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Probability

TOPIC 1 Multiplication Theorem on Probability, Independent events, Conditional Probability, Baye's Theorem

1. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is : **[Sep. 04, 2020 (II)]**

(a) $\frac{5}{31}$ (b) $\frac{31}{61}$

(c) $\frac{5}{6}$ (d) $\frac{30}{61}$

2. A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is : **[Sep. 03, 2020 (I)]**

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{8}$ (d) $\frac{1}{9}$

3. The probability that a randomly chosen 5-digit number is made from exactly two digits is : **[Sep. 03, 2020 (II)]**

(a) $\frac{135}{10^4}$ (b) $\frac{121}{10^4}$

(c) $\frac{150}{10^4}$ (d) $\frac{134}{10^4}$

4. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is : **[Sep. 02, 2020 (I)]**

(a) $\frac{2}{3}$ (b) $\frac{8}{17}$

(c) $\frac{4}{17}$ (d) $\frac{2}{5}$

5. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$.

Then $P(E_2^C \cap E_3^C / E_1)$ is equal to : **[Sep. 02, 2020 (II)]**

(a) $P(E_2^C) + P(E_3)$ (b) $P(E_3^C) - P(E_2^C)$

(c) $P(E_3) - P(E_2^C)$ (d) $P(E_3^C) - P(E_2)$

6. In a box, there are 20 cards, out of which 10 are labelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is : **[Jan. 9, 2020 (I)]**

(a) $\frac{9}{16}$ (b) $\frac{11}{16}$

(c) $\frac{13}{16}$ (d) $\frac{15}{16}$

7. Let A and B be two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$. Then, which of the following is TRUE ? **[Jan. 8, 2020 (I)]**

(a) $P(A/B) = \frac{2}{3}$ (b) $P(A/B') = \frac{1}{3}$

(c) $P(A'/B') = \frac{1}{3}$ (d) $P(A/(A \cup B)) = \frac{1}{4}$

8. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for $k = 3, 4, 5$, otherwise X takes the value -1. Then the expected value of X, is: **[Jan. 7, 2020 (I)]**

(a) $\frac{3}{16}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{16}$ (d) $-\frac{1}{8}$

9. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to: **[Jan. 7, 2020 (II)]**

(a) $\frac{17}{8}$ (b) $\frac{17}{4}$

(c) $\frac{17}{2}$ (d) 4

10. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is : **[April 12, 2019 (II)]**

(a) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$ (b) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$

(c) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$ (d) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

11. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is: **[April 10, 2019 (I)]**

(a) $\frac{1}{11}$ (b) $\frac{1}{10}$

(c) $\frac{1}{12}$ (d) $\frac{1}{17}$

12. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is : **[April 10, 2019 (II)]**

(a) 5 (b) 6

(c) 8 (d) 7

13. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is: **[April 09, 2019 (I)]**

(a) $\frac{25}{192}$ (b) $\frac{7}{32}$

(c) $\frac{1}{192}$ (d) $\frac{25}{32}$

14. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct? **[April 08, 2019 (I)]**

(a) $P(A|B) = P(B) - P(A)$ (b) $P(A|B) \geq P(A)$

(c) $P(A|B) \leq P(A)$ (d) $P(A|B) = 1$

15. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is : **[April. 08, 2019 (II)]**

(a) 5 (b) 3

(c) 4 (d) 2

16. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to : **[Jan. 12, 2019 (I)]**

(a) $\frac{200}{6^5}$ (b) $\frac{150}{6^5}$

(c) $\frac{225}{6^5}$ (d) $\frac{175}{6^5}$

17. In a game, a man wins ₹ 100 if he gets 5 or 6 on a throw of a fair die and loses ₹ 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is : **[Jan. 12, 2019 (II)]**

(a) $\frac{400}{9}$ loss (b) 0

(c) $\frac{400}{3}$ gain (d) $\frac{400}{3}$ loss

18. Two integers are selected at random from the set $\{1, 2, ..., 11\}$. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is : **[Jan. 11, 2019 (I)]**

(a) $\frac{7}{10}$ (b) $\frac{1}{2}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

19. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1, 2, 3, ..., 9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is: **[Jan 10, 2019 (I)]**

(a) $\frac{13}{36}$ (b) $\frac{15}{72}$ (c) $\frac{19}{72}$ (d) $\frac{19}{36}$

20. If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is: **[Jan. 10, 2019 (II)]**

(a) 3 (b) 6

(c) 5 (d) 4

21. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X=1)+P(X=2)$ equals: **[Jan 09, 2019 (I)]**

(a) 49/169 (b) 52/169
(c) 24/169 (d) 25/169

22. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red is: **[Jan. 09, 2019 (II)]**

(a) $\frac{21}{49}$ (b) $\frac{27}{49}$
(c) $\frac{26}{49}$ (d) $\frac{32}{49}$

23. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is : **[2018]**

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$
(c) $\frac{3}{4}$ (d) $\frac{3}{10}$

24. Let A, B and C be three events, which are pair-wise independence and $\overline{E}$ denotes the complement of an event E. If $P(A \cap B \cap C)=0$ and $P(C)>0$, then $P[(\overline{A} \cap \overline{B})|C]$ is equal to. **[Online April 16, 2018]**

(a) $P(A)+P(\overline{B})$ (b) $P(\overline{A})-P(\overline{B})$
(c) $P(\overline{A})-P(B)$ (d) $P(\overline{A})+P(\overline{B})$

25. A player *X* has a biased coin whose probability of showing heads is *p* and a player *Y* has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If *X* starts the game, and the probability of winning the game by both the players is equal, then the value of '*p*' is **[Online April 15, 2018]**

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$
(c) $\frac{1}{4}$ (d) $\frac{2}{5}$

26. If two different numbers are taken from the set (0, 1, 2, 3, ..., 10), then the probability that their sum as well as absolute difference are both multiple of 4, is : **[2017]**

(a) $\frac{7}{55}$ (b) $\frac{6}{55}$
(c) $\frac{12}{55}$ (d) $\frac{14}{55}$

27. Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then a value of $\frac{P(E)}{P(F)}$ is : **[Online April 9, 2017]**

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$
(c) $\frac{1}{3}$ (d) $\frac{5}{12}$

28. Three persons P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4}, \frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is : **[Online April 8, 2017]**

(a) $\frac{21}{64}$ (b) $\frac{9}{64}$
(c) $\frac{15}{64}$ (d) $\frac{39}{64}$

29. An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is : **[Online April 8, 2017]**

(a) $\frac{255}{256}$ (b) $\frac{127}{128}$
(c) $\frac{63}{64}$ (d) $\frac{1}{2}$

30. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true ? **[2016]**

(a) E_1 and E_3 are independent.
(b) E_1, E_2 and E_3 are independent.
(c) E_1 and E_2 are independent.
(d) E_2 and E_3 are independent.

31. If A and B are any two events such that $P(A)=\frac{2}{5}$ and $P(A \cap B)=\frac{3}{20}$, then the conditional probability, $P(A \mid A' \cup B'))$, where A' denotes the complement of A, is equal to : **[Online April 9, 2016]**

(a) $11/20$ (b) $5/17$
(c) $8/17$ (d) $1/4$

32. Let X be a set containing 10 elements and P(X) be its power set. If A and B are picked up at random from P(X), with replacement, then the probability that A and B have equal number elements, is : **[Online April 10, 2015]**

(a) $\frac{\left(2^{10}-1\right)}{2^{10}}$ (b) $\frac{{}^{20}C_{10}}{2^{10}}$

(c) $\frac{\left(2^{10}-1\right)}{2^{20}}$ (d) $\frac{{}^{20}C_{10}}{2^{20}}$

33. Let A and B be two events such that $P\left(\overline{A \cup B}\right)=\frac{1}{6}$, $P\left(\overline{A \cap B}\right)=\frac{1}{4}$ and $P\left(\overline{A}\right)=\frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are **[2014]**

(a) independent but not equally likely.
(b) independent and equally likely.
(c) mutually exclusive and independent.
(d) equally likely but not independent.

34. Let A and E be any two events with positive probabilities:

Statement - 1: P(E/A) ≥ P(A/E) P(E)

Statement - 2: P(A/E) ≥ $P(A \cap E)$

[Online April 19, 2014]

(a) Both the statements are true
(b) Both the statements are false
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

35. A, B, C try to hit a target simultaneously but independently. Their respective probabilities of hitting the targets are $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}$. The probability that the target is hit by A or B but not by C is : **[Online April 23, 2013]**

(a) 21/64 (b) 7/8
(c) 7/32 (d) 9/64

36. Given two independent events, if the probability that exactly one of them occurs is $\frac{26}{49}$ and the probability that none of them occurs is $\frac{15}{49}$, then the probability of more probable of the two events is : **[Online April 22, 2013]**

(a) 4/7 (b) 6/7
(c) 3/7 (d) 5/7

37. The probability of a man hitting a target is $\frac{2}{5}$. He fires at the target k times (k, a given number). Then the minimum k, so that the probability of hitting the target at least once is more than $\frac{7}{10}$, is : **[Online April 9, 2013]**

(a) 3 (b) 5
(c) 2 (d) 4

38. Three numbers are chosen at random without replacement from $\{1,2,3,..8\}$. The probability that their minimum is 3, given that their maximum is 6, is : **[2012]**

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$

(c) $\frac{1}{4}$ (d) $\frac{2}{5}$

39. Let A, B, C, be pairwise independent events with $P(C) > 0$ and $P(A \cap B \cap C)=0$. Then $P\left(A^c \cap B^c / C\right)$. **[2011RS]**

(a) $P\left(B^c\right)-P(B)$ (b) $P\left(A^c\right)+P\left(B^c\right)$

(c) $P\left(A^c\right)-P\left(B^c\right)$ (d) $P\left(A^c\right)-P(B)$

40. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is **[2011]**

(a) $P(C \mid D) \geq P(C)$ (b) $P(C \mid D) < P(C)$

(c) $P(C \mid D)=\frac{P(D)}{P(C)}$ (d) $P(C \mid D)=P(C)$

41. One ticket is selected at random from 50 tickets numbered 00,01,02,...,49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals: **[2009]**

(a) $\frac{1}{7}$ (b) $\frac{5}{14}$

(c) $\frac{1}{50}$ (d) $\frac{1}{14}$

42. It is given that the events A and B are such that $P(A)=\frac{1}{4}, P(A \mid B)=\frac{1}{2}$ and $P(B \mid A)=\frac{2}{3}$. Then $P(B)$ is **[2008]**

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

43. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is **[2007]**

(a) 0.2 (b) 0.7
(c) 0.06 (d) 0.14.

44. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is **[2005]**

(a) $\frac{2}{9}$ (b) $\frac{1}{9}$

(c) $\frac{8}{9}$ (d) $\frac{7}{9}$

45. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are **[2005]**

(a) equally likely and mutually exclusive
(b) equally likely but not independent
(c) independent but not equally likely
(d) mutually exclusive and independent

46. The probability that A speaks truth is $\frac{4}{5}$, while the probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is **[2004]**

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$

(c) $\frac{7}{20}$ (d) $\frac{3}{20}$

47. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is **[2002]**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

TOPIC 2 Random Variables, Probability Distribution, Bernoulli Trails, Binomial Distribution, Poisson Distribution

48. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ______. **[NA Sep. 05, 2020 (I)]**

49. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _________. **[NA Sep. 05, 2020 (II)]**

50. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ___________. **[NA Sep. 04, 2020 (I)]**

51. A random variable X has the following probability distribution:

X	:	1	2	3	4	5
P(X)	:	K^2	2K	K	2K	$5K^2$

Then, $P(X > 2)$ is equal to: **[Jan. 9, 2020 (II)]**

(a) $\frac{7}{12}$ (b) $\frac{1}{36}$

(c) $\frac{1}{6}$ (d) $\frac{23}{36}$

52. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X\, d''\, 2) = \frac{k}{2^{16}}$, then k is equal to: **[April 12, 2019 (I)]**

(a) 17 (b) 121

(c) 1 (d) 137

53. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is : **[April 12, 2019 (II)]**

(a) $\frac{1}{2}$ gain (b) $\frac{1}{4}$ loss

(c) $\frac{1}{2}$ loss (d) 2 gain

54. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, then $\left(\frac{\text{mean of X}}{\text{standard deviation of X}}\right)$ is equal to: **[Jan. 11, 2019 (II)]**

(a) 4 (b) $4\sqrt{3}$ (c) $3\sqrt{2}$ (d) $\frac{4\sqrt{3}}{3}$

55. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is: **[2017]**

(a) $\frac{6}{25}$ (b) $\frac{12}{5}$ (c) 6 (d) 4

56. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is : **[Online April 10, 2016]**

(a) $\frac{496}{729}$ (b) $\frac{192}{729}$

(c) $\frac{240}{729}$ (d) $\frac{256}{729}$

57. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than or equal to one is : **[Online April 11, 2015]**

(a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{1}{16}$ (d) $\frac{15}{16}$

58. If X has a binomial distribution, B(n, p) with parameters n and p such that P(X = 2) = P (X = 3), then E(X), the mean of variable X, is **[Online April 11, 2014]**

(a) $2-p$ (b) $3-p$ (c) $\frac{p}{2}$ (d) $\frac{p}{3}$

59. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is: **[2013]**

(a) $\frac{17}{3^5}$ (b) $\frac{13}{3^5}$ (c) $\frac{11}{3^5}$ (d) $\frac{10}{3^5}$

60. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval **[2011]**

(a) $\left(\frac{3}{4},\frac{11}{12}\right]$ (b) $\left[0,\frac{1}{2}\right]$

(c) $\left(\frac{11}{12},1\right]$ (d) $\left(\frac{1}{2},\frac{3}{4}\right]$

61. In a binomial distribution $B\left(n, p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than: **[2009]**

(a) $\frac{1}{\log_{10}4+\log_{10}3}$ (b) $\frac{9}{\log_{10}4-\log_{10}3}$

(c) $\frac{4}{\log_{10}4-\log_{10}3}$ (d) $\frac{1}{\log_{10}4-\log_{10}3}$

62. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is **[2007]**

(a) 8/729 (b) 8/243 (c) 1/729 (d) 8/9.

63. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10 minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is **[2006]**

(a) $\frac{6}{5^e}$ (b) $\frac{5}{6}$ (c) $\frac{6}{55}$ (d) $\frac{6}{e^5}$

64. A random variable X has Poisson distribution with mean 2. Then $P(X > 1.5)$ equals **[2005]**

(a) $\frac{2}{e^2}$ (b) 0 (c) $1-\frac{3}{e^2}$ (d) $\frac{3}{e^2}$

65. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is **[2004]**

(a) $\frac{28}{256}$ (b) $\frac{219}{256}$ (c) $\frac{128}{256}$ (d) $\frac{37}{256}$

66. A random variable X has the probability distribution:

X:	1	2	3	4	5	6	7	8
p(X):	0.2	0.2	0.1	0.1	0.2	0.1	0.1	0.1

For the events $E = \{X \text{ is a prime number }\}$ and $F = \{X < 4\}$, the $P(E \cup F)$ is **[2004]**

(a) 0.50 (b) 0.77 (c) 0.35 (d) 0.87

67. The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is **[2003]**

(a) $\frac{1}{4}$ (b) $\frac{1}{32}$

(c) $\frac{1}{16}$ (d) $\frac{1}{8}$

68. A dice is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is **[2002]**

(a) 8/3 (b) 3/8 (c) 4/5 (d) 5/4

Hints & Solutions

1. **(d)** Probability of sum getting 6, $P(A)=\frac{5}{36}$

Probability of sum getting 7, $P(B)=\frac{6}{36}=\frac{1}{6}$

$P(A \text{ wins})=P(A)+P(\bar{A})P(\bar{B})P(A)$

$+P(\bar{A})\cdot P(\bar{B})P(\bar{A})P(\bar{B})P(A)+.....$

$$\Rightarrow \frac{5}{36}+\left(\frac{31}{36}\right)\left(\frac{30}{36}\right)\left(\frac{5}{36}\right)+.....\infty$$

$$\Rightarrow \frac{5}{36}\left(1+\frac{155}{216}+\left(\frac{155}{216}\right)^2+........\infty\right)$$

$$\Rightarrow \frac{\frac{5}{36}}{\frac{61}{216}}=\frac{30}{61} \qquad \left(\because S_\infty=\frac{a}{1-r}\right)$$

2. **(d)** E_1 [the event for getting score a multiple of 4]
$=(1,3),(3,1),(2,2),(2,6),(6,2),(3,5),(5,3),(4,4)\ \&\ (6,6)$
E_2 [4 has appeared atleast once]
$=(1,4),(2,4),(3,4),(4,4),(5,4),(6,4),(4,1),(4,2),(4,3),(4,5)\ \&\ (4,6)$

$E_1 \cap E_2=(4,4)$

$$P\left(\frac{E_2}{E_1}\right)=\frac{1}{9}$$

3. **(a)** Total outcomes $=9(10^4)$
Favourable outcomes

$={}^9C_2(2^5-2)+{}^9C_1(2^4-1)=36(30)+9(15)$

$$\text{Probability}=\frac{36\times30+9\times15}{9\times10^4}=\frac{4\times30+15}{10^4}=\frac{135}{10^4}$$

4. **(b)** Let B_1 and B_2 be the boxes and N be the number of non-prime number.

$$\because P(B_1)=P(B_2)=\frac{1}{2}$$

and P (non-prime number)

$$=P(B_1)\times P\left(\frac{N}{B_1}\right)+P(B_2)\times P\left(\frac{N}{B_2}\right)$$

$$=\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}$$

So,

$$P\left(\frac{B_1}{N}\right)=\frac{P(B_1)\times P\left(\frac{N}{B_1}\right)}{P(B_1)\times P\left(\frac{N}{B_1}\right)+P(B_2)\times P\left(\frac{N}{B_2}\right)}$$

$$=\frac{\frac{1}{2}\times\frac{20}{30}}{\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}}=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{15}{40}}=\frac{8}{17}.$$

5. **(d)** $P\left(\frac{E_2^C\cap E_3^C}{E_1}\right)=\frac{P\left[E_1\cap\left(E_2^C\cap E_3^C\right)\right]}{P(E_1)}$

$$=\frac{P(E_1)-P[E_1\cap(E_2\cup E_3)]}{P(E_1)}$$

$[\because P(A\cap B^C)=P(A)-P(A\cap B)]$

$$=\frac{P(E_1)-P[(E_1\cap E_2)\cup(E_1\cap E_3)]}{P(E_1)}$$

$$=\frac{P(E_1)-[P(E_1\cap E_2)+P(E_1\cap E_3)-P(E_1\cap E_2\cap E_3)]}{P(E_1)}$$

$$=\frac{P(E_1)-P(E_1\cap E_2)-P(E_1\cap E_3)+0}{P(E_1)}$$

$=1-P(E_2)-P(E_3) \qquad [\because P(A\cap B)=P(A)\cdot P(B)]$

$=P(E_2^C)-P(E_3)$ or $P(E_3^C)-P(E_2)$

6. **(b)** P(second A – card appears before the third B– card)
$=P(AA)+P(ABA)+P(BAA)+P(ABBA)+P(BBAA)$
$+P(BABA)$

$$=\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{11}{16}$$

7. **(b)** A and B are independent events.

$$\text{So, } P\left(\frac{A}{B'}\right)=\frac{P(A\cap B')}{P(B')}=\frac{\frac{1}{3}-\frac{1}{3}\cdot\frac{1}{6}}{\frac{1}{6}}=\frac{1}{3}$$

8. **(b)**

k	0	1	2	3	4	5
$P(k)$	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

$k=$ No. of times head occur consecutively

Now expectation

$$=\sum xP(k)=(-1)\times\frac{1}{32}+(-1)\times\frac{12}{32}+(-1)\times\frac{11}{32}$$

$$+3\times\frac{5}{32}+4\times\frac{2}{32}+5\times\frac{1}{32}=\frac{1}{8}$$

9. **(a)** Required probability = when no machine has fault + when only one machine has fault + when only two machines have fault.

$$={}^5C_0\left(\frac{3}{4}\right)^5+{}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4+{}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$$

$$=\frac{243}{1024}+\frac{405}{1024}+\frac{270}{1024}=\frac{918}{1024}=\frac{459}{512}=\frac{27\times17}{64\times8}$$

$$=\left(\frac{3}{4}\right)^3\times k=\left(\frac{3}{4}\right)^3\times\frac{17}{8}$$

$$\therefore\quad k=\frac{17}{8}$$

10. **(c)** Let p is the probability that candidate can solve a problem and q is the probability that candidate can not not solve a problem.

$$p=\frac{4}{5}\text{ and }q=\frac{1}{5}\qquad(\because\ p+q=1)$$

Probability of solving either 50 or 49 problem by the candidate

$$={}^{50}C_{50}\cdot p^{50}\cdot q^0+{}^{50}C_{49}\cdot p^{49}\cdot q^1=p^{49}[p+50q]$$

$$=\left(\frac{4}{5}\right)^{49}\cdot\left(\frac{4}{5}+\frac{50}{5}\right)=\frac{54}{5}\cdot\left(\frac{4}{5}\right)^{49}$$

11. **(a)** Let, A = At least two girls

B = All girls

$$P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{P(B)}{P(A)}$$

$$=\frac{\left(\frac{1}{4}\right)^4}{1-{}^4C_0\left(\frac{1}{2}\right)^4-{}^4C_1\left(\frac{1}{2}\right)^4}=\frac{1}{16-1-4}=\frac{1}{11}$$

12. **(d)** According to the question,

$$1-\left(\frac{1}{2}\right)^n>\frac{99}{100}\Rightarrow\left(\frac{1}{2}\right)^n<\frac{1}{100}\Rightarrow n\ge7$$

Hence, minimum value is 7.

13. **(d)** P (at least one hits the target)

= 1 – P (none of them hits the target)

$$=1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{8}\right)$$

$$=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{7}{8}=1-\frac{7}{32}=\frac{25}{32}$$

14. **(b)** $\because\ A\subset B$; so $A\cap B=A$

Now, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$

$$\Rightarrow\quad P\left(\frac{A}{B}\right)=\frac{P(A)}{P(B)}$$

$\because\quad P(B)\le1$

$$\Rightarrow P\left(\frac{A}{B}\right)\ge P(A)$$

15. **(c)** Let, p is probability for getting head and is probability for getting tail.

$$p=P(H)=\frac{1}{2},q=1-p=\frac{1}{2}$$

$$P(x\ge1)\ge\frac{9}{10}\Rightarrow1-P(x=0)\ge\frac{9}{10}$$

$$1-{}^nC_0\left(\frac{1}{2}\right)^n\ge\frac{9}{10}\Rightarrow\frac{1}{2^n}\le1-\frac{9}{10}\Rightarrow\frac{1}{2^n}\le\frac{1}{10}$$

$$2^n\ge10\ \Rightarrow n\ge4\ \Rightarrow n_{\min}=4$$

16. **(d)** Since, the experiment will end in the fifth throw. Hence, the possibilities are 4 * * 4 4, * 4 * 4 4, * * * 4 4 (where * is any number except 4)

Required Probability $=\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$

$$+\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^2$$

$$=\frac{25+25+125}{6^5}=\frac{175}{6^5}$$

17. **(b)** Probability of getting 5 or 6 $=P(E)=\frac{2}{6}=\frac{1}{3}$

Probability of not getting 5 or 6 $=P(E)=1-\frac{1}{3}=\frac{2}{3}$

E will consider as success.

Event	Success in Ist attempt	Success in IInd attempt	Success in IIIrd attempt	No success in IIIrd attempt
Probability	$\frac{1}{3}$	$\frac{2}{3}\times\frac{1}{3}$	$\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}$	$\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}$
Gain/loss	100	50	0	–150

His expected gain/loss

$$=\frac{1}{3}\times100+\frac{2}{9}\times50+\frac{8}{27}\times(-150)$$

$$=\frac{900+300-1200}{27}=0$$

18. **(c)** Probability of getting sum of selected two numbers is even

$= P(E_1) = \dfrac{{}^5C_2 + {}^5C_2}{{}^{11}C_2}$

Probability of getting sum is even and selected numbers are also even $P(E_2) = \dfrac{{}^5C_2}{{}^{11}C_2}$

Hence, $P\left(\dfrac{E_2}{E_1}\right) = \dfrac{{}^5C_2}{{}^6C_2 + {}^5C_2} = \dfrac{10}{15+10} = \dfrac{2}{5}$.

19. **(c)** $P(\text{Outcome is head}) = \dfrac{1}{2}$

$P(\text{Outcome is tail}) = \dfrac{1}{2}$

$P(7 \text{ or } 8 \text{ is the sum of two dice}) = \dfrac{6}{36} + \dfrac{5}{36} = \dfrac{11}{36}$

$P(7 \text{ or } 8 \text{ is the number of card}) = \dfrac{1}{9} + \dfrac{1}{9} = \dfrac{2}{9}$

Required probability $= \dfrac{1}{2} \times \dfrac{11}{36} + \dfrac{1}{2} \times \dfrac{2}{9}$

$= \dfrac{1}{2}\left(\dfrac{11+8}{36}\right) = \dfrac{19}{72}$

20. **(c)** Let the number of independent shots required to hit the target at least once be n, then

$1 - \left(\dfrac{2}{3}\right)^n > \dfrac{5}{6}$ $\left(\dfrac{2}{3}\right)^n < \dfrac{1}{6}$

Hence, the above inequality holds when least value of n is 5.

21. **(d)** X = number of aces drawn

$\therefore$ $P(X = 1) + P(X = 2)$

$= \left\{\dfrac{4}{52} \times \dfrac{48}{52} + \dfrac{48}{52} \times \dfrac{4}{52}\right\} + \left\{\dfrac{4}{52} \times \dfrac{4}{52}\right\}$

$= \dfrac{24}{169} + \dfrac{1}{169} = \dfrac{25}{169}$

22. **(d)** Let G represents drawing a green ball and R represents drawing a red ball

So, the probability that second drawn ball is red

$= P(G) \cdot P\left(\dfrac{R}{G}\right) + P(R)P\left(\dfrac{R}{R}\right)$

$= \dfrac{2}{7} \times \dfrac{6}{7} + \dfrac{5}{7} \times \dfrac{4}{7}$

$= \dfrac{12+20}{49}$

$= \dfrac{32}{49}$

23. **(a)** Let R_t be the even of drawing red ball in t^{th} draw and B_t be the event of drawing black ball in t^{th} draw.

Now, in the given bag there are 4 red and 6 black balls.

$\therefore$ $P(R_1) = \dfrac{4}{10}$ and $P(B_1) = \dfrac{6}{10}$

And, $P\left(\dfrac{R_2}{R_1}\right) = \dfrac{6}{12}$ and $P\left(\dfrac{R_2}{B_1}\right) = \dfrac{4}{12}$

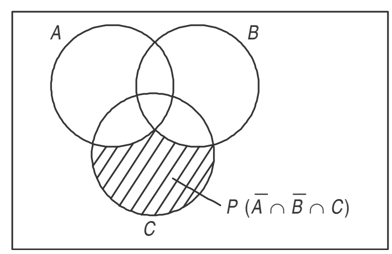
Now, required probability

$= P(R_1) \times P\left(\dfrac{R_2}{R_1}\right) + P(B_1) \times P\left(\dfrac{R_2}{B_1}\right)$

$= \left(\dfrac{4}{10} \times \dfrac{6}{12}\right) + \left(\dfrac{6}{10} \times \dfrac{4}{12}\right) = \dfrac{2}{5}$

24. **(c)** Here, $P(\bar{A} \cap \bar{B} \mid C) = \dfrac{P(\bar{A} \cap \bar{B} \cap C)}{P(C)}$.

$= \dfrac{P(C) - P(A \cap C - P(B \cap C) + P(A \cap B \cap C))}{P(C)}$



$= 1 - \left[\dfrac{P(A).P(C) + P(B).P(C)}{P(C)}\right]$

$= 1 - P(A) - P(B) = P(\bar{A}) - P(B) (\because P(A \cap B \cap C) = 0)$

25. **(a)** If the outcome is one of the following: $H, TTH, TTTTH, \ldots$, then X wins.

As subsequent tosses are independent, so the probability that X wins is

$p + \dfrac{p}{4} + \dfrac{p}{16} + \ldots = \dfrac{4p}{3}$.

Similarly Y wins if the outcome is one of the following: $TH, TTTH, TTTTTH, \ldots$

Therefore, the probability that Y wins is

$\dfrac{1-p}{2} + \dfrac{1-p}{8} + \dfrac{1-p}{32} = \dfrac{2(1-p)}{3}$

Since, the probability of winning the game by both the players is equal then, we have

$\dfrac{4p}{3} = \dfrac{2(1-p)}{3} \Rightarrow p = \dfrac{1}{3}$

26. **(b)** Let $A \equiv \{0, 1, 2, 3, 4, \ldots\ldots, 10\}$

$n(S) = {}^{11}C_2 = 55$ where 'S' denotes sample space

Let E be the given event

$\therefore E \equiv \{(0, 4), (0, 8), (2, 6), (2, 10), (4, 8), (6, 10)\}$

$\Rightarrow n(E) = 6$

$\therefore P(E) = \dfrac{n(E)}{n(S)} = \dfrac{6}{55}$

27. **(a)** $P(E \cap F) = P(E).P(F) = \dfrac{1}{12}$

$P(\bar{E} \cap \bar{F}) = P(\bar{E}).P(\bar{F}) = \dfrac{1}{2}$

$\Rightarrow (1 - P(E))(1 - P(F)) = \dfrac{1}{2}$

Let $P(E) = x$

$P(F) = y$

$\Rightarrow 1 - x - y + xy = \dfrac{1}{2}$

$\Rightarrow 1 - x - y = \dfrac{1}{2} - \dfrac{1}{12} = \dfrac{5}{12}$

$\Rightarrow \boxed{x + y = \dfrac{7}{12}}$

$\Rightarrow x + \dfrac{1}{12x} = \dfrac{7}{12} \qquad \left[\because x.y = \dfrac{1}{12}\right]$

$\Rightarrow 12x^2 - 7x + 1 = 0$

$\Rightarrow 12x^2 - 4x - 3x + 1 = 0$

$\Rightarrow (4x - 1)(3x - 1) = 0$

$\Rightarrow x = \dfrac{1}{3}, x = \dfrac{1}{4}$

and $y = \dfrac{1}{4}, y = \dfrac{1}{3}$

$\therefore \dfrac{x}{y} = \dfrac{1/3}{1/4} = \dfrac{4}{3}$ *or* $\dfrac{1/4}{1/3} = \dfrac{3}{4}$

28. **(a)** Required probability =

$\left(\dfrac{3}{4}\right)\left(\dfrac{1}{2}\right)\left(\dfrac{3}{8}\right) + \left(\dfrac{1}{4}\right)\left(\dfrac{1}{2}\right)\left(\dfrac{3}{8}\right) + \left(\dfrac{3}{4}\right)\left(\dfrac{1}{2}\right)\left(\dfrac{3}{8}\right)$

$= \dfrac{12 + 9}{64} = \dfrac{21}{64}$

29. **(b)** Required probability $= 1 - \{P(\text{All Head}) + P(\text{All Tail})\}$

$= 1 - \left\{\dfrac{1}{2^8} + \dfrac{1}{2^8}\right\}$

$= 1 - \left\{\dfrac{1}{2^7}\right\}$

$= 1 - \left\{\dfrac{1}{128}\right\} = \dfrac{127}{128}$

30. **(b)** $P(E_1) = \dfrac{1}{6}$; $P(E_2) = \dfrac{1}{6}$; $P(E_3) = \dfrac{1}{2}$

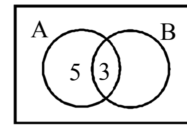
$P(E_1 \cap E_2) = \dfrac{1}{36}, P(E_2 \cap E_3) = \dfrac{1}{12}, P(E_1 \cap E_3) = \dfrac{1}{12}$

And $P(E_1 \cap E_2 \cap E_3) = 0 \neq P(E_1).P(E_2).P(E_3)$

$\Rightarrow E_1, E_2, E_3$ are not independent.

31. **(b)** $P(A) = \dfrac{2}{5} = \dfrac{8}{20}$; $P(A \cap B) = \dfrac{3}{20}$

$P(\overline{A \cap B}) = 1 - \dfrac{3}{20}$



$\Rightarrow P(\bar{A} \cup \bar{B}) = \dfrac{17}{20}$

$A \cap (A' \cup B')$

$= A - (A \cap B)$

$\therefore P(A - (A \cap B)) = \dfrac{5}{20}$

$\therefore P(A/(A' \cap B')) = \dfrac{P(A - (A \cap B))}{P(\bar{A} \cup \bar{B})} = \dfrac{5}{17}$

32. **(d)** Required probability is

$\dfrac{({}^{10}C_0)^2 + ({}^{10}C_1)^2 + ({}^{10}C_2)^2 + ... + ({}^{10}C_{10})^2}{2^{10}}$

$= \dfrac{{}^{20}C_{10}}{2^{20}}$

33. **(a)** Given, $P(\overline{A \cup B}) = \dfrac{1}{6} \Rightarrow P(A \cup B) = 1 - \dfrac{1}{6} = \dfrac{5}{6}$

$P(\bar{A}) = \dfrac{1}{4} \Rightarrow P(A) = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

We know,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \dfrac{5}{6} = \dfrac{3}{4} + P(B) - \dfrac{1}{4}$

$\left(\because P(A \cap B) = \dfrac{1}{4}\right)$

$\Rightarrow P(B) = \dfrac{1}{3}$

$\because P(A) \neq P(B)$ so they are not equally likely.

Also $P(A) \times P(B) = \dfrac{3}{4} \times \dfrac{1}{3} = \dfrac{1}{4} = P(A \cap B)$

So A & B are independent.

34. **(a)** Let A and E be any two events with positive probabilities.

Consider statement-1 :

$P(E/A) \geq P(A/E)P(E)$

LHS : $P(E/A) = \dfrac{P(E \cap A)}{P(A)}$ ...(1)

RHS : $P(A/E).P(E) = \dfrac{P(E \cap A)}{P(E)} \cdot P(E)$

$= P(A \cap E)$ ...(2)

Clearly, from (1) and (2), we have

P(E/A) $\geq$ P(A $\cap$ E)
Thus, statement-1 is true.
Similarly, statement-2 is also true.

35. **(a)** P(A or B but not by C) = P((A $\cup$ B) $\cap \bar{C}$)

$= P(A \cup B) \times P(\bar{C})$

$= [P(A) + P(B) - P(A \cap B)] \times P(\bar{C})$

$$= \left[\frac{3}{4}+\frac{1}{2}-\frac{3}{4}\times\frac{1}{2}\right]\times\frac{3}{8} = \left(\frac{6+4-3}{8}\right)\times\frac{3}{8} = \frac{21}{64}$$

36. **(a)** Let the probability of occurrence of first event A, be 'a'

i.e., P(A) = a

$\therefore$ P(not A) = 1 – a

And also suppose that probability of occurrence of second event B, P(B) = b,

$\therefore$ P(not B) = 1 – b

Now, P(A and not B) + P(not A and B) = $\frac{26}{49}$

$\Rightarrow$ P(A) × P(not B) + P(not A) × P(B) = $\frac{26}{49}$

$\Rightarrow$ $a \times (1-b) + (1-a)\, b = \frac{26}{49}$

$\Rightarrow$ $a + b - 2ab = \frac{26}{49}$ ...(i)

And P(not A and not B) = $\frac{15}{49}$

$\Rightarrow$ P (not A) × P(not B) = $\frac{15}{49}$

$\Rightarrow$ $(1-a)\times(1-b) = \frac{15}{49}$

$\Rightarrow$ $1 - b - a + ab = \frac{15}{49}$

$\Rightarrow$ $a + b - ab = \frac{34}{49}$ ...(ii)

From (i) and (ii),

$a + b = \frac{42}{49}$ ...(iii)

and $ab = \frac{8}{49}$

$(a-b)^2 = (a+b)^2 - 4ab = \frac{42}{49}\times\frac{42}{49} - \frac{4\times 8}{49} = \frac{196}{2401}$

$\therefore$ $a - b = \frac{14}{49}$ ...(iv)

From (iii) and (iv),

$a = \frac{4}{7}, b = \frac{2}{7}$

Hence probability of more probable of the two events = $\frac{4}{7}$

37. **(a)** $\frac{2}{5}+\frac{3}{5}\times\frac{2}{5}+\left(\frac{3}{5}\right)^2\times\frac{2}{5}+\ldots\ldots+\left(\frac{3}{5}\right)^k.\frac{2}{5} > \frac{7}{10}$

$\Rightarrow \frac{2}{5}\left[1+\frac{3}{5}+\left(\frac{3}{5}\right)^2+\ldots\ldots+\left(\frac{3}{5}\right)^k\right] > \frac{7}{10}$

$\Rightarrow \frac{2}{5}\times\frac{1-\left(\frac{3}{5}\right)^k}{1-\frac{3}{5}} > \frac{7}{10} \Rightarrow 1-\left(\frac{3}{5}\right)^k > \frac{7}{10}$

$\Rightarrow \left(\frac{3}{5}\right)^k < \frac{3}{10} \Rightarrow k \geq 3$

Hence minimum value of k = 3

38. **(b)** Given three numbers are chosen without replacement from = {1,2,3,.....,8}

Let Event

F : Maximum of three numbers is 6.

E : Minimum of three numbers is 3.

This is the case of conditional probability

We have to find P (minimum) is 3 when it is given that P (maximum) is 6.

s$\therefore$ $P\left(\frac{E}{F}\right) = \frac{P(E\cap F)}{P(F)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$

39. **(d)** $P(A^c \cap B^c/C) =$

$\frac{P\left(\left(A^c\cap B^c\right)\cap C\right)}{P(C)} = \frac{P\left(\left(A\cup B\right)^c\cap C\right)}{P(C)}$

$= \frac{P\left(\left(S-A\cup B\right)\cap C\right)}{P(C)}$

$= \frac{P\left(\left(S-A-B+A\cap B\right)\cap C\right)}{P(C)}$

$= \frac{P(C)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)}{P(C)}$

$= \frac{P(C)-P(A).P(C)-P(B)P(C)+0}{P(C)}$

$= 1 - P(A) - P(B)$ [$\because P(A^C) = 1 - P(A)$]

$= P(A^c) - P(B)$

40. **(a)** We know,

$P\left(\frac{C}{D}\right) = \frac{P(C\cap D)}{P(D)} = \frac{P(C)}{P(D)}$ [$\because C \subset D$]

Where, $0 \leq P(D) \leq 1$, hence

$P\left(\frac{C}{D}\right) \geq P(C)$

41. **(d)** Given that

$P = \frac{1}{4} \Rightarrow q = 1-\frac{1}{4} = \frac{3}{4}$

and $P(x \geq 1) \geq \frac{9}{10}$

$\Rightarrow \quad 1 - P(x=0) \geq \frac{9}{10}$

$\Rightarrow \quad 1 - {}^nC_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^n \geq \frac{9}{10}$

$\Rightarrow \quad 1 - \frac{9}{10} \geq \left(\frac{3}{4}\right)^n$

$\Rightarrow \quad \left(\frac{3}{4}\right)^n \leq \left(\frac{1}{10}\right)$

Taking log at the base 3/4, on both sides, we get

$n \log_{3/4}\left(\frac{3}{4}\right) \geq \log_{3/4}\left(\frac{1}{10}\right)$

$\Rightarrow \quad n \geq -\log_{3/4} 10 = \frac{-\log_{10} 10}{\log_{10}\left(\frac{3}{4}\right)} = \frac{-1}{\log_{10} 3 - \log_{10} 4}$

$\Rightarrow \quad n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$

42. **(b)** Given that $P(A) = 1/4, P(A/B) = \frac{1}{2}, P(B/A) = 2/3$

By conditional probability,

$P(A \cap B) = P(A)\,P(B/A) = P(B)P(A/B)$

$\Rightarrow \frac{1}{4} \times \frac{2}{3} = P(B) \times \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$

43. **(d)** Given that P(I) = 0.3 and P(II) = 0.2

$\therefore\ P(\overline{I}) = 1 - 0.3 = 0.7$

$\therefore$ The required probability

$= P(\overline{I} \cap II) = P(\overline{I}).P(II) = 0.7 \times 0.2 = 0.14$

44. **(b)** Probability of particular house being selected = $\frac{1}{3}$

P (all the persons apply for the same house)

$= \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right)3 = \frac{1}{9}.$

45. **(c)** $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$

$\Rightarrow P(A \cup B) = \frac{5}{6},\ P(A) = \frac{3}{4}$

Also $\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(B) = \frac{5}{6} - \frac{3}{4} + \frac{1}{4} = \frac{1}{3}$

$\Rightarrow P(A)\ P(B) = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4} = P(A \cap B)$

Hence A and B are independent but not equally likely.

46. **(c)** A and B will contradict each other if one speaks truth and other false . So , the required probability

$P(A \cap \overline{B}) + P(\overline{A} \cap B) = \frac{4}{5}\left(1 - \frac{3}{4}\right) + \left(1 - \frac{4}{5}\right)\frac{3}{4}$

$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{7}{20}$

47. **(a)** Given that $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and

$P(C) = \frac{1}{4}$; $P(AUBUC) = 1 - P(\bar{A})P(\bar{B})P(\bar{C})$

$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{4}$

48. **(11)**

Probability of getting at least two 3's or 5's in one trial

$= {}^4C_2\left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {}^4C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right) + {}^4C_4\left(\frac{2}{6}\right)^4$

$= \frac{33}{3^4} = \frac{11}{27}$

$E(x) = np = 27\left(\frac{11}{27}\right) = 11.$

49. **(11.00)**

Let 'n' bombs are required, then

$1 - {}^nC_1 \cdot \left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1} - {}^nC_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n \geq \frac{99}{100}$

$\Rightarrow \frac{1}{100} \geq \frac{n+1}{2^n} \Rightarrow 2^n \geq 100(n+1) \Rightarrow n \geq 11$

50. **(3.00)**

$p = \frac{1}{10}, q = \frac{9}{10}$

P (not hitting target in n trials) $= \left(\frac{9}{10}\right)^n$

P (at least one hit) $= 1 - \left(\frac{9}{10}\right)^n$

$\because 1 - \left(\frac{9}{10}\right)^n > \frac{1}{4} \Rightarrow (0.9)^n < 0.75$

$\therefore n_{\text{minimum}} = 3.$

51. **(d)** $\sum P(K) = 1 \quad \Rightarrow \quad 6K^2 + 5K = 1$

$6K^2 + 5K - 1 = 0$

$6K^2 + 6K - K - 1 = 0$

$\Rightarrow \quad (6K - 1)(K + 1) = 0$

$\Rightarrow \quad K = \frac{1}{6}$ $(K = -1$ rejected$)$

$P(X > 2) = K + 2K + 5K^2$

$= \frac{1}{6} + \frac{2}{6} + \frac{5}{36} = \frac{6 + 12 + 5}{36} = \frac{23}{36}$

52. **(d)** Given mean $\mu = 8$ and variance $\sigma^2 = 4$

$\Rightarrow \mu = np = 8$ and $\sigma^2 = npq = 4$.

$p + q = 1 \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}$ and $n = 16$

$\because P(X \le 2) = \frac{k}{2^{16}}$

$\therefore \quad {}^{16}C_0\left(\frac{1}{2}\right)^{16} + {}^{16}C_1\left(\frac{1}{2}\right)^{16} + {}^{16}C_2\left(\frac{1}{2}\right)^{16} = \frac{k}{2^{16}}$

$\Rightarrow k = (1 + 16 + 120) = 137$

53. **(c)** Let X be the random variable which denotes the Rs gained by the person.

Total cases $= 6 \times 6 = 36$.

Favorable cases for the person on winning ₹ 15 are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) i.e., 6 cases.

$\therefore \quad P(X = 15) = \frac{6}{36} = \frac{1}{6}$

Favorable cases for the person on winning ₹ 12 are (6, 3), (5, 4), (4, 5), (3, 6) i.e., 4.

$\therefore \quad P(X = 12) = \frac{4}{36} = \frac{1}{9}$

Remaining cases $= 36 - 6 - 4 = 26$

$\therefore \quad P(X = -6) = \frac{26}{36} = \frac{13}{18}$

X	15	12	-6
$P(X)$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{13}{18}$
$X.P(X)$	$\frac{5}{2}$	$\frac{4}{3}$	$\frac{-13}{3}$

Hence, $E(X) = \sum X.P(X) = \frac{5}{2} + \frac{4}{3} - \frac{13}{3} = -\frac{1}{2}$

54. **(b)** $P(\text{white ball}) = \frac{30}{40} = \frac{3}{4}, Q(\text{red ball}) = \frac{10}{40} = \frac{1}{4}, n = 16$

$\frac{\text{Mean of } X}{\text{standard deviation of } X} = \frac{nP}{\sqrt{nPQ}} = \frac{\sqrt{nP}}{\sqrt{Q}}$

$= \sqrt{\frac{16 \times \frac{3}{4}}{\frac{1}{4}}} = \sqrt{48} = 4\sqrt{3}$

55. **(b)** We can apply binomial probability distribution

We have n = 10

p = Probability of drawing a green ball $= \frac{15}{25} = \frac{3}{5}$

Also q $= 1 - \frac{3}{5} = \frac{2}{5}$

Variance = npq $= 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

56. **(d)** Let $p(F) = p \Rightarrow p(S) = 2p$

$\because \quad p + 2p = 1 \Rightarrow p = \frac{1}{3}$

$p(x \ge 5) = p(x = 5) + p(x = 6)$

$= {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1 + {}^6C_5\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^0$

$= \left(\frac{2}{3}\right)^5\left(6 \times \frac{1}{3} + \frac{2}{3}\right) = \frac{256}{729}$

57. **(d)** Let mean = np = 2 ...(1)

and variance = npq = 1 ...(2)

On solving eqn (1) and (2), we get

$q = \frac{1}{2}$ and $p = \frac{1}{2}$

From eqn (1), we have

n = 4

$P(x \ge 1) = {}^4C_1p^1q^3 + {}^4C_2p^2q^2 + {}^4C_3p^3q + {}^4C_4p^4$

$= 1 - {}^4C_0p^0q^4 = 1 - \left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}$

58. **(b)** Since X has a binomial distribution, B (n, p)

$\therefore \quad P(X = 2) = {}^nC_2\,(p)^2\,(1 - p)^{n-2}$

and $P(X = 3) = {}^nC_3\,(p)^3\,(1 - p)^{n-3}$

Given $P(X = 2) = P(X = 3)$

$\Rightarrow {}^nC_2\,p^2\,(1 - p)^{n-2} = {}^nC_3\,(p)^3\,(1 - p)^{n-3}$

$\Rightarrow \frac{n!}{2!(n-2)!}.\frac{p^2(1-p)^n}{(1-p)^2} = \frac{n!}{3!(n-3)!}.\frac{p^3(1-p)^n}{(1-p)^3}$

$\Rightarrow \frac{1}{n-2} = \frac{1}{3}.\frac{p}{1-p} \Rightarrow 3(1 - p) = p(n - 2)$

$\Rightarrow 3 - 3p = np - 2p$

$\Rightarrow np = 3 - p$

$\Rightarrow E(X) = \text{mean} = 3 - p$

($\because$ mean of B $(n, p) = np$)

59. **(c)** $p = p$ (correct answer), $q = p$ (wrong answer)

$\Rightarrow P = \frac{1}{3}, q = \frac{2}{3}, n = 5$

By using Binomial distribution

Required probability

$P(x \ge 4) = {}^5C_4\left(\frac{1}{3}\right)^4 \cdot \frac{2}{3} + {}^5C_5\left(\frac{1}{3}\right)^5$

$= 5 \cdot \frac{2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$

60. **(b)** Given that p (at least one failure) $\ge \frac{31}{32}$

$\Rightarrow 1 - p$ (no failure) $\ge \frac{31}{32}$

$\Rightarrow 1 - p^5 \ge \frac{31}{32}$

$\Rightarrow p^5 \le \frac{1}{32} \quad \Rightarrow \quad p \le \frac{1}{2}$

But $p \ge 0$

Hence p lies in the interval $\left[0, \frac{1}{2}\right]$.

61. **(d)** Given that

$P = \frac{1}{4} \Rightarrow q = 1 - \frac{1}{4} = \frac{3}{4}$

and $P(x \ge 1) \ge \frac{9}{10}$

$\Rightarrow \quad 1 - P(x=0) \ge \frac{9}{10}$

$\Rightarrow \quad 1 - {}^nC_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^n \ge \frac{9}{10}$

$\Rightarrow \quad 1 - \frac{9}{10} \ge \left(\frac{3}{4}\right)^n$

$\Rightarrow \quad \left(\frac{3}{4}\right)^n \le \left(\frac{1}{10}\right)$

Taking log at the base 3/4, on both sides, we get

$n \log_{3/4}\left(\frac{3}{4}\right) \ge \log_{3/4}\left(\frac{1}{10}\right)$

$\Rightarrow \quad n \ge -\log_{3/4} 10 = \frac{-\log_{10} 10}{\log_{10}\left(\frac{3}{4}\right)} = \frac{-1}{\log_{10} 3 - \log_{10} 4}$

$\Rightarrow \quad n \ge \frac{1}{\log_{10} 4 - \log_{10} 3}$

62. **(b)** The sample space of pair of fair dice is thrown,

$S = (1, 1), (1, 2)\,(1, 3) .... = 36$

Sum 9 are (5, 4), (4, 5), (6, 3), (3, 6)

$\text{P(score 9)} = \frac{4}{36} = \frac{1}{9}$

Number of trial = 3

$\therefore$ Probability of getting score 9 exactly twice

$= {}^3C_2 \times \left(\frac{1}{9}\right)^2 . \left(1 - \frac{1}{9}\right) = \frac{3!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9}$

$= \frac{3.2!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9} = \frac{8}{243}$

63. **(d)** From poission distribution

$P(X = r) = \frac{e^{-m} m^r}{r!}$

Given mean $(m) = 5$

P (at most 1 phone call)

$= P(X \le 1) = P(X = 0) + P(X = 1)$

$= e^{-5} + 5 \times e^{-5} = \frac{6}{e^5}$

64. **(c)** From poission distribution, probability of getting k successes is

$P(x = k) = e^{-\lambda}\frac{\lambda^k}{k!}$

Given mean $(\lambda) = 2$

$P(x \ge 2) = 1 - P(x = 0) - P(x = 1)$

$= 1 - e^{-\lambda} - e^{-\lambda}\left(\frac{\lambda}{1!}\right) = 1 - \frac{3}{e^2}$.

65. **(a)** Given that mean = $np = 4$ and variance = $npq = 2$

$\Rightarrow p = q = \frac{1}{2}$ and $n = 8$

$\therefore \quad P(2 \text{ success}) = {}^8C_2\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2$

$= \frac{28}{2^8} = \frac{28}{256}$

66. **(b)** $P(E) = P(\,2 \text{ or } 3 \text{ or } 5 \text{ or } 7)$

$= 0.23 + 0.12 + 0.20 + 0.07 = 0.62$

$P(F) = P(1 \text{ or } 2 \text{ or } 3) = 0.15 + 0.23 + 0.12 = 0.50$

$P(E \cap F) = P(2 \text{ or } 3) = 0.23 + 0.12 = 0.35$

We know that

$P(EUF) = P(E) + P(F) - P(E \cap F)$

$= 0.62 + 0.50 - 0.35 = 0.77$

67. **(b)** Given that $np = 4$ and

$npq = 2 \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$P(X = 1) = {}^8C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7 = 8.\frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}$

68. **(d)** The experiment follows binomial distribution with $n = 5, p = 3/6 = 1/2$.

$q = 1 - p = 1/2.$;

$\therefore$ Variance $= npq = 5/4$.