frac{y^3}{3} + e^y + c$ (b) $e^y = \frac{x^2}{3} + e^x + c$

(c) $e^y = \frac{x^3}{3} + e^x + c$ (d) None of these

6. Solution of differential equation $(x^2 - 2x + 2y^2)\,dx + 2xy\,dy = 0$ is

(a) $y^2 = 2x - \frac{1}{4}x^2 + \frac{c}{x^2}$ (b) $y^2 = \frac{2}{3}x - x^2 + \frac{c}{x^2}$

(c) $y^2 = \frac{2}{3}x - \frac{x^2}{4} + \frac{c}{x^2}$ (d) None of these

7. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$, respectively, are

(a) 2 and not defined (b) 2 and 2

(c) 2 and 3 (d) 3 and 3

8. $\tan^{-1} x + \tan^{-1} y = c$ is the general solution of the differential equation

(a) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

(b) $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$

(c) $(1 + x^2)\,dy + (1 + y^2)\,dx = 0$

(d) $(1 + x^2)\,dx + (1 + y^2)\,dy = 0$

9. Integrating factor of the differential equation $\frac{dy}{dx} + y \tan x - \sec x = 0$ is

(a) $\cos x$ (b) $\sec x$

(c) $e^{\cos x}$ (d) $e^{\sec x}$

10. General solution of $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{\left(1+x^2\right)^2}$ is

(a) $y(1 + x^2) = c + \tan^{-1} x$

(b) $\frac{y}{1+x^2} = c + \tan^{-1} x$

(c) $y \log (1 + x^2) = c + \tan^{-1} x$

(d) $y\,(1 + x^2) = c + \sin^{-1} x$

11. A homogeneous differential equation of the $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution

(a) $y = vx$ (b) $v = yx$

(c) $x = vy$ (d) $x = v$

12. Which of the following equation has $y = c_1 e^x + c_2 e^{-x}$ as the general solution?

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{d^2y}{dx^2} + 1 = 0$ (d) $\frac{d^2y}{dx^2} - 1 = 0$

13. The particular solution of $\log \frac{dy}{dx} = 3x + 4y$, $y(0) = 0$ is

(a) $e^{3x} + 3e^{-4y} = 4$ (b) $4e^{3x} - 3^{-4y} = 3$

(c) $3e^{3x} + 4e^{4y} = 7$ (d) $4e^{3x} + 3e^{-4y} = 7$

14. The solution of the equation $\frac{dy}{dx} = \frac{3x-4y-2}{3x-4y-3}$ is
(a) $(x-y^2)+c = \log(3x-4y+1)$
(b) $x-y+c = \log(3x-4y+4)$
(c) $(x-y+c) = \log(3x-4y-3)$
(d) $x-y+c = \log(3x-4y+1)$

15. The order of the differential equation of a family of curves represented by an equation containing four arbitrary constants, will be
(a) 2 (b) 4 (c) 6 (d) None of these

16. The order and degree of the differential equation
$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/3}+x^{1/4}=0$ is
(a) order = 3, degree = 2 (b) order = 2, degree = 3
(c) order = 2, degree = 2 (d) order = 3, degree = 3

17. The differential equation representing the family of curves $y = A\cos(x+B)$, where A, B are parameters, is
(a) $\frac{d^2y}{dx^2}+y=0$ (b) $\frac{d^2y}{dx^2}-y=0$
(c) $\frac{d^2y}{dx^2}=\frac{dy}{dx}+y$ (d) $\frac{dy}{dx}+y=0$

18. The order and degree of the differential equation
$y = x\frac{dy}{dx}+\sqrt{a^2\left(\frac{dy}{dx}\right)^2+b^2}$ is
(a) order = 1, degree = 2 (b) order = 2, degree = 1
(c) order = 2, degree = 2 (d) None of these

19. The order and degree of the differential equation whose solution is $y = cx+c^2-3c^{3/2}+2$, where c is a parameter, is
(a) order = 4, degree = 4 (b) order = 4, degree = 1
(c) order = 1, degree = 4 (d) None of these

20. An equation which involves variables as well as derivative of the dependent variable with respect to independent variable, is known as
(a) differential equation (b) integral equation
(c) linear equation (d) quadratic equation

21. For the differential equation $\frac{d^2y}{dx^2}+y=0$, if there is a function $y = \phi(x)$ that will satisfy it, then the function $y = \phi(x)$ is called
(a) solution curve only
(b) integral curve only
(c) solution curve or integral curve
(d) None of the above

22. The differential equation obtained by eliminating the arbitrary constants a and b from $xy = ae^x + be^{-x}$ is
(a) $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$ (b) $\frac{d^2y}{dx^2}+2y\frac{dy}{dx}-xy=0$
(c) $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+xy=0$ (d) $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=0$

23. A first order-first degree differential equation is of the form
(a) $\frac{d^2y}{dx^2}=F(x,y)$ (b) $\frac{d^3y}{dx^2}=F(x,y)$
(c) $\left(\frac{dy}{dx}\right)^2=F(x,y)$ (d) $\frac{dy}{dx}=F(x,y)$

24. The equation of a curve whose tangent at any point on it different from origin has slope $y+\frac{y}{x}$, is
(a) $y = e^x$ (b) $y = kx.\, e^x$
(c) $y = kx$ (d) $y = k.\, e^{x^2}$

25. The solution of the differential equation
$(1+e^{\frac{x}{y}})\,dx + e^{\frac{x}{y}}\left(1+\frac{x}{y}\right)dy = 0$ is
(a) $ye^x + x = C$ (b) $xe^y + y = C$
(c) $ye^{\frac{y}{x}} + y = C$ (d) $ye^{\frac{x}{y}} + y = C$

26. The differential equation of the form
$\frac{dy}{dx}+Py=Q$
where, P and Q are constants or functions of x only, is known as a
(a) first order differential equation
(b) linear differential equation
(c) first order linear differential equation
(d) None of the above

27. Which of the following is/are first order linear differential equation?
(a) $\frac{dy}{dx}+y=\sin x$ (b) $\frac{dy}{dx}+\left(\frac{1}{x}\right)y=e^x$
(c) $\frac{dy}{dx}+\left(\frac{y}{x\log x}\right)=\frac{1}{x}$ (d) All the above

28. If p and q are the order and degree of the differential equation $y\frac{dy}{dx}+x^3\frac{d^2y}{dx^2}+xy=\cos x$, then
(a) $p < q$ (b) $p = q$
(c) $p > q$ (d) None of these

29. The differential equation obtained by eliminating arbitrary constants from $y = ae^{bx}$ is
(a) $y\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$ (b) $\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$
(c) $y\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2=0$ (d) $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

30. The elimination of constants A, B and C from $y = A + Bx - Ce^{-x}$ leads the differential equation:
(a) $y'' + y''' = 0$ (b) $y'' - y''' = 0$
(c) $y' + e^x = 0$ (d) $y'' + e^x = 0$

31. The integrating factor of the differential equation
$x\frac{dy}{dx}-y=2x^2$ is
(a) e^{-x} (b) e^{-y} (c) $\frac{1}{x}$ (d) x

32. The degree of the differential equation

$y_3^{2/3} + 2 + 3y_2 + y_1 = 0$ is :

(a) 1 (b) 2 (c) 3 (d) None of these

33. In order to solve the differential equation

$$x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$$

the integrating factor is:

(a) x cos x (b) x sec x
(c) x sin x (d) x cosec x

34. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c > 0, is a parameter, is of order and degree as follows :

(a) order 1, degree 2 (b) order 1, degree 1
(c) order 1, degree 3 (d) order 2, degree 2

35. Consider the differential equation

$y^2 dx + \left(x - \frac{1}{y}\right)dy = 0$. If y (1) = 1, then x is given by :

(a) $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$ (b) $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

(c) $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$ (d) $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

36. The equation of the curve through the point (1, 2) and whose slope is $\frac{y-1}{x^2+x}$, is

(a) $(y-1)(x+1) - 2x = 0$ (b) $2x(y-1) + x + 1 = 0$

(c) $x(y-1)(x+1) + 2 = 0$ (d) None of these

37. Differential equation of all straight lines which are at a constant distance from the origin is

(a) $(y + xy_1)^2 = p^2(1 + y_1^2)$ (b) $(y - xy_1)^2 = p^2(1 - y_1^2)$

(c) $(y - xy_1)^2 = p^2(1 + y_1^2)$ (d) None of these

38. General solution of the differential equation

$\frac{dy}{dx} + y\,g'(x) = g(x).\,g'(x)$, where g(x) is a function of x is

(a) $g(x) - \log[1 - y - g(x)] = C$

(b) $g(x) - \log[1 + y - g(x)] = C$

(c) $g(x) + [1 + y - \log g(x)] = C$

(d) $g(x) + \log[1 + y - g(x)] = C$

39. If $x\frac{dy}{dx}$ = y (log y – log x + 1), then the solution of the equation is

(a) $y\log\left(\frac{x}{y}\right) = cx$ (b) $x\log\left(\frac{y}{x}\right) = cy$

(c) $\log\left(\frac{y}{x}\right) = cx$ (d) $\log\left(\frac{x}{y}\right) = cy$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

40. Consider the following statements

I. The order of the differential equation $\frac{dy}{dx} = e^x$ is 1.

II. The order of the differential equation $\frac{d^2y}{dx^2} + y = 0$ is 2.

III. The order of the differential equation

$$\left(\frac{d^3y}{dx^3}\right) + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0 \text{ is } 3.$$

Choose correct option.

(a) I and II are true (b) II and III are true
(c) I and III are true (d) All are true

41. To solve first order linear differential equation, we use following steps

I. Write the solution of the given differential equation as

$y\,(IF) = \int (Q \times IF)\,dx + C$

II. Write the given differential equation in the from $\frac{dy}{dx} + Py = Q$, where P and Q are constants or functions of x only.

III. Find the integrating factor (IF) $e^{\int P\,dx}$

The correct order of the above steps is

(a) II, III, I (b) II, I, III
(c) III, I, II (d) I, III, II

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

42.

	Column-I Differential equations	Column-II Degree
A.	$\frac{dy}{dx} = e^x$	1. 1
B.	$\frac{d^2y}{dx^2} + y = 0$	2. 2
C.	$\frac{d^3y}{dx^3} + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$	3. not defined
D.	$\frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx} + y = 0$	4. 3
E.	$\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right) - \sin^2 y = 0$	
F.	$\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$	

Codes

	A	B	C	D	E	F
(a)	2	2	4	3	1	1
(b)	1	1	1	1	2	3
(c)	3	4	1	1	2	3
(d)	1	1	1	3	4	2

43.

Column-I (Differential equations)	**Column-II (Order and degree respectively)**
A. $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$	1. 2,1
B. $y''' + 2y'' + y' = 0$	2. 1, 1
C. $y' + y = e^x$	3. 3, 1
D. $y'' + (y')^2 + 2y = 0$	4. 3, 2
	5. 1, not defined

Codes

	A	B	C	D
(a)	5	4	1	3
(b)	2	4	1	3
(c)	4	2	3	1
(d)	4	3	2	1

44.

Column-I (Solutions)	**Column-II Differential equations**
A. $y = e^x + 1$	1. $y' + \sin x = 0$
B. $y = x^2 + 2x + C$	2. $xy' = y + x\sqrt{x^2 - y^2}$ $(x \neq 0 \text{ and } x > y \text{ or } x < -y)$
C. $y = \cos x + C$	3. $y'' - y' = 0$
D. $y = \sqrt{1+x^2}$	4. $xy' = y (x \neq 0)$
E. $y = Ax$	5. $y' - 2x - 2 = 0$
F. $y = x \sin x$	6. $y' = \dfrac{xy}{1+x^2}$

Codes

	A	B	C	D	E	F
(a)	2	1	4	3	6	5
(b)	2	1	4	6	5	3
(c)	5	6	1	4	3	2
(d)	3	5	1	6	4	2

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

45. The order of the differential equation of all tangent lines to the parabola $y = x^2$, is
(a) 1 (b) 2 (c) 3 (d) 4

46. The order of the differential equation whose general solution is given by

$$y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x+C_5}$$

where C_1, C_2, C_3, C_4, C_5 are arbitrary constant, is
(a) 5 (b) 4 (c) 3 (d) 2

47. Family $y = Ax + A^3$ of curves will correspond to a differential equation of order
(a) 3 (b) 2 (c) 1 (d) not infinite

48. The degree of the differential equation satisfied by the curve $\sqrt{1+x} - a\sqrt{1+y} = 1$, is
(a) 0 (b) 1 (c) 2 (d) 3

49. If $y = e^x(\sin x + \cos x)$, then the value of $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y$, is
(a) 0 (b) 1 (c) 2 (d) 3

50. A differential equation of the form $\dfrac{dy}{dx} = F(x, y)$ is said to be homogeneous, if F(x, y) is a homogeneous function of degree,
(a) 0 (b) 1 (c) 2 (d) 3

51. The order of the differential equation whose solution is : $y = a\cos x + b\sin x + ce^{-x}$ is :
(a) 3 (b) 2 (c) 1 (d) None of these

52. The degree of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^5 + \frac{dy}{dx} + y = 0 \text{ is}$$

(a) 2 (b) 4 (c) 6 (d) 8

53. If the I.F. of the differential equation $\dfrac{dy}{dx} + 5y = \cos x$ is $\int e^{Adx}$, then A =
(a) 0 (b) 1 (c) 3 (d) 5

54. For $y = \cos kx$ to be a solution of differential equation $\dfrac{d^2y}{dx^2} + 4y = 0$, the value of k is
(a) 2 (b) 4 (c) 6 (d) 8

55. For the function $y = Bx^2$ to be the solution of differential equation $\left(\dfrac{dy}{dx}\right)^3 - 15x^2\dfrac{dy}{dx} - 2xy = 0$, the value of B is __________, given that $B \neq 0$.
(a) 2 (b) 4
(c) 6 (d) 8

56. In the particular solution of differential equation $\dfrac{dy}{dx} = \dfrac{1}{x(3y^2 - 1)}$, the value of constant term is ________, given that y = 2 when x = 1.
(a) 2 (b) 4 (c) 6 (d) 8

57. A family of curves is given by the equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. The differential equation representing this family of curves is given by $xy\dfrac{d^2y}{dx^2} + Ax\left(\dfrac{dy}{dx}\right)^2 - y\dfrac{dy}{dx} = 0$. The value of A is
(a) 0 (b) 1 (c) 3 (d) 5

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

58. **Assertion:** Order of the differential equation whose solution is $y = c_1e^{x+c_2} + c_3e^{x+c_4}$ is 4.
Reason: Order of the differential equation is equal to the number of independent arbitrary constant mentioned in the solution of differential equation.

59. **Assertion:** $x \sin x \frac{dy}{dx} + (x + x \cos x + \sin x)\, y = \sin x,\ y\left(\frac{\pi}{2}\right) = 1 - \frac{2}{\pi} \Rightarrow \lim_{x\to 0} y(x) = \frac{1}{3}$
Reason: The differential equation is linear with integrating factor $x(1 - \cos x)$.

60. **Assertion:** The differential equation of all circles in a plane must be of order 3.
Reason: If three points are non-collinear, then only one circle always passing through these points.

61. **Assertion:** The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is not defined.
Reason : If the differential equation is a polynomial in terms of its derivatives, then its degree is defined.

62. **Assertion:** The differential equation $x^2 = y^2 + xy\frac{dy}{dx}$ is an ordinary differential equation.
Reason: An ordinary differential equation involves derivatives of the dependent variable with respect to only one dependent variable.

63. For the differential equation $\frac{d^2y}{dx^2} + y = 0$, let its solution be $y = \phi_1(x) = 2\sin\left(x + \frac{\pi}{4}\right)$.
Assertion: The function $y = \phi_1(x)$ is called the particular solution.
Reason: The solution which is free from arbitrary constant, is called a particular solution.

64. **Assertion :** The differential equation
$$\frac{dx}{dy} + x = \cos y \text{ and } \frac{dx}{dy} + \frac{-2x}{y} = y^2 e^{-y}$$
are first order linear differential equations.
Reason : The differential equation of the form
$$\frac{dx}{dy} + P_1x = Q_1$$
where, P_1 and Q_1 are constants or functions of y only, is called first order linear differential equation.

65. **Assertion:** The differential equation $y^3\, dy + (x + y^2)\, dx = 0$ becomes homogeneous if we put $y^2 = t$.
Reason: All differential equation of first order first degree becomes homogeneous if we put $y = tx$.

66. Let a solution $y = y(x)$ of the differential equation $x\sqrt{x^2-1}\, dy - y\sqrt{y^2-1}\, dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$.
Assertion: $y(x) = \sec\left(\sec^{-1} x - \frac{\pi}{6}\right)$
Reason: $y(x)$ is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$

67. **Assertion :** The number of arbitrary constants in the solution of differential equation $\frac{d^2y}{dx^2} = 0$ are 2.
Reason: The solution of a differential equation contains as many arbitrary constants as is the order of differential equation.

68. **Assertion :** The degree of the differential equation
$$\frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} + 2y = 0 \text{ is zero}$$
Reason: The degree of a differential equation is not defined if it is not a polynomial eq in its derivatives.

69. **Assertion :** $\frac{dy}{dx} = \frac{x^3 - xy^2 + y^3}{x^2y - x^3}$ is a homogeneous differential equation.
Reason: The function $F(x,y) = \frac{x^3 - xy^2 + y^3}{x^2y - x^3}$ is homogeneous.

70. **Assertion :** $\frac{dy}{dx} + x^2y = 5$ is a first order linear differential equation.
Reason: If P and Q are functions of x only or constant then differential equation of the form $\frac{dy}{dx} + Py = Q$ is a first order linear differential equation.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

71. Which of the following differential equation has $y = x$ as one of its particular solution ?

(a) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$ (b) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x$

(c) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$ (d) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$

72. In a culture the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000 if the rate of growth of bacteria is proportional to the number present.

(a) $\dfrac{2}{\log\frac{11}{10}}$ (b) $\dfrac{2\log 2}{\log\left(\frac{11}{10}\right)}$

(c) $\dfrac{\log 2}{\log 11}$ (d) $\dfrac{\log 2}{\log\left(\frac{11}{10}\right)}$

73. The equation of the curve passing through the point $\left(0, \frac{\pi}{4}\right)$ whose differential equation is
$\sin x \cos y\, dx + \cos x \sin y\, dy = 0$, is

(a) $\sec x \sec y = \sqrt{2}$ (b) $\cos x \cos y = \sqrt{2}$

(c) $\sec x = \sqrt{2} \cos y$ (d) $\cos y = \sqrt{2} \sec y$

74. The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20, 000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009?

(a) 3125 (b) 31250

(c) 21350 (d) 12350

75. Solution of the differential equation

$$\frac{dx}{dy} - \frac{x\log x}{1+\log x} = \frac{e^y}{1+\log x},\ \text{if } y(1) = 0, \text{ is}$$

(a) $x^x = e^{ye^y}$ (b) $e^y = x^{e^y}$

(c) $x^x = ye^y$ (d) None of these

76. If the solution of the differential equation $\dfrac{dy}{dx} = \dfrac{ax+3}{2y+f}$ represents a circle, then the value of 'a' is

(a) 2 (b) – 2

(c) 3 (d) – 4

77. If $\left(1+e^{x/y}\right)dx + \left(1-\frac{x}{y}\right)e^{x/y}dy = 0$, then

(a) $x - ye^{x/y} = c$ (b) $y - xe^{x/y} = c$

(c) $x + ye^{x/y} = c$ (d) $y + xe^{x/y} = c$

78. The solution of the differential equation

$$\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2} \text{ is}$$

(a) $\dfrac{y}{4} + \dfrac{1}{x^2+y^2} = c$ (b) $\dfrac{y}{x} - \dfrac{1}{x^2+y^2} = c$

(c) $\dfrac{x}{y} - \dfrac{1}{x^2+y^2} = c$ (d) None of these

79. The equation of the curve satisfying the differential equation $y_2(x^2 + 1) = 2xy$, passing through the point (0, 1) and having slope of tangent at x = 0 as 3, is

(a) $y = x^3 + 3x + 1$ (b) $y = x^2 + 3x + 1$

(c) $y = x^3 + 3x$ (d) $y = x^3 + 1$

80. If y(t) is a solution of $(1+t)\dfrac{dy}{dt} - ty = 1$ and $y(0) = -1$, then the value of y (1) is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) 1

81. The female-male ratio of a village decreases continuously at the rate proportional to their ratio at any time. If the ratio of female : male of the villages was 980 : 1000 in 2001 and 920 : 1000 in 2011. What will be the ratio in 2021 ?

(a) 864 : 1000 (b) 864 : 100

(c) 1000 : 864 (d) 100 : 864

82. Solution of the differential equation

$x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$ is

(a) $y = cx^2$ (b) $y = cx^2 + \sqrt{x^2+y^2}$

(c) $y + \sqrt{x^2+y^2} = cx^2$ (d) $y - \sqrt{x^2-y^2} = c$

83. The order and degree of the differential equation $\dfrac{d^4y}{dx^4} + \sin(y''') = 0$ are respectively

(a) 4 and 1 (b) 1 and 2

(c) 4 and 4 (d) 4 and not defined

84. The degree of the equation $e^x\dfrac{d^2y}{dx^2} + \sin\left(\dfrac{dy}{dx}\right) = 3$ is

(a) 2 (b) 0

(c) not defined (d) 1

85. If $y = \left(x+\sqrt{1+x^2}\right)^n$, then $(1 + x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is

(a) n^2y (b) $-n^2y$ (c) $0 - y$ (d) $2x^2y$

86. The equation of the curve passing through the point (1, 1) whose differential equation is $x\,dy = (2x^2 + 1)\,dx$ $(x \neq 0)$ is

(a) $x^2 = y + \log|x|$ (b) $y = x^2 + \log|x|$

(c) $y^2 = x + \log|x|$ (d) $y = x + \log|x|$

87. At any point (x, y) of a curve, the slope of tangent is twice the slope of the line segment joining the point of contact to the point (–4, –3). The equation of the curve given that it passes through (–2, 1) is

(a) $x + 4 = (y + 3)^2$ (b) $(x + 4)^2 = (y - 3)$

(c) $x - 4 = (y - 3)^2$ (d) $(x + 4)^2 = |y + 3|$

88. In a bank, principal increases continuously at the rate of 5% per year. In how many years ` 1000 double itself?

(a) 2 (b) 20

(c) $20 \log_e 2$ (d) $2 \log_e 20$

89. The equation of curve through the point (1, 0), if the slope of the tangent to the curve at any point (x, y) is $\dfrac{y-1}{x^2+x}$, is

(a) $(y + 1)(x - 1) + 2x = 0$

(b) $(y + 1)(x - 1) - 2x = 0$

(c) $(y - 1)(x - 1) + 2x = 0$

(d) $(y - 1)(x + 1) + 2x = 0$

90. The general solution of the homogeneous differential equation of the type.

$\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, when y = v : x is

(a) $\int \frac{dv}{g(v)+v} = \int \frac{1}{x} dx + C$

(b) $\int \frac{dv}{g(v)-v} = \int \frac{1}{x} dx + C$

(c) $\int \frac{dv}{g(v)} = \int \frac{1}{x} dx + C$

(d) $\int \frac{dv}{vg(v)} = \int \frac{1}{x} dx + C$

91. If dx + dy = (x + y) (dx – dy), then log (x + y) is equal to

(a) $x + y + C$ (b) $x + 2y + C$

(c) $x - y + C$ (d) $2x + y + C$

92. If the slope of the tangent to the curve at any point P(x, y) is $\frac{y}{x} - \cos^2 \frac{y}{x}$, then the equation of a curve passing through $\left(1, \frac{\pi}{4}\right)$ is

(a) $\tan\left(\frac{y}{x}\right) + \log x = 1$ (b) $\tan\left(\frac{y}{x}\right) + \log y = 1$

(c) $\tan\left(\frac{x}{y}\right) + \log x = 1$ (d) $\tan\left(\frac{x}{y}\right) + \log y = 1$

93. The general solution of the differential equation $(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$ is

(a) $x = (\tan^{-1} y + 1) + Ce^{-\tan^{-1} y}$

(b) $x = (\tan^{-1} y - 1) + Ce^{-\tan^{-1} y}$

(c) $x = (\tan^{-1} x - 1) + Ce^{-\tan^{-1} x}$

(d) $x = (\tan^{-1} x + 1) + Ce^{-\tan^{-1} x}$

94. The solution of differential equation

$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1; x \neq 0$ is

(a) $ye^{2\sqrt{x}} = 2\sqrt{x} + C$ (b) $ye^{\sqrt{x}} = \sqrt{x} + C$

(c) $ye^{2\sqrt{x}} = \sqrt{y} + C$ (d) $ye^{2\sqrt{x}} = 2\sqrt{x} + C$

95. Solution of differential equation xdy – ydx = 0 represents:

(a) rectangular hyperbola.

(b) parabola whose vertex is at origin.

(c) circle whose centre is at origin.

(d) straight line passing through origin.

96. The order and degree of the differential equation

$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ are respectively

(a) 2, 2 (b) 2, 3

(c) 2, 1 (d) None of these

97. The solution of the differential equation $y' = \frac{y}{x} + \frac{\phi(y/x)}{\phi'(y/x)}$ is

(a) $x\,\phi(y/x) = k$ (b) $\phi(y/x) = kx$

(c) $y\,\phi(y/x) = k$ (d) $\phi(y/x) = ky$

98. The differential equation

$(1+y^2)x\,dx - (1+x^2)y\,dy = 0$

represents a family of :

(a) ellipses of constant eccentricity

(b) ellipses of variable eccentricity

(c) hyperbolas of constant eccentricity

(d) hyperbolas of variable eccentricity

99. The solution of the differential equation

$ydx - x\,dy + 3x^2y^2e^{x^3}\,dx = 0$ is

(a) $\frac{x}{y} + e^{x^3} = c$ (b) $\frac{x}{y} - e^{x^3} = c$

(c) $\frac{y}{x} + e^{x^3} = c$ (d) $\frac{y}{x} - e^{x^3} = c$

100. The solution of $x^3\frac{dy}{dx} + 4x^2 \tan y = e^x \sec y$ satisfying y (1) = 0 is :

(a) $\tan y = (x-2)e^x \log x$ (b) $\sin y = e^x(x-1)x^{-4}$

(c) $\tan y = (x-1)e^x x^{-3}$ (d) $\sin y = e^x(x-1)x^{-3}$

101. The differential equations of all conics whose axes coincide with the co-ordinate axis is

(a) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = 0$

(b) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} = 0$

(c) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(d) $xy\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = 0$

102. The expression satisfying the differential equation $(x^2-1)\frac{dy}{dx} + 2xy = 1$ is

(a) $x^2y - xy^2 = c$ (b) $(y^2-1)x = y + c$

(c) $(x^2-1)y = x + c$ (d) None of these

103. The diffrerential equation $\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^3\cos^2 y$ represents a family of curves given by the equation

(a) $x^6 + 6x^2 = C\tan y$ (b) $6x^2 \tan y = x^6 + C$

(c) $\sin 2y = x^3\cos^2 y + C$ (d) none of these

104. A steam boat is moving at velocity V when steam is shut off. Given that the retardation at any subsequent time is equal to the magnitude of the velocity at that time. The velocity v in time t after steam is shut off is

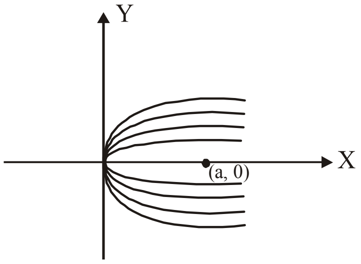
(a) $v = Vt$ (b) $v = Vt - V$

(c) $v = Ve^t$ (d) $v = Ve^{-t}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(c)** Family of parabola satisfying given conditions can be represented graphically as shown below:



Equation is $y^2 = 4ax$ … (i)

Differentiating w.r.t. x, we get $2yy' = 4a$

Substituting 4a from equation (i)

$$2yy' = \frac{y^2}{x}$$

$\Rightarrow$ $y^2 - 2xyy' = 0$

2. **(b)** The general equation of all non-horizontal lines in a plane is $ax + by = 1$, where $a \neq 0$.

Now, $ax + by = 1$

$\Rightarrow$ $a\frac{dx}{dy} + b = 0$ [Differentiating w.r.t. y]

$\Rightarrow$ $a\frac{d^2x}{dy^2} = 0$ [Differentiating w.r.t. y]

$\Rightarrow$ $\frac{d^2x}{dy^2} = 0$ [$\because a \neq 0$]

Hence, the required differential equation is $\frac{d^2x}{dy^2} = 0$

3. **(d)** $\ln y = \ln a + bx$

Differentiating w.r.t. x, we get

$$\frac{1}{y}y' = b$$

Again differentiating w.r.t. x, we get

$$\frac{y''}{y} - \frac{1}{y^2}(y')^2 = 0$$

$\Rightarrow$ $yy'' = (y')^2$

4. **(c)** $\frac{ydy}{\sqrt{1-y^2}} = dx$

On integration, we get $-\sqrt{1-y^2} = x + c$

$1 - y^2 = (x + c)^2 \Rightarrow (x + c)^2 + y^2 = 1$, radius 1 and centre on the x-axis.

5. **(c)** From given differential equation

$$\frac{dy}{dx} = \frac{e^x + x^2}{e^y}$$

Using variable separable form, we have

$e^y dy = (e^x + x^2)\,dx$

Integrating, we get

$$e^y = e^x + \frac{x^3}{3} + c$$

6. **(c)** As $(x^2 - 2x + 2y^2)\,dx = -2xy\,dy$

$\Rightarrow$ $2xy\frac{dy}{dx} + 2y^2 + x^2 - 2x = 0$

$\Rightarrow$ $2xy\frac{dy}{dx} + 2y^2 = 2x - x^2$

$\Rightarrow$ $x\left(2y\frac{dy}{dx}\right) + 2y^2 = 2x - x^2$

By putting $y^2 = v$

$\because$ $2y.\frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow$ $x\frac{dv}{dx} + 2v = 2x - x^2.$

$\Rightarrow$ $\frac{dv}{dx} + v\left(\frac{2}{x}\right) = \frac{2x - x^2}{x}$

I.F.$= e^{\int \frac{2}{x}dx} = x^2$

Now, required solution is

$$v.x^2 = \int \frac{(2x - x^2)x^2 dx}{x} = \int x^2(2 - x)\,dx$$

$\Rightarrow$ $v.x^2 = \frac{2x^3}{3} - \frac{x^4}{4} + c$

$\Rightarrow$ $v = \frac{2x}{3} - \frac{1}{4}x^2 + \frac{c}{x^2}$

$\therefore$ $y^2 = \frac{2x}{3} - \frac{1}{4}x^2 + \frac{c}{x^2}$ which is required solution.

7. **(a)** $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$

Clearly, order of given differential equation is 2 and degree is not defined.

8. **(c)** $\tan^{-1} x + \tan^{-1} y = c$

differentiating w.r.t. x

$$\frac{1}{1+x^2} + \frac{1}{1+y^2}\frac{dy}{dx} = 0$$

$\Rightarrow$ $\frac{dx}{1+x^2} + \frac{dy}{1+y^2} = 0$

$\Rightarrow$ $(1 + y^2)dx + (1 + x^2)\,dy = 0$

9. **(b)** $\frac{dy}{dx} + y\tan x - \sec x = 0$

$\frac{dy}{dx} + (\tan x)\,y = \sec x$

I.F. $= e^{\int \tan x\,dx} = e^{\log \sec x} = \sec x$

10. **(a)** $\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{1}{(1+x^2)^2}$

It is linear differential equation with

$\text{I.F.}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$

Now, solution is

$y(1+x^2)=\int 1+x^2.\frac{1}{(1+x^2)^2}dx+c=\int\frac{dx}{1+x^2}+c$

$y(1+x^2)=\tan^{-1}x+c$

11. **(c)** For solving the homogeneous equation of the form

$\frac{dx}{dy}=h\left(\frac{x}{y}\right)$, to make the substitution $x=vy$

12. **(b)** Family of curves is $y=c_1e^x+c_2e^{-x}$ ... (i)

Differentiating w.r.t. x

$y'=c_1e^x-c_2e^{-x}$, $y''=c_1e^x+c_2e^{-x}=y$

$\therefore y''-y=0$

Solution is $\frac{d^2y}{dx^2}-y=0$

13. **(d)** $\frac{dy}{dx}=e^{3x+4y}=e^{3x}\cdot e^{4y}$

$\Rightarrow e^{-4y}dy=e^{3x}dx\Rightarrow\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}+c$

put $x=0$

We have $-\frac{1}{4}-\frac{1}{3}=c\Rightarrow c=-\frac{7}{12}$,

$\therefore\ \frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}-\frac{7}{12}\Rightarrow 7=3e^{-4y}+4e^{3x}$

14. **(d)** Hint : Put $3x-4y=X$

$\Rightarrow 3-4\frac{dy}{dx}=\frac{dX}{dx}\Rightarrow\frac{dy}{dx}=\frac{1}{4}\left(3-\frac{dX}{dx}\right)$

$\Rightarrow\frac{3}{4}-\frac{1}{4}\frac{dX}{dx}=\frac{X-2}{X-3}$

$\Rightarrow-\frac{1}{4}\frac{dX}{dx}=\frac{4X-8-3(X-3)}{4(X-3)}$

$=-\frac{1}{4}\frac{dX}{dx}=\frac{X+1}{4(X-3)}$

15. **(b)** It is obvious.

16. **(b)** Clearly order of the differential equation is 2.

Again $\frac{d^2y}{dx^2}+x^{1/4}=-\left(\frac{dy}{dx}\right)^{1/3}$

$\Rightarrow\left(\frac{d^2y}{dx^2}+x^{1/4}\right)^3=-\frac{dy}{dx}$

which shows that degree of the differential equation is 3.

17. **(a)** Since $y=A\cos(x+B)$

$\therefore\ \frac{dy}{dx}=-A\sin(x+B)$

$\Rightarrow\frac{d^2y}{dx^2}=-A\cos(x+B)=-y$

$\Rightarrow\frac{d^2y}{dx^2}+y=0$

18. **(a)** Given differential equation can be written as

$y^2+x^2\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}=a^2\left(\frac{dy}{dx}\right)^2+b^2$

Clearly, it is a 1st order and 2nd degree differential equation.

19. **(c)** $y=cx+c^2-3c^{3/2}+2$ ... (i)

Differentiating above with respect to x, we get

$\frac{dy}{dx}=c$.

Putting this value of c in (i), we get

$y=x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2-3\left(\frac{dy}{dx}\right)^{3/2}+2$

Clearly its order is ONE and after removing the fractional power we get the degree FOUR.

20. **(a)** In general, an equation involving derivative of the dependent variable with respect to independent variable (variables) is called a differential equation.

21. **(c)** Consider the differential equation $\frac{d^2y}{dx^2}+y=0$

The solution of this differential equation is a function ϕ that will satisfy it i.e., when the function ϕ is substituted for the unknown y (dependent variable) in the given differential equation, LHS becomes equal to RHS.

The curve $y=\phi(x)$ is called the solution curve (integral curve) of the given differential equation.

22. **(a)** The given function is

$xy=ae^x+be^{-x}$ …(i)

On differentiating equation (i) w.r.t. x, we get

$x\frac{dy}{dx}+y=ae^x-be^{-x}$

Again, differentiating w.r.t. x, we get

$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=ae^x-be^{-x}$

$\Rightarrow x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$ [using eq. (i)]

23. **(d)** A first order-first degree differential equation is of the form

$\frac{dy}{dx}=F(x,y)$

24. **(b)** According to the equation,

$\frac{dy}{dx}=y+\frac{y}{x}=y\left(1+\frac{1}{x}\right)$

$\Rightarrow\frac{dy}{y}=\left(1+\frac{1}{x}\right)dx$

On integrating both sides, we get

$$\int \frac{dy}{y} = \int\left(1+\frac{1}{x}\right)dx$$

$\Rightarrow$ $\log y = x + \log x + C$

$\Rightarrow$ $\log\left(\frac{y}{x}\right) = x + C$

$\Rightarrow$ $\frac{y}{x} = e^{x+C} = e^{x}.e^{C}$

$\Rightarrow$ $\frac{y}{x} = ke^{x}$

$\Rightarrow$ $y = kx.\, e^{x}$

25. (d) The given differential equation is

$$(1+e^{\frac{x}{y}})\,dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$$

$$\frac{dx}{dy} = \frac{e^{\frac{x}{y}}\left(\frac{x}{y}-1\right)}{\left(e^{x/y}+1\right)} \quad \text{...(i)}$$

$$= g\left(\frac{x}{y}\right)$$

$\because$ $\frac{dx}{dy} = g\left(\frac{x}{y}\right)$

$\therefore$ eq. (i) is the homogeneous differential equation

so, put $\frac{x}{y} = v$

i.e., $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

Then, eq. (i) becomes

$$v + y\frac{dv}{dy} = \frac{e^{v}(v-1)}{e^{v}+1}$$

$\Rightarrow$ $y\frac{dv}{dy} = \frac{e^{v}(v-1)}{e^{v}+1} - v$

$\Rightarrow$ $y\frac{dv}{dy} = \frac{ve^{v}-e^{v}-ve^{v}-v}{e^{v}+1} - v$

$\Rightarrow$ $\frac{e^{v}+1}{e^{v}+v}dv = -\frac{1}{y}dy$

On integrating both sides, we get

$$\int\frac{e^{v}+1}{e^{v}+v}dv = -\int\frac{1}{y}dy$$

Put $e^{v} + v = t$

$\Rightarrow$ $e^{v}+1 = \frac{dt}{dv}$

$\Rightarrow$ $dv = \frac{dt}{e^{v}+1}$

$\therefore$ $\int\frac{e^{v}+1}{t}\frac{dt}{e^{v}+1} - \log|y| + \log C$

$\Rightarrow$ $\log|t| + \log|y| = \log C$

$\Rightarrow$ $\log|e^{v}+v| + \log|y| = \log C$ $\quad (\because\ t = e^{v}+v)$

$\Rightarrow$ $\log|(e^{v}+v)y| = C \Rightarrow |(e^{v}+v)\,y| = C$

$\Rightarrow$ $(e^{v}+v)y = C$

So, put $v = \frac{x}{y}$, we get

$$\left(e^{x/y}+\frac{x}{y}\right)y = C \Rightarrow ye^{x/y} + x = C$$

This is the required solution of the given differential equation.

26. (c) A differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

where, P and Q are constants or functions of x only, is known as a first order linear differential equation.

27. (d) The differential equation

$$\frac{dy}{dx} + y = \sin x$$

$\Rightarrow$ $\frac{dy}{dx} + \left(\frac{1}{x}\right)y = e^{x}$

$\Rightarrow$ $\frac{dy}{dx} + \left(\frac{y}{x\log x}\right) = \frac{1}{x}$

are the first order linear different equation since, all the equation are of the type $\frac{dy}{dx} + Py = Q,$ where P and Q, are constants or functions of x only.

28. (c) The given differential equation is

$$y\frac{dy}{dx} + x^{3}\frac{d^{2}y}{dx^{2}} + xy = \cos x$$

Its order is 2 and its degree is 1.

$\therefore$ $p = 2$ and $q = 1$

Hence, $p > q$

29. (c) The given equation is

$y = ae^{bx}$

$\Rightarrow$ $\frac{dy}{dx} = abe^{bx}$ ...(i)

$\Rightarrow$ $\frac{d^{2}y}{dx^{2}} = ab^{2}e^{bx}$ ...(ii)

$\Rightarrow$ $ae^{bx}\frac{d^{2}y}{dx^{2}} = a^{2}b^{2}e^{bx}$

$\Rightarrow$ $y\frac{d^{2}y}{dx^{2}} = \left(\frac{dy}{dx}\right)^{2}$ [from eq. (ii)]

$\Rightarrow$ $y\frac{d^{2}y}{dx^{2}} - \left(\frac{dy}{dx}\right)^{2} = 0$

30. (a) Given, $y = A + Bx - Ce^{-x}$

$\Rightarrow$ $y' = B + C.e^{-x}$ ..... (i)

and $y'' = -Ce^{-x}$ .... (ii)

and $y''' = Ce^{-x}$ ... (iii)

From eqs. (ii) and (iii),

$y''' = -y'' \Rightarrow y''' + y'' = 0$

31. (c) $x\frac{dy}{dx} - y = 2x^{2}$ or $\frac{dy}{dx} - \frac{y}{x} = 2x$

$$\text{I.F.} = e^{\int\frac{-1}{x}dx} = e^{-\log x} = e^{\log\frac{1}{x}} = \frac{1}{x}$$

32. **(b)** Given : Differential equation is

$$y_3^{\frac{2}{3}} + 2 + 3y_2 + y_1 = 0$$

We know that the degree of a differential equation is the degree of highest order derivative.

$\therefore$ degree = 2.

33. **(b)** Given differential equation is :

x cos x dy/dx + y (x sin x + cos x) = 1

Dividing both the sides by x cos x,

$$\Rightarrow \frac{dy}{dx} + \frac{xy\sin x}{x\cos x} + \frac{y\cos x}{x\cos x} = \frac{1}{x\cos x}$$

$$\Rightarrow \frac{dy}{dx} + y\tan x + \frac{y}{x} = \frac{1}{x\cos x}$$

$$\Rightarrow \frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right)y = \frac{\sec x}{x}$$

which is of the form $\frac{dy}{dx} + Py = Q$

Here, $P = \tan x + \frac{1}{x}$ and $Q = \frac{\sec x}{x}$

Integrating factor $= e^{\int P dx}$

$$= e^{\int \tan x + \frac{1}{x} dx}$$

$$= e^{(\log \sec x + \log x)} = e^{\log (\sec x \,.\, x)}$$

$$= x \sec x$$

34. **(c)** $y^2 = 2c(x + \sqrt{c})$ … (i)

$2yy' = 2c.1$ or $yy' = c$ … (ii)

$\Rightarrow y^2 = 2yy'(x + \sqrt{yy'})$

[On putting value of c from (ii) in (i)]

On simplifying, we get

$(y - 2xy')^2 = 4yy'^3$ … (iii)

Hence equation (iii) is of order 1 and degree 3.

35. **(c)** $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

$$\text{I.F.} = e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$$

$$\text{So, } x.e^{-\frac{1}{y}} = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$$

$$\Rightarrow x.e^{-\frac{1}{y}} = I$$

$$\text{where } I = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$$

$$\text{Let } \frac{-1}{y} = t \Rightarrow \frac{1}{y^2} dy = dt$$

$$\Rightarrow I = -\int te^t dt = e^t - te^t = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$$

$$\Rightarrow xe^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$$

$$\Rightarrow x = 1 + \frac{1}{y} + c.e^{1/y}$$

Since $y(1) = 1$

$$\therefore c = -\frac{1}{e}$$

$$\Rightarrow x = 1 + \frac{1}{y} - \frac{1}{e}.e^{1/y}$$

36. **(a)** Given $\frac{dy}{dx} = \frac{y-1}{x^2 + x} \Rightarrow \frac{dy}{y-1} = \frac{dx}{x(x+1)}$

Integrating we get,

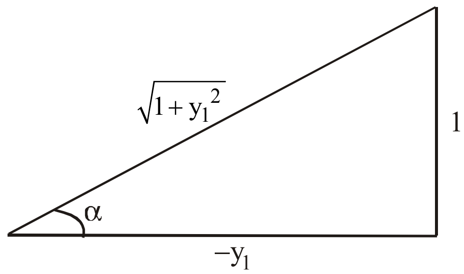
$$\ln(y-1) = 2\ln\left(\frac{x}{x+1}\right) + c$$

It passes through (1, 2), so, c = log 2

Required equation is $\ln(y-1) = \ln\left(\frac{2x}{x+1}\right)$

$\Rightarrow (y-1)(x+1) - 2x = 0$

37. **(c)** Any straight lines which is at a constant distance p from the origin is



x cos α + y sin α = p … (i)

Diff. both sides w.r.t.'x', we get

$$\cos\alpha + \sin\alpha \frac{dy}{dx} = 0$$

$$\Rightarrow \tan\alpha = -\frac{1}{y_1} \qquad \left(\text{where } y_1 = \frac{dy}{dx}\right)$$

$$\therefore \sin\alpha = \frac{1}{\sqrt{1+y_1^2}}; \cos\alpha = -\frac{y_1}{\sqrt{1+y_1^2}}$$

Putting the value of sin α and cos α in (i), we get

$$x.\frac{-y_1}{\sqrt{1+y_1^2}} + y\frac{1}{\sqrt{1+y_1^2}} = p$$

$\Rightarrow (y - xy_1)^2 = p^2(1 + y_1{}^2)$

38. **(d)** We have, $\frac{dy}{dx} = [g(x) - y]\, g'(x)$

Put $g(x) - y = V \Rightarrow g'(x) - \frac{dy}{dx} = \frac{dV}{dx}$

Hence, $g'(x) - \frac{dV}{dx} = V.g'(x)$

$$\Rightarrow \frac{dV}{dx} = (1-V).g'(x) \Rightarrow \frac{dV}{1-V} = g'(x)\,dx$$

$$\Rightarrow \int \frac{dV}{1-V} = \int g'(x)\ dx \Rightarrow -\log(1-V) = g(x) - C$$

$\Rightarrow g(x) + \log(1 - V) = C$

$\therefore$ g (x) + log [1 + y – g (x)] = C

39. **(c)** $\frac{xdy}{dx} = y(\log y - \log x + 1)$

$\frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$

Put y = vx

$\frac{dy}{dx} = v + \frac{xdv}{dx} \Rightarrow v + \frac{xdv}{dx} = v(\log v + 1)$

$\frac{xdv}{dx} = v \log v \Rightarrow \frac{dv}{v \log v} = \frac{dx}{x}$

Put log v = z

$\frac{1}{v} dv = dz \Rightarrow \frac{dz}{z} = \frac{dx}{x}$

ln z = ln x + ln c

x = cx or log v = cx or $\log\left(\frac{y}{x}\right) = cx$.

STATEMENT TYPE QUESTIONS

40. **(d)** I. The differential equation $\frac{dy}{dx} = e^x$ involves the highest derivative of first order.

$\therefore$ Its order is 1.

II. The order of the differential equation $\frac{d^2y}{dx^2} + y = 0$ is 2 since it involves highest derivative of second order.

III. The differential equation $\left(\frac{d^3y}{dx^3}\right) + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$ involves highest derivative of third order

$\therefore$ Its order is 3.

41. **(a)** Steps involved to solve first order linear differential equation.

I. Write the given differential equation in the form

$\frac{dy}{dx} + Py = Q$

where P, Q are constants or functions of x only.

II. Find the Integrating Factor (IF) = $e^{\int P\,dx}$.

III. Write the solution of the given differential equation as

$y(IF) = \int (Q \times IF)\,dx + C$

MATCHING TYPE QUESTIONS

42. **(b)** The degree of the differential equation

$\frac{dy}{dx} = e^x$

$\Rightarrow \frac{d^2y}{dx^2} + y = 0$

$\Rightarrow \left(\frac{d^3y}{dx^3}\right) + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$

$\Rightarrow \frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx} + y = 0$

is 1 since, the highest power of highest order derivative is 1.

The degree of the differential equation

$\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right) - \sin^2 y = 0$

is 2 since, the highest power of highest order derivative is 2.

The degree of the differential equation

$\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$

cannot defined since, this differential equation is not a polynomial in y', y'', y''',...etc.

43. **(d)** A. The highest order derivative which occurs in the given differential equation is y'''. Therefore, its order is three. The given differential equation is a polynomial equation in y''', y'' and y'.
The highest power raised to y''' is 2. Hence, its degree is 2.

B. The highest order derivative which occurs in the given differential equation is y'''. Therefore, its order is three. It is a polynomial equation in y''', y'' and y'. The highest power raised to y''' is 1. Hence, its degree is 1.

C. The highest order derivative present in the given differential equation is y'. Therefore, its order is one. The given differential equation is a polynomial equation in y'. The highest power raised to y' is 1. Hence, its degree is 1.

D. The highest order derivative present in the differential equation is y''. Therefore, its order is two. The given differential equation is a polynomial equation in y'' and y' and the highest power raised to y'' is 1. Hence, its degree is 1.

44. **(d)** A. Given, $y = e^x + 1$ … (i)

On differentiating both sides of this equation w.r.t. x, we get

$y' = \frac{d}{dx}(e^x + 1) = e^x$

Again, differentiating both sides w.r.t. x, we get

$y'' = \frac{d}{dx}(e^x) = e^x$

$\Rightarrow y'' = e^x \Rightarrow y'' - y' = e^x - e^x = 0$

Hence $y = e^x + 1$ is a solution of the differential equation

$y'' - y' = 0$.

B. Given $y = x^2 + 2x + C$ … (i)

On differentiating both sides w.r.t. x, we get

$\Rightarrow y' - 2x - 2 = 0$

Hence, $y = x^2 + 2x + C$ is a solution of the differential equation $y' - 2x - 2 = 0$

C. Given, $y = \cos x + C$ … (i)
On differnetiating both sides w.r.t. x, we get
$y' = -\sin x$
$\Rightarrow y' + \sin x = 0$
Hence, $y = \cos x + C$ is a solution of the differential equation $y' + \sin x = 0$

D. Given $y = \sqrt{1+x^2}$ … (i)
On differentiating both sides of eq. (i) w.r.t. x, we get

$$y' = \frac{d}{dx}\left(\sqrt{1+x^2}\right)$$

$$\Rightarrow y' = \frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}(2x)$$

$$\Rightarrow y' = \frac{2x}{2\sqrt{1+x^2}}$$

$$\Rightarrow y' = \frac{x}{\sqrt{1+x^2}} = \frac{xy}{\sqrt{1+x^2}.\sqrt{1+x^2}}$$

$$\Rightarrow y' = \frac{xy}{\sqrt{1+x^2}}$$

Hence, $y = \sqrt{1+x^2}$ is the solution of the differential equation $y' = \frac{xy}{\left(1+x^2\right)}$.

E. The given function is $y = Ax$ … (i)
On differentiating both sides, we get
$y' = A$

$\Rightarrow y' = \frac{y}{x}$ $(\because y = Ax, x \neq o)$
$\Rightarrow xy' = y$
Hence, $y = Ax$ is the solution of the differential equation
$xy' = y'$ $(x \neq 0)$

F. Given $y = x \sin x$ ....(i)
On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = y' = \frac{d}{dx}(x \sin x)$$

$$\Rightarrow y' = \sin x.\frac{d}{dx}x + x\frac{d}{dx}(\sin x)$$

(using product rule of differentiation)
$\Rightarrow y' = x\cos x + \sin x$
On substituting the value of y and y' in the equation

$xy' = y + x\sqrt{x^2 - y^2}$, we get
LHS $= xy' = x(x\cos x + \sin x)$
$= x^2 \cos x + x \sin x$

$$\Rightarrow xy' = x^3\sqrt{1-\sin^2 x + y}$$
$(\because \cos^2 x = 1 - \sin^2 x$ and $x \sin x = y)$

$$\Rightarrow xy' = x^2\sqrt{1-\left(\frac{y}{x}\right)^2} + y$$

$$\Rightarrow xy' = x\sqrt{x^2 - y^2} + y = \text{RHS}$$

Hence, $y = x \sin x$ is a solution of the differential equation
$(x \neq 0, x > y \text{ or } x < -y)$
$xy' = y + x\sqrt{x^2 - y^2}$

INTEGER TYPE QUESTIONS

45. (a) The parametric form of the given equation is $x = t, y = t^2$.
The equation of any tangent at t is $2xt = y + t^2$.
On differentiating, we get $2t = y_1$.
On putting this value in the above equation, we get

$$xy_1 = y + \left(\frac{y_1}{2}\right)^2 \Rightarrow 4xy_1 = 4y + y_1^2$$

The order of this equation is 1.

46. (c) The given equation can be written as
$y = A\cos(x + C_3) - Be^x$.
where $A = C_1 + C_2$ and $B = C_4e^{C_5}$
Here, there are three independent variables, (A, B, C_3).
Hence, the differential equation will be of order 3.

47. (b) $y = Ax + A^3$
differentiating w.r.t. x

$$\frac{dy}{dx} = A$$

Again differentiating w.r.t.x

$$\frac{d^2y}{dx^2} = 0$$

which is differential equation of order 2.

48. (b) Differentiate the given equation

$$\frac{1}{2}(1+x)^{-1/2} - \frac{a}{2}(1+y)^{-1/2}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{1}{\sqrt{1+x}} = \frac{a}{\sqrt{1+y}}\frac{dy}{dx}$$

$$\Rightarrow a = \frac{\sqrt{1+y}}{\sqrt{1+x}}\frac{1}{dy/dx}$$

Putting this value in the given equation

$$\frac{dy}{dx}\sqrt{1+x} - \frac{1+y}{\sqrt{1+x}} = \frac{dy}{dx}$$

$$\Rightarrow (1+x)\frac{dy}{dx} = 1 + y\sqrt{1+x}\frac{dy}{dx}$$

The degree of this equation is one.

49. (a) Hint : $y = e^x(\sin x + \cos x)$

$$\Rightarrow \frac{dy}{dx} = 2e^x \cos x$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2e^x(\cos x - \sin x)$$

L.H.S. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$
$= 2e^x[\cos x - \sin x - 2\cos x + \sin x + \cos x]$
$= 2e^x \times 0 = 0 = $ R.H.S.

50. (a) A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if F(x, y) is a homogeneous function of degree zero.

51. (a) Given : $y = a \cos x + b \sin x + ce^{-x}$
This equation has three parameters.
$\therefore$ The order of differential equation is 3.

52. (b) The degree of a differential equations is the exponent of the highest order in the differential equation. Therefore the degree of the given differential equation is 4.

53. (d) The I.F. of the differential equation $\frac{dy}{dx} + Py = Q$ is $e^{\int P dx}$. Here P = 5 therefore I.F. = $e^{\int 5 dx}$. Hence A = 5.

54. (a) Given that y = cos k x, therefore $\frac{dy}{dx} = -k \sin kx$ and $\frac{d^2y}{dx^2} = -k^2 \cos kx$ Putting this value of $\frac{d^2y}{dx^2}$ and y = cos kx in $\frac{d^2y}{dx^2} + 4y = 0$, we get

$-k^2 \cos kx + 4 \cos kx = 0$
or $k^2 = 4$
or $k = \pm 2$, or k = 2.

55. (a) $\frac{dy}{dx} = B.2x$, Putting this value of $\frac{dy}{dx}$ in equation

$\left(\frac{dy}{dx}\right)^3 - 15x^2 \frac{dy}{dx} - 2xy = 0$, we get

$(B.2x)^3 - 15x^2 (B.2x) - 2x (Bx^2) = 0$
or $B^3.8x^3 - B.30x^3 - B.2x^3 = 0$
or $B^3 . 8x^3 - B.32x^3 = 0$
or $B (B^2.8x^3 - 32x^3) = 0$
$B \neq 0$
$\therefore B^2 8x^3 - 32x^3 = 0$
or $B^2 8x^3 = 32x^3$
or $B^2 = 4$
or $B = \pm 2$ or B = 2

56. (c) $\because \frac{dy}{dx} = \frac{1}{x(3y^2 - 1)}$

$\therefore \int (3y^2 - 1)dy = \int \frac{dx}{x}$

$y^3 - y = \ln x + c$
when x = 1, y = 2
$\therefore 2^3 - 2 = \ln 1 + c$
$8 - 2 = 0 + c$ or c = 6

57. (b) Differentiating the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ w.r.t. x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

or $\frac{x^2}{a^2} + \frac{xy}{b^2}\frac{dy}{dx} = 0$

or $1 - \frac{y^2}{b^2} + \frac{xy}{b^2}\frac{dy}{dx} = 0 \qquad \left[\because \frac{x^2}{a^2} = 1 - \frac{y^2}{b^2}\right]$

Differentiating again w.r.t. x, we get

$$\frac{-2y}{b^2}\frac{dy}{dx} + \frac{y}{b^2}\frac{dy}{dx} + \frac{x}{b^2}\frac{dy}{dx}.\frac{dy}{dx} + \frac{xy}{b^2}\frac{d^2y}{dx^2} = 0$$

or $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

comparing with the given differential equation, we get A = 1.

ASSERTION - REASON TYPE QUESTIONS

58. (d) $\because y = \left(c_1 e^{c_2} + c_3 e^{c_4}\right)e^x = ce^x$ (say)

$\therefore \frac{dy}{dx} = ce^x = y$

$\therefore$ Order is 1.

59. (a) $\frac{dy}{dx} + \left(\frac{1}{\sin x} + \cot x + \frac{1}{x}\right)y = \frac{1}{x}$

$$\text{I.F.} = \exp\int\left(\frac{1}{\sin x} + \cot x + \frac{1}{x}\right)dx$$

$$= \exp \ell n\left(x \tan\frac{x}{2} \sin x\right)$$

$$= x \tan\frac{x}{2} \times 2\sin\frac{x}{2}\cos\frac{x}{2} = x(1 - \cos x)$$

Solution, yx(1 – cos x)

$$= \int \frac{1}{x}.x(1 - \cos x)\, dx = x - \sin x + c$$

$$y\left(\frac{\pi}{2}\right) = 1 - \frac{2}{\pi} \Rightarrow c = 0$$

$$\therefore y(x) = \frac{x - \sin x}{x(1 - \cos x)}$$

$$y = \frac{x - \left(x - \frac{x^3}{6}...\right)}{x\left(1 - \left(1 - \frac{x^2}{2}...\right)\right)} = \frac{x^2}{6}\frac{1}{\frac{x^2}{2}} \text{ as } x \to 0, y \to \frac{1}{3}$$

60. (b) Let $x^2 + y^2 + 2gx + 2fy + c = 0$
Here in this equation, there are three constants.
$\therefore$ Order = 3
Reason is also true.

61. (a) The given differential equation is
$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$$
This is not a polynomial equation in terms of its derivatives.
$\therefore$ Its degree is not defined.

62. (a) The given differential equation is
$$x^2 = y^2 + xy\frac{dy}{dx}$$
Since, this equation involves the derivative of the dependent variable y with respect to only one independent variable x.
$\therefore$ It is an ordinary differential equation.

63. (a) The solution free from arbitrary constants, i.e., the solution obtained from the general solution by giving particular values to the arbitrary constant is called a particular solution of the differential equation.
Here, function ϕ_1 contains no arbitrary constants but only the particular values of the parameters a and b and hence is called a particular solution of the given differential equation.

64. (a) Another form of first order linear differential equation is
$$\frac{dx}{dy} + P_1 y = Q_1$$
where P_1 and Q_1 are constants or function of y only.
This type of differential equation are
$$\frac{dx}{dy} + x = \cos y$$
$$\frac{dx}{dy} + \frac{-2x}{y} = y^2 e^{-y}$$

65. (c) **Assertion:** $y^3\frac{dy}{dx} + (x + y^2) = 0$
$y^2 = t$
$$2y\frac{dy}{dx} = \frac{dt}{dx}$$
$\frac{1}{2}\frac{dt}{dx}.t + x + t = 0$ is homogeneous equation.
Reason is obviously false.

66. (c) **67. (a)**

68. (d) The given differential equaiton is not a polynomial equation in its derivatives so its degree is not defined.

69. (a)

70. (a) Here $P = x^2$ and $Q = 5$. P is a function of x only and Q is a constant.

CRITICALTHINKING TYPE QUESTIONS

71. (c) $y = x \Rightarrow \frac{dy}{dx} = 1, \frac{d^2y}{dx^2} = 0$

Now $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$

72. (b) Let y denote the number of bacteria at any instant t · then according to the question
$$\frac{dy}{dt} \alpha y \Rightarrow \frac{dy}{y} = k\,dt \quad \text{... (i)}$$
k is the constant of proportionality, taken to be + ve on integrating (i), we get
$$\log y = kt + c \quad \text{... (ii)}$$
c is a parameter. let y_0 be the initial number of bacteria i.e., at t = 0 using this in (ii), $c = \log y_0$
$\Rightarrow \log y = kt + \log y_0$
$$\Rightarrow \log\frac{y}{y_0} = kt \quad \text{...(iii)}$$
$$y = \left(y_0 + \frac{10}{100}y_0\right) = \frac{11y_0}{10}, \text{ when } t = 2$$
So, from (iii), we get $\log\frac{\frac{11y_0}{10}}{y_0} = k\ (2)$
$$\Rightarrow k = \frac{1}{2}\log\frac{11}{10} \quad \text{... (iv)}$$
Using (iv) in (iii) $\log\frac{y}{y_0} = \frac{1}{2}\left(\log\frac{11}{10}\right)t$ ... (v)
let the number of bacteria become 1, 00, 000 to 2,00,000 in t_1 hours. i.e., $y = 2y_0$ when $t = t_1$ hours. from (v)
$$\log\frac{2y_0}{y_0} = \frac{1}{2}\left(\log\frac{11}{10}\right)t_1 \Rightarrow t_1 = \frac{2\log 2}{\log\frac{11}{10}}$$
Hence, the reqd. no. of hours $= \frac{2\log 2}{\log\frac{11}{10}}$

73. (a) The given differential equation is
$\sin x \cos y\, dx + \cos x \sin y\, dy = 0$
dividing by $\cos x \cos y \Rightarrow \frac{\sin x}{\cos x}dx + \frac{\sin y}{\cos y}dy = 0$
Integrating, $\int \tan x\, dx + \int \tan y\, dy = \log c$
or $\log \sec x \sec y = \log c$ or $\sec x \sec y = c$
curve passes through the point $\left(0, \frac{\pi}{4}\right)$
$$\sec 0 \sec\frac{\pi}{4} = c = \sqrt{2}$$
Hence, the reqd. equ. of the curve is $\sec x \sec y = \sqrt{2}$

74. (b) Let y be the population at an instant t. Now population increase at a rate α no. of inhabitants
$$\therefore \frac{dy}{dt} \alpha\, y \text{ or } \frac{dy}{dt} = ky$$
$$\therefore \frac{dy}{y} = kdt \text{ Integrating } \int\frac{dy}{y} = \int kdt + c$$
or $\log y = kt + c$ ...(i)
In 1999, t = 0, population = 20,000
$\therefore$ log 20,000 = c Put the value of c in (i)

log y = kt + log 20,000 or log y – log 20000 = kt

or $\log \dfrac{y}{20000} = kt$ ...(ii)

In 2004, t = 5, y = 25000

$\log \dfrac{25000}{20000} = k \times 5 \Rightarrow k = \dfrac{1}{5} \log \dfrac{5}{4}$

Equ (ii) as $\log \dfrac{y}{20000} = \left(\dfrac{1}{5} \log \dfrac{5}{4}\right) t$

In 2009, t = 10

$\Rightarrow \log \dfrac{y}{20000} = \left(\dfrac{1}{5} \log \dfrac{5}{4}\right) \times 10 = 2 \log \dfrac{5}{4}$

$\Rightarrow \log \left(\dfrac{5}{4}\right)^2 = \log \dfrac{25}{16} \Rightarrow \dfrac{y}{20000} = \dfrac{25}{16}$

$\Rightarrow y = \dfrac{25}{16} \times 20000 = 25 \times 1250 = 31250$

75. **(a)** $(1 + \log x)\dfrac{dx}{dy} - x \log x = e^y$

putting $x \log x = t \Rightarrow (1 + \log x)\, dx = dt$

$\therefore \dfrac{dt}{dy} - t = e^y$

Now, I.F. $= e^{\int -1 dy} = e^{-y}$

$\Rightarrow te^{-y} = \int e^{-y} e^y dy + C$

$\Rightarrow t = Ce^y + ye^y$

$\Rightarrow x \log x = (C + y)\, e^y$,

Since, y(1) = 0, then

$0 = (C + 0)\, 1 \Rightarrow C = 0$

$\therefore ye^y = x \log x$

$\Rightarrow x^x = e^{ye^y}$

76. **(b)** We have,

$\dfrac{dy}{dx} = \dfrac{ax+3}{2y+f} \Rightarrow (ax + 3)\, dx = (2y + f)\, dy$

$\Rightarrow a\dfrac{x^2}{2} + 3x = y^2 + fy + C$ (Integrating)

$\Rightarrow -\dfrac{a}{2}x^2 + y^2 - 3x + fy + C = 0$

This will represent a circle, if

$-\dfrac{a}{2} = 1$ [$\because$ Coeff. of x^2 = Coeff. of y^2]

and, $\dfrac{9}{4} + \dfrac{f^2}{4} - C > 0$ [Using : $g^2 + f^2 - c > 0$]

$\Rightarrow a = -2$ and $9 + f^2 - 4C > 0$

77. **(c)** $\dfrac{dx}{dy} = \dfrac{\left(\dfrac{x}{y} - 1\right) e^{\frac{x}{y}}}{1 + e^{\frac{x}{y}}}$

Substitute $x = vy \Rightarrow \dfrac{dx}{dy} = \dfrac{ydv}{dy} + v$

Now given equation becomes

$\dfrac{ydv}{dy} + v = \dfrac{(v-1)e^v}{1+e^v}$

$\Rightarrow \dfrac{ydv}{dy} = \dfrac{(v-1)e^v}{1+e^v} - v = \dfrac{-(v+e^v)}{1+e^v}$

$\Rightarrow \dfrac{(1+e^v)dv}{v+e^v} + \dfrac{dy}{y} = 0$

$\Rightarrow \ln (v + e^v)\, y = \ln c \Rightarrow (v + e^v)y = c$

$\Rightarrow x + ye^{x/y} = c$

78. **(b)** Given $\dfrac{x + y\dfrac{dy}{dx}}{y - x\dfrac{dy}{dx}} = x^2 + 2y^2 + \dfrac{y^4}{x^2}$

$\Rightarrow \dfrac{d(x^2+y^2)}{(x^2+y^2)^2} = 2\dfrac{d\left(\dfrac{x}{y}\right)}{\left(\dfrac{x}{y}\right)^2}$

Integrating, we get

$-\dfrac{1}{x^2+y^2} = \dfrac{-1}{x/y} + c \Rightarrow c = \dfrac{y}{x} - \dfrac{1}{x^2+y^2}$

79. **(a)** The given differential equation can be written as $y_2/y_1 = 2x/(x^2 + 1)$.

Integrating both the sides we have

$\log y_1 = \log (x^2 + 1) + c$

which implies $\log y_1 (0) = \log 1 + c$, i.e., $c = \log 3$.

Therefore, $\log y_1 = \log (x^2 + 1) + \log 3$ which implies $y_1 = 3 (x^2 + 1)$ or $y = x^3 + 3x + A$,

so, $1 = y (0) = 0 + 0 + A$, i.e., A = 1.

Hence the required equation of curve is

$y = x^3 + 3x + 1$.

80. **(b)** By multiplying e^{-t} and rearranging the terms, we get

$e^{-t}(1+t)dy + y(e^{-t} - (1+t)e^{-t})dt = e^{-t}dt$

$\Rightarrow d(e^{-t}(1+t)y) = d(-e^{-t}) \Rightarrow ye^{-t}(1+t) = -e^{-t} + c.$

Also $y_0 = -1 \Rightarrow c = 0 \Rightarrow y(1) = -1/2$

81. **(a)** Let female-male ratio at any time be r

$\dfrac{dr}{dt} \propto r \Rightarrow \dfrac{dr}{dt} = -k\, r$

where k is the constant of proportionality and k > 0

We have $\dfrac{dr}{r} = -k\, dt$

Integrating both sides, we have

$\int \dfrac{dr}{r} = -k\int dt$

log r = –kt + log C

$\log r - \log C = -kt \Rightarrow \log\left(\dfrac{r}{C}\right) = -kt$

where log C is the constant of integration

$\Rightarrow r = Ce^{-kt}$ ...(i)

Let us start time from the year 2001,

So in 2001, $t = 0, r = \dfrac{980}{1000} = \dfrac{49}{50}$

Putting t = 0 in (i), we have

$\frac{49}{50} = C \Rightarrow r = \frac{49}{50}e^{-kt}$ ...(ii)

Also in the year 2011, t = 10 and

$r = \frac{920}{1000} = \frac{23}{25}$

Putting in (ii), we have

$\frac{23}{25} = \frac{49}{50}e^{-10k} \Rightarrow e^{10k} = \frac{49}{50} \times \frac{25}{23} = \frac{49}{46}$

or $e^{-10k} = \frac{46}{49}$

Hence, $r = \frac{49}{50}e^{-10k \times \frac{t}{10}} \Rightarrow r = \frac{49}{50}\left(\frac{46}{49}\right)^{\frac{t}{10}}$

In the year 2021, $t = 20 \therefore r = \frac{49}{50}\left(\frac{46}{49}\right)^{\frac{20}{10}}$

$= \frac{49}{50} \times \frac{46}{49} \times \frac{46}{49} = 0.864$

Thus, at this trend female : male $\simeq$ 864 : 1000

82. (c) Given equation can be written as

$xdy = \left(\sqrt{x^2 + y^2} + y\right)dx$, i.e.,

$\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x}$ …(i)

Substituting y = vx, we get from (i)

$v + x\frac{dv}{dx} = \frac{\sqrt{x^2 + v^2 + x^2} + vx}{x}$

$v + x\frac{dv}{dx} = \sqrt{1 + v^2} + v$

$x\frac{dv}{dx} = \sqrt{1 + v^2} \Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$ ....(ii)

Intergrating both sides of (ii), we get

$\log\ (v + \sqrt{1 + v^2}) = \log x + \log c$

$\Rightarrow v + \sqrt{1 + v^2} = cx \Rightarrow \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = cx$

$\Rightarrow y + \sqrt{x^2 + y^2} = cx^2$

83. (d) $\frac{d^4y}{dx^4} + \sin(y''') = 0$

$\Rightarrow y'''' + \sin(y''') = 0$

The highest order derivative which occurs in the given differential equation is y'''', therefore its order is 4. As the given differential equation is not a polynomial equation in derivatives of y w.r.t. x (i.e., y'''), therefore its degree is not defined.

84. (c) The given differential equation is

$e^x\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 3$

Since, this differential equation is not a polynomial in terms of its derivatives.

$\therefore$ Its degree is not defined

85. (a) Given, $y = \left(x + \sqrt{1 + x^2}\right)^n$ …(i)

$\Rightarrow \frac{dy}{dx} = n\left[x + \sqrt{1 + x^2}\right]^{n-1}\left(1 + \frac{x}{\sqrt{x^2 + 1}}\right)$

$= \frac{n\left[x + \sqrt{1 + x^2}\right]^n}{\sqrt{1 + x^2}}$

$\Rightarrow \left(\frac{dy}{dx}\right)^2(1 + x^2) = n^2y^2$ [using eq. (i) and squaring]

Again, differentiating, we get

$2\frac{dy}{dx}.\frac{d^2y}{dx^2}(1 + x^2) + 2x\left(\frac{dy}{dx}\right)^2 = 2n^2y\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2}(1 + x^2) + x\frac{dy}{dx} = n^2y \left(\text{divide by } 2\frac{dy}{dx}\right)$

86. (b) The given differential equation can be expressed as

$dy = \frac{2x^2 + 1}{x}dx$

or $dy = \left(2x + \frac{1}{x}\right)dx$ … (i)

On integrating both sides of eq. (i), we get

$\int dy = \left(2x + \frac{1}{x}\right)dx$

$\Rightarrow y = x^2 + \log|x| + C$ … (ii)

Eq. (ii) represents the family of solution curves of the given differential equation but we are interested in finding the equation of a particular member of the family which passes through the point (1, 1). Therefore, substituting x = 1, y = 1 in eq. (ii), we get C = 0

87. (d) It is given that (x, y) is the point of contact of the curve and its tangent.

The slope of the line segment joining the points. $(x_2, y_2) \to (x, y)$ and $(x_1, y_1) \to (-4, -3)$

$= \frac{y - (-3)}{x - (-4)} = \frac{y + 3}{x + 4} \quad \left(\because \text{slope of a tangent} = \frac{y_2 - y_1}{x_2 - x_1}\right)$

According to the question, (slope of tangent is twice the slope of the line), we must have

$\frac{dy}{dx} = 2\left(\frac{y + 3}{x + 4}\right)$

Now, separating the variable, we get

$\frac{dy}{y + 3} = \left(\frac{2}{x + 4}\right)dx$

On integrating both sides, we get

$\int\frac{dy}{y + 3} = \int\left(\frac{2}{x + 4}\right)dx$

$\Rightarrow \log|y + 3| = 2\log|x + 4| + \log|C|$

$\Rightarrow \log|y + 3| = \log|x + 4|^2 + \log|C|$

$\Rightarrow \log\frac{|y + 3|}{|x + 4|^2} = \log|C| \left(\because \log m - \log n = \log\frac{m}{n}\right)$

$\Rightarrow \frac{|y + 3|}{|x + 4|^2} = C$ …(i)

The curve passes through the point (–2, 1) therefore

$$\frac{|1+3|}{|-2+4|^2} = C \Rightarrow C = 1$$

On substituting C = 1 in eq. (i), we get

$$\frac{|y+3|}{|x+4|^2} = 1$$

$\Rightarrow |y + 3| = (x + 4)^2$

Which is the required equation of curve

88. (c) Let P be the principal at any time T.

According to the given problem,

$$\frac{dP}{dt} = \left(\frac{5}{100}\right) \times P$$

$$\Rightarrow \frac{dP}{dt} = \frac{P}{20} \quad \ldots (i)$$

On separating the variables in eq. (i), we get

$$\frac{dP}{P} = \frac{dt}{20} \quad \ldots (ii)$$

On integrating both sides of eq.(ii), we get

$$\log P = \frac{t}{20} + C_1$$

$$\Rightarrow P = e^{\frac{t}{20}} . e^{C_1}$$

$$\Rightarrow P = Ce^{\frac{t}{20}} \left(\text{where, } e^{C_1} = C\right) \quad \ldots (iii)$$

Now, P = 1000, when t = 0

On substituting the values of P and t in eq. (iii), we get C = 1000. Therefore, eq. (iii), gives

$$P = 1000e^{\frac{t}{20}}$$

Let t years be the time required to double the principal. Then,

$$2000 = 1000e^{\frac{t}{20}}$$

$\Rightarrow t = 20 \log_e 2$

89. (d) Here, the slope of the tangent to the curve at any point (x, y) is $\frac{y-1}{x^2+x}$.

$$\therefore \frac{dy}{dx} = \frac{y-1}{x^2+x}$$

$$\Rightarrow \frac{dy}{y-1} = \frac{dx}{x^2+x}$$

On integrating both sides, we get

$$\int \frac{dy}{y-1} = \int \frac{dx}{x(x-1)}$$

$\Rightarrow \log (y – 1) = \log x – \log (x + 1) + \log C$

$$\log (y-1) = \log\left(\frac{xC}{x+1}\right)$$

$\Rightarrow (y – 1) (x + 1) = xC$

Since, the above curve passes through (1, 0)

$\Rightarrow (–1) (2) = 1.C$

$\Rightarrow C = –2$

$\therefore$ Required equation of the curve is

$(y – 1) (x + 1) + 2x = 0$

90. (b) To solve a homogenous differential equation of the type

$$\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right) \quad \ldots (i)$$

We make the substitution

$$y = v . x \quad \ldots (ii)$$

On differntiating eq. (ii) w.r.t. x, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \ldots (iii)$$

On substituting the value of $\frac{dy}{dx}$ from eq. (iii) in eq. (i), we get

$$v + x\frac{dv}{dx} = g(v)$$

$$\text{or} \quad x\frac{dv}{dx} = g(v) - v \quad \ldots (iv)$$

On separating the variables in eq. (iv) we get

$$\frac{dv}{g(v)-v} = \frac{dx}{x} \quad \ldots (v)$$

On integrating both sides of Eq. (v), we get

$$\int \frac{dv}{g(v)-v} = \int \frac{1}{x}dx + C \quad \ldots (vi)$$

eq (vi) gives general solution (primitive) of the differential eq. (i) when we replace v by $\frac{v}{x}$.

91. (c) The given differential equation is

dx + dy = (x + y) (dx – dy)

$$\Rightarrow \frac{dy}{dx} = \frac{x+y-1}{x+y+1} \quad \ldots (i)$$

Put x + y = t

$$\Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$$

So, from equation (i), we have

$$\frac{dt}{dx} - 1 = \frac{t-1}{t+1} \Rightarrow \frac{dt}{dx} = \frac{t-1}{t+1} + 1$$

$$\Rightarrow \frac{dt}{dx} = \frac{t-1+t+1}{t+1} \Rightarrow \frac{1}{2}\left(1+\frac{1}{t}\right)dt = dx$$

On integrating both sides, we get

$$\int \frac{1}{2}\left(1+\frac{1}{t}\right)dt = \int dx \Rightarrow \frac{1}{2}.(t + \log t) = x + \frac{C}{2}$$

$\Rightarrow t + \log t = 2x + C$

$\Rightarrow \log (x + y) = x – y + C$

92. (a) According to the condition,

$$\frac{dy}{dx}=\frac{y}{x}-\cos^2\frac{y}{x} \quad \ldots(i)$$

This is a homogeneous differential equation
Substituting y = vx, we get

$$v+x\frac{dv}{dx}=v-\cos^2 v$$

$$\Rightarrow x\frac{dv}{dx}=-\cos^2 v$$

$$\Rightarrow \int \sec^2 v\, dv=-\int\frac{dx}{x}$$

$$\Rightarrow \tan v=-\log x+C$$

$$\Rightarrow \tan\frac{y}{x}+\log x=C \quad \ldots(ii)$$

Substituting $x = 1, y=\frac{\pi}{4}$, we get C = 1. Thus, we get

$$\tan\left(\frac{y}{x}\right)+\log x=1$$

which is the required solution

93. (b) The given differential equation can be written as

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2} \quad \ldots(i)$$

Now, eq. (i) is a linear differential equation of the form $\frac{dx}{dy}+P_1 x=Q_1$

where $P_1=\frac{1}{1+y^2}$ and $Q_1=\frac{\tan^{-1}y}{1+y^2}$

Therefore, $I.F=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$

Thus, the solution of the given differential equation is given by

$$x\,e^{\tan^{-1}y}=\int\left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy+C \quad \ldots(ii)$$

Let $I=\int\left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy$

On substituting $\tan^{-1}y=t$, so that $\left(\frac{1}{1+y^2}\right)dy=dt$,

we get

$$I=\int te^t dt=te^t-\int 1.e^t dt$$

$$=te^t-e^t=e^t(t-1)$$

or $I=e^{\tan^{-1}y}\left(\tan^{-1}y-1\right)$

On substituting the value of I in equation (ii), we get

$$x.e^{\tan^{-1}y}=.e^{\tan^{-1}y}\left(\tan^{-1}y-1\right)+C$$

or $x=(\tan^{-1}y-1)+Ce^{\tan^{-1}y}$

which is the general solution of the given differntial equation.

94. (a) The given differential equation is

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1 \quad \ldots(i)$$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}=\frac{y}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{x}}y=\frac{e^{-2\sqrt{x}}}{\sqrt{x}} \quad \ldots(ii)$$

On comparing with the form $\frac{dy}{dx}+Py=Q$, we get

$$P=\frac{1}{\sqrt{x}} \text{ and } Q=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$\therefore$ $I.F=e^{\int\frac{1}{\sqrt{x}}dx} \Rightarrow I.F=e^{2\sqrt{x}}$

The general solution of the given differential equation is given by

$$y.I.F=\int(Q\times I.F)\,dx+C$$

$$\Rightarrow ye^{2\sqrt{x}}=\int e^{2\sqrt{x}}\times\frac{e^{-2\sqrt{x}}}{\sqrt{x}}dx+C$$

$$\Rightarrow ye^{2\sqrt{x}}=\int\frac{1}{\sqrt{x}}dx+C$$

$$\Rightarrow ye^{2\sqrt{x}}=2\sqrt{x}+C$$

95. (d) Given: xdy – ydx = 0
Dividing by xy on both sides,
we get:

$$\frac{dy}{y}-\frac{dx}{x}=0$$

$$\Rightarrow \frac{dy}{y}=\frac{dx}{x}$$

By integrating on both sides, we get,
log y = log x + log c

$$\Rightarrow \log\frac{y}{x}=\log c \Rightarrow y=cx \text{ or } y-cx=0$$

which represents a straight line passing through origin.

96. (a) Differential equation is given by

$$\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\rho\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

order of a differential equation is the order of the highest derivative appearing in the equation. Hence the order is 2.

To find the degree of the differential equation, it has to be expressed as a polynomial in derivatives. For this we square both the sides of differential eq^n.

$$\rho\left(\frac{d^2y}{dx^2}\right)^2=\left[1+\left(\frac{dy}{dx}\right)^2\right]^3$$

Here power of highest derivative is 2, hence order is 2 and degree is also 2.

97. (b) Put $\frac{y}{x}=u$ we have $\frac{dy}{dx}=u+x\frac{du}{dx}$

$$u+x\frac{du}{dx}=u+\frac{\phi(u)}{\phi'(u)}\Rightarrow x\frac{dy}{dx}=\frac{\phi(u)}{\phi'(u)}$$

$\Rightarrow \frac{\phi'(u)}{\phi(u)}du=\frac{dx}{x}$ and Integrate it

Required solution is $\phi\left(\frac{y}{x}\right)=kx$

98. (d) Given $\frac{x\,dx}{1+x^2}=\frac{y\,dy}{1+y^2}$

Integrating we get,

$$\frac{1}{2}\log(1+x^2)=\frac{1}{2}\log(1+y^2)+a$$

$\Rightarrow 1+x^2=c(1+y^2)$,

Where $c=e^{2a}$

$$x^2-cy^2=c-1\Rightarrow\frac{x^2}{c-1}-\frac{y^2}{\left(\frac{c-1}{c}\right)}=1 \quad \ldots (i)$$

Clearly $c>0$ as $c=e^{2a}$

Hence, the equation (i) gives a family of hyperbolas with eccentricity

$$=\sqrt{\frac{c-1+\frac{c-1}{c}}{c-1}}=\sqrt{\frac{c^2-1}{c-1}}=\sqrt{c+1}\ \text{if}\ c\neq 1$$

Thus ecentricity varies from member to member of the family as it depends on c.

99. (a) Divide the equation by y^2, we get

$$\frac{ydx-xdy}{y^2}=-3x^2e^{x^3}dx\Rightarrow\frac{d}{dx}\left(\frac{x}{y}\right)=-\frac{d}{dx}\left(e^{x^3}\right)$$

On integrating we get,

$$\frac{x}{y}=-e^{x^3}+c\Rightarrow\frac{x}{y}+e^{x^3}=c$$

100. (b) Rewriting the given equation in the form

$$x^4\cos y\frac{dy}{dx}+4x^3\sin y=xe^x\Rightarrow\frac{d}{dx}(x^4\sin y)=xe^x$$

$$\Rightarrow x^4\sin y=\int xe^x dx+c=(x-1)e^x+c$$

Since, $y(1)=0$ so, $c=0$.

Hence, $\sin y=x^{-4}(x-1)e^x$

101. (c) Any conic whose axes coincide with co-ordinate axis is $ax^2+by^2=1$ ..(i)

Diff. both sides w.r.t. 'x', we get

$$2ax+2by\frac{dy}{dx}=0 \text{ i.e. } ax+by\frac{dy}{dx}=0 \quad \ldots (ii)$$

$$\text{Diff. again, } a+b\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)=0 \quad \ldots (iii)$$

From (ii), $\frac{a}{b}=-\frac{ydy/dx}{x}$

From (iii), $\frac{a}{b}=-\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)$

$$\therefore\ \frac{y\frac{dy}{dx}}{x}=y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$

102. (c) Rewrite the given differential equation as follows :

$\frac{dy}{dx}+\frac{2x}{x^2-1}y=\frac{1}{x^2-1}$, which is a linear form

The integrating factor

$$\text{I.F.}=e^{\int\frac{2x}{x^2-1}dx}=e^{\ell n(x^2-1)}=x^2-1$$

Thus multiplying the given equation by (x^2-1), we get

$$(x^2-1)\frac{dy}{dx}+2xy=1\Rightarrow\frac{d}{dx}[y(x^2-1)]=1$$

On integrating we get $y(x^2-1)=x+c$

103. (b) The given equation can be converted to linear form by dividing both the sides by $\cos^2 y$. We get

$$\sec^2 y\frac{dy}{dx}+\frac{1}{x}2\tan y=x^3\ ;$$

Put $\tan y=z\Rightarrow\sec^2 y\frac{dy}{dx}=\frac{dz}{dx}$

The equation becomes $\frac{dz}{dx}+\frac{2}{x}z=x^3$, which is linear in z

The integrating factor is

$$\text{I.F.}=e^{\int\frac{2}{x}dx}=e^{2\log x}=e^{\log x^2}=x^2$$

Hence, the solution is

$$z(x^2)=\int x^3(x^2)dx+a\Rightarrow zx^2=\frac{x^6}{6}+a,$$

a is constant of integration.

$$\therefore (6\tan y)x^2=x^6+6a\Rightarrow 6x^2\tan y=x^6+c,\quad [c=6a]$$

104. (d) The retardation at time $t=-\frac{dv}{dt}$. Hence, the differential equation is $-\frac{dv}{dt}=v\Rightarrow\frac{dv}{v}=-dt$ ...(i)

Integrating, we get $\log v=-t+c$ ...(ii)

When $t=0$, $v=V\Rightarrow C=\log V$

The equation (ii) becomes $\log v=-t+\log V$

$$\Rightarrow \log\frac{v}{V}=-t\Rightarrow\frac{v}{V}=e^{-t}\Rightarrow v=Ve^{-t}$$