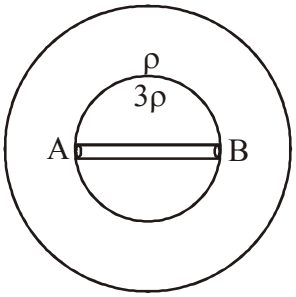
SECTION-I

Single Correct Answer Type 6 Q. [3 M (–1)]

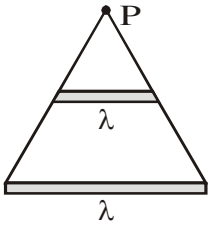
1. A planet of core density 3ρ and outer crust of density ρ has small tunnel in core. A small particle of mass m is released from end A then time required to reach end B :



(A) $\sqrt{\frac{\pi}{\rho G}}$ (B) $\frac{1}{2}\sqrt{\frac{\pi}{\rho G}}$

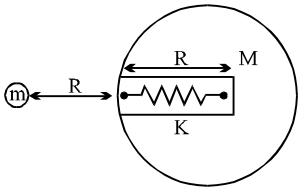
(C) $\pi\sqrt{\frac{1}{\rho G}}$ (D) $2\pi\sqrt{\frac{1}{\rho G}}$

2. The two sticks shown in diagram have same linear mass density λ, and they subtend same angle at point P. Their distances from point P are ℓ and 2ℓ. Which stick creates a larger force on a point mass placed at point P?



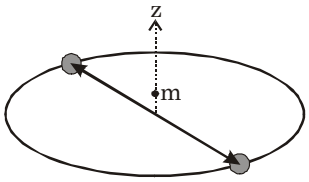
(A) The top stick
(B) The bottom stick
(C) They produce equal forces
(D) The relative magnitude of forces depends on ℓ

3. A small ball of mass 'm' is released at a height 'R' above the earth surface, as shown in the figure above. If the maximum depth of the ball to which it goes is R/2 inside the earth through a narrow grove before coming to rest momentarily. The grove, contain an ideal spring of spring constant K and natural length R, find the value of K if R is radius of earth and M mass of earth



(A) $\frac{3GMm}{R^3}$ (B) $\frac{6GMm}{R^3}$ (C) $\frac{9GMm}{R^3}$ (D) $\frac{7GMm}{R^3}$

4. Two stars, each of mass M and separated by a distance, orbit about their centre of mass. A planetoid of mass m(m << M) moves along the axis of this system perpendicular to the orbital plane. Let T_P be the period of simple harmonic motion for the planetoid for small displacement from the center of mass along the z-axis and let T_S be the period of motion for the two stars. Determine the ratio $\frac{T_P}{T_S}$:-



(A) The ratio of $\frac{T_P}{T_S} = 2$ (B) The ratio of $\frac{T_P}{T_S} = \frac{1}{2}$

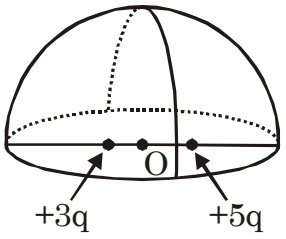
(C) The ratio of $\frac{T_P}{T_S} = \frac{1}{2\sqrt{2}}$ (D) The ratio of $\frac{T_P}{T_S} = \frac{1}{3}$

5. An electric field $\vec{E} = E_0\hat{i} + E_0\hat{j}$ exists in a space. Find the flux through a triangular loop with vertices at

$\left(\frac{a}{2}, 0, 0\right)$, $\left(\frac{a}{2}, 0, a\right)$ and $\left(\frac{a}{2}, a, \frac{a}{2}\right)$

(A) $\sqrt{2}\, E_0 a^2$ (B) $\frac{E_0 a^2}{2}$ (C) $\frac{\sqrt{3}}{2} E_0 a^2$ (D) $\frac{\sqrt{3}}{4} E_0 a^2$

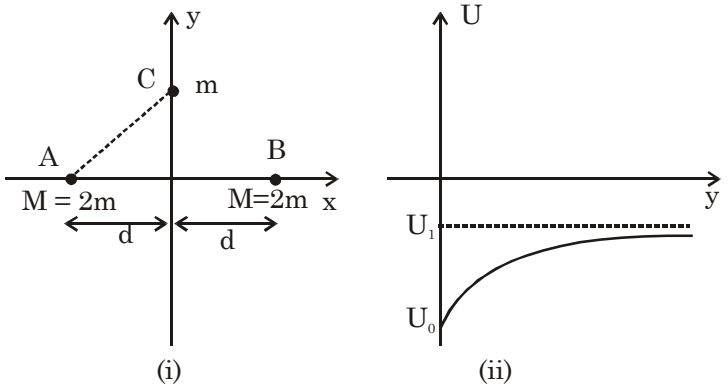
6. The figure shows a hollow hemisphere of radius R in which two charges 3q and 5q are placed symmetrically about the centre O on the planar surface. The electric flux over the curved surface is :



(A) $\frac{15}{2}\frac{q}{\varepsilon_0}$ (B) $\frac{4q}{\varepsilon_0}$ (C) $\frac{q}{\varepsilon_0}$ (D) $\frac{2q}{\varepsilon_0}$

Multiple Correct Answer Type 5 Q. [4 M (–1)]

7. Two point masses, each of mass M are fixed at points A and B respectively. A third point m is released from infinity, so that it can move along y-axis under the influence of mutual gravitational attraction on it due to point masses kept at A and B respectively as shown in the figure -1. Figure -2 represents the potential energy of system (includes m, M at A and M at B) with position of m at y-axis. (Neglect any forces other than gravity on paritcle C) (Given: Gm^2/d = 12 Joule, m = 6 kg). Choose the correct option(s) :-



(A) Point mass m will perform periodic motion

(B) Value of U_1 must be equal to –24 Joule if gravitational potential energy of two point mass system is taken to be zero when seperation between them is infinite.

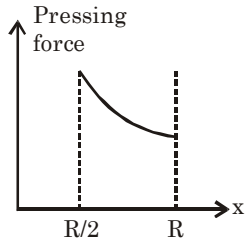
(C) Maximum speed of particle is 24 m/s

(D) Maximum speed of particle is 4 m/s

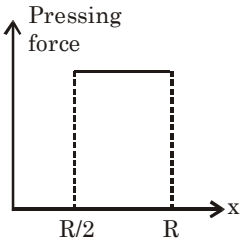
8. A smooth tunnel is dug along the chord of earth at a perpendicular distance $\frac{R}{2}$ from the centre of earth.

The pressing force by the particle on the wall and the acceleration of the particle varies with x (distance of particle from centre of earth) as :

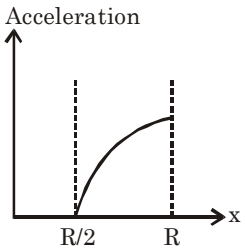
(A)



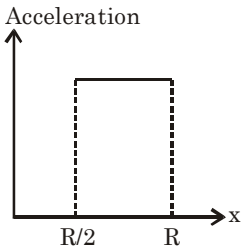
(B)



(C)



(D)



9. A spherical planet of radius R has spherically symmetrical distribution of mass density, varying as square of the distance from the centre, from zero at centre to maximum value ρ_0 at its surface.

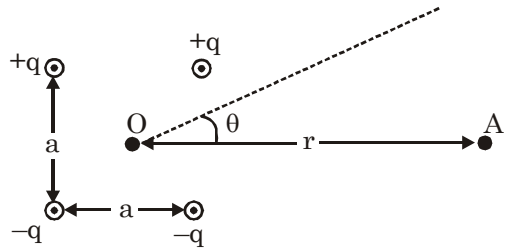
(A) The value of escape velocity of a mass m at the surface of planet is $\sqrt{\frac{4\pi G\rho_0 R^2}{5}}$.

(B) The value of acceleration due to gravity 'g' varies inside the planet as cube of the distance from centre.

(C) The value of escape velocity is same as the escape velocity from another planet of same total mass & radius but having uniform mass density.

(D) The energy required to impart escape velocity to particles of masses 'm' & '2m', at the surface of planet, will be in ratio 1:2.

10. The electric quadrupole consists of two positive and two negative point charges of equal magnitude, q, located at the vertices of a square. Find electric field at A and B in terms of polar coordinates at A (1000 a, 0°) and B (1000 a, 45°),



(A) $E_A \approx \frac{2kqa}{(1000a)^3}(-\hat{\theta})$

(B) $E_B \approx \frac{kq\sqrt{2}a}{(1000a)^3}[2\hat{r} - \hat{\theta}]$

(C) $E_A \approx \frac{2kqa}{(1000a)^3}(\hat{r})$

(D) $E_B \approx \frac{kq\sqrt{2}a}{(1000a)^3}(\hat{r} + \hat{\theta})$

11. Mark the **CORRECT** statement(s) :

(A) The electric field at a height z above the center of a square sheet of side a carrying a uniform surface charge σ is given by $\frac{\sigma}{2\in_0}\left[\frac{2}{\pi}\tan^{-1}\sqrt{1+\frac{a^2}{2z^2}}-1\right]$

(B) Potential difference between the top and the center of an inverted hemispherical bowl of radius R carrying a uniform surface charge density σ is $\frac{R\sigma}{2\in_0}\left(\sqrt{2}-1\right)$

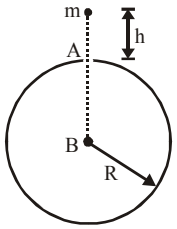
(C) A sphere of radius R carries a charge density $\rho(r) = kr$, k is some +ve constant, then energy of configuration is $\frac{\pi k^2 R^7}{7\in_0}$

(D) A point charge q is inside a cavity in an uncharged conductor, then the force on q will be necessarily zero.

Linked Comprehension Type (1 Para × 2Q.) [3 M (-1)]
(Single Correct Answer Type)

Paragraph for Question no. 12 and 13

A hollow spherical planet has mass M & radius 'R'. A small particle of mass m is released from rest from a height h above a small hole in the planet. Assume that the mass density of planet is constant.



12. What time will the particle take to move from A to B?

(A) $R\sqrt{\frac{R(R+h)}{GMh}}$ (B) $R\sqrt{\frac{2R(R+h)}{GMh}}$ (C) $R\sqrt{\frac{R(R+h)}{2GMh}}$ (D) $2R\sqrt{\frac{R(R+h)}{GMh}}$

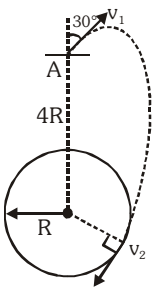
13. With what approximate speed will the particle reach B?

(A) $2\sqrt{\frac{GMh}{R(R+h)}}$ (B) $\sqrt{\frac{GMh}{2R(R+h)}}$ (C) $\sqrt{\frac{GMh}{R(R+h)}}$ (D) $\sqrt{\frac{2GMh}{R(R+h)}}$

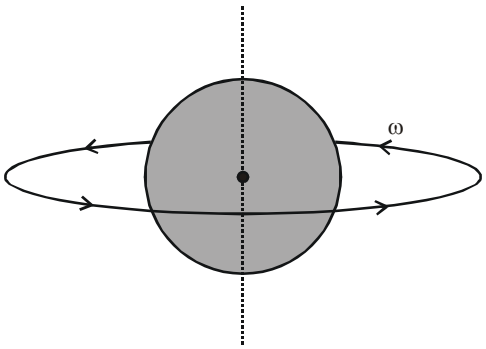
SECTION-III

Numerical Grid Type (Ranging from 0 to 9) 5 Q. [4 M (0)]

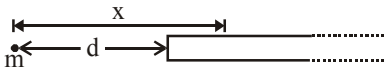
1. A particle is projected from point A, that is at a distance 4R from the centre of the Earth, with speed v_1 in a direction making 30° with the line joining the centre of the Earth and point A, as shown. Find the speed v_1 if particle passes grazing the surface of the earth. Consider gravitational interaction only between these two. (use $\frac{GM}{R} = 6.4 \times 10^7\ m^2/s^2$) Express you answer in the form $\left(500\sqrt{2}\ X\right)$ m/s and write the value of X.



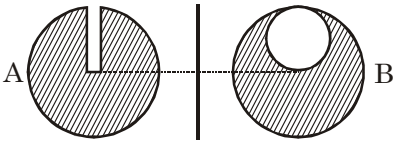
2. A ring of mass m and radius 3R is rotating with constant angular speed ω around a planet of mass M and radius R. Center of ring and planet concide with each other. Tension in the ring is given as $T = \frac{GMm}{3n\pi R^2}$. Find value of n $\left(\omega = \sqrt{\frac{GM}{9R^3}}\right)$



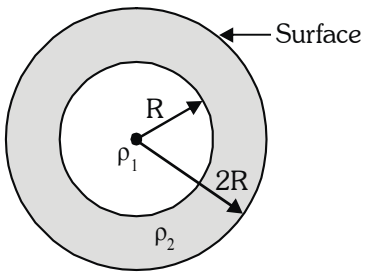
3. Find the gravitational force of interaction between the mass m and infinite rod of varying mass density λ such that $\lambda(x) = \frac{k}{x}$, where x is the distance from mass m. Given that mass m is placed at a distance d from the end of the rod on its axis as shown in the figure. If force is $\frac{Gmk}{nd^2}$, then find the value of n.



4. Nowadays ISRO is working on a space research program. In this program they discover spherical Asteroid made up titanium, a precious metal. They dig a tunnel from surface to centre to find depth of titanium. Titanium was uniformly distributed in the sphere. Now they want to mine this precious material out of the asteroid due to which cavity is formed as shown in figure B. If a piece of metal falls in the tunnel it takes time t_1 and when it falls in cavity it takes time t_2 to reach at the bottom. Ratio of time $\frac{t_1}{t_2} = \left(\frac{\pi}{n}\right)$. Find the value of 'n'.



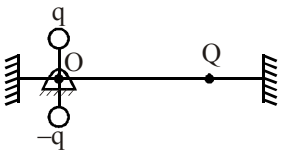
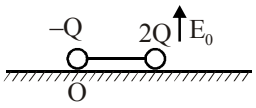
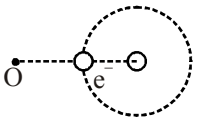
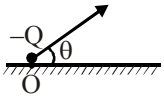
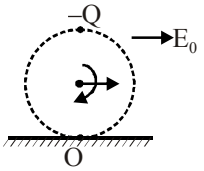
5. A planet is made of two materials of density ρ_1 and ρ_2 as shown in figure. The acceleration due to gravity at surface of planet is same as a depth 'R'. The ratio of $\frac{\rho_1}{\rho_2}$ is $\frac{a}{b}$. Find value of $\frac{a^b + 1}{86}$ (a and b are lowest possible integers)



SECTION-IV

Matrix Match Type (4 × 5) **1 Q. [8 M (for each entry +2(0)]**

1.

	Column-I		**Column-II**
(A)	Magnitude of angular momentum of system about O increases after the instant shown.	(P)	A dipole is hinged on a long smooth fixed horizontal wire. A charge Q (A) is threaded on wire and is free to move on the wire. The system comprises of the dipole and point charge. Gravity is absent. 
(B)	The net torque on body A is zero about O at the instant shown.	(Q)	A system of two charges –Q and 2Q (A) of mass m each are connected by a light rod and kept on smooth level ground. An external uniform electric field $E_0 = mg/Q$ is applied as shown. Gravity is present. 
(C)	The net force on negative charge increases in magnitude after the instant shown.	(R)	An electron (A) is revolving about a fixed proton in anticlockwise direction The system comprises of the electron and proton. Gravity is absent. 
(D)	Mechanical energy of the system increases after the instant shown.	(S)	System consists of a particle (A) of mass m and charge –Q. It is projected from level ground in presence of constant electric field $E = mg/Q$ in vertically upward direction. Gravity is present. 
		(T)	System consists of a uniform ring and particle (A) of mass m and charge –Q on its circumference as shown. The ring is in pure rolling -on a sufficiently rough horizontal surface. Gravity is present. 

CLASS TEST # 22 **ANSWER KEY**

SECTION-I

Single Correct Answer Type **6 Q. [3 M (–1)]**

1. Ans. (B) **2. Ans. (A)** **3. Ans. (D)** **4. Ans. (C)**

5. Ans. (B) **6. Ans. (B)**

Multiple Correct Answer Type **5 Q. [4 M (–1)]**

7. Ans. (A,B,D) **8. Ans. (B,C)** **9. Ans. (B, C, D)** **10. Ans. (A,B)**

11. Ans. (B,C)

Linked Comprehension Type **(1 Para × 2Q.) [3 M (-1)]**

(Single Correct Answer Type)

12. Ans. (C) **13. Ans. (D)**

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) **5 Q. [4 M (0)]**

1. Ans. 8 **2. Ans. 9** **3. Ans. 2** **4. Ans. 4**

5. Ans. 4

SECTION-IV

Matrix Match Type (4 × 5) **1 Q. [8 M (for each entry +2(0)]**

1. Ans. (A) P,Q,S (B) P,R,S (C) P,Q, (D) Q