Chapter 3

Algebra

Ex 3.1

Question 1.

Fill in the blanks.
1. $(p - q)^2 =$ _______
2. The product of $(x + 5)$ and $(x - 5)$ is _______
3. The factors of $x^2 - 4x + 4$ are _______
4. Express $24ab^2c^2$ as product of its factors is _______

Answers:
1. $p^2 - 2pq + q^2$
2. $x^2 - 25$
3. $(x - 2)$ and $(x - 2)$
4. $2 \times 2 \times 2 \times 3 \times a \times b \times b \times c \times c$

Question 2.
Say whether the following statements are True or False.
(i) $(7x + 3)(7x - 4) = 49x^2 - 7x - 12$
(ii) $(a - 1)^2 = a^2 - 1$.
(iii) $(x^2 + y^2)(y^2 + x^2) = (x^2 + y^2)^2$
(iv) 2p is the factor of 8pq.

Answers:
(i) True
(ii) False
(iii) True
(iv) True

Question 3.
Express the following as the product of its factors.
(i) $24ab^2c^2$
(ii) $36x^3y^2z$
(iii) $56mn^2p^2$

Solution:
(i) $24ab^2c^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b \times c \times c$

(ii) $36\,x^3y^2z = 2 \times 2 \times 3 \times 3 \times x \times x \times x \times y \times y \times z$
(iii) $56\,mn^2p^2 = 2 \times 2 \times 2 \times 7 \times m \times n \times n \times p \times p$

Question 4.
Using the identity (x + a)(x + b) – x2 + x(a + b) + ab, find the following product.
(i) $(x + 3)(x + 7)$
(ii) $(6a + 9)(6a – 5)$
(iii) $(4x + 3y)(4x + 5y)$
(iv) $(8 + pq)(pq + 7)$

Solution:
(i) $(x + 3)(x + 7)$
Let a = 3; b = 7, then
$(x + 3)(x + 7)$ is of the form $x^2 + x(a + b) + ab$
$(x + 3)(x + 7) = x^2 + x(3 + 7) + (3 \times 7) = x^2 + 10x + 21$

(ii) $(6a + 9)(6a – 5)$
Substituting x = 6a ; a = 9 and b = -5
In $(x + a)(x + b) = x^2 + x(a + b) + ab$, we get
$(6a + 9)(6a – 5) = (6a)^2 + 6a(9 + (-5)) + (9 \times (-5))$
$6^2 a^2 + 6a(4) + (-45) = 36a^2 + 24a – 45$
$(6a + 9)(6a – 5) = 36a^2 + 24a – 45$

(iii) $(4x + 3y)(4x + 5y)$
Substituting x = 4x ; a = 3y and b = 5y in
$(x + a)(x + b) = x^2 + x(a + b) + ab$, we get
$(4x + 3y)(4x – 5y) = (4x)^2 + 4x(3y + 5y) + (3y)(5y)$
$= 4^2 x^2 + 4x(8y) + 15y^2 = 16x^2 + 32xy + 15y^2$
$(4x + 3y)(4x + 5y) = 16x^2 + 32xy + 15y^2$

(iv) $(8 + pq)(pq + 7)$
Substituting x = pq ; a = 8 and b = 7 in
$(x + a)(x + b) = x^2 + x(a + b) + ab$, we get
$(pq + 8)(pq + 7) = (pq)^2 + pq(8 + 7) + (8)(7)$
$= p^2 q^2 + pq(15) + 56$
$(8 + pq)(pq + 7) = p^2 q^2 + 15pq + 56$

Question 5.
Expand the following squares, using suitable identities.
(i) $(2x + 5)^2$
(ii) $(b – 7)^2$
(iii) $(mn + 3p)^2$
(iv) $(xyz – 1)^2$

Solution:
(i) $(2x + 5)^2$
Comparing $(2x + 5)^2$ with $(a + b)^2$ we have a = 2x and b = 5
a = 2x and b = 5,
$(a + b)^2 = a^2 + 2ab + b^2$
$(2x + 5)^2 = (2x)^2 + 2(2x)(5) + 5^2 = 2^2 x^2 + 20x + 25$
$= 2^2 x^2 + 20x + 25$
$(2x + 5)^2 = 4x^2 + 20x + 25$

(ii) $(b - 7)^2$
Comparing $(b - 7)^2$ with $(a - b)^2$ we have a = b and b = 7
$(a - b)^2 = a^2 - 2ab + b^2$
$(b - 7)^2 = b^2 - 2(b)(7) + 7^2$
$(b - 7)^2 = b^2 - 14b + 49$

(iii) $(mn + 3p)^2$
Comparing $(mn + 3p)^2$ with $(a + b)^2$ we have
$(a + b)^2 = a^2 + 2ab + b^2$
$(mn + 3p)^2 = (mn)^2 + 2(mn)(3p) + (3p)^2$
$(mn + 3p)^2 = m^2 n^2 + 6mnp + 9p^2$

(iv) $(xyz - 1)^2$
Comparing $(xyz - 1)^2$ with $(a - b)^2$ we have = a + xyz and b = 1
a = xyz and b = 1
$(a - b)^2 = a^2 - 2ab + b^2$
$(xyz - 1)^2 = (xyz)^2 - 2(xyz)(1) + 1^2$
$(xyz -1)^2 = x^2 y^2 z^2 - 2xyz + 1$

Question 6.
Using the identity (a + b)(a – b) = a2 – b2, find the following product.
(i) (p + 2) (p – 2)
(ii) (1 + 3b) (3b – 1)
(iii) (4 – mn) (mn + 4)
(iv) (6x + 7y) (6x – 7y)

Solution:
(i) (p + 2) (p – 2)
Substituting a = p ; b = 2 in the identity $(a + b)(a - b) = a^2 - b^2$, we get
$(p + 2)(p - 2) = p^2 - 2^2$

(ii) (1 + 3b)(3b – 1)
(1 + 3b) (3b -1) can be written as (3b + 1) (3b – 1)
Substituting a = 36 and b = 1 in the identity
$(a + b)(a - b) = a^2 - b^2$, we get

$(3b + 1)(3b - 1) = (3b)^2 - 1^2 = 3^2 \times b^2 - 1^2$
$(3b + 1)(3b - 1) = 9b^2 - 1^2$

(iii) $(4 - mn)(mn + 4)$
$(4 - mn)(mn + 4)$ can be written as $(4 - mn)(4 + mn) = (4 + mn)(4 - mn)$
Substituting $a = 4$ and $b = mn$ is
$(a + b)(a - b) = a^2 - b^2$, we get
$(4 + mn)(4 - mn) = 4^2 - (mn)^2 = 16 - m^2 n^2$

(iv) $(6x + 7y)(6x - 7y)$
Substituting $a = 6x$ and $b = 7y$ in
$(a + b)(a - b) = a^2 - b^2$, We get
$(6x + 7y)(6x - 7y) = (6x)^2 - (7y)^2 = 6^2x^2 - 7^2y^2$
$(6x + 7y)(6x - 7y) = (6x)^2 - (7y)^2 = 6^2x^2 - 7^2y^2$
$(6x + 7y)(6x - 7y) = 36x^2 - 49y^2$

Question 7.
Evaluate the following, using suitable identity.
(i) 51^2
(ii) 103^2
(iii) 998^2
(iv) 47^2
(v) 297×303
(vi) 990×1010
(vii) 51×52

Solution:
51^2
$= (50 + 1)^2$
Taking $a = 50$ and $b = 1$ we get
$(a + b)^2 = a^2 + 2ab + b^2$
$(50 + 1)^2 = 50^2 + 2(50)(1) + 1^2 = 2500 + 100 + 1$
$51^2 = 2601$

(ii) 103^2
$103^2 = (100 + 3)^2$
Taking $a = 100$ and $b = 3$
$(a + b)^2 = a^2 + 2ab + b^2$ becomes
$(100 + 3)^2 = 100^2 + 2(100)(3) + 3^2 = 10000 + 600 + 9$
$103^2 = 10609$

(iii) 998^2
$998^2 = (1000 - 2)^2$
Taking $a = 1000$ and $b = 2$

$(a - b)^2 = a^2 + 2ab + b^2$ becomes
$(1000 - 2)^2 = 1000^2 - 2\ (1000)\ (2) + 2^2$
$= 1000000 - 4000 + 4$
$998^2 = 10,04,004$

(iv) 47^2
$47^2 = (50 - 3)^2$
Taking a = 50 and b = 3
$(a - b)^2 = a^2 - 2ab + b^2$ becomes
$(50 - 3)^2 = 50^2 - 2\ (50)\ (3) + 3^2$
$= 2500 - 300 + 9 = 2200 + 9$
$47^2 = 2209$

(v) 297×303
$297 \times 303 = (300 - 3)\ (300 + 3)$
Taking a = 300 and b = 3, then
$(a + b)\ (a - b) = a^2 - b^2$ becomes
$(300 + 3)\ (300 - 3) = 300^2 - 3^2$
$303 \times 297 = 90000 - 9$
$297 \times 303 = 89,991$

(vi) 990×1010
$990 \times 1010 = (1000 - 10)\ (1000 + 10)$
Taking a = 1000 and b = 10, then
$(a - b)\ (a + b) = a^2 - b^2$ becomes
$(1000 - 10)\ (1000 + 10) = 1000^2 - 10^2$
$990 \times 1010 = 1000000 - 100$
$990 \times 1010 = 999900$

(vii) 51×52
$= (50 + 1)\ (50 + 1)$
Taking x = 50, a = 1 and b = 2
then $(x + a)\ (x + b) = x^2 + (a + b)\ x + ab$ becomes
$(50 + 1)\ (50 + 2) = 50^2 + (1 + 2)\ 50 + (1 \times 2)$
$2500 + (3)\ 50 + 2 = 2500 + 150 + 2$
$51 \times 52 = 2652$

Question 8.
Simplify: (a + b)2 – 4ab
Solution:
$(a + b)^2 - 4ab = a^2 + b^2 + 2ab - 4ab = a^2 + b^2 - 2ab = (a - b)^2$

Question 9.
Show that (m – n)2 + (m + n)2 = 2(m2 + n2)

Solution:
Taking the LHS = $(m - n)^2 + (m + n)^2$

$$= m^2 - 2mn + n^2 + m^2 + 2mn + n^2 = m^2 + n^2 + m^2 + n^2$$
$$= 2m^2 + 2n^2 \qquad [\because (a+b)^2 - 4ab = a^2 + 2ab + b^2$$
$$= 2(m^2 + n^2) = \text{RHS} \qquad (a-b)^2 = a^2 - 2ab + b^2]$$
$$\therefore (m-n)^2 + (m+n)^2 = 2(m^2 + n^2)$$

Question 10.
If a + b = 10 , and ab = 18, find the value of a2 + b2.

Solution:
We have $(a + b)^2 = a^2 + 2ab + b^2$
$(a + b)^2 = a^2 + b^2 + 2ab$
given a + b = 0 and ab = 18
$10^2 = = a^2 + b^2 + 2(18)$
$100 = = a^2 + b^2 + 36$
$100 - 36 = a^2 + b^2$
$a^2 + b^2 = 64$

Question 11.
Factorise the following algebraic expressions by using the identity a2 – b2 = (a + b)(a – b).
(i) $z^2 - 16$
(ii) $9 - 4y^2$
(iii) $25a^2 - 49b^2$
(iv) $x^4 - y^4$

Solution:
(i) $z^2 - 16$
$z^2 - 16 = z^2 - 4^2$
We have $a^2 - b^2 = (a + b)(a - b)$
let a = z and b = 4,
$z^2 - 4^2 = (z + 4)(z - 4)$

(ii) $9 - 4y^2$
$9 - 4y^2 = 3^2 - 2^2 y^2 = 3^2 - (2y)^2$
let a = 3 and b = 2y, then
$a^2 - b^2 = (a + b)(a - b)$
$\therefore 3^2 - (2y)^2 = (3 + 2y)(3 - 2y)$
$9 - 4y^2 = (3 + 2y)(3 - 2y)$

(iii) $25a^2 - 49b^2$
25a2 – 49b2 = 52 – a2 – 72 = (5a)2 – (7b)2

let $A = 5a$ and $B = 7b$
A^2 B^2
$(5a)^2 - (7b)^2 = (5a + 7b)(5a - 7b)$

(iv) $x^4 - y^4$
Let $x^4 - y^4 = (x^2)^2 - (y^2)^2$
We have $a^2 - b^2 = (a + b)(a - b)$
$(x^2)^2 - (y^2)^2 = (x^2 + y^2)(x^2 - y^2)$
$x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$
Again we have $x^2 - y^2 = (x + y)(x - y)$
$\therefore x^4 - y^4 = (x^2 + y^2)(x + y)(x - y)$

Question 12.
Factorise the following using suitable identity.
(i) $x^2 - 8x + 16$
(ii) $y^2 + 20y + 100$
(iii) $36m^2 + 60m + 25$
(iv) $64x^2 - 112xy + 49y^2$
(v) $a^2 + 6ab + 9b^2 - c^2$

Solution:
(i) $x^2 - 8x + 16$
$x^2 - 8x + 16 = x^2 - (2 \times 4 \times x) + 4^2$
This expression is in the form of identity
$a^2 - 2ab + b^2 = (a - b)^2$
$x^2 - 2 \times 4 \times x + 4^2 = (x - 4)^2$
$\therefore x^2 - 8x + 16 = (x - 4)(x - 4)$

(ii) $y^2 + 20y + 100$
$y^2 + 20y + 100 = y^2 + (2 \times (10))\, y + (10 \times 10)$
$= y^2 + (2 \times 10 \times y) + 10^2$
This is of the form of identity
$a^2 + 2\,ab + b^2 = (a + b)^2$
$y^2 + (2 \times 10 \times y) + 10^2 = (y + 10)^2$
$y^2 + 20y + 100 = (y + 10)^2$
$y^2 + 20y + 100 = (y + 10)(y + 10)$

(iii) $36m^2 + 60m + 25$
$36m^2 + 60m + 25 = 62\, m^2 + 2 \times 6m \times 5 + 5^2$
This expression is of the form of identity
$a^2 + 2ab + b^2 = \{a + b)^2$
$(6m)^2 + (2 \times 6m \times 5) + 5^2$
$= (6m + 5)^2$
$36m^2 + 60m + 25 = (6m + 5)(6m + 5)$

(iv) $64x^2 – 112xy + 49y^2$
$64x^2 – 112xy + 49y^2 = 82\ x^2 – (2 \times 8x \times 7y) + 7^2y^2$
This expression is of the form of identity
$a^2 – 2ab + b^2 = (a- b)^2$
$(8x)^2 – (2 \times 8x \times 7y) + (7y)^2 = (8x – 7y)^2$
$64x^2 – 112xy + 49y^2 = (8x – 7y)\ (8x – 7y)$

(v) $a^2 + 6ab + 9b^2 – c^2$
$a^2 + 6ab + 9b^2 – c^2 = a^2 + 2 \times a \times 3b + 3^2\ b^2 – c^2$
$= a^2 + (2 \times a \times 3b) + (3b)^2 – c^2$

This expression is of the form of identity
$[a^2 + 2ab + b^2] – c^2 = (a + b)^2 – c^2$
$a^2 + (2 \times a \times 36) + (3b)^2 – c^2 = (a + 3b)^2 – c^2$

Again this RHS is of the form of identity
$a^2 – b^2 = (a + b)\ (a – b)$
$(a + 3b)^2 – c^2 = [(a + 3b) + c]\ [(a + 3b) – c]$
$a^2 + 6ab + 9b^2 – c^2 = (a + 3b + c)\ (a + 3b – c)$

Objective Type Questions

Question 1.
If a + b = 5 and a2 + b2 = 13, then ab = ?
(i) 12
(ii) 6
(iii) 5
(iv) 13

Answer:
(ii) 6

Hint: $(a + b)^2 = 25$
$13 + 2ab = 25$
$2ab = 12$
$ab = 6$

Question 2.
(5 + 20)(-20 – 5) = ?
(i) -425
(ii) 375
(iii) -625
(iv) 0

Answer:
(iii) -625

Hint: $(50 + 20) (-20 – 5) = -(5 + 20)^2 = – (25)^2 = – 625$

Question 3.
The factors of x2 – 6x + 9 are
(i) $(x – 3)(x – 3)$
(ii) $(x – 3)(x + 3)$
(iii) $(x + 3)(x + 3)$
(iv) $(x – 6)(x + 9)$

Answer:
(i) $(x – 3)(x – 3)$

Hint: $x^2 – 6x + 9 = x^2 – 2(x) (3) + 3^2$
$a^2 – 2ab + b^2 – (a- b)^2 = (x – 3)^2 = (x – 3) (x – 3)$

Question 4.
The common factors of the algebraic expression ax2y, bxy2 and cxyz is
(i) x^2y
(ii) xy^2
(iii) xyz
(iv) x

Ans :
(iv) xy

Hint: $ax^2y = a \times x \times x \times y$
$bxy^2 = b \times x \times y \times y$
$cxyz = C \times x \times y \times z$
Common factor = xy

Ex 3.2

Question 1.
Given that x > y. Fill in the blanks with suitable inequality signs.
(i) y [ ] x
(ii) x+ 6 [ ] y + 6
(iii) x^2 [ ] xy
(iv) -xy [ ] $– y^2$

(v) x – y [ ] 0

Answer:
(i) y [$\leq$] x
(ii) x+ 6 [$\geq$] y + 6
(iii) x^2 [$\geq$] xy
(iv) -xy [$\leq$] – y^2
(v) x – y [$\geq$] 0

Question 2.
Say True or False.

(i) Linear inequation has almost one solution.

Answer: False

(ii) When x is an integer, the solution set for x < 0 are -1, -2,..

Answer:
False

(iii) An inequation, -3 < x < -1, where x is an integer, cannot be represented in the number line.

Answer:
True

(iv) x < -y can be rewritten as – y < x

Ans :
False

Question 3.
Solve the following inequations.
(i) x $\leq$ 7, where x is a natural number.
(ii) x – 6 < 1, where x is a natural number.
(iii) 2a + 3 $\leq$ 13, where a is a whole number.
(iv) 6x – 7 $\geq$ 35, where x is an integer.
(v) 4x – 9 > -33, where x is a negative integer.

Solution:

(i) $x \leq 7$, where x is a natural number.

Since the solution belongs to the set of natural numbers, that are less than or equal to 7, we take the values of x as 1,2, 3, 4, 5, 6 and 7.

(ii) $x - 6 < 1$, where x is a natural number.

$x - 6 < 1$ Adding 6 on the both the sides $x - 6 + 6 < 1 + 6$

$x < 7$

Since the solutions belongs to the set of natural numbers that are less than 7, we take the values of x as 1,2, 3, 4, 5 and 6

(iii) $2a + 3 \leq 13$, where a is a whole number.

$2a + 3 \leq 13$

Subtracting 3 from both the sides $2a + 3 - 3 \leq 13 - 3$

$2a \leq 10$

Dividing both the side by 2. $2a2 \leq 102$

$a \leq 5$

Since the solutions belongs to the set of whole numbers that are less than or equal to 5 we take the values of a as 0, 1, 2, 3, 4 and 5

(iv) $6x - 7 \geq 35$, where x is an integer.

$6x - 7 \geq 35$ Adding 7 on both the sides

$6x - 7 + 7 \geq 35 + 7$

$6x \geq 42$

Dividing both the sides by 6 we get $6x6 \geq 426$

$x \geq 7$

Since the solution belongs to the set of integers that are greater than or equal to 7, we take the values of x as 7, 8, 9, 10…

(v) $4x - 9 > -33$, where x is a negative integer.

$4x - 9 > -33 + 9$ Adding 9 both the sides

$4x - 9 + 9 > -33 + 9$

$4x > -24$

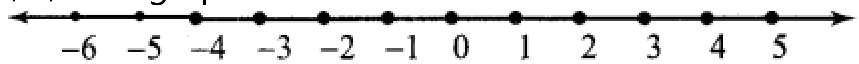
Dividing both the sides by 4

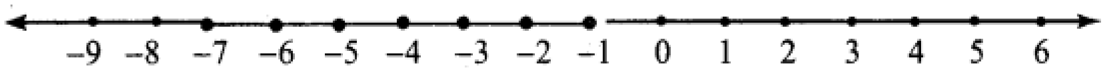
$4x4 > -244$

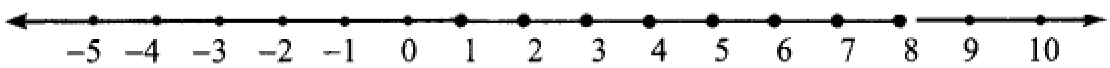
$x > -6$

Since the solution belongs to a negative integer that are greater than -6, we take values of u as -5, -4, -3, -2 and -1

Question 4.
Solve the following inequations and represent the solution on the number line:
(i) k > -5, k is an integer.
(ii) $-7 \leq y$, y is a negative integer.
(iii) $-4 \leq x \leq 8$, x is a natural number.
(iv) $3m – 5 \leq 2m + 1$, m is an integer.

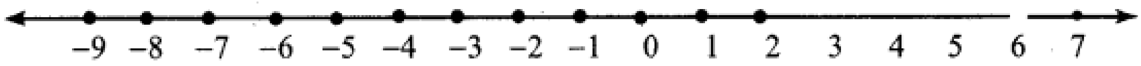
Solution:
(i) k > -5, k is an integer.
Since the solution belongs to the set of integers, the solution is -4, -3, -2, -1, 0,... It's graph on number line is shown below.


(ii) $-7 \leq y$, y is a negative integer.
$-7 \leq y$
Since the solution set belongs to the set of negative integers, the solution is -7, -6, -5, -4, -3, -2, -1.
Its graph on the number line is shown below


(iii) $-4 \leq x \leq 8$, x is a natural number.
$-4 \leq x \leq 8$
Since the solution belongs to the set of natural numbers, the solution is
1, 2, 3, 4, 5, 6, 7 and 8.
Its graph on number line is shown below


(iv) $3m – 5 \leq 2m + 1$, m is an integer.
$3m – 5 \leq 2m + 1$
Subtracting 1 on both the sides
$3m – 5 – 1 \leq 2m + 1 + 1$
$3m – 6 \leq 2m$
Subtracting 2m on both the sides $3m- 6 – 2m \leq 2m -2m$
$m – 6 \leq 0$
Adding 6 on both the sides $m – 6 + 6 \leq 0 + 6$

m ≤ 6
Since the solution belongs to the set of integers, the solution is
6, 5, 4, 3, 2, 1, 0,-1,...
Its graph on number line is shown below



Question 5.
An artist can spend any amount between ₹ 80 to ₹ 200 on brushes. If cost of each brush is ₹ 5 and there are 6 brushes in each packet, then how many packets of brush can the artist buy?

Solution:
Given the artist can spend any amount between ₹ 80 to ₹ 200
Let the number of packets of brush he can buy be x
Given cost of 1 brush = ₹ 5
Cost of 1 packet brush (6 brushes) = ₹ 5 × 6 = ₹ 30
∴ Cost of x packets of brushes = 30 x
∴ The inequation becomes 80 ≤ 30x ≤ 200
Dividing throughout by 30 we get 8030 ≤ 30x30 ≤ 20030
83 ≤ x ≤ 203 ;
2 23 ≤ x ≤ 6 23
brush packets cannot get in fractions.
∴ The artist can buy 3 ≤ x ≤ 6 packets of brushes,
or x = 3, 4, 5 and 6 packets of brushes.

Objective Type Questions

Question 1.
The solutions set of the inequation 3 < p < 6 are (where p is a natural number)
(i) 4,5 and 6
(ii) 3,4 and 5
(iii) 4 and 5
(iv) 3,4,5 and 6

Answer:
(iv) 3,4,5 and 6

Question 2.
The solution of the inequation 5x + 5 < 15 are (where x is a natural number)
(i) 1 and 2
(ii) 0,1 and 2
(iii) 2, 1,0, -1,-2
(iv) 1, 2, 3..

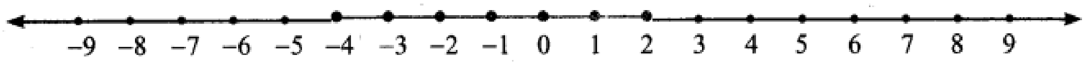
Answer:
(i) 1 and 2
Hint: $5x + 5 \leq 15$
$5x \leq 15 - 5 = 10$
$x \leq 105 = 2$

Question 3.
The cost of one pen is ₹ 8 and it is available in a sealed pack of 10 pens. If Swetha has only ₹ 500, how many packs of pens can she buy at the maximum?
(i) 10
(ii) 5
(iii) 6
(iv) 8

Answer:
(iii) 6

Hint:
Price of 1 pen = ₹ 8
Price of 1 pack = 10 × 8 = 80
Number of packs Swetha can buy = x
$80x \leq 500$
$8x \leq 50$
$x \leq 508 = 6.25$
x is a natural number x = 1, 2, 3, 4, 5, 6

Question 4.
The inequation that is represented on the number line as shown below is _______.



(i) $-4 < x < 0$
(ii) $-4 \le x \le 0$
(iii) $-4 < x \le 0$
(iv) $-4 \le x < 0$
(v) $-4 \le x \le 2$

Answer:
(v) $-4 \le x \le 2$

Ex 3.3

Miscellaneous Practice problems

Question 1.
Using identity, find the value of
(i) $(4.9)^2$
(ii) $(100.1)^2$
(iii) $(1.9) \times (2.1)$

Solution:
(i) $(4.9)^2$
$(4.9)^2 = (5 - 0.1)^2$
Substituting a = 5 and b = 0.1 in
$(a - b)^2 = a^2 - 2ab + b^2$, we have
$(5 - 0.1)^2 = 5^2 - 2(5)(0.1) + (0.1)^2$
$(4.9)^2 = 25 - 1 + 0.01 = 24 + 0.01$
$(4.9)^2 = 24.01$

(ii) $(100.1)^2$
$(100.1)^2 = (100 + 0.1)^2$
Substituting a = 100 and b = 0.1 in
$(a + b)^2 = a^2 + 2ab + b^2$, we have
$(100 + 0.1)^2 = (100)^2 + 2(100)(0.1) + (0.1)^2$
$(100.1)^2 = 10000 + 20 + 0.01$
$(100.1)^2 = 10020.01$

(iii) $(1.9) \times (2.1)$
$(1.9) \times (2.1) = (2 - 0.1) \times (2 + 0.1)$
Substituting a = 100 and b = 0.1 in

$(a - b)(a + b) = a^2 - b^2$ we have
$(2 - 0.1)(2 + 0.1) = 2^2 - (0.1)^2$
$(1.9) \times (2.1) = 4 - 0.01$
$(9.9)(2.1) = 3.99$

Question 2.
Factorise: 4x2 – 9y2

Solution:
$4x^2 - 9y^2 = 22\ x^2 - 3^2\ y^2 = (2x)^2 - (3y)^2$
Substituting $a = 2x$ and $b = 3y$ in
$(a^2 - b^2) = (a + b)(a - b)$, we have
$(2x)^2 - (3y)^2 = (2x + 3y)(2x - 3y)$
$\therefore$ Factors of $4x^2 - 9y^2$ are $(2x + 3y)$ and $(2x - 3y)$

Question 3.
Simplify using identities
(i) $(3p + q)(3p + r)$
(ii) $(3p + q)(3p - q)$

Solution:
(i) $(3p + q)(3p + r)$
Substitute $x = 3p, a = q$ and $b = r$ in
$(x + a)(x + b) = x^2 + x(a + b) + ab$
$(3p + q)(3p + r) = (3p)^2 + 3p(q + r) + (q \times r)$
$= 3^2\ p^2 + 3p(q + r) + qr$
$(3p + q)(3p + r) = 9p^2 + 3p(q + r) + qr$

(ii) $(3p + q)(3p - q)$
Substitute $a = 3p$ and $b = q$ in
$(a + b)(a - b) = a^2 - b^2$, we have
$(3p + q)(3p - q) = (3p)^2 - q^2 = 32\ p^2 - q^2$
$(3P + q)(3p - q) = 9p^2 - q^2$

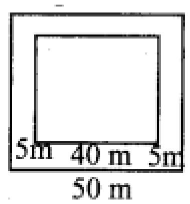
Question 4.
Show that (x + 2y)2 – (x – 2y)2 = 8xy.

Solution:
$LHS = (x + 2y)^2 - (x - 2y)^2$
$= x^2 + (2 \times x \times 2y) + (2y)^2 - [x^2 - (2 \times x \times 2y) + (2y)^2]$
$= x^2 + 4xy + 4y^2 - [x^2 - 4xy + 2^2y^2]$
$= x^2 + 4xy + 4y^2 - x^2 + 4xy - 4y^2$
$= x^2 - x^2 + 4xy + 4xy + 4y^2 - 4y^2$
$= x^2(1 - 1) + xy(4 + 4) + y^2(4 - 4)$

$= 0x^2 + 8xy + 0y^2 = 8xy = RHS$
$\therefore (x + 2y)^2 - (x - 2y)^2 = 8xy$
$[\because (a + b)^2 = a^2 + 2ab + b^2 \; (a - b)^2 = a^2 - 2ab + b^2]$

Question 5
The pathway of a square paddy field has 5 m width and length of its side is 40 m. Find the total area of its pathway. (Note: Use suitable identity)

Solution:
Given side of the square = 40 m
Also width of the pathway = 5 m
∴ Side of the larger square = 40m + 2(5)m = 40m + 10m = 50m
Area of the path way = area of large square – area of smaller square
$= 50^2 - 40^2$



Substituting a = 50 and b = 40 in
$a^2 - b^2 = (a + b)(a - b)$ we have
$50^2 - 40^2 = (50 + 40)(50 - 40)$
Area of pathway = 90×10
Area of the pathway = 900 m^2

Challenge Problems

Question 1.
If X = a2 – 1 and Y = 1 – b2, then find X + Y and factorize the same.

Solution:
Given $X = a^2 - 1$
$Y = I - b^2$
$X + Y = (a^2 - 1) + (1 - b^2)$
$= a^2 - 1 + 1 - b^2$
We know the identity that $a^2 - b^2 = (a + b)(a - b)$
$\therefore X + Y = (a + b)(a - b)$

Question 2.
Find the value of (x – y) (x + y) (x2 + y2).
Solution:
We know that $(a - b)(a + b) = a^2 - b^2$

Put a = x and b = y in the identity (1) then
$(x - y)(x + y) = x^2 - y^2$
Now $(x - y)(x + y)(x^2 + y^2) = (x^2 - y^2)(x^2 + y^2)$
Again put $a = x^2$ and $b = y^2$ in (1)
We have $(x^2 - y^2)(x^2 + y^2) = (x^2)^2 - (y^2)^2 = x^4 - y^4$
So $(x - y)(x + y)(x^2 + y^2) = x^4 - y^4$

Question 3.
Simplify (5x – 3y)2 – (5x + 3y)2.

Solution:
We have the identities
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$
So $(5x - 3y)^2 - (5x + 3y)^2 = (5x)^2 - (2 \times 5x \times 3y) + (3y)^2$
$= 5^2x^2 - 30xy + 3^2 y^2 - [5^2x^2 - 30xy + 3^2 y^2]$
$= 25x^2 - 30xy + 9y^2 - [25x^2 + 30xy + 9y^2]$
$= 25x^2 - 30xy + 9y^2 - 25x^2 - 30xy - 9y^2$
$= x^2 (25 - 25) - xy (30 + 30) + y^2 (9 - 9)$
$= 0x^2 - 60xy + 0y^2 = - 60 xy$
$\therefore (5x - 3y)^2 - (5x + 3y)^2 = -60xy$

Question 4.
Simplify : (i) (a + b)2 – (a – b)2
(ii) (a + b)2 + (a – b)2

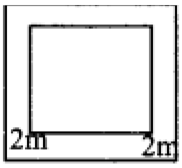
Solution:
Applying the identities
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$

(i) $(a + b)^2 - (a - b)^2 = a^2 + 2ab + b^2 - [a^2 - 2ab + b^2]$
$= a^2 + 2ab + b^2 - a^2 + 2ab - b^2$
$= a^2 (1 - 1) + ab (2 + 2) + b^2 (1 - 1)$
$= 0a^2 + 4 ab + 0b^2 = 4ab$
$(a + b)^2 - (a - b)^2 = 4ab$

(ii) $(a + b)^2 + (a - b)^2 = a^2 + 2ab + b^2 + (a^2 - 2ab + b^2)$
$= a^2 + 2ab + b^2 + a^2 - 2ab + b^2$
$= a^2 (1 + 1) + ab (2 - 2) + b^2 (1 + 1)$
$= 2a^2 + 0 ab + 2b^2 = 2a^2 + 2b^2 = 2 (a^2 + b^2)$
$\therefore (a + b)^2 - (a - b)^2 = 2 (a^2 + b^2)$

Question 5.
A square lawn has a 2 m wide path surrounding it. If the area of the path is 136 m2, find the area of lawn.

Solution:
Let the side of the lawn = a m
then side Of big square = (a + 2(2)) m
= (a + 4)m



Area of the path – Area Of large square – Area of smaller square
136 = (a + 4)2 – a2
$136 = a^2 + (2 \times a \times 4) + 4^2 - a^2$
$136 = a^2 + 8a + 16 - a^2$
136 = 8a + 16
136 = 8 (a + 2)
Dividing by 8
17 = a + 2
Subtracting 2 on both sides
17 – 3 = a + 2 – 2
15 = a
∴ side of small square = 15 m
Area of square = (side × side) Sq. units
∴ Area of the lawn = $(15 \times 15)m^2 = 225\ m^2$
∴ Area of the lawn = $225\ m^2$

Question 6.
Solve the following inequalities.
(i) $4n + 7 \geq 3n + 10$, n is an integer
(ii) $6(x + 6) \geq 5 (x - 3)$, x is a whole number.
(iii) $-13 \leq 5x + 2 \leq 32$, x is an integer.

Solution:
(i) $4n + 7 \geq 3n + 10$, n is an integer.
$4n + 7 - 3n \geq 3n + 10 - 3n$
$n(4 - 3) + 7 \geq 3n + 10 - 3n$
$n (4 - 3) + 7 \geq n (3 - 3) + 10$
$n + 7 \geq 10$
Subtracting 7 on both sides
$n + 7 - 7 \geq 10 - 7$
$n \geq 3$

Since the solution is an integer and is greater than or equal to 3, the solution will be 3,
4, 5, 6, 7, …..
n = 3, 4, 5, 6,7, ….

(ii) 6 (x + 6) ≥ 5 (x – 3), x is a whole number.
6x + 36 ≥ 5x – 15
Subtracting 5x on both sides
6x + 36 – 5x ≥ 5x – 15 – 5x
x (6 – 5) + 36 ≥ x(5 – 5) – 15
x + 36 ≥ -15
Subtracting 36 on both sides
x + 36 – 36 ≥ -15 -36
x ≥ -51
The solution is a whole number and which is greater than or equal to -51
∴ The solution is 0, 1, 2, 3, 4,…
x = 0,1,2, 3,4,…

(iii) -13 ≤ 5x + 2 ≤ 32, x is an integer.
Subtracting throughout by 2
-13 – 2 ≤ 5x + 2 – 2 ≤ 32 – 2
-15 ≤ 5x ≤ 30
Dividing throughout by 5
−155 ≤ 5x5 ≤ 305
– 3 ≤ x ≤ 6
∴ Since the solution is an integer between -3 and 6 both inclusive, we have the

solution
as -3, -2, -1,0, 1,2, 3, 4, 5, 6.
i.e. x = -3, -2, 0, 1, 2, 3,4, 5 and 6.