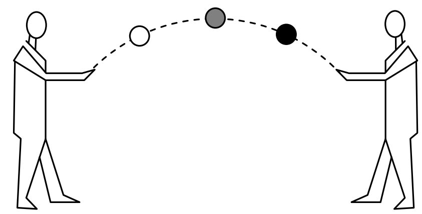
Centre of Mass and Collision

1. Introduction



Suppose a spin bowler throws a cricket ball vertically upward. Being a spinner, his fingers turn while throwing the ball and the ball goes up spinning rapidly. Focus your attention to a particular point on the surface of ball. How does it move in the space ?

All the points of the ball do not go in parabolic paths. If the ball is spinning, the paths of most of the particles of the ball are complicated. But the centre of the ball always goes in a parabola irrespective of how the ball is thrown.

The centre of the ball is a very special point which is called the *centre of mass* of the ball. Its motion is just like the motion of a single particle thrown.

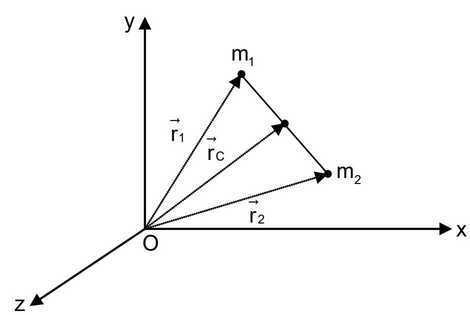
For a system of particles centre of mass is that point at which its total mass is supposed to be concentrated. The centre of mass of an object is a point that represents the entire body and moves in the same way as a point mass having mass equal to that of the object, when subjected to the same external forces that act on the object.

2. Centre of Mass

Through observations it has been revealed that every physical system or body has associated with a certain point whose motion represents the translatory motion of the system or body. This point is called centre of mass, i.e., centre of mass of a body or system is a point where the whole mass of body or system is supposed to be concentrated for dealing its translatory motion.

3. Centre of Mass of Discrete Particle System

(A) Centre of Mass of a two Particles System (Vector form)



Consider two particles of masses m_1 and m_2 with position vectors $\vec{r}_1$ and $\vec{r}_2$ respectively. Let their centre of mass C have position vector $\vec{r}_c$.

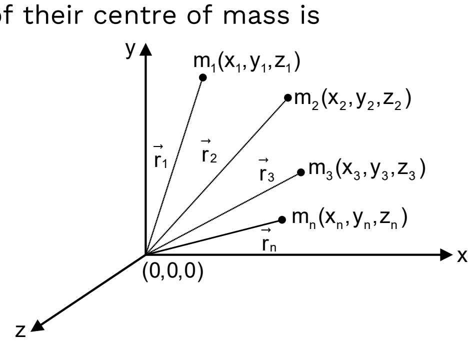
From definition, we have

$$\vec{r}_c = \frac{\Sigma m_1 \vec{r}_1}{M} \Rightarrow \vec{r}_c = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

Coordinate Form : If the co-ordinates of particles of mass m_1, m_2, ... are respectively

(x_1, y_1, z_1), (x_2, y_2, z_2)....

then position vector of their centre of mass is



$$\vec{R}_{CM} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$= \frac{m_1(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})+m_2(x_2\hat{i}+y_2\hat{j}+z_2\hat{k})+m_3(x_3\hat{i}+y_3\hat{j}+z_3\hat{k})}{m_1+m_2+m_3+....}$$

$$= \frac{(m_1x_1+m_2x_2+...)\hat{i}+(m_1y_1+m_2y_2....)\hat{j}+(m_1z_1+m_2z_2+....)\hat{k}}{m_1+m_2+m_3+....}$$

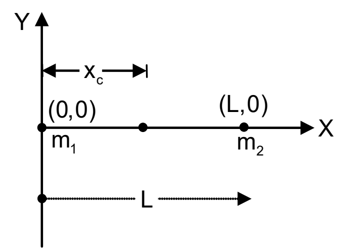
So, $x_{cm} = \left(\frac{m_1x_1+m_2x_2+.......}{m_1+m_2+m_3+......}\right)$,

$y_{cm} = \left(\frac{m_1y_1+m_2y_2+.......}{m_1+m_2+............}\right)$,

$z_{cm} = \left(\frac{m_1z_1+m_2z_2+.......}{m_1+m_2+............}\right)$

Example 1:

Calculate the position of the centre of mass of a system consisting of two particles of masses m_1 and m_2 separated by a distance L, in relative to m_1.



Solution:

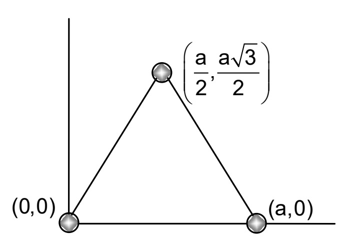
Treating the line joining the two particles as x axis

$x_{CM} = \frac{m_1\times 0+m_2\times L}{m_1+m_2} = \frac{m_2L}{m_1+m_2}$, $y_{CM} = 0$

$z_{CM} = 0$

Example 2:

Three bodies of equal masses are placed at (0, 0), (a, 0) and at $\left(\frac{a}{2},\frac{a\sqrt{3}}{2}\right)$. Find out the co-ordinates of centre of mass.



Solution:

$$x_{CM} = \frac{0\times m+a\times m+\frac{a}{2}\times m}{m+m+m} = \frac{a}{2},$$

$$y_{CM} = \frac{0\times m+0\times m+\frac{a\sqrt{3}}{2}\times m}{m+m+m} = \frac{a\sqrt{3}}{6}$$

Example 3:

Find the position vector of centre of mass of a system of three particles of masses 1kg, 2kg and 3kg located at position vectors $\vec{r}_1(4\hat{i}+2\hat{j}-3\hat{k})m$, $\vec{r}_2(\hat{i}-4\hat{j}+2\hat{k})m$ and $\vec{r}_3(2\hat{i}-2\hat{j}+\hat{k})m$ respectively.

Solution:

From eq. corresponding to com, we have

$$\vec{r}_c = \frac{\sum m_i\vec{r}_i}{M}$$

$$\vec{r}_c = \frac{1(4\hat{i}+2\hat{j}-3\hat{k})+2(\hat{i}-4\hat{j}+2\hat{k})+3(2\hat{i}-2\hat{j}+\hat{k})}{1+2+3}$$

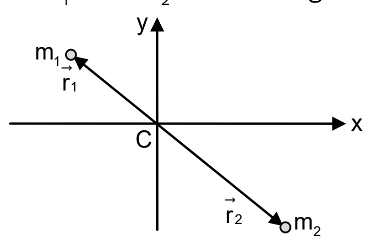
$$= \left(2\hat{i}-2\hat{j}+\frac{2}{3}\hat{k}\right)m$$

(B) Centre of Mass of two Particle System

- If Centre of Mass of two particle system is assumed to be at origin. If we assume origin to be at the centre of mass, then the vector $\vec{r}_c$ vanishes and we have

  $$\vec{r}_c = \frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2} \quad \text{so,} \qquad m_1\vec{r}_1+m_2\vec{r}_2 = \vec{0}$$

  Since neither of the masses m_1 and m_2 can be negative, to satisfy

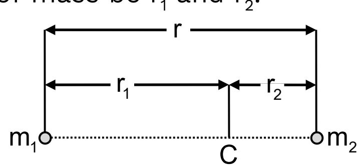
  the above equation, vectors $\vec{r}_1$ and $\vec{r}_2$ must have opposite signs. It is geometrically possible only when the centre of mass C lies between the two particles on the line joining them as shown in the figure.

  If we substitute magnitudes r_1 and r_2 of vectors $\vec{r}_1$ and $\vec{r}_2$ in the above equation, we have

  $$m_1r_1 = m_2r_2 \text{ or } \frac{r_1}{r_2} = \frac{m_2}{m_1}.$$

  We conclude that the centre of mass of the two particles system lies between the two particles on the line joining them which divides the distance between them in the inverse ratio of their respective masses.

  Consider two particles of masses m_1 and m_2 at a distance r from each other. Their centre of mass C must lie in between them on the line joining them. Let the distances of these particles from the centre of mass be r_1 and r_2.

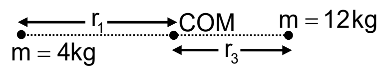
  Since centre of mass of a two particles system lies between the two particles on the line joining them which divides the distance between them in the inverse ratio of masses of the particles, we can write

  $$\boxed{r_1 = \frac{m_2r}{m_1+m_2}} \text{ and } \boxed{r_2 = \frac{m_1r}{m_1+m_2}}$$

Example 4:

Two masses 4 kg & 12 kg are placed 20 m apart. What is the distance of COM from 12 kg mass

Solution:



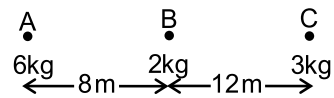
$4r_1 = 12r_2 \Rightarrow \frac{r_1}{r_2} = 3,$

also $r_1 + r_2 = 20$ m $\Rightarrow 3r_2 + r_2 = 20$ m

or $r_2 = 5$m & $r_1 = 15$ m.

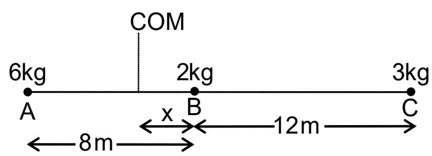
Example 5:

For the following arrangement, what is the distance of COM from 3 kg mass.



Solution:

Let us assume COM is xm towards A from B then



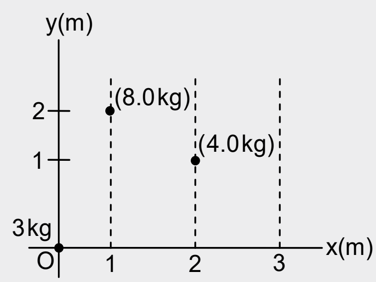
$6(8 - x) = 2(x) + 3(12 + x)$

$\Rightarrow \quad x = \frac{12}{11}$ m

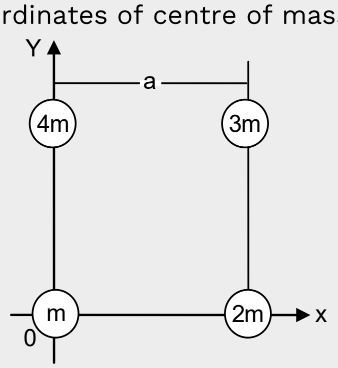
So distance from 3kg mass $= 12 + \frac{12}{11} = \frac{144}{11}$ m

Concept Builder-1

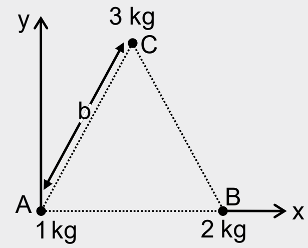
Q.1 What are the co-ordinates of the centre of mass of the three particles system shown in figure?



Q.2 Four particles of masses m, 2m, 3m, 4m are placed at the corners of a square of side 'a' as shown in fig. Find out the co-ordinates of centre of mass.

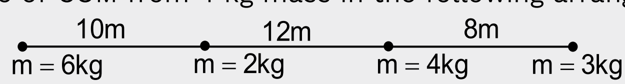


Q.3 Locate the centre of mass of a system of particles of masses 1kg, 2kg and 3kg situated at the corners of an equilateral triangle of side b as shown in figure.

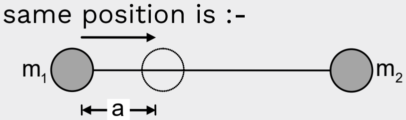


Q.4 A rigid body consists of a 3 kg mass connected to a 2 kg mass by a massless rod. The 3 kg mass is located at $\vec{r}_1\left(2\hat{i}+5\hat{j}\right)$ m and the 2 kg mass at $\vec{r}_2\left(4\hat{i}+2\hat{j}\right)$ m. Find the coordinate of the centre of mass.

Q.5 What is the distance of COM from 4 kg mass in the following arrangement ?



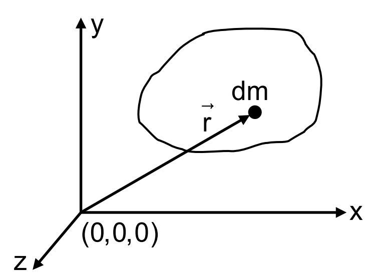
Q.6 Consider a system having two masses m_1 and m_2 in which first mass is pushed towards the centre of mass by a distance a. The distance by which the second mass should be moved to keep the centre of mass at same position is :-



(1) $\frac{m_1}{m_2}a$ (2) $\frac{m_1}{(m_1+m_2)}a$ (3) $\frac{m_2}{m_1}a$ (4) $\left(\frac{m_2}{m_1+m_2}\right)a$

4. Centre of Mass of Continuous Distribution of Mass

If a system has continuous distribution of mass, treating the mass element dm at position $\vec{r}$ as a point mass and replacing summation by integration.

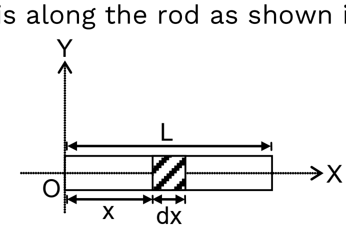


$$\vec{R}_{CM} = \frac{1}{M}\int \vec{r}dm \; ; \text{ where } M = \int dm$$

So that $x_{cm} = \frac{1}{M}\int xdm$, $y_{cm} = \frac{1}{M}\int ydm$ and $z_{cm} = \frac{1}{M}\int zdm$

- **Centre of Mass of a Uniform Straight Rod**

  Let M and L be the mass and the length of the rod respectively. Take the left end of the rod as the origin and the X-axis along the rod as shown in figure.



Consider an element of the rod between the positions x and x + dx. If x = 0, the element is at the left end. So as x varies from 0 through L, the elements cover the entire rod.

The x-coordinate of the centre of mass of the rod is

$$X = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^L x\left(\frac{M}{L}dx\right)$$

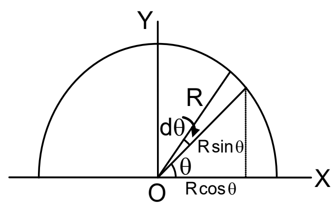
$$= \frac{1}{L}\left[\frac{x^2}{2}\right]_0^L = \frac{L}{2}$$

The y-coordinate is

$$Y = \frac{1}{M}\int y\,dm = 0$$

and similarly Z = 0. The centre of mass is at $\left(\frac{L}{2}, 0, 0\right)$, i.e., at the middle point of the rod.

- **Centre of Mass of a Uniform Semicircular Wire**



Let M be the mass and R the radius of a uniform semicircular wire. Take its centre as the origin, the line joining the ends as the X-axis, and the Y-axis in the plane of the wire (figure). The centre of mass must be in plane of the wire i.e., in the X-Y plane.

As the wire is uniform, the mass per unit length of the wire is $\frac{M}{\pi R}$. The mass of the element is, therefore,

$$dm = \left(\frac{M}{\pi R}\right)(R\,d\theta) = \frac{M}{\pi}d\theta$$

The coordinates of the centre of mass are

$$X = \frac{1}{M}\int x\,dm$$

$$= \frac{1}{M}\int_0^\pi (R\cos\theta)\left(\frac{M}{\pi}\right)d\theta = 0$$

and

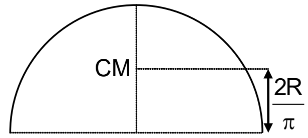
$$Y = \frac{1}{M}\int y\,dm$$

$$= \frac{1}{M}\int_0^\pi (R\sin\theta)\left(\frac{M}{\pi}\right)d\theta = \frac{2R}{\pi}$$

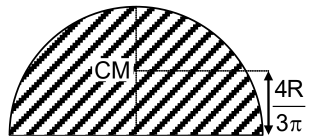
The centre of mass is at $\left(0, \frac{2R}{\pi}\right)$

Centre of mass of some uniform symmetric bodies are

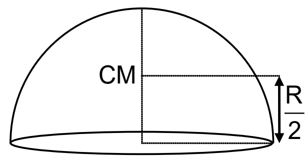
(i) Semicircular ring of radius R



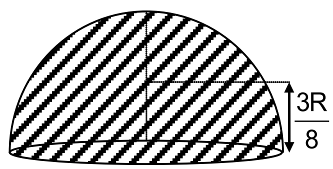
(ii) Semicircular disc



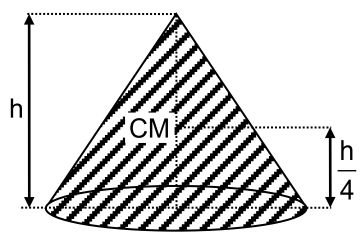
(iii) Hemispherical shell



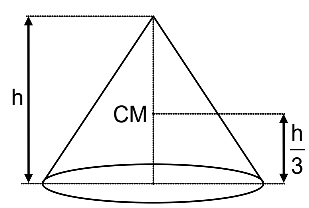
(iv) Solid hemisphere



(v) Solid cone

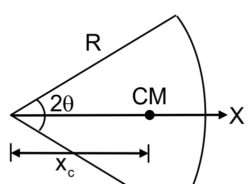


(vi) Hollow cone



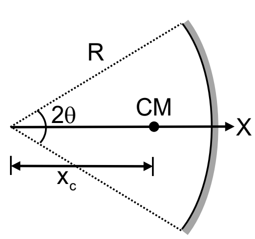
(vii) Circular arc

$$x_c = \frac{R\sin\theta}{\theta}$$



(viii) Sector of a circular plate

$$x_c = \frac{2R\sin\theta}{3\theta}$$



Note : Here θ is in radians.

Example 6:

If the linear density of a rod of length L varies as $\lambda = A + Bx$, determine the position of its centre of mass. (where x is the distance from one of its ends)

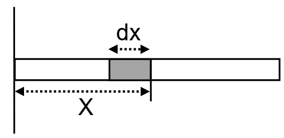
Solution:

Let the x-axis be along the length of the rod with origin at one of its end as shown in figure. As the rod is along x-axis, so $y_{CM} = 0$ and $z_{CM} = 0$ i.e., centre of mass will be on the rod.

Now consider an element of rod of length dx at a distance x from the origin, mass of this element

$dm = \lambda dx = (A + Bx)dx$ so,

$$x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(A+Bx)\,dx}{\int_0^L (A+Bx)\,dx} = \frac{\frac{AL^2}{2} + \frac{BL^3}{3}}{AL + \frac{BL^2}{2}}$$



$$= \frac{L(3A+2BL)}{3(2A+BL)}$$

Note : (i) If the rod is of uniform density then $\lambda = A =$ constant & $B = 0$ then $x_{CM} = L/2$

(ii) If the density of rod varies linearly with x, then $\lambda = Bx$ and $A = 0$ then $x_{CM} = 2L/3$

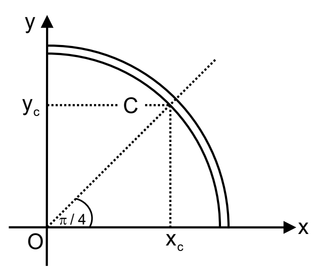
Example 7:

Find coordinates of centre of mass of a quarter ring of radius r placed in the first quadrant of a Cartesian coordinate system, with centre at origin.

Solution:

Making use of the result of circular arc, distance OC of the centre of mass from the centre is

$OC = \frac{r\sin(\pi/4)}{\pi/4} = \frac{2\sqrt{2}r}{\pi}$. Coordinates of the centre of mass (x_c, y_c) are $\left(\frac{2r}{\pi}, \frac{2r}{\pi}\right)$.



Example 8:

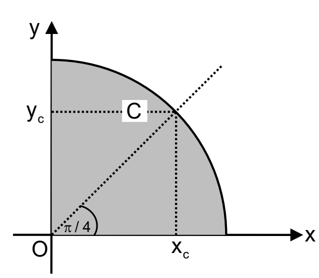
Find coordinates of centre of mass of a quarter sector of a uniform disc of radius r placed in the first quadrant of a Cartesian coordinate system with centre at origin.

Solution:

From the result obtained for sector of circular plate distance OC of the centre of mass form the centre is

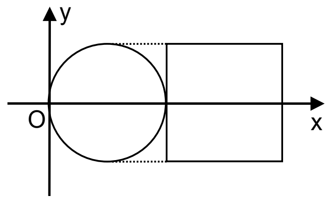
$$OC = \frac{2r\sin(\pi/4)}{3\pi/4} = \frac{4\sqrt{2}r}{3\pi}$$

Coordinates of the centre of mass (x_c, y_c) are $\left(\frac{4r}{3\pi}, \frac{4r}{3\pi}\right)$



5. Centre of Mass of Composite Bodies

In order to find the centre of mass, the component bodies are assumed to be particles of masses equal to the corresponding bodies located at their respective centres of masses. Then we use the equation to find the coordinates of the centre of mass of the composite body.



To find the centre of mass of the composite body, we first have to calculate the masses of the bodies, because their mass distribution is given.

If we denote the surface mass density (mass per unit area) by σ then the masses of the bodies assumed to be uniform are

Mass of the disc

m_d = Mass per unit area × Area = $\sigma(A_d)$

Mass of the square plate

m_s = Mass per unit area × Area = $\sigma(A_s)$

Location of centre of mass of the disc = (x_d, y_d)

Location of centre of mass of the square plate = (x_s, y_s)

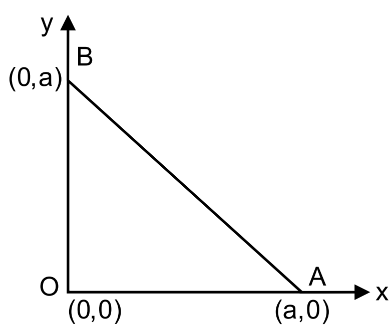
Using eq. corresponding to centre of mass, we obtain its coordinates (x_c, y_c) of the composite body.

$$x_c = \frac{m_d x_d + m_s x_s}{m_d + m_s} = \frac{A_d x_d + A_s x_s}{A_d + A_s} \quad [\text{if } \sigma = \text{same}]$$

and $$y_c = \frac{m_d y_d + m_s y_s}{m_d + m_s} = \frac{A_d y_d + A_s y_s}{A_d + A_s}$$

Example 9:

Three rods of the same mass are placed as shown in the figure. Calculate the coordinates of the centre of mass of the system.



Solution:

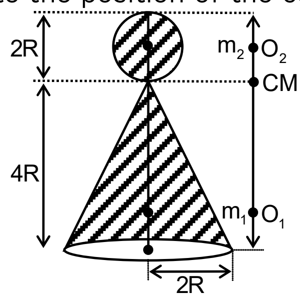
COM of rod OA is at $\left(\frac{a}{2}, 0\right)$, COM of rod OB is at $\left(0, \frac{a}{2}\right)$ and COM of rod AB is at $\left(\frac{a}{2}, \frac{a}{2}\right)$

For the system,

$$x_{cm} = \frac{m \times \frac{a}{2} + m \times 0 + m \times \frac{a}{2}}{m + m + m} = \frac{a}{3} \qquad \Rightarrow y_{cm} = \frac{m \times 0 + m \times \frac{a}{2} + m \times \frac{a}{2}}{m + m + m} = \frac{a}{3}$$

Example 10:

A carpenter has constructed a toy as shown in figure. If the density of the material of the sphere is 12 times that of cone, compute the position of the centre of mass of the toy.



Solution:

If the density of cone be ρ, then its mass will be $m_1 = \frac{1}{3}\pi\ (2R)^2\ (4R)\rho = \frac{16}{3}\pi R^3\rho$ and its centre of mass O_1 will be at a height $\left(\frac{h}{4}\right) = \left(\frac{4R}{4}\right) = R$ from O on the line of symmetry, i.e., $y_1 = R$.

Similarly, the mass of the sphere $m_2 = \frac{4}{3}\pi R^3\ (12\rho) = 16\pi R^3\rho = 3m_1$ and its centre of mass will be at its centre O_2, i.e., $y_2 = 5R$.

Now treating sphere and cone as point mass with their masses concentrated at their centre of mass respectively and taking the line of symmetry as y-axis with origin at O, for the centre of mass of the toy

$$Y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{m_1R + 3m_1 \times 5R}{m_1 + 3m_1}$$

i.e., centre of mass of the toy is at a distance 4R from O on the line of symmetry, i.e., at the apex of the cone.

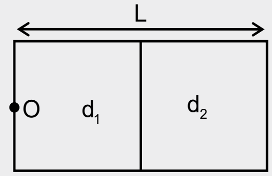
Concept Builder-2

Q.1 The centre of mass of a non-uniform rod of length L whose mass per unit length ρ varies as $\rho = \frac{k \cdot x^2}{L}$ where k is a constant and x is the distance of any point from one end is :

(1) $\frac{3}{4}L$ (2) $\frac{1}{4}L$ (3) $\frac{kL}{4}$ (4) $\frac{3kL}{4}$

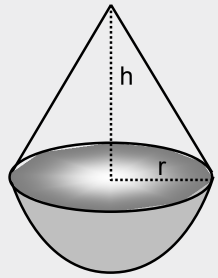
Q.2 Find coordinates of centre of mass of a semicircular ring of radius r placed symmetric to the y-axis of a Cartesian coordinate system.

Q.3 Shows a rectangular plate of length L, the half of which is made of material of density d_1 and another half of density d_2. The distance of centre of mass from the origin O is :



(1) $\frac{d_1 + 2d_2}{2(d_1 + d_2)}L$ (2) $\frac{(d_1 + 3d_2)L}{4(d_1 + d_2)}$ (3) $\frac{(d_1 + 3d_2)L}{2(d_1 + d_2)}$ (4) $\frac{(3d_1 + d_2)L}{4(d_1 + d_2)}$

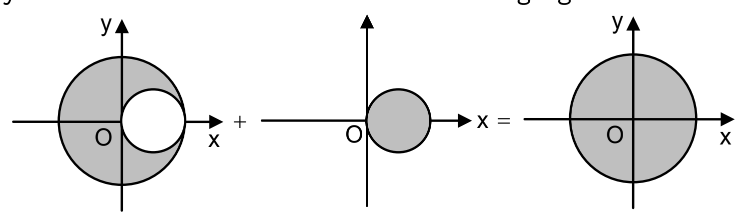
Q.4 A uniform solid right circular cone of base radius r is joined to a uniform solid hemisphere of radius r and of the same density, so as to have a common face. The centre of mass of the composite solid lies on the common face. The height of the cone is :



(1) $\sqrt{3}r$ (2) 4r (3) 3r (4) $r\sqrt{6}$

6. Centre of Mass of Truncated Bodies

To find the centre of mass of truncated bodies or bodies with cavities, we can make use of superposition principle that is, if we restore the removed portion in the same place we obtain the original body. The idea is illustrated in the following figure.



The removed portion is added to the truncated body keeping their location unchanged relative to the coordinate frame.

If a portion of a body is taken out, the remaining portion may be considered as,

[Original mass (M) – mass of the removed part (m)] = {original mass (M)} + {– mass of the removed part (m)}

The formula changes to :

$$x_{cm} = \frac{Mx - mx'}{M - m} \; ; \; y_{cm} = \frac{My - my'}{M - m} ; \; z_{cm} = \frac{Mz - mz'}{M - m}$$

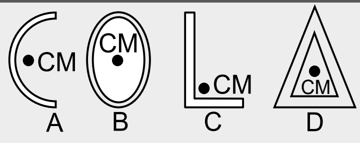
Where x', y' and z' represent the coordinates of the centre of mass of the removed part.

Centre of gravity

Centre of gravity of a body is that point where it is assumed that the gravitational force of earth on body i.e. weight of body acts on it.

In normal cases, if the acceleration due to gravity remains the same throughout the mass distribution then centre of gravity coincides with the centre of mass and both in turn coincide with the geometrical centre of the body.

Key Points



- There may or may not be any mass present physically at the centre of mass (See figure A, B, C)
- Centre of mass may be inside or outside a body (See figure A, B, C)
- Position of centre of mass depends on the shape of the body. (See figure A, B, C)

- For a given shape, it depends on the distribution of mass within the body and is closer to massive portion. (See figure A, C)
- For symmetrical bodies having homogeneous distribution of mass it coincides with the centre of symmetry or the geometrical centre. (See figure B, D).
- If we know the centre of mass of parts of the system and their masses, we can get the combined centre of mass by treating the parts as particles placed at their respective centre of masses.
- It is independent of the co-ordinate system, e.g., the centre of mass of a ring is at its centre whatever be the co-ordinate system.

Example 11:

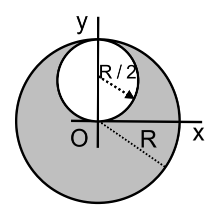
A disc of radius R is cut off from a uniform thin sheet of metal. A circular hole of radius $\frac{R}{2}$ is now cut out from the disc, with the hole being tangent to the rim of the disc. Find the distance of the centre of mass from the centre of the original disc.

Solution:

We treat the hole as a 'negative mass' object that is combined with the original uncut disc. (When the two are overlapped together, the hole region then has zero mass). By symmetry, the CM lies along the +y-axis in figure, so $x_{CM} = 0$. With the origin at the centre of the original circle whose mass is assumed to be m. Mass of original uncut circle $m_1 = m$ & Location of CM = (0, 0)

Mass of hole of negative mass : $m_2 = \frac{m}{4}$; Location of CM = $\left(0, \frac{R}{2}\right)$

Thus



$$y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{m(0) + \left(-\frac{m}{4}\right)\frac{R}{2}}{m + \left(-\frac{m}{4}\right)} = -\frac{R}{6}$$

So the centre of mass is at the point $\left(0, -\frac{R}{6}\right)$

Thus, the required distance is R/6.

7. Motion of Centre of Mass

As for a system of particles, position of centre of mass is given by

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3 + ...}$$

So, $\frac{d}{dt}(\vec{R}_{CM})$

$$= \frac{m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + m_3\frac{d\vec{r}_3}{dt} + ...}{m_1 + m_2 + m_3 + ...}$$

- **Velocity of centre of mass**

$$\vec{v}_{CM} = \frac{d\vec{R}_{CM}}{dt} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ...}{m_1 + m_2 + ...}$$

Similarly

- **Acceleration of centre of mass**

$$\vec{a}_{CM} = \frac{d}{dt}(\vec{v}_{CM}) = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...}$$

We can write

$$M\vec{v}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + ...$$

$$= \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ... [\because \vec{p} = m\vec{v}]$$

$$M\vec{v}_{CM} = \vec{p}_{CM} \qquad [\because \Sigma\vec{p} = \vec{p}_{CM}]$$

Linear momentum of a system of particles is equal to the product of mass of the system with velocity of its centre of mass. From Newton's second law

$$\vec{F}_{ext.} = \frac{d(M\vec{v}_{CM})}{dt}$$

If $\vec{F}_{ext} = \vec{0}$ then $\vec{v}_{CM}$ = constant

If no net external force acts on a system the velocity of its centre of mass remains constant, i.e., velocity of centre of mass is unaffected by internal forces.

Example 12:

Two particles of masses 1 kg and 0.5 kg are moving in the same direction with speeds of 2 m/s and 6 m/s, respectively, on a smooth horizontal surface. Find the speed of the centre of mass of the system.

Solution:

Velocity of centre of mass of the system $\vec{v}_{CM} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$. Since the two particles are moving in same direction, $m_1\vec{v}_1$ and $m_2\vec{v}_2$ are parallel.

$$\Rightarrow \quad |m_1\vec{v}_1 + m_2\vec{v}_2| = m_1v_1 + m_2v_2$$

Therefore,

$$v_{cm} = \frac{|m_1\vec{v}_1 + m_2\vec{v}_2|}{m_1 + m_2} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(1)(2) + \left(\frac{1}{2}\right)(6)}{\left(1 + \frac{1}{2}\right)} = 3.33 \text{ ms}^{-1}$$

Example 13:

Two bodies of masses 10 kg and 2 kg are moving with velocities $\left(2\hat{i} - 7\hat{j} + 3\hat{k}\right)$ m/s and $\left(-10\hat{i} + 35\hat{j} - 3\hat{k}\right)$ m/s respectively. Find the velocity of their centre of mass.

Solution:

$$V_{CM} = \frac{10(2\hat{i} - 7\hat{j} + 3\hat{k}) + 2(-10\hat{i} + 35\hat{j} - 3\hat{k})}{10 + 2} = 2\hat{k} \text{ m/s}$$

Example 14:

Two particles of masses 2 kg and 4 kg are approaching towards each other with accelerations of 1 m/s^2 and 2 m/s^2 respectively, on a smooth horizontal surface. Find the acceleration of centre of mass of the system.

Solution:

The acceleration of centre of mass of the system

$$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$$

$$\Rightarrow a_{CM} = \frac{|m_1\vec{a}_1 + m_2\vec{a}_2|}{m_1 + m_2}$$

Since $\vec{a}_1$ and $\vec{a}_2$ are anti-parallel, so

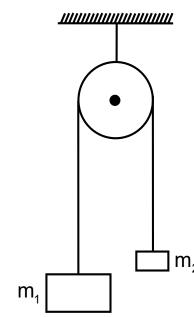
$$a_{CM} = \frac{|m_1a_1 - m_2a_2|}{m_1 + m_2} = \frac{|(2)(1)-(4)(2)|}{2+4}$$

Since $m_2a_2 > m_1a_1$ so the direction of acceleration of centre of mass is along in the direction of a_2.

Example 15:

Two bodies of masses m_1 and m_2 ($<m_1$) are connected to the ends of a massless cord and allowed to move as shown in figure. The pulley is massless and frictionless. Calculate the acceleration of the centre of mass.

Solution:



If $\vec{a}$ is the acceleration of m_1, then $-\vec{a}$ is the acceleration of m_2 then

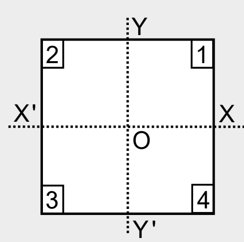
$$\vec{a}_{cm} = \frac{m_1\vec{a} + m_2(-\vec{a})}{m_1 + m_2} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{a}$$

But $\vec{a} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{g}$ so $\vec{a}_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2\vec{g}$

Concept Builder-3

Q.1 Find the centre of mass of a uniform disc of radius 'a' from which a circular section of radius 'b' has been removed. The centre of the hole is at a distance c from the centre of the disc.

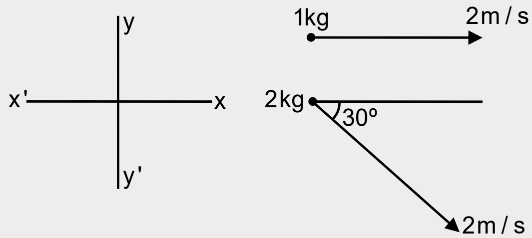
Q.2 Figure shows a uniform square plate from which one or more of the four identical squares at the corners will be removed.



(a) Where is the centre of mass of the plate originally.
(b) Where is the C.M. after square 1 is removed.
(c) Where is the C.M. after square 1 and 2 removed.
(d) Where is the C.M. after square 1 and 3 are removed.
(e) Where is the C.M. after square 1, 2 and 3 are removed.
(f) Where is the C.M. after all the four squares are removed.
Give your answers in terms of quadrants and axis.

Q.3 Two blocks of masses 5 kg and 2 kg placed on a frictionless surface are connected by a spring. An external kick gives a velocity of 14 m/s to the heavier block in the direction of the lighter one. Calculate the velocity gained by the centre of mass.

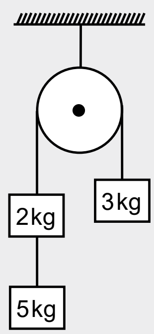
Q.4 The velocity of centre of mass of the system as shown in the figure is :-



(1) $\left(\frac{2-2\sqrt{3}}{3}\right)\ \hat{i}-\frac{1}{3}\hat{j}$ (2) $\left(\frac{2+2\sqrt{3}}{3}\right)\hat{i}-\frac{2}{3}\hat{j}$ (3) $4\hat{i}$ (4) None of these

Q.5 Three particles of masses 1 kg, 2 kg and 3 kg are subjected to forces $\left(3\hat{i}-2\hat{j}+2\hat{k}\right)$ N, $\left(-\hat{i}+2\hat{j}-\hat{k}\right)$ N and $\left(\hat{i}+\hat{j}+\hat{k}\right)$ N respectively. Find the magnitude of the acceleration of the CM of the system.

Q.6 Calculate the acceleration of Centre of Mass of system.



8. Momentum & Impulse, Conservation of Linear Momentum

8.1 Momentum

Linear momentum of a particle is a vector physical quantity associated with state of motion and is defined as product of mass of the particle (m) with its velocity (not speed), i.e.,

$$\vec{p} = m\vec{v}$$

Regarding linear momentum it is worth noting that :

(i) Linear momentum is vector physical quantity having direction of velocity, dimensions $[MLT^{-1}]$ and units kg m/s or N-s.

(ii) Linear momentum depends on frame of reference e.g., the linear momentum of a body at rest in a moving train is zero relative to a person sitting in the train while is not zero for a person standing on the ground.

(iii) Two bodies of same mass and moving with same speed will have different momenta unless their directions of motion are same.

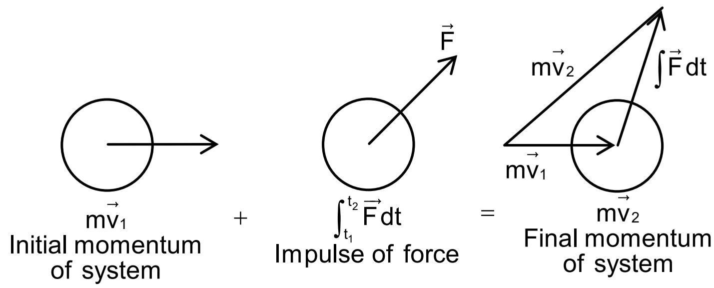
8.2 Impulse – Momentum Theorem

Newton's law may be expressed in the form

$$\vec{F} = \frac{d}{dt}(m\vec{v}) \text{ or } \vec{F}\,dt = d(m\vec{v})$$

or $$\int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$$

Above equation is referred to as impulse momentum equation.



It reveals that: "The impulse exerted on an object equals its change in momentum."

8.3 Conservation of Linear Momentum

Total linear momentum of a system of particles remains conserved in a time interval in which impulse of external forces is zero.

Total momentum of a system of particles cannot change under the action of internal forces and if net impulse of the external forces in a time interval is zero, the total momentum of the system in that time interval will remain conserved.

$$\vec{p}_{final} = \vec{p}_{initial}$$

The above statement is known as the principle of conservation of momentum.

Since force, impulse and momentum are vectors, component of momentum of a system in a particular direction is conserved, if net impulse of all external forces in that direction vanishes.

No external force ⇒ Stationary mass relative to an inertial frame remains at rest

8.4 Application of Conservation of Linear Momentum

(a) Firing of Bullet from Gun

If the bullet is the system, the force exerted by trigger will be external and so the linear momentum of the bullet will change from 0 to mv. This is not the violation of the law of conservation of linear momentum as linear momentum is conserved only in the absence of external force.

If the bullet and gun is the system, then the force exerted by trigger will be internal so. total momentum of the system

$\vec{p}_S = \vec{p}_B + \vec{p}_G$ = constant ....(i)

Now, as initially both bullet and gun are at rest so $\vec{p}_B + \vec{p}_G = \vec{0}$. From this it is evident that : $\vec{p}_G = -\vec{p}_B$, i.e., if bullet acquires forward momentum, the gun will acquire equal and opposite (backward) momentum.

From (i) $m\vec{v} + M\vec{V} = \vec{0}$ i.e., $\vec{V} = -\frac{m}{M}\vec{v}$ i.e., if the bullet moves forward, the gun 'recoils' or 'kicks backwards'. Heavier the gun lesser will be the recoil velocity V.

Kinetic energy K = $\frac{p^2}{2m}$ and $|\vec{p}_B| = |\vec{p}_G| = p$. Kinetic energy of gun $K_G = \frac{p^2}{2M}$

Kinetic energy of bullet

$K_B = \frac{p^2}{2m} \therefore \frac{K_G}{K_B} = \frac{m}{M} < 1$ ($\because$ M >> m).

Thus kinetic energy of gun is lesser than that of bullet i.e., kinetic energy of bullet and gun will not be equal. Initial kinetic energy of the system is zero as both are at rest. Final kinetic energy of the system is greater than zero.

So, here kinetic energy of the system is not constant but increases. If PE is assumed to be constant then Mechanical energy = (kinetic energy + potential energy) will also increase. However, energy is always conserved. Here chemical energy of gun powder is converted into KE.

Example 16:

Two stars of mass M & 4M, initially at rest, start moving towards each other. Due to their mutual attraction. At any instant the velocity of first star is v_0. Then find out

(A) Velocity of second star

(B) Velocity of their COM

(C) Total increase in their kinetic energy

(D) Ratio of their kinetic energies.

Solution:

Since both stars moves due to mutual attraction so F_{ext} on the system is zero Hence

(A) $M v_0 = 4M v' \Rightarrow v' = \frac{v_0}{4}$

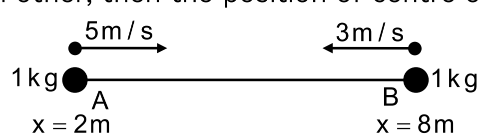
(B) v_{COM} is zero as the COM was initially at rest

(C) $K_{Total} = \frac{1}{2} M v_0^2 + \frac{1}{2} (4M) \left(\frac{v_0}{4}\right)^2 = \frac{5}{8} M v_0^2$

(D) $\frac{K_1}{K_2} = \frac{\frac{1}{2} M (v_0)^2}{\frac{1}{2}(4M)\left(\frac{v_0}{4}\right)^2} = \frac{4}{1}$

Example 17:

The figure shows the positions and velocities of two particles. If the particles move under the mutual attraction of each other, then the position of centre of mass at t = 1 s is :-



(1) x = 5 m (2) x = 6 m (3) x = 3 m (4) x = 2 m

Solution:

After 1 second.

$x_A = 2 + 5(1) = 7$ m

$x_B = 8 - 3(1) = 5$ m

Location of COM

$x_{CM} = \frac{1(7) + 1(5)}{2} = 6$ m

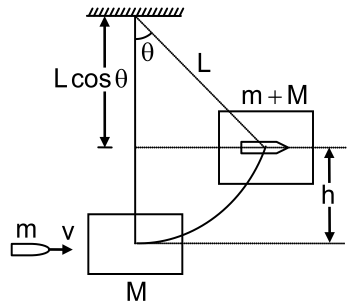
(b) Bullet Block System

- **When bullet remains embedded in the block**

  Conserving momentum of bullet and block

  mv +0 = (M + m) V

  Velocity of block $V = \frac{mv}{M+m}$ ......(i)



by conservation of mechanical energy

$\frac{1}{2}$ (M + m) V^2 = (M + m)gh ⇒ $V = \sqrt{2gh}$ .....(ii)

From equation (i) and equation (ii)

$$\frac{mv}{M+m} = \sqrt{2gh}$$

Speed of bullet v = $\frac{(M+m)\sqrt{2gh}}{m}$,

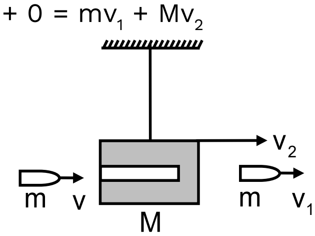
Maximum height gained by block

$$h = \frac{V^2}{2g} = \frac{m^2v^2}{2g(M+m)^2} \qquad \because h = L - L\cos\theta$$

$\therefore \cos\theta = 1 - \frac{h}{L} \quad \Rightarrow \theta = \cos^{-1}\left(1 - \frac{h}{L}\right)$

- **If bullet emerges out of the block**

  Conserving momentum mv + 0 = $mv_1 + Mv_2$



$m(v - v_1) = Mv_2$ ....(i)

Conserving energy $\frac{1}{2}Mv_2^2 = Mgh$

⇒ $v_2 = \sqrt{2gh}$ ....(ii)

From equation (i) and equation (ii)

$m(v - v_1) = M\sqrt{2gh}$

⇒ $h = \frac{m^2(v-v_1)^2}{2gM^2}$

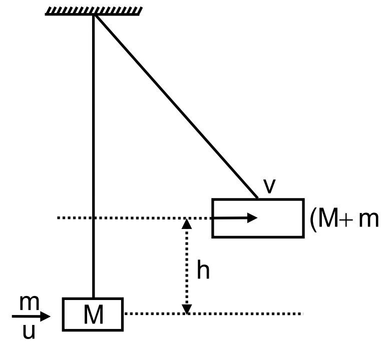
Example 18:

A simple pendulum of length 1m has a wooden bob of mass 1kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of 2×10^2 m/s. The bullet gets embedded within the bob. Obtain the height to which the bob rises before swinging back.

Solution:

Applying principle of conservation of linear momentum

mu = (M + m) v



$\Rightarrow \quad 10^{-2} \times (2 \times 10^2) = (1 + .01)\ v$

$\Rightarrow \quad v = \dfrac{2}{1.01}$ m/s

Initial KE of the block with bullet in it, is fully converted into PE as it rises through a height h, given by

$\dfrac{1}{2}(M + m)v^2 = (M + m)gh$

$\Rightarrow \quad v^2 = 2gh$

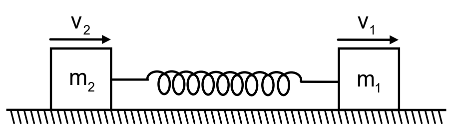
$\Rightarrow \quad h = \dfrac{v^2}{2g} = \left(\dfrac{2}{1.01}\right)^2 \times \dfrac{1}{2 \times 9.8} = 0.2$ m

(c) Two Masses Connected with a Spring

Consider two blocks resting on a frictionless surface and connected by a massless spring as shown in figure. If the spring is stretched (or compressed) and then released from rest,

Then $F_{ext} = 0$ so $\vec{p}_S = \vec{p}_1 + \vec{p}_2 =$ constant

However initially both the blocks were at rest so, $\vec{p}_1 + \vec{p}_2 = \vec{0}$



It is clear that :

- $\vec{p}_2 = -\vec{p}_1$, i.e., at any instant the two blocks will have momentum equal in magnitude but opposite in direction (though they have different values of momentum at different positions).
- As momentum $m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{0}$

  $\Rightarrow \vec{v}_2 = -\left(\dfrac{m_1}{m_2}\right)\vec{v}_1$

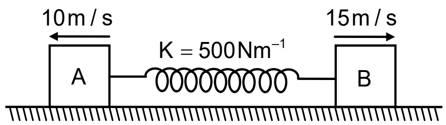
  The two blocks always move in opposite direction with lighter block moving faster.
- Kinetic energy KE $= \dfrac{p^2}{2m}$ and

  $|\vec{p}_1| = |\vec{p}_2|$, $\dfrac{KE_1}{KE_2} = \dfrac{m_2}{m_1}$ or the kinetic energy of two blocks will not be equal but in the inverse ratio of their masses and so lighter block will have greater kinetic energy.
- Initially kinetic energy of the blocks is zero (as both are at rest) but after some time kinetic energy of the blocks is not zero (as both are in motion). So, kinetic energy is not constant but changes. Here during the motion of the blocks KE is converted into elastic potential energy of the spring and vice versa but total mechanical energy of the system remains constant.

  Kinetic energy + Potential energy = Mechanical Energy = Constant

Example 19:

Two blocks A and B are joined together with a compressed spring. When the system is released the two blocks appear to be moving with unequal speeds in opposite directions as shown in figure. Select the correct statement :



(1) The centre of mass of the system will remain stationary.
(2) Mass of block A is equal to that of block B
(3) The centre of mass of the system will move towards right.
(4) It is an impossible physical situation.

Ans. **(1)**

Solution:

As net force on the system = 0 (after being released) So centre of mass of the system will remain unaccelerated as initially COM was at rest so finally the COM remains stationary.

(d) Explosion of a Bomb at Rest

Conserving momentum

$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{0}$

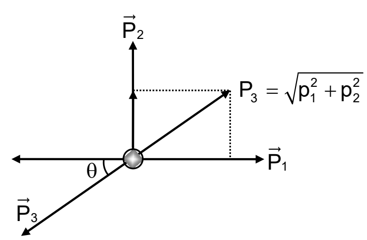
$\Rightarrow \quad \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$

$\Rightarrow \quad p_3 = \sqrt{p_1^2 + p_2^2}$ as $\vec{p}_1 \perp \vec{p}_2$

Angle made by $\vec{p}_3$ with $\vec{p}_1 = \pi - \theta$

Angle made by $\vec{p}_3$ with $\vec{p}_2 = \dfrac{\pi}{2} + \theta$

where $\theta = \tan^{-1}\left(\dfrac{p_2}{p_1}\right)$



Energy released in explosion = $K_f - K_i$

$= \dfrac{p_1^2}{2m_1} + \dfrac{p_2^2}{2m_2} + \dfrac{p_3^2}{2m_3} - 0$

$= \dfrac{p_1^2}{2m_1} + \dfrac{p_2^2}{2m_2} + \dfrac{p_3^2}{2m_3}$

Example 20:

A bomb at rest explodes into two pieces of mass ratio 1 : 3. What will be the ratio of their kinetic energies.

Solution:

Let the two pieces be A & B

$|\vec{P}_A| = |\vec{P}_B|$ & $K = \dfrac{P^2}{2m}$

so $\dfrac{K_A}{K_B} = \dfrac{m_B}{m_A} = \dfrac{3}{1}$

Example 21:

A projectile is fired at a speed of 100 m/s at an angle of 37° above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1 : 3, the smaller coming to rest. Find the distance from the launching point to the point where the heavier piece lands.

Solution:

At the highest point, by momentum conservation the COM will not get deviated from its path

also $m_A \frac{R}{2} = m_B x$

So $\quad x = \frac{R}{6}$

distance of heavier piece from launching

$P_{ad} = \frac{7R}{6}$

$R = \frac{u^2 \sin 2\theta}{g} = 960$ m

so distance = $\frac{7}{6}(960)$ m = 1120 m

Men-Plank Problems:

In such cases once again considering both the bodies as a system, $\vec{F}_{ext} = \vec{0}$

So $\vec{a}_{COM} = \vec{0}$

This denotes two possibilities

(a) If COM is at rest, it will remain at rest

(b) If COM is initially in motion then will remain in constant motion.

For condition (a)

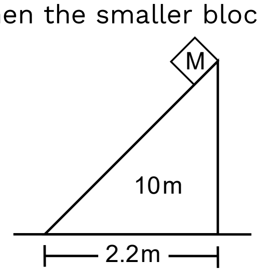
$$m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n = \vec{0}$$

For Condition (b)

$$m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n = (m_1 + m_2 + ...)\vec{v}_{cm}$$

Example 22:

A block of mass M is placed on the top of a bigger block of mass 10 M as shown in the figure. All the surfaces are frictionless. The system is released from rest. Find the distance moved by the bigger block at the instant when the smaller block reaches the ground.



Solution:

If the bigger block moves towards right by a distance (x) then the smaller block will move toward left by a distance (2.2 – x).

Now considering both the blocks together as a system horizontal position of CM remains same.

As the sum of mass moments about centre of mass is zero i.e., $\sum m_i x_{i/cm} = 0$

M(2.2 – x) = 10 Mx $\quad \Rightarrow \quad$ x = 0.2 m.

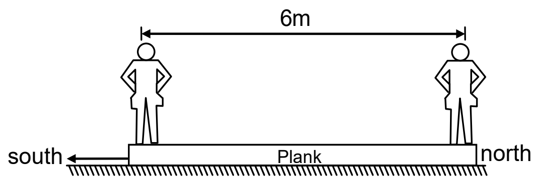
Example 23:

A man of mass 80 kg stands on a plank of mass 40 kg. The plank is lying on a smooth horizontal floor. Initially both are at rest. The man starts walking on the plank towards north and stops after moving a distance of 6 m on the plank. Then

(1) the centre of mass of plank-man system remains stationary
(2) the plank will slide to the north by a distance of 4 m
(3) the plank will slide to the south by a distance of 4 m
(4) the plank will slide to the south by a distance of 12 m

Solution:

Since net force is zero so centre of mass remains stationary
Let x be the displacement of the plank.



Since CM of the system remains stationary
so 80 (6 – x) = 40 x

$\Rightarrow \quad 12 - 2x = x \quad \Rightarrow \quad x = 4$ m.

Ans. (1, 3)

Example 24:

In a gravity free room a man of mass m_1 is standing at a height h above the floor. He throws a ball of mass m_2 vertically downwards with a speed u. Find the distance of the man from the floor when the ball reaches the ground.

Solution:

Time taken by ball to reach the ground $t = \frac{h}{u}$

By conservation of linear momentum, speed of man $v = \left(\frac{m_2 u}{m_1}\right)$

Therefore, the man will move upward by a distance $= vt = \left(\frac{h}{u}\right)\left(\frac{m_2 u}{m_1}\right) = \frac{m_2}{m_1}h$

Total distance of the man from the floor

$= h + \frac{m_2}{m_1}h = \left(1 + \frac{m_2}{m_1}\right)h$

Concept Builder-4

Q.1 A bullet weighing 50 g leaves the gun with a velocity of 30 m/s. If the recoil speed imparted to the gun is 1m/s, the mass of the gun :

(1) 15 kg (2) 30 kg (3) 1.5 kg (4) 20 kg

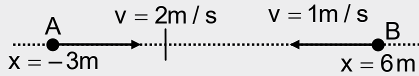
Q.2 In case of rifle shooting the kick will be minimum when:

(1) a light rifle is held loosely against shoulder
(2) a light rifle is held tightly against shoulder
(3) a heavy rifle is held loosely against shoulder
(4) a heavy rifle is held tightly against shoulder

Q.3 Two objects initially some distance apart are released from rest. They move towards each other under mutual gravitational force . At some instant, speed of one of them is v and that of other is v/4. Then speed of their centre of mass at this instant is:

(1) $\frac{3v}{4}$ (2) $\frac{v}{4}$ (3) $\frac{5v}{4}$ (4) Zero

Q.4 Two particles A(m_A = 3 kg) & B(m_B = 3 kg) are connected with a gravitational interaction between them. If at any instant their position & velocities are as shown.



Find out the position of COM after 2 seconds.

Q.5 A bullet weighing 10g and moving at 300 m/s strikes a 5kg block of ice and drops dead. The ice block is sitting on frictionless level surface. The speed of the block, after the collision is :

(1) 6 cm sec^{-1} (2) 6 m sec^{-1} (3) 60 cm sec^{-1} (4) 60 m sec^{-1}

Q.6 A nucleus at rest emits an α-particle. In this process :

(1) α-particle moves with large velocity and the nucleus remains at rest
(2) both α-particle and nucleus move with equal speed in opposite directions
(3) both move in opposite directions but nucleus with greater speed
(4) both move in opposite directions but α-particle with greater speed

Q.7 A bomb of mass 9 kg explodes into two pieces of masses 3 kg and 6 kg. The velocity of mass 3 kg is 16 m/s. The KE of mass 6kg in joule is

(1) 96 (2) 384 (3) 192 (4) 768

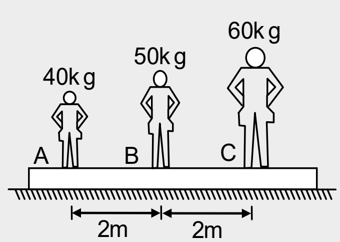
Q.8 A projectile is moving at 20 m/s at its highest point, where it breaks into two equal parts due to an internal explosion. One part moves vertically up at 30 m/s with respect to the ground. Then the other part will move at

(1) 20 m/s (2) $10\sqrt{13}$ m/s (3) 50 m/s (4) 30 m/s

Q.9 An isolated particle of mass m is moving in a horizontal plane (x–y) along the x-axis at a certain height above the ground. It suddenly explodes into two fragments of masses $\frac{m}{4}$ and $\frac{3m}{4}$. An instant later, the smaller fragment is at y = + 15 cm. The larger fragment at this instant is at :-

(1) y = – 5 cm (2) y = + 20 cm (3) y = +5 cm (4) y = – 20 cm

Q.10 Three man A, B & C of masses 40 kg, 50 kg and 60 kg are standing on a plank of mass 90 kg, which is kept on a smooth horizontal plane. If A and C exchange their positions then mass B will shift



(1) 1/3 m towards left (2) 1/3 m towards right
(3) will not move w.r.t. ground (4) 5/3 m towards left

9. Collision

Impact or collision is the interaction between two bodies during very small duration in which they exert relatively large forces on each other. Interaction forces during an impact are created either due to direct contact or strong repulsive force fields or some connecting links. These are so large as compared to other external forces acting on either of the bodies that the effects of later can be neglected. The duration of the interaction is short enough to permit us only to consider the states of motion just before and after the event and not during the impact. Duration of a impact ranges from 10^{-23} s for impacts between elementary particles to millions of years for impacts between galaxies. The impacts we observe in our everyday life such as that between two balls last from 10^{-3} s to few seconds.

For example, when an α - particle passes by the nucleus of a gold atom in Rutherford's experiment, it gets deflected in a very short time. Deflection means a change in the direction of motion a change in velocity. In this process, the particles do not touch each other.

Let us take another example, when a rubber ball strikes a floor, it remains in contact with the floor for very short time in which it changes its velocity. This is an example of collision where physical contact takes place between the colliding bodies.

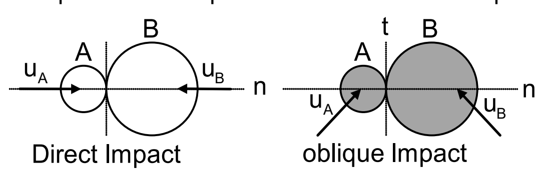
As a result of collision, the momentum and kinetic energy of the interacting bodies change.

Forces involved in a collision are action-reaction forces, i.e., the internal forces of the system.

The total momentum remains conserved in any type of collision.

Head-on (Direct) and Oblique Collision (Impact)

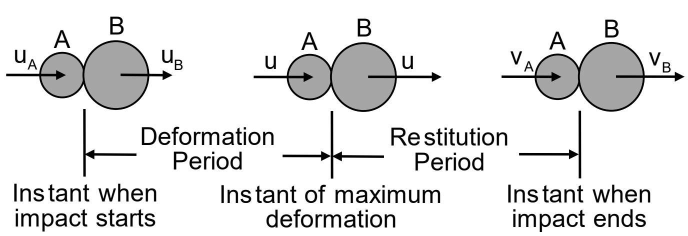
If velocity vectors of the colliding bodies are directed along the line of impact the impact is called a direct or head-on impact; and if velocity vectors of both or of any one of the bodies are not along the line of impact the impact is called on oblique impact.



10. Head on Collision

(i) Head-on (Direct) Impact

To understand what happens in a head-on impact let us consider two balls A and B of masses m_A and m_B moving with velocities u_A and u_B in the same direction as shown. Velocity u_A is larger than u_B so the ball A hits the ball B. During the impact both the bodies push each other and first they get deformed till the deformation reaches a maximum value and then they try to regain their original shapes due to elastic behavior of the materials forming the balls.

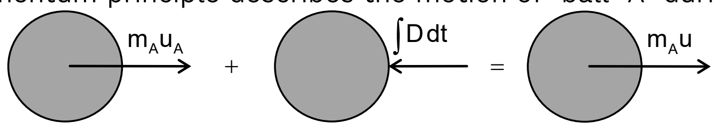


The time interval during which deformation takes place is called the **deformation period** and the time interval in which the bodies try to regain their original shapes is called the **restitution period**. Due to push applied by the balls on each other during period of deformation speed of ball A decreases and that of ball B increases and at the end of the deformation period, when the deformation is maximum both the balls move with the same velocity say it is u.

Thereafter, the balls will either move together with this velocity or follow the period of restitution. During the period of restitution due to push applied by the balls on each other, speed of the ball A decreases further and that of ball B increases further till they separate from each other. Let us denote the velocities of the balls A and B after the impact by v_A and v_B respectively.

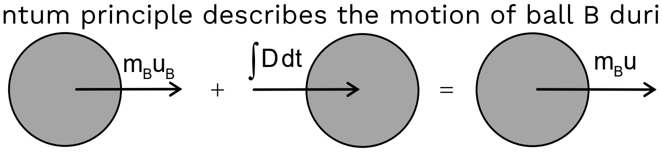
Equation of Impulse and Momentum During Impact

Impulse momentum principle describes the motion of ball A during deformation period



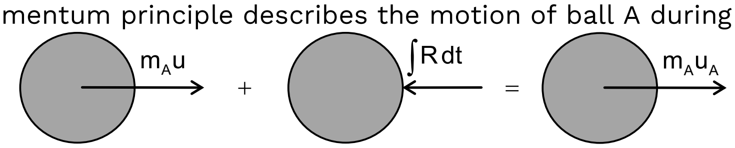
$$m_A u_A - \int D\,dt = m_A u \qquad .....(i)$$

Impulse momentum principle describes the motion of ball B during deformation period.



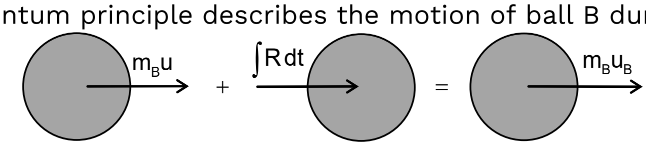
$$m_B u_B + \int D\,dt = m_B u \qquad ....(ii)$$

Impulse momentum principle describes the motion of ball A during restitution period.



$$m_A u - \int R\,dt = m_A v_A \qquad ....(iii)$$

Impulse momentum principle describes the motion of ball B during restitution period.



$$m_B u + \int R\,dt = m_B v_B \qquad ......(iv)$$

Conservation of Momentum During impact

From equations, (i) and (ii) we have

$$m_A u_A + m_B u_B = (m_A + m_B)u \qquad ......(v)$$

From equations (iii) and (iv) we have

$$(m_A + m_B)u = m_A v_A + m_B v_B \qquad ......(vi)$$

From equations, (v) and (vi) we obtain the following equation.

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B \qquad ......(vii)$$

The above equation elucidates the principle of conservation of momentum.

Coefficient of Restitution

Usually the force D applied by the bodies A and B on each other during the period of deformation differs from the force R applied by the bodies on each other during the period of restitution. Therefore, it is not necessary that the magnitude of impulse $\int D\,dt$ due to deformation equals to that of impulse $\int R\,dt$ due to restitution. The ratio of magnitudes of impulse of restitution to that of deformation is called the coefficient of restitution and is denoted by e.

$$e = \frac{\text{impulse of recovery}}{\text{impulse of deformation}} = \frac{\int R\,dt}{\int D\,dt}$$

$$e = \frac{\text{velocity of separation along line of impact}}{\text{velocity of approach along line of impact}}$$

$$= \frac{\left|\vec{v}_B - \vec{v}_A\right|}{\left|\vec{u}_A - \vec{u}_B\right|}$$

From equations (i), (ii), (iii) and (iv) we have

coefficient of restitution depends on various factors as elastic properties of materials forming the bodies, velocities of the contact points before impact state of rotation of the bodies and temperature of the bodies. In general, its value ranges from zero to one but in collisions where additional kinetic energy is generated, its value may exceed one.

Depending on the values of coefficient of restitution, two particular cases are of special interest.

Perfectly Plastic or Inelastic Impact

For these impacts e = 0, and bodies undergoing impact stick to each other after the impact.

Perfectly Elastic Impact

For these impacts e = 1.

Strategy to Solve Problems of Head-on Impact:

Write the momentum conservation equation

$m_A v_A + m_B v_B = m_A u_A + m_B u_B$ ....(A)

Write the equation involving coefficient of restitution

$v_B - v_A = e\,(u_A - u_B)$ ....(B)

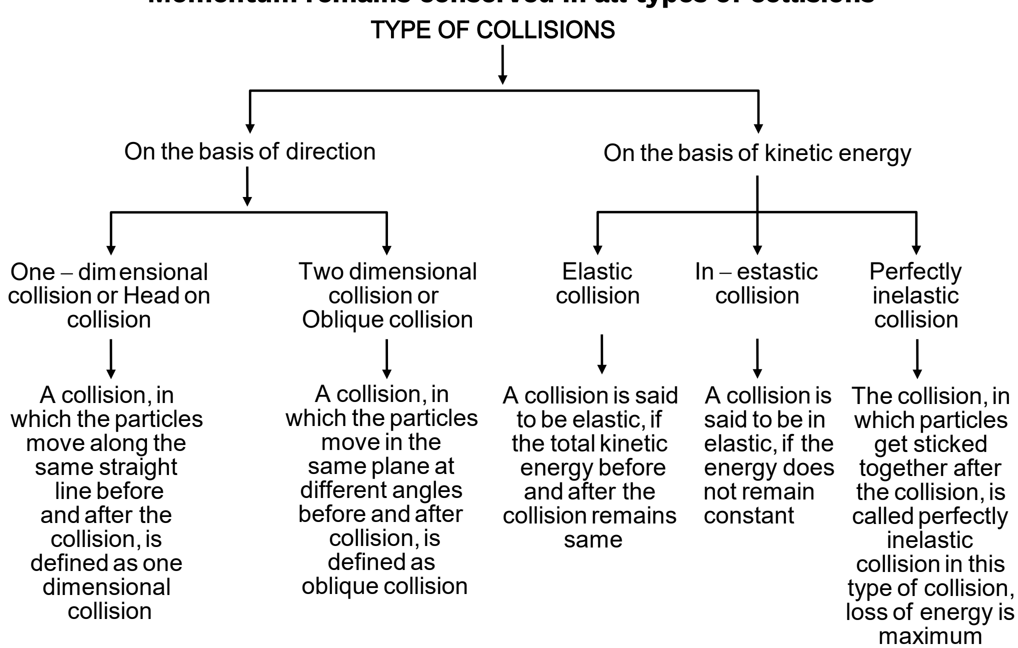
(ii) Type of Collisions According to the Conservation Law of Kinetic Energy

(a) **Elastic Collision :** KE before collision = KE after collision

(b) **Inelastic Collision :** Kinetic energy is not conserved.
Some energy is lost in collision; Therefore KE before collision > KE after collision

(c) **Perfect Inelastic Collision :** Both the bodies stick together after collision.

Momentum remains conserved in all types of collisions



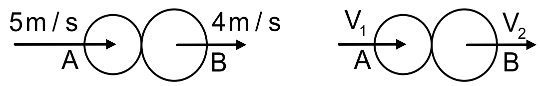
Example 25:

A ball of mass 2 kg moving with a speed of 5 m/s collides directly with another ball of mass 3 kg moving in the same direction with a speed of 4 m/s. The coefficient of restitution is 2/3. Find the velocities after collision.

Solution:

Denoting the first ball by A and the second ball by B, velocities immediately before and after the impact are shown in the figure.

By COLM : $2(5) + 3(4) = 2v_1 + 3v_2$



Before collision After collision

$\Rightarrow \quad 2v_1 + 3v_2 = 22 \quad$ ......(i)

By definition of e : $e = \dfrac{v_2 - v_1}{u_1 - u_2} \Rightarrow \dfrac{2}{3} = \dfrac{v_2 - v_1}{5 - 4}$

$\Rightarrow \quad 3v_2 - 3v_1 = 2 \quad$ ......(ii)

by solving equations (i) and (ii), we have $v_1 = 4$ m/s and $v_2 = 4.67$ m/s

Example 26:

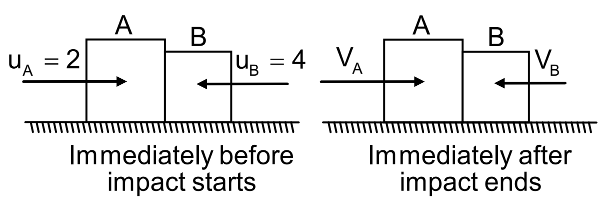
A block of mass 5 kg moves from left to right with a velocity of 2 m/s and collides with another block of mass 3 kg moving along the same line in the opposite direction with velocity 4 m/s.

(a) If the collision is perfectly elastic, determine the velocities of both the blocks after their collision.

(b) If coefficient of restitution is 0.6 determine the velocities of both the blocks after their collision.

Solution:

Denoting the first block by A and the second block by B, velocities immediately before and after the impact are shown in the figure.



Applying principle of conservation of momentum,

$m_B v_B + m_A v_A = m_B u_B + m_A u_A$

we have

$3v_B + 5v_A = 5 \times 2 + 3 \times (-4)$

$\Rightarrow \quad 3v_B + 5v_A = -2$ .....(i)

Applying equation of coefficient of restitution,

$v_B - v_A = e(u_A - u_B)$

we have $v_B - v_A = e(2-(-4))$

$\Rightarrow \quad v_B - v_A = 6e \quad$ .....(ii)

(a) For perfectly elastic impact e = 1. Using this value in equation (ii), we have

$v_B - v_A = 6$

Solving equations (i) and (ii), we obtain

$\Rightarrow \quad v_A = -2.5$ m/s and $v_B = 3.5$ m/s

(b) For value e = 0.6 equation 2 is modified as

$\Rightarrow \quad v_B - v_A = 3.6$ ......(iii)

Solving equations (i) and (iii), we obtain

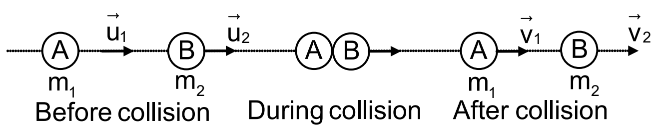
$\Rightarrow \quad v_A = -1.6$ m/s and $v_B = 2.0$ m/s

Block A reverses back with speed 1.6 m/s and B too moves in a direction opposite to its original direction with speed 2.0 m/s.

11. Special Cases of Head on Collision

(a) Head on Elastic Collision

The head on elastic collision is one in which the colliding bodies move along the same straight line path before and after the collision.



Assuming initial direction of motion to be positive and $u_1 > u_2$ (so that collision may take place) and applying law of conservation of linear momentum.

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ......(i)

For elastic collision, kinetic energy before collision must be equal to kinetic energy after collision, i.e.,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$\Rightarrow m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$ ......(ii)

Dividing equation (ii) by (i) $u_1 + v_1 = v_2 + u_2$

$\Rightarrow \quad (u_1 - u_2) = (v_2 - v_1)$ ......(iii)

In 1-D elastic collision velocity of approach before collision is equal to the velocity of separation after collision no matter what the masses of the colliding particles are.

This law is called **Newton's law for elastic collision.**

If we multiply equation (iii) by m_2 and subtract if from (i)

$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$

$$\Rightarrow \quad \boxed{v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2} \quad ....(iv)$$

Similarly, multiplying equation (iii) by m_1 and adding it to equation (i)

$2m_1u_1 + (m_2 - u_2)u_2 = (m_2 + m_1)v_2$

$$\Rightarrow \quad \boxed{v_2 = \frac{m_2 - m_1}{m_1 + m_2}u_2 + \frac{2m_1}{m_1 + m_2}u_1} \quad ...(v)$$

Example 27:

A 0.1 kg ball makes an elastic head on collision with a ball of unknown mass which is initially at rest. If the 0.1 kg ball rebounds with one third of its original speed, what is the mass of other ball?

Solution:

Here $m_1 = 0.1$ kg, $m_2 = ?$, $u_2 = 0$, $u_1 = u$,

$v_1 = -u/3$

Using, $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$

$\Rightarrow \quad -\frac{u}{3} = \left(\frac{0.1 - m_2}{0.1 + m_2}\right)u \Rightarrow m_2 = 0.2\text{kg}.$

(b) Head on Elastic Collision of Identical Bodies

- **If the two bodies are of equal masses.**

  $m_1 = m_2 = m$

  By momentum conservation

  $m(u_1 + u_2) = m(v_1 + v_2)$

  $\Rightarrow v_1 + v_2 = u_1 + u_2$ ...(i)

  Also

  $v_{sepration} = e\, u_{approach}$

  $\Rightarrow v_2 - v_1 = (1)(u_1 - u_2)$ ...(ii)

  By eq (i) & (ii)

  $\Rightarrow 2v_2 = 2u_1 \Rightarrow v_2 = u_1$

  Also

  $v_1 = u_2$

  Thus, if two bodies of equal masses undergo elastic collision in one dimension, then the bodies exchange their velocities after the collision.

- **If the two bodies are of equal masses and second body is at rest.**

  $m_1 = m_2$ and initial velocity of second body

  $u_2 = 0$, $v_1 = 0$, $v_2 = u_1$

  When body A collides against body B of equal mass at rest, then body A comes to rest and body B moves with the velocity of body A. In this case transfer of energy is hundred percent e.g., Billiard's Ball, Nuclear moderation.

Example 28:

Two balls each of mass 5 kg moving in opposite directions with equal speeds 5 m/s collide head on with each other. Find out the final velocities of the ball is the collision is perfectly elastic.

Solution:

Here $m_1 = m_2 = 5$ kg, $u_1 = 5$ m/s,

$u_2 = -5$ m/s

In such a condition velocities get interchanged so

$v_2 = u_1 = 5$ m/s and $v_1 = u_2 = -5$ m/s

(c) Head on Inelastic Collision of Bodies:

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ......(i)

By definition of coefficient of restitution

$v_2 - v_1 = e\,(u_1 - u_2)$ ....(ii)

$$\boxed{v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2}$$

$$\text{Loss in KE} = \frac{1}{2}\ \frac{m_1m_2}{(m_1+m_2)}\ (1 - e^2)\ (u_1 - u_2)^2$$

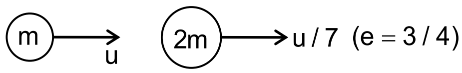
$$\boxed{v_2 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 + \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1}$$

Example 29:

If a ball moving with velocity u collides another ball of twice its own mass moving with one-seventh of its velocity and if the coefficient of restitution between them is $\frac{3}{4}$, the velocity of the first ball after striking the second ball is :

(1) u/2 (2) Zero (3) u/4 (4) 3u/4

Solution:



$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2$$

$$v_1 = -\frac{u}{6} + \frac{u}{6} = 0$$

(d) Head-On Perfectly Inelastic Collision of Bodies

In perfectly inelastic collision e = 0

So, $V_{separation} = 0\ (V_{approach}) = 0$

This means bodies will stick to each other & will move with a common velocity

Applying COLM $m_1\vec{u}_1 + m_2\vec{u}_2 = (m_1 + m_2)$

$$\Rightarrow \quad \vec{v} = \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1 + m_2} = \vec{v}_{COM}$$

After such collision, Bodies will stick to each other & will move a common velocity which is $\vec{v}_{COM}$

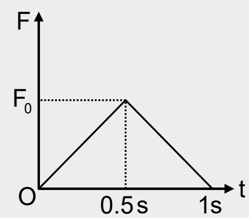
Concept Builder-5

Q.1 Two balls of equal masses undergo a head-on collision with speeds 6 m/s moving in opposite direction. If the coefficient of restitution is $\frac{1}{3}$, find the speed of each ball after impact in m/s.

Q.2 A particle of mass 2 kg moving with a velocity $5\hat{i}\ m/s$ collides head-on with another particle of mass 3 kg moving with a velocity $-2\hat{i}\ m/s$. After the collision the first particle has speed of 1.6 m/s in negative x direction. Find the :
(a) velocity of the centre of mass after the collision
(b) velocity of the second particle after the collision
(c) coefficient of restitution.

Q.3 A body of mass 2 kg makes an elastic collision with another body at rest and continues to move in the original direction with one fourth of its original speed. Find the mass of the second body.

Q.4 A body of mass 1 kg moving with velocity 1 m/s makes an elastic one dimensional collision with an identical stationary body. They are in contact for a brief period 1 s.



Their force of interaction increases from zero to F_0 linearly in 0.5 s and decreases linearly to zero in a further 0.5 s as shown in figure. Find the magnitude of force F_0 in newtons.

Q.5 A particle of mass m moving with a velocity v makes a head on elastic collision with another particle of same mass initially at rest. Find the velocity of the first particle after the collision.

Q.6 A body moving towards a body of finite mass at rest, collides with it. It is impossible that
(1) both bodies come to rest
(2) both bodies move after collision
(3) the moving body stops and body at rest starts moving
(4) the stationary body remains stationary and the moving body rebounds

Q.7 A body of 2 kg mass having velocity 3 m/s collides with a body of 1 kg mass moving with a velocity of 4m/s in the opposite direction. After collision both bodies stick together and move with a common velocity. Find the velocity in m/s.

12. Collision with Very Heavy Mass, Collision with Wall

(a) Collision with Very Heavy Mass

If a body of mass m, velocity u, collides with another body of mass M (M >>> m) moving with velocity v_0 then

$$mu + Mv_0 = mu' + Mv_0'$$

$$\Rightarrow \quad v_0' = \frac{m}{M}(u - u') + v_0$$

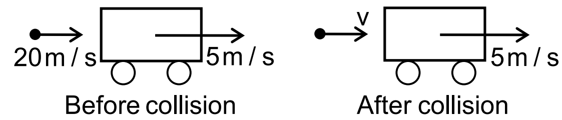
since m <<< M, $\boxed{v'_0 \approx v_0}$

i.e., velocity of very heavy mass remain unaltered.

Example 30:

A ball of mass 10kg moving with a velocity of 20m/s collides with a truck moving in same direction with a speed of 5m/s. If coefficient of restitution is $\frac{1}{3}$. What is the velocity of ball after collision.

Solution:



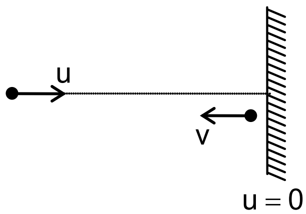
$v_{sep} = e\ v_{app}$

$5 - v = \frac{1}{3}(20 - 5) = 5$

$\Rightarrow \quad v = 0$

(b) Collision with Stationary Wall

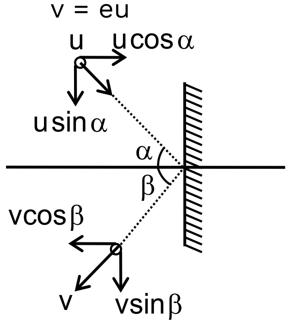
(i)



Here $v_{separation} = e\ u_{approach}$

$v = eu$

(ii)



If surface is frictionless then the velocity of ball along the wall will remain unchanged i.e.,

$v \sin\beta = u \sin\alpha$

Also along the normal of Wall

$v \cos\beta = e\, u \cos\alpha$

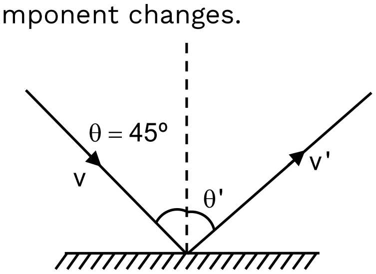
Note : If $e = 1$, $v = u$ & $\alpha = \beta$

Example 31:

A ball of mass m hits a floor with a speed v making an angle of incidence $\theta = 45°$ with the normal to the floor. If the coefficient of restitution is $\frac{1}{\sqrt{2}}$, find the speed of the reflected ball and the angle of reflection.

Solution:

Since the floor exerts a force one the ball along the normal during the collision so horizontal component of velocity remains same and only the vertical component changes.



Therefore, $v' \sin\theta' = v\sin\theta = \frac{v}{\sqrt{2}}$

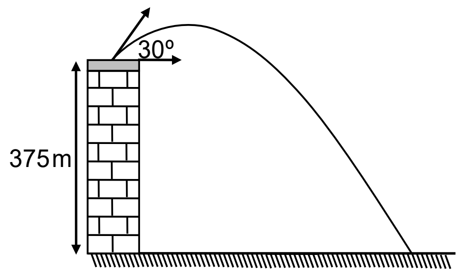
and $v' \cos\theta' = ev\cos\theta = \frac{1}{\sqrt{2}} v \times \frac{1}{\sqrt{2}} = \frac{v}{2}$

$\Rightarrow \quad v'^2 = \frac{v^2}{2} + \frac{v^2}{4} = \frac{3}{4}v^2 \quad \Rightarrow \quad v' = \frac{\sqrt{3}}{2}v$

and $\quad \tan\theta' = \sqrt{2} \quad \Rightarrow \quad \theta' = \tan^{-1}\sqrt{2}$

Example 32:

A particle of mass 1 kg is projected from a tower of height 375 m with an initial velocity of 100 m/s at an angle 30° with the horizontal. Find its kinetic energy in joules just after the collision with ground if the collision is inelastic with e = $\frac{1}{2}$ (take g = 10 ms^{-2})



Solution:

$v_y^2 = u_y^2 + 2gh$

$\Rightarrow \quad v_y = \sqrt{(50)^2 + 2 \times 10 \times 375} = 100ms^{-1}$

Horizontal velocity just after collision

$= 50\sqrt{3}\,ms^{-1}$

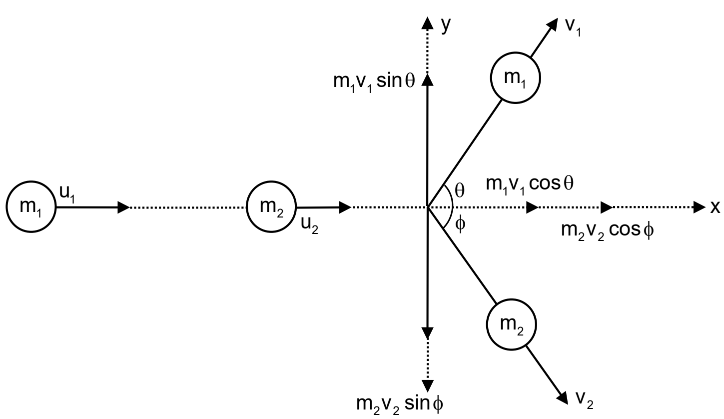
Vertical velocity just after collision

$= 100 \times \frac{1}{2} = 50\ ms^{-1}$

Kinetic energy just after the collision

$= \frac{1}{2} \times 1 \times \left[(50\sqrt{3})^2 + (50)^2\right] = 5000\ J$

13. Oblique Elastic Collision



By COLM along x-axis

$m_1u_1 + m_2u_2 = m_1v_1\cos\theta + m_2v_2\cos\phi$

By COLM along y-axis

$0 + 0 = m_1v_1\sin\theta - m_2v_2\sin\phi$

If collision is elastic then,

By conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2$$

Oblique elastic collision of identical bodies

Conservation of linear momentum in x-direction gives

$mu_1 = mv_1\cos\theta_1 + mv_2\cos\theta_2$

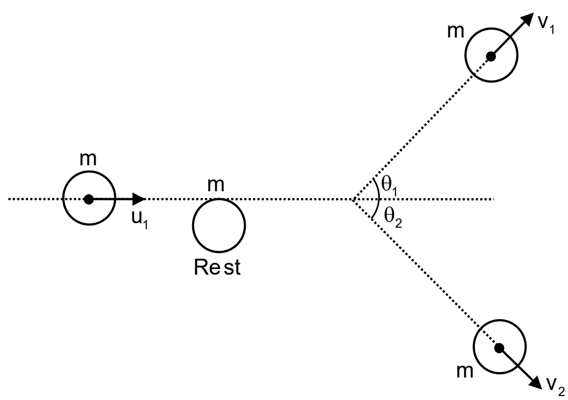
$\Rightarrow$ $u_1 = v_1\cos\theta_1 + v_2\cos\theta_2$ ....(i)

Conservation of linear momentum in y-direction gives

$0 = mv_1\sin\theta_1 - mv_2\sin\theta_2$

$\Rightarrow$ $0 = v_1\sin\theta_1 - v_2\sin\theta_2$ ......(ii)

Conservation of kinetic energy yields,



$$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$\Rightarrow$ $u_1^2 = v_1^2 + v_2^2$ ......(iii)

Squaring and adding equations (i) and (ii)

$\Rightarrow$ $u_1^2 + 0 = v_1^2\cos^2\theta_1 + v_2^2\cos^2\theta_2 + 2v_1v_2\cos\theta_1\cos\theta_2 + v_1^2\sin^2\theta_1 + v_2^2\sin^2\theta_2 - 2v_1v_2\sin\theta_1\sin\theta_2$

$\Rightarrow$ $u_1^2 = v_1^2(\cos^2\theta_1 + \sin^2\theta_1) + v_2^2(\cos^2\theta_2 + \sin^2\theta_2) + 2v_1v_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)$

$\Rightarrow$ $u_1^2 = v_1^2 + v_2^2 + 2v_1v_2\cos(\theta_1 + \theta_2)$ $\{\therefore u_1^2 = v_1^2 + v_2^2\}$

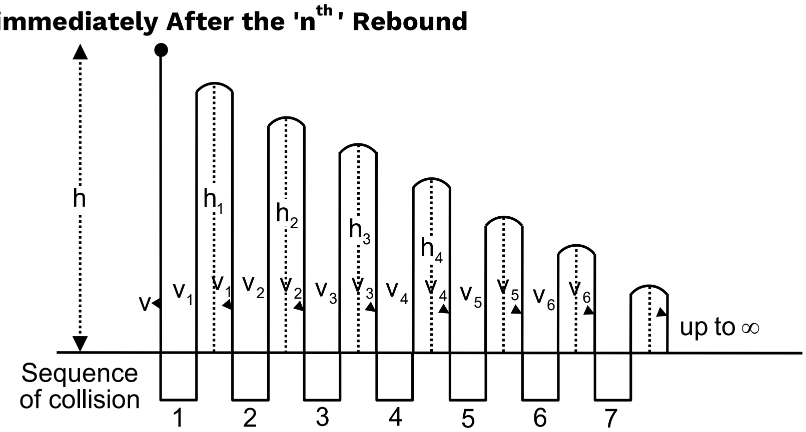
$\Rightarrow$ $\cos(\theta_1 + \theta_2) = 0$

$\Rightarrow$ $\theta_1 + \theta_2 = 90°$

14. Bouncing of Ball

Let a ball fall from a height (h) and let it touches the ground with a velocity v taking time (t) to reach the ground. Let v_1, v_2, v_3 ....be the velocities immediately after first, second, third ....collisions with the ground

- **Velocity immediately After the 'nth' Rebound**



By Newton's formula $(\vec{v}_2 - \vec{v}_1) = e(\vec{u}_1 - \vec{u}_2)$

$v = \sqrt{2gh}$ here $\vec{v}_2 = \vec{0}$, $\vec{u}_2 = \vec{0}$ (surface at rest)

$v_1 = ev$ (opposite direction)

$(\because v = u_1)$

$v_1 = ev$ ......(1), $v_2 = ev_1$ ....(2),

$v_2 = e(ev)$ $\Rightarrow$ $v_2 = e^2 v$

Similarly

$v_3 = e^3 v$, $v_4 = e^4 v$ ...., $\boxed{v_n = e^n\sqrt{2gh} = e^n v}$

- **Height Attained by the Ball After the 'n^{th}' Rebound**

$v_1 = ev$ $\Rightarrow$ $\sqrt{2gh_1} = e\sqrt{2gh}$ $\Rightarrow h_1 = e^2 h$,

$v_2 = e^2 v$ $\Rightarrow$ $\sqrt{2gh_2} = e^2\sqrt{2gh}$ $\Rightarrow h_2 = e^4 h$,

Similarly $\boxed{h_n = e^{2n} h}$

- **Time Taken in n^{th} Rebound**

$h_1 = e^2 h$

$$\frac{1}{2}gt_1^2 = e^2\,\frac{1}{2}\,gt^2$$

$\Rightarrow t_1 = et$

$$t_1 = e\sqrt{\frac{2h}{g}}$$

$h_2 = e^4 h$

$$\frac{1}{2}gt_2^2 = e^4\left(\frac{1}{2}gt^2\right)$$

$$\Rightarrow t_2^2 = e^4 t^2,\ t_2 = e^2 t,\ t_2 = e^2\sqrt{\frac{2h}{g}}$$

Similarly

$$\boxed{t_n = e^n\sqrt{\frac{2h}{g}}},\quad \boxed{t_n = e^n t}$$

Total time taken in bouncing. (i.e., total time elapsed before the ball stops)

$T = t + 2t_1 + 2t_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= t + 2et + 2e^2 t + 2e^3 t + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= t + 2t\,(e + e^2 + e^3 + \ldots..)$

$$= t + 2t\left(\frac{e}{1-e}\right) = t\left(\frac{1+e}{1-e}\right) = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$$

$$\boxed{T = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)}$$

- **Distance Covered by the Ball Before it Stops**

$s = h + 2h_1 + 2h_2 + \ldots\ldots\ldots\ldots + \infty$

$= h + 2e^2h + 2e^4h + 2e^6h + \ldots\ldots\ldots\ldots$

$= h + 2e^2h\,(1 + e^2 + e^4 + e^6 + \ldots\ldots\ldots\ldots)$

$= h + 2e^2h \left(\frac{1}{1-e^2}\right)$

$= h\left[1+\frac{2e^2}{1-e^2}\right],$

$$\boxed{s = h\left(\frac{1+e^2}{1-e^2}\right)}$$

- **Average Speed**

$$v_{av} = \frac{\text{Total distance}}{\text{Total time}} = \frac{h\left(\frac{1+e^2}{1-e^2}\right)}{\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)}$$

$$\boxed{v_{av} = \sqrt{\frac{gh}{2}}\left[\frac{1+e^2}{(1+e)^2}\right]}$$

- **Average Velocity**

$$v_{av} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{h}{\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)};$$

$$\boxed{v_{av} = \sqrt{\frac{gh}{2}}\left(\frac{1-e}{1+e}\right)}$$

Example 33:

A body falling on the ground from a height of 10 m, rebounds to a height 2.5 m calculate the :

(i) percentage loss in K.E.

(ii) Ratio of the velocities of the body just before and after the collision.

Solution:

Let v_1 and v_2 be the velocities of the body just before and just after the collision.

$KE_1 = \frac{1}{2}\,mv_1^2 = mgh$ ...(i)

and $KE_2 = \frac{1}{2}\,mv_2^2 = mgh_2$ ...(ii)

$\Rightarrow \quad \frac{v_1^2}{v_2^2} = \frac{h_1}{h_2} = \frac{10}{2.5} = 4$

$\Rightarrow \quad \frac{v_1}{v_2} = 2$

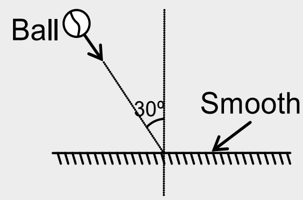
Percentage loss in

$KE = \frac{mg(h_1 - h_2)}{mgh_1} \times 100 = \frac{10 - 2.5}{10} \times 100 = 75\%$

Concept Builder-6

Q.1 What will be the coefficient of restitution if a ball ($v = 12$ m/s) collides with another very heavy ball moving in opposite direction with a speed of 3m/s & the light ball rebounds with a velocity of 12m/s.

Q.2 The ball moving with a speed of 20 m/s strikes the floor as shown if $e = \frac{1}{2}$, then calculate



(a) Angle of reflection
(b) Velocity of ball after striking

Q.3 A ball of mass 2kg moving with speed 10 m/sec. collide obliquely with a stationary identical ball. It is observe that after collision first ball moves with speed 6 m/s then find final momentum of second ball.

Q.4 A base of mass 1 kg is dropped from 20 m height. Find:-
(i) Velocity of ball just after second collision?
(ii) Maximum height attained by the ball agter second collision
(iii) Average speed for whole interval
(If e = 0.5) ($g = 10$ m/s^2)

ANSWER KEY FOR CONCEPT BUILDERS

CONCEPT BUILDER-1

1. 1.1 m, 1.3 m

2. $\left(\frac{a}{2}, \frac{7}{10}a\right)$

3. $\left[\frac{7b}{12}, \frac{3\sqrt{3}b}{12}, 0\right]$

4. $\left(\frac{14}{5}, \frac{19}{5}\right)$

5. 8.8 m towards 2 kg mass

6. (1)

CONCEPT BUILDER-2

1. (1)

2. $\left(0, \frac{2r}{\pi}\right)$

3. (2)

4. (1)

CONCEPT BUILDER-3

1. $x = \frac{-cb^2}{(a^2 - b^2)}$

2. (a) at O, (b) III quadrant;
(c) on OY' axis (d) at O,
(e) iv quadrant; (f) at O

3. 10 m/s

4. (2)

5. $\frac{\sqrt{14}}{6}\ ms^{-2}$

6. 1.6 m/s^2 downwards

CONCEPT BUILDER-4

1. (3)

2. (4)

3. (4)

4. x_{CM} = 2 m

5. (3)

6. (4)

7. (3)

8. (3)

9. (1)

10. (2)

CONCEPT BUILDER-5

1. 2 m/s each, but they will move in opposite directions.

2. (a) $0.8\,\hat{i}$ m/s (b) $2.4\ \hat{i}$ m/s
(c) 4/7

3. 1.2 kg

4. 2 N

5. Zero

6. (1) & (4)

7. $\frac{2}{3}$ m/s along the initial direction of motion of 2 kg mass.

CONCEPT BUILDER-6

1. $e = \frac{3}{5}$

2. (a) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$; (b) $10\hat{i} + 5\sqrt{3}\hat{j}$

3. 16 kg-m/s

4. (i) 5 m/s (ii) $\frac{5}{4}$ m

(iii) $\frac{50}{9}$ m/s

Exercise - I

Centre of Mass of System of Discrete Particles, Centre of Mass of Continuous Body System of Discrete Particles

1. The centre of mass of a system of particles does not depend on :
(1) masses of the particles
(2) Internal forces on the particles
(3) position of the particles
(4) relative distance between the particles

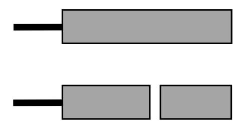
2. The centre of mass of a system of two particles divides the distance between them
(1) In inverse ratio of square of masses of particles
(2) In direct ratio of square of masses of particles
(3) In inverse ratio of masses of particles
(4) In direct ratio of masses of particles

3. The centre of mass of a body :-
(1) Lies always outside the body
(2) May lie within, outside of the surface of the body
(3) Lies always inside the body
(4) Lies always on the surface of the body

4. A system consists of mass M and m (<<M). The centre of mass of the system is :-
(1) at the middle
(2) nearer to M
(3) nearer to m
(4) at the position of larger mass

5. In the HCl molecule the separation between the nuclei of the two atoms is about 1.27 Å (1Å = 10^{-10} m). The approximate location of the centre of mass of the molecule from hydrogen atom, assuming the chlorine atom to be about 35.5 times massive hydrogen is :
(1) 1Å (2) 2.5 Å
(3) 1.24 Å (4) 1.5 Å

6. A cricket bat is cut at the location of its centre of mass as shown. Then :-



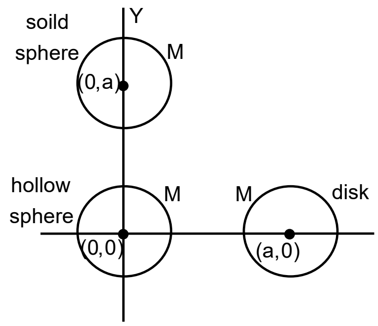
(1) The two pieces will have the same mass
(2) The bottom piece will have larger mass
(3) The handle piece will have larger mass
(4) Mass of handle piece is double the mass of bottom piece

7. If linear density of a rod of length 3 m varies as $\lambda = 2 + x$, then the position of the centre of gravity of the rod is :
(1) $\frac{7}{3}$ m
(2) $\frac{12}{7}$ m
(3) $\frac{10}{7}$ m
(4) $\frac{9}{7}$ m

Centre of Mass of Composite Body, COM of Truncated Body

8. A uniform metal disc of radius R is taken and out of it a disc of diameter $\frac{R}{2}$ is cut off from the end. The centre of mass of the remaining part will be :
(1) $\frac{R}{10}$ from the centre
(2) $\frac{R}{15}$ from the centre
(3) $\frac{R}{5}$ from the centre
(4) $\frac{R}{20}$ from the centre

9. The coordinate of the centre of mass of a system as shown in figure :-



(1) $\left(\frac{a}{3},0\right)$ (2) $\left(\frac{a}{2},\frac{a}{2}\right)$

(3) $\left(\frac{a}{3},\frac{a}{3}\right)$ (4) $\left(0,\frac{a}{3}\right)$

10. Three identical metal balls, each of radius r, are placed touching each other on a horizontal surface such that an equilateral triangle is formed when the centres of the three balls are joined. The centre of mass of the system is located at :-

(1)horizontal surface

(2) centre of one of the balls

(3) line joining centres of any two balls

(4) point of intersection of their medians

11. The centre of mass of a system of three particles of masses 1 g, 2 g and 3 g is taken as the origin of a co-ordinate system. The position vector of a fourth particle of mass 4 g such that the centre of mass of the four particle system lies at the point (1, 2, 3) is $\alpha(\hat{i}+2\hat{j}+3\hat{k})$, where α is a constant. The value of α is :

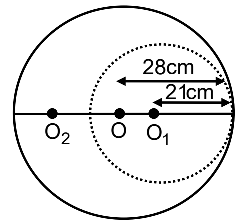
(1) $\frac{10}{3}$

(2) $\frac{5}{2}$

(3) $\frac{1}{2}$

(4) $\frac{2}{5}$

12. A circular plate of uniform thickness has a diameter 56 cm. A circular portion of diameter 42 cm is removed from one edge as shown in the figure. The centre of mass of the remaining portion from the centre of plate will be :



(1) 5 cm (2) 7 cm

(3) 9 cm (4) 11 cm

13. A uniform rod of length 1.0 meter is bent at its midpoint to make 90º angle. The distance of the centre of mass from the centre of the rod is :-

(1) 35.3 cm (2) 25.2 cm

(3) 17.7 cm (4) zero

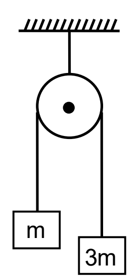
Motion of Centre of Mass

14. A person of mass m is standing on one end of a plank of mass M and length L and floating in water. The person moves from one end to another and stops. The displacement of the plank is -

(1) $\frac{Lm}{(m+M)}$ (2) Lm (M + m)

(3) $\frac{(M+m)}{Lm}$ (4) $\frac{LM}{(m+M)}$

15. If the system is released then the acceleration of the centre of mass of the system :



(1) $\frac{g}{4}$ (2) $\frac{g}{2}$

(3) g (4) 2g

16. Initially two stable particles x and y start moving towards each other under mutual attraction. If at one time the velocities of x and y are V and 2V respectively what will be the velocity of centre of mass of the system?

(1) V (2) Zero

(3) $\frac{V}{3}$ (4) $\frac{V}{5}$

17. A 2 kg body and a 3 kg body are moving along the x-axis. At a particular instant the 2 kg body has a velocity of 3 ms^{-1} and the 3 kg body has the velocity of 2 ms^{-1}. The velocity of the centre of mass at that instant is :-

(1) 5 ms^{-1} (2) 1 ms^{-1}

(3) 0 (4) $\frac{12}{5}ms^{-1}$

18. Two objects of masses 200 gram and 500 gram possess velocities $10\hat{i}\,m/s$ and $3\hat{i}+5\hat{j}$ respectively. The velocity of their centre of mass in m/s is :-

(1) $5\hat{i}-25\hat{j}$ (2) $\frac{5}{7}\hat{i}-25\hat{j}$

(3) $5\hat{i}+\frac{25}{7}\hat{j}$ (4) $25\hat{i}-\frac{5}{7}\hat{j}$

Conservation of Linear Momentum

19. The law of conservation of momentum for a system is based on Newton's :-

(1) First law of motion

(2) Second law of motion

(3) Third law of motion

(4) Law of gravitation

20. Bullets of mass 40 g each are fired from a machine gun with a velocity of 10^3 m/s. If the person firing the bullets experience an average force of 200 N, then the number of bullets fired per minute will be -

(1) 300 (2) 600

(3) 150 (4) 75

21. Which of the following is true :

(1) Momentum is conserved in all collision but kinetic energy is conserved only in inelastic collision

(2) Neither momentum nor kinetic energy is conserved in inelastic collisions.

(3) Momentum is conserved in all collisions but not kinetic energy

(4) Both momentum and kinetic energy are conserved in all collisions.

22. A bullet of mass m is fired into a large block of wood of mass M with velocity v. The final velocity of the system is :

(1) $\left(\frac{m}{M-m}\right)v$ (2) $\left(\frac{m+M}{M}\right)v$

(3) $\left(\frac{M-m}{M}\right)v$ (4) $\left(\frac{m}{m+M}\right)v$

23. Identify the wrong statement.

(1) A body can have momentum without mechanical energy

(2) A body can have energy without momentum

(3) The momentum is conserved in an elastic collision only.

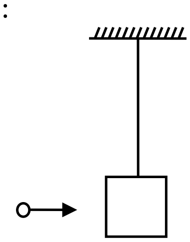
(4) Kinetic energy is not conserved in an inelastic collision.

24. A 50 gm bullet moving with a velocity 10 m/s gets embedded into a 950 gm stationary body. The loss in kinetic energy of the system will be :

(1) 5% (2) 50%

(3) 100% (4) 95%

25. A bullet of mass m moving with a speed v strikes a wooden block of mass M and gets embedded into the block. The final speed is :



(1) $\sqrt{\frac{M}{M+m}}v$ (2) $\sqrt{\frac{m}{M+m}}v$

(3) $\frac{m}{M+m}v$ (4) $\frac{v}{2}$

26. A 10g bullet, moving with velocity of 500 m/s enters a stationary piece of ice of mass 10 kg and stops. If the piece of ice is lying on a frictionless plane, then its velocity will be :

(1) 5 cm/s (2) 5 m/s
(3) 0.5 m/s (4) 0.5 cm/s

27. A nucleus of mass number A originally at rest, emits alpha particle with speed v. The recoil speed of the daughter nucleus is :-

(1) $\frac{4v}{A-4}$ (2) $\frac{4v}{A+4}$
(3) $\frac{v}{A-4}$ (4) $\frac{v}{A+4}$

Explosion

28. A bomb of mass 9 kg explodes into two pieces of 3kg and 6 kg. The velocity of 3 kg piece is 16 m/s. The kinetic energy of 6 kg piece is :-

(1) 768 joule (2) 786 joule
(3) 192 joule (4) 687 joule

29. A bomb initially at rest explodes by it self into three equal mass fragments. The velocities of two fragments are $(3\hat{i}+2\hat{j})$m/s and $(-\hat{i}-4\hat{j})$. The velocity of the third fragment is (in m/s) :-

(1) $2\hat{i}+2\hat{j}$ (2) $2\hat{i}-2\hat{j}$
(3) $-2\hat{i}+2\hat{j}$ (4) $-2\hat{i}-2\hat{j}$

30. A bomb of 50 kg is fired from a cannon with a velocity 600 m/s. If the mass of the cannon is 10^3 kg, then its velocity will be -

(1) 30 m/s (2) –30 m/s
(3) 0.30 m/s (4) –0.30 m/s

31. A 1 kg stationary bomb is exploded in three parts having mass ratio 1 : 1 : 3. Parts having same mass move in perpendicular directions with velocity 30 m/s, then the velocity of bigger part will be :-

(1) $10\sqrt{2}$ m/s (2) $\frac{10}{\sqrt{2}}$ m/s
(3) $15\sqrt{2}$ m/s (4) $\frac{15}{\sqrt{2}}$ m/s

32. A heavy nucleus at rest breaks into two fragments which fly off with velocities 8 : 1. The ratio of radii of the fragments is:

(1) 1 : 2 (2) 1 : 4
(3) 4 : 1 (4) 2 : 1

33. A stationary particle explodes into two particles of masses m_1 and m_2 which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies E_1/E_2 is :

(1) m_1/m_2 (2) m_2/m_1
(3) 1 (4) $m_1 v_2 / m_2 v_1$

34. A body of mass 4m at rest explodes into three pieces. Two of the pieces each of mass m move with a speed v each in mutually perpendicular directions. The total kinetic energy released is :

(1) $\frac{1}{2}mv^2$ (2) mv^2
(3) $\frac{3}{2}mv^2$ (4) $\frac{5}{2}mv^2$

35. A bomb of mass 3.0 kg explodes in air into two pieces of masses 2.0 kg and 1.0 kg. The smaller mass goes at a speed of 80 m/s. The total energy imparted to the two fragments is :

(1) 1.07 kJ (2) 2.14 kJ
(3) 2.4 kJ (4) 4.8 kJ

36. A bomb of mass 30 kg at rest explodes into two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is 6 ms^{-1}. The kinetic energy of the other mass is :

(1) 524 J (2) 256 J
(3) 486 J (4) 324 J

Collision : Head on Collision, Elastic Collision & Inelastic Collision

37. A sphere of mass m moving with a constant velocity collides with another stationary sphere of same mass. The ratio of velocities of two spheres after collision will be, if the coefficient of restitution is e:

(1) $\frac{1-e}{1+e}$ (2) $\frac{e-1}{e+1}$

(3) $\frac{1+e}{1-e}$ (4) $\frac{e+1}{e-1}$

38. Two elastic bodies P and Q having equal masses are moving along the same line with velocities 16m/s and 10 m/s respectively. Their velocities after the elastic collision will be in m/s :

(1) 0 and 25
(2) 5 and 20
(3) 10 and 16
(4) 20 and 5

39. The unit of the coefficient of restitution is:

(1) m/s
(2) s/m
(3) m × s
(4) None of the above

40. Two solid balls of rubber A and B whose masses are 200 gm and 400 gm respectively, are moving in mutually opposite directions. If the velocity of ball A is 0.3 m/s and both the balls come to rest after collision, then the velocity of ball B is :

(1) 0.5
(2) – 0.15 m/s
(3) 1.5 m/s
(4) None of the above

41. Two bodies of same mass moving with speed V in mutually opposite directions. They collide and stick together. The elastic collision will be in m/s:

(1) Zero (2) $\frac{V}{2}$

(3) V (4) From zero to ∞

42. A 5 kg body collides with another stationary body. After the collision, the bodies move in the same direction with one-third of the velocity of the first body. The mass of the second body will be :

(1) 5 kg (2) 10 kg
(3) 15 kg (4) 20 kg

43. A heavy body moving with a velocity 20 ms^{-1} and another small object at rest undergo an elastic collision. The latter will move with a velocity of

(1) 20 m/s (2) 40 m/s
(3) 60 m/s (4) Zero

44. A 5gm lump of clay moving with velocity of 10 cm/s towards east, collides head -on with another 2 gm lump of clay moving with 15 cm/s towards west. After collision, the two lumps stick together. The velocity of the compound lump will be :

(1) 5 cm/s towards east
(2) 5 cm/s towards west
(3) 2.88 cm/s towards east
(4) 25 cm/s towards west

45. In an inelastic collision between two bodies, the physical quantity that is conserved :

(1) Kinetic energy
(2) Momentum
(3) Potential energy
(4) Kinetic energy and momentum

46. A mass of 20 kg moving with a speed of 10 m/s collides with another stationary mass of 5 kg. As a result of the collision, the two masses stick together. The kinetic energy of the composite mass will be :

(1) 600 Joule (2) 800 Joule
(3) 1000 Joule (4) 1200 Joule

47. A body of mass m having an initial velocity v makes head on collision with a stationary body of mass M. After the collision, the body of mass m comes to rest and only the body having mass M moves. This will happen only when :

(1) m > > M (2) m < < M

(3) m = M (4) m = $\frac{M}{2}$

48. A body A experiences perfectly elastic collision with a stationary body B. If after collision the bodies fly apart in the opposite direction with equal speeds, the mass ratio of A and B is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

(3) $\frac{1}{4}$ (4) $\frac{1}{5}$

49. A collision is said to be perfectly inelastic when:

(1) Coefficient of restitution = 0

(2) Coefficient of restitution = 1

(3) Coefficient of restitution = ∞

(4) Coefficient of restitution < 1

50. If two masses m_1 and m_2 collide, the ratio of the changes in their respective velocities is proportional to :

(1) $\frac{m_1}{m_2}$ (2) $\sqrt{\frac{m_1}{m_2}}$

(3) $\frac{m_2}{m_1}$ (4) $\sqrt{\frac{m_2}{m_1}}$

51. Two identical balls, one moves with 12 m/s and second is at rest, collides elastically. After collision velocity of second and first ball will be

(1) 6 m/s, 6 m/s (2) 12 m/s, 12 m/s

(3) 12 m/s, 0 m/s (4) 0 m/s, 12 m/s

52. A neutron makes a head on elastic collision with a stationary deuteron. The fractional energy loss of the neutron in the collision is :-

(1) 16/82 (2) 8/9

(3) 8/27 (4) 2/3

53. Two particles each of mass m travelling with velocities u_1 and u_2 collide perfectly inelastically. The loss of kinetic energy will be :

(1) $\frac{1}{2}m(u_1 - u_2)^2$ (2) $\frac{1}{4}m(u_1 - u_2)^2$

(3) $m\,(u_1-u_2)^2$ (4) $2m\,(u_1-u_2)^2$

54. Two ice skaters A and B approach each other at right angles. Skater A has a mass 30 kg and velocity 1m/s and skater B has a mass 20 kg and velocity 2 m/s. They meet and cling together. Their final velocity of the couple is :

(1) 2 m/s (2) 1.5 m/s

(3) 1 m/s (4) 2.5 m/s

55. The bob (mass m) of a simple pendulum of length L is held horizontal and then released. It collides elastically with a block of equal mass lying on a frictionless table. The kinetic energy of the block will be :

(1) Zero

(2) mgL

(3) 2mgL

(4) mgL/2

Collision : Collision with Very Heavy Bodies, Bouncing of Balls and Oblique Collision

56. A metal ball does not rebound when struck on a wall, whereas a rubber ball of same mass when thrown with the same velocity on the wall rebounds. From this it is inferred that -

(1) Change in momentum is same in both

(2) Change in momentum in rubber ball is more

(3) Change in momentum in metal ball is more

(4) Initial momentum of metal ball is more than that of rubber ball

57. A particles of mass m moving with speed v towards east strikes another particle of same mass moving with same speed v towards north. After striking the two particles fuse together. With what speed this new particle of mass 2m will move in north-east direction ?

(1) v

(2) $\frac{v}{2}$

(3) $\frac{v}{\sqrt{2}}$

(4) $v\sqrt{2}$

58. A ball strikers the floor and after collision rebounds back. In this state :
(1) Momentum of the ball is conserved
(2) Mechanical energy of the ball is conserved
(3) Momentum of ball-earth system is conserved
(4) The kinetic energy of ball-earth system is not conserved.

59. A 1 Kg ball falls from a height of 25 cm and rebounds upto a height of 9 cm. The coefficient of restitution is
(1) 0.6 (2) 0.32
(3) 0.40 (4) 0.56

60. A ball is dropped from height h on the ground level. If the coefficient of restitution is e then the height upto which the ball will go after n^{th} jump will be :
(1) $\frac{h}{e^{2n}}$ (2) $\frac{e^{2n}}{h}$
(3) he^{n} (4) he^{2n}

61. A sphere P of mass m and velocity undergoes an oblique and perfectly elastic collision with an identical sphere Q initially at rest. The angle θ between the velocities of the spheres after the collision shall be :
(1) 0 (2) 45°
(3) 90° (4) 180°

62. A ball is dropped from a height of 10 m. If 40% of its energy is lost on collision with the earth then after collision the ball will rebound to a height of
(1) 10 m (2) 8 m
(3) 4 m (4) 6 m

63. A rubber ball is dropped from a height of 5 m on a plane, where the acceleration due to the gravity is not shown. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of :
(1) $\frac{16}{25}$ (2) $\frac{2}{5}$
(3) $\frac{3}{5}$ (4) $\frac{9}{25}$

64. A ball falls from a height of 5 m and strikes the roof of a lift. If at the time of collision, lift is moving in the upward direction with a velocity of 1m/s, then the velocity with which the ball rebounds after collision will be (e = 1)
(1) 11 m/s downwards
(2) 12 m/s upwards
(3) 13 m/s upwards
(4) 12 m/s downwards

65. A big ball of mass M, moving with velocity u strikes a small ball of mass m, which is at rest. Finally small ball attains velocity u and big ball v. What is the value of v :
(1) $\frac{M-m}{M}u$ (2) $\frac{m}{M+m}u$
(3) $\frac{2m}{M+m}$ (4) $\frac{M}{M+m}v$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	2	3	2	2	3	2	2	4	3	4	2	3	3	1	1	2	4	3	3	1	3	4	3	4	3
Que.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Ans.	3	1	3	3	2	1	1	2	3	4	3	1	3	4	2	1	2	2	3	2	2	3	2	1	3
Que.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65										
Ans.	3	2	2	3	2	2	3	3	1	4	3	4	2	2	1										

Exercise - II

1. A sphere of diameter r is cut from a solid sphere of radius r such that the centre of mass of remaining part be at maximum distance from original centre, then this distance is :

(1) $\frac{r}{2}$ (2) $\frac{r}{3}$

(3) $\frac{r}{14}$ (4) None of these

2. A bullet of mass m is fired from a gun of mass M. The recoiling gun compresses a spring of force constant k by a distance d. Then the velocity of the bullet is :

(1) $kd\sqrt{M/m}$ (2) $\frac{d}{m}\sqrt{km}$

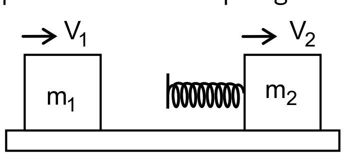
(3) $\frac{d}{m}\sqrt{kM}$ (4) $\frac{kM}{m}\sqrt{d}$

3. A Shell is fired from a canon with velocity V m/s at an angle θ with the horizontal direction. At the highest point in its path with same speed it explodes into two pieces of equal masses. One of the pieces retraces its path to the cannon. The speed in m/sec of the other piece immediately after the explosion is

(1) $\left(\frac{\sqrt{3}}{2}\right)V\cos\theta$ (2) $3V\cos\theta$

(3) $2V\cos\theta$ (4) $\left(\frac{3}{2}\right)V\cos\theta$

4. Two masses m_1 = 2 kg and m_2 = 5 kg are moving on a frictionless surface with velocities 10 m/s and 3 m/s respectively. m_2 is ahead of m_1. An ideal spring of spring constant k = 1120 N/m is attached on the back side of m_2. The maximum compression of the spring will be :-



(1) 0.51 m (2) 0.062 m
(3) 0.25 m (4) 0.72 m

5. A bomb of mass m = 1 kg thrown vertically upwards with a speed u = 100 m/s explodes into two parts after t = 5 s. A fragment of mass m_1 = 400 g moves downwards with a speed v_1 = 25 m/s, then speed v_2 and direction of another mass m_2 will be :-
(1) 40 m/s downwards
(2) 40 m/s upwards
(3) 60 m/s upwards
(4) 100 m/s upwards

6. A frictionless steel ball of radius 2 cm, moving on a horizontal plane with a velocity of 5 cm/s, collides head-on with another stationary steel ball of radius 3 cm respectively be (in cm/s) (e = 1) :-
(1) 2.7, 2.3 (2) −2.7, 2.3
(3) 2.7, −2.3 (4) −2.7, −2.3

7. A body of mass 2.0 kg makes an elastic collision with another body at rest and continues to move in the original direction but with one-fourth of its original speed v. What is the mass of other body and the speed of the two body center of mass ?

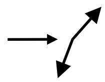
(1) 1.0 kg and $\frac{2}{3}v$

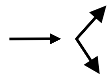
(2) 1.2 kg and $\frac{5}{8}v$

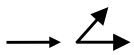
(3) 1.4 kg and $\frac{10}{17}v$

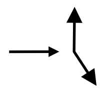
(4) 1.5 kg and $\frac{4}{7}v$

8. In the diagrams given below the horizontal line represents the path of a ball coming from left and hitting another ball which is initially at rest. The other two lines represents the paths of the two balls after the collision. Which is initially at rest. The other two lines represents the paths of the two balls after the collision. Which of the diagram shows a physically impossible situation ?

(1) 

(2) 

(3) 

(4) 

9. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i}+2\hat{j}+\hat{k}$ and $-3\hat{i}-2\hat{j}+\hat{k}$, respectively. The centre of mass of this system has a position vector :

(1) $-\hat{i}+\hat{j}+\hat{k}$
(2) $-2\hat{i}+2\hat{k}$
(3) $-2\hat{i}-\hat{j}+\hat{k}$
(4) $2\hat{i}-\hat{j}-2\hat{k}$

10. An explosion blows a rock into three parts. Two parts go off at right angles to each other. These two are, 1 kg first part moving with a velocity of 12 ms^{-1} and 2 kg second part moving with a velocity of 8 ms^{-1}. If the third part files off with a velocity of 4 ms^{-1}, its mass would be :

(1) 3 kg
(2) 5 kg
(3) 7 kg
(4) 17 kg

11. A ball moving with velocity 2 m/s collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0.5, then their velocities (in m/s) after collision will be :

(1) 0, 2
(2) 0, 1
(3) 1, 1
(4) 1, 0.5

12. Two particles which are initially at rest, move towards each other under the action of their internal attraction. If their speeds are v and 2v at any instant, then the speed of centre of mass of the system will be :

(1) v
(2) 2v
(3) Zero
(4) 1.5 v

13. A ball of mass 100 g is projected vertically upwards from the ground with a velocity of 49 m/s. At the same time another identical ball is dropped from a height of 98 m to fall freely along the same path as followed by the first ball. After sometime the two balls collide and stick together. The velocity of the combined mass just after the collision is :-

(1) 4.9 m/s upward
(2) 4.9 m/s downward
(3) 9.8 m/s upward
(4) 9.8 m/s downward

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	3	3	2	3	4	2	2	3	3	2	2	3	1

Exercise – III (Previous Year Question)

1. Two persons of masses 55 kg and 65 kg respectively, are at the opposite ends of a boat. The length of the boat is 3.0 m and weights 100 kg. The 55 kg man weight up to the 65 kg man and sits with him. If the boat is in still water the centre of mass of the system shifts by : **[AIPMT (Pre)- 2012]**

(1) zero (2) 0.75 m

(3) 3.0 m (4) 2.3 m

2. Two sphere A and B of masses m_1 and m_2 respectively collide. A is at rest initially and B is moving with velocity v along x-axis. After collision B has a velocity $\frac{v}{2}$ in a direction perpendicular to the original direction. The mass A moves after collision in the direction.

[AIPMT (Pre)- 2012]

(1) $\theta = \tan^{-1}(1/2)$ to the x-axis

(2) $\theta = \tan^{-1}(-1/2)$ to the x-axis

(3) same as that of B

(4) opposite to that of B

3. Two particles of masses m_1, m_2 move with initial velocities u_1 and u_2. On collision, one of the particles get excited to higher level, after absorbing energy ε. If final velocities of particles be v_1 and v_2 then we must have: **[AIPMT - 2015]**

(1) $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \varepsilon$

(2) $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \varepsilon = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

(3) $\frac{1}{2}m_1^2u_1^2 + \frac{1}{2}m_2^2u_2 - \varepsilon = m_1^2v_1 + m_2^2v_2$

(4) $m_1^2u_1 + m_2^2u_2 - \varepsilon = m_1^2v_1 + m_2^2v_2$

4. Two spherical bodies of mass M and 5M and radii R and 2R are released in free space with initial separation between their centres space with initial separation between their centres equal to 12R. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is :

[AIPMT - 2015]

(1) 4.5R (2) 7.5R

(3) 1.5R (4) 2.5R

5. A ball is thrown vertically downwards from a height of 20 m with an initial velocity v_0 and losses 50% of its energy. After collision ball again reaches to same height. Find initial velocity of ball: (Take $g = 10\ ms^{-2}$). **[Re-AIPMT - 2015]**

(1) $10\ ms^{-1}$ (2) $14\ ms^{-1}$

(3) $20\ ms^{-1}$ (4) $28\ ms^{-1}$

6. On a frictionless surface, a block of mass. M moving at speed v collides elastically with another block of same mass M which is initially at rest. After collision the first block moves at an angle θ to its initial direction and has a speed $\frac{v}{3}$. The second block's speed after the collision is :

[Re-AIPMT - 2015]

(1) $\frac{\sqrt{3}}{2}v$ (2) $\frac{2\sqrt{2}}{3}v$

(3) $\frac{3}{4}v$ (4) $\frac{3}{\sqrt{2}}v$

7. A bullet of mass 10g moving horizontally with a velocity of 400 m/s strikes a wooden block of mass 2 kg which is suspended by a light inextensible string of length 5 m. As a result, the centre of gravity of the block is found to rise a vertical distance of 10 cm. The speed of the bullet after it emerges out horizontally from the block will be : **[NEET - 2016]**

(1) 120 m/s (2) 160 m/s

(3) 100 m/s (4) 80 m/s

8. Two identical balls A and B having velocities of 0.5 m/s and –0.3 m/s respectively collide elastically in one dimension. The velocities of B and A after the collision respectively will be :

[NEET - 2016]

(1) –0.3 m/s and 0.5 m/s
(2) 0.3 m/s and 0.5 m/s
(3) –0.5 m/s and 0.3 m/s
(4) 0.5 m/s and –0.3 m/s

9. A moving block having mass m, collides with another stationary block having mass 4m. The lighter block comes to rest after collision. When the initial velocity of the lighter block is v, then the value of coefficient of restitution (e) will be :

[NEET - 2018]

(1) 0.5 (2) 0.25
(3) 0.8 (4) 0.4

10. Body A of mass 4m moving with speed u collides with another body B of mass 2m, at rest. The collision is head on and elastic in nature. After the collision the fraction of energy lost by the colliding body A is :

[NEET - 2019]

(1) $\frac{1}{9}$ (2) $\frac{8}{9}$
(3) $\frac{4}{9}$ (4) $\frac{5}{9}$

11. An object flying in air with velocity $\left(20\hat{i}+25\hat{j}-12\hat{k}\right)$ suddenly breaks in two pieces whose masses are in the ratio 1 : 5. The smaller mass flies off with a velocity $\left(100\hat{i}+35\hat{j}+8\hat{k}\right)$. The velocity of the larger piece will be : **[NEET - 2019 (Odisha)]**

(1) $4\hat{i}+23\hat{j}-16\hat{k}$ (2) $-100\hat{i}-35\hat{j}-8\hat{k}$
(3) $20\hat{i}+15\hat{j}-80\hat{k}$ (4) $-20\hat{i}-15\hat{j}-80\hat{k}$

12. A particle of mass 5 m at rest suddenly breaks on its own into three fragments. Two fragments of mass m each move along mutually perpendicular direction with speed v each. The energy released during the process is :

[NEET - 2019 (Odisha)]

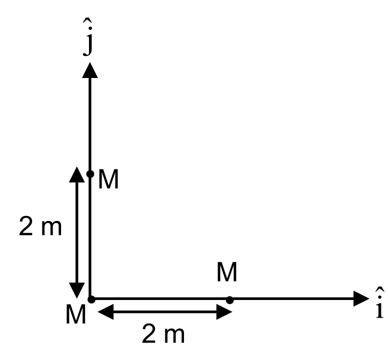
(1) $\frac{3}{5}mv^2$ (2) $\frac{5}{3}mv^2$
(3) $\frac{3}{2}mv^2$ (4) $\frac{4}{3}mv^2$

13. Two particles of mass 5 kg and 10 kg respectively are attached to the two ends of a rigid rod of length 1 m with negligible mass. The centre of mass of the system from the 5 kg particle is nearly at a distance of : **[NEET - 2020]**

(1) 67 cm (2) 80 cm
(3) 33 cm (4) 50 cm

14. Three identical spheres, each of mass M, are placed at the corners of a right angle triangle with mutually perpendicular sides equal to 2 m (see figure). Taking the point of intersection of the two mutually perpendicular sides as the origin, find the position vector of centre of mass.

[NEET - Covid - 2020]



(1) $2\left(\hat{i}+\hat{j}\right)$ (2) $\left(\hat{i}+\hat{j}\right)$
(3) $\frac{2}{3}\left(\hat{i}+\hat{j}\right)$ (4) $\frac{4}{3}\left(\hat{i}+\hat{j}\right)$

15. A shell of mass m is at rest initially. It explodes into three fragments having mass in the ratio 2 : 2 : 1. If the fragments having equal mass fly off along mutually perpendicular directions with speed ν, the speed of the third (lighter) fragment is: **[NEET - 2022]**

(1) υ (2) $\sqrt{2}\,\upsilon$

(3) $2\sqrt{2}\,\upsilon$ (4) $3\sqrt{2}\,\upsilon$

16. Two objects of mass 10kg and 20kg respectively are connected to the two ends of a rigid rod of length 10m with negligible mass. The distance of the center of mass of the system from the 10kg mass is: **[NEET - 2022]**

(1) $\frac{10}{3}$m (2) $\frac{20}{3}$m

(3) 10m (4) 5m

ANSWER KEY																
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Ans.	1	1,2	2	2	3	2	1	4	2	2	1	4	1	3	3	2