23 Chapter

DIFFERENTIAL EQUATIONS

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. The order 'O' and degree D of the differential equation

$$y = 1 + x\left(\frac{dy}{dx}\right) + \frac{x^2}{2!}\left(\frac{dy}{dx}\right)^2 + ... + \frac{x^n}{n!}\left(\frac{dy}{dx}\right)^n + ...\infty$$ are given

(a) $O = 1, D = 1$
(b) $O = 1, D = 0$
(c) $O = 1$, D is not defined
(d) $O = 1$, D cannot be determined

2. Solution of the differential equation,

$2y\sin x\frac{dy}{dx} = 2\sin x\cos x - y^2\cos x$ satisfying

$y\left(\frac{\pi}{2}\right) = 1$ is given by

(a) $y^2 = \sin x$ (b) $y = \sin^2 x$
(c) $y^2 = \cos x + 1$ (d) $y^2 \sin x = 4\cos^2 x$

3. The solution of $x^3\frac{dy}{dx} + 4x^2\tan y = e^x \sec y$ satisfying $y(1) = 0$, is

(a) $\tan y = (x-2)e^x \ln x$
(b) $\sin y = e^x(x-1)x^{-4}$
(c) $\tan y = (x-1)e^x x^{-3}$
(d) $\sin y = e^x(x-1)x^{-3}$

4. The solution of $y(2x^2y + e^x)dx - (e^x + y^3)dy = 0$, if $y(0) = 1$, is

(a) $6e^x + 4x^3y - 3y^3 - 3y = 0$
(b) $y^2e^x - 4xy - 3x^3 - 3 = 0$
(c) $x^2e^x - 4x^3y - 3xy^3 - 3x = 0$
(d) None of these

5. The solution of $\frac{xdy}{x^2+y^2} = \left(\frac{y}{x^2+y^2} - 1\right)dx$ is

(a) $y = x\cot(c-x)$ (b) $\cos^{-1}\left(\frac{y}{x}\right) = -x + c$
(c) $y = x\tan(c-x)$ (d) $\frac{y^2}{x^2} = x\tan(c-x)$

6. The solution of $ye^{-x/y}dx - (xe^{(-x/y)} + y^3)dy = 0$ is

(a) $e^{-x/y} + y^2 = C$ (b) $xe^{-x/y} + y = C$
(c) $2e^{-x/y} + y^2 = C$ (d) $e^{-x/y} + 2y^2 = C$

7. If $\phi(x)$ is a differentiable function then the solution of $dy + (y\,\phi'(x) - \phi(x)\phi'(x))dx = 0$ is

(a) $y = (\phi(x) - 1) + Ce^{-\phi(x)}$
(b) $y\,\phi(x) = (\phi(x))^2 + C$
(c) $ye^{\phi(x)} = \phi(x)e^{\phi(x)} + C$
(d) $(y - \phi(x)) = \phi(x)e^{-\phi(x)} + C$

8. The solution of

$(y(1 + x^{-1}) + \sin y)dx + (x + \ln x + x\cos y)dy = 0$ is

(a) $(1 + y^{-1}\sin y) + x^{-1}\ln x = C$
(b) $(y + \sin y) + xy\ln x = C$
(c) $xy + y\ln x + x\sin y = C$
(d) None of these

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ		

9. The solution of $y^5 x + y - x\frac{dy}{dx} = 0$ is

(a) $\frac{x^4}{4} + \frac{x^5}{5y^5} = C$ (b) $\frac{x^5}{5} + \frac{x^4}{4y^4} = C$

(c) $\frac{x^5}{y^5} + \frac{x^5}{5y^5} = C$ (d) $x^4 y^4 + \frac{x^5}{5} = C$

10. The solution of the equation

$\frac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1$ is

(a) $\frac{1}{x+y} = x^2 + 1 + Ce^x$

(b) $\frac{1}{(x+y)^2} = x^2 + 1 + Ce^{x^2}$

(c) $\frac{1}{(x+y)^2} = x + 1 + Ce^x$

(d) $\frac{1}{x+y} = x + 1 + Ce^{x^2}$

11. The solution of $\frac{xdx + ydy}{xdy - ydx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2 + y^2}}$ is

(a) $\sqrt{x^2 + y^2} = a\left\{\sin\left(\tan^{-1}\frac{y}{x} + C\right)\right\}$

(b) $\sqrt{x^2 + y^2} = a\cos\left\{\left(\tan^{-1}\frac{y}{x} + C\right)\right\}$

(c) $\sqrt{x^2 + y^2} = a\left\{\tan\left(\sin^{-1}\frac{y}{x} + C\right)\right\}$

(d) None of these

12. A particle of mass m is moving in a straight line is acted on by an attractive force $\frac{mk^2a^2}{x^2}$ for $x \geq a$ and $\frac{2mk^2x}{a}$ for $x < a$. If the particle starts from rest at the point $x = 2a$, then it will reach the point $x = 0$ with a speed

(a) $k\sqrt{a}$ (b) $k\sqrt{2a}$

(c) $k\sqrt{3a}$ (d) $\frac{1}{k}\sqrt{a}$

13. The real value of m for which the substitution $y = u^m$ will transform the differential equation

$2x^4 y\frac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation is

(a) $m = 0$ (b) $m = 1$

(c) $m = \frac{3}{2}$ (d) $m = \frac{2}{3}$

14. Solution of the differential equation

$x = 1 + xy\frac{dy}{dx} + \frac{x^2y^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{x^3y^3}{3!}\left(\frac{dy}{dx}\right)^3 + ...$ is

(a) $y = \ln(x) + c$ (b) $y = (\ln x)^2 + c$

(c) $y = \pm\ln(x) + c$ (d) $xy = x^y + c$

15. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{xy[x^2 \sin y^2 + 1]}$ is

(a) $x^2(\cos y^2 - \sin y^2 - 2Ce^{-y^2}) = 2$

(b) $y^2(\cos x^2 - \sin y^2 - 2Ce^{-y^2}) = 2$

(c) $x^2(\cos y^2 - \sin y^2 - e^{-y^2}) = 4C$

(d) None of these

16. The solution of $y = 2x\left(\frac{dy}{dx}\right) + x^2\left(\frac{dy}{dx}\right)^4$ is

(a) $y = 2c^{1/2}x^{1/4} + c$ (b) $y = 2\sqrt{c}x^2 + c^2$

(c) $y = 2\sqrt{c}(x+1)$ (d) $y = 2\sqrt{cx} + c^2$

17. Solution of the differential equation $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ is

(a) $e^y = e^x - 1 + ce^{-x}$ (b) $e^x = e^y + ce^{-y} + 1$

(c) $e^y = e^x - 1 + ce^{-e^x}$ (d) None of these

18. Solution of the differental equation

$x\cos\left(\frac{y}{x}\right)(ydx + xdy) = y\sin\left(\frac{y}{x}\right)(xdy - ydx)$ is

(a) $y = cx\cos\left(\frac{x}{y}\right)$ (b) $\sec\left(\frac{y}{x}\right) = cxy$

(c) $\left(\frac{y}{x}\right)\sec\left(\frac{y}{x}\right) = c$ (d) none of these

MARK YOUR RESPONSE					
	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ
	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ

19. The solution of $\sin y\left(\frac{dy}{dx}\right) = \cos y(1 - x\cos y)$ is

(a) $\sec y = x + 1 + ce^{x}$ (b) $\sec y = c(x+1) + e^{-x}$

(c) $e^{x} + \sec y = cx$ (d) All correct

20. Solution of the differential equation

$$\left\{\frac{1}{x} - \frac{y^2}{(x-y)^2}\right\}dx + \left\{\frac{x^2}{(x-y)^2} - \frac{1}{y}\right\}dy = 0 \text{ is}$$

(a) $\ln\left|\frac{x}{y}\right| + \frac{xy}{x-y} = c$ (b) $\frac{xy}{x-y} = ce^{x/y}$

(c) $\ln |xy| = c + \frac{xy}{x-y}$ (d) None of these

21. The solution of the differential equation

$$x^3 \frac{dy}{dx} = y^3 + y^2\sqrt{y^2 - x^2} \text{ is}$$

(a) $y + \sqrt{y^2 - x^2} = cxy$ (b) $y - \sqrt{y^2 - x^2} = cxy$

(c) $y\sqrt{y^2 - x^2} = cx + y$ (d) $x\sqrt{y^2 - x^2} = cx + y$

22. The family of curves whose tangents form an angle of $\frac{\pi}{4}$ with the hyperbolas $xy = C$ are

(a) pair of straight lines (b) $y^2 - xy - x^2 = C$

(c) $y^2 - 2xy - x^2 = C$ (d) $y^2 + 2xy - x^2 = C$

23. Through any point (x, y) of a curve which passes through the origin, lines are drawn parallel to the co-ordinate axes. The curve, given that it divides the rectangle formed by the two lines and the axes into two areas, one of which is twice the other, represents a family of
(a) circles
(b) pair of straight lines
(c) parabolas
(d) rectangular hyperbolas

24. A curve $f(x)$ passes through the point $P(1, 1)$. The normal to the curve at point P is $a(y-1) + (x-1) = 0$. If the slope of the tangent at any point on the curve is proportional to the ordinate at that point, then the equation of the curve is

(a) $y = e^{ax} - 1$ (b) $y - 1 = e^{ax}$

(c) $y = e^{a(x-1)}$ (d) $y - a = e^{ax}$

25. If the length of the portion of the normal intercepted between a curve and the x-axis varies as the square of the ordinate, then the curve is given by

(a) $kx + \sqrt{1 - k^2x^2} = Ce^{ky}$ (b) $ky + \sqrt{k^2y^2 - 1} = ce^{kx}$

(c) $ky + \sqrt{k^2x^2 - 1} = ce^{xy}$ (d) None of these

[k is the constant of proportionality and c is any orbitrary constant]

26. The general solution of the differential equation :

$$\frac{dy}{dx} = xy\,(x^2y^2 - 1) \text{ is}$$

(a) $e^{x^2}y^2 = (x^2 + 1) + ce^{x^2}$ (b) $y^{-2} = x^2 + 1 + ce^{x^2}$

(c) $cy^2 = (x^2 + 1)e^{-x^2}$ (d) none of these

27. A tangent and a normal to a curve at any point P meet the x and y axes at A, B and C, D respectively. If the centre of circle through O, C, P and B lies on the line $y = x$ (O is the origin) then the differential equation of all such curves is :

(a) $\frac{dy}{dx} = \frac{y-x}{y+x}$ (b) $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$

(c) $\frac{dy}{dx} = \frac{x-y}{xy}$ (d) none of these

28. If $y(x)$ is the solution of the differential equation $\frac{dy}{dx} = -2x(y-1)$ with $y(0) = 1$ then $\lim\limits_{x \to \infty} y(x)$ equals

(a) 0 (b) 1
(c) e (d) ∞

29. Let $y = f(x)$ be a curve passing through (e, e^e), which satisfy the differential equation $(2ny + xy\log_e x)\,dx - x\log_e x\,dy = 0$, $x > 0, y > 0$. If $g(x) = \lim\limits_{n \to \infty} f(x)$ then $\int_{1/e}^{e} g(x)\,dx$ equals to

(a) e (b) 1
(c) 0 (d) none of these

30. The curve, which satisfies the differential equation $\frac{xdy - ydx}{xdy + ydx} = y^2 \sin(xy)$ and passes through (0, 1), is given by

(a) $y(1 - \cos xy) + x = 0$ (b) $\sin xy - x = 0$
(c) $\sin y + y = 0$ (d) $\cos xy - 2y = 0$

MARK YOUR RESPONSE					
	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ
	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ
	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ			

31. The solution of the differential equation

$x\frac{dy}{dx} = -\frac{y}{2} - \frac{\sin 2x}{2y}$ is given by

(a) $xy^2 = \cos^2 x + c$ (b) $xy^2 = \sin^2 x + c$
(c) $yx^2 = \cos^2 x + c$ (d) None of these

32. If x-intercept of any tangent is 3 times the x-coordinate of the point of tangency, then the equation of the curve lying in the first quadrant, given that it passes through (1, 1) is

(a) $y = \frac{1}{x}$ (b) $y = \frac{1}{x^2}$

(c) $y = \frac{1}{\sqrt{x}}$ (d) $y = \sqrt{x}$

33. The radius of a right circular cylinder increases at a constant rate $1/3\pi$ cm/s. Its altitude is a linear function of the radius and increases three times as fast as radius. When the radius is 1 cm then the altitude is 6 cm. When the radius is 3 cm, the volume is increasing at a rate of

(a) 12 cm^3/sec (b) 22 cm^3/sec
(c) 30 cm^3/sec (d) 33 cm^3/sec

34. If a curve C has the property that if the tangent drawn at any point P on C meets the coordinate axes at A and B, and P is the mid-point of AB, then the curve if it passes through (1, 1), is

(a) $xy = 1$ (b) $x^2y = 1$
(c) $xy^2 = 1$ (d) $y^2 = x$

35. The solution of $\frac{dy}{dx}\left(x^2y^3 + xy\right) = 1$ is

(a) $\frac{1}{x} = 2 - y^2 + ce^{-y^2/2}$

(b) $xy^2 + 2 = cxe^{-y^2/2}$

(c) $\frac{c}{x} = 1 - y^2 + e^{-y/2}$

(d) $\frac{1-2x}{2} = -y^2 + ce^{-y^2/2}$

36. The solution of the differential equation

$\left\{y\left(1+\frac{1}{x}\right) + \cos y\right\}dx + (x + \log x - x\sin y)\,dy = 0$ is

(a) $xy - y\log x + x\cos y = c$
(b) $xy + y\log x + x\cos y = c$
(c) $xy - y\log x - x\log y = c$
(d) none of these

37. The solution of the differential equation

$xy\log\left(\frac{x}{y}\right)dx + \left\{y^2 - x^2\log\left(\frac{x}{y}\right)\right\}dy = 0$ is

(a) $\frac{x^2}{2y^2}\log\left(\frac{x}{y}\right) - \frac{x^2}{4y^2} = -\log y + \log c$

(b) $\frac{x^2}{2y^2}\log\left(\frac{x}{y}\right) + \frac{x^2}{4y^2} = \log y + \log c$

(c) $\log\left(\frac{x}{y}\right) - \frac{x^2}{4y^2} + \frac{x^2}{2y^2}\log y + \log c = 0$

(d) None of these

38. Lef $f(x)$ be a positive, continuous and differentiable function on the interval (a,b). If $\lim_{x\to a^+} f(x) = 1$ and $\lim_{x\to b^-} f(x) = 3^{1/4}$. Also $f'(x) \ge f^3(x) + \frac{1}{f(x)}$ then

(a) $b - a \ge \frac{\pi}{4}$ (b) $b - a \le \frac{\pi}{4}$

(c) $b - a \le \frac{\pi}{24}$ (d) None of these

39. The solution of the differential equation

$\frac{dy}{dx} = \frac{1}{xy\left[x^2\sin y^2 + 1\right]}$ is

(a) $x^2(\cos y^2 - \sin y^2 - 2ce^{-y^2}) = 2$

(b) $y^2(\cos x^2 - \sin y^2 - 2ce^{-y^2}) = 2$

(c) $x^2(\cos y^2 - \sin y^2 - e^{-y^2}) = 4c$

(d) None of these

40. Solution of the differential equation

$\left\{\frac{1}{x} - \frac{y^2}{(x-y)^2}\right\}dx + \left\{\frac{x^2}{(x-y)^2} - \frac{1}{y}\right\}dy = 0$ is

(a) $\ell n\left|\frac{x}{y}\right| + \frac{xy}{x-y} = c$ (b) $\frac{xy}{x-y} = ce^{x/y}$

(c) $\ell n|xy| = c + \frac{xy}{x-y}$ (d) $\frac{xy}{x+y} = ce^{\left|\frac{x}{y}\right|}$

MARK YOUR RESPONSE					
	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ
	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

The ORDER of a differential equation is defined as the order of the highest order derivative occuring in the equation. For example, the order of equation $\frac{d^2y}{dx^2}+y=0$ is 2, as it contains second order derivative as the highest order derivative. For a given family of curves the order of its differential equation will be equal to the number of arbitrary constants in its equation. For example, the family of curves given by $y = A\cos x + B\sin x$; has two arbitrary constants, so its differential equation will be second order.

The DEGREE of a differential equation is the exponent of the highest order derivative in the equation after the equation is expressed as a polynomial in various order derivatives. For example, the equation $\frac{d^2y}{dx^2}+y=0$ is in polynomial form so its degree is the exponent of $\frac{d^2y}{dx^2}$, which is 1.

1. The order and degree of the differential equation whose solution is $y = cx + c^2 - 3c^{3/2} + 2$, where c is a parameter, are respectively

(a) 1 and 4 (b) 1 and 3
(c) 2 and 2 (d) 1 and 2

2. The order and degree of the differential equation $\frac{d^2y}{dx^2} = \sin\left(\frac{dy}{dx}\right) + xy$ are respectively

(a) 2, 1 (b) 2, infinite
(c) 2, 0 (d) 2, not defined

3. The order of the differential equation whose general solution is given by

$y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x+c_5}$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is

(a) 5 (b) 4
(c) 3 (d) 2

4. The order of the differential equation formed by differentiating and eliminating the constants from $y = a\sin^2 x + b\cos^2 x + c\sin 2x + d\cos 2x$, where a, b, c, d are arbitrary constants; is

(a) 1 (b) 2
(c) 3 (d) 4

PASSAGE-2

Let us represent the derivative $\frac{dy}{dx}$ by p. An equation of the form

$$y = px + f(p) \quad .....(1)$$

is known as Clairut's equation where $f(p)$ is a function of p.

To solve equation (1), we differentiate the equation with respect to x, we get

$$p = p + x\frac{dp}{dx} + f'(p)\frac{dp}{dx}$$

$$\Rightarrow [x + f'(p)]\frac{dp}{dx} = 0 \Rightarrow \frac{dp}{dx} = 0 \quad ....(2)$$

or, $x + f'(p) = 0$ ....(3)

Now, (2) gives p = constant = c, say.

Then eliminating p from (1) we get $y = cx + f(c)$ ....(4)

Which is a solution of equation (1).

If we eliminate p between (1) and (3) we will obtain another solution not contained in the general solution (4). This solution is known as the singular solution.

5. The general equation of the differential equation $y = px + \log p$ which does not contain the singular solution, is

(a) $y = cx + \log c$ (b) $y = cx + \frac{1}{c}$
(c) $y = \log x + c$ (d) $y = -\log x + c$

6. Singular solution of the differential equation $x\frac{dy}{dx} = y - \left(\frac{dy}{dx}\right)^2$ is

(a) $y = \frac{x}{4}$ (b) $y = \frac{x^2}{4}$
(c) $y = -\frac{x^2}{4}$ (d) $y = x$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ				

7. Solution of the differential equation

$$x^2\left(y - x\frac{dy}{dx}\right) = y\left(\frac{dy}{dx}\right)^2$$

which does not contain singular solution is

(a) $x^2(y - xc) = yc^2$ (b) $y = cx + c^2$

(c) $y^2 = cx^2 + c^2$ (d) $xy = cx^2 + c$

PASSAGE-3

Let $y = f(x)$ and $y = g(x)$ be the pair of curves such that the tangents at point with equal abscissae intersect on y-axis, and the normals drawn at points with equal abscissae intersect on x-axis. One curve passes through (1, 1) and the other passes through (2, 3). Then

8. The curve $f(x)$, is given by

(a) $\frac{2}{x} - x$ (b) $2x^2 - \frac{1}{x}$

(c) $\frac{2}{x^2} - x$ (d) $\frac{1}{x}$

9. The curve $g(x)$, is given by

(a) $x^2 - \frac{2}{x}$ (b) $x + \frac{2}{x}$

(c) $x^2 - \frac{4}{x^2}$ (d) $y^2 = \frac{9}{2}x$

10. The number of positive integral solutions for $f(x) = g(x)$, are

(a) 4 (b) 5

(c) 6 (d) none of these

MARK YOUR RESPONSE	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options:

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.

(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.

(c) Statement-1 is true but Statement-2 is false.

(d) Statement-1 is false but Statement-2 is true.

1. Statement-1 : The general solution of $\frac{dy}{dx} + y = 1$ is $ye^x = e^x + c$

Statement-2 : The number of arbitrary constants in the general solution of the differential equation is equal to the order of differential equation.

2. Statement-1 : If the lengths of subtangent and subnormal at point (x, y) on $y = f(x)$ are respectively 9 and 4. Then $x = \pm 6$

Statement-2 : Product of sub tangent and sub normal is square of the ordinate of the point.

3. Statement-1 : The differential equation whose general solution is $y = C_1 x + \frac{C_2}{x}$ for all values of C_1 and C_2 is a linear equation

Statement-2 : The equation $y = C_1 x + \frac{C_2}{x}$ has two arbitrary constants, so the corresponding differential equation is of second order.

4. Statement-1 : The differential equation of all non-hroizontal lines in a plane is $\frac{d^2y}{dx^2} = 0$

Statement-2 : The general equation of all non-horizontal lines in xy plane is $ax + by = 1$, $a \neq 0$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. The solution of $\left(\frac{dy}{dx}\right)^2 + 2y\cot x\frac{dy}{dx} = y^2$ is

(a) $y - \frac{c}{1+\cos x} = 0$ (b) $y = \frac{c}{1-\cos x}$

(c) $x = 2\sin^{-1}\sqrt{\frac{c}{2y}}$ (d) $x = 2\cos^{-1}\sqrt{\frac{c}{2y}}$

2. The solution of $\frac{dy}{dx} + x = xe^{(n-1)y}$ is

(a) $\frac{1}{n-1}\log\left(\frac{e^{(n-1)y}-1}{e^{(n-1)y}}\right) = \frac{x^2}{2} + C$

(b) $e^{(n-1)y} = Ce^{(n-1)y+(n-1)x^2/2} + 1$

(c) $\log\left(\frac{e^{(n-1)y}-1}{(n-1)e^{(n-1)y}}\right) = x^2 + C$

(d) $e^{(n-1)y} = Ce^{(n-1)x^2/2+x} + 1$

3. The differential equation of the curve for which the initial ordinate of any tangent is equal to the corresponding subnormal

(a) is linear

(b) is homogeneous of first degree

(c) has degree 2

(d) is second order

4. The orthogonal trajectories of the family of coaxial circle $x^2 + y^2 + 2gx + C = 0$, where g is a parameter are

(a) family of circles with center on y-axis

(b) system of coaxial parabolas

(c) $x^2 + y^2 - C'x - Cy = 0$, where C' is an arbitrary constant

(d) system of coaxial circles with radical axis along x-axis

5. The curve for which the area of the triangle formed by the x-axis, the tangent line and radius vector of the point of tangency is equal to a^2 is

(a) $x = cy + \frac{a^2}{y}$ (b) $y = cx + \frac{a^2}{x}$

(c) $xy = cx + \frac{a^2}{x}$ (d) $x = cy - \frac{a^2}{y}$

6. The curve $y = f(x)$ is such that the area of the trapezium formed by the coordinate axes ordinate of an arbitrary point and the tangent at this point equals half the square of its abscissa. The equation of the curve can be

(a) $y = cx^2 + x$ (b) $y = cx^2 - x$

(c) $y = cx + x^2$ (d) $y = cx^2 \pm x \pm 1$

7. Let $f(x)$ be a non-zero function, whose all successive derivatives exist and are non-zero.If $f(x)$, $f'(x)$ and $f''(x)$ are in G. P. and $f(0) = f'(0) = 1$ then

(a) $f(x) > 0\ \forall x \in R$ (b) $f'(x) > 0 \forall x \in R$

(c) $f''(0) = 1$ (d) $f(x) \le 1 \forall x \in R$

8. Given a function 'g' which has a derivative $g'(x)$ for every real x and satisfies $g'(0) = 2$ and $g(x + y) = e^y g(x) + e^x g(y)$ for all x and y then

(a) $g(x)$ is increasing for all $x \in [-1, \infty)$

(b) Range of $g(x)$ is $\left[-\frac{2}{e}, \infty\right)$

(c) $g''(x) > 0 \forall x$

(d) $\lim\limits_{x\to 0} \frac{g(x)}{x} = 2$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ		

9. If the length of subnormal is equal to the length of subtangent at any point (3, 4) on the curve $y = f(x)$ and the tangent at (3, 4) to $y = f(x)$ meets the coordinate areas at A and B, then the area of the triangle OAB, where O is origin, is

(a) $\frac{25}{2}$ (b) $\frac{49}{2}$

(c) $\frac{1}{2}$ (d) $\frac{9}{2}$

10. The tangent at any point P on $y = f(x)$ meets x and y-axis at A and B. If $PA : PB = 2 : 1$ then the equation of the curve, is :

(a) $|x|y = c$ (b) $x^2|y| = c$

(c) $|x|y^2 = c$ (d) $x = cy^2$

11. Solution of the differential equation

$\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx}(e^x + e^{-x}) + 1 = 0$ are given by

(a) $y + e^{-x} = k$ (b) $y - e^{-x} = k$

(c) $y + e^x = k$ (d) $y - e^x = k$

12. The function $f(x)$ satisfying $\{f(x)\}^2 + 4f(x)\,f'(x) + \{f'(x)\}^2 = 0$ is given by

(a) $ke^{(2+\sqrt{3})x}$ (b) $ke^{(-2+\sqrt{3})x}$

(c) $ke^{-(2+\sqrt{3})x}$ (d) $k\log(2+\sqrt{3})x$

13. A curve passing through (1, 2) has its slope at any point (x, y) equal to $\frac{2}{y-2}$. If the curve has the equation $y = f(x)$ then

(a) The curve intersects y-axis at two distinct points

(b) The curve intersects x-axis at unique point

(c) The curve is a parabola

(d) The area bounded by $y = f(x)$ and the line $2x - y - 4 = 0$ is 9.

MARK YOUR RESPONSE	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. Let a function $y = f(x)$ satisfies the following conditions

1. $\frac{dy}{dx} = y + \int_0^1 y\,dx$ 2. $f(0) = 1$

Column-I	**Column-II**
A. $f''(0)$ is equal to	p. $f(0)$
B. $f(1)$ is equal to	q. $\frac{2}{3-e}$
C. $\lim_{x\to 0}\frac{f(x)-1}{x}$ is equal to	r. $\frac{e+1}{3-e}$
D. $\frac{1}{2}f'(\ln(3\text{-}e))$ is equal to	s. $f'(0)$
	t. 1

MARK YOUR RESPONSE

1.

	p	q	r	s	t
A	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
B	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
C	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
D	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ

2. **Observe the following column :**

	Column-I		Column-II
(A)	The order of the differential equation of all conics whose centre lie at the origin is equal to	p.	1
(B)	The order of the differential equation of all circles of radius a is equal to	q.	2
(C)	The order of the differential equation of all parabolas whose axis of symmetry is parallel to x-axis is equal to	r.	3
(D)	The order of the differential equation of all conics whose axes coincide with the axes of coordinates is equal to	s.	4
		t.	Can't be determined

3. The slope of tangent to curve $y = f(x)$ at the point $(x, f(x))$ is $2x + 1$. The curve passes through the point (1, 2). If the curve also passes through (x_1, y_1) then, match the entries of column I and column II

	Column I		Column II
(A)	x_1 can be equal to	p.	1
(B)	y_1 can be equal to	q.	$\frac{5}{6}$
(C)	Area bounded by curve $y = f(x)$, x axis and $x = 1$ is	r	$\frac{5\sqrt{5}-7}{12}$
(D)	Area bounded by curve $y = f(x)$, y axis and $y = 1$ is	s.	π
		t	0

MARK YOUR RESPONSE

2.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

3.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

F NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010 etc.) or a single-digit integer, ranging from 0 to 9.

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to a question is 6092, then the correct darkening of bubbles will look like the given.

For single digit integer answer darken the extreme right bubble only.

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

1. If the solution of the differential equation

$$\frac{x\,dx - y\,dy}{x\,dy - y\,dx} = \sqrt{\left(\frac{1+x^2-y^2}{x^2-y^2}\right)}$$ be

$$\sqrt{f(x, y)} + \sqrt{1+f(x, y)} = c\left(\frac{x+y}{\sqrt{f(x, y)}}\right)$$ where c is an arbitrary constant then $f(3, 2)$ is equal to

2. If the equation of a curve $y = y(x)$ satisfies the differential equation $x\int_0^x y(t)dt = (x+1)\int_0^x ty(t)dt,\ x > 0$, and $y(1) = e$, then $y\left(\frac{1}{2}\right)$ is equal to

3. The population of a country increases at a rate proportional to the number of inhabitants. If the population doubles in 30 years then the number of years in the nearest integer when the population will triple is equal to

4. A curve $y = f(x)$ is such that $f(x) \geq 0$ and $f(0) = 0$ and bounds a curvilinear trapezoid with the base $[0, x]$ whose area is proportional to $(n + 1)^{\text{th}}$ power of $f(x)$. If $f(1) = 1$, then $\{f(10)\}^n$ is equal to

5. If the differential equation corresponding to $y = \sum_{i=1}^{3} C_i e^{m_i x}$ where C_i's are arbitrary constants and m_1, m_2, m_3 are roots of $m^3 - 7m + 6 = 0$ is $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} + k = 0$ then k is equal to

6. If $y = C_1e^{2x} + C_2e^{x} + C_3e^{-x}$ satisfies the differential equation $\frac{d^3y}{dx^3} + a\frac{dy^2}{dx^2} + b\frac{dy}{dx} + cy = 0$ then $a^2 + b^2 + c^2$ is equal to

MARK YOUR RESPONSE

	1.	2.	3.	4.	5.	6.
	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2
	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3
	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4
	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5
	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6
	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7
	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8
	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9

Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(a)	**11**	(a)	**21**	(a)	**31**	(a)
2	(a)	**12**	(c)	**22**	(c)	**32**	(c)
3	(b)	**13**	(c)	**23**	(c)	**33**	(d)
4	(a)	**14**	(c)	**24**	(c)	**34**	(a)
5	(c)	**15**	(a)	**25**	(b)	**35**	(a)
6	(c)	**16**	(d)	**26**	(b)	**36**	(b)
7	(a)	**17**	(c)	**27**	(a)	**37**	(a)
8	(c)	**18**	(b)	**28**	(b)	**38**	(c)
9	(b)	**19**	(a)	**29**	(c)	**39**	(a)
10	(b)	**20**	(a)	**30**	(a)	**40**	(a)

B COMPREHENSION TYPE

1	(a)	**3**	(c)	**5**	(a)	**7**	(c)	**9**	(b)
2	(d)	**4**	(c)	**6**	(c)	**8**	(a)	**10**	(d)

C REASONING TYPE

1	(b)	**2**	(d)	**3**	(b)	**4**	(d)

D MULTIPLE CORRECT CHOICE TYPE

1	(a,b,c,d)	**4**	(a, d)	**7**	(a, b, c)	**10**	(c, d)	**13**	(b, c, d)
2	(a, b)	**5**	(a, d)	**8**	(a, b, d)	**11**	(a, d)		
3	(a, b)	**6**	(a, b)	**9**	(b, c)	**12**	(b, c)		

E MATRIX-MATCH TYPE

1. **A-q,s ; B-r ; C-q,s ; D-p,t**

2. **A-r ; B-q ; C-r ; D-q**

3. **A - p, q, r, s, t, ; B – p, q, r, s, t ; C - q ; D – r**

F NUMERIC/INTEGER ANSWER TYPE

1	5	**2**	8	**3**	48	**4**	10	**5**	6	**6**	9

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(a)** The equation is $y = e^{x\frac{dy}{dx}} \Rightarrow x\frac{dy}{dx} = \ell n\ y$

2. **(a)** The given equation can be written as

$$2y\sin x\frac{dy}{dx} + y^2\cos x = \sin 2x$$

$$\Rightarrow \frac{d}{dx}(y^2\sin x) = \sin 2x$$

$$\Rightarrow y^2\sin x = -\frac{1}{2}\cos 2x + C$$

At $x = \frac{\pi}{2}$, $(1)^2\sin\frac{\pi}{2} = -\frac{1}{2}\cos\frac{2\pi}{2} + C \Rightarrow C = -\frac{1}{2}$

Hence $y^2\sin x = \frac{1}{2}(1-\cos 2x) = \sin^2 x$

$$\Rightarrow y^2 = \sin x.$$

3. **(b)** $x^3\frac{dy}{dx} + 4x^2\tan y = e^x\sec y$

$$\Rightarrow \cos y\frac{dy}{dx} + \frac{4}{x}\sin y = \frac{e^x}{x^3} \qquad ....(1)$$

Put $\sin y = z \Rightarrow \cos y\frac{dy}{dx} = \frac{dz}{dx}$, the equation (1) becomes

$\frac{dz}{dx} + \frac{4}{x}z = \frac{e^x}{x^3}$, which is linear differential equation with respect to variable z.

Integrating factor $= e^{\int\frac{4}{x}dx} = x^4$

$\therefore$ Solution is $z(x^4) = \int (x^4)\frac{e^x}{x^3}dx$

$$\Rightarrow z(x^4) = (x-1)e^x + C$$

$\therefore$ $x^4\sin y = (x-1)e^x + C$.

Put $x = 1, y = 0 \Rightarrow C = 0$

$\therefore$ $\sin y = e^x(x-1)x^{-4}$

4. **(a)** We have, $ye^x dx - e^x dy + 2x^2y^2dx - y^3dy = 0$

$$\Rightarrow \frac{ye^x dx - e^x dy}{y^2} + 2x^2dx - ydy = 0 \qquad ....(1)$$

Let $\frac{e^x}{y} = z \Rightarrow \frac{ye^x - e^x\frac{dy}{dx}}{y^2} = \frac{dz}{dx}$

$$\Rightarrow \frac{ye^x dx - e^x dy}{y^2} = dz$$

$\therefore$ (1) become $dz + 2x^2dx - ydy = 0$.

Integrating we get, $z + \frac{2x^3}{3} - \frac{y^2}{2} = c$

$$\Rightarrow \frac{e^x}{y} + \frac{2}{3}x^3 - \frac{y^2}{2} = c$$

Put $x = 0, y = 1$ we get, $c = \frac{1}{2}$.

Hence, the solution is $6e^x + 4x^3y - 3y^3 - 3y = 0$

5. **(c)** $\frac{xdy}{x^2+y^2} = \left(\frac{y}{x^2+y^2} - 1\right)dx$

$$\Rightarrow \frac{xdy - ydx}{x^2+y^2} = -dx$$

$$\Rightarrow \int d\left(\tan^{-1}\left(\frac{y}{x}\right)\right) = -\int dx$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = -x + c \Rightarrow \frac{y}{x} = \tan(c-x)$$

6. **(c)** The given equation can be rewritten as

$(ydx - xdy)e^{-x/y} - y^3dy = 0$

$$\Rightarrow \frac{ydx - xdy}{y^2}e^{-x/y} = ydy$$

$$\Rightarrow d(x/y)e^{-x/y} = ydy$$

$$\Rightarrow -e^{-x/y} = \frac{y^2}{2} + \text{constant} \Rightarrow 2e^{-x/y} + y^2 = C$$

7. **(a)** The equation is $dy+(y\phi'(x)-\phi(x)\phi'(x))dx=0$

$$\Rightarrow \frac{dy}{dx}=-[y-\phi(x)]\phi'(x).$$

Put $y-\phi(x)=z \Rightarrow \frac{dy}{dx}-\phi'(x)=\frac{dz}{dx}$

The equation becomes, $\phi'(x)+\frac{dz}{dx}=-z\phi'(x)$

$$\Rightarrow \frac{dz}{1+z}=\phi'(x)dx$$

Intergrating, we have $\int\frac{dz}{1+z}=-\int\phi'(x)dx$

$$\Rightarrow \ln(1+z)=-\phi(x)+k$$

$$\therefore\ 1+z=e^{k-\phi(x)}$$

$$\Rightarrow 1+y-\phi(x)=Ce^{-\phi(x)}$$

$$\Rightarrow y=\phi(x)-1+Ce^{-\phi(x)}$$

8. **(c)** The given equation can be written as $y(1+x^{-1})dx+(x+\log x)dy+\sin y dx+x\cos y dy=0$

$$\Rightarrow d(y(x+\log x))+d(x\sin y)=0$$

$$\Rightarrow y(x+\log x)+x\sin y=C$$

9. **(b)** The given differential equation can be written as $y^5xdx+ydx-xdy=0$

Multiplying by $\frac{x^3}{y^5}$, we have

$$x^4dx+\frac{x^3}{y^3}\left(\frac{ydx-xdy}{y^2}\right)=0$$

Integrating, we get $\frac{x^5}{5}+\frac{1}{4}\left(\frac{x}{y}\right)^4=C$

10. **(b)** Putting $u=\frac{1}{(x+y)^2}$, we have

$$\frac{du}{dx}=\frac{-2}{(x+y)^3}\left(1+\frac{dy}{dx}\right)$$

The given equation can be rewritten as

$$\frac{1}{(x+y)^3}\left(\frac{dy}{dx}+1\right)=-\frac{x}{(x+y)^2}+x^3$$

$$\Rightarrow -\frac{1}{2}\frac{du}{dx}=-ux+x^3 \Rightarrow \frac{du}{dx}-2ux=-2x^3.$$

This is a linear equation whose I.F. is e^{-x^2}. Hence

$$\frac{d}{dx}(ue^{-x^2})=-2x^3e^{-x^2}$$

$$\Rightarrow ue^{-x^2}=-\int te^t dt+C \quad (\text{Putting } -x^2=t)$$

$$\Rightarrow \frac{1}{(x+y)^2}e^{-x^2}=-te^t+e^t+C$$

$$\Rightarrow \frac{1}{(x+y)^2}=x^2+1+Ce^{x^2}$$

11. **(a)** Taking $x=r\cos\theta$ and $y=r\sin\theta$, so that $x^2+y^2=r^2$ and $y/x=\tan\theta$, we have

$xdx+ydx=rdr$ and

$xdy-ydx=x^2\sec^2\theta d\theta=r^2d\theta$.

The given equation can be transformed into

$$\frac{rdr}{r^2d\theta}=\sqrt{\frac{a^2-r^2}{r^2}} \Rightarrow \frac{dr}{d\theta}=\sqrt{a^2-r^2}$$

$$\Rightarrow C+\sin^{-1}r/a=\theta=\tan^{-1}y/x$$

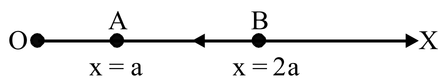
$$\Rightarrow y=x\tan\left(C+\sin^{-1}\frac{1}{a}\sqrt{x^2+y^2}\right)$$

or $\sqrt{x^2+y^2}=a\sin\left(\text{const.}+\tan^{-1}\frac{y}{x}\right)$

12. **(c)** For the path BA, the equation of the motion of the particle is

$$mv\frac{dv}{dx}=-\frac{mk^2a^2}{x^2} \Rightarrow \int_0^v vdv=-k^2a^2\int_{2a}^{a}\frac{dx}{x^2}$$

$$\Rightarrow v=k\sqrt{a}$$



For the path AO, The equation of motion of the particle is

$$mv\frac{dv}{dx}=-\frac{2mk^2x}{a} \Rightarrow \int_v^V v\,dv=-\frac{2k^2}{a}\int_a^0 xdx,$$

Where V is velocity at $x=0$

$$\Rightarrow \frac{V^2-v^2}{2}=k^2a \Rightarrow V^2=2k^2a+k^2a=3k^2a$$

$[\therefore\ v=k\sqrt{a}\,]$

$$\therefore\ v=k\sqrt{3a}$$

13. **(c)** The substitution $y = u^m$

$\Rightarrow \frac{dy}{dx} = mu^{m-1}\frac{du}{dx}$ changes the equation to,

$2.x^4.u^m.mu^{m-1}\frac{du}{dx} + u^{4m} = 4x^6$

$\Rightarrow \frac{du}{dx} = \frac{4x^6 - u^{4m}}{2mx^4x^{2m-1}}$. Since it is homogeneous, the degree of $4x^6 - u^{4m}$ and $2mx^4x^{2m-1}$ must be same.

$\Rightarrow 6 = 4m = 4 + 2m - 1 \Rightarrow m = \frac{3}{2}$

The correct answer is (c).

14. **(c)** The given equation is reduced to

$x = e^{xy(dy/dx)}$

$\Rightarrow \ln x = xy\frac{dy}{dx} \Rightarrow \int y dy = \int \frac{1}{x}\ln x\, dx$

$\Rightarrow \frac{y^2}{2} = \frac{(\ln x)^2}{2} + C$

$\Rightarrow y = \pm\sqrt{(\ln x)^2 + C} = \pm \ell\text{n}\, x + c$

15. **(a)** The given differential equation can be written as $\frac{dx}{dy} = xy[x^2 \sin y^2 + 1] \Rightarrow \frac{1}{x^3}\frac{dx}{dy} - \frac{1}{x^2}y = y\sin y^2$

This equation is reducible to linear equation, so putting $-\frac{1}{x^2} = u$, the last equation can be written as

$\frac{du}{dy} + 2uy = 2y\sin y^2$

The integrating factor of this equation is e^{y^2}. So required solution is

$ue^{y^2} = \int 2y\sin y^2.e^{y^2}dy + C$

$= \int (\sin t)e^t dt + C \qquad (t = y^2)$

$\Rightarrow ue^{y^2} = \frac{1}{2}e^{y^2}(\sin y^2 - \cos y^2) + C$

$\Rightarrow 2u = (\sin y^2 - \cos y^2) + Ce^{-y^2}$

$\Rightarrow 2 = x^2[\cos y^2 - \sin y^2 - 2Ce^{-y^2}]$

16. **(d)** Writing $p = \frac{dy}{dx}$ and differentiating w.r.t. x, we have

$p = 2p + 2x\frac{dp}{dx} + 2xp^4 + 4p^3x^2\frac{dp}{dx}$

$\Rightarrow 0 = p(1 + 2xp^3) + 2x\frac{dp}{dx}(1 + 2p^3x)$

$\Rightarrow p + 2x\frac{dp}{dx} = 0 \Rightarrow 2\frac{dp}{p} = -\frac{dx}{x}$

$\Rightarrow 2\ln p + \ln x = \text{const.} \Rightarrow p^2x = c$ or $p = \sqrt{\frac{c}{x}}$

Substituting this value in the given equation, we get $y = 2\sqrt{cx} + c^2$.

One more solution will be obtained with $1 + 2xp^3 = 0$

$\Rightarrow p = \left(-\frac{1}{2x}\right)^{1/3}$, which is singular solution.

17. **(c)** The given equation can be written as

$e^y\frac{dy}{dx} = e^{2x} - e^xe^y \Rightarrow e^y\frac{dy}{dx} + e^xe^y = e^{2x}$ .... (i)

Let $v = e^y$

$\therefore \frac{dv}{dx} = e^y\frac{dy}{dx}$ then from (i) $\frac{dv}{dx} + ve^x = e^{2x}$

which is linear differential equation and I.F. = e^{e^x}

$\therefore$ The solution is

$v(e^{e^x}) = \int e^{2x}e^{e^x}dx + c = \int e^{e^x}e^x.e^x dx + c$

Put $e^x = t \;\therefore\; e^x dx = dt$ then

$v(e^{e^x}) = \int e^t.t\,dt + c = te^t - e^t + c = e^{e^x}(e^x - 1) + c$

$\therefore\; v = e^x - 1 + ce^{-e^x}$ or $e^y = e^x - 1 + ce^{-e^x}$

18. **(b)** $x\left(\cos\frac{y}{x}\right)(y\,dx + xdy) = y\sin\left(\frac{y}{x}\right)(x\,dy - y\,dx)$

Dividing both sides by x^2dx we get

$\Rightarrow \left(\cos\frac{y}{x}\right)\left(\frac{y}{x} + \frac{dy}{dx}\right) = \frac{y}{x}\sin\left(\frac{y}{x}\right)\left(\frac{dy}{dx} - \frac{y}{x}\right)$

Which is homogeneous equation

Putting $y = vx$ we get $\frac{dy}{dx} = v + x\frac{dv}{dx}$

or $\cos v\left(v+v+x\frac{dv}{dx}\right)=v\sin v\left(x\frac{dv}{dx}\right)$

$\Rightarrow\ 2v\cos v=x\frac{dv}{dx}(v\sin v-\cos v)$

Separating the variables, we get

$\Rightarrow\ \frac{2dx}{x}=\left(\frac{v\sin v-\cos v}{v\cos v}\right)dv$

Integrating $2\ln x+\ln c=\ln\sec v-\ln v$

$\Rightarrow\ cx^2=\frac{\sec v}{v}\ \Rightarrow\ \sec v=cx^2v\ \Rightarrow\ \sec\frac{y}{x}=cxy$

Where 'c' is an arbitrary constant.

19. **(a)** The given equation can be written as

$\tan y\frac{dy}{dx}=1-x\cos y$

or $\sec y\tan y\frac{dy}{dx}+x=\sec y$ .... (1)

Let $\sec y=v\ \therefore\ \sec y\tan y\frac{dy}{dx}=\frac{dv}{dx}$ then from (1),

$\frac{dv}{dx}+x=v$ or $\frac{dv}{dx}-v=-x$

Which is linear differential equation, its I.F. $=e^{-x}$

$\therefore$ The solution is

$v(e^{-x})=\int(-x)e^{-x}dx+c=xe^{-x}+e^{-x}+c$

$\therefore\ v=x+1+ce^{x}\qquad\therefore\ \sec y=x+1+ce^{x}$

20. **(a)** The given equation can be written as

$$\left(\frac{dx}{x}-\frac{dy}{y}\right)+\left(\frac{x^2dy-y^2dx}{(x-y)^2}\right)=0$$

or $\left(\frac{dx}{x}-\frac{dy}{y}\right)+\left(\frac{\frac{dy}{y^2}-\frac{dx}{x^2}}{\left(\frac{1}{y}-\frac{1}{x}\right)^2}\right)=0$

or $\left(\frac{dx}{x}-\frac{dy}{y}\right)+\left(\frac{\frac{dy}{y^2}-\frac{dx}{x^2}}{\left(\frac{1}{x}-\frac{1}{y}\right)^2}\right)=0$

Integrating we get $\ln|x|-\ln|y|-\frac{1}{\left(\frac{1}{x}-\frac{1}{y}\right)}=c$

or $\ln\left|\frac{x}{y}\right|+\frac{xy}{(x-y)}=c$

Where 'c'is an arbitrary constant

21. **(a)** Put $y=x\sec\theta\qquad\therefore\ \frac{dy}{dx}=x\sec\theta\tan\theta\frac{d\theta}{dx}+\sec\theta$

From the given equation, $x^3\left(x\sec\theta\tan\theta\frac{d\theta}{dx}+\sec\theta\right)$

$=x^3\sec^3\theta+x^3\sec^2\theta\tan\theta$

Dividing both sides by $x^3\sec\theta$, we get

$x\tan\theta\frac{d\theta}{dx}+1=\sec^2\theta+\sec\theta\tan\theta$

$\Rightarrow\ x\tan\theta\frac{d\theta}{dx}=\tan^2\theta+\sec\theta\tan\theta$

or $x\frac{d\theta}{dx}=(\tan\theta+\sec\theta)$

or $\frac{d\theta}{\sec\theta+\tan\theta}=\frac{dx}{x}$

or $(\sec\theta-\tan\theta)d\theta=\frac{dx}{x}$

Integrating we get

$\ln(\sec\theta+\tan\theta)-\ln\sec\theta=\ln x+\ln c$

or $1+\frac{\sqrt{y^2-x^2}}{y}=cx\ \Rightarrow\ y+\sqrt{y^2-x^2}=cxy$ where

c is an arbitrary constant.

ALTERNATE SOL.

$x^3\frac{dy}{dx}=y^3+y^2\sqrt{y^2-x^2}$

$\Rightarrow\ \frac{dy}{dx}=\left(\frac{y}{x}\right)^3+\sqrt{\left(\frac{y}{x}\right)^2-1}\ \left(\frac{y}{x}\right)^2$

Let $\frac{y}{x}=v$

(the equation is homogeneous differential equation)

$\Rightarrow\ \frac{dy}{dx}=v+x\frac{dv}{dx}$, so

$v+x\frac{dv}{dx}=v^3+v^2\sqrt{v^2-1}$

$\Rightarrow\ x\frac{dv}{dx}=v(v^2-1)+v^2\sqrt{v^2-1}$

$$\Rightarrow \; x\frac{dv}{dx} = v\sqrt{v^2-1}\;\sqrt{v^2-1}+v$$

$$\therefore \; \frac{1}{v\sqrt{v^2-1}\left(\sqrt{v^2-1}+v\right)}dv = \frac{dx}{x}$$

$$\Rightarrow \left(\frac{v-\sqrt{v^2-1}}{v\sqrt{v^2-1}}\right)dv = \frac{dx}{x}$$

$$\Rightarrow \left(\frac{1}{\sqrt{v^2-1}}-\frac{1}{v}\right)dv = \frac{dx}{x}$$

$$\Rightarrow \ell n\left(v+\sqrt{v^2-1}\right)-\ell n\; v = \ell n\; x + \ell n\; c$$

$$\Rightarrow \; 1+\sqrt{1-\frac{1}{v^2}} = cx \;\Rightarrow\; 1+\sqrt{1-\frac{x^2}{y^2}} = cx$$

$$\Rightarrow \; y+\sqrt{y^2-x^2} = cxy$$

22. **(c)** Differentiating $xy = C$, we get $y + xp = 0$ where $p = \frac{dy}{dx}$.

Replacing p by $\frac{p+\tan\pi/4}{1-p\tan\pi/4} = \frac{p+1}{1-p}$, we have

$$y + \frac{p+1}{1-p}x = 0$$

$\Rightarrow \frac{dy}{dx} = \frac{y/x+1}{y/x-1}$ Putting $y/x = v$, the last equation reduces to

$$x\frac{dv}{dx} = \frac{v+1}{v-1} - v = -\frac{v^2-2v-1}{v-1}$$

$$\Rightarrow 2\frac{dx}{x} = -\frac{2(v-1)}{v^2-2v-1}dv$$

$$\Rightarrow \; 2ln\; x = -ln(v^2-2v-1)+ln\; C$$

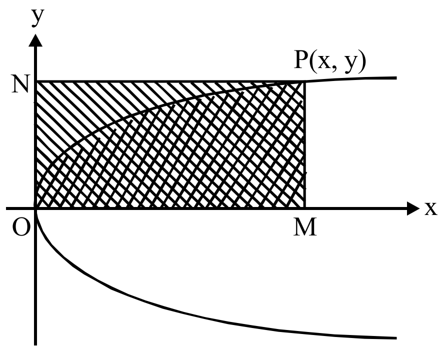
$$\Rightarrow \; ln\, x^2(y^2/x^2-2y/x-1) = ln\; C$$

$\Rightarrow \; y^2-2xy-x^2 = C$, which does not represent a pair of straight lines if $C \neq 0$.

23. **(c)** Let $P(x, y)$ be the point on the curve passing through the origin $O(0, 0)$ and let PN and PM be the lines parallel to the x-and y-axis, respectively. If the equation of the curve is $y = y(x)$, the area POM equals $\int_0^x y dx$

and the area PON equals $xy - \int_0^x y dx$. Assuming that $2(POM) = PON$, we therefore have

$$2\int_0^x y\, dx = xy - \int_0^x y\, dx \;\Rightarrow\; 3\int_0^x y\, dx = xy$$



Differentiating both sides of this gives

$$3y = x\frac{dy}{dx}+y \;\Rightarrow\; 2y = x\frac{dy}{dx} \;\Rightarrow\; \frac{dy}{y} = 2\frac{dx}{x}$$

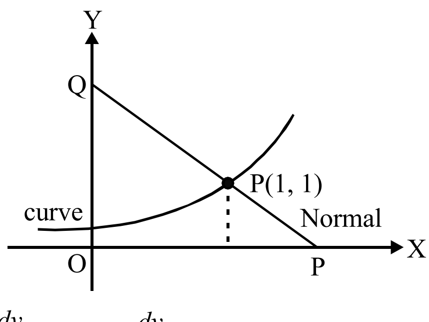
$$\Rightarrow \; \log y = 2\log x + C \;\Rightarrow\; y = Cx^2$$

With C being a constant. This solution represents a parabola. We will get a similar result if we have started instead with $2(PON) = POM$

24. **(c)** Given, equation of normal at $P(1,1)$ is $ay + x = a + 1$

$\therefore$ Slope of tangent at $P = a \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = a$ Given

$$\frac{dy}{dx} \propto y \;\Rightarrow\; \frac{dy}{dx} = ky \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = k = a$$



$\frac{dy}{dx} = ay \Rightarrow \frac{dy}{y} = adx$ (variable being separated)

$$\Rightarrow \ln y = ax + c$$

It is passing through (1, 1) then $c = -a \Rightarrow$ equation of the curve is $y = e^{a(x-1)}$

25. **(b)** Let $y = y(x)$ be equation of the curve. Then the equation of the normal to it at (x, y) is $Y - y = -\frac{dx}{dy}(X - x)$.

This normal meets the x-axis ($Y = 0$) at the point $\left(y\frac{dy}{dx} + x, 0\right)$

The length of the normal between (x, y) and this point is

$$\sqrt{\left(y\frac{dy}{dx} + x - x\right)^2 + (0 - y)^2} = y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Given that this length is proportional to the square of the ordinate y, so

$$y\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = ky^2 \Rightarrow \frac{dy}{dx} = \sqrt{k^2y^2 - 1}$$

$$\Rightarrow \frac{dy}{\sqrt{k^2y^2 - 1}} = dy$$

$$\Rightarrow \log\left(y + \sqrt{y^2 - \frac{1}{k^2}}\right) = kx + \text{const.}$$

$$\Rightarrow ky + \sqrt{k^2y^2 - 1} = ce^{kx}$$

26. **(b)** $\frac{dy}{dx} + xy = x^3y^3 \Rightarrow \frac{1}{y^3}\frac{dy}{dx} + \frac{x}{y^2} = x^3$ ....(1)

$$\Rightarrow -\frac{2}{y^3}\frac{dy}{dx} - \frac{2}{y^2}x\,dx = -2x^3dx$$

$$\Rightarrow e^{-x^2}\left(-\frac{2}{y^3}\right)\frac{dy}{dx} + e^{-x^2}(-2x)\frac{dx}{y^2}$$

$$= (-2x)x^2\,e^{-x^2}dx$$

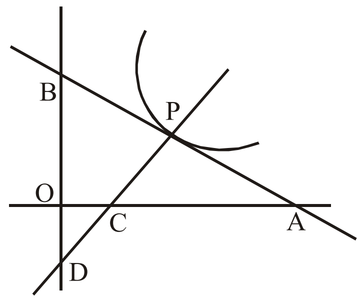
$\Rightarrow d\left(e^{-x^2}.\frac{1}{y^2}\right) = x^2(-2x)e^{-x^2}dx$. Integrating we get

$$e^{-x^2}.\frac{1}{y^2} = (x^2 + 1)e^{-x^2} + c \;\; or \;\; y^{-2} = x^2 + 1 + ce^{x^2}$$

[**NOTE :** the equation will be converted to linear form if we put $\frac{1}{y^2} = z$ in the equation (1)]

27. **(a)** Let P be (x, y). C is $\left(x + y\frac{dy}{dx}, 0\right)$ and B is $\left(0, y - x\frac{dy}{dx}\right)$. Centre of the circle through O, C, P and B has its centre at the mid-point of BC. Let it be (α, β) then

$$2\alpha = x + y\frac{dy}{dx} \text{ and } 2\beta = y - x\frac{dy}{dx}$$



Now, (α, β) lies on $y = x$

so, $y - x\frac{dy}{dx} = x + y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{y - x}{x + y}$

28. **(b)** $\frac{dy}{dx} = -2x(y - 1) \Rightarrow \frac{dy}{dx} + 2xy = 2x$

Integrating factor $= e^{\int 2x\,dx} = e^{x^2}$

$\therefore$ Solution of the d.e. is

$$ye^{x^2} = \int 2xe^{x^2}dx + c \Rightarrow ye^{x^2} = e^{x^2} + c$$

Given $y(0) = 1 \Rightarrow c = 0$. So, $ye^{x^2} = e^{x^2} \Rightarrow y = 1$

$\therefore \lim_{x\to\infty} y(x) = \lim_{x\to\infty}(1) = 1$

29. **(c)** $(2ny + xy\log_e x)\,dx = x\log_e x\,dy$

$$\Rightarrow \frac{dy}{y} = \left(\frac{2n}{x\log_e x} + 1\right)dx$$

$\Rightarrow \log(y) = 2n\log|\log x| + x + c$ and $c = 0$

($\because$ curve passes through (e, e^e))

$$\therefore y = e^{x + \log(\log x)^{2n}} = e^x(\log x)^{2n}$$

$$\Rightarrow f(x) = e^x(\log x)^{2n}$$

$$\text{Now, } g(x) = \lim_{n\to\infty} f(x) = \begin{cases} \to\infty & if \quad x < \frac{1}{e} \\ 0 & if \quad \frac{1}{e} < x < e \\ \to\infty & if \quad x > e \end{cases}$$

$$\therefore \int_{1/e}^{e} g(x)\,dx = 0$$

30. (a) Differential equation can be rewritten as

$$\left(\frac{xdy-ydx}{x^2}\right)\left(\frac{x^2}{y^2}\right)=(xdy+ydx)\sin xy$$

or, $\left(d\left(\frac{y}{x}\right)\right)\left(\frac{x^2}{y^2}\right)=(d(xy))\sin xy$

Integrating both sides, we get

$$-\frac{1}{y/x}=-\cos(xy)+c \Rightarrow \frac{x}{y}=\cos(xy)-c$$

Put $x=0, y=1$, we get $c=1$

31. (a) The differential equation can be rewritten as

$$2y\frac{dy}{dx}x=-y^2-\sin 2x$$

$$\Rightarrow\ y^2+2yx\frac{dy}{dx}=-\sin 2x$$

$$\Rightarrow\ \frac{d}{dx}(xy^2)=\frac{d}{dx}(\cos^2 x)$$

$$\Rightarrow\ xy^2=\cos^2 x+c$$

32. (c) Equation of tangent is $Y-y=\frac{dy}{dx}(X-x)$

For $Y=0, X=3x$, we get $\frac{dy}{dx}=-\frac{y}{2x}$

$$\Rightarrow\ \frac{dy}{y}=-\frac{1}{2}\frac{dx}{x}$$

$$\Rightarrow \ell n|y|=-\frac{1}{2}\ell n|x|+\ell n\, c$$

$$\Rightarrow\ y=\frac{c}{\sqrt{x}}$$

$\Rightarrow\ y=\frac{1}{\sqrt{x}}$ ($c=1$, as the curve passes through (1, 1))

33. (d) Let $h=Ar+B$

Given that $6=A+B$ and $\frac{dh}{dt}=A\frac{dr}{dt}\Rightarrow A=3$

$\therefore\ h=3r+3$

So volume $V=\pi r^2 h=\pi r^2(3r+3)$

$$\frac{dV}{dt}=3\pi\left[3r^2+2r\right]\frac{dr}{dt}$$

$$\Rightarrow\ n=3\pi(27+6)\frac{1}{3\pi}=33\,\text{cm}^3/s$$

34. (a) $Y-y=\frac{dy}{dx}(X-x)\Rightarrow A\equiv\left(x-y\frac{dx}{dy},0\right)$,

$B\equiv\left(0, y-x\frac{dy}{dx}\right)$

$P(x,y)$ is mid-point of AB

$$\Rightarrow\ 2x=x-y\frac{dx}{dy}\Rightarrow x\frac{dy}{dx}=-y$$

and $2y=y-x\frac{dy}{dx}\Rightarrow x\frac{dy}{dx}=-y$

$$\Rightarrow\ \frac{dy}{y}+\frac{dx}{x}=0\Rightarrow\ \ell n|y|+\ell n|x|=\ell n|c|$$

$\Rightarrow\ |xy|=|c|\Rightarrow xy=1$ as it passes through (1, 1)

35. (a) $\frac{dx}{dy}=x^2y^3+xy\Rightarrow x^{-2}\frac{dx}{dy}-x^{-1}y=y^3$

Put $x^{-1}=t\ \Rightarrow\ \frac{dt}{dy}+t.y=-y^3$

which is linear differential equation.

$$\Rightarrow\ t.e^{y^2/2}=-y^2e^{y^2/2}-2e^{y^2/2}+c$$

$$\Rightarrow\ \frac{1}{x}=(2-y^2)+ce^{-y^2/2}$$

or $\left(\frac{1-2x}{x}\right)=-y^2+ce^{-y^2/2}$

36. (b) The equation is

$$ydx+xdy+\frac{y}{x}dx+\log x\,dy+\cos y\,dx-x\sin y\,dy=0$$

$$\Rightarrow\ d(xy)+\{yd(\log x)+\log x\,dy\}+\{\cos y\,dx+xd(\cos y)\}=0$$

$$\Rightarrow\ d(xy)+d(y\log x)+d(x\cos y)=0$$

On integration,

$xy+y\log x+x\cos y=c$

37. (a) $$\frac{dx}{dy}=\frac{x^2\log\left(\frac{x}{y}\right)-y^2}{xy\log\left(\frac{x}{y}\right)}$$

Let $x=vy$ and $\frac{dx}{dy}=v+y\frac{dv}{dy}$

$$\therefore\ v+y\frac{dy}{dy}=\frac{v^2\log v-1}{v\log v}\Rightarrow\ v+y\frac{dv}{dy}=v-\frac{1}{v\log v}$$

$$\Rightarrow y\frac{dv}{dy}=-\frac{1}{v\log v}\quad\Rightarrow\ v\log v dv=-\frac{dy}{y}$$

$$\int \log v . v dv = -\int \frac{1}{y} dy$$

$$\Rightarrow (\log v)\frac{v^2}{2} - \int \frac{1}{v} \times \frac{v^2}{2} dv = -\log y + \log c$$

$$\Rightarrow \frac{v^2}{2}\log v - \frac{v^2}{4} = -\log y + \log c$$

$$\Rightarrow \frac{x^2}{2y^2}\log\left(\frac{x}{y}\right) - \frac{x^2}{4y^2} = -\log y + \log c$$

38. (c) $f'(x) \geq f^3(x) + \frac{1}{f(x)}$

or $f'(x).f(x) \geq 1 + f^4(x)$

or $\frac{f(x).f'(x)}{1+f^4(x)} \geq 1$

Integrating with respect to x, from $x = a$ to $x = b$

$$\frac{1}{2}\left(\tan^{-1}(f^2(x))\right)_a^b \geq b - a$$

or $(b-a) \leq \frac{1}{2}\left\{\lim_{x\to b^-} (\tan^{-1}(f^2(x))\right.$

$$\left. - \lim_{x\to a^+}\left(\tan^{-1}(f^2(x))\right)\right\}$$

$\Rightarrow b - a \leq \frac{\pi}{24}$

39. (a) The given differential equation is

$$\frac{dx}{dy} = xy\left[x^2 \sin y^2 + 1\right]$$

$$\Rightarrow \frac{1}{x^3}\frac{dx}{dy} - \frac{1}{x^2} y = y \sin y^2$$

Put $-\frac{1}{x^2} = u$, $\therefore \frac{2}{x^3}\frac{dx}{dy} = \frac{du}{dy}$

$$\frac{du}{dy} + 2uy = 2y \sin y^2$$

$$\text{I.F.} = e^{\int 2y dy} = e^{y^2}$$

$$\therefore u.e^{y^2} = \int 2y \sin y^2 . e^{y^2} dy + c$$

$$= \int (\sin t)e^t dt + c \qquad [t = y^2]$$

$$= \frac{1}{2}e^{y^2}(\sin y^2 - \cos y^2) + c$$

$$\Rightarrow 2u = (\sin y^2 - \cos y^2) + ce^{-y^2}$$

$$2 = x^2 [\cos y^2 - \sin y^2 - 2ce^{-y^2}]$$

40. (a) $\left(\frac{dx}{x} - \frac{dy}{y}\right) + \left(\frac{x^2dy - y^2dx}{(x-y)^2}\right) = 0$

or $\left(\frac{dx}{x} - \frac{dy}{y}\right) + \left(\frac{\frac{dy}{y^2} - \frac{dx}{x^2}}{\left(\frac{1}{y} - \frac{1}{x}\right)^2}\right) = 0$

or $\left(\frac{dx}{x} - \frac{dy}{y}\right) + \left(\frac{\frac{dy}{y^2} - \frac{dx}{x^2}}{\left(\frac{1}{x} - \frac{1}{y}\right)^2}\right) = 0$

Integrating,

$$\ell n |x| - \ell n |y| - \frac{1}{\left(\frac{1}{x} - \frac{1}{y}\right)} = c$$

$\therefore \ell n \left|\frac{x}{y}\right| + \frac{xy}{x-y} = c$. where c is arbitrary constant.

B COMPREHENSION TYPE

1.(a) Differentiating the equation, we get $\frac{dy}{dx} = c$. So, eliminating c we get $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 - 3\left(\frac{dy}{dx}\right)^{3/2} + 2$. Clearly its order is 1 and removing the fractional power, it will result into 4th degree.

2.(d) Order is 2 but degree can not be determine because the equation is not expressible as polynomial.

3.(c) The equation can be written as $y = a \cos(x + b) + ce^x$

4.(c) $y = a\left(\frac{1-\cos 2x}{2}\right) + b\left(\frac{1+\cos 2x}{2}\right)$

$+c \sin 2x + d \cos 2x = A + B \sin 2x + C \cos 2x$

5.(a) The general solution will be obtained by replacing p by c, where c is an arbitrary constant. So, the solution is $y = cx + \log c$.

6.(c) Putting $\frac{dy}{dx} = p$ the equation becomes $y = xp + p^2$, which is the Clairut's equation. So the solution is obtained by replacing p by c, so $y = cx + c^2$, where c is the arbitrary constant.

Singular form is $x + f'(p) = 0 \Rightarrow x + 2p = 0$

Therefore $p = -\frac{x}{2}$ putting in the given equation,

We get $y = x\left(-\frac{x}{2}\right) + \frac{x^2}{4} \Rightarrow y = -\frac{x^2}{4}$

7.(c) Put $x^2 = u$ and $y^2 = v$, then $\frac{dy}{dx} = \frac{x}{y}\left(\frac{dv}{du}\right)$

The equation then becomes,

$$x^2\left(y - \frac{x^2}{y}\frac{dv}{du}\right) = y.\frac{x^2}{y^2}\left(\frac{dv}{du}\right)^2 \Rightarrow y^2 - x^2\frac{dv}{du} = \left(\frac{dv}{du}\right)^2$$

or $v = u\frac{dv}{du} + \left(\frac{dv}{du}\right)^2$

which is Clairut's equation in variables u and v, so the solution is $v = uc + c^2 \Rightarrow y^2 = cx^2 + c^2$.

8. **(a) ; 9. (b); 10. (d) .**

Let $y = f(x)$ and $y = g(x)$ be the required curves. The equation of the tangents to these two curves at points with equal abscissae x are

$Y - f(x) = f'(x)(X - x)$ and $Y - g(x) = g'(x)(X - x)$

These two lines intersect on y-axis

$\therefore \quad Y - f(x) = -xf'(x)$ and $Y - g(x) = -xg'(x)$

$\Rightarrow \quad Y = f(x) - xf'(x) = g(x) - xg'(x)$

$\Rightarrow \quad f(x) - g(x) = x\{f'(x) - g'(x)\}$

$\Rightarrow \quad f(x) - g(x) = x\frac{d}{dx}\{f(x) - g(x)\}$

$\Rightarrow \quad \frac{d\{f(x) - g(x)\}}{f(x) - g(x)} = \frac{dx}{x}$

On integration.

$\log\{f(x) - g(x)\} = \log x + \log c$

$f(x) - g(x) = cx$ ...(1)

The equation of the normals at points with equal abscissae x to the two curves are

$Y - f(x) = -\frac{1}{f'(x)}(X - x)$ and

$Y - g(x) = -\frac{1}{g'(x)}(X - x)$

These intersect on x-axis

$0 - f(x) = \frac{1}{f'(x)}(X - x)$ and $0 - g(x) = -\frac{1}{g'(x)}(X - x)$

$\Rightarrow \quad X = x + f(x)f'(x)$ and $X = x + g(x).g'(x)$

$\Rightarrow \quad x + f(x)f'(x) = x + g(x)g'(x)$

$\Rightarrow \quad f(x)f'(x) = g(x)g'(x)$

On integration,

$\{f(x)\}^2 = \{g(x)\}^2 + c_1 \quad \Rightarrow \{f(x)^2 - \{g(x)\}^2 = c_1$

$\Rightarrow \quad \{f(x) + g(x)\}\,cx = c_1 \Rightarrow f(x) + g(x) = \frac{c_1}{cx}$ ...(2)

Solving (1) and (2),

$f(x) = \frac{1}{2}\left(cx + \frac{c_1}{x}\right)$ ....(3)

and $g(x) = \frac{1}{2}\left(\frac{c_1}{cx} - cx\right)$ ...(4)

Since (3) passes through (1, 1) and (4) passes through (2, 3)

$\therefore \quad 2 = c + \frac{c_1}{c}$ and $6 = \frac{c_1}{2c} - 2c \Rightarrow c = -2$ anc $c_1 = -8$

$\therefore \quad f(x) = -x + \frac{2}{x}$ and $g(x) = \frac{2}{x} + x$

For number of positive integral solutions for $f(x) = g(x)$

$-x + \frac{2}{x} = \frac{2}{x} + x$

$\therefore \quad x = 0$, no solution.

C REASONING TYPE

1. **(b)** $\frac{dy}{dx} + y = 1 \Rightarrow \frac{dy}{1 - y} = dx$

$\int\frac{dy}{1 - y} = \int dx \Rightarrow \log(1 - y) = x$

$1 - y = e^{-x}$, $ye^x = e^x + c$

Order of differential equation is the number of orbitarary constants.

Both are true but statement - II is not correct reason.

2. **(d)** $\left|\frac{y_1}{m}\right| = 9$ and $|y_1 m| = 4 \Rightarrow |y_1|^2 = 36$

$\Rightarrow y_1 = \pm 6$

Product of subtangent and sub normal is y_1^2.

Statement - I is false.

3. **(b)** $y = C_1 x + \frac{C_2}{x}$

$\Rightarrow \frac{dy}{dx} = C_1 - \frac{C_2}{x^2}$

$\Rightarrow \frac{d^2 y}{dx^2} = \frac{2C_2}{x^3}$

Eliminating C_1 & C_2 from the above three equations.

We get $\frac{d^2 y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0$

4. **(d)** Equation of non-horizontal lines in a plane is $ax + by = 1$ $(a \neq 0)$

or $a\frac{dx}{dy} + b = 0$ $(a \neq 0$ and $b \in$ R)

or $a\frac{d^2 x}{dy^2} = 0$ or $\frac{d^2 x}{dy^2} = 0$

D MULTIPLE CORRECT CHOICE TYPE

1. **(a,b,c,d)** Solving for $\frac{dy}{dx}$, we obtain

$$\frac{dy}{dx} = \frac{-2y\cot x \pm \sqrt{4y^2\cot^2 x + 4y^2}}{2}$$

$$= y(-\cot x \pm \text{cosec } x)$$

Thus, we have $\frac{dy}{y} = (-\cot x + \text{cosec } x)dx$

$\Rightarrow \ln y = -\ln \sin x + \ln \tan\frac{x}{2} + \ln c$

$$\Rightarrow y = \frac{c\tan\frac{x}{2}}{\sin x} = \frac{c}{2\cos^2\frac{x}{2}} = \frac{c}{1+\cos x}$$

Solving $\frac{dy}{y} = -(\cot x + \text{cosec } x)dx$,

we get $y = \frac{c}{1-\cos x} \Rightarrow x = 2\sin^{-1}\sqrt{\frac{c}{2y}}$

2. **(a,b)** Rewriting the given equation, we get

$$\frac{dy}{dx} = x(e^{(n-1)y} - 1) \Rightarrow \frac{dy}{e^{(n-1)y} - 1} = xdx$$

$$\Rightarrow \frac{1}{n-1}\int \frac{(n-1)e^{(n-1)y}}{(e^{(n-1)y} - 1)e^{(n-1)y}}dy = \frac{x^2}{2} + C$$

$$\Rightarrow \frac{1}{n-1}\int \frac{du}{u(u-1)} = \frac{x^2}{2} + C \text{ (where } u = e^{(n-1)y})$$

$$\Rightarrow \frac{1}{n-1}\log\frac{u-1}{u} = \frac{x^2}{2} + C$$

$$\Rightarrow \frac{1}{n-1}\log\left(\frac{e^{(n-1)y} - 1}{e^{(n-1)y}}\right) = \frac{x^2}{2} + C$$

$$\Rightarrow e^{(n-1)y} = Ce^{(n-1)y + (n-1)x^2/2} + 1$$

3. **(a,b)** If $y = f(x)$ is the curve, $Y - y = f'(x)(X - x)$ is the equation of the tangent at (x, y), with $f'(x) = \frac{dy}{dx}$.

Putting $X = 0$, the initial ordinate of the tangent is therefore $y - xf'(x)$. The subnormal at this point is given by $y\frac{dy}{dx}$, so we have

$$y\frac{dy}{dx} = y - x\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{y}{x+y}$$

This is a homogeneous equation and, by rewriting it as

$$\frac{dx}{dy} = \frac{x+y}{y} = \frac{x}{y} + 1 \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 1$$

we see that it is also a linear equation.

4. **(a, d)** Differentiating the given equation, we have

$$2x + 2y\frac{dy}{dx} + 2g = 0 \Rightarrow g = -\left(x + y\frac{dy}{dx}\right)$$

Putting this value in $x^2 + y^2 + 2gx + C = 0$, we have $x^2 + y^2 - 2x\left(x + y\frac{dy}{dx}\right) + C = 0$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, we have the differential equation of orthogonal trajectories as

$$y^2 - x^2 + 2xy\frac{dy}{dx} + C = 0$$

$$\Rightarrow \quad 2x\frac{dx}{dy} - \frac{1}{y}x^2 = -\frac{C}{y} - y$$

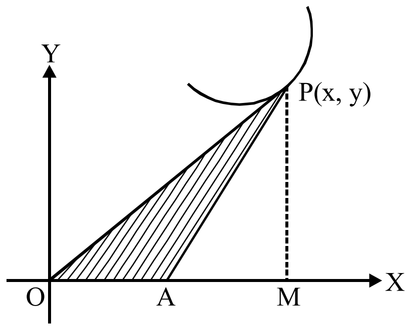
Putting $x^2 = v$, we have $\frac{dv}{dy} - \frac{1}{y}v = -\frac{C}{y} - y$,

Which is linear in v and y whose I.F. is $\frac{1}{y}$. Hence

$$\frac{v}{y} = \int\left(-\frac{C}{y^2} - 1\right)dy + C' = \frac{C}{y} - y + C'$$

$\Rightarrow \quad x^2 + y^2 - C'y - C = 0$ which represent system of circles with center on y-axis.

5. **(a, d)** Equation of tangent at (x,y), $Y - y = \frac{dy}{dx}(X - x)$



$\therefore$ Co-ordinate of A is $\left(x - y\frac{dx}{dy}, 0\right)$

Radius vector $OP = \sqrt{(x^2 + y^2)}$

Area of $\Delta OAP = \Delta OPM - \Delta APM$

$$= \frac{1}{2}xy - \frac{1}{2}\left(y\frac{dx}{dy}\right)y = \pm a^2 \text{ (given)}$$

$$\Rightarrow \quad \frac{dx}{dy} - \frac{x}{y} = \mp\frac{2a^2}{y^2}$$

This is a linear equation and I.F. $= e^{-\ln y} = \frac{1}{y}$

$\therefore$ Solution is $x\left(\frac{1}{y}\right) = \mp 2a^2\int\frac{1}{y^3}dy + c$

$$\Rightarrow \quad \frac{x}{y} = \pm\frac{a^2}{y^2} + c \quad \Rightarrow \quad x = cy \pm \frac{a^2}{y}$$

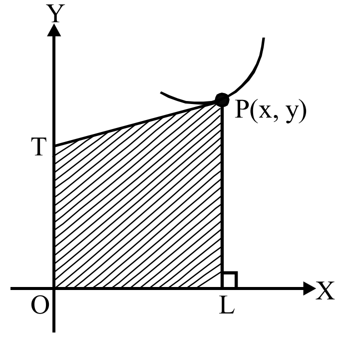
C is arbitrary constant

6. **(a, b)** Let $P\ (x, y)$ be any point on the curve. Length of intercept on y-axis by any tangent at

$$P(x, y) = OT = y - x\frac{dy}{dx}$$

$\therefore$ Area of trapezium $OLPTO = \frac{1}{2}(PL + OT)OL$

$$= \frac{1}{2}\left(y + y - x\frac{dy}{dx}\right)x = \frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x$$



According to question, Area of trapezium $OLPTO = \frac{1}{2}x^2$

i.e., $\frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x = \pm\frac{1}{2}x^2$

$\Rightarrow \ 2y - x\frac{dy}{dx} = \pm x$ or $\frac{dy}{dx} - \frac{2y}{x} = \pm 1$

Which is linear differential equation and I.F. $e^{-2\ln x} = \frac{1}{x^2}$

$\therefore$ The solution is $\frac{y}{x^2} = \int \pm\frac{1}{x^2}dx + c = \pm\frac{1}{x} + c$

$\therefore \ \ y = \pm x + cx^2$ or $y = cx^2 \pm x$

Where c is an arbitrary constant.

7. **(a, b, c)** According to question

$$\{f'(x)\}^2 = f(x)f''(x) \Rightarrow \frac{f'(x)}{f(x)} = \frac{f''(x)}{f'(x)}$$

Integrating we get,

$\ln|f(x)| + \ln A = \ln|f'(x)| \Rightarrow |f'(x)| = A|f(x)|$

$\because f'(0) = f(0) = 1 \Rightarrow A = 1 \quad so,\ f'(x) = \pm f(x)$

Integrating again we get $\ln|f(x)| = \pm x + c$ or $|f(x)| = ke^{\pm x}$

$\because \ \ f'(0) = 1 \Rightarrow k = 1$. So, $f(x) = \pm e^{\pm x}$

Now $f(0) = 1 \Rightarrow f(x) = e^x \ \ or \ \ e^{-x}$, but $f(x) \neq e^{-x}$ otherwise $f'(x) = -1$

$\therefore f(x) = e^x \Rightarrow f(x) = f'(x) = f''(x) = ... = e^x$

8. **(a,b,d)** Put $x = y = 0$ we get $g(0) = 0$. Differentiating the given equation with respect to x, we get

$$g'(x+y)\left[1+\frac{dy}{dx}\right]$$

$$= e^y g'(x) + e^y g(x)\frac{dy}{dx} + e^x g(y) + e^x g'(y)\frac{dy}{dx}$$

$\because$ x and y are independent

so $\frac{dy}{dx} = 0$ and we have

$$g'(x+y) = e^y g'(x) + e^x g(y)$$

Putting, $x = 0$, $g'(y) = 2e^y + g(y)$

or $g'(y) - g(y) = 2e^y$

which is a linear differential equation I.F. $= e^{-y}$

$\therefore$ Solution is $g(y)e^{-y} = \int 2\,dy + c$

$$\Rightarrow g(y)e^{-y} = 2y + C$$

$\because$ $g(0) = 0 \Rightarrow C = 0$ $\therefore$ $g(y) = 2ye^y$

or $g(x) = 2xe^x$

Now, $g'(x) = 2(x+1)e^x > \forall\ x > -1$.

Also, $g(x)$ attains absolute minimum at $x = -1$ and

$$f(-1) = \frac{-2}{e}$$

$\therefore$ Range $= \left[-\frac{2}{e}, \infty\right]$.

Further $g''(x) = 2(x+2)e^x \ngtr 0\,\forall x$

and $\lim\limits_{x \to 0} \frac{g(x)}{x} = 2$

9. **(b,c)** Length of subtangent = Length of subnormal

$$\Rightarrow \frac{dy}{dx} = \pm 1$$

If $\frac{dy}{dx} = 1$, then equation of tangent is $y - 4 = x - 3$

$\Rightarrow y - x = 1$ $\therefore$ $ar(\Delta AOB) = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

If $\frac{dy}{dx} = -1$, then eqn of tangent is $y - 4 = -(x - 3)$

$\Rightarrow x + y = 7$ $\therefore$ $ar(\Delta AOB) = \frac{1}{2} \times 7 \times 7 = \frac{49}{2}$

10. **(c,d)** The equation of tangent at (x, y) is

$$Y - y = \frac{dy}{dx}(X - x)$$

Points A and B are respectively $\left(x - \frac{y}{\frac{dy}{dx}}, 0\right)$

and $\left(0, y - x\frac{dy}{dx}\right)$

Now, $\frac{PA}{PB} = \frac{2}{1}$

$$\Rightarrow \left(\frac{y}{\frac{dy}{dx}}\right)^2 + y^2 = 4\left[x^2 + x^2\left(\frac{dy}{dx}\right)^2\right]$$

$$\therefore\ 4x^2\left(\frac{dy}{dx}\right)^4 + (4x^2 - y^2)\left(\frac{dy}{dx}\right)^2 - y^2 = 0$$

$$\therefore\ \left(\frac{dy}{dx}\right)^2 = \frac{y^2 - 4x^2 \pm \sqrt{(4x^2 - y^2)^2 + 16x^2y^2}}{8x^2}$$

$$= \frac{(y^2 - 4x^2) \pm (4x^2 + y^2)}{8x^2}$$

$$\therefore\ \left(\frac{dy}{dx}\right)^2 = \frac{y^2}{4x^2} \text{ or } \left(\frac{dy}{dx}\right)^2 = -1$$

$$\therefore\ \frac{dy}{dx} = \pm\frac{y}{2x} \Rightarrow 2\ln|y| = \pm\ln|x| + c$$

$$\Rightarrow \ln\frac{y^2}{|x|} = c \text{ or } \ln y^2|x| = c$$

or $y^2 = k_1|x|$ or $y^2|x| = k_2$

11. **(a,d)** $\left(\frac{dy}{dx} - e^{-x}\right)\left(\frac{dy}{dx} - e^x\right) = 0$

$\Rightarrow dy = e^{-x}\,dx$, $\quad dy = e^x dx$

$\Rightarrow y + e^{-x} = c$ or $y - e^x = k$

12. **(b,c)** $\{f(x)\}^2 + 4f(x)\,f'(x) + \{f'(x)^2\} = 0$

$$\therefore f'(x) = \frac{-4f(x) \pm \sqrt{16\{f(x)\}^2 - 4\{f(x)\}^2}}{2}$$

$$= -2f(x) \pm \sqrt{3}f(x)$$

$\therefore \frac{f'(x)}{f(x)} = -2 \pm \sqrt{3}$

$\therefore \log f(x) = \left(-2 \pm \sqrt{3}\right)x + c$

$\therefore f(x) = ke^{\left(-2\pm\sqrt{3}\right)x}$, where $k = e^c$.

13. **(b,c,d)** $\frac{dy}{dx} = \frac{2}{y-2} \Rightarrow (y-2)dy = 2dx$

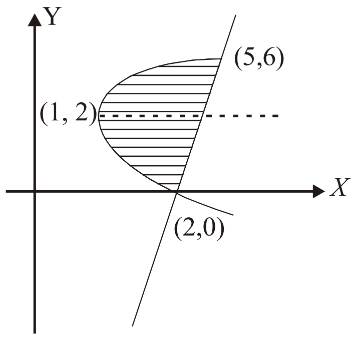
$\Rightarrow \frac{y^2}{2} - 2y = 2x + c$

It passes through $(1, 2) \Rightarrow C = -4$

So the equation of the curve is $\frac{y^2}{2} - 2y = 2x - 4$

$\Rightarrow (y-2)^2 = 4(x-1)$

Desired area $= \int_0^6 \frac{y+4}{2} - \frac{(y-2)^2+4}{4} = 9$ sq. units



E MATRIX-MATCH TYPE

1. **A-q,s ; B-r ; C-q,s ; D-p,t**

We have

$$\frac{dy}{dx} = y + \int_0^1 y\, dx \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dy}{dx} + 0 \Rightarrow \frac{d}{dx}p = p$$

(where $p = \frac{dy}{dx}$)

Integrating we get $\ln p = x + \ln k \Rightarrow p = k e^x$.

$\therefore \frac{dy}{dx} = k\, e^x$ .....(1)

Integrating again $y = k e^x + c$ .....(2)

Now $f(0) = 1 \Rightarrow 1 = c + k$ or $c = 1 - k$

Also $\frac{dy}{dx} = y + \int_0^1 y\, dx \Rightarrow ke^x = ke^x + 1 - k + \int_0^1 (ke^x + 1 - k)\, dx$

$\therefore 0 = 1 - k + ke + 1 - k - k \Rightarrow k = \frac{2}{3-e}$

Clearly $\left.\frac{dy}{dx}\right|_{x=0} = f'(0) = k = \frac{2}{3-e}$

$\left.\frac{d^2y}{dx^2}\right|_{x=0} = f''(0) = k = \frac{2}{3-e}$

Also, $y = f(x) = \frac{2e^x + 1 - e}{3-e} \Rightarrow f(1) = \frac{e+1}{3-e}$

$$\lim_{x\to 0}\frac{f(x)-1}{x} = \lim_{x\to 0}\frac{2e^x + 1 - e - 3 + e}{x(3-e)}$$

$$= \frac{2}{3-e}\lim_{x\to 0}\frac{e^x-1}{x} = \frac{2}{3-e}$$

2. **A-r ; B-q ; C-r ; D-q**

(A) The general equation of all such conic is $ax^2 + 2hxy + by^2 = 1$, which has three arbitrary constants.

(B) The general equation of all such circles is $(x-h)^2 + (y-k)^2 = a^2$, which has two arbitrary constants.

(C) The general equation of all such parabolas is $x = ay^2 + by + c$, which has three arbitrary constants.

(D) The general equation of all such conics is $ax^2 + by^2 = 1$, which has two arbitrary constants.

3. **A - p, q, r, s, t, ; B – p, q, r, s, t ; C - q ; D – r**

$f'(x) = 2x + 1 \Rightarrow f(x) = x^2 + x + C$

$\because f(1) = 2 \Rightarrow C = 0 \Rightarrow f(x) = x^2 + x$

$\because f(x)$ is polynomial, so x can take any value

Also, $x^2 + x = a$ has real solution if $1 + 4a \geq 0 \Rightarrow a \geq -\frac{1}{4}$

So, y can take any value $\geq \frac{-1}{4}$

(C) Area $= \int_0^1 \left(x^2 + x\right)dx = \frac{5}{6}$

(D) Area $= \int_0^1 \left[-\frac{1}{2} + \sqrt{\frac{1+4y}{2}}dy\right] = \frac{5\sqrt{5}-7}{12}$

F NUMERIC/INTEGER ANSWER TYPE

1. **Ans. : 5**

Put $x = r\sec\theta$ and $y = r\tan\theta$ So, $x^2 - y^2 = r^2$ .... (1)

and $\sin\theta = \frac{y}{x}$ ....(2)

then differentiating (1) we get, $2xdx - 2ydy = 2rdr$

or $xdx - ydy = rdr$ ...... (3)

and differentiaing (2) we get, $\frac{xdy - ydx}{x^2} = \cos\theta d\theta$

or $xdy - ydx = x^2\cos\theta d\theta$

$= r^2\sec^2\theta\cos\theta d\theta = r^2\sec\theta d\theta$ ....(4)

Substituting values from (3) and (4) in the given differetial equation, we get

$$\frac{rdr}{r^2\sec^2\theta d\theta} = \sqrt{\left(\frac{1+r^2}{r^2}\right)} = \frac{\sqrt{1+r^2}}{r}$$

or $\frac{dr}{\sqrt{(1+r^2)}} = \sec\theta d\theta$

Intergrating both sides,

$\ell n(r + \sqrt{(1+r^2)}) = \ell n(\sec\theta + \tan\theta) + \ell \text{n}\, c$

Where c is an arbitrary constant.

or $(r + \sqrt{(1+r^2)}) = c(\sec\theta + \tan\theta)$

or $(\sqrt{(x^2 - y^2)} + \sqrt{(1 + x^2 - y^2)} = c\left(\frac{x+y}{\sqrt{(x^2 - y^2)}}\right)$

2. **Ans. : 8**

Differentiating both sides w.r.t., x of the given equation

$$x.y(x) + \int_0^x y(t)dt.1 = (x+1)x.y(x) + \int_0^x ty(t)dt$$

or $\int_0^x y(t)dt = x^2 y(x) + \int_0^x ty(t)dt$

Again differentiating both sides w.r.t. x

$y(x) = x^2 y'(x) + y(x)2x + xy(x)$

or $(1-3x)y(x) = x^2 y'(x)$ or $\frac{y'(x)}{y(x)} = \left(\frac{1}{x^2} - \frac{3}{x}\right)$

Integrating, we get $\ell \text{n}\, y(x) = -\frac{1}{x} - 3\ell \text{n}\, x + \ell \text{n}\, c$

or $\ell \text{n}\left(\frac{x^3 y(x)}{c}\right) = -\frac{1}{x}$ or $\frac{x^3 y(x)}{c} = e^{-1/x}$

or $y(x) = \frac{ce^{-1/x}}{x^3}$

So, $y(1) = e \Rightarrow c = e^2$

$\therefore\ y\left(\frac{1}{2}\right) = 8$

3. **Ans. 48**

Let population = x, at time t years

Give $\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = kx$

Where k is constant of proportionality or $\frac{dx}{x} = kdt$

Integrating, we get $\ln x = kt + \ln c \Rightarrow \frac{x}{c} = e^{kt}$

or $x = ce^{kt}$

If initially i.e., when time $t = 0, x = x_0$ then $x_0 = ce^0 = c$

$\therefore\ x = x_0 e^{kt}$

Given $x = 2x_0$ when $t = 30$ then $2x_0 = x_0 e^{30k}$

$\Rightarrow\ 2 = e^{30k}$ .... (1)

$\therefore\ \ln 2 = 30k$

To find t, when it tripples, $x = 3x_0$

$\therefore\ 3x_0 = x_0 e^{kt} \Rightarrow 3 = e^{kt}$ ....(2)

$\therefore\ \ln 3 = kt$

Diving (2) by (1) then $\frac{t}{30} = \frac{\ln 3}{\ln 2}$

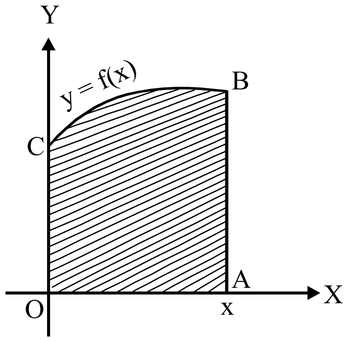
or $t = 30 \times \frac{\ln 3}{\ln 2} = 30 \times 1.5849 = 48$ years. (approx.)

4. **Ans. : 10**

Area of curvilinear trapezoid $OABCO = \int_0^x f(x)dx$

according to question

$$\int_0^x f(x)dx \propto \{f(x)\}^{n+1} \text{ or } \int_0^x f(x)\,dx = k\{f(x)\}^{n+1}$$



Where k is constant of proportionality.
Differentiating both sides w.r.t. x,

$$f(x) = k(n+1)(f(x))^n f'(x)$$

or $\{f(x)^{n-1}\}f'(x) = \dfrac{1}{k(n+1)'}$

Integrating both sides w.r.t. x, $\dfrac{\{f(x)\}^n}{n} = \dfrac{x}{k(n+1)} + c$

Putting x = 0 $\dfrac{\{f(0)\}^n}{n} = 0 + c \Rightarrow 0 = 0 + c$

$(\because f(0) = 0)$

$$\therefore \frac{\{f(x)\}^n}{n} = \frac{x}{k(n+1)} \Rightarrow \{f(x)\}^n = \frac{nx}{k(n+1)} \quad .... (1)$$

Again putting x = 1 then $\{f(x)\}^n = \dfrac{n}{k(n+1)} = 1$

$(\because f(1) = 1)$

From (1), $(f(x))^n = x$ or $f(x) = x^{1/n}$

5. Ans. : 6

Given $y = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x}$ ...... (1)

so, $y_1 = C_1 m_1 e^{m_1 x} + C_2 m_2 e^{m_2 x} + C_3 m_3 e^{m_3 x}$

$$= m_1(y - C_2 e^{m_2 x} - C_3 e^{m_3 x}) + C_2 m_2 e^{m_2 x} + C_3 m_3 e^{m_3 x}$$

{from (1)}

$$= m_1 y + C_2(m_2 - m_1)e^{m_2} + C_3(m_3 - m_1)e^{m_3 x} \text{ ...(2)}$$

Next $y_2 = m_1 y_1 + C_2 m_2 (m_2 - m_1)e^{m_2 x}$

$$+ C_3 m_3 (m_3 - m_1)e^{m_3 x}$$

$$= m_1 y_1 + m_2[y_1 - m_1 y - C_3(m_3 - m_1)e^{m_3 x}]$$

$$+ C_3 m_3 (m_3 - m_1)e^{m_3 x}$$

[from (2)]

$$= (m_1 + m_2)y_1 - m_1 m_2 y$$

$$+ C_3(m_3 - m_1)(m_3 - m_2)e^{m_3 x} \quad ......(3)$$

Further, $y_3 = (m_1 + m_2)y_2 - m_1 m_2 y_1$

$$+ C_3 m_3 (m_3 - m_1)(m_3 - m_2)e^{m_2 x}$$

$$= (m_1 + m_2)y_2 - m_1 m_2 y_1$$

$$+ m_3[y_2 - (m_1 + m_2)y_1 + m_1 m_2 y] \text{ [from (3)]}$$

$$= (m_1 + m_2 + m_3)y_2 - (m_1 m_2 + m_1 m_3 + m_2 m_3)y_1$$

$$+ m_1 m_2 m_3 y$$

$$= 0.y_2 - (-7)y_1 - 6y \Rightarrow y_3 - 7y_1 + 6y = 0$$

6. Ans. : 9

$$\frac{dy}{dx} = 2C_1 e^{2x} + C_2 e^x - C_3 e^{-x}, \frac{d^2 y}{dx^2}$$

$$= 4C_1 e^{2x} + C_2 e^x + C_3 e^{-x}$$

$$\frac{d^3 y}{dx^3} = 8C_1 e^{2x} + C_2 e^x - C_3 e^{-x}.$$

Putting into the given differential equation.
We get, $8 + 4a + 2b + c = 0,$
$1 + a + b + c = 0, -1 + a - b + c = 0$
$\Rightarrow a = -2, b = -1, c = 2.$

Thus $\dfrac{a^3 + b^3 + c^3}{abc} = -\dfrac{1}{4}.$