Chapter 2

Fluid Pressure and Buoyancy

CHAPTER HIGHLIGHTS

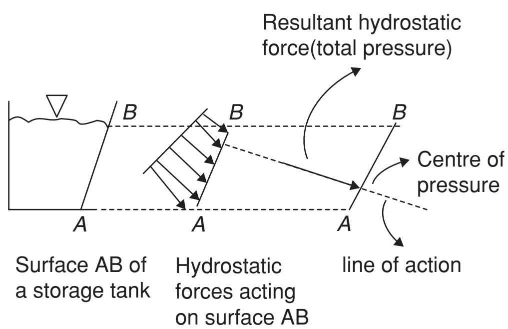
- Total Pressure and Center of Pressure
- Hydrostatic Forces on a Submerged Inclined Plane Surface
- Buoyancy
- Archimedes Principle
- Centre of Buoyancy
- Buoyant Force–layered Fluid
- Stability (rotational Stability) of a Submerged Body
- Stability (rotational Stability) of a Floating Body
- Oscillation of a Floating Body

Fluid Pressure and Buoyancy

A surface exposed to a static fluid will be subjected to a distribution of fluid pressure over the exposed area; the pressure distribution is called *hydrostatic pressure distribution.* The hydrostatic pressure distribution gives rise to a system of hydrostatic forces that act on the surface's exposed area. The determination of the hydrostatic forces along with their locations are important in the design of structures such as storage tanks, dams, ships, etc., which surfaces have exposed to fluids at rest.

Total Pressure and Centre of Pressure

Consider a storage tank where a surface *AB* is exposed to the fluid in the tank as shown in the figure below. The surface *AB* will thus be subjected to a hydrostatic pressure distribution and hence a system of hydrostatic forces will be acting on the surface. This system of hydrostatic forces can be represented by a resultant hydrostatic force, referred to as 'total pressure force' or simply as *total pressure*, as shown in the figure below. The unit of total pressure is Newtons.

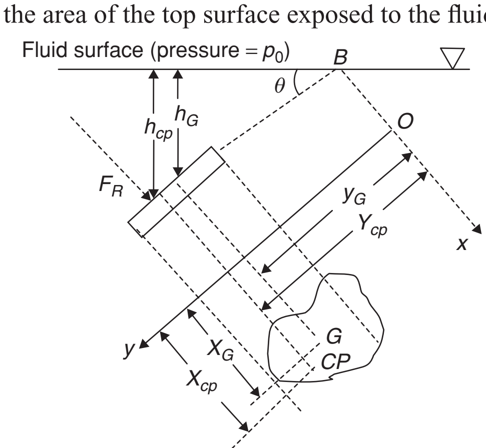


Total pressure is defined as the force exerted by a static fluid on a surface that is in contact with the fluid. The total pressure, i.e., the resultant hydrostatic force, always acts normal to the surface.

The point of intersection of the line of action of the resultant hydrostatic force and the corresponding surface is called *centre of pressure*. Centre of pressure is also defined as the point of application of the total pressure on the corresponding surface.

Hydrostatic Forces on a Submerged Inclined Plane Surface

Consider the top plane surface of an inclined plate(of arbitrary shape) submerged in a fluid as illustrated in the figure below, where the fluid is assumed to have a constant density ρ and the area of the top surface exposed to the fluid is *A*.



The plane, containing the top surface, intersects the fluid surface (having a pressure P_0) at B making an angle θ with it. The orthogonal x and y axes with the origin are defined such that the top surface lies in the xy-plane generated by the axes and the x-axis lies in the intersection of the fluid surface and the plane containing the top surface.

The points G (x_G, y_G) and CP (x_{CP}, y_{CP}) represent the centroid of the top surface area exposed to the fluid and the centre of pressure respectively. If F_R is the resultant hydrostatic force or the total pressure, then

$$F_R = (p_0 + \rho g h_G)\,A = (p_0 + \rho g y_G \sin\theta)\,A$$

Where h_G ($= y_G \sin\theta$) is the vertical distance of the centroid G from the fluid surface. 'p_0' being the pressure acting at the fluid surface usually atmospheric pressure. The coordinates of the centre of pressure are given by

$$X_{cp} = X_G + \frac{I_{xyG}}{\left(Y_G + \dfrac{p_0}{\rho g \sin\theta}\right)A}$$

$$Y_{cp} = Y_G + \frac{I \times G}{\left(Y_G + \dfrac{p_0}{pg \sin\theta}\right)A}$$

Where I_{XYG} is the product of inertia with respect to an orthogonal coordinate system passing through the centroid G and formed by a translation of the x-y coordinate system and I_{XG} is the moment of inertia about an axis passing through the centroid G and parallel to the x axis.

$$I_{xyG} = I_{xy} - AX_G Y_G$$
$$I_{xy} = \int_A^{xy} dA$$

$$I_{XG} = I_X - Ay_G^2$$
$$I_X - \int_A y^2 dA$$

Where I_{XY} is the product of inertia with respect to the x and y axes and I_X is the moment of inertia with respect to the x-axis.

NOTE

The inertia terms I_{XYG}, I_{XG}, I_{XY} and I_X are defined with respect to the area of the surface that is exposed to the fluid.

The centre of pressure is generally expressed only in terms of its vertical distance (h_{cp}) from the fluid surface, where

$$H_{cp} = y_{cp}\sin\theta = Y_G \sin\theta + \frac{I_{XG}\sin\theta}{\left(Y_G + \dfrac{p_0}{\rho g \sin\theta}\right)A} = h_G + \frac{I_{XG}\sin^2\theta}{\left(h_G + \dfrac{p_0}{\rho g}\right)A}$$

NOTES

1. If the fluid surface pressure P_0 also acts at the bottom surface of the inclined plate. Then the variable P_0 can be ignored (i.e., P_0 can be set to zero) in the equation for total pressure and centre of pressure to yield the following equations

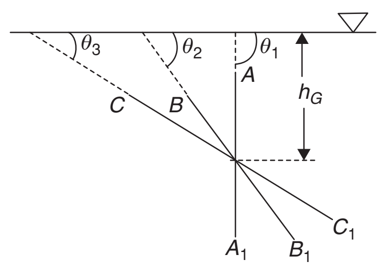
$$F_R = \rho g h_G A = \rho g y_G \sin\theta A$$

$$X_{cp} = X_G + \frac{I_{xyG}}{Y_G A}$$

$$Y_{cp} = Y_G + \frac{I_{xG}}{Y_G A}$$

$$h_{cp} = Y_G \sin\theta + \frac{I_{xG}\sin\theta}{Y_G A} = h_G + \frac{I \times G \sin^2\theta}{h_G A}$$

2. Consider an inclined plate in different orientations, i.e., along AA_1', BB_1' and CC_1' as shown in the following figure, such that the centre of gravity depth h_G is the same for all the orientations.



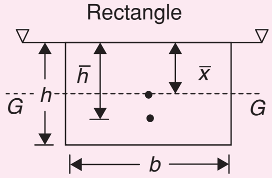
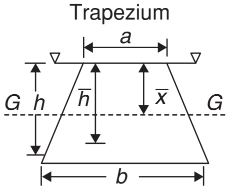
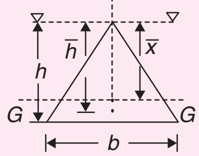
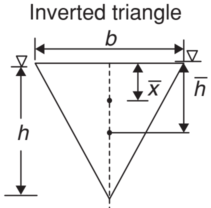
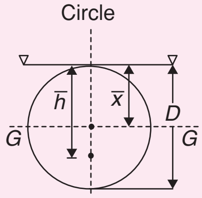
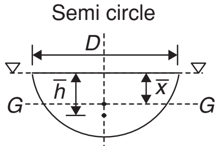
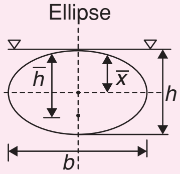
For all the orientations, the total pressure acting at the top surface remains the same, i.e.

For an inclined plate, the total pressure does not depend on the angle of inclination θ as long as the depth of centre of gravity hG does not change.

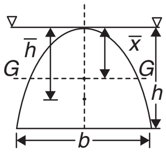
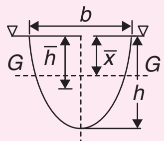
NOTE

As the inclined plate is submerged deeper deeper and deeper from the fluid surface, the distance between th centre of pressure and the centre of gravity decreases hyperbolically. At very large depths, for practical calculations, the centre of pressure and the centre of gravity are the same. It should be however noted that the centre of pressure can coincide or to be below the centre of gravity but can never be above it.

Table 1 *Geometric Properties of Some Important Plane Surfaces*

Sl	Plane surface	CG from free Surface	CP from free Surface	Area	Moment of inertia about an axis passing through CG and parallel to free surface (IG)
1	Rectangle 	$\frac{h}{2}$	$\frac{2h}{3}$	bh	$\frac{bh^3}{12}$
2	Trapezium 	$\left(\frac{a+2b}{a+b}\right)\frac{h}{3}$	$\left(\frac{a+3b}{a+2b}\right)\frac{h}{2}$	$\frac{(a+b)}{2}h$	$\frac{(a^2+4ab+b^2)}{(a+b)}\frac{h^3}{36}$
3	Triangle 	$\frac{2h}{3}$	$\frac{3h}{4}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$
4	Inverted triangle 	$\frac{h}{3}$	$\frac{h}{2}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$
5	Circle 	$\frac{D}{2}$	$\frac{5D}{8}$	$\frac{\pi D^2}{4}$	$\frac{\pi D^4}{64}$
6	Semi circle 	$\frac{2D}{3\pi}$	$\frac{3\pi D}{32}$	$\frac{\pi D^2}{8}$	$0.1098\left(\frac{D}{2}\right)^4 = 0.11R^4$ $\left(R=\frac{D}{2}\right)$
7	Ellipse 	$\frac{h}{2}$	$\frac{5h}{8}$	$\frac{\pi bh}{4}$	$\frac{\pi bh^3}{64}$

(*Continued*)

8	Parabola 	$\frac{3}{5}h$	$\frac{5h}{7}$	$\frac{2}{3}bh$	$\left(\frac{8}{175}\right)bh^3$
9	Inverted Parabola 	$\frac{2}{5}h$	$\frac{4h}{7}$	$\frac{2}{3}bh$	$\left(\frac{8}{175}\right)bh^3$

Solved Examples

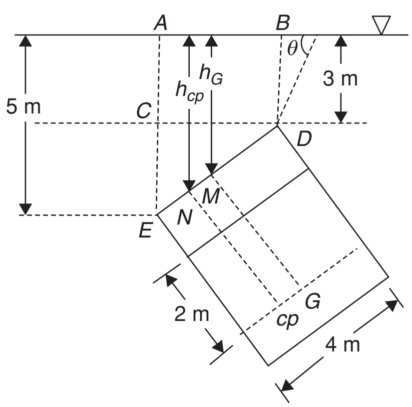
Direction for questions 1 and 2: A rectangular flat thin plate (length = 4 m, breadth = 2 m, breadth = 2 m) is submerged in water such that the plate's largest and smallest depths from the water surface are 5 m ad 3 m respectively.

The breadth of the plate is parallel to the water surface

Example 1: The total pressure force on the top surface of the plate is

(A) 1125 kN (B) 314 kN
(C) 141 kN (D) 40 kN

Solution:



Area of the plate, $A = 4 \times 2 = 8\ \text{m}^2$

$$\sin\theta = \frac{CE}{ED} = \frac{5-3}{4} = 0.5$$

Vertical distance of the centroid G from the water surface, $h_G = BD + MD\,\sin\theta = 3 + 2 \times 0.5 = 4$ m

Since the fluid surface pressure p_0 will act at the top and bottom surfaces, total pressure force, $FR = \rho g h_G A$ (ρ is density of water = 1000 Kg/m^3

$$= 1000 \times 9.81 \times 4 \times 8$$

$$= 313.92\ \text{Kn}.$$

Example 2: The vertical distance of the centre of pressure, on the top surface of the plate, from the water surface is

(A) 3.917 m (B) 4.333 m
(C) 4 M (D) 4.083 M

Solution:
The vertical distance of the centre of manure from the water surface,

$$h_{cp} = h_G + \frac{I \times_G \sin^2\theta}{h_G A}$$

Moment of inertia, $I_{XG} = \dfrac{b \times l^3}{12}$

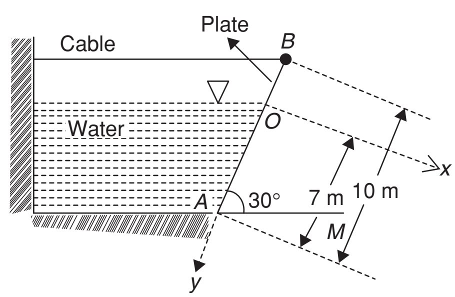
(b is breadth, l is length)

$$= \frac{2 \times 4^3}{12} = 10.667\ \text{m}^4$$

$$\therefore\ h_{cp} = 4 + \frac{10.667 \times (0.5)^2}{4 \times 8}$$

$$= 4.083\ \text{m}.$$

Example 3: On a homogenous rectangular plate of weight 7.5 kN, length 10 m, width 5 m, and hanged at point A, a body of water acts as shown in the figure below.



The plate is held in place, inclined at an angle of 30° to the horizontal, by a horizontal flexible cable of negligible weight attached to the plate at point B. If the friction in the hinge is negligible, then the tension in the cable is

(A) 286.5 kN (B) 847.7 kN
(C) 280 kN (D) 284.55

Solution:

Vertical distance of the centroid of the plate surface area exposed to the fluid, $h_G = \dfrac{l_e \sin 30°}{2}$

Where l_e (= 7 m) is the length of the plate exposed to the fluid.

$\therefore \quad h_G = 1.75$ m

Area of the plate exposed to the fluid,

$A = l_e \times w$ (w is the width)

$= 7 \times 5 = 35 \text{ m}^2$

Since the fluid surface pressure p_0 (= atmospheric pressure) acts on both the surface of the plate,

Total pressure force, $F_R = h_G\, \rho g A$ (ρ is the density of water $= 1000 \dfrac{\text{Kg}}{\text{M}^3}$)

$= 1.75 \times 1000 \times 9.81 \times 35$

$= 600862.5$ N

y – coordinate of the centroid of the plate surface area exposed to the fluid,

$y_G = 3.5$ m

Moment of inertia, $I \times G = \dfrac{w l_e^3}{12}$

$$= \frac{1}{12} \times 5 \times 7^3$$

$$= 142.92 \text{ m}^4$$

y – coordinate of the centre of pressure,

$$Y_{cp} = y_G + \frac{I_{XG}}{y_G A}$$

$$= 3.5 + \frac{142.92}{3.5 \times 35}$$

$$= 4.67 \text{ m}$$

Taking moment of the forces about the point A for equilibrium, we have

$$T \times 1 \times \sin 30°$$

$$= W \times \frac{l \times \cos 30°}{2} + F_R \times (l_e - y_{cp})$$

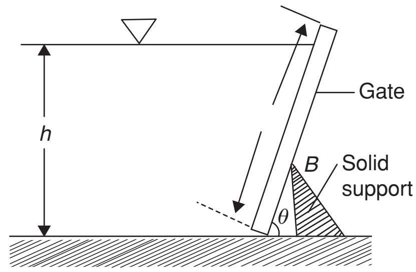
Where T is the tension in the cable, W (= 7.5 kN) and l (= 10 m) are the weight and length of the plate

$$\therefore \quad T \times 10 \times \sin 30°$$

$$= \frac{7.5 \times 10^3 \times 10 \times \cos 30°}{2} + 600862.5 \times (7 - 4.67)$$

$\therefore \quad T = 286.5$ kN.

Example 4: A rectangular gate, of length l metres, width w metres and negligible weight, is inclined at an angle θ to the horizontal and supports a water body of height h metres as shown in the following figure.



The height of the water body is such that the gate tips about the point B at which it is hinged against a solid support. If the vertical distance of the centre of pressure, on the rectangular gate, from the water surface is h_{CP}, then the ratio $\dfrac{h}{h_{CP}}$ is equal to,

(A) $\dfrac{2}{3}$ (B) $\dfrac{2\sin\theta}{3}$

(C) $\dfrac{\sin\theta}{3}$ (D) $\dfrac{1}{3}$

Solution:

At water body height h, the gate just tips about the point B, i.e., the total pressure force is acting at point B.

Area of the gate exposed to water,

$$A = w \times \frac{h}{\sin\theta}$$

Vertical distance of the centroid, of the area exposed to water, from the water surface,

$$h_G = \frac{h}{2}$$

Since the fluid surface pressure P_0 (= atmospheric pressure) acts on the top and bottom surface of the gate, vertical distance of the centre of pressure from the water surface,

$$h_{cp} = h_G + \frac{I_{XG} \sin^2\theta}{A h_G}$$

Moment of inertia, $I_{XG} = \dfrac{\omega \times \left(\dfrac{h}{\sin\theta}\right)^3}{12}$

$$\therefore \quad h_{cp} = \frac{h}{2} + \frac{\omega h^3 \times \sin^2\theta \times \sin\theta}{12 \sin^3\theta \times \omega \times h \times h} \times 2$$

$$= \frac{2h}{3}$$

$$\therefore \quad \frac{h_{cp}}{h_G} = \frac{2}{3}.$$

Hydrostatic Forces on a Submerged Vertical Plane Surface

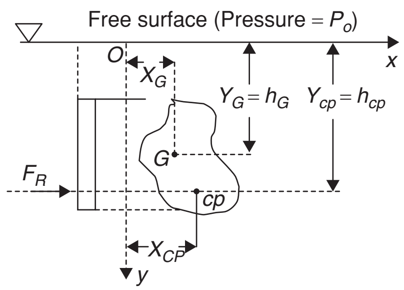
The total pressure and centre of pressure equations are

$$\boxed{F_R = (\rho_0 + \rho g h_G)A}$$

$$\boxed{X_{cp} = X_G + \frac{I_{xyg}}{\left(h_G + \frac{P_o}{\rho g}\right)A}}$$

$$\boxed{h_{cp} = h_G + \frac{I_{xG}}{\left(h_G + \frac{P_o}{\rho g}\right)A}}$$

Here also, the fluid surface P_0 can be set to zero in the above equations if it acts on both sides of the submerged vertical plane surface.



Example 5: For a horizontal surface of negligible thickness submerged in a fluid of constant density, let points *G* and *CP* represent the centroid and centre of pressure of the area exposed to the fluid. Then, which one of the following statement is ONLY correct about the points *G* and *CP*?
(A) Point1 *G* is always above point *CP*.
(B) Point *CP* is always above point *G*.
(C) Points *G* and *CP* will always coincide
(D) Points *G* and *CP* may coincide.

Solution:
Considering that the fluid surface pressure P_0 is acting on both the sides of the submerged surface, we have

$$h_{cp} = h_G + \frac{I_{XG}\sin^2\theta}{h_G A}$$

$(0 < \theta \le 90°)$

Since $\frac{I_{XG}\sin^2\theta}{h_G A}$ is always greater than zero, h_{cp} is always greater than h_G. i.e., point *G* is always above point *CP.*

However, one would find the points *G* and *CP* coinciding for non-horizontal surfaces when one considers the fluid contacting the surface to be a gas at constant pressure.

Example 6: An equilateral triangular thin plate of side *b* meters in length is immersed vertically in a liquid such that one side of the plate coincides with the free surface of the liquid. The vertical distance between the centre of pressure on a surface of the plate from the corner of the plate, that away from the free surface, is *h* meters. The ratio *h*:*b* is equal to

(A) $\sqrt{3} : 4$ (B) 1 : 2
(C) $13\sqrt{3} : 36$ (D) $\sqrt{3} : 12$

Solution:
Since the fluid surface pressure P_0 acts on both sides of the immersed surface,

$$h_{cp} = h_G + \frac{I_{XG}}{h_{GA}} \quad (1)$$

Here, $h_G = \frac{b\sin 60°}{3}$

$$A = \frac{1}{2}\times b \times b\sin 60° \text{ and}$$

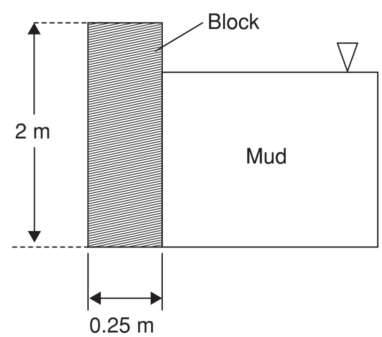
$$I_{XG} = \frac{b\times(b\sin 60°)^3}{36} \quad (2)$$

Now, $h_{cp} + h = b \sin 60°$
i.e., $hcp = b \sin 60° - h$ (3)
From equations (1), (2) and (3), one could write

$$b\sin 60° \quad h = \frac{b\sin 60°}{3} + \frac{b\times b^3 \times \sin^3 60° \times 3\times 2}{36\times b\times \sin 60° \times b \times b\sin 60°}$$

$$\text{i.e., } \frac{h}{b} = \frac{\sqrt{3}}{4}.$$

Example 7: A concrete block 2 m high, 0.25 m wide and 1 m long, is used for holding mud at one side of the block as shown in the figure below.



The density of concrete is 2600 kg/m^3 while the density of the mud is 1700 kg/m^3.If the coefficient of friction between the ground and the concrete block is 0.4, then the mud height at which the block will start to slide is
(A) 1.237 m (B) 5 m
(C) 0.782 m (D) 0.553 m

Solution:
Weight of the concrete block,

$$W = (1 \times 0.25 \times 2) \times 2600 \times 9.81$$
$$= 12753 \text{ N}$$

Frictional force acting between the block and the ground,

$$F_{\text{fric}} = \mu W$$
$$= 0.4 \times 12753$$
$$= 5101.2 \text{ N}$$

Let 'h' be the mud height at which the block will start to slide.

Here, $A = h \times 1 = h$ m^2

$$h_G = \frac{h}{2}$$

Total pressure force exerted by the mud on the block,

$F_R = h_G \times \rho \times g \times A$ (ρ is the density of the mud)

$$= \frac{h^2}{2} \times 1700 \times 9.81$$

$$= 8338.5 \text{ h}^2$$

Just before the blocks starts to slide,

$F_R = F_{\text{fric}}$

i.e., $8338.5\, h^2 = 5101.2$

i.e., $h = 0.782$ m.

Direction for example 8 and 9: A tank 12 m high contains a liquid (specific gravity = 0.8) up to a height of 11 m. The air space above the free surface of the liquid is at a pressure of 1.2 atm. A circular opening (diameter = 2 m) present in the vertical side of the tank is closed by a disc of 2 m diameter. The disc can rotate about a horizontal diameter that is at a height of 7 m from the bottom of the tank.

Example 8: The total pressure force on the disc is

(A) 98.7 kN (B) 162.2 kN
(C) 480.36 kN (D) 186.85 kN

Solution:
Diameter of the circular opening, $d = 2$ m

Area of the circular opening, $A = \frac{\pi}{4} \times d^2$

$$= \frac{\pi}{4} \times (2)^2 = 3.14 \text{ m}^2$$

Assuming that atmospheric pressure acts on the outside of the disc.

$$Po = (1.2 - 1) \times 101325 = 20265 \text{ Pa}$$

Total pressure force on the disc,

$F_R = (P_o + h_G \rho g)\, A$ (ρ is density of the liquid).

h_G = liquid height – height of the disc centre from the bottom of the tank.

i.e., $hG = 11 - 7 = 4$ m

$$\therefore F_R = (20265 + 4 \times 0.8 \times 1000 \times 9.8) \times 3.14$$
$$= 162202.98 \text{ N} \approx 162.2 \text{ kN}.$$

Example 9: The torque required to maintain the disc in equilibrium in the vertical position is

(A) 7100.3 Nm (B) 18253.7 Nm
(C) 3745.7 Nm (D) 6163.7 Nm

Solution: Here, $hcp = h_G + \dfrac{I_{xG}}{\left(h_G + \dfrac{P_o}{\rho g}\right) A}$

Here $I_{xG} = \frac{\pi}{64} \times d^4$

$$= \frac{\pi}{64} \times (2)^4$$

$$= 0.7854 \text{ m}^4$$

$$\therefore\ h_{cp} = 4 + \frac{0.7854}{\left(4 + \dfrac{20265}{0.8 \times 1000 \times 9.81}\right) \times 3.14}$$

$$= 4.038 \text{ m}$$

Moment of the total pressure force about the horizontal diameter

$$= F_R \times (h_{cp} - h_G)$$
$$= 162202.98 \times (4.038 - 4)$$
$$= 6163.713 \text{ Nm (anticlockwise)}$$

Here the torque required = 6163.713 Nm (clockwise)

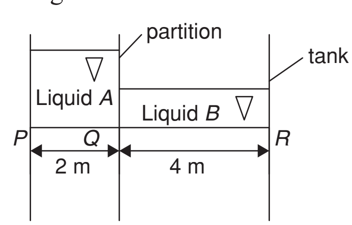
Hydrostatic Force on a Submerged Horizontal Plane Surface

If the horizontal surface is at a distance of h^* from the liquid surface, then the total pressure and centre of pressure equations are:

$$F_R = (P_o + \rho g h^*)\, A$$
$$X_{cp} = x_G$$
$$Y_{cp} = y_G$$
$$h_{cp} = h_G = h^*$$

From the above equation, one can see that for horizontal surfaces submerged in a fluid of constant density, the centre of pressure always coincides with the centroid of the horizontal surface area. This is because the fluid pressure is constant and uniformly distributed over the surface. The fluid surface pressure P_o can be set to zero in the above equation if it acts on both sides of the horizontal surface.

Example 10: A rectangular tank has a rotatable bottom PR that is hinged at Q. The tank is partitioned into two volumes as shown in the figure below.



The volume of the tank to the right of the partition is filled to a height of 2 m with liquid B. of density 1500 kg/m^3. When the volume of the tank to the left of the partition is filled to a height of h metres with liquid A of density 900 kg/m^3, the bottom of the tank remains in a stationary horizontal position, If the hinge is assumed to be frictionless, then the value of h for the bottom just to tilt is

(A) 13.33 m (B) 3.33 m
(C) 6.67 m (D) 1.67 m

Solution:
Let the length of the tank into the plane of the paper be l metres.
Total pressure force on the bottom of the tank due to liquid A,

$$F_{RA} = h \times \rho_A \times g \times A_{pq}$$
$$= h \times 900 \times g \times 2 \times \ell$$

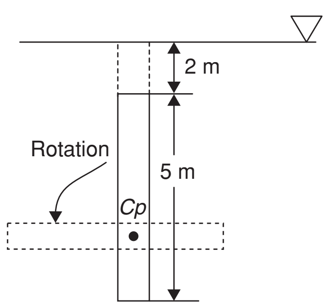
Similarly, the total pressure force on the bottom of the tank due to liquid B

$$F_{RB} = 2 \times 1500 \times g \times 4 \times \ell$$

For equilibrium, taking moments about B we have $F_{RA} \times 1 = F_{RB} \times 2$

$$h \times 900 \times g \times 2 \times \ell \times 1$$
$$= 2 \times 1500 \times g \times 4 \times \ell \times 2$$
$$\therefore \quad h = 6.67 \text{ m.}$$

Example 11: A rectangular thin plate of height (h) 5 m and width (w) 2 m is immersed in water, vertically along its height such that the plate is at a distance of 2 m from the water surface as shown in the figure below. Alter determining the centre of pressure for the right surface of the plate, if the plate is rotated 90° anticlockwise about an axis parallel to the water surface and on which the centre of pressure lies, then the resultant hydrostatic force acting on the plate would be:



(A) 441.45 kN (B) 686.70 kN
(C) 486. 47 kN (D) 327 kN

Solution:
Since the fluid surface pressure P_o acts on both the surfaces.

$$h_{cp} = h_G + \frac{I_{XG}}{h_G A}$$

Area of the plate, $A = h \times w = 5 \times 2$
$$= 10 \text{ m}^2$$

Moment of Inertia,

$$I_{xG} = \frac{w \times h^3}{12} = \frac{2 \times 5^3}{12}$$
$$= 20.833 \text{ m}^4$$

Now, $h_G = 2 + \frac{5}{2} = 4.5$ m

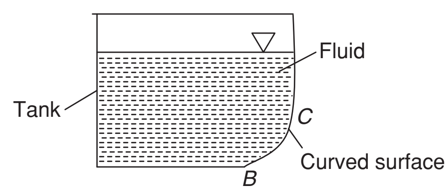
$$\therefore \quad h_{cp} = 4.5 + \frac{20.833}{4.5 \times 10}$$
$$= 4.963 \text{ m}$$

The rotated surface will be a horizontal surface located at a distance h_{cp} from the water surface.

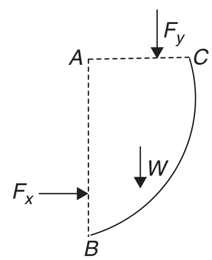
$\therefore \quad h^* = h_{cp} = 4.963$ m

Resultant hydrostatic force,
$F_R = \rho g h^* A$ (ρ is density of water = 1000 Kg/m^3)
1000 × 9.81 × 4.963 × 10 = 486.87 kN.

Hydrostatic Force on a Submerged Curved Surface
Consider the curved surface BC of a tank filled with a fluid as shown in the figure below.

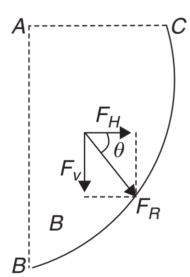


Let f_x and f_y be the respective hydrostatic forces acting on the planar surfaces AB and AC which form the respective vertical and horizontal projections of the curved surface BC is illustrated in the free body diagram I ($FBDI$)



FBD I

In $FBDI$, W corresponds to the weight of the fluid block enclosed by the curved surface and the two planer surfaces. Let F_R (horizontal component = F_{HS}, vertical component = F_v) be the total pressure force or the resultant hydrostatic force acting on the curved surface BC as illustrated in free body diagram (FBD)

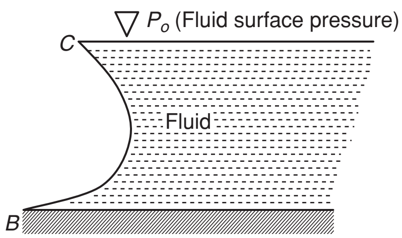


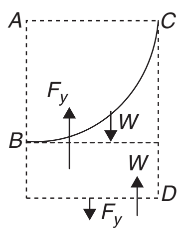
$$F_R = \sqrt{F_H{}^2 + F_v{}^2}$$
$$F_H = F_x$$
$$F_v = F_y + w$$
$$\tan\theta = \frac{F_v}{F_H}$$

The location of the line of action of the total pressure force (for example, with respect to any of the ends of the curved surface) can be determined by taking moments about an appropriate point.

NOTE

When the fluid is present on the convex side of a curved surface *BC* as shown below, then the vertical component (F_v) of the total pressure force F_R acting on the curved surface is given as follows.

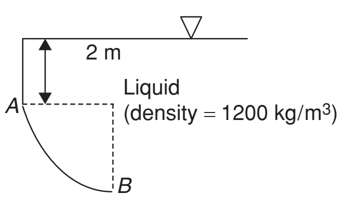




$$F_v = F_y - W$$
$$P_o + W_{IV},$$

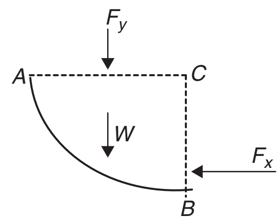
Where W_{IV} is the weight of the fluid enclosed by the imaginary volume *ABCA*.
Here W_{IV} = Weight of the fluid enclosed by the imaginary volume *ABCDA* – weight of the fluid enclosed by the volume *CBDC*

Example 12: A gate has a curved surface *AB* in the form of a quadrant of a circle of radius 3 m as shown in the figure below. If the width of the gate is 1.5 m, then the total pressure force acting on the curved surface *AB* is



(A) 384608.4 N (B) 309015 N
(C) 493370 N (D) Not determinable.

Solution:
Given, $\rho = 1200$ Kg/m^3
$R = 3$ m
$h = 2$ m and
$W = 2.5$ m
The free body diagram is



Let F_R be the total pressure force action on the curved surface *AB*
Area of the planar surface *BC*,

$$A_{BC} = r \times w$$
$$= 3 \times 2.5 = 7.5 \text{ m}^2$$

Area of the planar surface *AC*,

$$A_{AC} = r \times w = 7.5 \text{ m}^2$$

Horizontal component of F_R,

$$F_A = P_o + \left(h + \frac{x}{2}\right)\rho \times g \times A_{BC}$$

Vertical component of F_R,

$$F_v = F_y + W$$

Where $F_y = P_o + h\rho g \times A_{AC}$ and

$$W = \frac{\pi}{4} r^2 \times w \times \rho \times g$$

Since the fluid surface pressure P_o also acts on the bottom surface of the gate, it can be ignored.

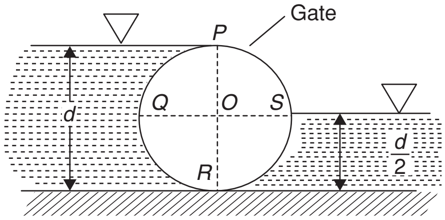
$$\therefore \quad F_H = \left(2 + \frac{3}{2}\right) \times 1200 \times 9.81 \times 7.5$$
$$= 309015 \text{ N}$$

$$F_v = 2 \times 1200 \times 9.81 \times 7.5 + \frac{\pi}{4} \times 3^2 \times 2.5 \times 1200 \times 9.81$$
$$= 384608.4 \text{ N}$$

$$F_R = \sqrt{F_H^2 + F_v^2}$$
$$= \sqrt{(309015)^2 + (384608.4)^2}$$
$$= 493370 \text{ N.}$$

Example 13: A cylindrical gate 5 m long has a liquid, of density 900 Kg/m^3, on both sides as shown in the following figure. The gate has a diameter of *d* metres and width of *W* metres. If it is to be ensured that the gate is in contact with the floor, then the density of the material making up the gate should be at least

(A) 169 kg/m^3 (B) 675 kg/m^3
(C) 1200 kg/m^3 (D) 900 kg/m^3



Solution:

Let ρ_f (= 900 kg/m^3) be the density of the fluid.

Here the pressures at the surfaces of the fluid are ignored. i.e., *Po* is ignored.

Vertical force acting upward on *QO*

F_{v1} = weight of the fluid enclosed by the imaginary volume *PQRP*

$$= \frac{\pi}{8} d^2 \times w \times \rho_f \times g$$

Vertical force acting upward on *OS*,

F_{v2} = Weight of the fluid enclosed by the imaginary volume *SORS*:

$$\frac{\pi}{16} d^2 \times w \times \rho_f \times g$$

Total vertical force acting upward,

$$F_v = F_{v1} + F_{v2}$$

$$= \pi d^2 w \rho f g \left(\frac{1}{8} + \frac{1}{16}\right)$$

$$= \frac{3}{16} \pi d^2 w \rho j g$$

Weight of the gate,

$$W = \frac{\pi d^2}{4} \times w \times \rho_s \times g$$

Where ρ_s is the density of the material making up the gate.
For the gate to remain in contact with the floor.

$$W \geq F_v$$

$$\text{i.e.,} \frac{\pi d^2}{4} w \rho_s g \geq \frac{3}{16} \pi d^2 w \rho f g$$

$$\text{i.e.,} \rho_s \geq \frac{3}{4} \rho$$

$$\geq \frac{3}{4} \times 900$$

$$\geq 675 \text{ Kg/m}^3.$$

Buoyancy

Buoyancy is the tendency of a body to be lifted (or buoyed) up in a fluid in which it is immersed wholly or partially. The force, acting opposite to the gravity force (i.e., acting vertically upward), that tends to lift the body is called the *buoyant force* or *force of buoyancy* or *up thrust*

Archimedes Principle

The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body.

Centre of Buoyancy

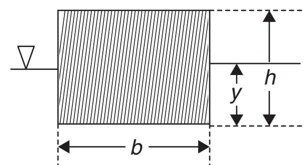
The point of application of the buoyant force on a body is known as the *centre of buoyancy*. It is always located at the centroid of the fluid volume displaced by the body.

NOTE

In the above definition, the fluid is always assumed to be of constant specific weight.

Buoyant Force – Single Fluid

Consider a rectangular block (density = ρ_s) of length ℓ metres in to the plane of the paper, immersed in a fluid of density ρ_j as shown in the figure below.



Force of buoyancy,

$$\boxed{F_B = V_s \times \rho_j \times g}$$

Where V_s is the volume of the body.
Submerged in the fluid, i.e, the displaced volume $(= b \times y \times \ell)$

Here $F_B = b \times y \times \ell \times \rho_j \times g$

For the body to be in static equilibrium,

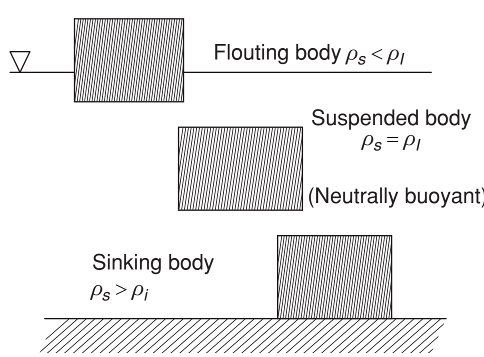
$$F_B = W,$$

Where $W = b \times h \times \ell \times \rho_s \times g$, is the weight of the block.

$$\therefore \quad b \times y \times \ell \times \rho_j \times g = b \times h \times \ell \times \rho_s \times g$$

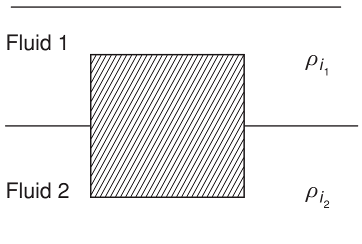
$$\text{or} \quad \boxed{\frac{V_s}{V_t} = \frac{\rho_s}{\rho_j}}$$

Where $V_t (= b \times h \times \ell)$ is the total volume of the block.



Buoyant Force – Layered Fluid

If the rectangular block considered above is present in a layered fluid (as shown in the figure below.).Where the ith layer of fluid has the density ρ_{ji}, then



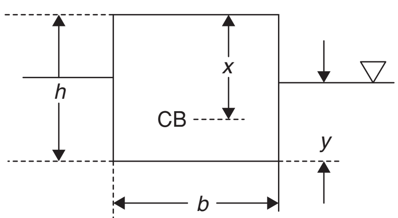
Force of buoyancy,

$$F_B = \left(\sum_i V_{si}\rho_{ji}\right)g$$

Example 14: A rectangular block of width = 5 m, height = 3 m and length = 10 m (in to the plane of the paper) is floating in a liquid of density 1500 kg/m^3. If the centre of buoyancy is located at a vertical distance of 2.5 metres from he top edge of the block, then the density of the material making up the block is:

(A) 1500 kg /m^3 (B) 500 kg/m^3
(C) 750 kg / m^3 (D) 1700 kg/m^3

Solution:



$CB \rightarrow$ centre of buoyancy

Let ℓ be the length of the block into the plane of the paper.

Given, $\ell = 10$ m
$h = 3$ m
$b = 5$ m
$x = 2.5$ m

Since point CB form the centroid of the displaced fluid volume,

$$x + \frac{y}{2} = h$$

i.e., $y = 2 \times (3 - 2.5) = 1$ m

Let $\rho\ell$ and ρ_s be the densities of the liquid and the material of the block respectively.

Given, $\rho\ell = 1500$ kg/m^3

Now, $\frac{V_s}{V_t} = \frac{\rho_s}{\rho_\ell}$

i.e., $\frac{y}{h} = \frac{\rho_s}{\rho_\ell}$

or $\rho_s = \frac{1500 \times 1}{3} = 500\,\text{kg/m}^3$.

Example 15: A spherical object (density = ρ_s) is submerged in a tank of liquid (density = ρ_j). The object, which does not touch the tank's bottom, is held in place by chaining it to the bottom of the tank. The tension in the chain would be zero when

(A) $\rho_s = 2\rho_j$ (B) $2\rho_s = \rho_j$
(C) $\rho_s = \rho_j$ (D) $\rho_j >>> \rho_s$

Solution:

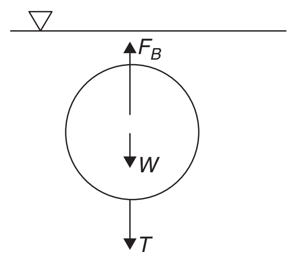
Let the radius of the spherical object be r

Weight of the object,

$$W = \frac{4}{3}\pi r^3 \times \rho_s \times g$$

Buoyant force acting on the object

$$F_B = \frac{4}{3}\pi r^3 \times \rho_g \times g$$



If T is the tension in the chain, then from the equilibrium of force we have

$$T = W - F_B$$
$$= \frac{4}{3}\pi r^3 g \times (\rho_s - \rho_j)$$

So when $\rho_s = \rho_j$, $T = 0$

This problem can also be solved without the above mathematical steps. If the tension in the chain is zero, then the object would be a suspended body in the tank. For suspended body, $\rho_s = \rho_j$.

Example 16: A body of unknown shape and density ρ kg/m^3 floats at the interface of two invisible liquids A and B having the respective densities of ρ_A and $\rho_B(\rho_A < \rho_B)$. The ratio of the volume of the block submerged in liquid B to the total volume of the block is equal to

(A) $(\rho - \rho_A)/(\rho_B - \rho_A)$
(B) $(\rho_A - \rho)/(\rho_B - \rho_A)$
(C) $(\rho_B - \rho_A)/(\rho - \rho_A)$
(D) $(\rho_B - \rho_A)/(\rho_A - \rho)$

Solution:

Let V_{sa} and V_{SB} be the volumes of the block submerged respectively in liquid A and B. Force of buoyancy,

$$F_B = (V_{SA}\,\rho_A + V_{SB}\,\rho_B)\,g$$

If V_t is the total volume of the block then

$$V_t = V_{SA} + V_{SB}$$

$\therefore \quad F_B = (V_t \rho_A + V_{SB} (\rho_B - \rho_A)\, g$

Under static equilibrium,

$F_B = W = V_t \rho g$ (W is weight of the block)

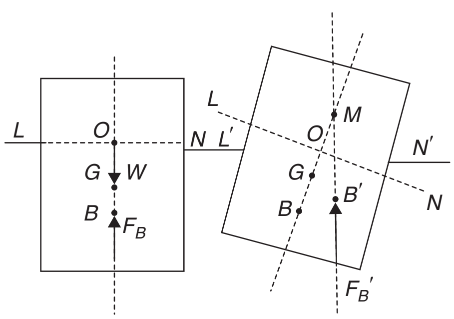
i.e., $[V_t\rho_A + V_{SB} (\rho_B - \rho_{A)}] = V_t \rho$

or $\dfrac{V_{SB}}{V_t} = \dfrac{\rho - \rho_A}{\rho_B - \rho_A}.$

Stability (Rotational Stability) of a Submerged Body A body is said to be *stable* i.e., in *stable* equilibrium if the centre of gravity is directly below the centre of buoyancy for the body. However if the centre of gravity is directly above the centre of buoyancy, then the submerged body is said to be *unstable* i.e., in unstable *equilibrium*. When the centre of gravity and the centre of buoyancy coincide, the submerged body is said to be *neutrally stable* i.e., in *neutral equilibrium.*

Stability (Rotational Stability) of a Floating Body A floating body is always stable when the centre of gravity is directly below the centre of buoyancy. When the centre of gravity is directly above the centre of buoyancy, metacentre plays a role in determining the stability of the floating body.

Consider a body being rotated by a small angle, along an axis that passes through the point O and that is perpendicular to the plane of the paper as shown in the following figures.



Metacentre (point M) is the point of intersection of the line passing through the centre of gravity (point G) and the original centre of buoyancy (point B) and a vertical line passing through the centre of buoyancy of the rotated position of the body (point B')

The distance between the metacentre and the centre of gravity of a floating body is called as the *metacentric height*. (GM)

If BM and BG represent the distance between the centre of buoyancy (point B) and the metacentre and the centre of gravity points respectively, then

$$\boxed{Gm = BM - BG}$$

$$\boxed{BM = \frac{\rho g \text{I}}{W} = \frac{\text{I}}{V_s}}$$

$$\frac{\rho g \text{I}}{W} = \frac{\text{I}}{\frac{W}{\rho g}} = \frac{\text{I}}{V_s}$$

Where ρ is the density of the fluid, W is the weight of the body, I is the moment of inertia of the sectional area of the body at the fluid surface (i.e., LN) about the axis at point O and V_s is the volume of the body submerged in the fluid. A floating body is said to be *stable*

If point M is above point G (i.e., GM is positive) and *unstable* if point M is below point G. The floating body is *neutrally stable* if point M coincides with point G. Larger the metacentric height, more stable the floating body will be

Example 17: A rectangular block of width W, height h and length ℓ (perpendicular to the plane of the paper) is floating in a liquid. The height of the block submerged in the liquid is b.

If the centre of gravity of the body is located at the liquid surface, then which one of the following condition when satisfied will ensure that the block is stable?

(A) $W > \sqrt{\dfrac{3}{2}}b$ (B) $\ell > \sqrt{6}b$

(C) $W > \sqrt{6}b$ (D) $\ell > \sqrt{\dfrac{3}{2}}b$

Solution:

Volume of the block that is submerged in the liquid'

$$V_S = b \times W \times \ell$$

The cross-sectional area of the block at the water surface will have a base ℓ and height W. relative to the axis (Perpendicular to the plane of the paper) at point O.

$\therefore$ Moment of inertia,

$$I = \frac{1}{12} w^3 \ell$$

Metacentric height, $GM = BM - BG$

Here $BM = \dfrac{I}{V_s}$

$$\frac{\frac{1}{12} w^3 \ell}{bw\ell} = \frac{w^2}{12b}$$

Also, $BG = \dfrac{b}{2}$

$\therefore \quad GM = \dfrac{w^2}{12b} - \dfrac{b}{2}$

for a stable body GM should be positive,

i.e., $\dfrac{w^2}{12b} > \dfrac{b}{2}$

or $w > \sqrt{6}b.$

Example 18: Cubes A, B, C and D all have the same side length of ℓ metres and are floating in the same water body. The cubes A, B, C and D have the respective constant specific gravities of 0.152, 0.561, 0.789 and 0.923 respectively. Which one of the following statements is ONLY not correct?

(A) Cube C is instable (B) Cube B is unstable
(C) Cube A is stable (D) Cube D is stable.

Solution:
Let us consider a floating cube in general with specific gravity V. Let the height of the floating cube submerged in water be h.
For the floating cube,
Buoyant force experienced = weight of the cube

i.e., $h \times \ell^2 \times 1000 \times g = \ell^3 \times r \times 1000 \times g$

or $h = \ell r$ (1)

Volume of the cube submerged,

$$V_s = h \times \ell^2$$

Moment of inertia,

$$I = \frac{1}{12} \times \ell^3 \times \ell$$

$$BM = \frac{I}{V} = \frac{\frac{1}{12}\ell^4}{h\ell^2} = \frac{\ell^2}{12h}$$

Here, $BG = \frac{\ell}{2} - \frac{h}{2}$ point B is below point G)

$GM = BM - BG$

$$GM = \frac{\ell^2}{12h} - \frac{\ell}{2} + \frac{h}{2} \qquad (2)$$

Substituting equation (1) in equation (2)
We get

$$GM = \frac{\ell}{12r}(1 - 6\gamma + 6\gamma^2)$$

When $GM = 0$, we have

$$6\Upsilon^2 - 6\Upsilon + 1 = 0$$

i.e., $\Upsilon = 0.789$ or 0.211
It can be shown that when
$0.211 < \Upsilon < 0.789$, GM is negative
∴ Cube C is neutrally stable (or stable if higher precision is considered in the values of Υ).

Oscillation of a Floating Body

When a floating body is given a small angular displacement, the body will oscillate about its metacentre. If T denotes the time period of Oscillation or rolling (i.e., time for one complete oscillation) of the floating body, then

$$\boxed{T = 2\pi\sqrt{\frac{k^2}{GM \cdot g}}}$$

Where GM is the metacentric height, K is the radius of gyration and T is in seconds.

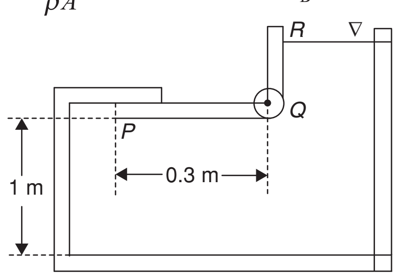
Exercises

Practice Problems I

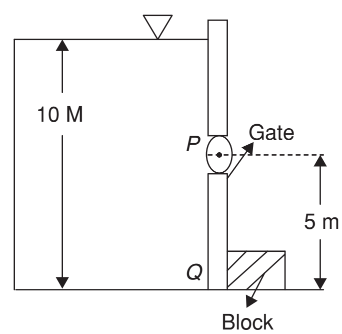
Direction for questions 1 to 20: Select the correct alternative from the given choices.

1. The gate PQR (with rectangular surfaces) shown in the figure below, is hinged permanently at Q. The gate operates to release the liquid A when the liquid depth h on the right side of the gate is large enough. For liquid A (density = ρ_A), the required liquid depth to open the gate was found to be h_A and for liquid B (density = $\rho_\beta \neq \rho_{A)}$ the required liquid depth was found to be h_B. One could then conclude that

   (A) $h_A = \frac{\rho_A}{\rho_B} h_B$ (B) $h_A = 1.27$ m
   (C) $h_A = \frac{\rho\beta}{\rho A} h_B$ (D) $h_B = 1.52$ m



2. A rectangular gate PQ has a length of 7 m into the plane of the paper and a height of 5 m. The gate is hinged at p and is prevented from opening by a block at Q as shown in the figure below.



   When the height of the body of liquid (present on the left side of the gate) reaches 10 m, the hinge breaks, as the reaction at the hinge becomes 1029 kN. The density of the liquid is

   (A) 257 kg/m^3
   (B) 900 kg/m^3
   (C) 2700 kg/m^3
   (D) 400 kg/m^3

3. A vertical rectangular gate, having a width of 3 m and a height of 1.5 m, is hinged of the bottom. In the upstream side of the gate, it is observed that a liquid of density 1500 kg/m^3, whose free surface extends 2 m above the top of the gate, is present. While in the downstream side, a liquid of density 900 kg/m^3 is present such that its free surface touches the top of the gate. From the hinge, the line of action of the resultant hydrostatic force acting on the gate is situated at a distance of

(A) 0.7174 m (B) 0.5 m
(C) 0.75 m (D) 0.6818 m

Direction for questions 4 and 5: In a tank, having a square base of side length 2 m, two layers of immiscible liquids are present. The less dense liquid layer is 0.6 m thick. One of the liquids has a specific gravity of 0.7 .While the other has a specific gravity of 0.9.The total pressure force acting on a side of the tank is 4502.8 N

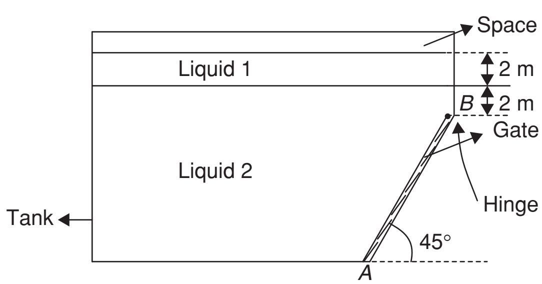
4. The thickness of the bottom liquid layer is

(A) 0.481 m (B) 0.251 m
(C) 0.202 m (D) 0.608 m

5. The vertical distance of the centre of pressure from the top liquid free surface is:

(A) 0.6 m (B) 0.548 m
(C) 0.75 m (D) 0.317 m

6. A 4 m square gate *AB* is hinged at *B* as shown in the figure below. The tank contains 2 m thick layer of a liquid having a specific gravity of 0.5. The tank also contains another liquid having a specific gravity of 0.8 which is filled to a height of 2 m above the top edge of the gate. The space above the top liquid layer is subjected to a Negative pressure of 10000 N/m^2. The vertical pull force, applied at *A*, that is required to open the gate is equal to:



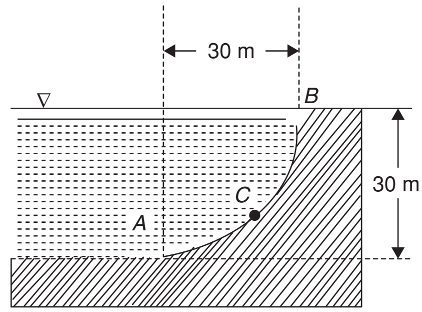
(A) 768.45 kN (B) 342.8 kN
(C) 3073.78 kN (D) 82.85 kN

7. An inclined annular plate of external diameter *D* metres ($D = 2d$) is submerged in water such that the greatest and least depths of the plate below the water surface are h_1 and h_2 metres ($h_1 = kh_2$).The vertical distance of the centre of pressure from water surface is *h* meters. If the ratio $h{:}h_2$ is equal to 69:32, then the value of *k* is:

(A) 3 (B) 4
(C) 5 (D) 1.5

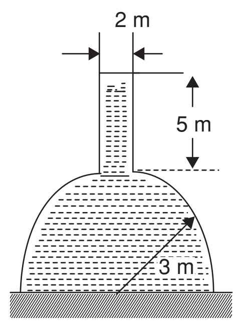
8. The dam shown in the figure below is a quarter of a circle of radius 30 m and has a width of 60 m into the plane of the paper. The point *C* is *x* meters away to the right of point *A* and *y* meters up from the same point *A*. If point *C* is the point at which the total pressure force (due to water) strikes the dam, then the respective values of *x* and *y* are:

(A) 16.11 m and 25.3 m
(B) 12.73 m and 4.7 m
(C) 12.73 m and 25.3 m
(D) 16.11 m and 4.7 m



9. The hemispherical dome shown in the figure below has a weight of 40 kN and is filled with water. The dome can be attached to the floor using equally spaced bolts. If the maximum force that can be allowed through a bolt is 183.8 kN, then the least number of bolts required to hold the dome onto the floor is

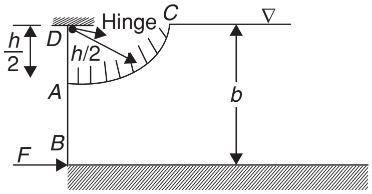
(A) 4 (B) 9
(C) 8 (D) 0



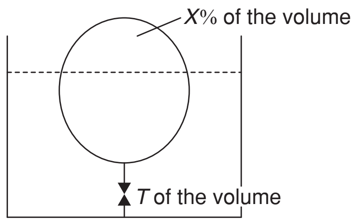
10. A water in a channel of width 6 metres held back using a hinged gate of negligible weight a shown in the following figure (surface *AC* is a quarter of circle). A horizontal force *F* newtons is applied at point *B* on the gate to hold it in place. If the friction in the hinge can be neglected, then the force *F* per unit specific weight of the fluid is equal to

(A) $\frac{7}{24}h^2b$ (B) $\frac{9}{32}h^2b$

(C) $\frac{1}{96}h^2b$ (D) $\frac{7}{9}h^2b$



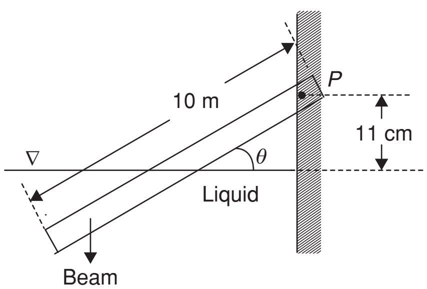
11. A sphere of radius 1.5 m and of material specific gravity is immersed in a liquid of S.G.1.8. The sphere is attached to the bottom of the vessel containing the liquid by means of a string which experiences a tension of 6 kN. The sphere is such that x% of its volume protrudes out of the liquid as shown. The density of the solid sphere shall be



(A) 27.9% (B) 36.5%
(C) 48.2% (D) 40.5%

12. A beam (density = 800 kg/m^3), having the dimensions of 1.5 cm × 1.5 cm × 10 m, is hinged at the point P as shown in the following figure. If the beam floats in a liquid of density 1100 kg/m^3 at an angle θ to the horizontal, then the value of θ is,

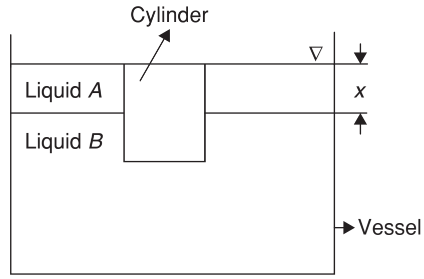
(A) 1.26° (B) 1.32°
(C) 60° (D) 30°



13. A cylinder (density = ρ) is held down in a vessel containing two liquids using a string such that two – thirds of the cylinder's height is in liquid B (density = $\rho_{B)}$ and the rest is in liquid A (density = ρ_A) as shown in the following figure. The pressure at the cylinder's bottom is twice that of the atmospheric pressure (p_{atm}). If the ratio of the densities ρ:ρ_A:ρ_B is equal to 2: 1: 2, then x is equal to

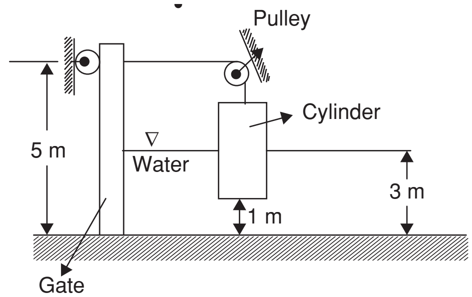
(A) $\dfrac{2P_{atm}}{\rho_A g}$ (B) $\dfrac{P_{atm}}{\rho g}$

(C) $\dfrac{P_{atm}}{\rho_B g}$ (D) $\dfrac{P_{atm}}{\rho_A g}$



14. A cylinder of diameter 2 m and height 4 m is connected to a gate of width 3 m as shown in the following figure. When the water level drops to a height of 3 m, the gate opens up. If the friction of the gate and pulley can be neglected, then the density of the cylinder is

(A) 500 kg/m^3 (B) 715 kg/m^3
(C) 822 kg/m^3 (D) 965 kg/m^3



15. Two right circular cones A (density = ρ_A) and B (density = ρ_B) have the same radii and height of 1 m and 3 m respectively. Both the cones are floating in a liquid of specific gravity 1.2 with their apex downwards. When a mass of 31.42 kg is placed on the base of cone A, the submerged heights become equal for both the floating cones. The relationship between the densities of the cones can be approximately described by

(A) $\rho_B = 1 + \rho_A$ (B) $\rho_B = 10 + \rho_A$
(C) $\rho_B = \rho_A$ (D) $\rho_A = 10 + \rho_{B.}$

16. A neutrally stable solid cylinder of radius r, height h and density ρ_s is floating in a liquid of density $\rho\ell$ with its axis vertical. If the height of the cylinder submerged in the liquid is b, then the difference in the densities of the liquid and the cylinder is equal to

(A) $\dfrac{r^2\rho_\ell}{2b^2}$ (B) $\dfrac{r^2\rho_\ell}{b^2}$

(C) $\dfrac{r^2\rho_s}{b^2}$ (D) $\dfrac{r^2\rho_s}{2b^2}$

17. A hollow cylinder of outer diameter 750 mm and inner diameter 300 mm is floating with its axis vertical in a liquid whose specific gravity is 1.5 times the specific gravity of the cylinder. The maximum length the cylinder can have to stably float in the liquid is

(A) 0.55 m (B) 0.61 m (C) 0.78 m (D) 0.92 m

18. A cube (side length = 1 m) floating in water has a base that is 0.2 m thick and made of a material with specific

gravity 7. The rest of the cube, excluding the base, has a specific gravity of 0.6 .The metacentric height of the floating cube is
(A) 0.668 m (B) 1.6524 m
(B) 1.608 m (D) 0.757 m

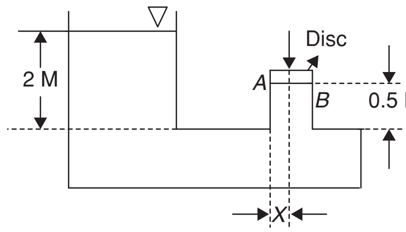
19. A sphere having a uniform density throughout and submerged in a liquid
(A) is always stable
(B) is always unstable.
(C) always neutrally stable.
(D) could be stable or unstable.

20. A ship is floating in sea water $\left(\text{specific weight} = 10.2\dfrac{\text{kN}}{\text{m}^3}\right)$ such that its centre of buoyancy is 1.5 m below its centre of gravity. The moment of inertia of the ship area at the sea water level is 9000 m^4. The radius of gyration of the ship is 5 m. If the ship takes 7 secs to complete one oscillation, then the ship has a mass of:
(A) 51209 kN (B) 25837 kN
(C) 17779 kN (D) 64329 kN

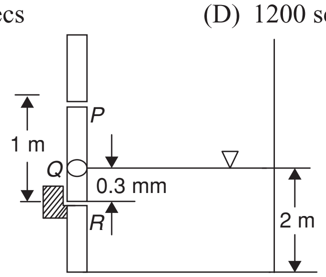
Practice Problems 2

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. Water is filled in a structure, as shown in the figure below, which has a circular opening *AB* closed by a disc. Both the opening and the disc have the same diameter of 0.6 m. When a force *F* (in Newtons) is applied at a distance of *x* metres from the point *A*, then the disc is held at stationary horizontal position.
The values of *F* and *x* are:
(A) 5546.6 N and 0.3 m (B) 5546.N and 0.6 m
(C) 4160 N and 0.6 m (D) 4160 N and 0.3 m



2. A rectangular tank of base area 40 m^2 contain water up to a height of 2 m. A square gate *PR*, of side length of 1 m, is present on the vertical side of the tank and is hinged at *Q* as shown in the figure below. A stopwatch is started and water is added in to the tank at the rate of 0.4 m^3/s. The gate *PR* will open after a time duration of
(A) 262 secs. (B) 20 secs
(C) 62 secs (D) 1200 secs

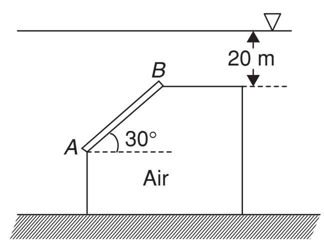


3. A tank having a length of 6 m and width of 5 m is partitioned along its length into two volumes. The length of the tank on the left side of the partition is 4 m while the length on the right side of the partition is 2 m. In the left partition volume, water is added at the rate of 10 m^3/s while simultaneously in the right partition volume, a liquid of density 1500 kg/m^3 is added at the rate of 2.5 m^3/s. When the net force acting on the partition is 1533 kN, the partition is seen to break and this would happen aftrer a time duration of
(A) 20 secs (B) 13.48 secs
(C) 666.75 secs (D) 15.81 sec

4. A tank contains a liquid up to a certain level. When the tank is stationary the free liquid surface is horizontal. If the tank is moving with a constant acceleration 2 m/s^2 in the horizontal direction, the slope of the free liquid surface will become
(A) $\tan\theta = 0.2$ (B) $\tan\theta = 0.4$
(C) $\tan\theta = 0.6$ (D) $\tan\theta = 0.8$

5. Three layers of liquids, having specific gravities of 0.2, 0.4 and 0.8 and an equal thickness of 6 metres, are present in a rectangular tank. If the vertical distance of the centre of pressure points, with respect to the individual layers, on a side of the tank are h_1, h_2 and h_3 metres fro m the top liquid surface.
(A) $29\,h_1 = 8\,h_2$ (B) $154\,h_2 = 95\,h_3$
(C) $h_1 + h_3 = \dfrac{16}{5}\times 6$ (D) $h_3 : h_2 : h_1 = 4 : 2 : 1$

6. A container filled with air and having a circular hatch *AB* (hinged at *B*) of 2.5 m diameter, is present at the bottom of an ocean as shown in the below figure. The ocean water has a specific gravity of 1.03 and the atmospheric pressure is 1 atm. If the friction in the hinge and the weight of the hatch can be considered to be negligible, then the air pressure within the container required to open the hatch is



(A) 667.5 kPa (B) 10246.2 kPa
(C) 335 kPa (D) 418.75 kPa

7. A trapezoidal plate, having a bottom edge length twice that of the top edge length is submerged in a liquid such that the plan makes an angle of 30° to the free surface of the liquid. From the liquid surface the depth at which the bottom edge lies, is equal to thrice the depth at which the top edge lies. The perpendicular distance between the top and the bottom edges of the plate is 2 m. If the total pressure force acting on the plate per unit area is 9319.5N, then the density of the liquid is

(A) 3109 kg/m^3 (B) 1267 kg/m^3
(C) 1800 kg/m^3 (D) 900 kg/m^3

8. An inclined submerged rectangular plate is rotated such that the angle θ the plane containing the top surface makes with the fluid surface is increased from 30° to 60°. The centre of pressure depth from the fluid surface remains unchanged after rotation. If the total pressure force acting on the top surface of the plate is F, then after rotation it is

(A) F (B) $\frac{F}{\sqrt{3}}$ (C) $\sqrt{3}F$ (D) $3F$

9. On a top surface (area = 10 m^2) of an inclined submerged rectangular plate, the pressures acting at a point on the top edge, at the centroid and at a point on the bottom edge are 120 kPa, 150 kPa and 190 kPa respectively. The total pressure force acting on the top surface is

(A) 1.2 MN (B) 1.5 MN
(C) 1.9 MN (D) 1.53 MN

10. A liquid of density ρ is pumped in to a rectangular tank up to a height of h metres. For a particular side, the vertical distances of the centroid and centre of pressure from the surface are determined to be h_G and h_{CP} respectively. The ratio $\left(\frac{h_G}{h_{CP}}\right)$

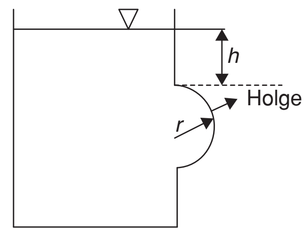
(A) increase as the density ρ increases.
(B) increases as the density ρ decreases.
(C) depends on the height h
(D) is always equal to $\frac{3}{4}$

11. On the top surface of a given submerged horizontal plate in a given fluid, the distance between the centroid and the cente of pressure can be

(A) changed by changing the fluid surface pressure.
(B) increased by submerging the plate further deep.
(C) never changed by changing the shape of the plate.
(D) Changed by changing the acceleration due to gravity.

12. A semicircular shaped bulge is present in the vertical side of an open rectangular tank containing water as shown in the following figure. The water surface in the tank is present h metres above the top edge of the bulge which has a radius of r metres. The total pressure force acting on the bulge makes an angle of θ with the horizontal. If the value of the height h and the radius r are tripled, then the value of θ will be:

(A) doubled (B) tripled
(C) quadrupled (D) unchanged

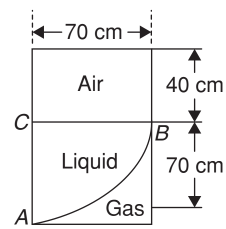


13. A fluid is present on the convex side of a circular arc. The circular arc is a part of the circle where centre is denoted by point O. With respect to the pressure forces acting on the circular arc, which one of the following statements is NOT correct?

(A) Pressure force act normal to the arc surface.
(B) Total pressure force does not pass through point O.
(C) Pressure forces form a concurrent force system.
(D) Total pressure force acts normal to the arc surface.

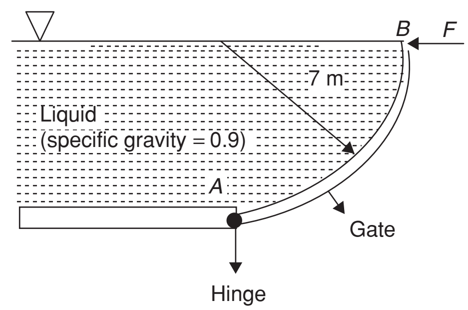
14. The tank shown in the following figure contains a liquid (specific gravity = 0.7) on top of which the air present is pressurized to 150 kPa (gauge). The bottom of the curved surface AB is exposed to a gas present at a pressure of 150 kPa. The perpendicular distance between the line of action of total force on the curved surface AB and the point A is

(A) 0.35 m (B) 0.297 m
(C) 0 m (D) 0.335 m



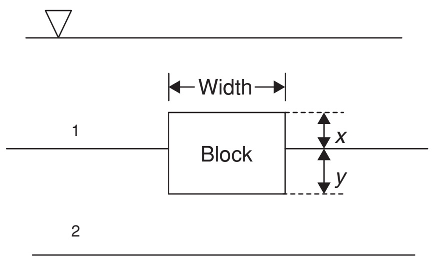
15. The gate, shown in the figure below is a quarter circle having a length of 50 m into the plane of the paper. A horizontal force F is applied at point B to hold the gate in place .The gate is hinged at point A. If the friction with hinge and the weight of the gate are neglected, then the force F is equal to:

(A) 13.9 MN (B) 3.6 MN
(C) 10.81 MN (D) 16.9 MN



16. A rectangular block (density = 7000 kg/m^3) floats at the interface of two immiscible liquids having the densities of 800 kg/m^3 and 9000 kg/m^3 as shown in the following figure. The ratio x:y is equal to

(A) 41 : 31 (B) 10 : 31 (C) 31 : 10 (D) 31 : 41



17. A cylindrical drum of diameter 50 cm and height 100 cm weighs 20 N. It contains some wine (density = 700 kG/m^3 and is seen to be the floating (along its height) in a large water tank. When the drum is completely filled with wine, it will

(A) still be floating
(B) Sink to the bottom of the tank.
(C) be just submerged
(D) be a suspended body in the water.

18. A square pole (Density = 700 Kg/m^3) having a cross sectional area of 0.01 m^2 is suspended by a wire so that 7 metres of its length is above the surface of a liquid of density 1200 kg/m^3 with the rest being immersed in the liquid. The tension in the wire is T Newtons.When the pole is lifted completely out of the liquid, the tension in the wire increases to 2.06 T Newtons. The length of the pole is:

(A) 7 m (B) 10 m (C) 14 m (D) 17 m

19. A tank of cross sectional area A_1 is filled to a height h_1 with a liquid of density ρ_1. When a cylinder of cross sectional area A_2 (= $0.5A_1$), height h_2 and density ρ_2 (= $2\rho_1$) floats in the liquid, the rise in the liquid level is:

(A) $2h_2$ (B) 0.5 h_2 (C) $4h_2$ (D) h_2

20. When a concrete weight of mass 2800 kg is placed on a cubical block (Specific gravity = 0.85), the block is completely immersed in a liquid of specific gravity 1.2 such that the top edge of the block coincides with the liquid surface. The concrete weight is slowly removed off such that the block moves to a stationary floating position. Then the height of the block that will be protruding out of the liquid surface is

(A) 0.27 m (B) 1.42 m
(C) 0.58 m (D) 0.66 m

21. A large boulder is present in a boat floating on a lake, if the boulder is thrown overboard and it sinks to the bottom of the lake, then the water level, with respect to the shore,

(A) could rise or drop
(B) would remain the same
(C) would drop
(D) would rise

22. Two identical glasses A and B have water filled up to the same level. Glass B however has some ice cubes floating in it. The ratio of the weight of glass B to that of glass A (along with their contents.)

(A) could be equal to one.
(B) would be greater than one.
(C) would be lesser than one.
(D) could be greater or lesser than or equal to one.

23. An uniform right circular cone, floating in a liquid with its apex vertically downwards has a base radius of 30 cm and height of 60 cm.The variable r is the radius of the circle formed by the intersection of the cone with the liquid surface. From the given set of values for r (in cm) such as {16, 20, 24, 28}, the maximum value of r for which the cone will be unstable is

(A) 16 (B) 28 (C) 24 (D) 20

24. Two hollow cylinder A and B having the same outer diameter of 800 mm, inner diameter of 320 mm and length of 50 cm are floating in a stable manner in a liquid of specific gravity 0.9. The cylinders are made of a material having a density of 800 Kg/m^3. If the lengths of the cylinder A and B are extended by an amount of 40 cm and 30 cm respectively., then when they float in the liquid,

(A) Cylinder A and B are unstable
(B) Cylinder A and B are stable
(C) Cylinder A is only stable.
(D) Cylinder B is only stable

25. A right circular cylinder is of radius R, height H and specific gravity 0.9 is floating in water with its axis being vertical. The minimum value the fraction $\frac{R}{H}$ can take so that the floating cylinder will be stable is

(A) 0.7846 (B) 0.18
(C) 0.4243 (D) 0.9

26. A body is set on the surface of a liquid to float. If the liquid and the body have the same density, then the body is in

(A) Stable equilibrium
(B) Unstable equilibrium
(C) Neutral equilibrium
(D) Stable, unstable or neutral equilibrium.

27. The relation among the metacentric height of a floating cube, its length 'ℓ' and S.G's' is given by $GM = \frac{\ell}{12s} + \frac{\ell}{2}(s-1)$

Two cubes A and B are floating in a body of water. Cube A has a side length of 0.1 m and specific gravity 0.9. Cube B has a side length of 0.2 m and specific gravity 0.8. Which one of the following statement is ONLY correct?

(A) Cube A is unstable
(B) Cube A is more stable than cube B
(C) Cube B is unstable
(D) Cube B is more stable than cube A.

28. If points G, M and B denote the centre of gravity meta-centre and centre of buoyancy for a body floating in a liquid, the sufficient condition for the body to be stable is
(A) point M being above point G
(B) point M being above point B.
(C) point B being below point G
(D) point M being below point B

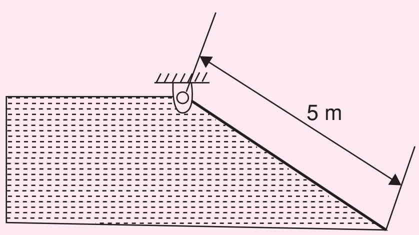
29. The time period of rolling of a ship of weight 25000 kN in sea water is half a minute. Along a line joining the metacentre (point M), centre of gravity (point G) and Centre of buoyancy (point B), the distance between points M and B is 3.5 m while the distance between points B and G is 1.5 m. Point M is above point G which a is above point B along this line. If the specific weight of sea water is 10.1 kN/m^3. Then the radius of gyration of the ship is:
(A) 21.5 m (B) 10.63 m
(C) 0.352 m (D) 8.42 m

30. If the metacentric height for a ship is x metres and the radius of gyration is equal to $\sqrt{xg}$, where g is the acceleration due to gravity, then the time taken by the ship to complete 3 oscillations is:
(A) 2 π secs (B) 0.1 π hrs
(C) 6 π min (D) 0.1 π min

Previous Years' Questions

1. For the stability of a floating body, under the influence of gravity alone, which of the following is TRUE? **[2010]**
(A) Metacentre should be below centre of gravity
(B) Metacentre should be above centre of gravity
(C) Metacentre and centre of gravity must lie on the same horizontal line
(D) Metacentre and centre of gravity must lie on the same vertical line

2. A hinged gate of length 5 m inclined at 30° with the horizontal and with water mass on its left, is shown in the figure below. Density of water is 1000 kg/m^3. The minimum mass of the gate in kg per unit width (perpendicular to the plane of paper), required to keep it closed is **[2013]**



(A) 5000 (B) 6600
(C) 7546 (D) 9623

3. For a completely submerged body with centre of gravity 'G' and centre of buoyancy 'B', the condition of stability will be **[2014]**
(A) G is located below B
(B) G is located above B
(C) G and B are coincident
(D) independent of the locations of G and B

4. A flow field which has only convective acceleration is **[2014]**
(A) a steady uniform flow
(B) an unsteady uniform flow
(C) a steady non-uniform flow
(D) an unsteady non-uniform flow

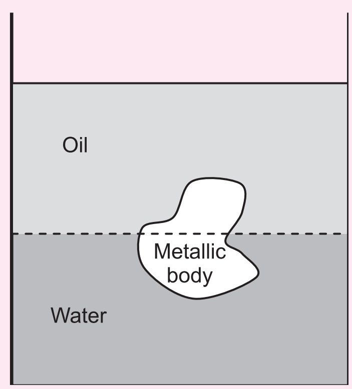
5. For a floating body, buoyant force acts at the: **[2016]**
(A) centroid of the floating body
(B) center of gravity of the body
(C) centroid of the fluid vertically below the body
(D) centroid of the displaced fluid

6. Assuming constant temperature condition and air to be an ideal gas, the variation in atmospheric pressure with height calculated from fluid static is: **[2016]**
(A) Linear (B) Exponential
(C) Quadratic (D) Cubic

7. The large vessel shown in the figure contains oil and water. A body is submerged at the interface of oil and water such that 45 percent of its volume is in oil while the rest is in water. The density of the body is _______ kg/m^3.

The specific gravity of oil is 0.7 and density of water is 1000 kg/m^3. **[2016]**

Acceleration due to gravity g = 10 m/s^2.

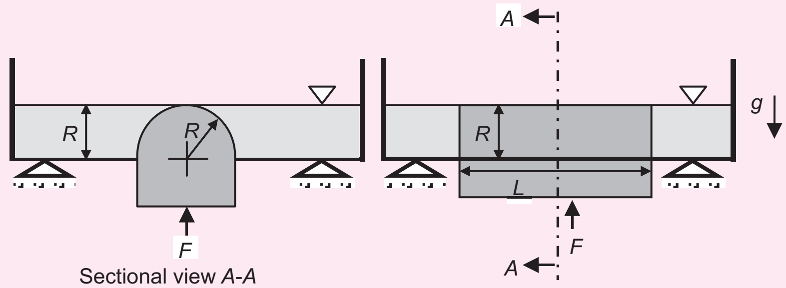


8. Consider a frictionless, mass less and leak-proof plug blocking a rectangular hole of dimensions $2R \times L$ at the bottom of an open tank as shown in the figure. The head of the plug has the shape of a semi-cylinder of radius R. The tank is filled with a liquid of density ρ

up to the tip of the plug. The gravitational acceleration is g. Neglect the effect of the atmospheric pressure (see figure given on next page).
The force F required to hold the plug in its position is: **[2016]**

(A) $2\rho R^2 gL\left(1-\frac{\pi}{4}\right)$ (B) $2\rho R^2 gL\left(1+\frac{\pi}{4}\right)$

(C) $\pi R^2 \rho gL$ (D) $=\frac{\pi}{2}\rho R^2 gL$



Answer Keys

Exercises

Practice Problems 1

1. D	**2.** B	**3.** A	**4.** C	**5.** B	**6.** B	**7.** A	**8.** D	**9.** C	**10.** A
11. B	**12.** D	**13.** D	**14.** B	**15.** B	**16.** D	**17.** A	**18.** D	**19.** C	**20.** B

Practice Problems 2

1. D	**2.** C	**3.** A	**4.** A	**5.** B	**6.** C	**7.** D	**8.** A	**9.** B	**10.** D
11. C	**12.** D	**13.** B	**14.** D	**15.** C	**16.** B	**17.** A	**18.** B	**19.** D	**20.** C
21. C	**22.** A	**23.** D	**24.** B	**25.** C	**26.** D	**27.** B	**28.** A	**29.** A	**30.** D

Previous Years' Questions

1. B	**2.** D	**3.** A	**4.** C	**5.** D	**6.** B	**7.** 865	**8.** A