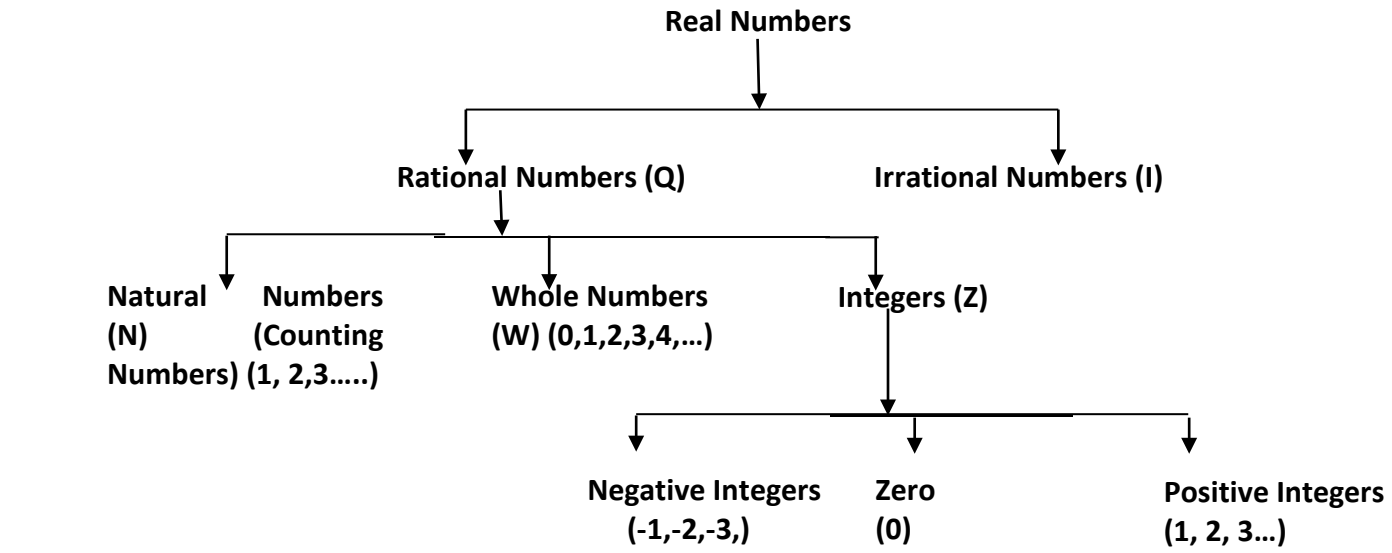
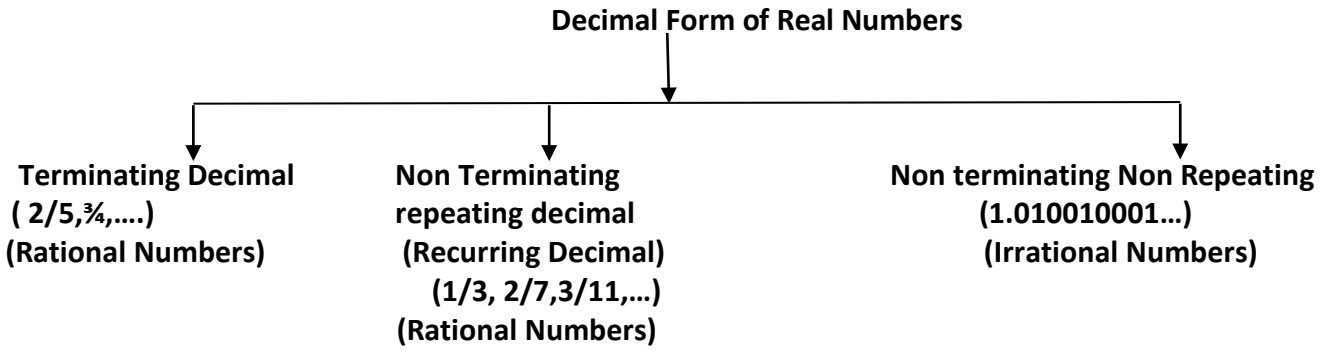
Real Numbers

(Key Points)





1. **Euclid's Division lemma:-G**iven Positive integers and b there exist unique integer's q and r satisfying
   a=bq +r, where 0 $\leq$ r<b, where a ,b, q and r are respectively called as dividend, divisor, quotient and remainder.

2. **Euclid's division Algorithm:-**To obtain the HCF of two positive integers say c and d, with c>d, follow the steps below:

   Step I: Apply Euclid's division lemma, to c and d, so we find whole numbers, q and r such that c =dq +r,0 $\leq r < d.$

   Step II: If r=0,d is the HCF of c and d. If r $\neq 0, apply\ the$ division lemma to d and r.

   Step III: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF

Note:- Let a and b be positive integers .If a=bq +r, 0≤r<b, then HCF(a,b)= HCF(b,r)

3. **The Fundamental theorem of Arithmetic:-**
   Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, a part from the order in which the prime factors occur.
   Ex.: $24 = 2\ X\ 2\ X\ 2\ X\ 3 = 3\ X\ 2\ X\ 2\ X\ 2$

Theorem: LET x be a rational number whose decimal expansion terminates. Then x can be expressed in the form

Of p/q where $p \text{ and } q$ are co-prime and the prime factorization of q is of the form of $2^n . 5^m$, where n, m are non-negative integers.

Ex. $\frac{7}{10} = \frac{7}{2 X 5} = 0.7$

Theorem: Let $x = \frac{p}{q}$ be a rational number such that the prime factorization of q is not of the form of $2^n . 5^m$, where n ,m are non-negative integers. Then x has a decimal expansion which is none terminating repeating (recurring).

Ex. $\frac{7}{6} = \frac{7}{2 X 3} = 1.1666 \ldots \ldots$

Theorem: For any two positive integers a and b, HCF

(a,b) XLCM(a,b)=aXb

Ex.:4&6; HCF (4,6) =2, LCM(4,6) =12;HCFXLCM=2X12=24

Ans.: aXb=24

LEVEL-I

1. If $\frac{p}{q}$ is a rational number $(q \neq 0)$.What is the condition on q so that the decimal representation of is $\frac{p}{q}$ terminating?
2. Write a rational number between $\sqrt{2} \text{ and } \sqrt{3}$.
3. The decimal expansion oftherationalno.43/$2^4 5^3$ will terminate after how many places of decimal?
4. Find the $(HCF \ X \ LCM) for \ the \ numbers \ 100 \ and \ 190.$
5. State whether the number $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3}) \ is$ rational or irrational justify.
6. Write one rational and one irrational number lying between 0.25and 0.32.
7. Express 107 in the form of 4q+3 for some positive integer q.
8. Write whether the rational number $\frac{51}{1500}$ will have a terminating decimal expansion or a non Terminating repeating decimal expansion.
9. Show that any positive odd integer is of the form 6q+1 or 6q+3 or 6q+5, where q is some integer.
10. Express 0.2545454……..As a fraction in simplest form.

LEVEL-II

1. Use Euclid's division algorithm to find the HCF of 1288 and 575.
2. Check whether 5 x 3 x 11+11 and 5x7+7X3 are composite number and justify.
3. Check whether 6^n can end with the digit 0, where n is any natural number.

4. Given that LCM (26,169) = 338, write HCF (26,169).]
5. Find the HCF and LCM of 6, 72and 120 using the prime factorization method.
6. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m+1 for some integer m.
7. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m+1 or 9m+8 for some integer m.

LEVEL-III

1. Show that $\sqrt{3}$ is an irrational number.
2. Show that $5 + 3\sqrt{2}$ is an irrational number.
3. Show that square of an odd positive integer is of the form 8m+1, for some integer m.
4. Find the LCM &HCF of 26 and 91 and verify that
5. Prove that $\sqrt[3]{7}$ is irrational.
6. Show that one and only one out of n, n+2, n+4 is divisible by 3, where n is any positive integer.
7. Find the HCF of 65 & 117 and express it in the form of 65m + 117n.

(PROBLEMS FOR SELF EVALUATION/HOTS)

1. State the fundamental theorem of Arithmetic.
2. Express 2658 as a product of its prime factors.
3. Find the LCM and HCF of 17, 23 and 29.
4. Prove that $\sqrt{2}$ is not a rational number.
5. Find the largest positive integer that will divide 122, 150 and 115 leaving remainder 5,7 and 11 respectively.
6. Show that there is no positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.
7. Using prime factorization method, find the HCF and LCM of 72, 126 and 168. Also show that

   HCF X LCM ≠ product of three numbers.
8. Three sets of English, Mathematics and Science books containing 336, 240 and 96 books respectively have to be stacked in such a way that all the books are stored subject wise and the height of each stack is the same. How many stacks will be there?

Value Based Questions

Q.1 Aperson wanted to distribute 96 apples and 112 oranges among poor children in an orphanage. He packed all the fruits in boxes in such a way that each box contains fruits of the same variety, and also every box contains an equal number of fruits.

(i) Find the maximum number of boxes in which all the fruits can be packed.

(ii) Which concept have you used to find it?

(iii)Which values of this person have been reflected in above situation?

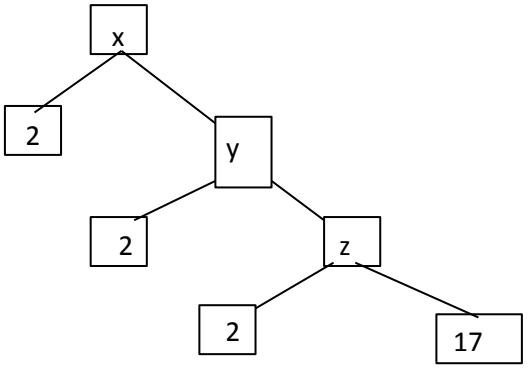
Q.2 A teacher draws the factor tree given in figure and ask the students to find the value of x without finding the value of y and z.

Shaurya gives the answer x=136

a) Is his answer correct?

b) Give reason for your answer.

c) Which value is depicted in this?



Answer

Level-I

1. q is of the form $2^n.5^m$, where m and n are non-negative integers.
2. 1.5
3. After 4 places of decimal.
4. 19000
5. Rational number
6. One rational number=26/100, one irrational no.=0.27010010001……
7. 4 X 26+3
8. Terminating

10.14/55

Level-II

1.23

2. Composite number

3. No, 6^n cannot end with the digit 0.

4.13

5. HCF=6 , LCM = 360

Level-III

4. LCM= 182 ,HCF = 13

7. m = 2 and n = -1.

Problems for self-evaluation

1. See textbook.
2. 2658 = 2 X 3 x 443
3. HCF = 1 , LCM = 11339

5. 13

8. Total no. of stacks = 14

Value based Questions

1. (i)No. of boxes = 16

   (ii)Number System & HCF
   (iii)The person is kind hearted and of helping attitude.
2. (a) Yes, his answer is correct.
   (b) Z =2 X 17 = 34, Y = 2 X 34 = 68, X = 2 x 68 = 136
   (c) Knowledge of prime factorization.