14

Statistics

TOPIC 1 Arithmetic Mean, Geometric Mean, Harmonic Mean, Median & Mode

1. Consider the data on x taking the values $0, 2, 4, 8, ..., 2^n$ with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to ______.

[NA Sep. 06, 2020 (II)]

2. The minimum value of $2^{\sin x} + 2^{\cos x}$ is : **[Sep. 04, 2020 (II)]**

(a) $2^{-1+\frac{1}{\sqrt{2}}}$ (b) $2^{-1+\sqrt{2}}$ (c) $2^{1-\sqrt{2}}$ (d) $2^{1-\frac{1}{\sqrt{2}}}$

3. If for some $x \in \mathbf{R}$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is : **[April 10, 2019 (I)]**

(a) 3.2 (b) 3.0 (c) 2.5 (d) 2.8

4. The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to: **[April. 09, 2019 (II)]**

(a) 9/4 (b) 7/2 (c) 8/3 (d) 7/3

5. The mean of a set of 30 observations is 75. If each other observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. The λ is equal to **[Online April 15, 2018]**

(a) $\frac{10}{3}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

6. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If now the mean age of the teachers in this school is 39 years, then the age (in years) of the newly appointed teacher is : **[Online April 8, 2017]**

(a) 25 (b) 30 (c) 35 (d) 40

7. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is: **[2015]**

(a) 15.8 (b) 14.0 (c) 16.8 (d) 16.0

8. Let the sum of the first three terms of an A. P, be 39 and the sum of its last four terms be 178. If the first term of this A.P. is 10, then the median of the A.P. is :

[Online April 10, 2015]

(a) 28 (b) 26.5 (c) 29.5 (d) 31

9. A factory is operating in two shifts, day and night, with 70 and 30 workers respectively. If per day mean wage of the day shift workers is ₹ 54 and per day mean wage of all the workers is ₹ 60, then per day mean wage of the night shift workers (in ₹) is : **[Online April 10, 2015]**

(a) 69 (b) 66 (c) 74 (d) 75

10. In a set of 2n distinct observations, each of the observations below the median of all the observations is increased by 5 and each of the remaining observations is decreased by 3. Then the mean of the new set of observations: **[Online April 9, 2014]**

(a) increases by 1 (b) decreases by 1
(c) decreases by 2 (d) increases by 2

11. If the median and the range of four numbers $\{x, y, 2x + y, x - y\}$, where $0 < y < x < 2y$, are 10 and 28 respectively, then the mean of the numbers is :

[Online April 23, 2013]

(a) 18 (b) 10 (c) 5 (d) 14

12. The mean of a data set consisting of 20 observations is 40. If one observation 53 was wrongly recorded as 33, then the correct mean will be : **[Online April 9, 2013]**

(a) 41 (b) 49 (c) 40.5 (d) 42.5

13. The median of 100 observations grouped in classes of equal width is 25. If the median class interval is 20 - 30 and the number of observations less than 20 is 45, then the frequency of median class is **[Online May 19, 2012]**

(a) 10 (b) 20 (c) 15 (d) 12

14. The frequency distribution of daily working expenditure of families in a locality is as follows:

Expenditure in ₹. (x):	0-50	50-100	100-150	150-200	200-250
No. of families (f):	24	33	37	b	25

If the mode of the distribution is ₹ 140, then the value of b is **[Online May 7, 2012]**

(a) 34 (b) 31 (c) 26 (d) 36

15. The average marks of boys in class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is **[2007]**

(a) 80 (b) 60 (c) 40 (d) 20

16. Let $x_1, x_2, ... x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then the possible value of n among the following is **[2005]**

(a) 15 (b) 18 (c) 9 (d) 12

17. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately **[2005]**

(a) 22.0 (b) 20.5 (c) 25.5 (d) 24.0

18. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set **[2003]**

(a) remains the same as that of the original set
(b) is increased by 2
(c) is decreased by 2
(d) is two times the original median.

19. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls? **[2002]**

(a) 73 (b) 65 (c) 68 (d) 74

TOPIC 2 Quartile, Measures of Dispersion, Quartile Deviation, Mean Deviation, Variance & Standard Deviation, Coefficient of Variation

20. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, $(n, a > 1)$, then the standard deviation of n observations $x_1, x_2, ..., x_n$ is : **[Sep. 06, 2020 (I)]**

(a) $a-1$ (b) $n\sqrt{a-1}$

(c) $\sqrt{n(a-1)}$ (d) $\sqrt{a-1}$

21. The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is : **[Sep. 05, 2020 (I)]**

(a) 1 (b) 4 (c) 2 (d) 3

22. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation : **[Sep. 05, 2020 (II)]**

(a) $x^2 - 10x + 18 = 0$ (b) $2x^2 - 20x + 19 = 0$

(c) $x^2 - 10x + 19 = 0$ (d) $x^2 - 20x + 18 = 0$

23. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is : **[Sep. 04, 2020 (I)]**

(a) 9 (b) 5 (c) 3 (d) 7

24. If a variance of the following frequency distribution :

Class	10-20	20-30	30-40
Frequency	2	x	2

is 50, then x is equal to ____________.

[NA Sep. 04, 2020 (II)]

25. For the frequency distribution :

Variate (x) :	x_1	x_2	$x_1 ... x_{15}$
Frequency (f) :	f_1	f_2	$f_3 ... f_{15}$

where $0 < x_1 < x_2 < x_3 < ... < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation **cannot** be : **[Sep. 03, 2020 (I)]**

(a) 4 (b) 1 (c) 6 (d) 2

26. Let $x_i (1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \ne p \in \mathbf{R}$, then the standard deviation of these observations is : **[Sep. 03, 2020 (II)]**

(a) $\sqrt{\frac{3}{5}}$ (b) $\frac{4}{5}$ (c) $\frac{9}{10}$ (d) $\frac{7}{10}$

27. Let $X = \{x \in \mathbf{N} : 1 \le x \le 17\}$ and $Y = \{ax + b : x \in X$ and $a, b \in \mathbf{R}, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then $a + b$ is equal to : **[Sep. 02, 2020 (I)]**

(a) 7 (b) –7 (c) –27 (d) 9

28. If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, ..., b_{11}$ is 90, then the common difference of this A.P. is ___________. **[NA Sep. 02, 2020 (II)]**

29. Let the observations $x_i (1 \le i \le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_i - 5) = 10$ and $\sum_{i=1}^{10}(x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, ..., x_{10} - 3$, then the ordered pair (μ, λ) is equal to: **[Jan. 9, 2020 (I)]**

(a) (3, 3) (b) (6, 3) (c) (6, 6) (d) (3, 6)

30. The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \ne 0$ and $q \ne 0$. If the new mean and new s.d. become half of their original values, then q is equal to: **[Jan. 8, 2020 (I)]**

(a) –5 (b) 10 (c) –20 (d) –10

31. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is: **[Jan. 8, 2020 (II)]**

(a) 3.99 (b) 4.01 (c) 4.02 (d) 3.98

32. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then $m + n$ is equal to———. **[NA Jan. 7, 2020 (I)]**

33. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then $x \cdot y$ is equal to _________. **[NA Jan. 7, 2020 (II)]**

34. If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000 ; then the standard deviation of this data is : **[April 12, 2019 (I)]**

(a) $2\sqrt{2}$ (b) 2 (c) 4 (d) $\sqrt{2}$

35. If both the mean and the standard deviation of 50 observations $x_1, x_2, ..., x_{50}$ are equal to 16, then the mean of $(x_1-4)^2, (x_2-4)^2, ..., (x_{50}-4)^2$ is : **[April 10, 2019 (II)]**

(a) 400 (b) 380 (c) 525 (d) 480

36. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to: **[April 09, 2019 (I)]**

(a) $2\sqrt{6}$ (b) $2\sqrt{\frac{10}{3}}$ (c) $4\sqrt{\frac{5}{3}}$ (d) $\sqrt{6}$

37. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is : **[April 08, 2019 (I)]**

(a) 45 (b) 49 (c) 48 (d) 40

38. A student scores the following marks in five tests: 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is : **[April. 08, 2019 (II)]**

(a) $\frac{10}{\sqrt{3}}$ (b) $\frac{100}{3}$ (c) $\frac{10}{3}$ (d) $\frac{100}{\sqrt{3}}$

39. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is : **[Jan. 12, 2019 (I)]**

(a) 30 (b) 51 (c) 50 (d) 31

40. The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then the absolute value of the difference of the other two observations, is : **[Jan. 12, 2019 (II)]**

(a) 7 (b) 5 (c) 1 (d) 3

41. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - \text{d}$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + \text{d}$ each. If the variance of this outcome data is $\frac{4}{3}$ then |d| equals : **[Jan. 11, 2019 (I)]**

(a) $\frac{2}{3}$ (b) 2 (c) $\frac{\sqrt{5}}{2}$ (d) $\sqrt{2}$

42. A data consists of n observations: $x_1, x_2, ..., x_n$. If $\sum_{i=1}^{n}(x_i + 1)^2 = 9\text{n}$ and $\sum_{i=1}^{n}(x_i - 1)^2 = 5\text{n}$ then the standard deviation of this data is: **[Jan. 09, 2019 (II)]**

(a) 2 (b) $\sqrt{5}$ (c) 5 (d) $\sqrt{7}$

43. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is: **[Jan. 10, 2019 (I)]**

(a) 10 : 3 (b) 4 : 9 (c) 5 : 8 (d) 6 : 7

44. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, ..., x_5$ and – 50 is equal to: **[Jan. 10, 2019 (II)]**

(a) 509.5 (b) 586.5 (c) 582.5 (d) 507.5

45. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is: **[Jan. 9, 2019 (I)]**

(a) 16 (b) 22 (c) 20 (d) 18

46. The mean and the standard deviation (s.d.) of five observations are 9 and 0, respectively.

If one of the observations is changed such that the mean of the new set of five observations becomes 10, then their s.d. is? **[Online April 16, 2018]**

(a) 0 (b) 4 (c) 2 (d) 1

47. If the mean of the data : 7, 8, 9, 7, 8, 7, λ, 8 is 8, then the variance of this data is **[Online April 15, 2018]**

(a) $\frac{9}{8}$ (b) 2 (c) $\frac{7}{8}$ (d) 1

48. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is : **[2018]**

(a) 4 (b) 2 (c) 3 (d) 9

49. The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3, 4 and 5, were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is : **[Online April 9, 2017]**

(a) 8.25 (b) 8.50 (c) 8.00 (d) 9.00

50. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true? **[2016]**

(a) $3a^2 - 34a + 91 = 0$ (b) $3a^2 - 23a + 44 = 0$

(c) $3a^2 - 26a + 55 = 0$ (d) $3a^2 - 32a + 84 = 0$

51. The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1, 2 and 6; then the mean deviation from the mean of the data is :

[Online April 10, 2016]

(a) 2.5 (b) 2.6 (c) 2.8 (d) 2.4

52. If the mean deviation of the numbers 1, 1 + d, ..., 1 + 100d from their mean is 255, then a value of d is :

[Online April 9, 2016]

(a) 10.1 (b) 5.05 (c) 20.2 (d) 10

53. The variance of first 50 even natural numbers is **[2014]**

(a) 437 (b) $\frac{437}{4}$ (c) $\frac{833}{4}$ (d) 833

54. Let $\bar{x}$, M and σ^2 be respectively the mean, mode and variance of n observations $x_1, x_2, ..., x_n$ and $d_i = -x_i - a$, $i = 1, 2, ..., n$, where a is any number.

Statement I: Variance of $d_1, d_2, ... d_n$ is σ^2.

Statement II: Mean and mode of $d_1, d_2, ... d_n$ are $-\bar{x} - a$ and $-M - a$, respectively. **[Online April 19, 2014]**

(a) Statement I and Statement II are both false

(b) Statement I and Statement II are both true

(c) Statement I is true and Statement II is false

(d) Statement I is false and Statement II is true

55. Let $\overline{X}$ and M.D. be the mean and the mean deviation about $\overline{X}$ of n observations x_i, i = 1, 2, ..., n. If each of the observations is increased by 5, then the new mean and the mean deviation about the new mean, respectively, are: **[Online April 12, 2014]**

(a) $\overline{X}$, M.D. (b) $\overline{X} + 5$, M.D.

(c) $\overline{X}$, M.D. + 5 (d) $\overline{X} + 5$, M.D. + 5

56. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ? **[2013]**

(a) mean (b) median (c) mode (d) variance

57. In a set of 2n observations, half of them are equal to 'a' and the remaining half are equal to '–a'. If the standard deviation of all the observations is 2 ; then the value of $|a|$ is :

[Online April 25, 2013]

(a) 2 (b) $\sqrt{2}$ (c) 4 (d) $2\sqrt{2}$

58. Mean of 5 observations is 7. If four of these observations are 6, 7, 8, 10 and one is missing then the variance of all the five observations is : **[Online April 22, 2013]**

(a) 4 (b) 6 (c) 8 (d) 2

59. Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance.

Statement-1 : Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.

Statement-2 : Arithmetic mean $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$.

[2012]

(a) Statement-1 is false, Statement-2 is true.

(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(c) Statement-1 is true, statement-2 is true; statement-2 is ***not*** a correct explanation for Statement-1.

(d) Statement-1 is true, statement-2 is false.

60. **Statement 1:** The variance of first n odd natural numbers is $\frac{n^2-1}{3}$

Statement 2: The sum of first n odd natural number is n^2 and the sum of square of first n odd natural numbers is $\frac{n(4n^2+1)}{3}$. **[Online May 26, 2012]**

(a) Statement 1 is true, Statement 2 is false.

(b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

(c) Statement 1 is false, Statement 2 is true.

(d) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

61. If the mean of 4, 7, 2, 8, 6 and a is 7, then the mean deviation from the median of these observations is **[Online May 12, 2012]**

(a) 8 (b) 5 (c) 1 (d) 3

62. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standarion deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively : **[2011RS]**

(a) 32, 2 (b) 32, 4 (c) 28, 2 (d) 28, 4

63. If the mean deviation about the median of the numbers a, $2a$,.......,$50a$ is 50, then $|a|$ equals **[2011]**

(a) 3 (b) 4 (c) 5 (d) 2

64. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is **[2010]**

(a) $\frac{11}{2}$ (b) 6 (c) $\frac{13}{2}$ (d) $\frac{5}{2}$

65. If the mean deviation of the numbers 1, 1 + d, 1 + 2d, ... 1 + 100d from their mean is 255, then d is equal to: **[2009]**

(a) 20.0 (b) 10.1 (c) 20.2 (d) 10.0

66. **Statement-1 :** The variance of first n even natural numbers is $\frac{n^2-1}{4}$.

Statement-2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$. **[2009]**

(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

67. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b? **[2008]**

(a) $a=0, b=7$ (b) $a=5, b=2$

(c) $a=1, b=6$ (d) $a=3, b=4$

68. Suppose a population A has 100 observations 101, 102, ..., 200 and another population B has 100 obsevrations 151, 152, ... 250. If V_A and V_B represent the variances of the two populations, respectively then $\frac{V_A}{V_B}$ is **[2006]**

(a) 1 (b) $\frac{9}{4}$ (c) $\frac{4}{9}$ (d) $\frac{2}{3}$

69. In a series of 2 n observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ equals. **[2004]**

(a) $\frac{\sqrt{2}}{n}$ (b) $\sqrt{2}$ (c) 2 (d) $\frac{1}{n}$

70. Consider the following statements :

(A) Mode can be computed from histogram

(B) Median is not independent of change of scale

(C) Variance is independent of change of origin and scale.

Which of these is / are correct ? **[2004]**

(a) (A), (B) and (C) (b) Only (B)

(c) Only (A) and (B) (d) Only (A)

71. In an experiment with 15 observations on x, the following results were available:

$\Sigma x^2 = 2830,\ \Sigma x = 170$

One observation that was 20 was found to be wrong and was replaced by the correct value 30. The corrected variance is **[2003]**

(a) 8.33 (b) 78.00 (c) 188.66 (d) 177.33

Hints & Solutions

1. **(6.00)**

Mean $=\frac{\Sigma x_i f_i}{\Sigma f_i}=\frac{0\cdot{}^nC_0+2\cdot{}^nC_1+2^2\cdot{}^nC_2+...+2^n\cdot{}^nC_n}{{}^nC_0+{}^nC_1+....+{}^nC_n}$

To find sum of numerator consider

$(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+...+{}^nC_nx^n$ ...(i)

Put $x=2\Rightarrow 3^n-1=2\cdot{}^nC_1+2^2\cdot{}^nC_2+....+2^n\cdot{}^nC_n$

To find sum of denominator, put $x=1$ in (i), we get

$2^n={}^nC_0+{}^nC_1+.....+{}^nC_n$

$\therefore \frac{3^n-1}{2^n}=\frac{728}{2^n}\Rightarrow 3^n=729\Rightarrow n=6$

2. **(d)** $\frac{2^{\sin x}+2^{\cos x}}{2}\ge(2^{\sin x+\cos x})^{\frac{1}{2}}$ $(\because \text{AM}\ge\text{GM})$

$\Rightarrow 2^{\sin x}+2^{\cos x}\ge 2\cdot 2^{\frac{\sin x+\cos x}{2}}$

Since, $-2\le\sin x+\cos x\le\sqrt{2}$

$\therefore$ Minimum value of $2^{\frac{\sin x+\cos x}{2}}=2^{-\frac{1}{\sqrt{2}}}$

$\Rightarrow 2^{\sin x}+2^{\cos x}\ge 2^{1-\frac{1}{\sqrt{2}}}$.

3. **(d)** Number of students are,

$(x+1)^2+(2x-5)+(x^2-3x)+x=20$

$\Rightarrow 2x^2+2x-4=20\Rightarrow x^2+x-12=0$

$\Rightarrow (x+4)(x-3)=0\Rightarrow x=3$

$\therefore$

Marks	2	3	5	7
No. of students	16	1	0	3

Average marks $=\frac{32+3+21}{20}=\frac{56}{20}=2.8$

4. **(d)** Ten numbers in increasing order are

$10, 22, 26, 29, 34, x, 42, 67, 70, y$

Mean $=\frac{\sum x_i}{n}=\frac{x+y+300}{10}=42\Rightarrow x+y=120$

Median $=\frac{T_5+T_6}{2}=35=\frac{34+x}{2}\Rightarrow x=36$ and $y=84$

Hence, $\frac{y}{x}=\frac{84}{36}=\frac{7}{3}$

5. **(b)** As mean is a linear operation, so if each observation is multiplied by λ and decreased by 25 then the mean becomes $75\quad\lambda-25$.

According to the question,

$75\lambda-25=75\Rightarrow\lambda=\frac{4}{3}$.

6. **(c)** Let; $\frac{x_1+x_2+.......+x_{25}}{25}=\bar{x}=40$

$\Rightarrow x_1+x_2+..........+x_{25}=1000$

$\therefore x_2+x_2+..........+x_{25}-60+A=39\times 25$

Let A be the age of new teacher.

$\Rightarrow 1000-60+A=975$

$\Rightarrow A=975-940=35$

7. **(b)** Sum of 16 observations $=16\times 16=256$

Sum of resultant 18 observations

$=256-16+(3+4+5)=252$

Mean of observations $=\frac{252}{18}=14$

8. **(c)** $a_1+a_2+a_3=39$

$\Rightarrow a_1+(a_1+d)+(a_1+2d)=39$

$\Rightarrow 3a_1+3d=39$ $[\because a_1=10]$

$\Rightarrow d=3$

Sum of last four term $=178$

Their mean $=\frac{178}{4}=44.5$

$a_n=44.5+1.5+3=49$

Median $=\frac{10+49}{2}=\frac{59}{2}=29.5$

9. **(c)** Let average wage of Night shift worker is x

$70\times 54+30\times x=60\times 100$

$x=74$

10. **(a)** There are $2n$ observations $x_1, x_2, ..., x_{2n}$

So, mean $=\sum_{i=1}^{2n}\frac{x_i}{2n}$

Let these observations be divided into two parts $x_1, x_2, ..., x_n$ and $x_{n+1}, ..., x_{2n}$

Each in 1st part 5 is added, so total of first part is $\sum_{i=1}^{n}x_i+5n\cdot$

In second part 3 is subtracted from each

So, total of second part is $\sum_{i=n+1}^{2n} x_i - 3n$

Total of $2n$ terms are

$$\sum_{i=1}^{n} x_i + 5n + \sum_{i=n+1}^{2n} x_i - 3n = \sum_{i=1}^{2n} x_i + 2n$$

$$\text{Mean} = \sum_{i=1}^{2n} \frac{x_i + 2n}{2n} = \sum_{i=1}^{2n} \frac{x_i}{2n} + 1$$

So, it increase by 1.

11. **(d)** Since $0 < y < x < 2y$

$\therefore\ y > \frac{x}{2} \Rightarrow x - y < \frac{x}{2}$

$\therefore\ x - y < y < x < 2x + y$

Hence median $= \frac{y+x}{2} = 10$

$\Rightarrow x + y = 20$ ...(i)

And range $= (2x + y) - (x - y) = x + 2y$

But range $= 28$

$\therefore x + 2y = 28$ ...(ii)

From equations (i) and (ii),

$x = 12, y = 8$

$\therefore$ Mean $= \frac{(x-y)+y+x+(2x+y)}{4} = \frac{4x+y}{4}$

$= x + \frac{y}{4} = 12 + \frac{8}{4} = 14$

12. **(a)** Correct mean $= \frac{20\times 40 - 33 + 55}{20} = 41.1$

Nearest option : (a) 41

13. **(a)** Median is given as

$$M = l + \frac{\frac{N}{2} - F}{f} \times C$$

where

l = lower limit of the median - class

f = frequency of the median class

N = total frequency

F = cumulative frequency of the class just before the median class

C = length of median class

Now, given, $M = 25, N = 100, F = 45,$

$C = 20 - 30 = 10,\ l = 20.$

$\therefore$ By using formula, we have

$$25 = 20 + \frac{50-45}{f} \times 10$$

$$25 - 20 = \frac{50}{f} \Rightarrow 5 = \frac{50}{f} \Rightarrow f = 10$$

14. **(d)** Frequency distribution is given as

Expenditure	No. of families (f)
0-50	24
50-100	33
100-150	37
150-200	b
200-250	25

Clearly, modal class is 100-150, as the maximum frequency occurs in this class.

Given, Mode = 140

We have

$$\text{Mode} = \ell + \frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \times i$$

where

$\ell = 100, f_0 = 37, f_{-1} = 33, f_1 = b$

$i = 50$

Thus, we get

$$140 = 100 + \left[\frac{37-33}{2(37) - 33 - b}\right] \times 50$$

$$= 100 + \left[\frac{4}{74 - 33 - b}\right] \times 50 = 100 + \frac{200}{41 - b}$$

$\Rightarrow\ 5740 = 4300 + 40b \Rightarrow b = 36$

15. **(a)** Let the number of boys be x and girls be y.

$\Rightarrow 52x + 42y = 50(x + y)$

$\Rightarrow 52x - 50x = 50y - 42y$

$\Rightarrow\ 2x = 8y \Rightarrow \frac{x}{y} = \frac{4}{1} \Rightarrow \frac{x}{x+y} = \frac{4}{5}$

$\therefore$ Required % of boys $= \frac{x}{x+y} \times 100$

$= \frac{4}{5} \times 100 = 80\,\%$

16. **(b)** We know that for positive real numbers $x_1, x_2, ..., x_n$, A.M. of k^{th} powers of $x_i \geq k^{\text{th}}$ the power of A.M. of x_i

$$\Rightarrow \frac{\sum x_1^2}{n} \geq \left(\frac{\sum x_1}{n}\right)^2 \Rightarrow \frac{400}{n} \geq \left(\frac{80}{n}\right)^2$$

$\Rightarrow n \geq 16$. So only possible value for $n = 18$

17. **(d)** We know that Mode = 3 Median – 2Mean

$3 \times 22 - 2 \times 21 = 66 - 42 = 24$

18. **(a)** $n = 9$ then median term $= \left(\frac{9+1}{2}\right)^{th} = 5^{th}$ term. That means four observation followed by it. If last four observations are increased by 2. The median is 5th observation which is remaining unchanged.

$\therefore$ There will be no change in median.

19. **(b)** Total student = 100

Total marks of 70 boys = 75 × 70 = 5250

$\Rightarrow$ Total marks of girls = 7200 – 5250 = 1950

Number of girls = 100 – 70 = 30

Average of girls = $\frac{1950}{30} = 65$

20. **(d)** Standard deviation

$$= \sqrt{\frac{\sum_{i=1}^{n}(x_i - a)^2}{n} - \left(\frac{\sum_{i=1}^{n}(x_i - a)}{n}\right)^2} \quad [\because n, a > 1]$$

$$= \sqrt{\frac{na}{n} - \left(\frac{n}{n}\right)^2} = \sqrt{a-1}$$

21. **(c)** Let two remaining observations are x_1, x_2.

So, $\bar{x} = \frac{2+4+10+12+14+x_1+x_2}{7} = 8$ (given)

$\Rightarrow x_1 + y_1 = 14$ ...(i)

Now, $\sigma^2 = \frac{\Sigma x_i^2}{N} - \left(\frac{\Sigma x_i}{N}\right)^2 = 16$ (given)

$= \frac{4+16+100+144+196+x_1^2+x_2^2}{7} - 64 = 16$

$\Rightarrow 460 + x_1^2 + x_2^2 = (16+64) \times 7$

$\Rightarrow x_1^2 + x_2^2 = 100$ ...(ii)

$\because (x+y)^2 = x^2 + y^2 + 2xy \Rightarrow xy = 48$ ...(iii)

$\because (x-y)^2 = (x+y)^2 - 4xy = 196 - 192 = 4$

$\Rightarrow x - y = 2 \Rightarrow |x-y| = 2$

22. **(c)** Mean $= \frac{3+5+7+a+b}{5} = 5 \Rightarrow a+b = 10$

Variance $= \frac{3^2+5^2+7^2+a^2+b^2}{5} - (5)^2 = 4$

$\Rightarrow a^2 + b^2 = 62$

$\Rightarrow (a+b)^2 - 2ab = 62$

$\Rightarrow ab = 19$

Hence, a and b are the roots of the equation,

$x^2 - 10x + 19 = 0$.

23. **(d)** Let the two remaining observations be x and y.

$\because \bar{x} = \frac{5+7+10+12+14+15+x+y}{8}$

$\Rightarrow 10 = \frac{63+x+y}{8}$

$\Rightarrow x + y = 80 - 63$

$\Rightarrow x + y = 17$ ...(i)

$\because \text{var}(x) = 13.5$

$= \frac{25+49+100+144+196+225+x^2+y^2}{8} - (10)^2$

$\Rightarrow x^2 + y^2 = 169$ ...(ii)

From (i) and (ii) we get

$(x, y) = (12, 5)$ or $(5, 12)$

So, $|x - y| = 7$.

24. **(4)**

x_i	15	25	35
f_i	2	x	2

$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{30+70+25x}{4+x} = 25$

$\sigma^2 = \frac{\Sigma f_i x_i^2}{\Sigma f_i} - (\bar{x})^2$

$\Rightarrow 50 = \frac{450+625x+2450}{4+x} - 625$

$\Rightarrow 675 = \frac{2900+625x}{4+x} \Rightarrow 50x = 200$

$\therefore x = 4$

25. **(c)** If variate varries from a to b then variance

$\text{var}(x) \le \left(\frac{b-a}{2}\right)^2$

$\Rightarrow \text{var}(x) < \left(\frac{10-0}{2}\right)^2$

$\Rightarrow \text{var}(x) < 25$

$\Rightarrow$ standard deviation < 5

It is clear that standard deviation cann't be 6.

26. **(c)** S.D. $= \sqrt{\frac{\sum_{i=1}^{10}(x_i - p)^2}{10} - \left(\frac{\sum_{i=1}^{10}(x_i - p)}{10}\right)^2}$

$$= \sqrt{\frac{9}{10} - \left(\frac{3}{10}\right)^2} = \frac{9}{10}.$$

27. **(b)** $\because \bar{x} = \frac{1+2+3+....+17}{17} = \frac{17 \times 18}{17 \times 2} = 9$

$\bar{y} = a\bar{x} + b = \frac{a(1+2+3+.....+17)}{17} + b = 17$

$$\Rightarrow \frac{a\cdot(17\cdot18)}{17\cdot2}+b=17 \Rightarrow 9a+b=17 \quad ...(i)$$

$$\text{Var}(x)=\sigma A^2=\frac{\Sigma x^2}{n}-(\bar{x})^2$$

$$=\frac{1^2+2^2+....+17^2}{17}-(9)^2$$

$$=\frac{17\cdot18\cdot35}{6\cdot17}-(9)^2=105-81=24$$

$\text{Var}(y)=a^2\,\text{Var}(x)=a^2\cdot24=216$

$$a^2=\frac{216}{24}=9\Rightarrow a=3$$

$\therefore$ From (i), $b=17-9a=17-27=-10$

$\therefore a+b=3+(-10)=-7$

28. **(3)**

$$\text{Variance}=\frac{\sum_{i=1}^{11}b_i^2}{11}-\left(\frac{\sum_{i=1}^{11}b_i}{11}\right)^2$$

Let common difference of A.P. be d

$$=\frac{\sum_{r=0}^{10}(b_1+rd)^2}{11}-\left(\frac{\sum_{r=0}^{10}(b_1+rd)}{11}\right)^2$$

$$=\frac{11b_1^2+2b_1d\left(\frac{10\times11}{2}\right)+d^2\left(\frac{10\times11\times21}{6}\right)}{11}-\left(\frac{11b_1+\frac{10\times11}{2}d}{11}\right)^2$$

$=(b_1^2+10b_1d+35d^2)-(b_1+5d)^2=10d^2$

$\because$ Variance $=90$ (Given)

$\Rightarrow 10d^2=90\Rightarrow d=3.$

29. **(a)** Mean of the observation $(x_i-5)=\frac{\Sigma(x_i-5)}{10}=1$

$\therefore\ \lambda=\{\text{Mean}(x_i-5)\}+2=3$

Variance of the observation

$$\mu=\text{var}(x_i-5)=\frac{\Sigma(x_i-5)^2}{10}-\frac{\Sigma(x_i-5)}{10}=3$$

30. **(c)** Let $\bar{x}$ and σ be the mean and standard deviations of given observations.

If each observation is multiplied with p and then q is subtracted.

New mean $(\bar{x}_1)=p\bar{x}-q$

$\Rightarrow\ 10=p(20)-q \quad ...(i)$

and new standard deviations $\sigma_1=|p|\,\sigma$

$\Rightarrow\ 1=|p|(2)\ \Rightarrow\ |p|=\frac{1}{2}\ \Rightarrow\ p=\pm\frac{1}{2}$

If $p=\frac{1}{2}$, then $q=0$ (from equation (i))

If $p=-\frac{1}{2}$, then $q=-20$

31. **(a)** Let $x_1, x_2, ..., x_{20}$ be 20 observations, then

$$\text{Mean}=\frac{x_1+x_2+.....+x_{20}}{20}=10$$

$$\Rightarrow \frac{\sum_{i=1}^{20}x_i}{20}=10 \quad ...(i)$$

$$\text{Variance}=\frac{\Sigma x_i^2}{n}-(\bar{x})^2$$

$$\Rightarrow \frac{\Sigma x_i^2}{20}-100=4 \quad ...(ii)$$

$\Sigma x_i^2=104\times20=2080$

$$\text{Actual mean}=\frac{200-9+11}{20}=\frac{202}{20}$$

$$\text{Variance}=\frac{2080-81+121}{20}-\left(\frac{202}{20}\right)^2$$

$$=\frac{2120}{20}-(10.1)^2=106-102.01=3.99$$

32. **(18)** $\text{Var}(1, 2, ..., n)=10$

$$\Rightarrow \frac{1^2+2^2+.....+n^2}{n}-\left(\frac{1+2+.....+n}{n}\right)^2=10$$

$$\Rightarrow \frac{(n+1)(2n+1)}{6}-\left(\frac{n+1}{2}\right)^2=10$$

$\Rightarrow\ n^2-1=120 \quad \Rightarrow\ n=11$

$\text{Var}(2, 4, 6, ..., 2m)=16\Rightarrow \text{Var}(1, 2, ..., m)=4$

$\Rightarrow\ m^2-1=48\Rightarrow m=7$

$\Rightarrow\ m+n=18$

33. **(52)** $\text{Mean}=\bar{x}=\frac{3+7+9+12+13+20+x+y}{8}=10$

$\Rightarrow\ x+y=16 \quad ...(i)$

Variance $= \sigma^2 = \frac{\Sigma(x_i)^2}{8} - (\bar{x})^2 = 25$

$$\sigma^2 = \frac{9+49+81+144+169+400+x^2+y^2}{8} - 100 = 25$$

$\Rightarrow x^2+y^2 = 148$ ...(ii)

From eqn. (i), $(x+y)^2 = (16)^2$

$\Rightarrow x^2+y^2+2xy = 256$

Using eqn. (ii), $148 + 2xy = 256$

$\Rightarrow xy = 52$

34. **(b)** According to the question,

$$\frac{x_1+x_2+x_3+x_4}{4} = 11 \Rightarrow x_1+x_2+x_3+x_4 = 44$$

$$\frac{x_5+x_6+...+x_{10}}{6} = 16 \Rightarrow x_5+x_6+...+x_{10} = 96$$

and $x_1^2+x_2^2+...x_{10}^2 = 2000$

$\because$ standard deviation, $\sigma^2 = \frac{\sum x_i^2}{N} - (\bar{x})^2$

$$= \frac{2000}{10} - \left(\frac{44+96}{10}\right)^2 = 4 \quad \Rightarrow \sigma = 2$$

35. **(a)** Given, mean and standard deviation are equal to 16.

$$\therefore \frac{x_1+x_2+....x_{50}}{50} = 16$$

and $16^2 = \frac{x_1^2+x_2^2...x_{50}^2}{50} - 16^2$

$\Rightarrow 2(16)^2 50 = x_1^2+x_2^2+....x_{50}^2$

Required mean $= \frac{(x_1-4)^2+(x_2-4)^2+...(x_{50}-4)^2}{50}$

$$= \frac{x_1^2+x_2^2+...+x_{50}^2+50\times16-8(x_1+x_2+...+x_{50})}{50}$$

$$= \frac{16^2(100)+(50)-8(16\times50)}{50} = 400$$

36. **(a)** Mean of given observation $= \frac{k}{4}$

$\because$ Standard deviation = 5

$\therefore \sigma^2 = 5$

$\Rightarrow \sigma^2 = \frac{1}{n}\Sigma(x_i-\bar{x})^2$

$$= \frac{\left(\frac{k}{4}+1\right)^2+\left(\frac{k}{4}\right)^2+\left(\frac{k}{4}-1\right)^2+\left(\frac{3k}{4}\right)^2}{4} = 5$$

$$\Rightarrow \frac{\frac{12k^2}{16}+2}{4} = 5 \Rightarrow k = 2\sqrt{6}$$

37. **(c)** Let the remaining numbers are a and b.

Mean $(\bar{x}) = \frac{\sum x_i}{N} = \frac{2+4+10+12+14+a+b}{7} = 8$

$\Rightarrow a+b = 14$ ...(i)

Variance $(\sigma^2) = \frac{\sum x_i^2}{N} - (\bar{x})^2 = 16$

$$\Rightarrow \frac{2^2+4^2+10^2+12^2+14^2+a^2+b^2}{7} - (8)^2 = 16$$

$\Rightarrow a^2+b^2 = 100$ ...(ii)

From (i) and (ii), $(14-b)^2+b^2 = 100$

$\Rightarrow 196+b^2-28b+b^2 = 100$

$\Rightarrow b^2-14b+48 = 0$

$\Rightarrow b = 6, 8$

$\therefore a = 8, 6.$

$\therefore (a, b) = (6, 8)$ or $(8, 6)$

Hence, the product of the remaining two observations $= ab = 48$

38. **(a)** $\because$ Mean score = 48

Let unknown score be x,

$$\therefore \bar{x} = \frac{41+45+54+57+43+x}{6} = 48$$

$\Rightarrow x+240 = 288 \Rightarrow x = 48$

Now, $\sigma^2 = \frac{1}{6}[(48-41)^2+(48-45)^2+(48-54)^2 + (48-57)^2+(48-43)^2+(48-48)^2$

$$= \frac{1}{6}(49+9+36+81+25) = \frac{200}{6} = \frac{100}{3}$$

$$\sigma = \frac{10}{\sqrt{3}}$$

39. **(d)** Given, $\sum_{i=1}^{50}(x_i-30) = 50$

$$\sum_{i=1}^{50}x_i - 50(30) = 50$$

$$\Rightarrow \sum_{i=1}^{50}x_i = 1550$$

Mean, $\bar{x} = \frac{\sum x_i}{50}$

$= \frac{1550}{50} = 31$

40. **(a)** Let two observations be x_1 and x_2, then

$\frac{x_1 + x_2 + 3 + 4 + 4}{5} = 4$

$\Rightarrow x_1 + x_2 = 9$ ...(i)

$\text{Variance} = \frac{\sum x_i^2}{N} - (\bar{x})^2$

$(5 \cdot 20) = \frac{9 + 16 + 16 + x_1^2 + x_2^2}{5} - 16$

$26 = 41 + x_1^2 + x_2^2 - 80$

$x_1^2 + x_2^2 = 65$ ...(ii)

From (i) and (ii);

$x_1 = 8, x_2 = 1$

Hence, the required value of the difference of other two observations $= |x_1 - x_2| = 7$

41. **(d)** Outcomes are $\left(\frac{1}{2} - d\right), \left(\frac{1}{2} - d\right), 0 ...,$ 10 times, $\frac{1}{2}, \frac{1}{2},$ ..., 10 times, $\frac{1}{2} + d, \frac{1}{2} + d, ...,$ 10 times

$\text{Mean} = \frac{1}{30}\left(\frac{1}{2} \times 30\right) = \frac{1}{2}$

Variance of the outcomes is,

$\sigma^2 = \frac{1}{30}\Sigma x_i^2 - (\bar{x})^2$

$= \frac{1}{30}\left[\left(\frac{1}{2} - d\right)^2 \times 10 + \left(\frac{1}{2}\right)^2 \times 10 + \left(\frac{1}{2} + d\right)^2 \times 10\right] - \frac{1}{4}$

$\Rightarrow \frac{4}{3} = \frac{1}{30}\left[30 \times \frac{1}{4} + 20d^2\right] - \frac{1}{4}$

$\Rightarrow \frac{4}{3} = \frac{1}{4} + \frac{2}{3}d^2 - \frac{1}{4}$

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

42. **(b)** Variance is given by,

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$

$\sigma^2 = \frac{1}{n}A - \frac{1}{n^2}B^2$ ...(i)

Here, $A = \sum_{i=1}^{n} x_i^2$ and $B = \sum_{i=1}^{n} x_i$

$\because \sum_{i=1}^{n}(x_i + 1)^2 = 9n$

$\Rightarrow A + n + 2B = 9n \Rightarrow A + 2B = 8n$ ...(ii)

$\because \sum_{i=1}^{n}(x_i - 1)^2 = 5n$

$\Rightarrow A + n - 2B = 5n \Rightarrow A - 2B = 4n$ ...(iii)

From (ii) and (iii),

$A = 6n, B = n$

$\Rightarrow \sigma^2 = \frac{1}{n} \times 6n - \frac{1}{n^2} \times n^2 = 6 - 1 = 5$

$\Rightarrow \sigma = \sqrt{5}$

43. **(b)** Since mean of x_1, x_2, x_3, x_4 and x_5 is 5

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 25$

$\Rightarrow 1 + 3 + 8 + x_4 + x_5 = 25$

$\Rightarrow x_4 + x_5 = 13$ ...(i)

$\because \frac{\sum_{i=1}^{5} x_i^2}{5} - (5)^2 = 9.2 \Rightarrow \sum_{i=1}^{5} x_i^2 = 5(25 + 9.2)$

$= 125 + 46 = 171$

$\Rightarrow (1)^2 + (3)^2 + (8)^2 + x_4^2 + x_5^2 = 171$

$\Rightarrow x_4^2 + x_5^2 = 97$ ...(ii)

$\Rightarrow (x_4 + x_5)^2 - 2x_4 x_5 = 97$

$\Rightarrow 2x_4 x_5 = 13^2 - 97 = 72 \Rightarrow x_4 x_5 = 36$ ...(iii)

(i) and (iii) $\Rightarrow x_4 : x_5 = \frac{4}{9}$ or $\frac{9}{4}$

44. **(d)** $\because \bar{x} = \frac{\sum_{i=1}^{5} x_i}{5} \Rightarrow \sum_{i=1}^{5} x_i = 10 \times 5 = 50 \Rightarrow \sum_{i=1}^{6} xi = 50 - 50 = 0$

$\frac{\sum_{i=1}^{5} x_i^2}{5} - (10)^2 = 3^2 = 9$

$\Rightarrow \sum_{i=1}^{5} x_i^2 = 545$

Then,

$\Rightarrow \sum_{i=1}^{6} x_i^2 = \sum_{i=1}^{5} x_i^2 + (-50)^2$

$= 545 + (-50)^2 = 3045$

$$\text{Variance} = \frac{\sum_{i=1}^{6} x_i^2}{6} - \left(\frac{\sum_{i=1}^{6} x_i}{6}\right)^2 = \frac{3045}{6} - 0 = 507.5$$

45. **(c)** $\because$ Variance $= \sigma^2 = \frac{\Sigma x_i^2}{N} - (\bar{x})^2$

$\Rightarrow \; 18 = \frac{\Sigma x_i^2}{5} - (150)^2$

$\Rightarrow \; \Sigma x_i^2 = 90 + 112590 = 112590$

Then, variance of the height of six students

$$V' = \frac{112590 + (156)^2}{6} - \left(\frac{750+156}{6}\right)^2$$

$= 22821 - 22801 = 20$

46. **(c)** Here mean $= \bar{x} = 9$

$\Rightarrow \bar{x} = \frac{\Sigma x_i}{n} = 9$

$\Rightarrow \Sigma x_i = 9 \times 5 = 45$

Now, standard deviation = 0

$\therefore$ all the five terms are same i.e.; 9.

Now for changed observation

$\bar{x}_{\text{new}} = \frac{36 + x_5}{5} = 10$

$\Rightarrow x_5 = 14$

$$\therefore \; \sigma_{\text{new}} = \sqrt{\frac{\Sigma (x_i - \bar{x}_{\text{new}})^2}{n}}$$

$$= \sqrt{\frac{4(9-10)^2 + (14-10)^2}{5}} = 2$$

47. **(d)** $\bar{x} = \frac{7+8+9+7+8+7+\lambda+8}{8} = 8$

$\Rightarrow \frac{54+\lambda}{8} = 8 \Rightarrow \lambda = 10$

Now variance $= \sigma^2$

$$= \frac{(7-8)^2 + (8-8)^2 + (9-8)^2 + (7-8)^2 + (8-8)^2 + (7-8)^2 + (10-8)^2 + (8-8)^2}{8}$$

$\Rightarrow \sigma^2 = \frac{1+0+1+1+0+1+4+0}{8} = \frac{8}{8} = 1$

Hence, the variance is 1.

48. **(b)** Given $\sum_{i=1}^{9} (x_i - 5) = 9 \Rightarrow \sum_{i=1}^{9} x_i = 54$ ...(i)

Also, $\sum_{i=1}^{9} (x_i - 5)^2 = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 - 10\sum_{i=1}^{9} x_i + 9(25) = 45$ ...(ii)

From (i) and (ii) we get,

$\sum_{i=1}^{9} x_i^2 = 360$

Since, variance $= \frac{\sum x_i^2}{9} - \left(\frac{\sum x_i}{9}\right)^2$

$= \frac{360}{9} - \left(\frac{54}{9}\right)^2 = 40 - 36 = 4$

$\therefore$ Standard deviation $= \sqrt{\text{Variance}} = 2$

49. **(d)** $\sum_{i=1}^{100} x_i = 400$ $\qquad \sum_{i=1}^{100} x_i^2 = 2475$

Variance $= \sigma^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$

$= \frac{2475}{97} - \left(\frac{388}{97}\right)^2$

$= \frac{2425 - 1552}{97} = \frac{873}{97} = 9$

50. **(d)** $\bar{x} = \frac{2+3+a+11}{4} = \frac{a}{4} + 4$

$\sigma = \sqrt{\sum \frac{x_i^2}{n} - (\bar{x})^2}$

$\Rightarrow 3.5 = \sqrt{\frac{4+9+a^2+121}{4} - \left(\frac{a}{4}+4\right)^2}$

$\Rightarrow \frac{49}{4} = \frac{4(134+a^2) - (a^2 + 256 + 32a)}{16}$

$\Rightarrow 3a^2 - 32a + 84 = 0$

51. **(c)** $n = 5$

$\bar{x} = 5$

variance = 124

$x_1 = 1,\ x_2 = 2,\ x_3 = 6$

$\bar{x} = 5$

$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 5$

$\Rightarrow\ x_4 + x_5 + 9 = 25$
$\Rightarrow\ x_4 + x_5 = 16$
$\Rightarrow\ x_4 + x_5 + 10 - 10 = 16$
$\Rightarrow\ (x_4 - 5) + (x_5 - 5) = 16 - 10$
$\Rightarrow\ (x_4 - 5) + (x_5 - 5) = 6$

Mean deviation $= \dfrac{\Sigma |x_i - \bar{x}|}{N}$

$= |x_1 - 5| + |x_2 - 5| + |x_3 - 5| + \dfrac{|x_4 - 5| + |x_5 - 5|}{5}$

$= \dfrac{4+3+1+6}{5} = \dfrac{14}{5} = 2.8$

52. **(a)** $\bar{x} = \dfrac{1}{101}\ [1 + (1+d) + (1+2d)]\ .....\ (1+100d)]$

$= \dfrac{1}{101} \times \dfrac{101}{2}\ [1 + (1+100d)] = 1 + 50d$

mean deviation from mean

$= \dfrac{1}{101}\ [|1 - (1+50d)| + |(1+d) - (1+50d)|.....|$
$[1+100d] - (1+50d)|]$

$= \dfrac{2|d|}{101}\ (1+2+3.....+50)$

$= \dfrac{2|d|}{101} \times \dfrac{50 \times 51}{2} = \dfrac{2550}{101}\ |d|$

$= \dfrac{2550}{101}\ |d| = 225 \Rightarrow |d| = 10.1$

53. **(d)** First 50 even natural numbers are 2, 4, 6 ....., 100

Variance $= \dfrac{\sum x_i^2}{N} - (\bar{x})^2$

$\Rightarrow\ \sigma^2 = \dfrac{2^2 + 4^2 + ... + 100^2}{50} - \left(\dfrac{2+4+...+100}{50}\right)^2$

$= \dfrac{4(1^2 + 2^2 + 3^2 + ... + 50^2)}{50} - (51)^2$

$= 4\left(\dfrac{50 \times 51 \times 101}{50 \times 6}\right) - (51)^2$

$= 3434 - 2601 \Rightarrow \sigma^2 = 833$

54. **(b)** $\bar{x} = \dfrac{x_1 + x_2 + x_3 + ... + x_n}{n}$

$\sigma^2 = \dfrac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Mean of $d_1, d_2, d_3, ..., d_n$

$= \dfrac{d_1 + d_2 + d_3 + ... + d_n}{n}$

$= \dfrac{(-x_1 - a) + (-x_2 - a) + (-x_3 - a) + ... + (-x_n - a)}{n}$

$= -\left[\dfrac{x_1 + x_2 + x_3 + ... + x_n}{n}\right] - \dfrac{na}{n} = -\bar{x} - a$

Since, $d_i = -x_i - a$ and we multiply or subtract each observation by any number the mode remains the same. Hence mode of $-x_i - a$ i.e. d_i and x_i are same.

Now variance of $d_1, d_2, ..., d_n$

$= \dfrac{1}{n}\sum_{i=1}^{n}\ [d_i - (-\bar{x} - a)]^2$

$= \dfrac{1}{n}\sum_{i=1}^{n}\ [-x_i - a + \bar{x} + a]^2$

$= \dfrac{1}{n}\sum_{i=1}^{n}\ (-x_i + \bar{x})^2 = \dfrac{1}{n}\sum_{i=1}^{n}\ (\bar{x} - x_i)^2 = \sigma^2$

55. **(b)** Let x_i be n observations, $i = 1, 2, ...n$

Let $\bar{X}$ be the mean and M.D be the mean deviation about $\bar{X}$.

If each observation is increased by 5 then new mean will be $\bar{X} + 5$ and new M.D. about new mean will be M.D.

$\left(\because \text{Mean} = \sum_{i=1}^{n} \dfrac{x_i}{n}\right)$

56. **(d)** If initially all marks were x_i then

$\sigma_1^2 = \dfrac{\sum_i (x_i - \bar{x})^2}{N}$

Now each is increased by 10

$\sigma_1^2 = \dfrac{\sum_i [(x_i + 10) - (\bar{x} + 10)]^2}{N} = \dfrac{\sum_i (x_i - \bar{x})^2}{N} = \sigma_1^2$

Hence, variance will not change even after the grace marks were given.

57. **(a)** Clearly mean A = 0

Now, standard deviation $\sigma = \sqrt{\dfrac{\Sigma(x - \text{A})^2}{2n}}$

$2 = \sqrt{\dfrac{(a-0)^2 + (a-0)^2 + ... + (0-a)^2 + ...}{2n}}$

$= \sqrt{\dfrac{a^2 . 2n}{2n}} = |a|$

Hence, $|a| = 2$

58. **(d)** Let 5th observation be x.

Given mean = 7

$\therefore\ 7 = \dfrac{6+7+8+10+x}{5}$

$\Rightarrow\ x = 4$

Now, Variance

$$=\sqrt{\frac{(6-7)^2+(7-7)^2+(8-7)^2+(10-7)^2+(4-7)^2}{5}}$$

$$=\sqrt{\frac{1^2+0^2+1^2+3^2+3^2}{5}}=\sqrt{\frac{20}{5}}=\sqrt{4}=2$$

59. (d) A.M. of $2x_1, 2x_2, ..., 2x_n$ is

$$\frac{2x_1+2x_2+...+2x_n}{n}=2\left(\frac{x_1+x_2+....+x_n}{n}\right)=2\bar{x}$$

$$\left(\because \text{ Mean}=\frac{\text{Sum of observations}}{\text{Number of observations}}\right)$$

So statement-2 is false.

If each observations is multiply by 2 then mean multiply by 2 and variance multiply by 2^2.

variance $(2x_i) = 2^2$ variance $(x_i) = 4\sigma^2$ where $i = 1, 2, ...n$

So statement-1 is true.

60. (a) Statement 2 : Sum of first n odd natural numbers is not equal to n^2.

So, statement - 2 is false.

61. (d) Given observations are 4, 7, 2, 8, 6, a and mean is 7.

We know

$$\text{Mean}=\frac{4+7+2+8+6+a}{6}$$

$$\Rightarrow \quad 7=\frac{4+7+2+8+6+a}{6} \Rightarrow a=15$$

Now, given observations can be written in ascending order which is 2, 4, 6, 7, 8, 15

Since, No. of observation is even

$\therefore$ Median

$$=\frac{\left(\frac{6}{2}\right)\text{th observation}+\left(\frac{6}{2}+1\right)\text{th observation}}{2}$$

$$=\frac{\text{3rd observation}+\text{4th observation}}{2}=\frac{6+7}{2}=\frac{13}{2}$$

$$\text{Now, Mean deviation}=\frac{\sum_{i=1}^{6}\left|x_i-\frac{13}{2}\right|}{6}$$

$$=\frac{\left|4-\frac{13}{2}\right|+\left|7-\frac{13}{2}\right|+\left|2-\frac{13}{2}\right|+\left|8-\frac{13}{2}\right|+\left|6-\frac{13}{2}\right|+\left|15-\frac{13}{2}\right|}{6}$$

$$=\frac{\frac{5}{2}+\frac{1}{2}+\frac{9}{2}+\frac{3}{2}+\frac{1}{2}+\frac{17}{2}}{6}=\frac{18}{6}=3$$

62. (a) We know that if each observation is increase by 2 then mean is increase by 2 but S.D. remains same.

Correct mean = observed mean + 2 $= 30 + 2 = 32$

Correct S. D. = observed S.D. = 2

63. (b) $\because n = 50$ (even)

$$\text{Median}=\frac{25^{\text{th}}\text{ obs.}+26^{\text{th}}\text{ obs.}}{2}$$

$$\therefore \quad M=\frac{25a+26a}{2}=25.5a$$

$$M.D\,(M)=\frac{\sum|x_i-M|}{N}$$

$$\Rightarrow \quad 50=\frac{1}{50}[2\times|a|\times(0.5+1.5+2.5+....24.5)]$$

$$\Rightarrow \quad 2500=2|a|\times\frac{25}{2}(25)$$

$$\Rightarrow \quad |a|=4$$

64. (a) $\sigma_x^2=4, \sigma_y^2=5, \bar{x}=2, \bar{y}=4$

$$\sigma_x^2=\frac{1}{5}\sum x_i^2-(2)^2=4 \Rightarrow \sum x_i^2=40;$$

$$\sigma_y^2=\frac{1}{5}\sum y_i^2-(4)^2=5 \Rightarrow \sum y_i^2=105$$

$$\Rightarrow \quad \sum x_i^2+\sum y_i^2=\sum\left(x_i^2+y_i^2\right)=145$$

$$\Rightarrow \quad \sum x_i+\sum y_i=\sum(x_i+y_i)=5(2)+5(4)=30$$

Variance of combined data

$$=\frac{1}{10}\sum\left(x_i^2+y_i^2\right)-\left(\frac{1}{10}\sum(x_i+y_i)\right)^2$$

$$=\frac{145}{10}-9=\frac{11}{2}$$

65. (b) $$\text{Mean}=\frac{101+d(1+2+3+......+100)}{101}$$

$$=1+\frac{d\times100\times101}{101\times2}=1+50\,d$$

Given that mean deviation from the mean = 255

$$\Rightarrow \quad \frac{1}{101}[|1-(1+50d)|+|(1+d)-(1+50d)|$$
$$+|(1+2d)-(1+50d)|+....+$$
$$|(1+100d)-(1+50d)|]=255$$

$$\Rightarrow \quad 2d[1+2+3+...+50]=101\times255$$

$$\Rightarrow \quad 2d\times\frac{50\times51}{2}=101\times255$$

$$\Rightarrow \quad d=\frac{101\times255}{50\times51}=10.1$$

66. (c) First n even natural numbers be $2, 4, 6, 8, ..., 2n$

$$\therefore \quad \bar{x}=\frac{2(1+2+3+...+n)}{n}=\frac{2[n(n+1)]}{2n}=(n+1)$$

And $Var = \frac{\Sigma(x-\bar{x})^2}{2n} = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$= \frac{4\Sigma n^2}{n} - (n+1)^2 = \frac{4n(n+1)(2n+1)}{6n} - (n+1)^2$

$= \frac{2(2n+1)(n+1)}{3} - (n+1)^2 = (n+1)\left[\frac{4n+2-3n-3}{3}\right]$

$= \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$

$\therefore$ Statement-1 is false. Clearly, statement - 2 is true.

67. **(d)** Mean of $a, b, 8, 5, 10$ is 6

$\Rightarrow \frac{a+b+8+5+10}{5} = 6$

$\Rightarrow a + b = 6$ ...(i)

Variance of $a, b, 8, 5, 10$ is 6.80

$\Rightarrow \frac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5}$

$= 6.80$

$\Rightarrow a^2 - 12a + 36 + (1-a)^2 + 21 = 34$ [using eq. (i)]

$\Rightarrow 2a^2 - 14a + 24 = 0 \Rightarrow a^2 - 7a + 12 = 0$

$\Rightarrow a = 3$ or $4 \Rightarrow b = 4$ or 3

$\therefore$ The possible values of a and b are $a = 3$ and $b = 4$ or, $a = 4$ and $b = 3$

68. **(a)** $\sigma_x^2 = \frac{\sum d_i^2}{n}$ (Here d_i = deviations are taken from the mean). Since population A and population B both have 100 consecutive integers, therefore both have same standard deviation and hence the variance is also same. $\therefore \frac{V_A}{V_B} = 1$

69. **(c)** Clearly sum of observations = 0,

$\therefore$ mean $A = 0$

Standard deviation $\sigma = \sqrt{\frac{\sum(x-A)^2}{2n}}$

$2 = \sqrt{\frac{(a-0)^2+(a-0)^2+...(0-a)^2+...}{2n}}$ [$\because \sigma = 2$]

$= \sqrt{\frac{a^2.2n}{2n}} = |a|$

Hence, $|a| = 2$

70. **(c)** Only first statement (A) and second statements (B) are correct.

71. **(b)** $\Sigma x = 170, \Sigma x^2 = 2830$

New, $\Sigma x' = 170 + (30-20) = 180$

New, $\Sigma x'^2 = 2830 + (900-400)$

$= 2830 + 500 = 3330$

Now, Variance $= \frac{1}{n}\Sigma x'^2 - \left(\frac{1}{n}\Sigma x'\right)^2$

$= \frac{1}{15} \times 3330 - \left(\frac{1}{15} \times 180\right)^2 = 222 - 144 = 78.$