19

Set Theory

Set Theory is an important concept of mathematics which is often asked in aptitude exams. There are two types of questions in this chapter:

(i) Numerical questions on set theory based on venn diagrams

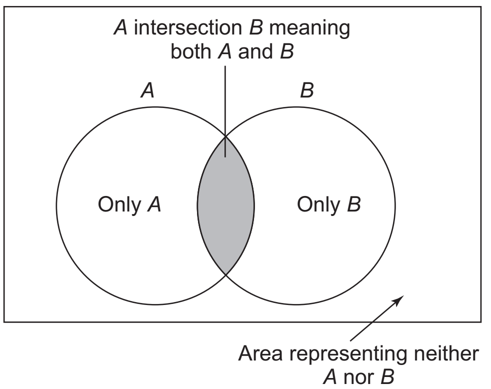
(ii) Logical questions based on set theory

Let us first take a look at some standard theoretical inputs related to set theory.

SET THEORY

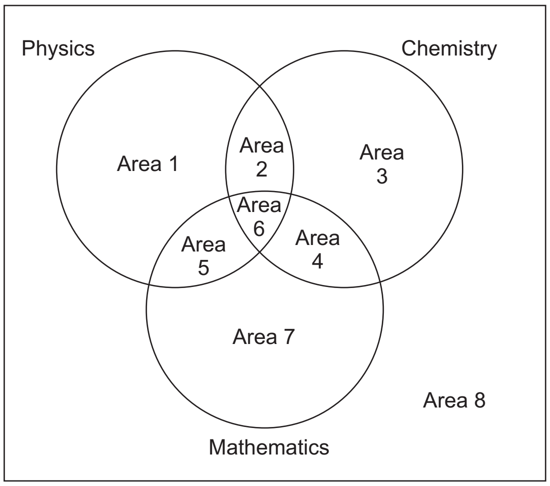
Look at the following diagrams:

Figure 1: Refers to the situation where there are two attributes *A* and *B*. (Let's say *A* refers to people who passed in Physics and *B* refers to people who passed in Chemistry.) Then the shaded area shows the people who passed both in Physics and Chemistry.



In mathematical terms, the situation is represented as:
Total number of people who passed at least 1 subject = $A + B - A \cap B$

Figure 2: Refers to the situation where there are three attributes being measured. In the figure below, we are talking about people who passed Physics, Chemistry and/or Mathematics.



In the above figure, the following explain the respective areas:

Area 1: People who passed in Physics only

Area 2: People who passed in Physics and Chemistry only (in other words—people who passed Physics and Chemistry but not Mathematics)

Area 3: People who passed Chemistry only

Area 4: People who passed Chemistry and Mathematics only (also, can be described as people who passed Chemistry and Mathematics but not Physics)

Area 5: People who passed Physics and Mathematics only (also, can be described as people who passed Physics and Mathematics but not Chemistry)

Area 6: People who passed Physics, Chemistry and Mathematics

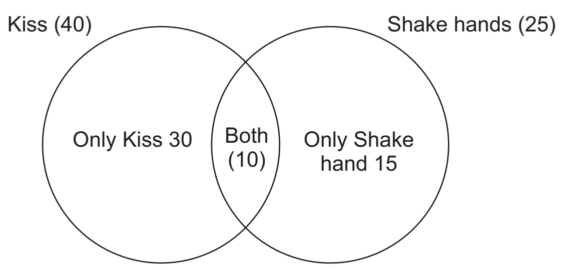
Area 7: People who passed Mathematics only
Area 8: People who passed in no subjects
Also take note of the following language which there is normally confusion about:
People passing Physics and Chemistry—Represented by the sum of areas 2 and 6
People passing Physics and Maths—Represented by the sum of areas 5 and 6
People passing Chemistry and Maths—Represented by the sum of areas 4 and 6
People passing Physics—Represented by the sum of the areas 1, 2, 5 and 6
In mathematical terms, this means:
Total number of people who passed at least 1 subject =
$P + C + M - P \cap C - P \cap M - C \cap M + P \cap C \cap M$
Let us consider the following questions and see how these figures work in terms of real time problem solving:

Illustration 1

At the birthday party of Sherry, a baby boy, 40 persons chose to kiss him and 25 chose to shake hands with him. 10 persons chose to both kiss him and shake hands with him. How many persons turned out at the party?

(a) 35 (b) 75
(c) 55 (d) 25

Solution:



From the figure, it is clear that the number of people at the party were 30 + 10 + 15 = 55.
We can of course solve this mathematically as below:
Let n(A) = No. of persons who kissed Sherry = 40
n(B) = No. of persons who shake hands with Sherry = 25
and $n(A \cap B)$ = No. of persons who shook hands with Sherry and kissed him both = 10
Then using the formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \cup B) = 40 + 25 - 10 = 55$

Illustration 2

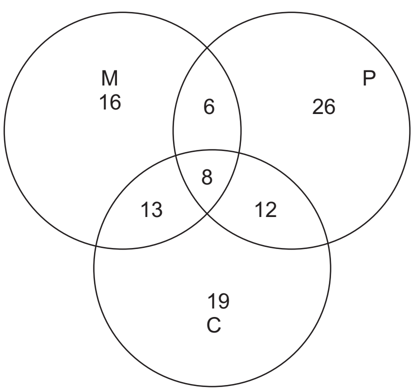
Directions for Questions 1 to 4: Refer to the data below and answer the questions that follow:

In an examination 43% passed in Math, 52% passed in Physics and 52% passed in Chemistry. Only 8% students passed in all the three. 14% passed in Math and Physics and 21% passed in Math and Chemistry and 20% passed in Physics and Chemistry. Number of students who took the exam is 200.

Let Set P, Set C and Set M denotes the students who passed in Physics, Chemistry and Math respectively. Then

1. How many students passed in Math only?
   (a) 16 (b) 32
   (c) 48 (d) 80
2. Find the ratio of students passing in Math only to the students passing in Chemistry only?
   (a) 16:37 (b) 29:32
   (c) 16:19 (d) 31:49
3. What is the ratio of the number of students passing in Physics only to the students passing in either Physics or Chemistry or both?
   (a) 34/46 (b) 26/84
   (c) 49/32 (d) None of these
4. A student is declared pass in the exam only if he/she clears at least two subjects. The number of students who were declared passed in this exam is?
   (a) 33 (b) 66
   (c) 39 (d) 78

Sol. Let P denote Physics, C denote Chemistry and M denote Maths.
% of students who passed in P and C only is given by % of students who passed in P and C - % of students who passed all three = 20% – 8% = 12%
% of students who passed in P and M only is given by % of students who passed in P and M - % of students who passed all three = 14% – 8% = 6%
% of students who passed in M and C only is:
% of students who passed in C and M - % of students who passed all three = 21% – 8% = 13%
So, % of students who passed in P only is given by:
Total no. passing in P – No. Passing in P & C only – No. Passing P & M only – No. Passing in all three→
52% – 12% – 6% – 8%- = 26%
% of students who passed in M only is:
Total no. passing in M – No. Passing in M & C only – No. Passing P & M only – No. Passing in all three→
43% – 13% – 6% – 8%- = 16%



% of students who passed in Chemistry only is
Total no. passing in C – No. Passing in P & C only – No. Passing C & M only – No. Passing in all three → 52% – 12% – 13% – 8%– = 19%

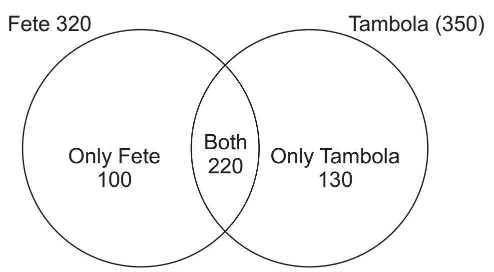
The answers are:

1. Only Math =16% = 32 people. Option (b) is correct.
2. Ratio of Only Math to Only Chemistry = 16:19. Option (c) is correct.
3. 26:84 is the required ratio. Option (b) is correct.
4. 39 % or 78 people. Option (d) is correct.

Illustration 3

In the Mindworkzz club all the members participate either in the Tambola or the Fete. 320 participate in the Fete, 350 participate in the Tambola and 220 participate in both. How many members does the club have?

(a) 410 (b) 550
(c) 440 (d) None of these

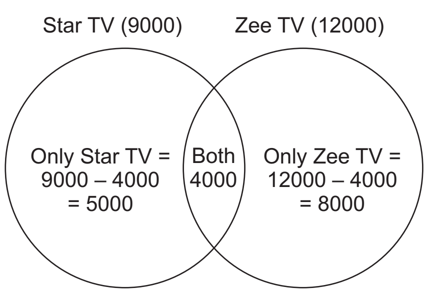


The total number of people = 100 + 220 + 130 = 450
Option (d) is correct.

Illustration 4

There are 20000 people living in Defence Colony, Gurgaon. Out of them 9000 subscribe to Star TV Network and 12000 to Zee TV Network. If 4000 subscribe to both, how many do not subscribe to any of the two?

(a) 3000 (b) 2000
(c) 1000 (d) 4000



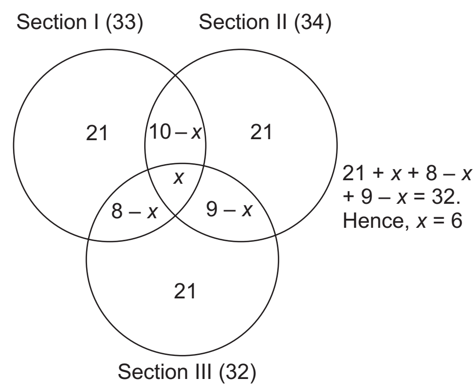
The required answer would be 20000 – 5000 – 4000 – 8000 = 3000.

Illustration 5

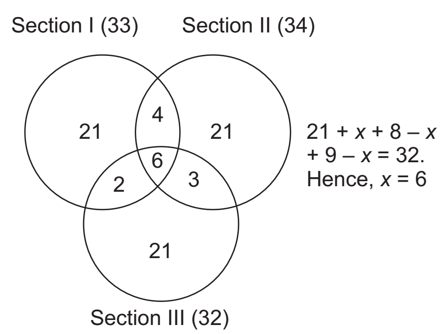
Directions for Questions 1 to 3: Refer to the data below and answer the questions that follow.

Last year, there were 3 sections in the Catalyst, a mock CAT paper. Out of them 33 students cleared the cut-off in Section 1, 34 students cleared the cut-off in Section 2 and 32 cleared the cut-off in Section 3. 10 students cleared the cut-off in Section 1 and Section 2, 9 cleared the cut-off in Section 2 and Section 3, 8 cleared the cut-off in Section 1 and Section 3. The number of people who cleared each section alone was equal and was 21 for each section.

1. How many cleared all the three sections?
   (a) 3 (b) 6
   (c) 5 (d) 7
2. How many cleared only one of the three sections?
   (a) 21 (b) 63
   (c) 42 (d) 52
3. The ratio of the number of students clearing the cut-off in one or more of the sections to the number of students clearing the cutoff in Section 1 alone is?
   (a) 78/21 (b) 3
   (c) 73/21 (d) None of these



Since, $x = 6$, the figure becomes:



The answers would be:

1. 6. Option (b) is correct.
2. 21+21+21=63. Option (b) is correct.
3. (21+21+21+6+4+3+2)/21 = 78/21. Option (a) is correct.

Illustration 6

In a locality having 1500 households, 1000 watch Zee TV, 300 watch NDTV and 750 watch Star Plus. Based on this information answer the questions that follow:

1. The minimum number of households watching Zee TV and Star Plus is:

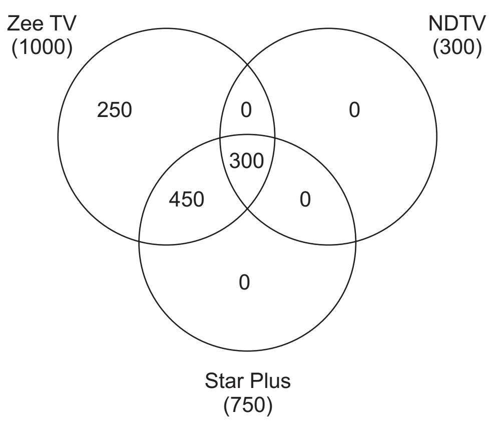
   Logic: If we try to consider each of the households watching Zee TV and Star Plus as independent of each other, we would get a total of 1000 + 750 = 1750 households. However, we have a total of only 1500 households in the locality and hence, there has to be a minimum interference of at least 250 households who would be watching both Zee TV and Star Plus. Hence, the answer to this question is 250.

2. The minimum number of households watching both Zee TV and NDTV is:

   In this case, the number of households watching Zee TV and NDTV can be separate from each other since there is no interference required between the households watching Zee TV and the households watching NDTV as their individual sum (1000 + 300) is smaller than the 1500 available households in the locality. Hence, the answer in this question is 0.

3. The maximum number of households who watch neither of the the three channels is:

   For this to occur the following situation would give us the required solution:



   As you can clearly see from the figure, all the requirements of each category of viewers is fulfilled by the given allocation of 1000 households. In this situation, the maximum number of households who do not watch any of the three channels is visible as 1500 –1000 = 500.

Illustration 7

1. In a school, 90% of the students faced problems in Mathematics, 80% of the students faced problems in Computers, 75% of the students faced problems in Sciences, and 70% of the students faced problems in Social Sciences. Find the minimum percent of the students who faced problems in all four subjects.

   Solution: In order to think about the minimum number of students who faced problems in all four subjects you would need to think of keeping the students who did not face a problem in any of the subjects separate from each other. We know that 30% of the students did not face problems in Social Sciences, 25% of the students did not face problems in Sciences, 20% students did not face problems in computers and 10% students did not face problems in Mathematics. If each of these were separate from each other, we would have 30+25+20+10=85% people who did not face a problem in one of the four subjects. Naturally, the remaining 15% would be students who faced problems in all four subjects. This represents the minimum percentage of students who faced problems in all the four subjects.

2. For the above question, find the maximum possible percentage of students who could have problems in all 4 subjects.

   In order to solve this, you need to consider the fact that 100 (%) people are counted 315 (%) times, which means that there is an extra count of 215 (%). When you put a student into the 'has problems in each of the four subjects' he is one student counted four times — an extra count of 3. Since, 215/3= 71 (quotient) we realise that if we have 71 students who have problems in all four subjects — we will have an extra count of 213 students. The remaining extra count of 2 more can be matched by putting 1 student in 'has problems in 3 subjects' or by putting 2 students in 'has problems in 2 subjects'. Thus, from the extra count angle, we have a limit of 71% students in the 'have problems in all four categories.'

   However, in this problem there is a constraint from another angle — i.e. there are only 70% students who have a problem in Social Sciences — and hence it is not possible for 71% students to have problems in all the four subjects. Hence, the maximum possible percentage of people who have a problem in all four subjects would be 70%.

3. In the above question if it is known that 10% of the students faced none of the above mentioned four problems, what would have been the minimum number of students who would have a problem in all four subjects?

   If there are 10% students who face none of the four problems, we realise that these 10% would be common to students who face no problems in Mathematics, students who face no problems in Sciences, students who face no problems in Computers and students who face no problems in Social Sciences.

   Now, we also know that overall there are 10% students who did not face a problem in Mathematics; 20%

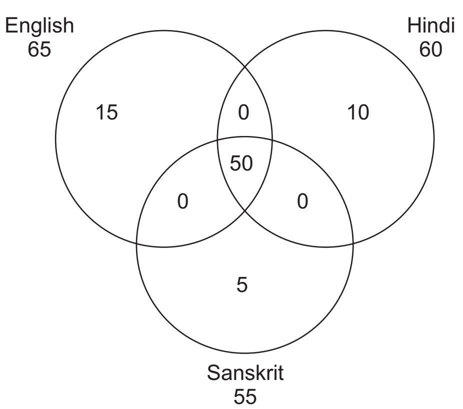
students who did not face a problem in computers; 25% students who did not face a problem in Sciences and 30% students who did not face a problem in Social Sciences. The 10% students who did not face a problem in any of the subjects would be common to each of these 4 counts. Out of the remaining 90% students, if we want to identify the minimum number of students who had a problem in all four subjects we will take the same approach as we took in the first question of this set — i.e. we try to keep the students having problems in the individual subjects separate from each other. This would result in: 0% additional students having no problem in Mathematics; 10% additional students having no problem in Computers; 15% additional students having no problem in Sciences and 20% additional students having no problem in Social Sciences. Thus, we would get a total of 45% (0+10+15+20=45) students who would have no problem in one of the four subjects. Thus, the minimum percentage of students who had a problem in all four subjects would be 90 – 45 = 45%.

Illustration 8

In a class of 80 students, each of them studies at least one language—English, Hindi and Sanskrit. It was found that 65 studied English, 60 studied Hindi and 55 studied Sanskrit.

1. Find the maximum number of people who study all three languages.

   This question again has to be dealt with from the perspective of extra counting. In this question, 80 students in the class are counted 65 + 60 + 55 = 180 times — an extra count of 100. If we put 50 people in the all three categories as shown below, we would get the maximum number of students who study all three languages.

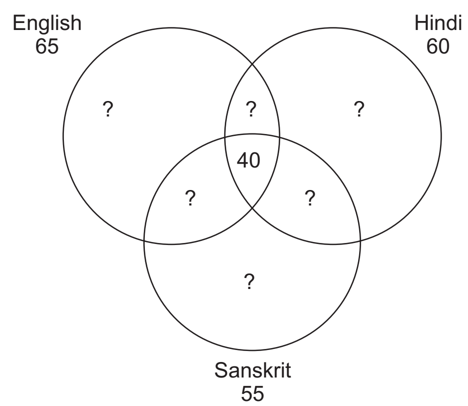


2. Find the minimum number of people who study all three languages.

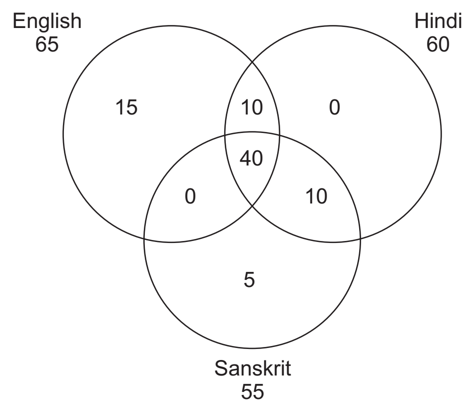
   In order to think about how many students are necessarily in the 'study all three languages' area of the figure (this thinking would lead us to the answer to the minimum number of people who study all three languages) we need to think about how many people we can shift out of the 'study all three category' for the previous question. When we try to do that, the following thought process emerges:

   Step 1: Let's take a random value for the all three categories (less than 50 of course) and see whether the numbers can be achieved. For this purpose we try to start with the value as 40 and see what happens. Before we move on, realise the basic situation in the question remains the same — 80 students have been counted 180 times — which means that there is an extra count of 100 students & also realise that when you put an individual student in the all three categories, you get an extra count of 2, while at the same time when you put an individual student into the 'exactly two languages category', he/she is counted twice — hence an extra count of 1.

   The starting figure we get looks something like this:



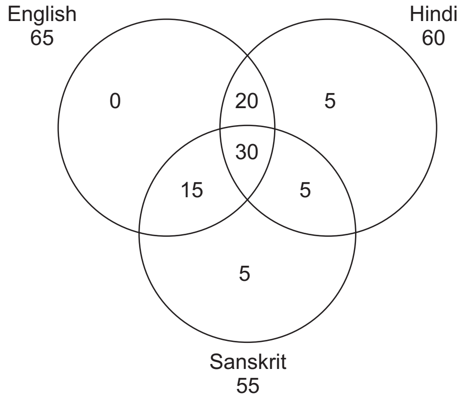
At this point, since we have placed 40 people in the all three categories, we have taken care of an extra count of $40 \times 2 = 80$. This leaves us with an extra count of 20 more to manage and as we can see in the above figure we have a lot of what can be described as 'slack' to achieve the required numbers. For instance, one solution we can think of from this point is as below:



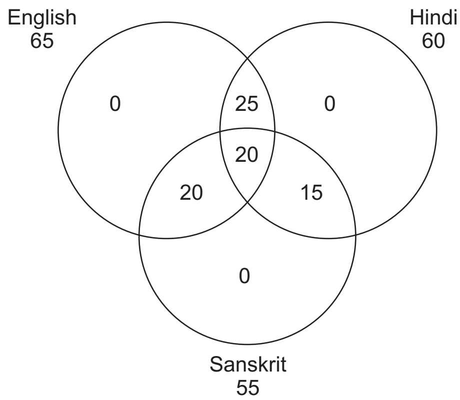
One look at this figure should tell you that the solution can be further optimised by reducing the middle value in the figure since there is still a lot of ‘slack’ in the figure — in the form of the number of students in the ‘exactly one language category’. Also, you can easily see that there are many ways in which this solution could have been achieved with 40 in the middle. Hence, we go in search of a lower value in the middle.

So, we try to take an arbitrary value of 30 to see whether this is still achievable.

In this case we see the following as one of the possible ways to achieve this (again there is a lot of slack in this solution as the ‘only Hindi’ or the ‘only Sanskrit’ areas can be reallocated):



Trying the same solution for 20 in the middle we get the optimum solution:



We realise that this is the optimum solution since there is no ‘slack’ in this solution and hence, there is no scope for re-allocating numbers from one area to another.

Author’s note: You might be justifiably thinking how do you do this kind of a random trial and error inside the exam? That’s not the point of this question at this place. What I am trying to convey to you is that this is a critical thought structure which you need to have in your mind. Learn it here and do not worry about how you would think inside the exam — remember you would need to check only the four options to choose the best one. We are talking about a multiple choice test here.

Illustration 9

In a group of 120 athletes, the number of athletes who can play Tennis, Badminton, Squash and Table Tennis is 70, 50, 60 and 30 respectively. What is the maximum number of athletes who can play none of the games?

In order to think of the maximum number of athletes who can play none of the games, we can think of the fact that since there are 70 athletes who play tennis, fundamentally there are a maximum of 50 athletes who would be possibly in the ‘can play none of the games’. No other constraint in the problem necessitates a reduction of this number and hence the answer to this question is 50.

LEVEL OF DIFFICULTY (I)

Directions for Questions 1 and 2: Refer to the data below and answer the questions that follow:

In the Indian athletic squad sent to the Olympics, 21 athletes were in the triathlon team; 26 were in the pentathlon team; and 29 were in the marathon team. 14 athletes can take part in triathlon and pentathlon; 12 can take part in marathon and triathlon; 15 can take part in pentathlon and marathon; and 8 can take part in all the three games.

1. How many players are there in all?
   (a) 35 (b) 43
   (c) 49 (d) none of these
2. How many were in the marathon team only?
   (a) 10 (b) 14
   (c) 18 (d) 15

Directions for Questions 3 and 4: Refer to the data below and answer the questions that follow.

In a test in which 120 students appeared, 90 passed in History, 65 passed in Sociology and 75 passed in Political Science. 30 students passed in only one subject and 55 students in only two. 5 students passed no subjects.

3. How many students passed in all the three subjects?
   (a) 25 (b) 30
   (c) 35 (d) Data insufficient
4. Find the number of students who passed in at least two subjects.
   (a) 85 (b) 95
   (c) 90 (d) Data insufficient

Directions for Questions 5 to 8: Refer to the data below and answer the questions that follow.

5% of the passengers who boarded Guwahati- New Delhi Rajdhani Express on 20th February, 2002 do not like coffee, tea and ice cream and 10% like all the three. 20% like coffee and tea, 25% like ice cream and coffee and 25% like ice cream and tea. 55% like coffee, 50% like tea and 50 % like ice cream.

5. The number of passengers who like only coffee is greater than the passengers who like only ice cream by
   (a) 50% (b) 100%
   (c) 25% (d) 0
6. The percentage of passengers who like both tea and ice cream but not coffee is
   (a) 15 (b) 5
   (c) 10 (d) 25
7. The percentage of passengers who like at least 2 of the 3 products is
   (a) 40 (b) 45
   (c) 50 (d) 60
8. If the number of passengers is 180, then the number of passengers who like ice cream only is
   (a) 10 (b) 18
   (c) 27 (d) 36

Directions for Questions 9 to 15: Refer to the data below and answer the questions that follow.

In a survey among students at all the IIMs, it was found that 48% preferred coffee, 54% liked tea and 64% smoked. Of the total, 28% liked coffee and tea, 32% smoked and drank tea and 30% smoked and drank coffee. Only 6% did none of these. If the total number of students is 2000 then find

9. The ratio of the number of students who like only coffee to the number who like only tea is
   (a) 5:3 (b) 8:9
   (c) 2:3 (d) 3:2
10. Number of students who like coffee and smoking but not tea is
    (a) 600 (b) 240
    (c) 280 (d) 360
11. The percentage of those who like coffee or tea but not smoking among those who like at least one of these is
    (a) more than 30 (b) less than 30
    (c) less than 25 (d) None of these
12. The percentage of those who like at least one of these is
    (a) 100 (b) 90
    (c) Nil (d) 94
13. The two items having the ratio 1:2 are
    (a) Tea only and tea and smoking only.
    (b) Coffee and smoking only and tea only.
    (c) Coffee and tea but not smoking and smoking but not coffee and tea.
    (d) None of these
14. The number of persons who like coffee and smoking only and the number who like tea only bear a ratio
    (a) 1:2 (b) 1:1
    (c) 5:1 (d) 2:1
15. Percentage of those who like tea and smoking but not coffee is
    (a) 14 (b) 14.9
    (c) less than 14 (d) more than 15
16. 30 monkeys went to a picnic. 25 monkeys chose to irritate cows while 20 chose to irritate buffaloes. How many chose to irritate both buffaloes and cows?
    (a) 10 (b) 15
    (c) 5 (d) 20

Directions for Questions 17 to 20: Refer to the data below and answer the questions that follow.

In the CBSE Board Exams last year, 53% passed in Biology, 61% passed in English, 60% in Social Studies, 24% in Biology & English, 35% in English & Social

Studies, 27% in Biology and Social Studies and 5% in none.

17. Percentage of passes in all subjects is
    (a) Nil (b) 12
    (c) 7 (d) 10
18. If the number of students in the class is 200, how many passed in only one subject?
    (a) 48 (b) 46
    (c) more than 50 (d) less than 40
19. If the number of students in the class is 300, what will be the % change in the number of passes in only two subjects, if the original number of students is 200?
    (a) more than 50% (b) less than 50%
    (c) 50% (d) None of these
20. What is the ratio of percentage of passes in Biology and Social Studies but not English in relation to the percentage of passes in Social Studies and English but not Biology?
    (a) 5:7 (b) 7:5
    (c) 4:5 (d) None of these

Directions for Questions 21 to 25: Refer to the data below and answer the questions that follow.

In the McGraw-Hill Mindworkzz Quiz held last year, participants were free to choose their respective areas from which they were asked questions. Out of 880 participants, 224 chose Mythology, 240 chose Science and 336 chose Sports, 64 chose both Sports and Science, 80 chose Mythology and Sports, 40 chose Mythology and Science and 24 chose all the three areas.

21. The percentage of participants who did not choose any area is
    (a) 23.59% (b) 30.25%
    (c) 37.46% (d) 27.27%
22. Of those participating, the percentage who choose only one area is
    (a) 60% (b) more than 60%
    (c) less than 60% (d) more than 75%
23. Number of participants who chose at least two areas is
    (a) 112 (b) 24
    (c) 136 (d) None of these
24. Which of the following areas shows a ratio of 1:8?
    (a) Mythology & Science but not Sports: Mythology only
    (b) Mythology & Sports but not Science: Science only
    (c) Science: Sports
    (d) None of these
25. The ratio of students choosing Sports & Science but not Mythology to Science but not Mythology & Sports is
    (a) 2:5 (b) 1:4
    (c) 1:5 (d) 1:2

Directions for Questions 26 to 30: Refer to the data below and answer the questions that follow.

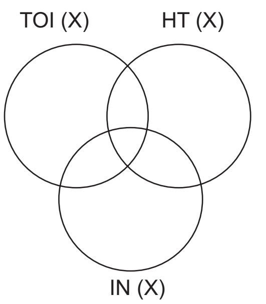
The table here gives the distribution of students according to professional courses.

Courses	*STUDENTS*			
	English		*Maths*	
	MALES	*FEMALES*	*MALES*	*FEMALES*
Part-time MBA	30	10	50	10
Full-time MBA only	150	8	16	6
CA only	90	10	37	3
Full time MBA & CA	70	2	7	1

26. What is the percentage of Math students over English students?
    (a) 50.4 (b) 61.4
    (c) 49.4 (d) None of these
27. The average number of females in all the courses is (count people doing full-time MBA and CA as a separate course)
    (a) less than 12 (b) greater than 12
    (c) 12 (d) None of these
28. The ratio of the number of girls to the number of boys is
    (a) 5:36 (b) 1:9
    (c) 1:7.2 (d) None of these
29. The percentage increase in students of full-time MBA only over CA only is
    (a) less than 20 (b) less than 25
    (c) less than 30 (d) more than 30
30. The number of students doing full-time MBA or CA is
    (a) 320 (b) 80
    (c) 160 (d) None of these.

Directions for Questions 31 to 34: Refer to the data below and answer the questions that follow:

A newspaper agent sells The TOI, HT and IN in equal numbers to 302 persons. Seven get HT & IN, twelve get The TOI & IN, nine get The TOI & HT and three get all the three newspapers. The details are given in the Venn diagram:



31. How many get only one paper?
    (a) 280 (b) 327
    (c) 109 (d) None of these
32. What percent get The TOI or The HT or both (but not The IN)?
    (a) more than 65% (b) less than 60%
    (c) ≅ 64% (d) None of these.

33. The number of persons buying The TOI and The HT only, The TOI and The IN only and The HT and The IN only are in the ratio of

(a) 6:4:9 (b) 6:9:4
(c) 4:9:6 (d) None of these

34. The difference between the number reading The HT and The IN only and HT only is

(a) 77 (b) 78
(c) 83 (d) None of these.

35. A group of 78 people watch Zee TV, Star Plus or Sony. Of these, 36 watch Zee TV, 48 watch Star Plus and 32 watch Sony. If 14 people watch both Zee TV and Star Plus, 20 people watch both Star Plus and Sony, and 12 people watch both Sony and Zee TV find the ratio of the number of people who watch only Zee TV to the number of people who watch only Sony.

(a) 9:4 (b) 3:2
(c) 5:3 (d) 7:4

Directions for Questions 36 and 37: Answer the questions based on the following information.

The following data was observed from a study of car complaints received from 180 respondents at Colonel Verma's car care workshop, viz., engine problem, transmission problem or mileage problem. Of those surveyed, there was no one who faced exactly two of these problems. There were 90 respondents who faced engine problems, 120 who faced transmission problems and 150 who faced mileage problems.

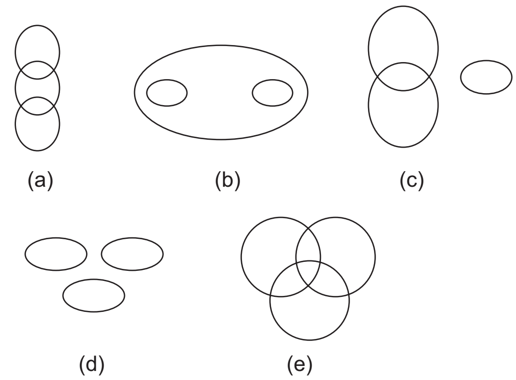
36. How many of them faced all the three problems?

(a) 45 (b) 60
(c) 90 (d) 20

37. How many of them faced either transmission problems or engine problems?

(a) 30 (b) 60
(c) 90 (d) 40

Directions for Questions 38 to 42: given below are five diagrams one of which describes the relationship among the three classes given in each of the five questions that follow. You have to decide which of the diagrams is the most suitable for a particular set of classes.



38. Elephants, tigers, animals
39. Administrators, Doctors, Authors
40. Platinum, Copper, Gold
41. Gold, Platinum, Ornaments
42. Television, Radio, Mediums of Entertainment
43. Seventy percent of the employees in a multinational corporation have VCD players, 75 percent have microwave ovens, 80 percent have ACs and 85 percent have washing machines. At least what percentage of employees has all four gadgets?

(a) 15 (b) 5
(c) 10 (d) Cannot be determined

Space for Rough Work

LEVEL OF DIFFICULTY (II)

1. At the Rosary Public School, there are 870 students in the senior secondary classes. The school is widely known for its science education and there is the facility for students to do practical training in any of the 4 sciences – viz: Physics, Chemistry, Biology or Social Sciences. Some students in the school however have no interest in the sciences and hence do not undertake practical training in any of the four sciences. While considering the popularity of various choices for opting for practical training for one or more of the choices offered, Mr. Arvindaksham, the school principal noticed something quite extraordinary. He noticed that for every student in the school who opts for practical training in at least M sciences, there are exactly three students who opt for practical training in at least (M – 1) sciences, for M = 2, 3 and 4. He also found that the number of students who opt for all four sciences was half the number of students who opt for none. Can you help him with the answer to "How many students in the school opt for exactly three sciences?"
   (a) 30 (b) 60
   (c) 90 (d) None of these
2. A bakery sells three kinds of pastries—pineappple, chocolate and black forest. On a particular day, the bakery owner sold the following number of pastries: 90 pineapple, 120 chocolate and 150 black forest. If none of the customers bought more than two pastries of each type, what is the minimum number of customers that must have visited the bakery that day?
   (a) 80 (b) 75
   (c) 60 (d) 90
3. Gauri Apartment housing society organised annual games, consisting of three games – viz: snooker, badminton and tennis. In all, 510 people were members in the apartments' society and they were invited to participate in the games — each person participating in as many games as he/she feels like. While viewing the statistics of the performance. Mr Capoor realised the following facts. The number of people who participated in at least two games was 52% more than those who participated in exactly one game. The number of people participating in 1, 2 or 3 games respectively was at least equal to 1. Being a numerically inclined person, he further noticed an interesting thing — the number of people who did not participate in any of the three games was the minimum possible integral value with these conditions. What was the maximum number of people who participated in exactly three games?
   (a) 298 (b) 300
   (c) 303 (d) 304
4. A school has 180 students in its senior section where foreign languages are offered to students as part of their syllabus. The foreign languages offered are: French, German and Chinese and the numbers of people studying each of these subjects are 80, 90 and 100 respectively. The number of students who study more than one of the three subjects is 50% more than the number of students who study all the three subjects. There are no students in the school who study none of the three subjects. Then how many students study all three foreign languages?
   (a) 18 (b) 24
   (c) 36 (d) 40

Directions for Questions 5 and 6: Answer the questions on the basis of the information given below.

In the second year, the Hampard Business School students are offered a choice of the specialisations they wish to study from amongst only three specialisations —Marketing, Finance and HR. The number of students who have specialised in only Marketing, only Finance and only HR are all numbers in an Arithmetic Progression— in no particular order. Similarly, the number of students specialising in exactly two of the three types of subjects are also numbers that form an Arithmetic Progression.

The number of students specialising in all three subjects is one-twentieth of the number of students specialising in only Finance which in turn is half of the number of students studying only HR. The number of students studying both Marketing and Finance is 15, whereas the number of students studying both Finance and HR is 19. The number of students studying HR is 120, which is more than the number of students studying Marketing (which is a 2 digit number above 50). It is known that there are exactly 4 students who opt for none of these specialisations and opt only for general subjects.

5. What is the total number of students in the batch?
   (a) 223 (b) 233
   (c) 237 (d) Cannot be determined
6. What is the number of students specialising in both Marketing and HR?
   (a) 11 (b) 21
   (c) 23 (d) Cannot be determined

Directions for Questions 7 to 9: In the Stafford Public School, students had an option to study none or one or more of three foreign languages viz: French, Spanish and German. The total student strength in the school was 2116 students out of which 1320 students studied French and 408 students studied both French and Spanish. The number of people who studied German was found to be 180 higher than the number of students who studied Spanish. It was also observed that 108 students studied all three subjects.

7. What is the maximum possible number of students who did not study any of the three languages?
   (a) 890 (b) 796
   (c) 720 (d) None of these
8. What is the minimum possible number of students who did not study any of the three languages?
   (a) 316 (b) 0
   (c) 158 (d) None of these
9. If the number of students who used to speak only French was 1 more than the number of people who used to speak only German, then what could be the maximum number of people who used to speak only Spanish?
   (a) 413 (b) 398
   (c) 403 (d) 431

Directions for Questions 10 to 13: In the Vijayant-khand sports stadium, athletes choose from four different racquet games (apart from athletics which is compulsory for all). These are Tennis, Table Tennis, Squash and Badminton. It is known that 20% of the athletes practising there are not choosing any of the racquet sports. The four games given here are played by 460, 360, 360 and 440 students respectively. The number of athletes playing exactly 2 racquet games for any combination of two racquet games is 40. There are 60 athletes who play all the four games but in a strange coincidence, it was noticed that the number of people playing exactly 3 games was also equal to 20 for each combination of 3 games.

10. What is the number of athletes in the stadium?
    (a) 1140 (b) 1040
    (c) 1200 (d) 1300
11. What is the number of athletes in the stadium who play either only squash or only Tennis?
    (a) 120 (b) 220
    (c) 340 (d) 440
12. How many athletes in the stadium participate in only athletics?
    (a) 160 (b) 1040
    (c) 260 (d) 220
13. If all the athletes were compulsorily asked to add one game to their existing list (except those who were already playing all the four games) — then what will be the number of athletes who would be playing all 4 games after this change?
    (a) 80 (b) 100
    (c) 120 (d) 140

Directions for Questions 14 and 15: Answer the questions on the basis of the following information.

In the Pattabhiraman family, a clan of 192 individuals, each person has at least one of the three Pattabhiraman characteristics—Blue eyes, Blonde hair, and sharp mind. It is also known that:

(i) The number of family members who have only blue eyes is equal to the number of family members who have only sharp minds and this number is also equal to twice the number of family members who have blue eyes and sharp minds but not blonde hair.
(ii) The number of family members who have exactly two of the three features is 50.
(iii) The number of family members who have blonde hair is 62.
(iv) Among those who have blonde hair, 26 family members have at least two of the three characteristics.

14. If the number of family members who have blue eyes is the maximum amongst the three characteristics, then what is the maximum possible number of family members who have both sharp minds and blonde hair but do not have blue eyes?
    (a) 11 (b) 10
    (c) 12 (d) Cannot be determined
15. Which additional piece of information is required to find the exact number of family members who have blonde hair and blue eyes but not sharp minds?
    (a) The number of family members who have exactly one of the three characterisitcs is 140.
    (b) Only two family members have all three characteristics.
    (c) The number of family members who have sharp minds is 89.
    (d) The number of family members who have only sharp minds is 52.
16. In a class of 97 students, each student plays at least one of the three games – Hockey, Cricket and Football. If 47 play Hockey, 53 play Cricket, 72 play Football and 15 play all the three games, what is the number of students who play exactly two games?
    (a) 38 (b) 40
    (c) 42 (d) 45

Directions for Questions 17 to 19: Answer the questions on the basis of the information given below.

In the ancient towns of Mohenjo Daro, a survey found that students were fond of three kinds of cold drinks (Pep, Cok and Thum). It was also found that there were three kinds of beverages that they liked (Tea, Cof and ColdCof).

The popluation of these towns was found to be 400000 people in all—and the survey was conducted on 10% of the population. Mr. Yadav, a data analyst observed the following things about the survey:

(i) The number of people in the survey who like exactly two cold drinks is five times the number of people who like all the three cold drinks.

(ii) The sum of the number of people in the survey who like Pep and 42% of those who like Cok but not Pep is equal to the number of people who like Tea.

(iii) The number of people in the survey who like Cof is equal to the sum of $3/8^{th}$ of those who like Cok and ½ of those who like Thum. This number is also equal to the number who like ColdCof.

(iv) 18500 people surveyed like Pep;

(v) 15000 like all the beverages and 3500 like all the cold drinks;

(vi) 14000 do not like Pep but like Thum;

(vii) 11000 like Pep and exactly one more cold drink;

(viii) 6000 like only Cok and the same number of people like Pep and Thum but not Cok.

17. The number of people in the survey who do like at least one of the three cold drinks?

(a) 38500 (b) 31500
(c) 32500 (d) 39500

18. What is the maximum number of people in the survey who like none of the three beverages?

(a) 24000 (b) 16000
(c) 12000 (d) Cannot be determined

19. What is the maximum number of people in the survey who like at least one of the three beverages?

(a) 7000 (b) 32,000
(c) 33000 (d) Cannot be determined

20. In a certain class of students, the number of students who drink only tea, only coffee, both tea and coffee and neither tea nor coffee are x, $2x$, $\frac{57}{x}$ and $\frac{57}{3x}$ respectively. The number of people who drink coffee can be

(a) 41 (b) 40
(c) 59 (d) Both a and c.

Space for Rough Work

ANSWER KEY

Level of Difficulty (I)

1. (b)	2. (a)	3. (b)	4. (a)
5. (b)	6. (a)	7. (c)	8. (b)
9. (c)	10. (b)	11. (a)	12. (d)
13. (c)	14. (b)	15. (a)	16. (b)
17. (c)	18. (b)	19. (c)	20. (a)
21. (d)	22. (c)	23. (c)	24. (a)
25. (b)	26. (d)	27. (b)	28. (b)
29. (c)	30. (a)	31. (a)	32. (c)
33. (b)	34. (d)	35. (a)	36. (c)
37. (b)	38. (b)	39. (e)	40. (d)
41. (a)	42. (b)	43. (c)	

Level of Difficulty (II)

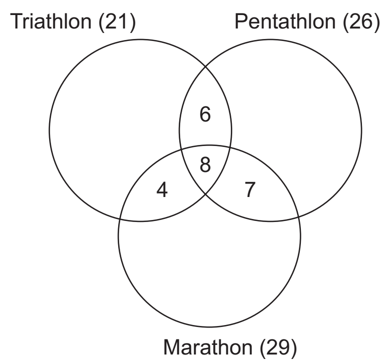
1. (b)	2. (b)	3. (c)	4. (c)
5. (c)	6. (c)	7. (b)	8. (b)
9. (d)	10. (d)	11. (c)	12. (c)
13. (d)	14. (a)	15. (c)	16. (d)
17. (a)	18. (b)	19. (c)	20. (d)

Solutions and Shortcuts

Level of Difficulty (I)

Solutions for Questions 1 and 2: Since there are 14 players who are in triathlon and pentathlon, and there are 8 who take part in all three games, there will be 6 who take part in only triathlon and pentathlon. Similarly,

Only triathlon and marathon = 12 – 8=4 & Only Pentathlon and Marathon = 15 – 8 = 7.



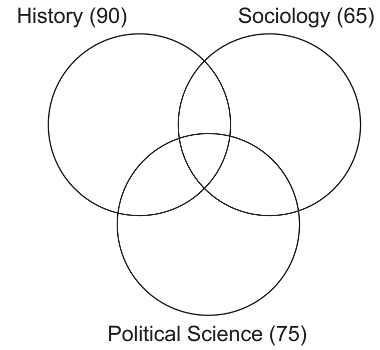
The figure above can be completed with values for each sport (only) plugged in:

The answers would be:

3 + 6 + 8 + 4 + 5 + 7 + 10 = 43. Option (b) is correct.

Option (a) is correct.

Solutions for Questions 3 and 4:



The given situation can be read as follows:

115 students are being counted 75+65+90= 230 times.

This means that there is an extra count of 115. This extra count of 115 can be created in 2 ways.

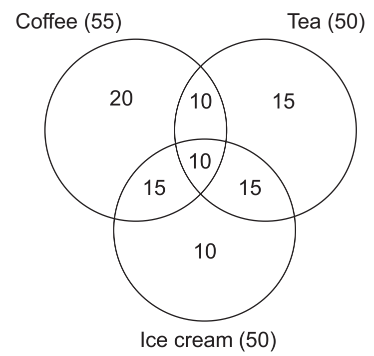
A. By putting people in the ‘passed exactly two subjects’ category. In such a case each person would get counted 2 times (double counted), i.e., an extra count of 1.
B. By putting people in the ‘all three’ category, each person put there would be triple counted. 1 person counted 3 times – meaning an extra count of 2 per person.

The problem tells us that there are 55 students who passed exactly two subjects. This means an extra count of 55 would be accounted for. This would leave an extra count of 115 – 55 = 60 more to be accounted for by ‘passed all three’ category. This can be done by using 30 people in the ‘all 3’ category.

Hence, the answers are:

3. Option (b)
4. Option (a)

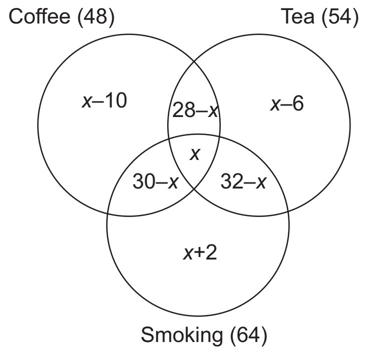
Solutions for Questions 5 to 8: Based on the information provided we would get the following figure:



The answers could be read off the figure as:

5. [(20 – 10)/10] * 100 = 100%. Option (b) is correct.
6. 15% (from the figure). Option (a) is correct.
7. 10+10+15+15=50%. Option (c) is correct.
8. Only ice cream is 10% of the total. Hence, 10% of 180 =18. Option (b) is correct.

Solutions for Questions 9 to 15: If you try to draw a figure for this question, the figure would be something like:



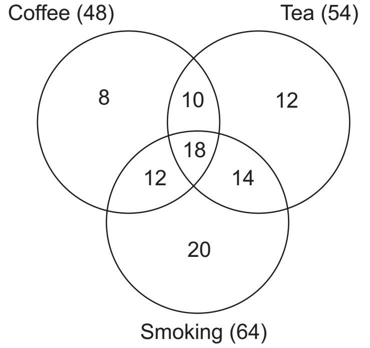
We can then solve this as:
$x - 10 + 28 - x + x + 30 - x + x + 2 + 32 - x + x - 6 = 94 \rightarrow x + 76 = 94 \rightarrow x = 18$.

Note: In this question, since all the values for the use of the set theory formula are given, we can find the missing value of students who liked all three as follows:

$94 = 48 + 54 + 64 - 28 - 32 - 30 + \text{All three} \rightarrow \text{All three} = 18$

As you can see this is a much more convenient way of solving this question, and the learning you take away for the 3 circle situation is that whenever you have all the values known and the only unknown value is the center value – it is wiser and more efficient to solve for the unknown using the formula rather than trying to solve through a venn diagram.

Based on this value of x we get the diagram completed as:



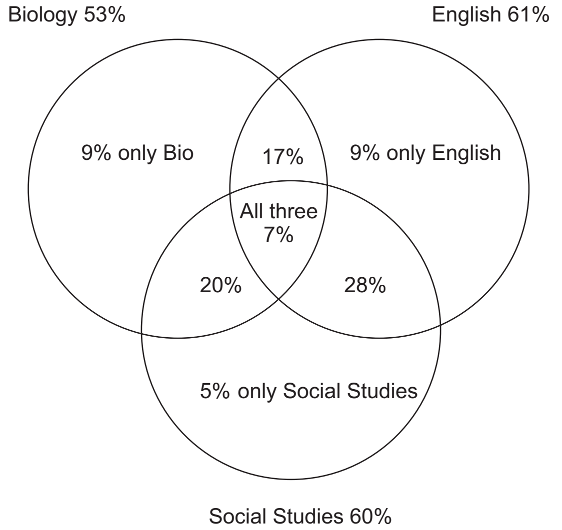
The answers then are:

9. 8:12 = 2:3 → Option (c) is correct.
10. 12 % of 2000 = 240. Option (b) is correct.
11. 30/94 → more than 30%. Option (a) is correct.
12. 94%. Option (d) is correct.
13. Option (c) is correct as the ratio turns out to be 10:20 in that case.
14. 12:12 = 1:1 → Option (b) is correct.
15. 14%. Option (a) is correct.
16. $30 = 25 + 20 - x \rightarrow x = 15$. Option (b) is correct.

Solutions for Questions 17 to 20:

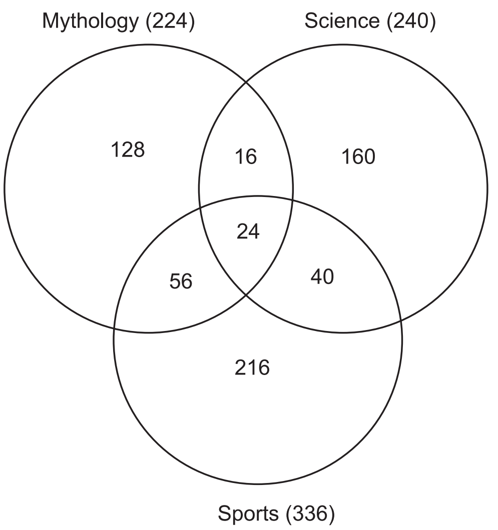
Let people who passed all three be x. Then:
$53 + 61 + 60 - 24 - 35 - 27 + x = 95$
$\rightarrow x = 7$.

The venn diagram in this case would become:



17. Option (c) is correct.
18. 23% of 200 = 46. Option (b) is correct.
19. If the number of students is increased by 50%, the number of students in each category would also be increased by 50%. Option (c) is correct.
20. 20:28 = 5:7. Option (a) is correct.

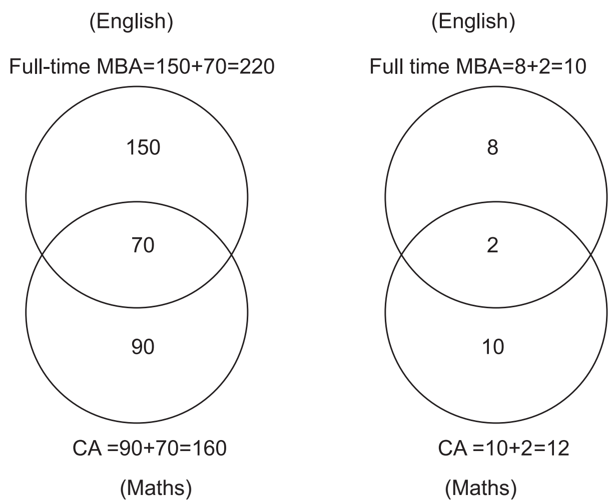
Solutions for Questions 21 to 25: The following figure would emerge on using all the information in the question:

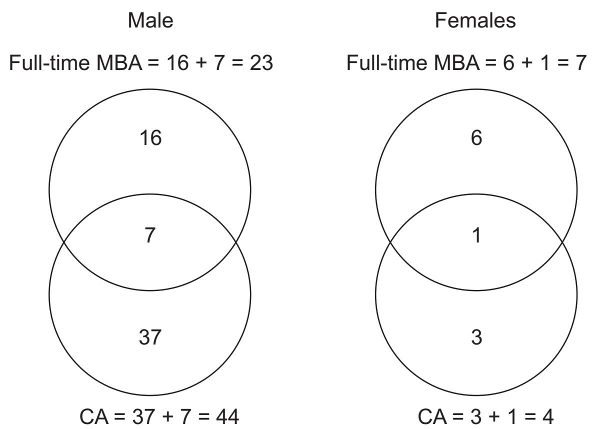


The answers would then be:

21. 240/880 = 27.27%. Option (d) is correct.
22. 504/880 = 57.27%. Hence, less than 60. Option (c) is correct.
23. 40 + 16 + 56 + 24 = 136. Option (c) is correct.
24. Option a gives us 16:128 = 1:8. Option (a) is hence correct.
25. 40:160 → 1:4. Option (b) is correct.

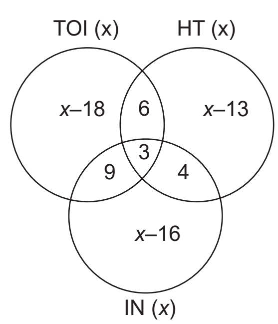
Solutions for Questions 26 to 30: The following Venn diagrams would emerge:





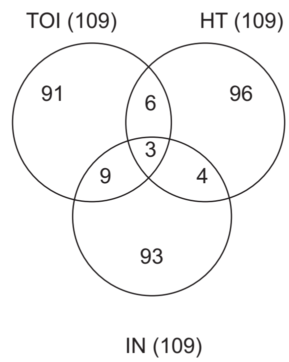
26. Math Students = 130. English Students =370
    130/370 = 35.13%. Option (d) is correct.
27. Number of Female Students = 10 + 8 + 10 + 2 + 10 + 6 + 3 + 1 = 50. Average number of females per course = 50/3 = 16.66. Option (b) is correct.
28. 50:450 = 1:9. Option (b) is correct.
29. 40/140 → 28.57%. Option (c) is correct.
30. From the figures, this value would be 150+8+ 90 + 10 + 16+6+37+3= 320. Option (a) is correct.

Solutions for Questions 31 to 34: The following figure would emerge-



Based on this figure we have:
$x + x - 13 + 4 + x - 16 = 302 \rightarrow 3x - 25 = 302 \rightarrow x = 327.$
Hence, $x = 109$.

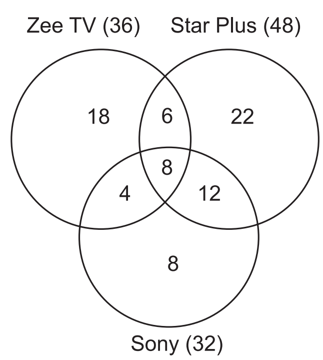
Consequently the figure becomes:



The answers are:

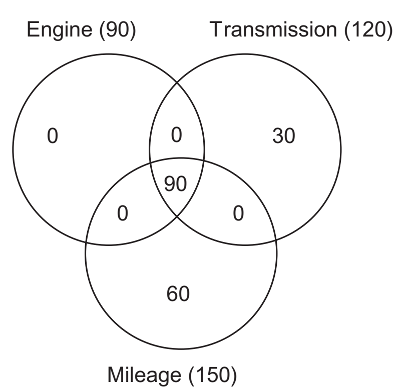
31. 91 + 93 + 96 = 280. Option (a) is correct.
32. 193/302 ≅ 64%
33. 6:9:4 is the required ratio. Option (b) is correct.
34. 96 – 4 = 92. Options (d) is correct.
35. $78 = 36 + 48 + 32 - 14 - 20 - 12 + x \rightarrow x = 8$.

The figure for this question would become:

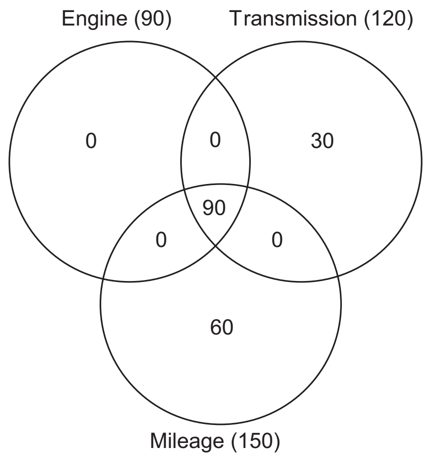


Required ratio is 18:8 → 9:4. Option (a) is correct.

36. Option (c)

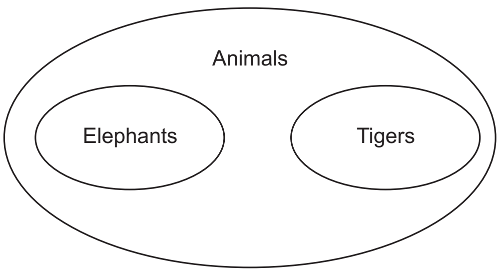


37. There are 30 such people. Option (b) is correct.

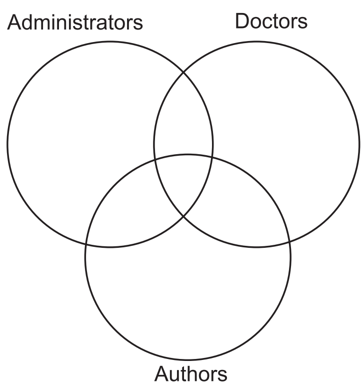


Solutions for Questions 38 to 42:

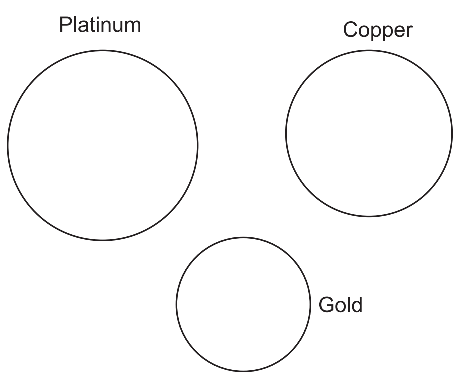
38. Option (b) is correct



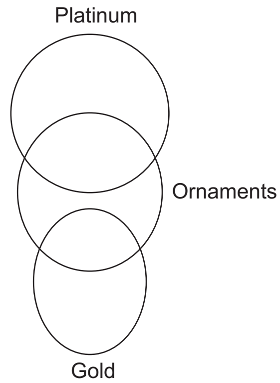
39. Option (e) is correct.



40. Option (d) is correct.



41. Option (a) is correct.



42. Option (b) is correct

Solution for Question 43:

43. The least percentage of people with all 4 gadgets would happen if all the employees who are not having any one of the four objects is mutually exclusive.
    Thus, 100 – 30 – 25 – 20 – 15 = 10
    Option (c) is correct

Level of Difficulty (II)

1. The key to think about this question is to understand what is meant by the statement —"for every student in the school who opts for practical training in at least M sciences, there are exactly three students who opt for practical training in at least (M – 1) sciences, for M = 2, 3 and 4"
   What this statement means is that if there are x students who opt for practical training in all 4 sciences, there would be $3x$ students who would opt for practical training in at least 3 sciences. Since opting for at least 3 sciences includes those who opted for exactly 3 sciences and those who opted for exactly 4 sciences—we can conclude from this that:
   The number of students who opted for exactly 3 sciences = Number of students who opted for at least 3 sciences – Number of students who opted for all 4 sciences = $3x - x = 2x$
   Thus, the number of students who opted for various number of science practicals can be summarised as below:

	Number of students who opted for at least n subjects	*Number of students who opted for exactly n subjects*
$n = 4$	x	x
$n = 3$	$3x$	$2x$
$n = 2$	$9x$	$6x$
$n = 1$	$27x$	$18x$

Also, number of students who opt for none of the sciences = twice the number of students who opt for exactly 4 sciences = $2x$.

Based on these deductions we can clearly identify that the number of students in the school would be: $x + 2x + 6x + 18x + 2x = 29x = 870 \rightarrow x = 30$.

Hence, number of students who opted for exactly three sciences = $2x = 60$

2. (b) In order to estimate the minimum number of customers we need to assume that each customer must have bought the maximum number of pastries possible for him to purchase.

   Since, the maximum number of pastries an individual could purchase is constrained by the information that no one bought more than two pastries of any one kind—this would occur under the following situation—First 45 people would buy 2 pastries of all three kinds, which would completely exhaust the 90 pineapple pastries and leave the bakery with 30 chocolate and 60 black forest pastries. The next 15 people would buy 2 pastries each of the available kinds and after this we would be left with 30 black forest pastries. 15 people would buy these pastries, each person buying 2 pastries each.

   Thus, the total number of people (minimum) would be: 45 + 15 + 15 = 75.

3. (c) Let the number of people who participated in 0, 1, 2 and 3 games be A, B, C, D respectively. Then from the information we have:

   $C+D = 1.52 \times B$ (Number of people who participate in at least 2 games is 52% higher than the number of people who participate in exactly one game)

   $A + B + C + D = 510$ (Number of people invited to participate in the games is 510)

   This gives us: $A + 2.52B = 510 \rightarrow B = \frac{25}{63}(510 - A)$

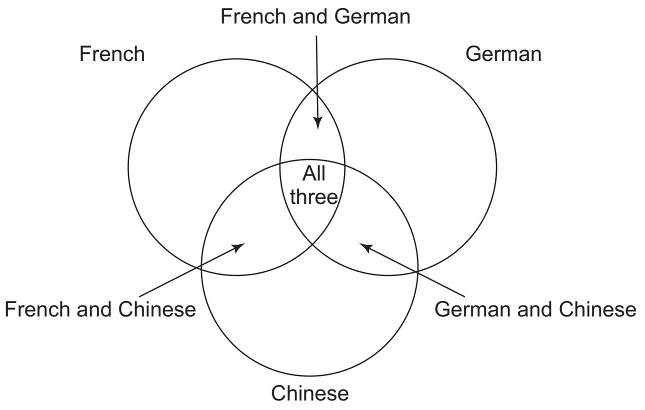
   For A to be minimum, 510 – A should give us the largest multiple of 63. Since, $63 \times 8 = 504$ we have A = 6.

   Also, 2.52B = 504, so B = 200 and C+D = 1.52B = 304.

   For number of people participating in exactly 3 games to be maximum, the number of people participating in exactly 2 games has to be minimised and made equal to 1. Thus, the required answer = 304 – 1 = 303.

4. (c) In order to think about this question, the best way is to use the process of slack thinking. In this question, we have 180 students counted 270 times. This means that there is an extra count of 90 students. In a three circle venn diagram, extra counting can occur only due to exactly two regions (where 1 individual student would be counted in two subjects leading to an extra count of 1) and the exactly three region (where 1 individual student would be counted in 3 subjects leading to an extra count of 2).

   This can be visualised in the figure below:



A student placed in the all three area will be counted three times when you count the number of students studying French, the number of students studying German and the number of students studying Chinese independently. Hence, HE/SHE would be counted three times—leading to an extra count of 2 for each individual places here.

A person placed in any of the three 'Exactly two' areas would be counted two times when we count the number of students studying French, the number of students studying German and the number of students studying Chinese independently. Hence, HE/SHE would be counted two times—leading to an extra count of 1 for each individual placed in any of these three areas.

The thought chain leading to the solution would go as follows:

(i) 180 students are counted 80 + 90 + 100 = 270 times.

(ii) This means that there is an extra count of 90 students.

(iii) Extra counts can fundamentally occur only from the 'exaclty two' areas or the all three area in the figure.

(iv) We also know that 'The number of students who study more than one of the three subjects is 50% more than the number of students who study all the three subjects' hence we know that if there are a total of 'n' students studying all three subjects, there would be $1.5n$ students studying more than one subject. This in turn means that there must be $0.5n$ students who study two subjects.

(Since, number of students studying more than 1 subject = number of students studying two subjects + number of students studying three subjects.

i.e. $1.5n = n$ + number of students studying 2 subjects $\rightarrow$ number of students studying 2 subjects = $1.5n - n = 0.5n$)

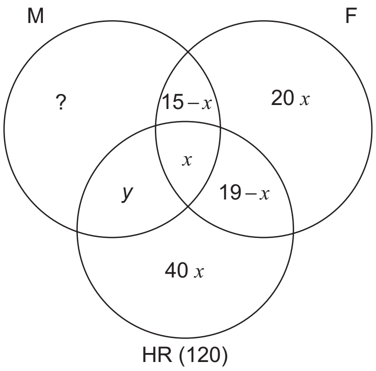
(v) The extra counts from the n students studying 3 subjects would amount to $n \times 2 = 2n$ – since each student is counted twice extra when he/she studies all three subjects.

(vi) The extra counts from the 0.5n students who study exactly two subjects would be equal to $0.5n \times 1 = 0.5\ n$.

(vii) Thus extra count = 90 = $2n + 0.5n \rightarrow n = 90/2.5 = 36$.

(viii) Hence, there must be 36 people studying all three subjects.

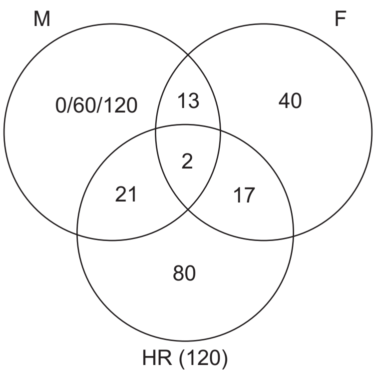
Solutions 5 and 6: The following would be the starting Venn diagram encapsulating the basic information:



From this figure we get the following equation: $40x + (19 - x) + x + y = 120$. This gives us $40x + y = 101$

Thinking about this equation, we can see that the value of x can be either 1 or 2. In case we put x as 1, we get $y = 61$ and then we have to also meet the additional condition that $15–x$, $19–x$ and y should form an AP which is obviously not possible (since it is not possible practically to build an AP having two positive terms below 19 and the third term being 61. Hence, this option is rejected.

Moving forward, the other possible value of x from the equation is $x = 2$ in which case we get, $y = 21$ and $15– x = 13$ and $19–x = 17$. Thus, we get the AP 13, 17, 21 which satisfies the given conditions. Putting $x = 2$ and $y = 21$ in the figure, the venn diagram evolves to:



In this figure the value that only Marketing takes can either be 0, 60 or 120 (to satisfy the AP condition). However, since the total number of students in Marketing is a two digit number above 50, the number of people studying only marketing would be narrowed down to the only possibility which remains – viz 60.

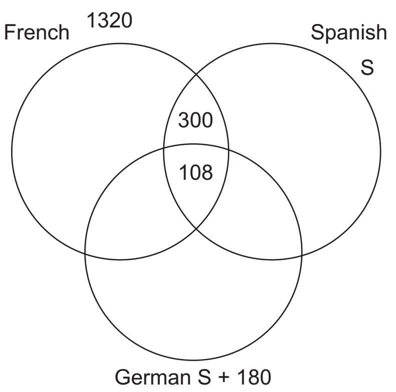
Thus, the number of students studying in the batch = 120 + 40 + 60 + 13 + 4 = 237.

The number of students specialising in both Marketing and HR is 21 + 2 = 23

5. (d) The total number of students is 237.
6. (c) The number of students studying both Marketing and HR is 23.

Solutions 7 to 9:

7 & 8: In order to think about the possibility of the maximum and/or the minimum number of people who could be studying none of the three languages, you need to first think of the basic information in the question. The basic information in the question can be encapsulated by the following Venn diagram:



At this point we have the flexibility to try to put the remaining numbers into this Venn diagram while maintaining the constraints the question has placed on the relative numbers in the figure. In order to do this, we need to think of the objective with which we have to fill in the remaining numbers in the figure. At this stage you have to keep two constraints in mind while filling the remaining numbers:

(a) The remaining part of the French circle has to total 1320 – 408 = 912;

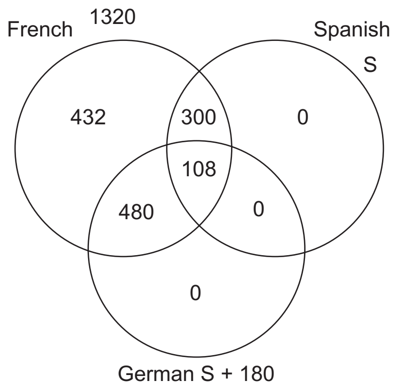
(b) The German circle has to be 180 more than the Spanish circle.

When we try to fill in the figure for making the number of students who did not study any of the three subjects maximum:

You can think of first filling the French circle by trying to think of how you would want to distribute the remaining 912 in that circle. When we want to maximise the number of students who study none of the three, we would need to use the minimum number of people inside the three circles—while making sure that all the constraints are met.

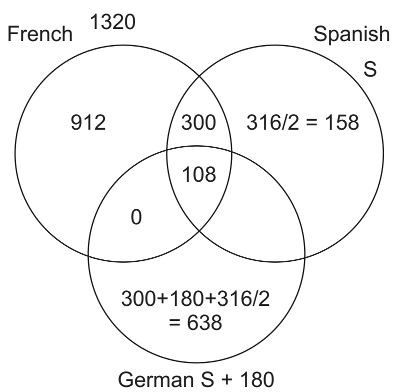
Since we have to forcefully fit in 912 into the remaining areas of the French circle, we need to see whether while doing the same we can also meet the second constraint.

This thinking would lead you to see the following solution possibility:



In this case we have ensured that the German total is 180 more than the Spanish total (as required) and at the same time the French circle has also reached the desired 1320. Hence, the number of students who study none of the three can be 2116–1320 = 796 (at maximum).

When minimising the number of students who have studied none of the three subjects, the objective would be to use the maximum number of students who can be used in order to meet the basic constraints. The answer in this case can be taken to as low as zero in the following case:

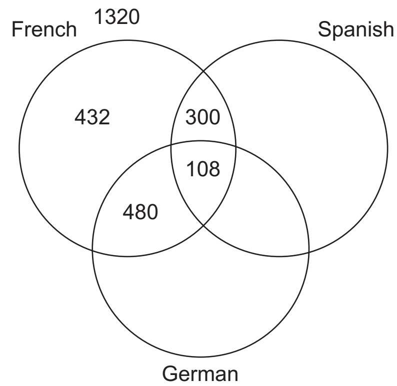


Note: While thinking about the numbers in this case, we first use the 912 in the 'only French' area. At this point we have 796 students left to be allocated. We first make the German circle 180 more than the Spanish circle (by taking the only German as 300 +180 to start with, this is accomplished). At this point, we are left with 316 more students, who can be allocated equally as 316 ÷ 2 for both the 'only German' and the 'only Spanish' areas.

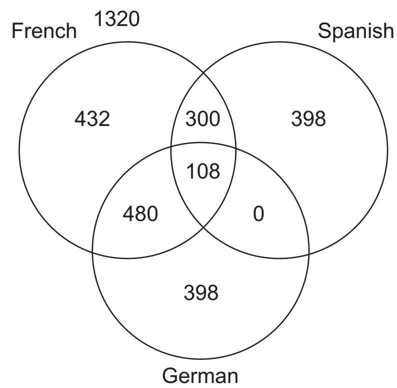
Thus, the minimum number of students who study none of the three is 0.

Solution 9:
In order to think about this question, let us first see the situation we had in order to maintain all constraints.

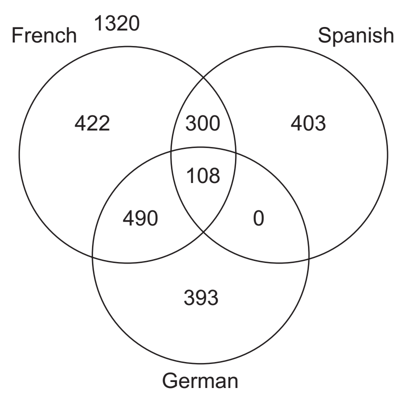
If we try to fit in the remaining constraints in this situation we would get:



This leaves us with a slack of 796 people which would need to be divided equally since we cannot disturb the equilibrium of German being exactly 180 more than Spanish.
This gives us the following figure:



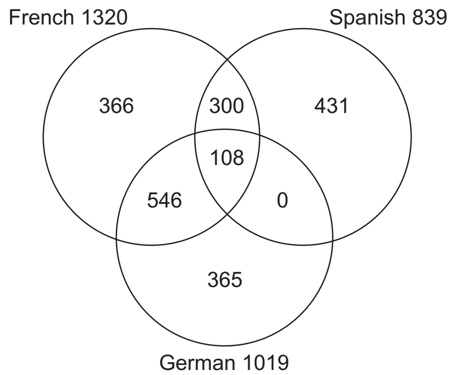
When you think about this situation, you realise that it is quite possible to increase Spanish if we reduce the only French area and reallocate the reduction into the 'only French' and German area. A reduction of 10 from the 'only French' area can be visualised as follows:



In this case, as you can see from the figure above, the number of students who study only Spanish has gone up by 5 (which is half of 10).

Since, there is still some gap between the 'only German' and the 'only French' areas in the figure, we should close that gap by reducing the 'only French' area as much as possible.

The following solution figure would emerge when we think that way:



Hence, the maximum possible for the only Spanish area is 431.

Solutions 10 to 13: The information given in the question can be encapsulated in the following way:

Game	*Only that game*	*2 games combination 1*	*2 games combination 2*	*2 games combination 3*	*3 games combination 1*	*3 games combination 2*	*3 games combination 3*	*All 4 games*
Tennis (460)	220	40	40	40	20	20	20	60
TT (360)	120	40	40	40	20	20	20	60
Squash (360)	120	40	40	40	20	20	20	60
Badminton (440)	200	40	40	40	20	20	20	60

From the above table, we can draw the following conclusions,, which can then be used to answer the questions asked.

The total number of athletes who play at least one of the four games = 220 + 120 + 120 + 200 + 40×6 + 20×4 + 60 = 1040

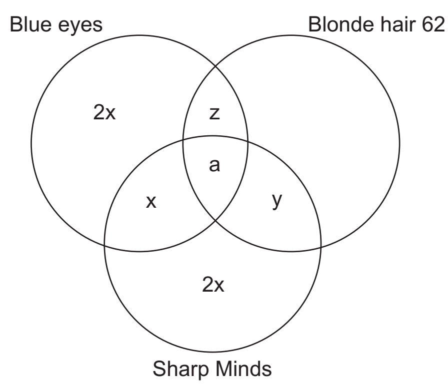
(Note : that in doing this calculation, we have used 40 × 6 for calculating how many unique people would be playing exactly two games—where 40 for each combination is given and there are ${}^4C_2$= 6 combinations of exactly two sports that exist. Similar logic applies to the 20 × 4 calculation for number of athletes playing exactly 3 sports.)

Also, since we know that the number of athletes who participate in none of these four games is 20% of the total number of athletes, we can calculate the total number of athletes who practise in the stadium as 5×1040 ÷ 4=1300.

Thus, the questions can be answered as follows:

10. The number of athletes in the stadium = 1300.
11. Only squash + only tennis = 120 + 220 = 340 (from the table)
12. Only athletics means none of the 4 games = total number of athletes – number of athletes who play at least one game = 1300 – 1040 = 260.
13. In case all the three game athletes would add one more game they would become 4 game athletes. Hence, the number of athletes who play all four games would be: Athletes playing 3 games earlier + athletes playing all 4 games earlier = 80+60=140

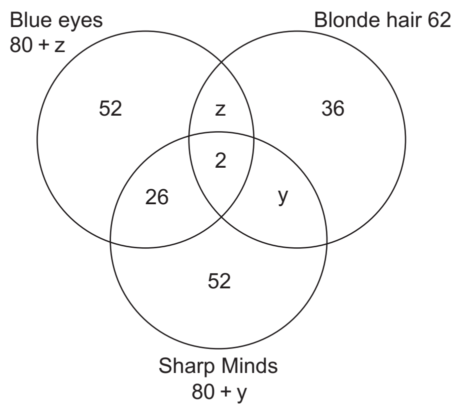
Solutions 14 and 15: The starting figure based on the information given in the question would look something as below:



From this figure we see a few equations:

$x + y + z = 50$; $a + y + z = 26 \rightarrow x - 24 = a$.

Also, since, $5x + 62 = 192$, we get the value of x as 26. The figure would evolve as follows.



Based on this we can deduce the answer to the two questions as:

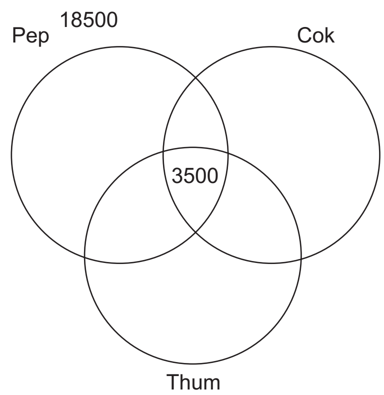
14. For the number of family members with blue eyes to be maximum, the family members with both sharp minds and blonde hair, but not blue eyes (represented by 'y' in the figure), would be at maximum 11 because we would need to keep $z > y$. Hence, Option (a) is the correct answer.
15. If we are given the information in Option (c) we know the value of y would be 9 and hence, the value of z would be determined as 15. Hence, Option (c) provides us the information to determine the exact number of family members who have blonde hair and blue eyes but not sharp minds. Notice here that the information in each of the other options is already known to us.
16. Solve this again using slack thinking by using the following thought process:

    97 students are counted 47 + 53 + 72 = 172 times—which means that there is an extra count of 75 students (172 – 97 = 75). Now, since there are 15 students who are playing all the three games, they would be counted 45 times—hence they take care of an extra count of 15 × 2 = 30. (**Note:** in a 3 circle venn diagram situation, any person placed in the all three areas is counted thrice—hence he/she is counted two extra times).

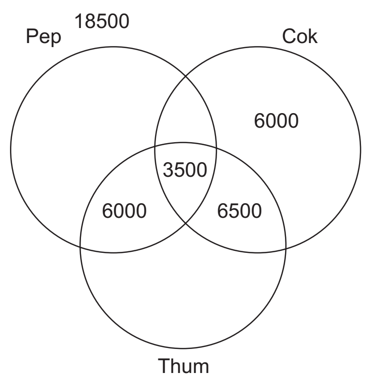
    This leaves us with an extra count of 45 to be managed—and the only way to do so is to place people in the exactly two areas. A person placed in the 'exactly two games area' would be counted once extra. Hence, with each student who goes into the 'exactly two games' areas it would be counted once extra. Thus, to manage an extra count of 45, we need to put 45 people in the 'exactly two' area. Option (d) is correct.

Solutions17 to 19: When you draw a Venn diagram for the three cold drinks, you realise as given here.

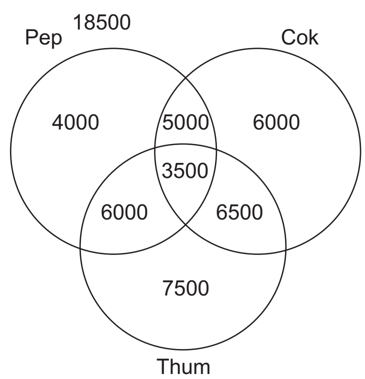
Once you fill in the basic information into the Venn diagram, you reach the following position:



At this point we know that since the 'all three area' is 3500, the value of the 'exactly two areas' would be 5 × 3500 = 17500. Also, we know that "11000 like Pep and exactly one more cold drink" which means that the area for Cok and Thum but not Pep is equal to 17500 – 11000 = 6500. Further, when you start adding the information: "6000 like only Cok and the same number of people like Pep and Thum but not Cok," the Venn Diagram transforms to the following:



Filling in the remaining gaps in the picture we get:



Note, we have used the following info here:

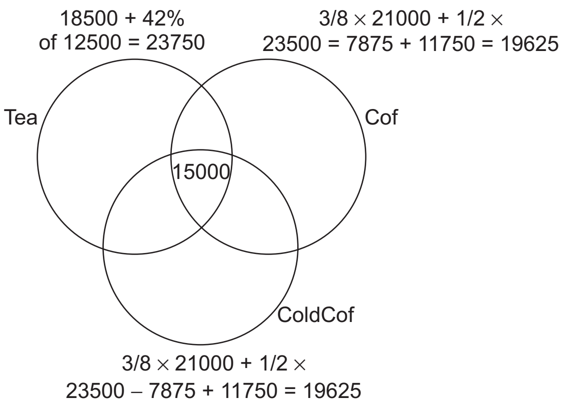
Thum but not Pep is 14000 and since we already know that Thum and Cok but not Pep is 6500, the value of 'only Thum' would be 14000 – 6500 = 7500.

We also know that the 'exactly two' areas add up to 17500 and we know that two of these three areas are 6500 and 6000 respectively. Thus, Pep and Cok but not Thum is 17500 – 6000 – 6500 = 5000.

Finally, the 'only Pep' area would be 18500 – 5000 – 6000 – 3500 = 4000.

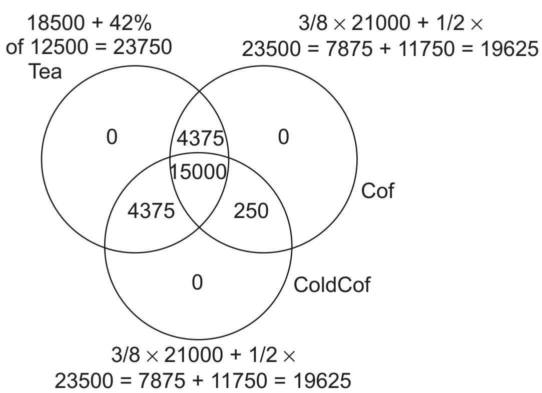
Once we have created the Venn diagram for the cold drinks, we can focus our attention to the Venn diagram for the beverages.

Based on the information provided, the following diagram can be created.



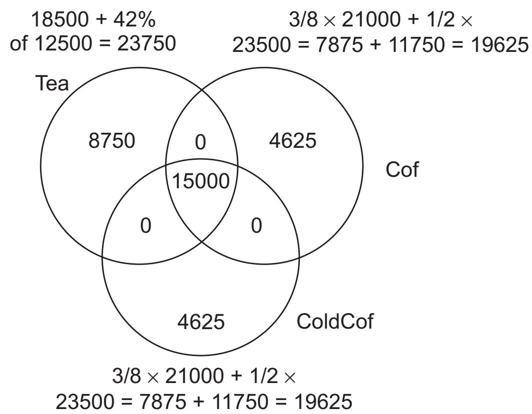
Based on these figures, the questions asked can be solved as follows:

17. Option (a) is correct as the number of people who like at least one of the cold drinks is the sum of 18500 + 6000 +6500 +7500 = 38500.
18. For the number of people who do not like any of the beverages to be maximum, we have to ensure that the number of people used in order to meet the situation described by the beverage's venn diagram should be minimum. This can be done by filling values in the inner areas of this venn diagram:



In this situation, the number of people used inside the Venn diagram to match upto all the values for this figure = 15000 + 4375 + 4375 + 250 = 24000. Naturally, in this case the number of people who do not like any of the beverages is maximised at 40000 – 24000 = 16000. Option (b) is correct.

19. The solution for this situation would be given by the following figure:



The number of people who like at least one of the three beverages is:

15000 + 8750 +4625 + 4625 = 33000. Option (c) is correct.

20. The number of people cannot be a fraction in any situation. We can deduce that the values of x and $3x$ have to be factors of 57. This gives us that the values of x can only be either 1 or 19 (for both x and $3x$ to be a factor of 57).

So, the number of people who drink coffee is equal to $2x + 57/x$ which can be 59 (if $x = 1$) or 41 (if $x = 19$).

Hence, Option (d) is correct.

Space for Rough Work

TRAINING GROUND FOR BLOCK VI

HOW TO THINK IN PROBLEMS ON BLOCK VI

1. The probability that a randomly chosen positive divisor of 10^{29} is an integer multiple of 10^{23} is : $a2/b2$, then '$b - a$' would be: **(XAT 2014)**

(a) 8 (b) 15
(c) 21 (d) 23
(e) 45

Solution: This question appeared in the XAT 2014 exam. The number $10^{29} = 2^{29} \times 5^{29}$

Factors or divisors of such a number would be of the form: $2^a \times 5^b$ where the values of a and b can be represented as $0 \le a, b \le 29$, i.e., there are $30 \times 30 = 900$ possibilities when we talk about randomly selecting a positive divisor of 10^{29}.

Next, we need to think of numbers which are integral multiples of 10^{23}. Such numbers would be of the form $2^x \times 5^y$ such that $x, y \ge 23$.

Hence, the number of values possible when the chosen divisor would also be an integer multiple of 10^{23} would be when $23 \le x, y \le 29$. There would be $7 \times 7 = 49$ such combinations.

Thus, the required probability is 49÷900. In the context of $a^2 \div b^2$, the values of a and b would come out as 7 and 30 respectively. The required difference between a and b is 23. Hence, Option (d) is correct.

2. Aditya has a total of 18 red and blue marbles in two bags (each bag has marbles of both colors). A marble is randomly drawn from the first bag followed by another randomly drawn from the second bag, the probability of both being red is 5/16. What is the probability of both marbles being blue? **(XAT 2014)**

(a) 1/16 (b) 2/16
(c) 3/16 (d) 4/16
(e) None of the above

Solution: This problem has again appeared in XAT 2014. The problem most students face in such situations is to understand how to place how many balls of each colour in each bag. Since there is no directive given in the question that tells us how many balls are there and/or how many balls are placed in any bag the next thing that a mathematically oriented mind would do would be to try to assume some variables to represent the number of balls in each bag. However, if you try to do so on your own you would realise that that would be the wrong way to solve this quesiton as it would lead to extreme complexity while solving the problem. So how can we think alternately? Is there a smarter way to think about this question?

Yes indeed there is. Let me explain it to you here. In order to think about this problem, you would need to first think about how a fraction like 5/16 would emerge. The value of 5/16 = 10/32 = 15/48 = 20/64 = 25/80 = 30/96 and so on. Next, you need to understand that there are a total of 18 balls and this 18 has to be broken into two parts such that their product is one of the above denominators. Scanning the denominators we see the opportunity that the number 80 = 10×8 and hence we realise that the probability of both balls being red would happen in a situation where the structure of the calculation would look something like: $(r_1/10) \times (r_2/8)$. Next, to get 25 as the corresponding numerator with 80 as the denominator the values of r_1 and r_2 should both be 5. This means that there are 5 red balls out of ten in the first bag and 5 red balls out of 8 in the second bag. This further means that the number of blue balls would be 5 out of 8 and 3 out of 8. Thus, the correct answer would be: (5/10) × (3/8) = 25/80 = 5/16. Hence, Option (c) is the correct answer.

3. The scheduling officer for a local police department is trying to schedule additional patrol units in each of two neighbourhoods – southern and northern. She knows that on any given day, the probabilities of major crimes and minor crimes being committed in the northern neighbourhood were 0.418 and 0.612, respectively, and that the corresponding probabilities in the southern neighbourhood were 0.355 and 0.520. Assuming that all crimes occur independent of each other and likewise that crime in the two neighbourhoods are independent of each other, what is the probability that no crime of either type is committed in either neighbourhood on any given day? **(XAT 2011)**

(a) 0.069 (b) 0.225
(c) 0.690 (d) 0.775
(e) None of the above

Solution: This question appeared in XAT 2011, and the key to solving this correctly is to look at the event definition. A major crime not occurring in the northern neighbourhood is the non-event for a major crime occurring in the nothern neighbourhood on any given day. Its probability would be (1–0.418) = 0.582.

The values of minor crime not occurring in the northern neighbourhood and a major crime not occurring in the southern neighbourhood and a minor crime not occurring in the northern neighbourhood would be (1–0.612); (1–0.355) and (1–0.520) respectively. The value of the required probability would be the probability of the event:

Major crime does not occur in the northern neighbourhood and minor crime does not occur in the northern neighbourhood and major crime does not occur in the southern neighbourhood and minor crime does not occur in the northern neighbourhood =

(1–0.418) × (1–0.612) × (1–0.355) × (1–0.520). Option (a) is the closest answer.

4. There are four machines in a factory. At exactly 8 pm, when the mechanic is about to leave the factory, he is informed that two of the four machines are not working properly. The mechanic is in a hurry, and decides that he will identify the two faulty machines before going home, and repair them next morning. It takes him twenty minutes to walk to the bus stop. The last bus leaves at 8 :32 pm. If it takes six minutes to identify whether a machine is defective or not, and if he decides to check the machines at random, what is the probability that the mechanic will be able to catch the last bus?

(a) 0 (b) 1/6
(c) 1/4 (d) 1/3
(e) 1

Solution: The first thing you look for in this question, is that obviously the mechanic has only 12 minutes to check the machines before he leaves to catch the bus. In 12 minutes, he can at best check two machines. He will be able to identify the two faulty machines under the following cases:

(The first machine checked is faulty AND the second machine checked is also faulty) OR (The first machine checked is working fine AND the second machine checked is also working fine)

Required probability = $\frac{2}{4}\times\frac{1}{3}+\frac{2}{4}\times\frac{1}{3}=\frac{1}{3}$

5. Little Pika who is five and half years old has just learnt addition. However, he does not know how to carry. For example, he can add 14 and 5, but he does not know how to add 14 and 7. How many pairs of consecutive integers between 1000 and 2000 (both 1000 and 2000 included) can Little Pika add?

(a) 150 (b) 155
(c) 156 (d) 258
(e) None of the above

Solution: This question again appeared in the XAT 2011 exam. If you try to observe the situations under which the addition of two consecutive four-digit numbers between 1000 and 2000 would come through without having a carry over value in the answer you would be able to identify the following situations – each of which differs from the other due to the way it is structured with respect to the values of the individual digits:

Category 1: 1000 + 1001; 1004 +1005, 1104 + 1105 and so on. A little bit of introspection should show you that in this case, the two numbers are 1abc and 1abd where $d = c + 1$. Also, for the sum to come out without any carry-overs, the values of a, b and c should be between 0 and 4 (including both). Thus, each of a, b and c gives us 5 values each – giving us a total of 5 × 5 × 5 = **125 such situations.**

Category 2: 1009 + 1010; 1019 + 1020; 1029 + 1030; 1409 + 1410. The general form of the first number here would be 1*ab* 9 with the values of a and b being between 0 and 4 (including both). Thus, each of a, b and c gives us 5 values each – giving us a total of 5 × 5 = **25 such situations**.

Category 3: 1099 + 1100; 1199 + 1200; 1299 + 1300; 1399 + 1400 and 1499 + 1500. There are only **5 such pairs.** Note that 1599 + 1600 would not work in this case as the addition of the hundreds' digit would become more than 10 and lead to a carry-over.

Category 4: 1999 + 2000 is the only other situation where the addition would not lead to a carry-over calculation. Hence, **1 more situation.**

The required answer = 125 + 25 + 5 + 1 = 156. Option (c) is the correct answer.

6. In the country of Twenty, there are exactly twenty cities, and there is exactly one direct road between any two cities. No two direct roads have an overlapping road segment. After the election dates are announced, candidates from their respective cities start visiting the other cities. The following are the rules that the election commission has laid down for the candidates:

- Each candidate must visit each of the other cities exactly once.
- Each candidate must use only the direct roads between two cities for going from one city to another.
- The candidate must return to his own city at the end of the campaign.
- No direct road between two cities would be used by more than one candidate.

The maximum possible number of candidates is

(a) 5 (b) 6
(c) 7 (d) 8
(e) 9

Solution: Again an XAT 2011 question. Although this question carried a very high weightage (it had 5 marks where 'normal questions' had 1 to 3 marks) it is not so difficult once you understand the logic of the question. The key to understanding this question is from two points.

(a) Since there is exactly one direct road between any pair of two cities – there would be a toal of $20C_2$ roads = 190 roads.

(b) The other key condition in this question is the one which talks about each candidate must visit each of the other cities exactly once and 'No direct road between two cities would be used by more than one candidate.' This means two things. i) Since each candidate visits each city exactly once, if there are 'c' candidates, there would be a total of 20c roads used and since no road is repeated it means that the 20 roads Candidate A uses will be different from the 20 roads Candidate B uses and so on. Thus, the value of $20c \leq 190$ should be an inequality that must be satisfied.This gives us a maximum possible value of c as 9. Hence, Option (e) is correct.

7. In a bank the account numbers are all 8 digit numbers, and they all start with the digit 2. So, an account number can be represented as $2x_1x_2x_3x_4x_5x_6x_7$. An account number is considered to be a 'magic' number if $x_1x_2x_3$ is exactly the same as $x_4x_5x_6$ or $x_5x_6x_7$ or both, x_i can take values from 0 to 9, but 2 followed by seven 0s is not a valid account number. What is the maximum possible number of customers having a 'magic' account number?

(a) 9989 (b) 19980
(c) 19989 (d) 19999
(e) 19990

Solution: This question appeared in XAT 2011. In order to solve this question, we need to think of the kinds of numbers which would qualify as magic numbers. Given the definition of a magic number in the question, a number of form 2mnpmnpq would be a magic number while at the same time a number of the form 2mnpqmnp would also qualify as a magic number. In this situation, each of m,n and p can take any of the ten digit values from 0 to 9. Also, q would also have ten different possibilities from 0 to 9. Thus, the total number of numbers of the form 2mnpmnpq would be 10^4 = 10000. Similarly, the total number of numbers of the form 2mnpqmnp would also be 10^4= 10000. This gives us a total of 20000 numbers. However, in this count the numbers like 21111111, 22222222, 23333333, 24444444 etc have been counted under both the categories. Hence we need to remove these numbers once each (a total of 9 reductions). Also, the number 20000000 is not a valid number according to the question. This number needs to be removed from both the counts.

Hence, the final answer = 20000 – 9 – 2 = 19989.

8. If all letters of the word "CHCJL" be arranged in an English dictionary, what will be the 50th word?

(a) HCCLJ (b) LCCHJ
(c) LCCJH (d) JHCLC
(e) None of the above

Solution: A Xat 2010 question. In the English dictionary the ordering of the words would be in alphabetical order. Thus, words starting with C would be followed by words starting with H, followed by words starting with J and finally words starting with L. Words starting with C = 4! = 24; Words starting with H = 4! ÷ 2! = 12 words. Words starting with J = 4! ÷ 2! = 12 words. This gives us a total of 48 words. The 49th and the 50th words would start with L. The 49th word would be the first word starting with L (=LCCHJ) and the 50th word would be the 2nd word starting with L – which would be LCCJH. Option (c) is correct.

9. The supervisor of a packaging unit of a milk plant is being pressurised to finish the job closer to the distribution time, thus giving the production staff more leeway to cater to last minute demand. He has the option of running the unit at normal speed or at 110% of normal – "fast speed". He estimates that he will be able to run at the higher speed 60% of time. The packet is twice as likely to be damaged at the higher speed which would mean temporarily stopping the process. If a packet on a randomly selected packaging runs has probability of 0.112 of damage, what is the probability that the packet will not be damaged at normal speed?

(a) 0.81 (b) 0.93
(c) 0.75 (d) 0.60
(e) None of the above

Solution: Again a XAT 2013 question. Let the probability of the package being damaged at normal speed be 'p'. This means that the probability of the damage of a package when the unit is running at a fast speed is '$2p$'. Since, he is under pressure to complete the production quickly, we would need to assume that he runs the unit at fast speed for the maximum possible time (60% of the time).

Then, we have

Probability of damaged packet in all packaging runs

$= 0.6 \times 2p + 0.4 \times p = 0.112.$

$\Rightarrow p = 0.07$

Probability of non damaged packets at normal speed

$= 1 - p = 1 - 0.07 = 0.93$. Option (b) is correct.

10. Let X be a four-digit positive integer such that the unit digit of X is prime and the product of all digits of X is also prime. How many such integers are possible?

(a) 4 (b) 8
(b) 12 (d) 24
(e) None of these

Solution: This one is an easy question as all you need to do is understand that given the unit digit is a prime number, it would mean that the number can only be of the form *abc*2; *abc*3 or *abc*5 or *abc*7. Further, for each of these, the product of the four digits $a \times b \times c \times$ units digit has to be prime. This can occur only if $a = b = c = 1$. Thus, there are only 4 such numbers viz: 1112, 1113, 1115 and 1117. Hence, Option (a) is correct.

11. The chance of India winning a cricket match against Australia is 1/6. What is the minimum number of matches India should play against Australia so that there is a fair chance of winning at least one match?

(a) 3 (b) 4
(c) 5 (d) 6
(e) None of the above

Solution: This is another question from the XAT 2009 test paper. A fair chance is defined when the probability of an event goes to above 0.5. If India plays 3 matches, the probability of at least one win will be given by the non-event of losing all matches. This would be:

$1 - (5/6)^3 = 1 - 125/216 = 91/216$ which is less than 0.5. Hence, Option (a) is rejected.

For four matches, the probability of winning at least 1 match would be:

$1 - (5/6)^4 = 1 - 625/1296 = 671/216$ which is more than 0.5. Hence, Option (b) is correct.

12. Two teams *Arrogant* and *Overconfident* are participating in a cricket tournament. The odds that team *Arrogant* will be champion is 5 to 3, and the odds that team *Overconfident* will be the champion is 1 to 4. What are the odds that either *Arrogant* or team *Overconfident* will become the champion?

(a) 3 to 2 (b) 5 to 2
(c) 6 to 1 (d) 7 to 1
(e) 33 to 7

Solution: You need to be clear about what odds for an event mean in order to solve this. Odds for team *Arrogant* to be champion being 5 to 3 means that the probability of team *Arrogant* being champion is 5/8. Similarly, the probability of team *Overconfident* being champion is 1/5 (based on odds of team *Overconfident* being champion being 1 to 4). Thus, the probability that either of the teams would be the champion would be

$$= \frac{5}{8} + \frac{1}{5} = \frac{33}{40}$$

This means that in 40 times, 33 times the event of one of the teams being champion would occur. Hence, the odds for one of the two given teams to be the chamion would be 33 to 7.

So required odds will be 33 to 7. Option (e) is correct.

13. Let X be a four-digit number with exactly three consecutive digits being same and is a multiple of 9. How many such X's are possible?

(a) 12 (b) 16
(c) 19 (d) 21
(e) None of the above

Solution: Since the number has to be a multiple of 9, the sum of the digits would be either 9 or 18 or 27. Also, the number would either be in the form aaab or baaa. For the sum of the digits to be 9, we would have the following cases:

$a = 1$ and $b = 6$ for the numbers 1116 and 6111;

$a = 2$ and $b = 3$ for the numbers 2223 and 3222;

$a = 3$ and $b = 0$ for the number 3330 and

$b = 9$ and $a = 0$ for the number 9000. We get a total of 6 such numbers.

Similarly for the sum of the digits to be 18 we will get:

3339, 9333; 4446, 6444; 5553, 3555; 6660. We get a total of 7 such numbers.

For the sum of the digits to be 27 we will get the numbers:

6669, 9666; 7776, 6777; 8883, 3888 and 9990. Thus, we get a total of 7 such numbers. Hence, the total number of numbers is 20. Option (e) is correct.

14. A shop sells two kinds of rolls—egg roll and mutton roll. Onion, tomato, carrot, chilli sauce and tomato sauce are the additional ingredients. You can have any combination of additional ingredients, or have standard rolls without any additional ingredients subject to the following constraints:

(a) You can have tomato sauce if you have an egg roll, but not if you have a mutton roll.
(b) If you have onion or tomato or both you can have chilli sauce, but not otherwise.

How many different rolls can be ordered according to these rules?

(a) 21 (b) 33
(c) 40 (d) 42
(e) None of the above.

Solution: Let the 5 additional ingredients onion, tomato, carrot, chilli sauce and tomato sauce are denoted by O, T, C, CS, TS respectively.

Number of ways of ordering the egg roll:

For the egg roll there are a total of 32 possibilities (with each ingredient being either present or not present – there being 5 ingredients the total number of possibilities of the combinations of the egg rolls would be equal to $2 \times 2 \times 2 \times 2 \times 2 = 32$ ways).

However, out of these 32 instances, the following combinations are not possible due to the constraint given in Statement (b) which tells us that to have CS in the roll either of onion or tomato must be present (or both should be present). The combinations which are not possible are:

(CS) (CS, TS) (CS, C) (CS, C, TS)

Total number of ways egg roll can be ordered

= 32 – 4 = 28.

Number of ways of ordering the mutton roll:

Total number of cases for mutton roll without any constraints = $2 \times 2 \times 2 \times 2 = 16$ ways. Cases rejected due to constraint given in statement (b): (CS); (CS,C) $\rightarrow$ 16 – 2 = 14 cases.

Total number of ways or ordering a roll = 28 + 14 = 42. Option (d) is correct.

15. Steel Express stops at six stations between Howrah and Jamshedpur. Five passengers board at Howrah. Each passenger can get down at any station till Jamshedpur. The probability that all five persons will get down at different stations is:

(a) $\frac{{}^6P_5}{6^5}$ (b) $\frac{{}^6C_5}{6^5}$

(c) $\frac{{}^7P_5}{7^5}$ (d) $\frac{{}^6C_5}{7^5}$

(e) None of the above.

Solution: The required probability would be given by:

$$\frac{\left(\begin{array}{c}\text{Total number of ways in which 5 people can get down}\\ \text{at 5 different stations from amongst 7 stations}\end{array}\right)}{\left(\begin{array}{c}\text{Total number of ways in which 5 people can get down}\\ \text{at 7 stations}\end{array}\right)}$$

The value of the numerator would be ${}^7P_5$, while the value of the denominator would be 7^5. The correct answer would be Option (c).

16. In how many ways can 53 identical chocolates be distributed amongst 3 children– C_1, C_2 and C_3 – such that C_1 gets more chocolates than C_2 and C_2 gets more chocolates than C_3?

(a) 468 (b) 344
(c) 1404 (d) 234

Solution: 53 identical chocolates can be distributed amongst 3 children in $^{55}C_2$ ways =1485 ways ($^{n+r-1}C_{r-1}$ formula). Out of these ways of distributing 53 chocolates, the following distributions methods are not possible as they would have two values equal to each other– (0, 0, 53); (1, 1, 51); (2, 2, 49)….(26, 26, 1).

There are 27 such distributions, but when allocated to C_1, C_2 and C_3 respectively, each of these distributions can be allocated in 3 ways amongst them. Thus, $C_1= 0$, $C_2 = 0$ and $C_3= 53$ is counted differently from $C_1 = 0$, $C_2 = 53$ and $C_3 = 0$ and also from $C_1 = 53$, $C_2 = 0$ and $C_3 = 0$. This will remove $27 \times 3 = 81$ distributions from 1485, leaving us with 1404 distributions. These 1404 distributions are those where all three numbers are different from each other. However, whenever we have three different values allocated to three children, there can be 3! = 6 ways of allocating the three different values amongst the three people. For instance, the distribution of 10, 15 and 48 can be seen as follows:

C_1	C_2	C_3	
48	15	10	Only case which meets the problems' requirement.
48	10	15	
15	48	10	
15	10	48	
10	15	48	
10	48	15	

Hence, out of every six distributions counted in the 1404 distributions we currently have, we need to count only one. The answer can be arrived at by dividing 1404 ÷ 6 = 234. Option (d) is correct.

17. In a chess tournament at the ancient Olympic Games of Reposia, it was found that the number of European participants was twice the number of non-European participants. In a round robin format, each player played every other player exactly once. The tournament rules were such that no match ended in a draw – any conventional draws in chess were resolved in favour of the player who had used up the lower time. While analysing the results of the tournament, K.Gopal the tournament referee observed that the number of matches won by the non-European players was equal to the number of matches won by the European players. Which of the following can be the total number of matches in which a European player defeated a non-European player?

(a) 57 (b) 58
(c) 59 (d) 60

Solution: If we assume the number of non-European players to be n, the number of European players would be 2n. Then there would be three kinds of matches played –

Matches between two Europan players – a total of $^{2n}C_2$ matches – which would yield a European winner.

Matches between two non-European players – a total of $^{n}C_2$ matches, – which would yield a non-European winner.

Matches, between a European and a non-European player = $2n^2$. These matches would have some European wins and some non-European wins. Let the number of European wins amongst these matches be x, then the number of non-European wins = $2n^2 - x$.

Now, the problem clearly states that the number of European wins = Number of non-European wins

$$\Rightarrow \quad \frac{2n(2n-1)}{2} + x = \frac{n(n-1)}{2} + 2n^2 - x$$

$$\Rightarrow \quad n(n+1) = 4x$$

This means that the value of 4 times the number of wins for a European player over a non-European player should be a product of two consecutive natural numbers (since n has to be a natural number).

Among the options, $n = 60$ is the only possible value as the value of $4 \times 60 = 15 \times 16$.

Hence, Option (d) is correct.

18. A man, starting from a point M in a park, takes exactly eight equal steps. Each step is in one of the four directions – East, West, North and South. What is the total number of ways in which the man ends up at point M after the eight steps?

(a) 4200 (b) 2520
(c) 4900 (d) 5120

Solution: For the man to reach back to his original point, the number of steps North should be equal to the number of steps South. Similarly, the number of steps East should be equal to the number of steps West.

The following cases would exist:

4 steps north and 4 steps south = $8!/(4! \times 4!) = 70$ ways;

3 steps north, 3 steps south, 1 step east and 1 step west = $8!/(3! \times 3!) = 1120$ ways;

2 steps north, 2 steps south, 2 steps east and 2 steps west = $8!/(2! \times 2! \times 2! \times 2!) = 2520$ ways;

1 step north, 1 step south, 3 steps east and 3 steps west = $8!/(3! \times 3!) = 1120$ ways;

4 Steps east and 4 steps west = $8!/(4! \times 4!) = 70$ ways;

Thus, the total number of ways = $70 \times 2 + 1120 \times 2 + 2520 = 140 + 2240 + 2520 = 4900$ ways.

Option (c) is correct.

BLOCK REVIEW TESTS

REVIEW TEST 1

1. 18 guests have to be seated, half on each side of a long table. 4 particular guests desire to sit on one particular side and 3 others on the other side. Determine the number of ways in which the sitting arrangements can be made
(a) ${}^{11}P_n \times (9!)^2$ (b) ${}^{11}C_5 \times (9!)^2$
(c) ${}^{11}P_6 \times (9!)^2$ (d) None of these
2. If m parallel lines in a plane are intersected by a family of n parallel lines, find the number of parallelograms that can be formed.
(a) $m^2 \times n^2$ (b) $m^{(m+1)}n^{(n+1)}/4$
(c) ${}^mC_2 \times {}^nC_2$ (d) None of these
3. A father with eight children takes three at a time to the zoological garden, as often as he can without taking the same three children together more than once. How often will he go and how often will each child go?
(a) ${}^8C_{3,}\ {}^7C_3$ (b) ${}^8C_{3,}\ {}^7C_2$
(c) ${}^8P_{3,}\ {}^7C_3$ (d) ${}^8P_{3,}\ {}^7C_2$
4. A candidate is required to answer 7 questions out of 2 questions which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. In how many different ways can he choose the 7 questions?
(a) 390 (b) 520
(c) 780 (d) None of these
5. Find the sum of all 5 digit numbers formed by the digits 1, 3, 5, 7, 9 when no digit is being repeated.
(a) 4444400 (b) 8888800
(c) 13333200 (d) 6666600
6. Consider a polygon of n sides. Find the number of triangles, none of whose sides is the side of the polygon.
(a) $nC_3 - n - n \times (n-4)C_1$
(b) $n(n-4)(n-5)/3$
(c) $n(n-4)(n-5)/6$
(d) $n(n-1l(n-2)/3$
7. The number of 4 digit numbers that can be formed using the digits 0, 2, 3, 5 without repetition is
(a) 18 (b) 20
(c) 24 (d) 20
8. Find the total number of words that can be made by using all the letters from the word MACHINE, using them only once.
(a) 7! (b) 5020
(c) 6040 (d) 7!/2
9. What is the total number of words that can be made by using all the letters of the word REKHA, using each letter only once?
(a) 240 (b) 4!
(c) 124 (d) 5!
10. How many different 5-digit numbers can be made from the first 5 natural numbers, using each digit only once?
(a) 240 (b) 4!
(c) 124 (d) 5!
11. There are 7 seats in a row. Three persons take seats at random. What is the probability that the middle seat is always occupied and no two persons are sitting on consecutive seats?
(a) 7/70 (b) 14/35
(c) 8/70 (d) 4/35
12. Let N = 33^x. where x is any natural no. What is the probability that the unit digit of N is 3?
(a) 1/4 (b) 1/3
(c) 1/5 (d) 1/2
13. Find the probability of drawing one ace in a single draw of one card out of 52 cards.
(a) $1/(52 \times 4)$ (b) 1/4
(c) 1/52 (d) 1/13
14. In how many ways can a committee of 4 persons be made from a group of 10 people?
(a) 10! / 4! (b) 210
(c) 10! / 6! (d) None of these
15. In Question 14, what is the number of ways of forming the committee, if a particular member must be there in the committee?
(a) 12 (b) 84
(c) 9!/ 3! (d) None of these
16. A polygon has 54 diagonals. The numbers of sides of this polygon are
(a) 12 (b) 84
(c) 3. 3! (d) 4. 4!
17. An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it, the probabilities of hitting the plane at first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane?
(a) 0.7654 (b) 0.6976
(c) 0.3024 (d) 0.2346
18. 7 white balls and 3 black balls are placed in a row at random. Find the probability that no two black balls are adjacent.
(a) 2/15 (b) 7/15
(c) 8/15 (d) 4/15

19. The probability that A can solve a problem is 3/10 and that B can solve is 5/7. If both of them attempt to solve the problem, what is the probability that the problem can be solved?
(a) 3/5 (b) 1/4
(c) 2/3 (d) 4/5

20. The sides *AB*, *BC*, *CA* of a triangle *ABC* have 3, 4 and 5 interior points respectively on them. Find the number of triangles that can be constructed using these points as vertices.
(a) 180 (b) 105
(c) 205 (d) 280

21. From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done if there is no restriction in its formation?
(a) 256 (b) 246
(c) 252 (d) 260

22. From 4 officers and 8 jawans in how many ways can 6 be chosen to include exactly one officer?
(a) $^{12}C_6$ (b) 1296
(c) 1344 (d) 224

23. From 4 officers and 8 jawans in how many ways can 6 be chosen to include atleast one officer?
(a) 868 (b) 924
(c) 896 (d) none of these

24. Two cards are drawn one after another from a pack of 52 ordinary cards. Find the probability that the first card is an ace and the second drawn is an honour card if the first card is not replaced while drawing the second.
(a) 12/13 (b) 12/51
(c) 1/663 (d) None of these

25. The probability that Andrews will be alive 15 years from now is 7/15 and that Bill will be alive 15 years from now is 7/10. What is the probability that both Andrews and Bill will be dead 15 years from now?
(a) 12/150 (b) 24/150
(c) 49/150 (d) 74/150

Space for Rough Work

REVIEW TEST 2

1. A group consists of 100 people; 25 of them are women and 75 men; 20 of them are rich and the remaining poor; 40 of them are employed. The probability of selecting an employed rich woman is:
   (a) 0.05 (b) 0.04
   (c) 0.02 (d) 0.08
2. Out of 13 job applicants, there are 5 boys and 8 men. It is desired to choose 2 applicants for the job. The probability that at least one of the selected applicant will be a boy is:
   (a) 5/13 (b) 14/39
   (c) 25/39 (d) 10/13
3. Fours dogs and three pups stand in a queue. The probability that they will stand in alternate positions is:
   (a) 1/34 (b) 1/35
   (c) 1/17 (d) 1/68
4. Asha and Vinay play a number game where each is asked to select a number from 1 to 5. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is:
   (a) 1/25 (b) 24/25
   (c) 2/25 (d) None of these
5. The number of ways in which 6 British and 5 French can dine at a round table if no two French are to sit together is given by:
   (a) $6! \times 5!$ (b) $5! \times 4!$
   (c) 30 (d) $7! \times 5!$
6. A cricket team of 11 players is to be formed from 20 players including 6 bowlers and 3 wicketkeepers. Find the number of ways in which a team can be formed having exactly 4 bowlers and 2 wicketkeepers:
   (a) 20790 (b) 6930
   (c) 10790 (d) 360
7. Three boys and three girls are to be seated around a circular table. Among them one particular boy Rohit does not want any girl neighbour and one particular girl Shaivya does not want any boy neighbour. How many such arrangements are possible?
   (a) 5 (b) 6
   (c) 4 (d) 2
8. Words with five letters are formed from ten different letters of an alphabet. Then the number of words which have at least one letter repeated is
   (a) 19670 (b) 39758
   (c) 69760 (d) 99748
9. Sunil and Kapil toss a coin alternatively till one of them gets a head and wins the game. If Sunil starts the game, the probability that he (Sunil) will win is:
   (a) 0.66 (b) 1
   (c) 0.33 (d) None of these
10. The number of pallelograms that can be formed if 7 parallel horizontal lines intersect 6 parallel vertical lines, is:
    (a) 42 (b) 294
    (c) 315 (d) None of these
11. $1.3.5\ldots(2n-1)/2.4.6\ldots(2n)$ is equal to:
    (a) $(2n)! \div 2^n(n!))^2$ (b) $(2n)! \div n!$
    (c) $(2n-1) \div)\ n-1)!$ (d) 2^n
12. How many four-digit numbers, each divisible by 4 can be formed using the digits 1, 2, 3, 4 and 5 (repetitions allowed)?
    (a) 100 (b) 150
    (c) 125 (d) 75
13. A student is to answer 10 out of 13 questions in a test such that he/she must choose at least 4 from the first five questions. The number of choices available to him is:
    (a) 140 (b) 280
    (c) 196 (d) 346
14. The number of ways in which a committee of 3 ladies and 4 gentlemen can be appointed from a meeting consisting of 8 ladies and 7 gentlemen, if Mrs. Pushkar refuses to serve in a committee if Mr.Modi is its member, is
    (a) 1960 (b) 3240
    (c) 1540 (d) None of these
15. A room has 3 lamps. From a collection of 10 light bulbs of which 6 are not good, a person selects 3 at random and puts them in a socket. The probability that he will have light, is:
    (a) 5/6 (b) 1/2
    (c) 1/6 (d) None of these
16. Two different series of a question booklet for an aptitude test are to be given to twelve students. In how many ways can the students be placed in two rows of six each so that there should be no identical series side by side and that the students sitting one behind the other should have the same series?
    (a) $2 \times {}^{12}C_6 \times (6!)^2$ (b) $6! \times 6!$
    (c) $7! \times 7 \times$ (d) None of these
17. The letters of the word PROMISE are arranged so that no two of the vowels should come together. The total number of arrangements is:
    (a) 49 (b) 1440
    (c) 7 (d) 1898
18. Find the remainder left after dividing 1! + 2! + 3! + … + 1000! by 7.
    (a) 0 (b) 5
    (c) 21 (d) 14
19. In the McGraw-Hill Mindworkzz mock test paper, there are two sections, each containing 4 questions. A candidate is required to attempt 5 questions but

not more than 3 questions from any section. In how many ways can 5 questions be selected?

(a) 24 (b) 48

(c) 72 (d) 96

20. A bag contains 10 balls out of which 3 are pink and rest are orange. In how many ways can a random sample of 6 balls be drawn from the bag so that at the most 2 pink balls are included in the sample and no sample has all the 6 balls of the same colour?

(a) 105 (b) 168

(c) 189 (d) 120

Space for Rough Work

ANSWER KEY

Review Test 1

1. (b)	2. (c)	3. (b)	4. (c)
5. (d)	6. (a)	7. (a)	8. (a)
9. (d)	10. (d)	11. (d)	12. (a)
13. (d)	14. (b)	15. (b)	16. (a)
17. (b)	18. (b)	19. (d)	20. (c)
21. (c)	22. (d)	23. (c)	24. (d)
25. (b)			

Review Test 2

1. (c)	2. (c)	3. (b)	4. (b)
5. (b)	6. (a)	7. (c)	8. (c)
9. (c)	10. (c)	11. (a)	12. (c)
13. (a)	14. (d)	15. (d)	16. (b)
17. (b)	18. (b)	19. (b)	20. (b)