CHAPTER

2

NUMBERS AND SEQUENCES

I. EUCLID'S DIVISION LEMMA AND ALGORITHM

Key Points

✓ Let a and b ($a > b$) be any two positive integers. Then, there exist unqiue integers q and r such that $a = bq + r$, $0 \leq r < b$. (Euclid's Division Lemma)

The remainder is always less than the divisor.

If $r = 0$ then $a = bq$ so b divides a.

Similarly, if b divides a then $a = bq$.

✓ If $a$and b are any two integers then there exist unique integers q and $r$such that $a = bq + r$, where $0 \leq r < | b |$.

✓ If a and b are positive integers such that $a = bq + r$, then every common divisor of a and b is a common divisor of b and r and vice-versa. (Euclid's Division Algorithm)

✓ If a, b are two positive integers with $a > b$ then G.C.D of (a, b) = GCD of $(a - b, b)$.

✓ Two positive integers are said to be relatively prime or co prime if their. Highest Common Factor is 1.

Example 2.2

Find the quotient and remainder when a is divided by b in the following cases (i) $a = -12, b = 5$ (ii) $a = 17, b = -3$ (iii) $a = -19, b = -4$.

Solution :

(i) $a = -12, b = 5$

By Euclid's division lemma

$$a = bq + r, \text{ where } 0 \leq r < | b |$$

$$-12 = 5 \times (-3) + 3 \quad 0 \leq r < | 5 |$$

Therefore, Quotient $q = -3$, Remainder r = 3.

(ii) $a = 17 \qquad b = -3$

By Eculid's division lemma

$$a = bq + r, \text{ where } 0 \leq r < | b |$$

$$17 = (-3) \times (-5) + 2, \quad 0 \leq r < | -3 |$$

Therefore, Quotient $q = -5$, Remainder r = 2.

(iii) $a = -19 \qquad b = -4$

By Eculid's division lemma

$$a = bq + r, \text{ where } 0 \leq r < | b |$$

$$-19 = (-4) \times (5) + 1, \quad 0 \leq r < | -4 |$$

Therefore, Quotient $q = 5$, Remainder r = 1.

Example 2.3

Show that the square of an odd integer is of the form $4q + 1$, for some integer q.

Solution :

Let x be any odd integer. Since any odd integer is one more than an even integer, we have $x = 2k + 1$, for some integer k.

$$\begin{aligned} x^2 &= (2k + 1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 4k (k + 1) + 1 \\ &= 4q + 1, \end{aligned}$$

where $q = k (k + 1)$ is some integer.

Example 2.4

If the Highest Common Factor of 210 and 55 is expressible in the form $55x - 325$, find x.

Solution :

Using Euclid's Division Algorithm, let us find the HCF of given numbers

$$\begin{aligned} 210 &= 55 \times 3 + 45 \\ 55 &= 45 \times 1 + 10 \\ 45 &= 10 \times 4 + 5 \\ 10 &= 5 \times 2 + 0 \end{aligned}$$

The remainder is zero.

So, the last divisor 5 is the Highest Common Factor (HCF) of 210 and 55.

Since, HCF is expressible in the form $55x - 325 = 5$

$$\text{gives } 55x = 330$$

$$\text{Hence } x = 6$$

Example 2.5

Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.

Solution :

Since the remainders are 4, 5 respectively the required number is the HCF of the number $445 - 4 = 441$, $572 - 5 = 567$.

Hence, we will determine the HCF of 441 and 567. Using Euclid's Division Algorithm, we have

$$\begin{aligned} 567 &= 441 \times 1 + 126 \\ 441 &= 125 \times 3 + 63 \\ 126 &= 63 \times 2 + 0 \end{aligned}$$

Therefore HCF of 441,567 = 63 and so the required number is 63.

Example 2.6

Find the HCF of 396, 504, 636.

Solution :

To find HCF of 396 and 504

Using Euclid's division algorithm we get
$504 = 396 \times 1 + 108$

The remainder is $108 \neq 0$

Again applying Euclid's division algorithm
$396 = 108 \times 3 + 72$

The remainder is $72 \neq 0$,

Again applying Euclid's division algorithm
$108 = 72 \times 1 + 36$

The remainder is $36 \neq 0$,

Again applying Euclid's division algorithm
$72 = 36 \times 2 + 0$

Here the remainder is zero. Therefore HCF of 396, 504 = 36.

To find the HCF of 636 and 36.

Using Euclid's division algorithm we get
$636 = 36 \times 17 + 24$

The remainder is $24 \neq 0$

Again applying Euclid's division algorithm
$36 = 24 \times 1 + 12$

The remainder is $12 \neq 0$

Again applying Euclid's division algorithm
$24 = 12 \times 2 + 0$

Here the remainder is zero. Therefore HCF of 636, 36 = 12

Therefore Highest Common Factor of 396, 504 and 636 is 12.

EXERCISE 2.1

1. Find all positive integers, when divided by 3 leaves remainder 2.

Solution:

To find all positive intergers which when divided by 3 leaves remainder 2.

i.e. $a \equiv 2 \pmod{3}$

$\Rightarrow$ $a - 2$ is divisible by 3

$\Rightarrow$ $a = 3K + 2$

$\therefore$ a takes the following values

a = 2, 5, 8, 11, 14, ... when K = 0, 1, 2, 3, 4, ...

2. A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over

Solution :

No. of flower pots = 532

All pots to be arranged in rows

& each row to contain 21 flower pots.

$\therefore\ 532 = 21q + r$

$$\begin{array}{r|r} & 25 \\ \hline 21 & 532 \\ & 42 \\ \hline & 112 \\ & 105 \\ \hline & 7 \end{array}$$

$\Rightarrow 532 = 21 \times 25 + 7$

$\therefore$ Number of completed rows = 25

Number of flower pots left out = 7

3. Prove that the product of two consecutive positive integers is divisible by 2.

Solution :

Let the 2 consecutive positive integers be

$x, x + 1$

$\therefore$ Product of 2 integers $= x(x + 1)$

$= x^2 + x$

Case (i)

If x is an even number

Let $x = 2k$

$\therefore\ x^2 + x = (2k)^2 + 2k$

$= 2k(2k + 1)$, divisible by 2

Case (ii)

If x is an odd number,

Let $x = 2k + 1$

$x^2 + x = (2k + 1)^2 + (2k + 1)$

$= 4k^2 + 4k + 1 + 2k + 1$

$= 4k^2 + 6k + 2$

$= 2(2k^2 + 3k + 2)$ divisible by 2

$\therefore$ Product of 2 consecutive positive integers is divisible by 2.

4. When the positive integers *a, b* and *c* are divided by 13, the respective remainders are 9,7 and 10. Show that $a + b + c$ is divisible by 13.

Solution :

When a is divided by 13, remainder is 9

i.e., $a = 13q + 9$ .... (1)

When b is divided by 13, remainder is 7

i.e., $b = 13q + 7$ .... (2)

When c is divided by 13, remainder is 11

i.e., $c = 13q + 11$ .... (3)

Adding (1), (2) & (3)

$a + b + c = 39q + 26$

$= 13 (2q + 2)$

= divisible by 13

Hence proved.

---

5. Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4.

Solution :

Case (i)

Let x be the even number

Let $x = 2n$

$x^2 = (2n)^2$

$= 4n^2$, which leaves remainder 0 when divided by 4.

Case (ii)

Let x be the odd number

$\therefore$ Let $x = 2n + 1$

$x^2 = (2n + 1)^2$

$= 4n^2 + 4n + 1$

$= 4 (n^2 + n) + 1$ leaves remainder 1 when divided by 4.

Hence proved.

---

6. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of

(i) 340 and 412 (ii) 867 and 255

(iii)10224 and 9648 (iv) 84, 90 and 120

Solution :

i) HCF of 340, 412 by Euclid's algorithm.

First we should divide 412 by 340.

$412 = 340 \times 1 + 72$

$340 = 72 \times 4 + 52$

$72 = 52 \times 1 + 20$

$52 = 20 \times 2 + 12$

$20 = 12 \times 1 + 8$

$12 = 8 \times 1 + 4$

$8 = (4) \times 2 + 0$

$\therefore$ The last divisor "4" is the HCF.

ii) HCF of 867 and 255.

$867 = 255 \times 3 + 102$

$255 = 102 \times 2 + 51$

$102 = (51) \times 2 + 0$

$\therefore$ The last divisor "51" is the HCF.

iii) HCF of 10224 and 9648

$10224 = 9648 \times 1 + 576$

$9648 = 576 \times 16 + 432$

$576 = 432 \times 1 + 144$

$432 = (144) \times 3 + 0$

$\therefore$ The last divisior "144" is the HCF.

iv) HCF of 84, 90, and 120

First we find HCF of 84 & 90

$90 = 84 \times 1 + 6$

84 = (6) × 14 + 0

∴ HCF of 84, 90 is 6

Next, we find HCF of 120 and 6.

120 = 6 × 20 + 0

∴ HCF of 84, 90 and 120 = 6

---

7. Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

Solution :

To find the largest number which dividies 1230 and 1926 leaving remainder 12

i.e., HCF of 1230 - 12 and 1926 - 12

i.e., HCF of 1218 and 1914

First we divide 1914 by 1218

1914 = 1218 × 1 + 696

1218 = 696 × 1 + 522

696 = 522 × 1 + 174

522 = (174) × 3 + 0

∴ HCF = 174

∴ The required largest number = 174.

---

8. If *d* is the Highest Common Factor of 32 and 60, find *x* and *y* satisfying d = 32*x* + 60*y*.

Solution :

Given d is the HCF of 32 and 60

∴ 60 = 32 × 1 + 28 .... (1)

32 = 28 × 1 + 4 .... (2)

28 = (4) × 7 + 0

∴ $d = 4$

= 32 − (28 × 1) [From (2)]

= 32 × 1 (60 − 32) [From (1)]

= 32 − (1 × 60 − 1 × 32)

= 32 − 1 × 60 + 1 × 32

= 32 (1 + 1) + 60 (−1)

d = 32 (2) + 60 (−1)

⇒ $32x + 60y$ = 32 (2) + 60 (−1)

∴ x = 2, y = −1

---

9. A positive integer when divided by 88 gives the remainder 61. What will be the remainder when the same number is divided by 11?

Solution :

The standard form is $a = bq + r$

$\Rightarrow a = 88q + 61$

$a = (11 \times 8q) + (55 + 6)$

$a = 11\,[8q + 5] + 6$

∴ When the same positive integer is divided by 11 the remainder is 6.

---

10. Prove that two consecutive positive integers are always coprime.

Solution :

Let x, $x + 1$ be two consecutive integers.

Let G.C.D. of (x, $x + 1$) be 'n'

Then 'n' divides $x + 1 - x$

i.e., $n = 1$

G.C.D. of (x, $x + 1$) = 1

x & $x + 1$ are Co-prime.

II. FUNDAMENTAL THEOREM OF ARITHMETIC :

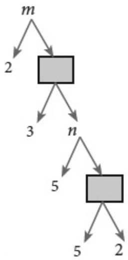
Key Points

- ✓ Every natural number except 1 can be factorized as a product of primes and this factorization is unique except for the order in which the prime factors are written.
- ✓ If a prime number p divides ab then either p divides a or p divides b. That is p divides at least one of them.
- ✓ If a composite number n divides ab, then n neither divide a nor b. For example, 6 divides 4×3 but 6 neither divide 4 nor 3.

Example 2.7

In the given factor tree, find the numbers m and n.

Solution :



Value of the first box from bottom = $5 \times 2 = 10$

Value of $n = 5 \times 10 = 50$

Value of the second box from bottom

$= 3\times50 = 150$

Value of $m = 2\times150 = 300$

Thus, the required numbers are $m = 300$, $n = 50$.

Example 2.8

Can the number 6^n, n being a natural number end with the digit 5 ? Give reason for your answer.

Solution :

Since $6^n = (2 \times 3)^n = 2^n \times 3^n$,

2 is a factor of 6^n. So, 6^n is always even.

But any number whose last digit is 5 is always odd.

Hence, 6^n cannot end with the digit 5.

Example 2.9

Is $7 \times 5 \times 3 \times 2 + 3$ a composite number ? Justify your answer.

Solution :

Yes, the given number is a composite number, because

$7 \times 5 \times 3 \times 2 + 3 = 3\times(7\times5\times2+1) = 3 \times 71$

Since the given number can be factorized in terms of two primes, it is a composite number.

Example 2.10

'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

Solution :

The number 800 can be factorized as

$800= 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^5 \times 5^2$

Hence, $a^b \times b^a = 2^5 \times 5^2$

This implies that $a = 2$ and $b = 5$ (or) $a = 5$ and $b = 2$.

EXERCISE 2.2

1. For what values of natural number n, 4^n can end with the digit 6?

Solution :

$4^n = (2 \times 2)^n$

$= 2^n \times 2^n$

Since 2 is a factor of 4, 4^n is always even.

∴ If 4^n is to be end with 6, n should be even.

∴ For the even powers of 'n', 4^n will be ended with even no.

---

2. If m, n are natural numbers, for what values of m, does $2^n \times 5^m$ ends in 5?

Solution :

Let $x = 2^n \times 5^m$

Since m and n are natural, numbers and 2^n is even.

for any value of m, $2^n \times 5^m$ will not be ended in 5.

∴ No value of m will make x true.

---

3. Find the HCF of 252525 and 363636.

Solution :

Given numbers are 252525, 363636

By using prime factorization

5	252525
5	50505
3	10101
7	3367
	481

2	363636
2	181818
3	90909
3	30303
3	10101
7	3367
	481

∴ $252525 = 5 \times 5 \times \underline{3} \times \underline{7} \times \underline{481}$

$363636 = 2 \times 2 \times \underline{3} \times 3 \times 3 \times \underline{7} \times \underline{481}$

∴ HCF $= 3 \times 7 \times 481$

$= 10,101$

---

4. If $13824 = 2^a \times 3^b$ then find a and b.

Solution :

Given $2^a \times 3^b = 13824$

$\Rightarrow$ $2^a \times 3^b = 2^9 \times 3^2$

$\therefore\ a = 9, b = 2$

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	3

---

5. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4.

Solution :

$p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$

2	113400
2	56700
2	28350
3	14175
3	4725
3	1575
3	525
5	175
5	35
	7

$\therefore\ 113400 = 2^3 \times 3^4 \times 5^2 \times 7^1$

$\therefore\ p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7$ and

$x_1 = 3, x_2 = 4, x_3 = 2, x_4 = 1$

---

6. Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.

Solution :

Given no's are 408, 170

$$\begin{array}{r|l} 2 & 408 \\ \hline 2 & 204 \\ \hline 2 & 102 \\ \hline 3 & 51 \\ \hline & 17 \end{array} \qquad \begin{array}{r|l} 2 & 170 \\ \hline 5 & 85 \\ \hline & 17 \end{array}$$

$\therefore\ 408 = 2^3 \times 3 \times 17$

$170 = 2 \times 5 \times 17$

$\therefore$ H.C.F $= 2 \times 17 = 34$

L.C.M $= 2^3 \times 17 \times 5 \times 3 = 2040$

---

7. Find the greatest number consisting of 6 digits which is exactly divisible by 24, 15, 36?

Solution :

First, we find the L.C.M of 24, 15, 36

$$\begin{array}{r|lll} 3 & 24, & 15, & 36 \\ \hline 2 & 8, & 5, & 12 \\ \hline 2 & 4, & 5, & 6 \\ \hline & 2, & 5, & 3 \end{array}$$

L.C.M $= 5 \times 3^2 \times 2^3$

$= 5 \times 9 \times 8$

$= 360$

The greatest 6 digit no. is 999999

$$\begin{array}{r|l|l} 360 & 999999 & 277 \\ & 720 & \\ \hline & 2799 & \\ & 2520 & \\ \hline & 2799 & \\ & 2520 & \\ \hline & 279 & \\ \hline \end{array}$$

$\therefore$ Required greatest number

$= 999999 - 279$ (remainder)

$= 999720$

---

8. What is the smallest number that when divided by three numbers such as 35, 56 and 91 leaves remainder 7 in each case?

Solution :

The required number is the LCM of (35, 56, 91) + remainder 7

$35 = 7 \times 5$

$56 = 7 \times 2 \times 2 \times 2$

$91 = 7 \times 13$

$\therefore$ L.C.M $= 7 \times 5 \times 13 \times 8$

$= 3640$

$\therefore$ The required number is $3640 + 7$

$= 3647$

---

9. Find the least number that is divisible by the first ten natural numbers.

Solution :

The required number is the LCM of

(1, 2, 3, ... 10)

$2 = \underline{2} \times 1$

$4 = \underline{2} \times 2$

$6 = 3 \times \underline{2}$

$8 = 2 \times 2 \times \underline{2}$

$9 = 3 \times 3$

$10 = 5 \times \underline{2}$ and 1, 3, 5, 7

L.C.M $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$= 2520$

III. MODULAR ARITHMETIC :

Key Points

- ✓ Two integers a and b are congruence modulo n if they differ by an integer multiple of n. That $b - a = kn$ for some integer k. This can also be written as $a \equiv b \pmod{n}$.
- ✓ Here the number n is called modulus. In other words, $a \equiv b \pmod{n}$ means $a - b$ is divisible by n.
- ✓ The equation $n = mq + r$ through Euclid's Division lemma can also be written as $n \equiv r$ (mod m).
- ✓ Two integers a and b are congruent modulo m, written as $a \equiv b \pmod{m}$, if they leave the same remainder when divided by m.
- ✓ a, b, c and d are integers and m is a positive integer such that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then

  (i) $(a + c) \equiv (b + d) \pmod{m}$ (ii) $(a - c) \equiv (b - d) \pmod{m}$ (iii) $(a \times c) \equiv (b \times d) \pmod{m}$
- ✓ If $a \equiv b \pmod{m}$ then

  (i) $ac \equiv bc \pmod{m}$ (ii) $a \pm c \equiv b \pm c \pmod{m}$ for any integer c

Example 2.11

Find the remainders when 70004 and 778 is divided by 7.

Solution :

Since 70000 is divisible by 7

$$\begin{aligned} 70000 &\equiv 0 \pmod{7} \\ 70000 + 4 &\equiv 0 + 4 \pmod{7} \\ 70004 &\equiv 4 \pmod{7} \end{aligned}$$

Therefore, the remainder when 70004 is divided by 7 is 4.

Since 777 is divisible by 7

$$\begin{aligned} 777 &\equiv 0 \pmod{7} \\ 777 + 1 &\equiv 0 + 1 \pmod{7} \\ 778 &\equiv 1 \pmod{7} \end{aligned}$$

Therefore, the remainder when 778 is divided by 7 is 1.

Example 2.12

Determine the value of d such that $15 \equiv 3 \pmod{d}$.

Solution :

$15 \equiv 3 \pmod{d}$ means $15 - 3 = kd$, for some integer k.

$12 = kd$. gives d divides 12.

The divisors of 12 are 1, 2, 3, 4, 6, 12. But d should be larger than 3 and so the possible values for d are 4, 6, 12.

Example 2.13

Find the least positive value of x such that

(i) $67 + x \equiv 1 \pmod{4}$ (ii) $98 \equiv (x + 4) \pmod{5}$

Solution :

(i) $67 + x \equiv 1 \pmod{4}$

$67 + x - 1 = 4n$, for some integer n

$66 + x = 4n$

$66 + x$ is a multiple of 4.

Therefore, the least positive value of x must be 2, since 68 is the nearest multiple of 4 more than 66.

(ii) $98 \equiv (x + 4) \pmod 5$

$98 - (x + 4) = 5n$, for some integer n.

$94 - x = 5n$

$94 - x$ is a multiple of 5.

Therefore, the least positive value of x must be 4

Since $94 - 4 = 90$ is the nearest multiple of 5 less than 94.

Example 2.14

Solve $8x \equiv 1 \pmod{11}$

Solution :

$8x \equiv 1 \pmod{11}$ can be written as $8x - 1 = 11k$, for some integer k.

$$x = \frac{11k + 1}{8}$$

When we put $k = 5, 13, 21, 29, \ldots$ then $11k + 1$ is divisible by 8.

$$x = \frac{11 \times 5 + 1}{8} = 7$$

$$x = \frac{11 \times 13 + 1}{8} = 18$$

Therefore, the solutions are 7, 18, 29, 40, ...

Example 2.15

Compute x, such that $10^4 \equiv x \pmod{19}$

Solution :

$10^2 = 100 \equiv 5 \pmod{19}$

$10^4 = (10^2)^2 \equiv 5^2 \pmod{19}$

$10^4 = 25 \pmod{19}$

$10^4 = 6 \pmod{19}$ (since $25 = 6 \pmod{19}$)

Therefore, $x = 6$.

Example 2.16

Find the number of integer solutions of $3x \equiv 1 \pmod{15}$.

Solution : $3x \equiv 1 \pmod{15}$ can be written as

$3x - 1 = 15k$ for some integer k

$$3x = 15k + 1$$

$$x = \frac{15k + 1}{3}$$

$$x = 5k + \frac{1}{3}$$

Since $5k$ is an integer, $5k + \frac{1}{3}$ cannot be an integer.

So there is no integer solution.

Example 2.17

A man starts his journey from Chennai to Delhi by train. He starts at 22.30 hours on Wednesday. If it takes 32 hours of travelling time and assuming that the train is not late, when will he reach Delhi ?

Solution :

Starting time 22.30, Travelling time 32 hours. Here we use modulo 24.

The reaching time is

$22.30 + 32 \pmod{24} \equiv 54.30 \pmod{24}$

$\equiv 6.30 \pmod{24}$

(Since $32 = (1 \times 24) + 8$

Thursday Friday)

Example 2.18

Kala and Vani are friends, Kala says, "Today is my birthday" and she asks Vani, "When will you celebrate your birthday?" Vani replies, "Today is Monday and I celebrated my birthday 75 days ago". Find the day when Vani celebrated her birthday.

Solution :

Let us associate the numbers 0, 1, 2, 3, 4, 5, 6 to represent the weekdays from Sunday to Saturday respecitvely.

Vani says today is Monday. So the number for Monday is 1. Since Vani's birthday was 75 days ago, we have to subtract 75 from 1 and take the modulo 7, since a week contain 7 days.

$-74 \pmod 7 \equiv -4 \pmod 7 \equiv 7-4 \pmod 7 \equiv 3 \pmod 7$

(Since, $-74-3=-77$ is divisible by 7)

Thus, $1-75 \equiv 3 \pmod 7$

The day for the number 3 is Wednesday.

Therefore, Vani's birthday must be on Wednesday.

EXERCISE 2.3

1. Find the least positive value of x such that

(i) $71 \equiv x \pmod 8$ (ii) $78 + x \equiv 3 \pmod 5$

(iii) $89 \equiv (x+3) \pmod 4$

(iv) $96 \equiv \frac{x}{7} \pmod 5$ (v) $5x \equiv 4 \pmod 6$

Solution :

i) $71 \equiv x \pmod 8$

$71 - x = 8n$

$\therefore$ $71 - x$ is a multiple of 8

$\therefore$ The least +ve value of x is 7

($\because$ $71 - 7 = 64$ is the nearest multiple of 8) less than 71.

ii) $78 + x = 3 \pmod 5$

$\Rightarrow 78 + x - 3 = 5n$

$\Rightarrow 75 + x$ is a multiple of 5

$\therefore$ The least +ve value of x is 5

($\because$ $75 + 5 = 80$, is the nearest multiple of 5 above 75)

iii) $89 \equiv (x + 3) \pmod 4$

$\Rightarrow 89 - x - 3 = 4n$

$\Rightarrow 86 - x$ is a multiple of 4

$\therefore$ The least +ve value is 2

($\because$ $86 - 2 = 84$ is the nearest multiple of 4 less than 86)

iv) $96 \equiv \frac{x}{7} \pmod 5$

$\Rightarrow 96 - \frac{x}{7} = 5n$

$\Rightarrow 96 - \frac{x}{7}$ is a multiple of 5

$\therefore$ The least +ve value of x is 7

($\because$ $96 - 1 = 95$ is a multiple of 5)

v) $5x \equiv 4 \pmod 6$

$\Rightarrow 5x - 4 = 6n$

$\therefore$ $5x - 4$ is a multiple of 6

$\therefore$ The least +ve value of x is 2

($\because$ $5(2) - 4 = 6$ is a multiple of 6)

2. If x is congruent to 13 modulo 17 then $7x - 3$ is congruent to which number modulo 17?

Solution :

Given $x \equiv 13 \pmod{17}$

$\Rightarrow$ $x - 13$ is a multiple of 17

$\therefore$ The least +ve value of x is 30

$\therefore$ $7x - 3 \equiv y \pmod{17}$

$\Rightarrow$ $7(30) - 3 \equiv y \pmod{17}$

$\Rightarrow$ $207 \equiv y \pmod{17}$

$\therefore$ $y = 3$ ($\because$ $207 - 3 = 204$ is divisible by 17)

3. Solve $5x \equiv 4 \pmod 6$

Solution :

Given $5x \equiv 4 \pmod 6$

$\Rightarrow$ $5x - 4$ is divisible by 6

$\therefore$ $x = 2, 8, 14, \ldots\ldots\ldots$ (by assumption)

4. Solve $3x - 2 \equiv 0 \pmod{11}$

Solution :

Given $3x - 2 \equiv 0 \quad (\text{mod } 11)$

$\therefore$ $3x - 2$ is divisible by 11

$\therefore$ The possible values of x are 8, 19, 30, ...

---

5. What is the time 100 hours after 7 a.m.?

Solution :

Formula :

$t + n = f \quad (\text{mod } 24)$

$t \rightarrow$ current time

$n \rightarrow$ no. of hrs.

$f \rightarrow$ future time

$100 + 7 = f\,(\text{mod } 24)$

$\Rightarrow$ $107 - f$ is divisible by 24

$\therefore$ $f = 11$ so that $107 - 11 = 96$ is divisible by 24.

$\therefore$ The time is 11 A.M.

---

6. What is the time 15 hours before 11 p.m.?

Solution :

Formula :

$t - n \equiv p \,(\text{mod } 24)$

$\Rightarrow 11 - 15 = -4 \equiv -1 \times 24 + 20$

$\equiv 20 \,(\text{mod } 24)$

$t \rightarrow$ Current time

$n \rightarrow$ no of hrs

$p \rightarrow$ past time

$\therefore$ The time 15 hours in the past was 8 p.m.

---

7. Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?

Solution :

Today is Tuesday

Day after 45 days = ?

When we divide 45 by 7, remainder is 3.

$\therefore$ The 3rd day from Tuesday is Friday

---

8. Prove that $2^n + 6 \times 9^n$ is always divisible by 7 for any positive integer *n*.

Solution :

When n = 1, 2n + 6 × 9n

$= 2 + (6 \times 9) = 56$, divisible by 7.

---

9. Find the remainder when 2^{81} is divided by 17.

Solution :

To find the remainder when 2^{81} is divided by 17.

$2^4 = 16 = -1 \,(\text{mod } 17)$

$\Rightarrow \quad 2^{80} = (2^4)^{20} = (-1)^{20} = 1$

$\therefore \quad 2^{81} = 2^{80} \times 2^1$

$= 1 \times 2$

$= 2$

---

10. The duration of flight travel from Chennai to London through British Airlines is approximately 11 hours. The airplane begins its journey on Sunday at 23:30 hours. If the time at Chennai is four and half hours ahead to that of London's time, then find the time at London, when will the flight lands at London Airport.

Solution :

Formula :

$t + n \equiv f\,(\text{mod } 24)$

$\Rightarrow 23.30 + 11 = f\,(\text{mod } 24)$

$\Rightarrow 34.30 = f\,(\text{mod } 24)$

$\therefore$ $34.30 - f$ is divisible by 24

$\Rightarrow \quad f = 10.30$ (a.m)

$t \rightarrow$ present time

$n \rightarrow$ no of hours

$f \rightarrow$ future time

But the time difference between London & Chennai is 4.30 hrs.

$\therefore$ Flight reaches London Airpot at

= 10.30 hrs − 4.30 hrs

= 6 AM next day   i.e. 6 AM on Monday

IV. SEQUENCES :

Key Points

- ✓ A real valued sequence is a function defined on the set of natural numbers and taking real values.
- ✓ A sequence can be considered as a function defined on the set of natural numbers.
- ✓ Though all the sequences are functions, not all the functions are sequences.

Example 2.19

Find the next three terms of the sequences.

(i) $\frac{1}{2}, \frac{1}{6}, \frac{1}{14}, ...$ (ii) 5, 2 – 1, – 4, ...

(iii) 1, 0.1, 0.01, ...

Solution :

(i)

$$\begin{tikzcd}
\frac{1}{2}, \arrow[r, bend right, "+4"'] & \frac{1}{6}, \arrow[r, bend right, "+4"'] & \frac{1}{10}, \arrow[r, bend right, "+4"'] & \frac{1}{14}, & \cdots
\end{tikzcd}$$

In the above sequence the numerators are same and the denominator is increased by 4.

So the next three terms are

$$a_5 = \frac{1}{14+4} = \frac{1}{18}$$

$$a_6 = \frac{1}{18+4} = \frac{1}{22}$$

$$a_7 = \frac{1}{22+4} = \frac{1}{26}$$

(ii)

$$\begin{tikzcd}
5, \arrow[r, bend right, "-3"'] & 2, \arrow[r, bend right, "-3"'] & -1, \arrow[r, bend right, "-3"'] & -4, & \cdots
\end{tikzcd}$$

Here each term is decreased by 3. So the next three terms are – 7, – 10, – 13.

(iii)

$$\begin{tikzcd}
1, \arrow[r, bend right, "+10"'] & 0.1, \arrow[r, bend right, "+10"'] & 0.01, \ldots
\end{tikzcd}$$

Here each term is divided by 10. Hence, the next three terms are

$$a_4 = \frac{0.01}{10} = 0.001$$

$$a_5 = \frac{0.001}{10} = 0.0001$$

$$a_6 = \frac{0.0001}{10} = 0.00001$$

Example 2.20

Find the general term for the following sequences

(i) 3, 6, 9, ... (ii) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, ...$

(iii) 5, – 25, 125. ....

Solution :

(i) 3, 6, 9, ...

Here the terms are multiples of 3. So the general term is

$a_n = 3n,$

(ii) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, ...$

$$a_1 = \frac{1}{2};\ a_2 = \frac{2}{3};\ a_3 = \frac{3}{4}$$

We see that the numerator of n^{th} terms is n, and the denominator is one more than the numerator. Hence, $a_n = \frac{n}{n+1}$, $n \in \mathrm{N}$

(iii) 5, –25, 125, ...

The terms of the sequence have + and – sign alternatively and also they are in powers of 5.

So the general terms $a_n = (-1)^{n+1} 5^n$, $n \in \mathrm{N}$

Example 2.21

The general term of a sequence is defined as

$$a_n = \begin{cases} n(n+3);\ n \in N \text{ is odd} \\ n^2+1 \quad ;\ n \in N \text{ is even} \end{cases}$$

Find the eleventh and eighteenth terms.

Solution :

To find a_{11}, since 11 is odd, we put

$n = 11$ in $a_n = n\ (n + 3)$

Thus, the eleventh term

$a_{11} = 11\ (11 + 3) = 154,$

To find a_{18}, since 18 is even we put

$n = 18$ in $a_n = n^2 + 1$

Thus, the eighteenth term

$a_{18} = 18^2 + 1 = 325.$

Example 2.22

Find the first five terms of the following sequence.

$$a_1 = 1,\ a_2 = 1,\ a_n = \frac{a_{n-1}}{a_{n-2}+3};\ n \geq 3,\ n \in N$$

Solution :

The first two terms of this sequence are given by $a_1 = 1$. $a_2 = 1$. The third term a_3 depends on the first and second terms.

$$a_3 = \frac{a_{3-1}}{a_{3-2}+3} = \frac{a_2}{a_1+3} = \frac{1}{1+3} = \frac{1}{4}$$

Similarly the fourth term a_4 depends upon a_2 and a_3.

$$a_4 = \frac{a_{4-1}}{a_{4-2}+3} = \frac{a_3}{a_2+3} = \frac{\frac{1}{4}}{1+3} = \frac{\frac{1}{4}}{4} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

In the same way, the fifth term a_5 can be calculated as

$$a_5 = \frac{a_{5-1}}{a_{5-2}+3} = \frac{a_4}{a_3+3} = \frac{\frac{1}{16}}{\frac{1}{4}+3} = \frac{1}{16} \times \frac{4}{13} = \frac{1}{52}$$

Therefore, the first five terms of the sequence are 1, 1, $\frac{1}{4}, \frac{1}{16}, \frac{1}{25}$

EXERCISE 2.4

1. **Find the next three terms of the following sequence.**

   (i) 8, 24, 72, … **(ii) 5, 1,-3,…**

   (iii) $\mathbf{\frac{1}{4}, \frac{2}{9}, \frac{3}{16}}$, **………**

Solution :

i) Given sequence is 8, 24, 72,..........

Each number is multiplied by 3

∴ The next 3 terms in the sequence are

72 × 3 = 216

216 × 3 = 648

648 × 3 = 1944

∴ 216, 648, 1944

ii) Given sequence is 5, 1, −3, ...

Each number is subtracted by 4

$-\ 3 - 4 = -\ 7$

$-\ 7 - 4 = -\ 11$

$-\ 11 - 4 = -\ 15$

∴ The next 3 terms in the sequence are − 7, − 11, − 15

---

iii) Given sequence is $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}$, ...

Each no. in Numerator is increased by 1 & all nos in denominator are consecutive square no's

∴ The next 3 terms are

$$\frac{4}{25}, \frac{5}{36}, \frac{6}{49}, \ldots\ldots\ldots$$

2. Find the first four terms of the sequences whose n^{th} terms are given by

(i) $a_n = n^3 - 2$ (ii) $a_n = (-1)^{n+1}$ $n\,(n+1)$ (iii) $a_n = 2n^2 - 6$

Solution :

i) Given $a_n = n^3 - 2$

$n = 1 \Rightarrow a_1 = 1^3 - 2 = 1 - 2 = -1$

$n = 2 \Rightarrow a_2 = 2^3 - 2 = 8 - 2 = 6$

$n = 3 \Rightarrow a_3 = 3^3 - 2 = 27 - 2 = 25$

$n = 4 \Rightarrow a_4 = 4^3 - 2 = 64 - 2 = 62$

$\therefore$ The first 4 terms are $-1, 6, 25, 62$

ii) Given $a_n = (-1)^{n+1}\, n\,(n + 1)$

$n = 1 \Rightarrow a_1 = (-1)^2 \,.\, 1\,(2) = 2$

$n = 2 \Rightarrow a_2 = (-1)^3 \,.\, 2\,(3) = -6$

$n = 3 \Rightarrow a_3 = (-1)^4 \,.\, 3\,(4) = 12$

$n = 4 \Rightarrow a_4 = (-1)^5 \,.\, 4\,(5) = -20$

$\therefore$ The first 4 terms are $2, -6, 12, -20$

iii) Given $a_n = 2n^2 - 6$

$n = 1 \Rightarrow a_1 = 2(1) - 6 = -4$

$n = 2 \Rightarrow a_2 = 2(4) - 6 = 2$

$n = 3 \Rightarrow a_3 = 2(9) - 6 = 12$

$n = 4 \Rightarrow a_4 = 2(16) - 6 = 26$

$\therefore$ The first 4 terms are $-4, 2, 12, 26$

---

3. Find the n^{th} term of the following sequences

(i) 2,5,10,17, ... (ii) 0, $\frac{1}{2}$, $\frac{2}{3}$, ...
(iii) 3,8,13,18, ...

Solution :

i) Given sequence is 2, 5, 10, 17, ...

$\Rightarrow 1^2 + 1, 2^2 + 1, 3^2 + 1, 4^2 + 1,$ ....

$\therefore$ The n^{th} term of the sequence is

$$a_n = n^2 + 1$$

ii) Given sequence is $0, \frac{1}{2}, \frac{1}{3},$ ....

$$\Rightarrow \frac{0}{1}, \frac{1}{2}, \frac{2}{3}, \text{..........}$$

$$\Rightarrow \frac{1-1}{1}, \frac{2-1}{2}, \frac{3-1}{3}, \text{..........}$$

$\therefore$ The n^{th} term of the sequence is

$$a_n = \frac{n-1}{n}$$

iii) Given sequence is 3, 8, 13, 18, ...

$\Rightarrow 5 - 2, 10 - 2, 15 - 2, 20 - 2,$ ....

$\Rightarrow 5\,(1) - 2, 5\,(2) - 2, 5(3) - 2, 5(4) - 2$

$\therefore$ The n^{th} term of the sequence is

$$a_n = 5n - 2$$

---

4. Find the indicated terms of the sequences whose n^{th} terms are given by

(i) $a_n = \frac{5n}{n+2}$; a_6 and a_{13}

(ii) $a_n = -(n^2 - 4)$; a_4 and a_{11}

Solution :

i) Given $a_n = \frac{5n}{n+2}, \quad a_6, \; a_{13} = ?$

$$a_6 = \frac{5(6)}{6+2} = \frac{30}{8} = \frac{15}{4}$$

$$a_{13} = \frac{5(13)}{13+2} = \frac{65}{15} = \frac{13}{3}$$

ii) $a_n = -(n^2 - 4)$; $a_4, a_{11} = ?$

$a_4 = -(16 - 4) = -12$

$a_{11} = -(121 - 4) = -117$

---

5. Find a_8 and a_{15} whose n^{th} term is $a_n =$

$$\begin{cases} \frac{n^2 - 1}{n+3} & ; n \text{ is even}, n \in N \\ \frac{n^2}{2n+1} & ; n \text{ is odd,} \end{cases}$$

Solution :

Given

$$a_n = \begin{cases} \dfrac{n^2 - 1}{n+3} & ; n \text{ is even, } n \in N \\ \dfrac{n^2}{2n+1} & ; n \text{ is odd,} \end{cases}$$

$$a_8 = \frac{8^2 - 1}{8+3} = \frac{63}{11}$$

$$a_{15} = \frac{15^2}{2(15)+1} = \frac{225}{31}$$

---

6. If $a_1 = 1$, $a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$, $n \geq 3$, $n \in$ N, then find the first six terms of the sequence.

Solution :

Given $a_1 = 1, a_2 = 1$

$a_n = 2\,a_{n-1} + a_{n-2}, \quad n \geq 3$

$a_3 = 2a_2 + a_1$

$= 2\,(1) + 1$

$= 3$

$a_4 = 2a_3 + a_2$

$= 2\,(3) + 1$

$= 7$

$a_5 = 2a_4 + a_3$

$= 2\,(7) + 3$

$= 17$

$a_6 = 2a_5 + a_4$

$= 2\,(17) + 7$

$= 41$

$\therefore$ The first 6 terms are 1, 1, 3, 7, 17, 41

V. ARITHMETIC PROGRESSION :

Key Points

- ✓ An Arithmetic Progression is a sequence whose successive terms differ by a constant number.
- ✓ Let a and d be real numbers. Then the numbers of the form $a, a + d, a + 2d, a + 3d, a + 4d,...$ is said to form Arithmetic Progression denoted by A.P. The number 'a' is called the first term and 'd ' is called the common difference.
- ✓ If there are finite numbers of terms in an A.P. then it is called Finite Arithmetic Progression. If there are infinitely many terms in an A.P. then it is called Infinite Arithmetic Progression.
- ✓ The n^{th} term denoted by t_n can be written as $t_n = a + (n - 1)d$.
- ✓ The common difference of an A.P. can be positive, negative or zero.
- ✓ An Arithmetic progression having a common difference of zero is called a constnat arithmetic progression.
- ✓ In a finite A.P. whose first term is a and last term l, then the number of terms in the A.P. is given by $l = a + (n - 1)d$ gives $n = \left(\dfrac{l-a}{d}\right) + 1$.
- ✓ If every term is added or subtracted by a constant, then the resulting sequence is also an A.P.

✓ In every term is multiplied or divided by a non-zero number, then the resulting sequence is also an A.P.

✓ If the sum of three consecutive terms of an A.P. is given, then they can be taken as $a - d$, a and $a + d$. Here the common difference is d.

✓ If the sum of four consecutive terms of an A.P. is given then, they can be taken as $a - 3d$, $a - d$, $a + d$ and $a + 3d$. Here common difference is $2d$.

✓ Three non-zero numbers a, b, c are in A.P. if and only if $\boxed{2b = a + c}$.

Example 2.23

Check whether the following sequences are in A.P. or not ?

(i) $x + 2, 2x + 3, 3x + 4, ...$

(ii) 2, 4, 8, 16, ...

(iii) $3\sqrt{2}$, $5\sqrt{2}$, $7\sqrt{2}$, $9\sqrt{2}$, ...

Solution :

To check that the given sequence is in A.P., it is enough to check if the differences between the consecutive terms are equal or not.

(i) $t_2 - t_1 = (2x + 3) - (x + 2) = x + 1$

$t_3 - t_2 = (3x + 4) - (2x + 3) = x + 1$

$t_2 - t_1 = t_3 - t_2$

Thus, the differnces between consecutive terms are equal.

Hence the sequence $x + 2, 2x + 3, 3x + 4, ...$ is in A.P.

(ii) $t_2 - t_1 = 4 - 2 = 2$

$t_3 - t_2 = 8 - 4 = 4$

$t_2 - t_1 = t_3 - t_2$

Thus, the differences between consecutive terms are not equal. Hence the terms of the sequence 2, 4, 8, 16, ... are not in A.P.

(iii) $t_2 - t_1 = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

$t_3 - t_2 = 7\sqrt{2} - 5\sqrt{2} = 2\sqrt{2}$

$t_4 - t_3 = 9\sqrt{2} - 7\sqrt{2} = 2\sqrt{2}$

Thus, the differences between consecutive terms are equal. Hence the terms of the sequence $3\sqrt{2}$, $5\sqrt{2}$, $7\sqrt{2}$, $9\sqrt{2}$, ... are in A.P.

Example 2.24

Write an A.P. whose first term is 20 and common difference is 8.

Solution :

First term = a = 20 ; common difference = d = 8

Arithmetic Progression is $a, a + d, a + 2d, a + 3d, ...$

In this case, we get 20, 20 + 8, 20 + 2(8), 20 + 3(8) ....

So, the required A.P. is 20, 28, 36, 44, ...

Example 2.25

Find the 15th, 24th and n^{th} term (general term) of an A.P. given by 3, 15, 27, 39, ...

Solution :

We have, first term = a = 3 and common difference = d = 15 – 3 = 12.

We know that n^{th} term (general term) of an A.P. with first term a and common difference d is given by $t_n = a + (n - 1)d$

$t_{15} = a + (15 - 1)\, d = a + 14d$

$= 3 + 14(12) = 171$

$t_{24} = a + 23\text{d} = 3 + 23(12) = 279$

(Here a = 3 and d = 12)

The n^{th} (general term) term is given by

$t_n = a + (n-1)d$

Thus, $t_n = 3 + (n-1)\ 12$

$t_n = 12n - 9$

Example 2.26

Fiond the number of terms in the A.P. 3, 6, 9, 12, ... 111.

Solution :

First term $a = 3$; common difference $d = 6 - 3 = 3$; last term $l = 111$

We know that, $n = \left(\frac{l-a}{d}\right) + 1$

$$n = \left(\frac{111-3}{3}\right) + 1 = 37$$

Thus the A.P. contain 37 terms.

Example 2.27

Determine the general term of an A.P. whose 7th term is – 1 and 16th term is 17.

Solution :

Let the A.P. be $t_1, t_2, t_3, t_4, \ldots$

It is given that $t_7 = -1$ and $t_{16} = 17$

$a + (7-1)d = -1$ and $a + (16-1)d = 17$

$$a + 6d = -1 \quad \ldots(1)$$

$$a + 15d = 17 \quad \ldots(2)$$

Subtracting equation (1) from equation (2), we get 9d = 18 gives d = 2

Putting d = 2 in equation (1), we get $a + 12 = -1$ so $a = -13$

Hence, general term $t_n = a + (n-1)d$

$= -13 + (n-1) \times 2 = 2n - 15$

Example 2.28

If l^{th}, m^{th} and n^{th} terms of an A.P. are x, y, z respectively, then show that

(i) $x(m-n) + y(n-1) + z(l-m) = 0$

(ii) $(x-y)n + (y-z)l + (z-x)m = 0$

Solution :

(i) Let a be the first term and d be the common difference. It is given that

$t_l = x, t_m = y, t_n = z$

Using the general term formula

$$a + (l-1)\,d = x \quad \ldots(1)$$

$$a + (m-1)\,d = y \quad \ldots(2)$$

$$a + (n-1)\,d = z \quad \ldots(3)$$

We have, $x(m-n) + y(n-l) + s(l-m)$

$= a[(m-n) + (n-l) + (l-m)] + d[(m-n) + (l-1) + (n-l)(m-1) + (l-m)(n-1)]$

$= a[0] + d[lm - ln - m + n + mn - lm - n + l + ln - mn - l + m]$

$= a(0) + d(0) = 0$

(ii) On subtracting equation (2) from equation (1), equation (3) from equation (2) and equation (1) from equation (3), we get

$$x - y = (l-m)\,d$$

$$y - z = (m-n)\,d$$

$$z - x = (n-l)\,d$$

$(x-y)n + (y-z)l + (z-x)m = [(l-m)n + (m-n)l + (n-l)m]\,d$

$= [ln - mn + lm - nl + nm - lm]\,d = 0$

Example 2.29

In an A.P., sum of four consective terms is 28 and their sum of their squares is 276. Find the four numbers.

Solution :

Let us take the four terms in the form $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$.

Since sum of the four terms is 28,

$$a - 3d + a - d + a + d + a + 3d = 28$$

$4a = 28$ gives $a = 7$

Similarly, since sum of their squares is 276,

$$(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 276.$$

$$a^2 - 6ad + 9d^2 + a^2 - 2ad + d^2 + a^2 + 2ad + d^2 + a^2 + 6ad + 9d^2 = 276$$

$$4a^2 + 20d^2 = 276 \Rightarrow 4(7)^2 + 20d^2 = 276.$$

$d^2 = 4$ gives $d = \pm 2$

If d = 2 then the four numbers are 7 – 3(2), 7 – 2, 7 + 2, 7 + 3(2)

That is the four numbers are 1, 5, 9 and 13.

If $a = 7$, d = –2 then the four numbers are 13, 9, 5 and 1

Therefore, the four consecutive terms of the A.P. are 1, 5, 9 and 13.

Example 2.30

A mother divides ₹207 into three parts such that the amount are in A.P. and gives it to her three children. The product of the two least amounts that the children had ₹4623. Find the amount received by each child.

Solution :

Let the amount received by the three children be in the form of A.P. is given by

$a - d, a, a + d$. Since, sum of the amount is ₹207, we have

$$(a - d) + a + (a + d) = 307$$

$3a = 207$ gives $a = 69$

It is given that product of the two least amounts is 4623

$$(a - d)\, a = 4623$$

$$(69 - d)\, 69 = 4623$$

$$d = 2$$

Therefore, amount given by the mother to her three children are

₹(69 –2), ₹69, ₹(69 + 2). That is, ₹67, ₹69 and ₹71.

EXERCISE 2.5

1. Check whether the following sequences are in A.P.

(i) $a - 3, a - 5, a - 7$, ...

(ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, ...

(iii) 9, 13, 17, 21, 25, ...

(iv) $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}$, ...

(v) 1, −1, 1, −1, 1, −1, ...

Solution :

i) $a - 3, a - 5, a - 7$, ...

$$t_2 - t_1 = (a - 5) - (a - 3)$$
$$= a - 5 - a + 3$$
$$= -2$$
$$t_3 - t_2 = (a - 7) - (a - 5)$$
$$= a - 7 - a + 5$$
$$= -2$$

$\therefore t_2 - t_1 = t_3 - t_2$

$\therefore$ The given sequence is in A.P.

ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\ldots$

$$t_2 - t_1 = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6} \qquad \Bigg| \qquad t_3 - t_2 = \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = \frac{-1}{12}$$

$\therefore\ t_2 - t_1 \neq t_3 - t_2$

$\therefore$ The given sequence is not in A.P.

iii) 9, 13, 17, 21, 25, ...

Each term of the sequence is increased by a constant number 4.

$\therefore$ The sequence is in A.P.

iv) $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots\ldots$

$$t_2 - t_1 = 0 - \left(\frac{-1}{3}\right) = \frac{1}{3}$$

$$t_3 - t_2 = \frac{1}{3} - 0 = \frac{1}{3}$$

$$t_4 - t_3 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$\therefore$ The sequence is in A.P.

v) $1, -1, 1, -1, 1, -1$ ....

$t_2 - t_1 = -1 - 1 = -2$

$t_3 - t_2 = 1 - (-1) = 1 + 1 = 2$

$\therefore$ The sequence is not in A.P.

2. First term *a* and common difference *d* are given below. Find the corresponding A.P.

(i) $a = 5, d = 6$ (ii) $a = 7, d = -5$

(iii) $a = \frac{3}{4},\ d = \frac{1}{2}$

Solution :

i) $a = 5$, d = 6

$\therefore$ The A.P. is $a, a + d, a + 2d, a + 3d$, ...

$= 5, 5 + 6, 5 + 2(6), 5 + 3(6)$, ...

$= 5, 11, 17, 23$, ...

ii) $a = 7, d = -5$

$\therefore$ The A.P is

$= 7, 7 + (-5), 7 + 2(-5), 7 + 3(-5)$, ...

$= 7, 2, -3, -8$, ...

iii) $a = \frac{3}{4},\ d = \frac{1}{2}$

$\therefore$ The A.P is

$$= \frac{3}{4}, \frac{3}{4} + \frac{1}{2}, \frac{3}{4} + 2\left(\frac{1}{2}\right), \frac{3}{4} + 3\left(\frac{1}{2}\right), \ldots\ldots$$

$$= \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots\ldots$$

3. Find the first term and common difference of the Arithmetic Progressions whose *n*th terms are given below

(i) $t_n = -3 + 2n$ (ii) $t_n = 4 - 7n$

Solution :

i) $t_n = -3 + 2n$

$n = 1 \Rightarrow t_1 = -3 + 2(1) = -1$

$n = 2 \Rightarrow t_2 = -3 + 2(2) = 1$

$n = 3 \Rightarrow t_3 = -3 + 2(3) = 3$ ....

$\therefore\ a = -1, \quad d = t_2 - t_1$

$= 1 - (-1)$

d = 2

ii) $t_n = 4 - 7n$

$n = 1 \Rightarrow t_1 = 4 - 7(1) = -3$

$n = 2 \Rightarrow t_2 = 4 - 7(2) = -10$

$n = 3 \Rightarrow t_3 = 4 - 7(3) = -17$

$\therefore\ a = -3, \quad d = t_2 - t_1$

$= -10 - (-3)$

$= -10 + 3$

d = −7

4. Find the 19th term of an A.P. −11, −15, −19,......

Solution :

Given A.P is −11, −15, −19, ...

$a = -11, \quad d = -15 - (-11)$

$= -15 + 11$

$= -4$

∴ The 19th term is

$t_{19} = a + (19 - 1)\,d$

$= a + 18\,d$

$= (-11) + 18\,(-4)$

$= -11 - 72$

$= -83$

---

5. Which term of an A.P. 16, 11, 6, 1,... is −54?

Solution :

Given A.P. is 16, 11, 6, 1, ... − 54

$a = 16, d = -5, t_n = -54$

$\Rightarrow \quad a + (n - 1)\,d = -54$

$\Rightarrow \quad 16 + (n - 1)(-5) = -54$

$\Rightarrow \quad 16 - 5n + 5 = -54$

$\Rightarrow \quad -5n + 21 = -54$

$\Rightarrow \quad -5n + 21 = -54$

$\Rightarrow \quad -5n = -54 - 21$

$\Rightarrow \quad -5n = -75$

$\therefore n = 15$

∴ 15th term of A.P. is − 54

---

6. Find the middle term(s) of an A.P. 9, 15, 21, 27,…,183.

Solution :

Given A.P is 9, 15, 21, 27, ... 183

$a = 9, \ d = 6, \ l = 183$

$n = \dfrac{l - a}{d} + 1$

$= \dfrac{183 - 9}{6} + 1$

$= \dfrac{174}{6} + 1$

$= 29 + 1$

$n = 30$

∴ Middle terms are $30/2, \ 30/2 + 1$

$= 15^{th}, \ 16^{th}$

$t_{15} = a + 14d$	$t_{16} = a + 15d$
$= 9 + 14(6)$	$= 9 + 15(6)$
$= 9 + 84$	$= 9 + 90$
$= 93$	$= 99$

∴ The 2 middle terms are 93, 99.

---

7. If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero.

Solution :

Given $9\,(t_9) = 15\,(t_{15})$

To Prove : $6\,(t_{24}) = 0$

$\Rightarrow \quad 9\,(t_9) = 15\,(t_{15})$

$\Rightarrow \quad 9\,(a + 8d) = 15\,(a + 14d)$

$\Rightarrow \quad 3\,(a + 8d) = 5\,(a + 14d)$

$\Rightarrow \quad 3a + 24d = 5a + 70d$

$\Rightarrow \quad 2a + 46d = 0$

$\Rightarrow \quad 2\,(a + 23d) = 0$

Multiplying 3 on both sides,

$\Rightarrow \quad 6\,(a + 23d) = 0$

$\Rightarrow \quad 6\,(t_{24}) = 0$

Hence proved.

8. If 3 + k, 18 − k, 5k + 1 are in A.P. then find k.

Solution :

Given 3 + k, 18 − k, 5k + 1 are in A.P.

$\Rightarrow (18 - k) - (3 + k) = (5k + 1) - (18 - k)$

$\Rightarrow \quad 15 - 2k = 6k - 17$

$\Rightarrow \quad -8k = -32$

$\Rightarrow \quad k = 4$

9. Find x, y and z, given that the numbers x, 10, y, 24, z are in A.P.

Solution :

Given that x, 10, y, 24, z are in A.P.

$\therefore$ y is the arithmetic mean of 10 & 24

$$\Rightarrow y = \frac{10+24}{2} = \frac{34}{2} = 17$$

$\therefore$ x, 10, y, 24, z are in A.P.

Clearly $d = 7$

$\therefore x = 10 - 7 = 3 \quad \& \; z = 24 + 7 = 31$

$\therefore x = 3, y = 17, z = 31$

10. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

Solution :

By the data given,

$a = 20, d = 2, n = 30$

$t_{30} = a + 29d$

$= 20 + 29(2)$

$= 20 + 58$

$= 78$

$\therefore$ The no. of seats in 30th row = 78

11. The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.

Solution :

Let the 3 consecutive terms in an A.P. be

$a - d, a, a + d$

i) Sum of 3 terms = 27

$\Rightarrow \quad a - d + a + a + d = 27$

$\Rightarrow \quad 3a = 27$

$a = 9$

ii) Product of 3 terms = 288

$\Rightarrow \quad (a - d) \, . \, a \, . \, (a + d) = 288$

$\Rightarrow \quad a^2 (a^2 - d^2) = 288$

$\Rightarrow \quad 9 (81 - d^2) = 288$

$\Rightarrow \quad 81 - d^2 = 32$

$\Rightarrow \quad d^2 = 49$

$\Rightarrow \quad d = \underline{+} 7$

a = 9, d = 7 $\Rightarrow$ the 3 terms are 2, 9, 16

a = 9, d = −7 $\Rightarrow$ the 2 terms are 16, 9, 2

12. The ratio of 6th and 8th term of an A.P. is 7 : 9. Find the ratio of 9th term to 13th term.

Solution :

Given $\frac{t_6}{t_8} = \frac{7}{9}$

$$\Rightarrow \frac{a+5d}{a+7d} = \frac{7}{9}$$

$\Rightarrow 9a + 45d = 7a + 49d$

$\Rightarrow \quad 2a = 4d$

$\Rightarrow \quad a = 2d \quad$ .....(1)

$$\therefore \frac{t_9}{t_{13}} = \frac{a+8d}{a+12d}$$

$$= \frac{2d+8d}{2d+12d} \quad \text{(from (1))}$$

$$= \frac{10d}{14d}$$

$$= \frac{5}{7}$$

$\therefore t_9 : t_{13} = 5 : 7$

13. In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is 0° C and the sum of the temperatures from Wednesday to Friday is 18° C. Find the temperature on each of the five days.

Solution :

Let the temperature from Monday to Friday respectively be

$$a,\ a+d,\ a+2d,\ a+3d,\ a+4d$$

i) Given $a + (a + d) + (a + 2d) = 0$

$$3a + 3d = 0$$
$$a + d = 0$$
$$a = -d$$

ii) Given $(a + 2d) + (a + 3d) + (a + 4d) = 18$

$$\Rightarrow \quad 3a + 9d = 18$$
$$\Rightarrow \quad -3d + 9d = 18$$
$$\Rightarrow \quad 6d = 18$$
$$\Rightarrow \quad d = 3$$
$$\therefore \quad a = -3$$

The temperature of each of the 5 days

-3^0 C, 0^0 C, 3^0 C, 6^0 C, 9^0 C

14. Priya earned ₹ 15,000 in the first year. Thereafter her salary increased by ₹ 1500 per year. Her expenses are ₹ 13,000 during the first year and the expenses increases by ₹ 900 per year. How long will it take for her to save ₹ 20,000.

Solution :

	1st year	**2nd year**
Salary :	₹15,000	₹16,500
Expense :	₹13,000	₹13,900
Savings :	₹2,000	₹2,600

$\therefore$ the yearly savings are

₹2,000, ₹2,600, ₹3,200, ... form an A.P with a = 2,000, d = 600, t_n = 20,000

$$a + (n-1)\,d = 20{,}000$$
$$\Rightarrow \quad 2{,}000 + (n-1)\,600 = 20{,}000$$
$$\Rightarrow \quad 600\,n - 600 = 18{,}000$$
$$\Rightarrow \quad 600\,n = 18{,}600$$
$$\Rightarrow \quad n = \frac{186}{6}$$
$$n = 31$$

After 31 years, her savings will be ₹20,000.

VI. ARITHMETIC SERIES :

Key Points

- ✓ The sum of terms of a sequence is called series.
- ✓ Let $a_1, a_2, a_3, ..., a_n, ...$ be the sequence of real numbers. Then the real numbers $a_1 + a_2 + a_3 + ...$ is defined as the series of real numbers.
- ✓ If a series has finite number of terms then it is called a Finite series. If a series has infinite number of terms then it is called Infinite series.
- ✓ Sum to n terms of an A.P. $S_n = \frac{n}{2}[2a + (n-1)d]$
- ✓ If the first term a, and the last term l (n^{th} term) are given then $S_n = \frac{n}{2}[a + l]$.

Example 2.31

Find the sum of first 15 terms of the A.P. $8, 7\frac{1}{4}, 6\frac{1}{2}, 5\frac{3}{4}, ...$

Solution :

Here the first term a = 8, common difference d = $7\frac{1}{4} - 8 = -\frac{3}{4}$,

Sum of first n terms of an A.P.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{15} = \frac{15}{2}\left[2\times 8+(15-1)(-\frac{3}{4})\right.$$

$$S_{15} = \frac{15}{2}\left[16-\frac{21}{2}\right] = \frac{165}{4}$$

Example 2.32

Find the sum of 0.40 + 0.43 + 0.46 + ... + 1.

Solution :

Here the value of n is not given. But the last term is given. From this, we can find the value of n.

Given a = 0.40 and l = 1, we find d = 0.43 – 0.40 = 0.03.

$$\text{Therefore, } n = \left(\frac{l-a}{d}\right)+1$$

$$= \left(\frac{1-0.40}{0.03}\right)+1 = 21$$

Sum of first n terms of an A.P.

$$S_n = \frac{n}{2}[a+l]$$

Here, $n = 21$.

$$\text{Therefore,} \quad S_{21} = \frac{21}{2}[0.40+1] = 14.7$$

So, the sum of 21 terms of the given series is 14.7.

Example 2.33

How many terms of the series 1 + 5 + 9 + ... must be taken so that their sum is 190 ?

Solution :

Here we have to find the value of n, such that $S_n = 190$.

First term a = 1, common difference

d = 5 – 1 = 4.

Sum of first n terms of an A.P.

$$S_n = \frac{n}{2}[2a+(n-1)d] = 190$$

$$\frac{n}{2}[2\times 1+(n-1)\times 4] = 190$$

$$n[4n-2] = 380$$

$$2n^2 - n - 190 = 0$$

$$(n-10)(2n+19) = 0$$

But n = 10 as $n = -\frac{19}{2}$ is impossible. Therefore, n = 10.

Example 2.34

The 13th term of an A.P. is 3 and the sum of first 13 terms is 234. Find the common difference and the sum of first 21 terms.

Solution :

Given the 13th term = 3 so,

$t_{13} = a + 12d = 3$ ...(1)

Sum of first 13 terms = 234 gives

$S_{13} = \frac{13}{2}[2a + 12d] = 234$

$2a + 12d = 36$ ...(2)

Solving (1) and (2) we get, $a = 33$, d = $\frac{-5}{2}$

Therefore, common difference is $\frac{-5}{2}$.

Sum of first 21 terms

$$S_{21} = \frac{21}{2}\left[2\times 33 + (21-1)\times\left(-\frac{5}{2}\right)\right]$$

$$= \frac{21}{2}[66-50] = 168$$

.

Example 2.35

In an AP. the sum of first n terms is $\frac{5n^2}{2}+\frac{3n}{2}$. Find the 17th term.

Solution :

The 17th term can be obtained by subtracting the sum of first 16 terms from the sum of first 17 terms.

$$S_{17} = \frac{5\times(17)^2}{2}+\frac{3\times 17}{2} = \frac{1445}{2}+\frac{51}{2} = 748$$

$$S_{16} = \frac{5\times(16)^2}{2}+\frac{3\times 16}{2} = \frac{1280}{2}+\frac{48}{2} = 664$$

Now, $t_{17} = S_{17} - S_{16} = 748 - 664 = 84$

Example 2.36

Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

Solution :

The natural numbres between 300 and 600 which are divisible by 7 are 301, 308, 315, ..., 595.

The sum of all natural numbers between 300 and 600 is 301 + 308 + 315 + ... + 595.

The terms of the above sereis are in A.P.

First term a = 301 ; common difference d = 7 ; Last term l = 595.

$$n = \left(\frac{l-a}{d}\right)+1 = \left(\frac{595-301}{7}\right)+1 = 43$$

Since, $S_n = \frac{n}{2}[a+l]$,

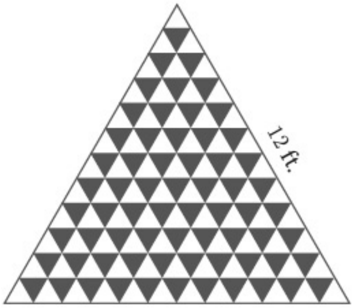
we have $S_{57} = \frac{43}{2}[301+595] = 19264.$

Example 2.37

A mosaic is designed in the shape of an equilateral triangle, 12 ft on each side. Each tile in the mosaic is in the shape of an equilateral triangle of 12 inch side. The tiles are alternate in colour as shown in the figure. Find the number of tiles of each colour and total number of tiles in the mosaic.

Solution :

Since the mosaic is in th eshape of an equilateral triangle of 12ft, and the tile is in the shape of an equilateral triangle of 12 inch (1 ft), there will be 12 rows in the mosaic.



From the figure, it is clear that number of white tiles in each row are 1, 2, 3, 4, ... 12 which clearly forms an Arithmetic Progression.

Similarly the number of blue tiles in each row are 0, 1, 2, 3, ... 11 which is also an Arithmetic Progression.

Number of white tiles

$= 1 + 2 + 3 + ... + 12 = \frac{12}{2}[1 + 12] = 78$

Number of blue tiles

$= 0 + 1 + 2 + 3 + ...+ 11 = \frac{12}{2}[0 + 11] = 66$

The total number of tiles in the mosaic

= 78 + 66 = 144

Example 2.38

The houses of a street are numbered from 1 to 49. Senthil's house is numbered such that the sum of numbers of the houses prior to Senthil's house is equal to the sum of numbers of the houses following Senthil's house. Find Senthil's house number ?

Solution :

Let Senthil's house number be x.

It is given that $1 + 2 + 3 + ... + (x - 1)$

$= (x - 1) + (x + 2) + ... + 49$

$1 + 2 + 3 + ... + (x - 1)$

$= [1 + 2 + 3 + ... + 49] - [1 + 2 + 3 +.... + x]$

$$\frac{x-1}{2}[1+(x-1)] = \frac{49}{2}[1+49] - \frac{x}{2}[1+x]$$

$$\frac{x(x-1)}{2} = \frac{49\times 50}{2} - \frac{x(x+1)}{2}$$

$$x^2 - x = 2450 - x^2 - x \Rightarrow 2x^2 = 2450$$

$$x^2 = 1225 \text{ gives } x = 35$$

Therefore, Senthil's hosue number is 35.

Example 2.39

The sum of first, n, 2n and 3n terms of an A.P. are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

Solution :

If S_1, S_2 and S_3 are sum of first n, 2n and 3n terms of an A.P. respectively then

$$S_1 = \frac{n}{2}[2a+(n-1)d],\ S_2 = \frac{2n}{2}[2a+(2n-1)d],$$

$$S_3 = \frac{3n}{2}[2a+(3n-1)d]$$

Consider

$$S_2 - S_1 = \frac{2n}{2}[2a+(2n-1)d] - \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{n}{2}[(4a+2(2n-1)d] - [2a+(n-1)d]$$

$$S_2 - S_1 = \frac{n}{2}\times[2a+(3n-1)d]$$

$$3(S_2 - S_1) = \frac{3n}{2}[2a+(3n-1)d]$$

$$3(S_2 - S_1) = S_3$$

EXERCISE 2.6

1. Find the sum of the following

(i) 3, 7, 11,… up to 40 terms.

(ii) 102, 97, 92,… up to 27 terms.

(iii) 6 + 13 + 20 + ... + 97

Solution :

i) Given A.P is 3, 7, 11, ... up to 40 terms

$a = 3, d = 4, n = 40$

$$S_n = {}^{n}\!/_{2}\,[2a + (n-1)\, d]$$

$$S_{40} = \frac{40}{2}[6+39(4)]$$

$$= 20\,[6+156]$$

$$= 20\times 162$$

$$= 3240$$

ii) Given A.P is 102, 97, 92, ... up to 27 terms

$a = 102, d = -5, n = 27$

$$S_n = {}^{n}\!/_{2}\,[2a + (n-1)\, d]$$

$$S_{27} = \frac{27}{2}[204+26(-5)]$$

$$= \frac{27}{2}[204-130]$$

$$= \frac{27}{2}\times 74$$

$$= 27\times 37$$

$$= 999$$

iii) Given 6 + 13 + 20 + ... + 97

$$a = 6, d = 7, l = 97$$

$$\therefore \quad n = \frac{l-a}{d} + 1$$

$$= \frac{97-6}{7} + 1$$

$$= \frac{91}{7} + 1$$

$$= 13 + 1$$

$$= 14$$

$$\therefore \; S_n = {}^{n}\!/_{2}\,(a+l)$$

$$S_{14} = {}^{14}\!/_{2}\,(6+97)$$

$$= 7 \times 103$$

$$= 721$$

---

2. How many consecutive odd integers beginning with 5 will sum to 480?

Solution :

By the data given,

The series is 5 + 7 + 9 + ... n = 480

$$\therefore \qquad a = 5, d = 2, S_n = 480$$

$$\Rightarrow {}^{n}\!/_{2}\,[2a+(n-1)d] = 480$$

$$\Rightarrow {}^{n}\!/_{2}\,[10+(n-1)d] = 480$$

$$\Rightarrow {}^{n}\!/_{2}\,[5+(n-1)] = 480$$

$$\Rightarrow n\,[n+4] = 480$$

$$\Rightarrow n^2 + 4n - 480 = 0$$

$$\Rightarrow (n+24)\,(n-20) = 0$$

$$\Rightarrow n = -24, \; n = 20$$

$$\therefore \; n = 20$$

---

3. Find the sum of first 28 terms of an A.P. whose n^{th} term is $4n - 3$.

Solution :

Given $\quad t_n = 4n - 3$

$$n = 1 \Rightarrow t_1 = 4 - 3 = 1$$

$$n = 2 \Rightarrow t_2 = 8 - 3 = 5$$

$$n = 3 \Rightarrow t_3 = 12 - 3 = 9$$

$$\therefore \; a = 1, d = 5 - 1 = 4$$

$$\therefore \; S_n = {}^{n}\!/_{2}\,[2a+(n-1)d]$$

$$S_{28} = {}^{28}\!/_{2}\,[2+27(4)] = 14\,[2+108]$$

$$= 14 \times 110 = 1540$$

---

4. The sum of first *n* terms of a certain series is given as $2n^2 - 3n$. Show that the series is an A.P.

Solution :

Given $\quad S_n = 2n^2 - 3n$

$$n = 1 \Rightarrow S_1 = 2 - 3 = -1$$

$$n = 2 \Rightarrow S_2 = 2(4) - 3\,(2) = 8 - 6 = 2$$

$$\therefore \; t_1 = a = -\,1, S_2 = 2$$

$$\Rightarrow \qquad t_2 + t_1 = 2$$

$$\Rightarrow \qquad t_2 - 1 = 2$$

$$\Rightarrow \qquad t_2 = 3$$

$$\therefore \qquad t_1 = -1, t_2 = 3, d = t_2 - t_1$$

$$= 4$$

$$\therefore \qquad a = -\,1, d = 4$$

$\therefore$ The series is − 1 + 3 + 7 + ... is an A.P.

---

5. The 104th term and 4th term of an A.P. are 125 and 0. Find the sum of first 35 terms.

Solution :

Given $t_{104} = 125$, $\qquad t_4 = 0$

To find : S_{35}

$$\begin{array}{rcr} a + 103\,d & = & 125 \\ a + \;\;\; 3\,d & = & 0 \\ \hline 100\,d & = & 125 \end{array}$$

$$d = \frac{5}{4}$$

$$a + 3\left(\frac{5}{4}\right) = 0$$

$$\Rightarrow \quad a + \frac{15}{4} = 0$$

$$\Rightarrow \quad a = \frac{-15}{4}$$

$$\therefore \ S_n = \frac{n}{2}\ [2a + (n-1)d]$$

$$S_{35} = \frac{35}{2}\left[\frac{-15}{2} + (34)\left(\frac{5}{4}\right)\right]$$

$$= \frac{35}{2}\left[\frac{-15}{2} + \frac{85}{2}\right]$$

$$= \frac{35}{2} \times 35$$

$$= \frac{1225}{2}$$

$$= 612.5$$

---

6. Find the sum of all odd positive integers less than 450.

Solution :

To find the sum :

1 + 3 + 5 + 7 + ... + 449

$a = 1,\ d = 2,\ l = 449$

$$\therefore \ n = \frac{l-a}{d} + 1$$

$$= \frac{449-1}{2} + 1$$

$$= \frac{448}{2} + 1$$

$$= 224 + 1$$

$$= 225$$

$$\therefore S_n = \frac{n}{2}[a + l]$$

$$S_{225} = \frac{225}{2}[450]$$

$$= 225 \times 225$$

$$= 50{,}625$$

---

7. Find the sum of all natural numbers between 602 and 902 which are not divisible by 4.

Solution :

First we take the sum of the numbers from 603 to 901

$a = 603,\ d = 1,\ l = 901$

$$\therefore \ n = \frac{l-a}{d} + 1$$

$$= \frac{901-603}{1} + 1$$

$$= 298 + 1$$

$$= 299$$

$$\therefore S_n = \frac{n}{2}[a + l]$$

$$= \frac{299}{2} \times 1504$$

$$= 299 \times 752$$

$$= 224848$$

Next we take sum of all the no's between 602 & 902 which are divi. by 4

$a = 604, \quad d = 4, \quad l = 900$

$$\therefore \ n = \frac{l-a}{d} + 1$$

$$= \frac{900-604}{4} + 1$$

$$= \frac{296}{4} + 1$$

$$= 74 + 1$$

$$= 75$$

$$\begin{array}{r} 150 \\ 4\overline{)602} \\ \underline{600} \\ \underline{2} \end{array}$$

602 + (4 – 2)

= 602 + 2

= 604

$$\therefore S_n = \frac{n}{2}[a + l]$$

$$S_{75} = \frac{75}{2} \times 1504$$

$$= 75 \times 752$$

$$= 56{,}400$$

$$\begin{array}{r} 225 \\ 4\overline{)902} \\ \underline{900} \\ \underline{2} \end{array}$$

= 902 – 2

= 900

$\therefore$ Sum of no's which are not div. by 4

$$= 224848 - 56400 = 168448$$

8. Raghu wish to buy a laptop. He can buy it by paying ₹ 40,000 cash or by giving it in 10 installments as ₹ 4800 in the first month, ₹ 4750 in the second month, ₹ 4700 in the third month and so on. If he pays the money in this fashion, find

(i) total amount paid in 10 installments.

(ii) how much extra amount that he has to pay than the cost?

Solution :

Installment in 1st month = Rs. 4800

Installment in 2nd month = Rs. 4750

Installment in 3rd month = Rs. 4700

i.e., 4800, 4750, 4700, ... forms an A.P. with

$a = 4800, d = -50, n = 10$

i) Total amount paid in 10 installments

$$\therefore \ S_n = {}^{n}\!/_{2} \ [2a + (n-1)d]$$

$$\therefore \ S_{10} = 5 \ \left[9600 + 9(-50)\right]$$

$$= 5 \ [9600 - 450]$$

$$= 5 \times 9150$$

$$= \text{Rs. } 45,750/-$$

ii) Amount he paid extra in installments

= 45,750 − 40,000

= Rs. 5,750/-

---

9. A man repays a loan of ₹ 65,000 by paying ₹ 400 in the first month and then increasing the payment by ₹ 300 every month. How long will it take for him to clear the loan?

Solution :

Amounts of repayment in successive months

400 + 700 + 1000 + ... n months = ₹ 65,000

$a = 400, d = 300, S_n = 65{,}000$

$${}^{n}\!/_{2} \ [2a + (n-1)\ d] = 65{,}000$$

$$\Rightarrow \ {}^{n}\!/_{2} \ [800 + (n-1)\ 300] = 65{,}000$$

$$\Rightarrow \ n \ [400 + (n-1)\ 150] = 65{,}000$$

$$\Rightarrow \ n \ [150n + 250] = 65{,}000$$

$$\Rightarrow \ n\,[3n + 5] = 1{,}300$$

$$\Rightarrow \ 3n^2 + 5n - 1300 = 0$$

$$\Rightarrow \ n = 20, \ {}^{-65}\!/_{3}$$

$$\boxed{\therefore \ n = 20}$$

$$\begin{array}{c|c} 5 & -3900 \\ \hline \frac{-65}{3} & \frac{-60}{3} \end{array}$$

$$\frac{65}{3}, \quad -20$$

---

10. A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two bricks less than the previous step.

(i) How many bricks are required for the top most step?

(ii) How many bricks are required to build the stair case?

Solution :

No. of bricks in bottom step = 100

No. of bricks in successive steps are

98, 96, 94, ...

∴ 100, 98, 96, 94, ... for 30 steps form an A.P. with

$a = 100, d = -2, n = 30$

i) No. of bricks used in the top most step

$$t_{30} = a + 29d$$

$$= 100 + 29\ (-2)$$

$$= 100 - 58$$

$$= 42$$

ii) Total no. of bricks used to build the stair case

$$S_{20} = \frac{30}{2}(100+42)$$
$$= 15 \times 142$$
$$= 2130$$

---

11. If S_1, S_2, S_3, ... S_m are the sums of n terms of m A.P.'s whose first terms are 1, 2, 3,...., m and whose common differences are 1, 3, 5, ..., ($2m - 1$) respectively, then show that

$$\boldsymbol{S_1 + S_2 + S_3 + \text{.........} + S_m = \frac{1}{2} mn\,(mn+1)}$$

Solution :

$$1^{st}\ A.P. \Rightarrow\ a = 1,\ d = 1$$
$$\Rightarrow S_1 = {}^{n}\!/_{2}\,[2 + (n-1)\,1]$$
$$= {}^{n}\!/_{2}\,[n+1]$$
$$2^{nd}\ A.P. \Rightarrow\ a = 2,\ d = 3$$
$$\Rightarrow S_2 = {}^{n}\!/_{2}\,[4 + (n-1)\,3]$$
$$= {}^{n}\!/_{2}\,[3n+1]$$
$$\text{m}^{\text{th}}\ \text{A.P.} \Rightarrow a = m,\ d = 2m-1$$
$$\Rightarrow S_m = {}^{n}\!/_{2}\,[2m + (n-1)\,(2m-1)]$$
$$= {}^{n}\!/_{2}\,[2m + 2mn - 2m - n + 1]$$
$$= {}^{n}\!/_{2}\,[2mn - n + 1]$$
$$= {}^{n}\!/_{2}\,[(2m-1)\,\text{n} + 1]$$

$$S_1 + S_2 + ... + S_m$$
$$= {}^{n}\!/_{2}\,(n+1) + {}^{n}\!/_{2}\,(3n+1) + ... + {}^{n}\!/_{2}\,((2m-1)\,n+1)$$
$$= {}^{n}\!/_{2}\,[(n + 3n + ... + (2m-1)n) + (1 + 1 + ...\ m\ \text{terms})]$$
$$= {}^{n}\!/_{2}\,[(n(1 + 3 + 5 + ... + (2m-1) + m]$$
$$= {}^{n}\!/_{2}\,[(n(m^2) + m]$$
$$= {}^{n}\!/_{2}\,[m(mn+1)]$$
$$= {}^{1}\!/_{2}\,mn\,(mn+1)$$

= RHS

Hence Proved

---

12. Find the sum $\left[\dfrac{\boldsymbol{a - b}}{\boldsymbol{a+b}} + \dfrac{\boldsymbol{3a - 2b}}{\boldsymbol{a+b}} + \dfrac{\boldsymbol{5a - 3b}}{\boldsymbol{a+b}} + \text{....... to 12 terms}\right]$

Solution :

Given series is

$$\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \text{......... to 12 terms}$$

$$1\text{st term} = \frac{a-b}{a+b}, \qquad \text{Common diff} = \frac{2a-b}{a+b}$$

$$S_{12} = \frac{12}{2}\left[2\left(\frac{a-b}{a+b}\right) + 11\left(\frac{2a-b}{a+b}\right)\right] (\because S_n = \frac{n}{2}[2a + (n-1)d])$$
$$= 6\left[\frac{2a - 2b + 22a - 11b}{a+b}\right]$$
$$= 6\left[\frac{24a - 13b}{a+b}\right]$$
$$= \frac{6}{a+b}[24a - 13b]$$

Hence proved.

VII. GEOMETRIC PROGRESSION :

Key Points

- ✓ A Geometric Progression is a sequence in which each term is obtained by multilplying a fixed non-zero number to the preceding term except the first term. The fixed number is called common ratio. The common ratio is usually denoted by *r*.
- ✓ Let *a* and $r \neq 0$ be real numbers. Then the numbers of the form $a, ar, ar^2, ... ar^{n-1}$ ... is called a Geometric Progression. The number '*a*' ios called the first term and number 'r' is called the common ratio.
- ✓ The general term or nth term of a G.P. is $t_n = ar^{n-1}$.
- ✓ When the product of three consecutive terms of a G.P. are given, we can take the three terms as $\frac{a}{r}, a, ar$.
- ✓ When the products of four consecutive terms are given for a G.P. then we can take the four terms as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.
- ✓ When each term of a Geometric Progression is multiplied or divided by a non-zero constant then the resulting sequence is also a Geometric Progression.
- ✓ Three non-zero numbers *a, b, c* are in G.P. if and only if $b^2 = ac$.

Example 2.40

Which of the following sequences form a Geometric Progression ?

(i) 7, 14, 21, 28, ... (ii) $\frac{1}{2}$, 1, 2, 4, ...

(iii) 5, 25, 50, 75, ...

Solution :

To check if a given sequence form a G.P. we have to see if the ratio between successive terms are equal.

(i) 7, 14, 21, 28, ...

$$\frac{t_2}{t_1} = \frac{14}{7} = 2; \quad \frac{t_3}{t_2} = \frac{21}{14} = \frac{3}{2}; \quad \frac{t_4}{t_3} = \frac{28}{21} = \frac{4}{3}$$

Since the ratios between successive terms are not equal, the sequence 7, 14, 21, 28, ... is not a Geometric Progression.

(ii) $\frac{1}{2}$, 1, 2, 4, ...

$$\frac{t_2}{t_1} = \frac{1}{\frac{1}{2}} = 2; \quad \frac{t_3}{t_2} = \frac{2}{1} = 2; \quad \frac{t_4}{t_3} = \frac{4}{2} = 2$$

Here the ratios between successive terms are equal. Therefore the sequence $\frac{1}{2}$, 1, 2, 4, ... is a Geometric Progression with common ratio r = 2.

(iii) 5, 25, 50, 75, ...

$$\frac{t_2}{t_1} = \frac{25}{5} = 5; \quad \frac{t_3}{t_2} = \frac{50}{25} = 2; \quad \frac{t_4}{t_3} = \frac{75}{50} = \frac{3}{2}$$

Since the ratios between successive terms are not equal, the sequence 5, 25, 50, 75, ... is not a Geometric Progression.

Example 2.41

Find the geometric progression whose first term and common ratios are given by (i) $a = -7$, r = 6 (ii) $a = 256$, r = 0.5

Solution :

(i) The general form of Geometric progression is $a, ar, ar^3, ...$

$a = -7$, $ar = -7 \times 6 = -42$, $ar^2 = -7 \times 6^2 = -252$

Therefore the required Geometric Progression is –7, – 42, – 252, ...

(ii) The general form of Geometric progression is $a, ar, ar^3, ...$

$a = 256$, $ar = 256 \times 0.5 = 128$, $ar^3 = 256 \times (0.5)^2 = 64$

Example 2.42

Find the 8th term of the G.P. 9, 3, 1, ...

Solution :

To find the 8th term we have to use the nth term formula $t_n = ar^{n-1}$

First term $a = 9$, common ratio $r = \frac{t_2}{t_1} = \frac{3}{9} = \frac{1}{3}$

$$t_8 = 9 \times \left(\frac{1}{3}\right)^{8-1} = 9 \times \left(\frac{1}{3}\right)^{7} = \frac{1}{243}$$

Therefore the 8th term of the G.P. is $\frac{1}{243}$.

Example 2.43

In a Geometric progression, the 4th term is $\frac{8}{9}$ and the 7th term is $\frac{64}{243}$. Find the Geometric Progression.

Solution :

$$4^{\text{th}} \text{ term, } t_4 = \frac{8}{9} \text{ gives } ar^3 = \frac{8}{9} \qquad ...(1)$$

$$7^{\text{th}} \text{ term, } t_7 = \frac{64}{243} \text{ gives } ar^6 = \frac{64}{243} \qquad ...(2)$$

Dividing (2) by (1) we get, $\frac{ar^6}{ar^3} = \frac{\frac{64}{243}}{\frac{8}{9}}$

$$r^3 = \frac{8}{27} \text{ gives } r = \frac{2}{3}$$

Substituting the value of r in (1), we get

$$a \times \left[\frac{2}{3}\right]^3 = \frac{8}{9} \Rightarrow a = 3$$

Therefore the Geometric Progression is $a, ar, ar^3, ...$ That is, 3, 2, $\frac{4}{3}$, ...

Example 2.44

The product of three consecutive terms of a Geometric Progression is 343 and their sum is $\frac{91}{3}$. Find the three terms.

Solution :

Since the product of 3 consecutive terms is given.

we can take them as $\frac{a}{r}, a, ar$.

Product of the terms = 343

$$\frac{a}{r} \times a \times ar = 343$$

$a^3 = 7^3$ gives $a = 7$

Sum of the terms = $\frac{91}{3}$

gives $7\left(\frac{1 + r + r^2}{r}\right) = \frac{91}{3}$

$3 + 3r + 3r^2 = 13r$ gives $3r^2 - 10r + 3 = 0$

$(3r - 1)(r - 3) = 0$ gives $r = 3$ or $r = \frac{1}{3}$

If $a = 7$, $r = 3$ then the three terms are $\frac{7}{3}$, 7, 21.

If $a = 7$, $r = \frac{1}{3}$ then the three terms are 21, 7, $\frac{7}{3}$.

Example 2.45

The present value of a machine is ₹40,000 and its value depreciates each year by 10%. Find the estimated value of the machine in the 6th year.

Solution :

The value of the machine at present is

₹40,000. Since it is depreciated at the rate of 10% after one year the value of the machine is 90% of the initial value.

That is the value of the machine at the end of the first year is 40,000 × $\frac{90}{100}$

After two years, the value of the machine is 90% of the value in the first year.

Value of the machine at the end of the 2nd year is 40,000 × $\left(\frac{90}{100}\right)^2$

Continuing this way, the value of the machine depreciates in the following way as

$$40000, 40000\times\frac{90}{100}, 40000\times\left(\frac{90}{100}\right)^2 \ldots$$

This sequence is in the form of G.P. with first term 40,000 and common ratio $\frac{90}{100}$. For finding the value of the machine at the end of 5th year (i.e. in 6th year), we need to find the sixth term of this G.P.

Thus, $n = 6,\ a = 40{,}000,\ r = \frac{90}{100}$.

Using $t_n = ar^{n-1}$, we have $t_6 = 40{,}000\times\left(\frac{90}{100}\right)^{n-1}$

$$= 40000\times\left(\frac{90}{100}\right)^5$$

$$t_6 = 40{,}000\times\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}$$

$$= 23619.6$$

Therefore the value of the machine in 6th year = ₹23619.60.

EXERCISE 2.7

1. Which of the following sequences are in G.P.?

(i) 3, 9, 27, 81,… (ii) 4, 44, 444, 4444,...

(iii) 0.5, 0.05, 0.005, …

iv) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}$, ... v) 1, −5, 25, −125 …

vi) 120, 60, 30, 18,… vii) $16, 4, 1, \frac{1}{4}$, ...

Solution :

i) Given sequence is 3, 9, 27, 81 ....

$$t_2/t_1 = 9/3 = 3$$

$$t_3/t_2 = 27/9 = 3$$

$$t_4/t_3 = 81/27 = 3$$

∴ The sequence is a G.P.

ii) Given sequence is 4, 44, 444, ...

$$t_2/t_1 = 44/4 = 11$$

$$t_3/t_2 = 444/44 = 111/11 \neq 11$$

$$\therefore t_2/t_1 \neq t_3/t_2$$

∴ The sequence is not a G.P.

iii) Given sequence is 0.5, 0.05, 0.005, ...

$$\frac{t_2}{t_1} = \frac{0.05}{0.5} = \frac{5}{50} = \frac{1}{10}$$

$$\frac{t_3}{t_2} = \frac{0.005}{0.05} = \frac{5}{50} = \frac{1}{10}$$

∴ The sequence is a G.P.

iv) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \ldots\ldots$

$$\frac{t_2}{t_1} = \frac{1/6}{1/3} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{t_3}{t_2} = \frac{1/2}{1/6} = \frac{6}{12} = \frac{1}{2}$$

$\therefore$ The sequence is a G.P.

v) 1, − 5, 25, − 125, ...

$$\frac{t_2}{t_1} = -5$$

$$\frac{t_3}{t_2} = \frac{25}{-5} = -5$$

$$\frac{t_4}{t_3} = \frac{-125}{25} = -5$$

$\therefore$ The sequence is a G.P.

vi) 120, 60, 30, 18, ...

$$\frac{t_2}{t_1} = \frac{60}{120} = \frac{1}{2}$$

$$\frac{t_3}{t_2} = \frac{30}{60} = \frac{1}{2}$$

$$\frac{t_4}{t_3} = \frac{18}{30} \neq \frac{1}{2}$$

$\therefore$ The sequence is not a G.P.

vii) $16, 4, 1, 1/4, \ldots\ldots.$

$$\frac{t_2}{t_1} = \frac{4}{16} = \frac{1}{4}$$

$$\frac{t_3}{t_2} = \frac{1}{4}$$

$$\frac{t_4}{t_3} = \frac{1}{4}$$

$\therefore$ The sequence is a G.P.

2. Write the first three terms of the G.P. whose first term and the common ratio are given below.

(i) $a = 6, r = 3$ (ii) $a = \sqrt{2}, r = \sqrt{2}$

(iii) $a = 1000, r = \frac{2}{5}$

Solution :

i) Given $a = 6, r = 3$

$\therefore$ The first 3 terms of the G.P. are

6, 18, 54, ...

ii) Given $a = \sqrt{2},\ r = \sqrt{2}$

$\therefore$ The first 3 terms of the G.P. are

$\sqrt{2}, 2, 2\sqrt{2}, \ldots\ldots\ldots$

iii) $a = 1000, r = 2/5$

$\therefore$ The first 3 terms are

$$1000,\ 1000 \times 2/5,\ 1000 \times 2/5 \times 2/5$$

= 1000, 400, 160, ...

---

3. In a G.P. 729, 243, 81,… find t_7.

Solution :

Given G.P is 729, 243, 21, ...

$$a = 729,\ r = 8/243 = 1/3$$

$$\therefore\ t_n = a\ .\ r^{n-1}$$

$$\Rightarrow t_7 = a\ .\ r^6$$

$$= 729 \times \left(1/3\right)^6$$

$$= 3^6 \times \frac{1}{3^6} = 1$$

---

4. Find x so that $x + 6, x + 12$ and $x + 15$ are consecutive terms of a Geometric Progression.

Solution :

Given $x + 6, x + 12, x + 15$ are consecutive terms of a G.P.

$$\Rightarrow \quad \frac{x+12}{x+6} = \frac{x+15}{x+12}$$

$$\Rightarrow \quad (x+12)^2 = (x+15)(x+6)$$

$$\Rightarrow \quad x^2 + 24x + 144 = x^2 + 21x + 90$$

$$\Rightarrow \quad 3x = -54$$

$$x = -18$$

5. Find the number of terms in the following G.P.

i) 4, 8, 16, … , 8192 ?

ii) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\ldots, \frac{1}{2187}$?

Solution :

i) Given G.P is 4, 8, 16, ... 8192

$$\Rightarrow \quad a = 4, r = 2, t_n = 8192$$

$$\Rightarrow \quad a \,.\, r^{n-1} = 8192$$

$$\Rightarrow \quad 4 \times 2^{n-1} = 8192$$

$$\Rightarrow \quad 2^{n-1} = 2048$$

$$\Rightarrow \quad 2^{n-1} = 2^{11}$$

$$\Rightarrow \quad n - 1 = 11$$

$$\therefore \; n = 12$$

$ii)$ Given G.P is $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\ldots, \frac{1}{2187}$

$$a = \frac{1}{3},\; r = \frac{1}{3},\; t_n = \frac{1}{2187}$$

$$\Rightarrow \quad a \,.\, r^{n-1} = \frac{1}{2187}$$

$$\Rightarrow \quad \frac{1}{3} \cdot \left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187}$$

$$\Rightarrow \quad \left(\frac{1}{3}\right)^{n-1} = \frac{3}{2187}$$

$$\Rightarrow \quad \left(\frac{1}{3}\right)^{n-1} = \frac{1}{729}$$

$$\Rightarrow \quad \left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^{6}$$

$$\therefore \; n - 1 = 6$$

---

6. In a G.P. the 9th term is 32805 and 6th term is 1215. Find the 12th term.

Solution :

Given $t_9 = 32805$, $t_6 = 1215$, $t_{12} = ?$

$$a \,.\, r^8 = 32805 \qquad \ldots\ldots\ldots\ldots (1)$$

$$a \,.\, r^5 = 1215 \qquad \ldots\ldots\ldots\ldots (2)$$

$$(1) \div (2) \Rightarrow r^3 = \frac{32805}{1215}$$

$$\Rightarrow r^3 = 27$$

$$\Rightarrow r = 3$$

Sub. $r = 3$ in (2)

$$a \times 3^5 = 1215$$

$$\Rightarrow \quad a \times 243 = 1215$$

$$\Rightarrow \quad a = \frac{1215}{243}$$

$$\Rightarrow \quad a = 5$$

$$\Rightarrow \quad \therefore\; t_{12} = a \,.\, r^{11}$$

$$= 5 \times 3^{11}$$

7. Find the 10th term of a G.P. whose 8th term is 768 and the common ratio is 2.

Solution :

Given $t_8 = 768$, $r = 2$

$$\Rightarrow \quad a \,.\, r^7 = 768$$

$$\Rightarrow \quad a \times 2^7 = 768$$

$$\Rightarrow \quad a \times 128 = 768$$

$$a = 6$$

$$\therefore \quad t_{10} = a \,.\, r^9$$

$$= 6 \times 2^9$$

$$= 6 \times 512$$

$$= 3072$$

8. If a, b, c are in A.P. then show that $3^a, 3^b, 3^c$ are in G.P.

Solution :

Given a, b, c are in A.P.

$$\Rightarrow b = \frac{a+c}{2} \qquad ......(1)$$

To Prove : $3^a, 3^b, 3^c$ are in G.P.

i.e. TP : $(3^b)^2 = 3^a \cdot 3^c$

$$\begin{aligned} \text{LHS :} \quad & (3^b)^2 \\ &= 3^{2b} \\ &= 3^{a+c} \qquad \text{(from (1))} \\ &= 3^a \cdot 3^c \\ &= \text{RHS} \end{aligned}$$

$\therefore$ $3^a, 3^b, 3^c$ are in G.P.

9. In a G.P. the product of three consecutive terms is 27 and the sum of the product of two terms taken at a time is $\frac{57}{2}$. Find the three terms.

Solution :

Let the 3 consecutive terms of a G.P be $\frac{a}{r}, a, ar$

i) Product of 3 terms $= 27$

$$\Rightarrow \frac{a}{r} \times a \times ar = 27$$

$$\Rightarrow a^3 = 27$$

$$\therefore a = 3$$

ii) Sum of product of terms taken 2 at a

$$\text{time} = \frac{57}{2}$$

$$i.e., \frac{a}{r} \times a + a \times ar + ar \times \frac{a}{r} = \frac{57}{2}$$

$$\Rightarrow a^2\left[\frac{1}{r} + r + 1\right] = \frac{57}{2}$$

$$\Rightarrow 9\left[\frac{1+r^2+r}{r}\right] = \frac{57}{2}$$

$$\Rightarrow \frac{1+r+r^2}{r} = \frac{\overset{19}{\cancel{57}}}{\underset{3}{\cancel{9}} \times 2}$$

$$\Rightarrow \frac{1+r+r^2}{r} = \frac{19}{6}$$

$$\Rightarrow 6r^2 + 6r + 6 = 19r$$

$$\Rightarrow 6r^2 - 13r + 6 = 0$$

$$\Rightarrow r = 3/2,\ 2/3$$

$$\therefore a = 3,\ r = 3/2 \Rightarrow \text{3 terms are } 2, 3, 9/2$$

&

$$\therefore a = 3,\ r = 3/2 \Rightarrow \text{3 terms are } 9/2, 3, 2$$

10. A man joined a company as Assistant Manager. The company gave him a starting salary of ₹ 60,000 and agreed to increase his salary 5% annually. What will be his salary after 5 years?

Solution :

Given, initial salary = Rs. 60,000

Annual increment = 5%

Salary increment at the end of 1 year =

$$60,0\cancel{00} \times \frac{5}{1\cancel{00}} = 3000$$

$\therefore$ Continuing this way, we want to find the total salary after 5 years.

$$\begin{aligned} A &= P\left(1 + r/100\right)^n \\ &= 60,000\left(1 + 5/100\right)^n \\ &= 60,000 \times \left(\frac{105}{100}\right)^5 \\ &= 60,000 \times (1.05)^5 \\ &= \text{Rs. } 76,600 \end{aligned}$$

log 60,000 = 4.7782

5 log (1.05) = 0.1060

5.8842

Antilog 76,600

---

11. Sivamani is attending an interview for a job and the company gave two offers to him.

Offer A: ₹ 20,000 to start with followed by a guaranteed annual increase of 6% for the first 5 years.

Offer B: ₹ 22,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years.

What is his salary in the 4th year with respect to the offers A and B?

Solution :

Offer A :

P = ₹ 20,000 $r = 6\%$

$n = 3$ (in the 4th year)

$$A = P\left(1 + {}^{r}\!/_{100}\right)^3$$

log 20,000 = 4.3010

3 log (1.06) = 0.0759

4.3769

Antilog 23820

$$= 20,000\ \left(1 + {}^{6}\!/_{100}\right)^3$$

$$= 20,000\ \left(\frac{106}{100}\right)^3$$

$$= 20,000\,(1.06)^3$$

$$= 23,820$$

Offer B :

P = ₹ 22,000 $r = 3\%$

$n = 3$ (in the 4th year)

$$A = P\left(1 + {}^{r}\!/_{100}\right)^3$$

log 22,000 = 4.3424

3 log (1.03) = 0.0384

$= 22,000 \times (1.03)^3$

4.3808

= ₹ 24040

Antilog 24040

---

12. If a, b, c are three consecutive terms of an A.P. and x, y, z are three consecutive terms of a G.P. then prove that $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$.

Solution :

Given a, b, c are consecutive terms of A.P.

$\Rightarrow a,\ a + d,\ a + 2d, ...$

x, y, z are consecutive terms of G.P.

$\Rightarrow x,\ xr,\ xr^2, ...$

T.P : $x^{b-c} \times y^{c-z} \times z^{a-b} = 1$

LHS : $x^{b-c} \times y^{c-a} \times z^{a-b} = x\!-d \times (xr)^{2d} \times (xr^2)^{-d}$

$= x^0 \times r^{2d} \times r^{-2d} = x^0 \times r^0 = 1$

= RHS. Hence proved.

VIII. GEOMETRIC SERIES :

Key Points

- ✓ A series whose terms are in Geometric progression is called Geometric series.
- ✓ The sum to n terms is $S_n = \frac{a(r^n - 1)}{r - 1}$, r ≠ 1.
- ✓ If r = 1, then $S_n = a + a + a + ... + a = na$.
- ✓ The sum of infinite terms of a G.P. is given by $a + ar + ar^2 + ar^3 + ... = \frac{a}{1 - r}$, $-1 < r < 1$.

Example 2.46

Find the sum of 8 terms of the G.P.

1, –3, 9, –27, ...

Solution :

Here the first term $a = 1$,

common ratio $r = \frac{-3}{1} = -3 < 1$, Here $n = 8$.

Sum to n terms of a G.P. is

$$S_n = \frac{a(r^n - 1)}{r - 1} \text{ if } r \neq 1$$

Hence, $$S_n = \frac{1((-3)^8 - 1)}{(-3) - 1} = \frac{6561 - 1}{-4} = -1640$$

Example 2.47

Find the first term of G.P. in which $S_6 = 4095$ and r = 4.

Solution :

Common ratio = 4 > 1, Sum of first 6 terms $S_6 = 4095$

Hence, $$S_n = \frac{a(r^n - 1)}{r - 1} = 4095$$

Since, r = 4, $$\frac{a(4^6 - 1)}{4 - 1} = 4095 \text{ gives}$$

$$a \times \frac{4095}{3} = 4095$$

First term $a = 3$.

Example 2.48

How many terms of the series 1 + 4 + 16 + ... make the sum 1365 ?

Solution :

Let n be the number of terms to be added to get the sum 1365

$$a = 1, r = \frac{4}{1} = 4 > 1$$

$$S_n = 1365 \text{ gives } \frac{a(r^n - 1)}{r - 1} = 1365$$

$$\frac{1(4^n - 1)}{4 - 1} = 1365 \text{ so, } (4^n - 1) = 4095$$

$$4^n = 4096 \text{ then } 4^n = 4^6$$

$$n = 6$$

Example 2.49

Find the sum $3 + 1 + \frac{1}{3} + ... \infty$

Solution :

Here $a = 3$, $r = \frac{t_2}{t_1} = \frac{1}{3}$

$$\text{Sum of infinite terms} = \frac{a}{1 - r} = \frac{3}{1 - \frac{1}{3}} = \frac{9}{2}$$

Example 2.50

Find the rational form of the number 0.6666 ...

Solution :

We can express the number 0.6666 ... as follows

0.6666... = 0.6 + 0.06+ 0.006 + 0.0006 + ...

We now see that numbers 0.6, 0.06, 0.006 ... forms an G.P. whose first term a = 0.6 and common ratio r = $\frac{0.06}{0.6}$ = 0.1. Also –1 < r = 0.1 < 1

Using the infinite G.P. formula, we have

$$0.6666... = 0.6 + 0.06 + 0.006 + 0.0006 + ...$$

$$= \frac{0.6}{1-0.1} = \frac{0.6}{0.9} = \frac{2}{3}$$

Thus the rational number equivalent of 0.6666 ... is $\frac{2}{3}$

Example 2.51

Find the sum to n terms of the series 5 + 55 + 555 + ...

Solution :

The series is neither Arithmetic nor Geometric series. So it can be split into two series and then find the sum.

5 + 55 + 555 + ... + n terms = 5 [1 + 11 + 111 + ... + n terms]

$$= \frac{5}{9}[9 + 99 + 999 + ... + n \text{ terms}]$$

$$= \frac{5}{9}[(10-1) + (100-1) + (1000-1) + ... + n \text{ terms}]$$

$$= \frac{5}{9}[(10 + 100 + 1000 + ... + n \text{ terms}) - n]$$

$$= \frac{5}{9}\left[\frac{10(10^n - 1)}{(10-1)} - n\right] = \frac{50(10^n - 1)}{81} - \frac{5n}{9}$$

Example 2.52

Find the least positive integer n such that 1 + 6 + 6^2 + ... + 6^n > 5000.

Solution :

We have to find the least number of terms for which the sum must be greater than 5000.

That is, to find the least value of n, such that S_n > 5000

We have $$S_n = \frac{a(r^n - 1)}{r-1} = \frac{1(6^n - 1)}{6-1} = \frac{6^n - 1}{5}$$

$$S_n > 5000 \text{ gives } \frac{6^n - 1}{5} > 5000$$

$$6^n - 1 > 25000 \text{ gives } 6^n > 25001$$

Since, $6^5 = 7776$ and $6^6 = 46656$

The least positive value of n is 6 such that 1 + 6 + 62 + ... + 6^n > 5000.

Example 2.53

A person saved money every year, half as much as he could in the previous year. If he had totally saved ₹ 7875 in 6 years then how much did he save in the first year ?

Solution :

Total amount saved in 6 years is S_6 = 7875

Since he saved half as much money as every year he saved in the previous year.

We have $r = \frac{1}{2} < 1$

$$\frac{a(1-r^n)}{1-r} = \frac{a\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}} = 7875$$

$$\frac{a\left(1-\frac{1}{64}\right)}{\frac{1}{2}} = 7875 \text{ gives } a \times \frac{63}{32} = 7875$$

$$a = \frac{7875 \times 32}{63} \text{so, } a = 4000$$

The amount saved in the first year is ₹4000.

EXERCISE 2.8

1. Find the sum of first *n* terms of the G.P.

(i) $5, -3, \frac{9}{5}, -\frac{27}{25}, ...$ (ii) 256, 64, 16, ...

Solution :

i) Given G.P is $5, -3, \frac{9}{5}, \frac{-27}{25}, ...$

$$a = 5,\ r = {}^{-3}\!/_5 < 1$$

$$\therefore\ S_n = a\,.\frac{1-r^n}{1-r}$$

$$= (5) \times \left(\frac{1-\left({}^{-3}\!/_5\right)^n}{1-\left({}^{-3}\!/_5\right)} \right)$$

$$= (5) \times \left(\frac{1-\left({}^{-3}\!/_5\right)^n}{{}^{8}\!/_5} \right)$$

$$= \frac{25}{8}\left(1-\left({}^{-3}\!/_5\right)^n\right)$$

ii) Given G.P is 256, 64, 16, ...

$$a = 256,\ r = {}^{1}\!/_4 < 1$$

$$\therefore\ S_n = a\,.\frac{1-r^n}{1-r}$$

$$= 256 \times \frac{1-\left({}^{1}\!/_4\right)^n}{1-{}^{1}\!/_4}$$

$$= 256 \times \frac{1-\left({}^{1}\!/_4\right)^n}{{}^{3}\!/_4}$$

$$= \frac{1024}{3}\left(1-\left({}^{1}\!/_4\right)^n\right)$$

---

2. Find the sum of first six terms of the G.P. 5, 15, 45, …

Solution :

Given G.P is 5, 15, 45, ...

$a = 5, r = 3 > 1$

$$S_n = a\,.\frac{r^n-1}{r-1}$$

$$\therefore\ S_6 = 5\,.\frac{3^6-1}{3-1}$$

$$= {}^{5}\!/_2 \times 728$$

$$= 5 \times 364$$

$$= 1820$$

---

3. Find the first term of the G.P. whose common ratio 5 and whose sum to first 6 terms is 46872.

Solution :

Given $r = 5$, $S_6 = 46872$

$$S_n = a\,.\frac{r^n-1}{r-1}$$

$$\Rightarrow\ a \times \frac{5^6-1}{4} = 46872$$

$$\Rightarrow\ a\,(5^6-1) = 46872 \times 4$$

$$\Rightarrow\ a\,(15624) = 46872 \times 4$$

$$\therefore\ a = \frac{46872 \times 4}{15624}$$

$$= 3 \times 4$$

$$a = 12$$

---

4. Find the sum to infinity of

(i) 9 + 3 + 1 + ...

(ii) $21 + 14 + \frac{28}{3} + ...$

Solution :

i) 9 + 3 + 1 + ... is a geometric series

with $a = 9,\ r = {}^{1}\!/_3 < 1$

$$S_\infty = \frac{a}{1-r} = \frac{9}{1-{}^{1}\!/_3}$$

$$= \frac{9}{{}^{2}\!/_3}$$

$$= {}^{27}\!/_2$$

ii) $21+14+\frac{28}{3}, \ldots\ldots\ldots$ is geo. series

with a = 21, $r=\frac{14}{21}=\frac{2}{3}<1$

$$\therefore S_\infty=\frac{a}{1-r}=\frac{21}{1-\frac{2}{3}}$$

$$=\frac{21}{1-\frac{2}{3}}$$

$$=\frac{21}{\frac{1}{3}}$$

$$=63$$

5. If the first term of an infinite G.P. is 8 and its sum to infinity is $\frac{32}{3}$ then find the common ratio.

Solution :

Given $a=8,\ S_\infty=\frac{32}{3},\ r=?$

$$\Rightarrow \frac{a}{1-r}=\frac{32}{3}$$

$$\Rightarrow \frac{\overset{1}{\cancel{8}}}{1-r}=\frac{\overset{4}{\cancel{32}}}{3}$$

$$\Rightarrow \quad 3=4-4r$$

$$\Rightarrow \quad 4r=1$$

$$\therefore \quad r=\frac{1}{4}$$

6. Find the sum to n terms of the series

(i) 0.4 + 0.44 + 0.444 + ... to *n* terms

(ii) 3 + 33 + 333 + ... to *n* terms

Solution :

i) 0.4 + 0.44 + 0.444 + ... to *n* terms

$$=\frac{4}{10}+\frac{44}{100}+\frac{444}{1000}+\ldots\ldots\ldots \text{ to } n \text{ terms}$$

$$=4\left[\frac{1}{10}+\frac{11}{100}+\frac{111}{1000}+\ldots\ldots\ n \text{ terms}\right]$$

$$=\frac{4}{9}\left[\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\ldots\ldots\ n \text{ terms}\right]$$

$$=\frac{4}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\left(1-\frac{1}{1000}\right)+\ldots\ldots\ n \text{ terms}\right]$$

$$=\frac{4}{9}\left[(1+1+1+\ldots\ldots\ n \text{ terms})-\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\ldots\ldots\ n \text{ terms}\right)\right]$$

$$=\frac{4}{9}\left[n-\frac{1}{10}\left(\frac{1-\left(\frac{1}{10}\right)^n}{1-\frac{1}{10}}\right)\right]$$

$$=\frac{4}{9}\left[n-\frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)\right]$$

ii) 3 + 33 + 333 + ... upto *n* terms

$$=3\,(1+11+111+\ldots\ldots\ldots+n \text{ terms})$$

$$=\frac{3}{9}\,(9+99+999+\ldots\ldots\ldots+n \text{ terms})$$

$$=\frac{3}{9}[(10-1)+(100-1)+(1000-1)+\ldots\ldots\ldots\ n \text{ terms}]$$

$$=\frac{3}{9}\left[(10+100+1000+\ldots\ldots\ n \text{ terms}\right]-\left[(1+1+1+\ldots\ldots\ n \text{ terms}\right]$$

$$=\frac{3}{9}\left[10.\left(\frac{10^n-1}{n}\right)-n\right]$$

$$=\frac{30}{81}(10^n-1)-\frac{3n}{9}$$

$$=\frac{10}{27}(10^n-1)-\frac{n}{3}$$

7. Find the sum of the Geometric series 3 + 6 + 12 + ... + 1536.

Solution :

Given 3 + 6 + 12 + ... + 1536 is a geometric series

$$a=3,\ r=2,\ t_n=1536$$

$$\Rightarrow \quad a \,.\, r^{n-1} = 1536$$

$$\Rightarrow \quad 3 \,.\, 2^{n-1} = 1536$$

$$\Rightarrow \quad 2^{n-1} = \frac{1536}{3}$$

$$\Rightarrow \quad 2^{n-1} = 512 = 2^9$$

$$\therefore \; n-1 = 9$$

$$n = 10$$

$$\therefore \quad S_n = a \,.\, \frac{r^n - 1}{r - 1}$$

$$S_{10} = 3 \times \frac{2^{10} - 1}{2 - 1}$$

$$= 3\,(1023)$$

$$= 3069$$

---

8. Kumar writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they continue the process similarly. Assuming that the process is unaltered and it costs ₹ 2 to mail one letter, find the amount spent on postage when 8th set of letters is mailed.

Solution :

By the data given,

The number of mails delivered are

4, 4 × 4, 4 × 4 × 4, ...

i.e., 4, 16, 64, ... 8th set of letters.

Each mail costs ₹ 2

∴ The total cost is

(4 × 2) + (16 × 2) + (64 × 2) + ... 8th set

= 8 + 32 + 28 + ... 8th set (which forms)

form a geometric series with $a = 8$, $r = 4$, $n = 8$

$$\therefore S_n = a \,.\, \frac{r^n - 1}{r - 1}$$

$$S_8 = 8 \,.\, \frac{4^8 - 1}{3}$$

$$= 8 \times \frac{65535}{3}$$

$$= 8 \times 21845$$

$$= ₹\,174760$$

---

9. Find the rational form of the number $0.\overline{123}$.

Solution :

Let $\quad x = 0.\overline{123}$

$x = 0.123123123$ .....(1)

$\Rightarrow 1000\,x = 123.123123$ ....

$\Rightarrow 1000\,x = 123.\overline{123}$ .....(2)

$\therefore$ (2) – (1) $\Rightarrow 999x = 123$

$$\Rightarrow x = \frac{123}{999}$$

$$\therefore x = \frac{41}{333}$$

---

10. If $S_n = (x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots\ldots$ n terms then prove that

$$(x - y)\, S_n = \left| \frac{x^2\,(x^n - 1)}{x - 1} - \frac{y^2\,(y^n - 1)}{y - 1} \right|$$

Solution :

Given

$S_n = (x + y)\,(x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots + n$ terms

$\Rightarrow \quad (x - y)\, S_n = (x - y)\,(x + y) + (x - y)$
$(x^2 + xy + y^2) + (x - y)\,(x^3 + x^2y + xy^2 + y^3)$
$+ \ldots\ldots\; n$ terms

$\Rightarrow \quad (x - y)\, S_n = (x^2 - y^2) + (x^3 + y^3) + (x^4 - y^4)$
$+ \ldots\ldots\; n$ terms

$(x^2 + x^3 + x^4 + \dots\dots \; n \text{ terms})$

$- (y^2 + y^3 + y^4 + \dots\dots \; n \text{ terms})$,

both of them are geometric series

$a = x^2, r = x$ & $a = y^2, r = y$

$$\therefore\ (x-y)\,S_n = \frac{x^2(x^n-1)}{x-1} - \frac{y^2(y^n-1)}{y-1}$$

$$\left(\because S_n = a\,.\,\frac{r^n-1}{r-1}\right)$$

Hence proved.

IX. SPECIAL SERIES :

Key Points

- ✓ The sum of first n natural numbers $1 + 2 + 3 + \dots.. + \text{n} = \dfrac{n(n+1)}{2}$
- ✓ The sum of squares of first n natural numbers

  $1^2 + 2^2 + 3^2 + \dots. + n^2 = \dfrac{n(n+1)(2n+1)}{6}$
- ✓ The sum of cubes of first n natural numbers $1^3 + 2^3 + 3^3 + \dots. + n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$
- ✓ The sum of first n odd natural numbers $1 + 3 + 5 + \dots + (2\text{n} - 1) = n^2$.

Example 2.54

Find the value of (i) $1 + 2 + 3 + \dots. + 50$ (ii) $16 + 17 + 18 + \dots. + 75$

Solution :

(i) $1 + 2 + 3 + \dots + 50$

Using, $1 + 2 + 3 + \dots. + \text{n} = \dfrac{n(n+1)}{2}$

$$1 + 2 + 3 + \dots. + 50 = \frac{50\times(50+1)}{2} = 1275$$

(ii) $16 + 17 + 18 + \dots + 75$

$= (1+2+3 + \dots + 75) - (1 + 2 + 3 + \dots + 15)$

$$= \frac{75(75+1)}{2} - \frac{15(15+1)}{2}$$

$= 2850 - 120 = 2730$

Example 2.55

Find the sum of (i) $1 + 3 + 5 + \dots. +$ to 40 terms (ii) $2 + 4 + 6 + \dots. + 80$ (iii) $1 + 3 + 5 + \dots. + 55$

Solution :

(i) $1 + 3 + 5 + \dots.. + 40 \text{ terms} = 40^2 = 1600$

(ii) $2 + 4 + 6 + \dots.. + 80$

$= 2\,(1 + 2 + 3 + \dots. + 40)$

$$= 2 \times \frac{40\times(40+1)}{2} = 1640$$

(iii) $1 + 3 + 5 + \dots. + 55$

Here the number of terms is not given. Now we have to find the number of terms using the formula, $n = \dfrac{(l-a)}{d} + 1$ gives $n = \dfrac{(55-1)}{2} + 1 = 28$.

Therefore, $1 + 3 + 5 + \dots. + 55$

$= (28)^2 = 784$.

Example 2.56

Find the sum of (i) $1^2 + 2^2 + ... + 19^2$

(ii) $5^2 + 10^2 + 15^2 + ... + 105^2$

(iii) $15^2 + 16^2 + 17^2 + ... 28^2$

Solution :

(i) $$1^2+2^2+.....+19^2 = \frac{19\times(19+1)\ (2\times19+1)}{6}$$

$$= \frac{19\times20\times39}{6} = 2470$$

(ii) $$5^2+10^2+15^2+....+105^2$$

$$= 5^2(1^2+2^2+3^2+....+21^2)$$

$$= 25\times\frac{25\times(21+1)\ (2\times21+1)}{6}$$

$$= \frac{25\times21\times22\times43}{6} = 82775$$

(iii) $$15^2+16^2+17^2+....28^2$$

$$= (1^2+2^2+3^2+....+28^2)$$

$$-(1^2+2^2+3^2+.....+14^2)$$

$$= \frac{28\times29\times57}{6} - \frac{14\times15\times29}{6}$$

$$= 7714-1015 = 6699$$

Example 2.57

Find the sum of (i) $1^3 + 2^3 + 3^3 + ... + 16^3$
(ii) $9^3 + 10^3 + ... + 21^3$

Solution :

(i) $$1^3+2^3+3^3+....+16^3 = \left[\frac{16\times(16+1)}{2}\right]^2$$

$$= (136)^2 = 18496$$

(ii) $$9^3+10^3+....+21^3 = (1^3+2^3+3^3+.....+21^3)$$

$$-(1^3+2^3+3^3+....+8^3)$$

$$= \left[\frac{21\times(21+1)}{2}\right]^2 = \left[\frac{8\times(8+1)}{2}\right]^2$$

$$= (231)^2-(36)^2 = 52065$$

Example 2.58

If 1 + 2 + 3 + ... + n = 666 then find *n*.

Solution :

Since, $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$,

we have $\frac{n(n+1)}{2} = 666$

$n^2 + n - 1332 = 0$ gives $(n + 37)(n - 36) = 0$

So, n = – 37 or n = 36

But n ≠ – 37 (Since *n* is a natural number) ; Hence *n* = 36.

EXERCISE 2.9

1. **Find the sum of the following series**

   i) 1 + 2 + 3 + ... + 60

   ii) 3 + 6 + 9 + ... + 96

   iii) 51 + 52 + 53 + ... + 92

   iv) 1 + 4 + 9 + 16 + ... + 225

   v) $6^2 + 7^2 + 8^2 + ... + 21^2$

   vi) $10^3 + 11^3 + 12^3 + ... + 20^3$

   vii) 1 + 3 + 5 + ... + 71

Solution :

i) 1 + 2 + 3 + ... + 60

$$\sum_{k=1}^{n} K = \frac{n(n+1)}{2}$$

$$= \frac{60\times61}{2}$$

$$= 30\times61$$

$$= 1830$$

ii) 3 + 6 + 9 + ... + 96

= 3 (1 + 2 + 3 + ... + 32)

$$= 3\left(\frac{32\times33}{2}\right)$$

$$= 3\times16\times33$$

$$= 1584$$

iii) $51 + 52 + 53 + \ldots\ldots.. + 92$

$= (1 + 2 + 3 + \ldots\ldots\ldots. + 92) - (1 + 2 + 3 + \ldots\ldots.. + 50)$

$$= \frac{92 \times 93}{2} - \frac{50 \times 51}{2}$$

$= 46 \times 93 - 25 \times 51$

$= 4278 - 1275$

$= 3003$

iv) $1 + 4 + 9 + 16 + \ldots\ldots.. + 225$

$= 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 15^2$

$$\sum_{k=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{15 \times 16 \times 31}{6}$$

$= 1240$

v) $6^2 + 7^2 + 8^2 + \ldots\ldots + 21^2$

$= (1^2 + 2^2 + 3^2 + \ldots\ldots. + 21^2) - (1^2 + 2^2 + \ldots\ldots. + 5^2)$

$$= \frac{21 \times 22 \times 43}{6} - \frac{5 \times 6 \times 11}{6}$$

$= 3311 - 55$

$= 3256$

vi) $10^3 + 11^3 + 12^3 + \ldots.. + 20^3$

$= (1^3 + 2^3 + \ldots\ldots. + 20^3) - (1^3 + 2^3 + \ldots\ldots. + 9^3)$

$$\sum_{k=1}^{n} K^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$= \left(\frac{20 \times 21}{2}\right)^2 - \left(\frac{9 \times 10}{2}\right)^2$$

$= (210)^2 - (45)^2$

$= 44100 - 2025$

$= 42075$

vii) $1 + 3 + 5 + \ldots\ldots + 71$

$a = 1, d = 2, l = 71$

$$\therefore n = \frac{l-a}{d} + 1$$

$$= \frac{71-1}{2} + 1$$

$= 36$

$\therefore\ 1 + 3 + 5 + \ldots\ldots. + 71 = (36)^2$

$(\because\ 1 + 3 + 5 + \ldots\ldots\ldots + n \text{ terms} = n^2)$

$= 1296$

2. If $1 + 2 + 3 + \ldots\ldots. + k = 325$, then find $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots.. + k^3$

Solution :

Given $1 + 2 + 3 + \ldots\ldots\ldots + k = 325$

$$\Rightarrow \frac{k(k+1)}{2} = 325$$

$\therefore\ 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots.. + k^3$

$$= \left(\frac{k(k+1)}{2}\right)^2$$

$= (325)^2$

$= 105625$

3. If $1^3 + 2^3 + 3^3 + \ldots\ldots. + k^3 = 44100$, then find $1 + 2 + 3 + \ldots\ldots\ldots.. + k$.

Solution :

Given $1^3 + 2^3 + 3^3 + \ldots\ldots.. + k^3 = 44100$

$$\Rightarrow \left(\frac{k(k+1)}{2}\right)^2 = 44100$$

$$\Rightarrow \frac{k(k+1)}{2} = 210$$

$\Rightarrow\ 1 + 2 + 3 + \ldots.. + k = 210$

4. How many terms of the series $1^3 + 2^3 + 3^3 + ...$ should be taken to get the sum 14400?

Solution :

Given $1^3 + 2^3 + 3^3 + ... + k^3 = 14400$

$$\Rightarrow \left(\frac{k(k+1)}{2}\right)^2 = 14400$$

$$\Rightarrow \frac{k(k+1)}{2} = 120$$

$$\Rightarrow k^2 + k - 240 = 0$$

$$\Rightarrow (k+16)(k-15) = 0$$

$$\therefore k = -16,\ k = 15$$

$$\text{But } k \neq -16$$

$$\therefore\ k = 15$$

---

5. The sum of the squares of the first n natural numbers is 285, while the sum of their cubes is 2025. Find the value of n.

Solution :

Give sum of the squares of first 'n' natural numbers = 285

$$i.e.,\ \frac{n(n+1)(2n+1)}{6} = 285 \qquad ....(1)$$

and Sum of their cubes = 2025

$$i.e.,\ \left(\frac{n(n+1)}{2}\right)^2 = 2025$$

$$\Rightarrow n\left(\frac{n+1}{2}\right) = 45 \qquad ....(2)$$

Sub (2) in (1)

$$(1) \Rightarrow \frac{n(n+1)}{2} \times \frac{2n+1}{3} = 285$$

$$\Rightarrow 45 \times \frac{2n+1}{3} = 285$$

$$\Rightarrow 2n+1 = \frac{285}{15} = 19$$

$$\Rightarrow 2n = 18$$

$$\therefore\ n = 9$$

---

6. Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm,…, 24 cm. How much area can be decorated with these colour papers?

Solution :

Given sides of 15 square Colour papers are

10 cm, 11 cm, 12 cm, ... 24 cm

$\therefore$ its area $= 10^2 + 11^2 + 12^2 + ... + 24^2$

$= (1^2 + 2^2 + 3^2 + ... + 24^2) - (1^2 + 2^2 + ... + 9^2)$

$$= \frac{24 \times 25 \times 49}{6} - \frac{9 \times 10 \times 19}{6}$$

$= 4900 - 285$

$= 4615$ cm^2

---

7. Find the sum of the series to $(2^3 - 1^3) + (4^3 - 3^3) + (6^3 - 5^3) + ...$ to

(i) n terms (ii) 8 terms

Solution :

To find the sum of the series :

i) $(2^3 - 1^3) + (4^3 - 3^3) + (6^3 - 5^3) + ... n$ terms

$= (2^3 + 4^3 + 6^3 + ... n$ terms)

$- (1^3 + 3^3 + 5^3 + ...$ n terms)

$$\sum_1^n (2n)^3 - \sum_1^n (2n-1)^3$$

$$= \sum_1^n [(2n)^3 - (2n-1)^3]$$

$$(\because a^3 - b^3 = (a-b)(a^2 + ab + b^2))$$

$$= \sum_1^n [(2n - 2n + 1)(4n^2 + 2n(2n-1) + (2n-1)^2]$$

$$= \sum_1^n [4n^2 + 4n^2 - 2n + 4n^2 - 4n + 1]$$

$$= \sum_1^n [12n^2 - 6n + 1]$$

$$= 12\sum n^2 - 6\sum n + \sum_1$$

$$= \overset{2}{\cancel{12}}\left[\frac{n(n+1)(2n+1)}{\cancel{6}}\right] - \overset{3}{\cancel{6}}\left[\frac{n(n+1)}{\cancel{2}}\right] + n$$

$$= n(n+1)[4n+2-3]+n$$

$$= (n^2+n)(4n-1)+n$$

$$= 4n^3+4n^2-n^2-n+n$$

$$= 4n^3+3n^2$$

ii) When n = 8,

$S_8 = 4(8^3) + 3(8^2)$

$= 4(512) + 3(64)$

$= 2048 + 192$

$= 2240$

EXERCISE 2.10

Multiple choice questions :

1. Euclid's division lemma states that for positive intgers *a* and *b*, there exist unique integers *q* and *r* such that *a* = *bq* + *r*, where *r* must satisfy.

(1) $1 < r < b$ (2) $0 < r < b$

(3) $0 \le r < b$ (4) $0 < r \le b$

Hint : **Ans : (3)**

By definition of Euclid's lemma $0 \le r < b$

---

2. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are

(1) 0, 1, 8 (2) 1, 4, 8

(3) 0, 1, 3 (4) 1, 3, 5

Hint : **Ans : (1)**

$x^3 \equiv y \pmod 9$

when $x = 3$, $y = 0$ (27 is div. by 9)

when $x = 4$, $y = 1$ (63 is div. by 9)

when $x = 5$, $y = 8$ (117 is div. by 9)

$\therefore$ The remainders are 0, 1, 8, ...

---

3. If the HCF of 65 and 117 is expressible in the form of 65m – 117, then the value of *m* is

(1) 4 (2) 2 (3) 1 (4) 3

Hint : **Ans : (2)**

HCF of 65, 117 is 13

65m – 117 = 13

$\Rightarrow$ 65m = 130

$\Rightarrow$ m = 2

---

4. The sum of the exponents of the prime factors in the prime factorization of 1729 is

(1) 1 (2) 2 (3) 3 (4) 4

Hint : **Ans : (3)**

$1729 = 7 \times 13 \times 19$

$= 7^1 \times 13^1 \times 19^1$

$\therefore$ Sum of the exponents = 1 + 1 + 1 = 3

---

5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is

(1) 2025 (2) 5220 (3) 5025 (d) 2520

Hint : **Ans : (4)**

Refer 9th sum in Ex. 2.2

---

6. $7^{4k} \equiv$ _______ (mod 100)

(1) 1 (2) 2 (3) 3 (4) 4

Hint : **Ans : (1)**

If k = 1, 7^4 leaves remainder 1 modulo 100.

---

7. Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is

(1) 3 (2) 5 (3) 8 (4) 11

Hint : **Ans : (4)**

$F_3 = F_2 + F_1 = 4$

$F_4 = F_3 + F_2 = 7$

$F_5 = F_4 + F_3 = 4 + 7 = 11$

8. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P.

(1) 4551 (2) 10091
(3) 7881 (4) 13531

Hint : **Ans : (3)**

$a = 1, d = 4$

$\therefore$ The A.P is 1, 5, 9, 13, ... leaves remainder 1 when divided by 4.

$\therefore$ 7881 leaves remainder 1 when divided by 4.

---

9. If 6 times 6th term of an A.P. is equal to 7 times the 7th term, then the 13th term of the A.P. is

(1) 0 (2) 6 (3) 7 (4) 13

Hint : **Ans : (1)**

$$6(t_6) = 7\,(t_7) \Rightarrow 6\,(a + 5d) = 7\,(a + 6d)$$
$$\Rightarrow 6a + 30d = 7a + 42d$$
$$\Rightarrow a + 12d = 0$$
$$\Rightarrow t_{13} = 0$$

---

10. An A.P. consists of 31 terms. If its 16th term is *m*, then the sum of all the terms of this A.P. is

(1) 16m (2) 62m (3) 31m (d) $\frac{31}{2}$m

Hint : **Ans : (3)**

$$n = 31,\ a + 15d = m \quad S_{31} = \frac{31}{2}\,[2a + 30d]$$
$$= 31\,(a + 15d)$$
$$= 31m$$

---

11. In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P. must be taken for their sum to be equal to 120?

(1) 6 (2) 7 (3) 8 (4) 9

Hint : **Ans : (3)**

$a = 1$, d = 4, $S_n = 120$

$$\Rightarrow \frac{n}{2}(2a + (n-1)d) = 120$$
$$\Rightarrow \frac{n}{2}(2 + (n-1)4) = 120$$
$$\Rightarrow n(1 + 2n - 2) = 120$$
$$\Rightarrow n(2n - 1) = 120$$
$$\Rightarrow 2n^2 - n - 120 = 0$$
$$\Rightarrow n = 8$$

$$\begin{array}{c|c} -1 & -240 \\ \hline \frac{-16}{2} & \frac{15}{2} \end{array}$$
$$-8,\ \frac{15}{2}$$

---

12. If A = 2^{65} and B = $2^{64} + 2^{63} + 2^{62} + ... + 2^0$ which of the following is true ?

(1) B is 2^{64} more than A

(2) A and B are equal

(3) B is larger than A by 1

(4) A is larger than B by 1

Hint : **Ans : (4)**

2^4 is greater than $2^0 + 2^1 + 2^2 + 2^3$ by 1

2^5 is greater than $2^0 + 2^1 + 2^2 + 2^3 + 2^4$ by 1

$\therefore$ 2^{65} is greater than $2^0 + 2^1 + ... + 2^{64}$ by 1

$\therefore$ A is larger than B by 1.

---

13. The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, ...$ is

(1) $\frac{1}{24}$ (2) $\frac{1}{27}$ (3) $\frac{2}{3}$ (4) $\frac{1}{81}$

Hint : **Ans : (2)**

$$r = \frac{1/8}{3/16} = \frac{1}{8} \times \frac{16}{3} = \frac{2}{3}$$

$$\therefore \text{Next term of the sequence} = \frac{1}{18} \times \frac{2}{3}$$
$$= \frac{1}{27}$$

14. If the sequence t_1, t_2, t_3, ... are in A.P. then the sequence t_6, t_{12}, t_{18}, ... is

(1) a Geometric Progression

(2) an Arithmetic Progression

(3) neither an Arithmetic Progression nor a Geometric Progression

(4) a constant sequence

Hint : **Ans : (3)**

Obivously they should be in A.P.

---

15. The vlaue of $(1^3 + 2^3 + 3^3 + ... + 15^3) - (1 + 2 + 3 + ... + 15)$ is

(1) 14400 (2) 14200

(3) 14280 (4) 14520

Hint : **Ans : (3)**

$$\left(\frac{15\times16}{2}\right)^2 - \frac{15\times16}{2}$$

$$=14400-120$$

$$=14280$$

UNIT EXERCISE - 2

1. Prove that $n^2 - n$ divisible by 2 for every positive interger *n*.

Solution :

Any positive integer is of the form $2q$ (or) $2q + 1$ for some integer q.

i) $n^2 - n = (2q)^2 - 2q$

$= 2q (2q - 1)$

which is divisible by 2.

ii) $n^2 - n = (2q + 1)^2 - (2q + 1)$

$= (2q + 1) (2q + 1 - 1)$

$= 2q (2q + 1)$, which is divisible by 2.

Hence proved.

---

2. A milk man has 175 litres of cow's milk and 105 litres of buffalow's milk. He wishes to sell the milk by filling the two types of milk in cans of equal capacity. Claculate the following (i) Capacity of a can (ii) Number of cans of cow's milk (iii) Number of cans of buffalow's milk.

Solution :

Cow's milk = 175 lrs.

Buffalow's milk = 105 lrs.

Since he wish to sell the milk by filling the 2 types of milk in cans of equal capacity,

i) Capacity of a can = HCF of 175 and 105

= 35 litres

ii) Number of cans of Cow's milk = $\frac{175}{35} = 5$

iii) Number of cans of buffalow's milk

$= \frac{105}{35} = 3$

---

3. When the positive integers *a, b* and *c* are divided by 13 the respective remainders are 9, 7 and 10. Find the remainder when *a* + 2b + 3c is divided by 13.

Solution :

Let $a = 13q + 9$

$b = 13q + 7 \Rightarrow 2b = 26q + 14$

$c = 13q + 10 \Rightarrow 3c = 39q + 30$

$a+ 2b + c = (13q + 9) + (26q + 14) + (39q + 30)$

$= 78q + 53$

$= 13 (6q) + 13(4) + 1$

$\therefore$ When $a + 2b + 3c$ is divided by 13, the remainder is 1.

4. **Show that 107 is of the form 4q + 3 for any integer *q*.**

Solution :

When 107 is divided by 4,

107 = 4(26) + 3

This is of the form

107 = 4q + 3 for q = 26.

5. **If (m + 1)th term of an A.P. is twice the (n + 1)th term, then prove that (3m + 1)th term is twice the (m + n + 1)th term.**

Solution :

Given $t_{m+1} = 2(t_{n+1})$

$a + (m + 1 - 1)\, d = 2\, (a + (n + 1 - 1)\, d)$

$a + md = 2\, (a + nd)$

$a + md = 2a + 2nd$ — (1)

To Prove : $t_{3m+1} = 2\, (t_{m+n+1})$

LHS : t_{3m+1}

$= a + (3m + 1 - 1)d$

$= a + 3md$

$= (a + md) + 2md$

$= 2a + 2nd + 2md$ (from (1))

$= 2\, [a + (m + n)d]$

$= 2\, [t_{m+n+1}]$

= RHS

Hence proved.

6. **Find the 12th term from the last term of the A.P – 2, – 4, – 6, ... –100.**

Solution :

Given A.P is – 2, – 4, – 6, ... – 100

To find : t_{12} from the last term

$a = -100$, d = 2

$t_{12} = a + 11d$

$= -100 + 11(2)$

$= -100 + 22$

$= -78$

7. **Two A.P.'s have the same common difference. The first term of one A.P. is 2 and that of the other is 7. Show that the difference between their 10th terms is the same as the difference between their 21st terms, which is the same as the difference between any two corresponding terms.**

Solution :

1st A.P	2nd A.P.
$a = 2$, d = d	$a = 7$, d = d
$t_{10} = a + 9d$	$T_{10} = a + 9d$
$= 2 + 9d$	$= 7 + 9d$
$t_{21} = a + 20d$	$T_{21} = a + 20d$
$= 2 + 20d$	$= 7 + 20d$

$\therefore\ T_{10} - t_{10} = 5$ and $T_{21} - t_{21} = 5 = T_n - t_n = 5$

8. **A man saved ₹16500 in ten years. In each year after the first he saved ₹100 more than he did in the preceding year. How much did he save in the first year ?**

Solution :

Given S_n = ₹ 16500, d = ₹ 100, n = 10 in A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \frac{10}{2}[2a + 9(100)] = 16500$$

$$\Rightarrow 2a + 900 = \frac{16500}{5}$$

$$\Rightarrow 2a + 900 = 3300$$

$$\Rightarrow 2a = 2400$$

$$\therefore a = 1200$$

∴ He saved Rs. 1200 in 1st year.

9. Find the G.P. in which the 2^{nd} term is $\sqrt{6}$ and the 6th term is $9\sqrt{6}$.

Solution :

Given $t_2 = \sqrt{6},\ t_6 = 9\sqrt{6}$ in G.P.

$$\Rightarrow a.r = \sqrt{6},\ \ a.r^5 = 9\sqrt{6}$$

$$\therefore r^4 = 9 \qquad \text{(when divide)}$$

$$\Rightarrow r = \sqrt{3}$$

$$\therefore a \times \sqrt{3} = \sqrt{6}$$

$$\therefore a = \sqrt{2}$$

$\therefore$ The G.P is

$\sqrt{2}, \sqrt{6}, \sqrt{18}, ...$

(or)

$\sqrt{2}, \sqrt{6}, 3\sqrt{2}, ...$

---

10. The value of a motor cycle depreciates at the rate of 15% per year. What will be the value of the motor cycle 3 year hence, which is now purchased for ₹ 45,000 ?

Solution :

P = ₹ 45000, n = 3, r = 15% (depreciation)

$$A = P\left(1 - \frac{r}{100}\right)^n$$

$$= 45{,}000\left(1 - \frac{15}{100}\right)^3$$

$$= 45{,}000 \times \frac{85}{100} \times \frac{85}{100} \times \frac{85}{100}$$

$$= 27{,}635.625$$

$$= 27636$$

PROBLEMS FOR PRACTICE

1. Express the number $0.\overline{3178}$ in the form of $\frac{a}{b}$.

   (Ans: $\frac{3178}{999}$)

2. A class of 20 boys and 15 girls is divided into *n* groups so that each group has *x* boys and *y* girls. Find *x, y* and *n*.

   (Ans : n = 7, *x* = 4, *y* = 3)

3. Show that 7^n cannot end with digit zero for any natural number.

4. Find the HCF of the following numbers by using Euclid's division algorithm.

   i) 867, 255 ii) 1656, 4025

   iii) 180, 252, 324 iv) 92690, 7378

   v) 134791, 6341, 6339

   (Ans : i) 51 ii) 23 iii) 36 iv) 31 v) 1)

5. Use Euclid's lemma, show that the square of any positive integer is either of the form 3m (or) 3m+1 for same integer m.

6. Find the largest number which divides 70 and 125 leaving remainder 5 and 8 respectively.

   (Ans : 13)

7. Find the largest positive integer that will divide 398, 436 and 542 that leaves remainders 7, 11, 15 respectively.

   (Ans : 17)

8. If HCF of 144 and 180 is expressed in the form 13m – 3, find m.

   (Ans : 3)

9. If d is the HCF of 56 and 72, find x and y satisfying $d = 56x + 72y$. Also show that x and y is not unique.

(Ans : 4 and –3, –68, 53)

10. Find the largest number of four digits exactly divisible by 12, 15, 18 and 27.

(Ans : 9720)

11. Write the first 6 terms of the sequence whose nth term is

i) $a_n = \begin{cases} n & , \text{ if } n = 1,2,3 \\ a_{n-1} + a_{n-2} + a_{n-3}, & \text{ if } n > 3 \end{cases}$

ii) $a = \dfrac{3n-2}{3^{n-1}}$

(Ans : i) 1, 2, 3, 5, 8, 13 ii) $1, \frac{4}{3}, \frac{7}{9}, \frac{10}{27}, ...$**)**

12. Find the indicated terms in each of the following :

i) $a_n = (-1)^n \cdot 2^{n+3} (n + 1)$; a_5, a_8

ii) $a_n = (-1)^n, (1 - n + n^2)$; a_2, a_9

(Ans : i) – 1536, 18432 ii) 3, –73)

13. How many temrs are there in the A.P.

$-1, \frac{-5}{6}, \frac{-2}{3}, ..., \frac{10}{3}$?

(Ans : 27)

14. Find the 40th term of an A.P whose 5th term is 41 and 11th term is 71.

(Ans : 216)

15. If 7th term of an A.P is $\frac{1}{9}$ and 9th term is $\frac{1}{7}$.

(Ans : 1)

16. Find the middle term of the A.P 213, 205, 197, ...37

(Ans :125)

17. The first term of an A.P is 5, the last term is 45. Sum of all its terms is 400. Find the number of terms and the common difference of A.P.

(Ans : n = 16, d = $\frac{8}{3}$**)**

18. The 24th term of an A.P is twice its 10th term. Show that its 72nd term is 4 times its 15th term.

19. Find the sum of all two digit odd positive numbers.

(Ans : 2475)

20. Which term of the sequence

$20, 19\frac{1}{4}, 18\frac{1}{2}$ .... is the first negative term ?

(Ans : 28)

21. Find the 18th term of the A.P from right end 3, 7, 11, ... 407.

(Ans : 339)

22. How many consecutive integers beginning with 10 must be taken for their sum to be 2035 ?

(Ans : 55)

23. Sum of 3 numbers in an A.P is 54 and their product is 5670. Find the 3 numbers.

(Ans : 15, 18, 21)

24. Find the sum of all natural numbers between 201 and 399 that are divisible by 5.

25. Find : $\left(4 - \frac{1}{n}\right) + \left(7 - \frac{2}{n}\right) + \left(10 - \frac{3}{n}\right) + ...$ up to n terms.

(Ans : $\frac{3n^2 + 4n - 1}{2}$**)**

26. The sum of first 'n' terms of an A.P is $5n - n^2$. Find the n^{th} term of the A.P.

(Ans : – 2 (n – 3))

27. If 7 times the 7^{th} term of an A.P is equal to 11 times its 11^{th} term, show that its 18^{th} term is 0.

28. Find the sum of 22 terms of the A.P $x + y$, $x - y$, $x - 3y$, ...

(Ans : 22 $(x - 20y)$)

29. If the ratio between the sums of n terms of AP's is (7n + 1) : (4n + 27), find the ratio of their 11th terms.

(Ans : 148 : 111)

30. A man gets the initial salary of 5200 p.m. and receive an automatic increase of 320 in the very next month and each month hereafter. Find i) his salary in 10th month ii) total earings during the 1st year.

(Ans : 8080, 83520)

31. Sum of 3 terms of a G.P is $\frac{39}{10}$ and their product is 1. Find the G.P.

(Ans : $\frac{5}{2},1,\frac{2}{5}$)

32. If 4^{th} and 7^{th} terms of a G.P are 54 and 1458 respectively, find G.P.

(Ans : 2, 6, 18, 54, ...)

33. In the series 18, – 12, 8, ..., which term is $\frac{512}{729}$?

(Ans : 9^{th} term)

34. Find the sum : 0.2 + 0.92 + 0.992 + ... to $n$terms.

35. Find the sum of the series :

$9^{1/3}\ 9^{1/9}\ 9^{1/27}$ .... ∞. **(Ans : 3)**

36. A farmer buys a used tractor for Rs. 12000. He pays Rs. 6000 and agress to pay the balance in annual instalment of Rs. 500 plus 12% interest on u paid amount. How much will the tractor cost him ?

(Ans : Rs. 16,680)

37. The sum of an infinite geometric series is 15 and the sum of the squares of these terms is 45. Find the series.

(Ans : $5+\frac{10}{3}+\frac{20}{9}+....$)

38. Evaluate : $1^2 + 3^2 + 5^2 +$ .... 29^2

(Ans : 4495)

39. Evaluate : $8^3 + 9^3 +$ ..... $+ 17^3$

40. The sum of the squares of first 'n' natural numbers is 285, while the sum of their cubes is 2025. Find 'n'.

(Ans : 9)

OBJECTIVE TYPE QUESTIONS

1. When the denominator number of $\frac{257}{500}$ writes in the form of $2^m \times 5^n$, then m + n is

(a) 6 (b) 5

(c) 23 (d) none of these

Ans : (b)

2. The common difference of the A.P $\frac{1}{2q},\frac{1-2q}{2q},\frac{1-4q}{2q},$.......

(a) 0 (b) 1 (c) – 1 (d) q

Ans : (c)

3. The first 3 terms of A.P are 3y – 1, 3y + 5 and 5y + 1, then y is

(a) 5 (b) 1 (c) – 5 (d) 4

Ans : (a)

4. For an A.P, $S_n = n^2 - n + 1$, the 2nd term is

(a) 2 (b) 3 (c) 4 (d) – 2

Ans : (b)

5. The next term of an A.P :

– 12, – 9, – 6, – 3, ... is

(a) 3 (b) 6

(c) 0 (d) none of these

Ans : (c)

6 The sum of 6 terms of the A.P 1, 3, 5, 7, ... is

(a) 25 (b) 49 (c) 36 (d) 30

Ans : (c)

7. Which term of the series – 3, – 1, 5, ... is 53 ?

(a) 12 (b) 13 (c) 14 (d) 15

Ans : (d)

8. The common ratio of the G.P $\frac{-5}{2}, \frac{25}{4}, \frac{-125}{8}$, ... is

(a) 15 (b) $\frac{35}{4}$ (c) $\frac{-5}{2}$ (d) $\frac{5}{2}$

Ans : (c)

9. If the 3rd term of G.P is 4, then the product of its first 5 terms is

(a) 4^3 (b) 4^5 (c) 4^4 (d) 4^2

Ans : (b)

10. If *a, b, c* are in A.P., *a, b, d*in G.P, then *a, a – b, d – c* will be in

(a) A.P (b) G.P

(c) A.P and G.P (d) none of these

Ans : (b)

11. The 3rd term of a G.P is the square of first term. If the 2nd term is 8, then the 6th term is

(a) 120 (b) 124 (c) 128 (d) 132

Ans : (d)

12. If $a_1 = a_2 = 2, a_n = a_{n-1} - 1$, then a_5 is

(a) 1 (b) – 1 (c) 0 (d) – 2

Ans : (b)

13. The 9th term of the series 27 + 9 + $5\frac{2}{5}$ + ... is

(a) $1\frac{10}{17}$ (b) $\frac{10}{17}$ (c) $\frac{16}{27}$ (d) $\frac{17}{27}$

(Ans : (a))

14. The nth term of the series 3.8 + 6.11 + 9.14 + 12.17 + ... will be

(a) 3n (n + 5) (b) n (n + 5)

(c) n (3n + 5) (d) 3n (3n + 5)

Ans : (d)

15. If the *n*th term of a G.P 5, $\frac{-5}{2}$, $\frac{5}{4}$, ... is $\frac{5}{1024}$, then *n* is

(a) 11 (b) 10 (c) 9 (d) 4

Ans : (a)

16. If 1 + 2 + 3 + ... + n = K, then $1^3 + 2^3 + 3^3 + ... + n^3$ is

(a) K^3 (b) K^2

(c) $\frac{K(K+1)}{2}$ (d) $(K + 1)^3$

Ans : (b)