Level-I

Chapter 10

Wave Optics

Solutions (Set-1)

Very Short Answer Type Questions :

1. Which phenomenon establish the wave nature of light?

Sol. Interference, diffraction and polarisation.

2. When a wave undergoes reflection at a denser medium, what happen to its phase?

Sol. Phase changes by π radian

3. Can two independent sources of light be coherent?

Sol. No

4. What is the ratio of slit widths when amplitudes of light waves from them have a ratio of $\sqrt{2}:1$.

Sol. $\frac{w_1}{w_2} = \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{\sqrt{2}}{1}\right)^2 = 2:1$

5. Is head light of a car plane polarized?

Sol. No, it is unpolarized.

6. Name two commonly used devices which use polarised light.

Sol. Sunglasses and liquid crystal display.

7. Does the value of polarizing angle depend on colour of light?

Sol. Yes, it depends.

8. What does red shift in the spectra of galaxies indicate?

Sol. The universe is expanding.

9. What is the angle between the reflected and refracted rays at polarising angle?

Sol. 90°

10. What is relationship between path difference and phase difference?

Sol. $\phi = \frac{2\pi}{\lambda}x$

Short Answer Type Questions :

11. Width of two slits in Young's double slit experiment are in the ratio 4 : 1. What is the ratio of amplitudes and intensities of light waves from them?

Sol. Here, $\frac{w_1}{w_2} = \frac{4}{1}$

$$\frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{4}{1}$$

$$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2, \frac{a_1}{a_2} = \frac{2}{1}$$

12. Write two applications of dopplers effect of light.

Sol. 1. Measuring speed of star or galaxy w.r.t. to us.

2. Estimation of velocities of satellite.

13. Write any four uses of polarised light.

Sol. 1. It is useful in three dimensional pictures *i.e.* in holography.

2. Improve colour contrast in old oil paintings.

3. Optical stress analysis.

4. In LCD calculators, electronic displays.

14. A slit 4.0 cm wide is irradiated with microwaves of wavelength 2.0 cm. Find the angular spread of central maximum, assuming incidence normal to the plane of slit.

Sol. $2\theta = \frac{2\lambda}{a} = \frac{2 \times 2 \times 10^{-2}}{4 \times 10^{-2}} = 1\,\text{rad}$

15. What speed should a galaxy move with respect to us so that the sodium line at 589.0 nm is observed at 589.6 nm?

Sol. $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

$\Rightarrow \quad v = 3.06 \times 10^5$ m/s

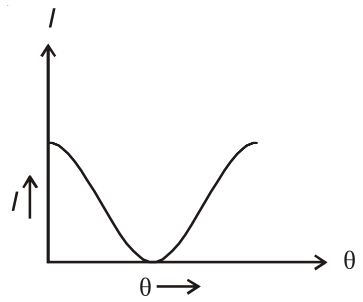
16. A slit or aperture diffracts light. Even then we say that light travels in a straight line. Comment.

Sol. Diffraction becomes significant only after Fresnel's distance $Z_F = \frac{a^2}{\lambda}$.

17. Name one device for producing polarised light. Draw a graph showing the dependence of intensity of transmitted light on the angle between polarizer and analyser.

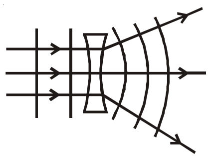
Sol. Nicole prism or calcite prism.

Intensity relation is given by Malus law $I = I_0\cos^2\theta$



18. Explain Huygens' principle. Also plot incident and refracted wavefronts for refraction through a concave lens.

Sol. According to Huygen's principle, each point on a wavefront is a source of secondary wavelets and wavelets emanating from these points spreads out in all directions with the speed of wave. The common envelop over the wavelets in same phase gives new wavefront.



19. Two incoherent sources of light each with intensity I_0 are used in Young's experiment. Derive expression for intensity observed on screen.

Sol. $I_R = I_0 + I_0 + 2I_0 \cos\phi$

$< I > = 4I_0 < \cos^2\left(\frac{\phi}{2}\right) >$. Time averaged value of $< \cos^2\frac{\phi}{2} >$ is $\frac{1}{2}$

$< I > = 2I_0$.

20. Find the maximum intensity in case of interference of n identical waves each of intensity I_0 if the sources are

(i) Coherent

(ii) Incoherent.

Sol. (i) Coherent sources

Let amplitudes be a for each wave.

$A_R = (a + a \ldots)_{n \text{ times}} = na$

$I_R \propto A_R^2 = n^2 I_0$

(ii) Incoherent sources

$I = (I_0 + I_0 \ldots)_{n \text{ times}} = nI_0$

21. In Young's experiment light waves of wavelength 5.4×10^{-7} m and 6.85×10^{-8} m are used in turn keeping the same geometry. Compare the fringe width in the two cases.

Sol. $\lambda_1 = 5.4 \times 10^{-7}$ m

$\lambda_2 = 6.85 \times 10^{-8}$ m

$\beta = \frac{\lambda D}{d}$

$\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2} = 8\ (\text{approx.})$

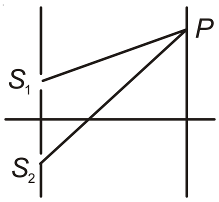
22. Why interference pattern is not detected, when the two coherent sources are far apart?

Sol. As fringe width $\beta \propto \frac{1}{d}$

Therefore, when d is so large, the width may reduce beyond the resolvable limit.

23. What is the shape of fringe pattern on the screen?

Sol. The locus of point P in plane of the screen, such that S_2P-S_1P is a constant, is a hyperbola. Thus fringe pattern are hyperbola.



24. What is corpuscular theory of light? Which prediction of this theory was rectified by Huygens' wave theory?

Sol. **Corpuscular model :** In 1637, Descartes gave the corpuscular model of light. According to this model, a luminous body emits a stream of particles in all directions. The particles are assumed to be very-very tiny. It explained the laws of reflection and refraction of light at an interface using concepts of elastic collisions and momentum conservation. Newton further developed this model and explained sensation of colours at the retina of the eye and spectrum formed due to prism. This model predicted that if the ray of light on refraction bends towards the normal, then the speed of light would be greater in the second medium. This prediction however proved wrong when Fermat formulated his famous principle of least time. Although this law could explain reflection and refraction, this law could not satisfactorily explain phenomenon like interference, polarization and diffraction.

25. Write two differences between interference and diffraction.

Sol. 1. Interference is due to superposition from two distinct sources. Unlike in diffraction in which infinite sources are involve.

2. In interference all fringes are of same width and intensity. In diffraction intensity decreases rapidly.

26. Explain briefly resolving power of an optical instrument.

Sol. It is defined as the ability of an optical instrument to produce distinctly separate images of two close objects. According to Rayleigh, two point object A and B will be just resolved, when central maximum of diffraction pattern of B lies on first minimum of diffraction pattern of A.

27. What are resolving power and limit of resolution of microscope. Explain how the two depend upon the wavelength of light used.

Sol. $\left[RP = \dfrac{2\mu\sin\theta}{1.22\lambda}, RP \propto \dfrac{1}{\lambda}\right]$

Limit of resolution = $d_{min} = \dfrac{1.22\lambda}{2\mu\sin\theta}$

$d_{min} \propto \lambda$ (For detail see text)

28. Discuss the Intensity of transmitted light when a polaroid sheet is rotated between the two crossed polaroids.

Sol. Suppose, I_0 is intensity of polarised light after passing through the first polariser P_1. If θ is the angle between axes of P_1 and P_2.

As P_1 and P_3 are crossed, the angle between P_1 and P_3 = $\dfrac{\pi}{2}$.

$\therefore$ Angle between axes of P_2 and $P_3 = 90 - \theta$.

Intensity of light from P_3

$I = (I_0\cos^2\theta)\cos^2(90 - \theta) = I_0\cos^2\theta\sin^2\theta$

$= \frac{I_0}{4}\sin^2 2\theta$

29. At what angle should the axes of two polaroids be placed so as to reduce the intensity of incident unpolarised light to $\frac{1}{3}$?

Sol. By Malus law,

$I = I_0\cos^2\theta$

$\cos^2\theta = \frac{I}{I_0} = \frac{1}{3}$

$\cos\theta = \frac{1}{\sqrt{3}}$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7°$

30. Why two independent sources of light cannot be coherent?

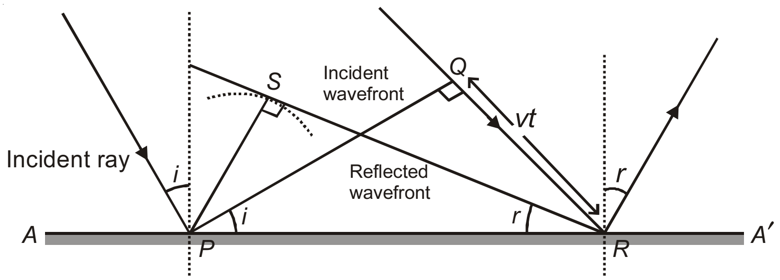
Sol. Production of light is based on de-excitation of electrons in atoms, photons radiated during de-exitation not necessarily in phase as well as photons radiated may of different frequencies so two independent light can not be coherent.

Long Answer Type Questions :

31. Prove reflection of light based on Huygens' wave theory.

Sol. Reflection of a Plane Wave by a Plane Reflecting Surface

To prove laws of reflection from Huygens' wave model. let us consider a plane wavefront *PQ* incident at an angle *i* on a reflecting surface *AA′*.



Time taken by the wave to advance to point *R* from point *Q* be *t*.

then $QR = vt$

Now, in order to construct the reflected wavefront we draw a sphere of radius *vt* from point *P*. Let *RS* represent a tangent drawn from *R* to wavefront from *P* to the spherical wavefront.

$\therefore \quad PS = vt$

Consider, ΔPSR and ΔRQP

$PR = PR$ (Common side)

$\angle PSR = \angle RQP$ (Each 90°)

$PS = QR$ (Each vt)

Hence, $\Delta PSR \cong \Delta RQP$

$\Rightarrow \quad \angle i = \angle r$

which proves law of reflection.

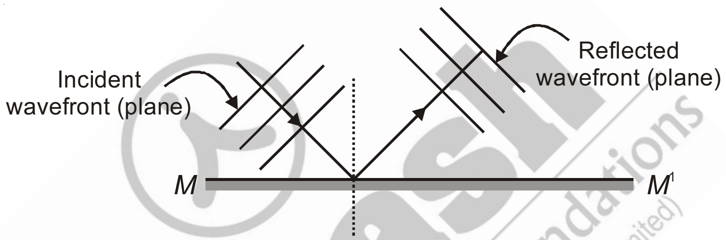
32. Make neat and labeled diagram showing wavefronts during following cases:

(a) Reflection through (i) Plane mirror (ii) Curved mirror

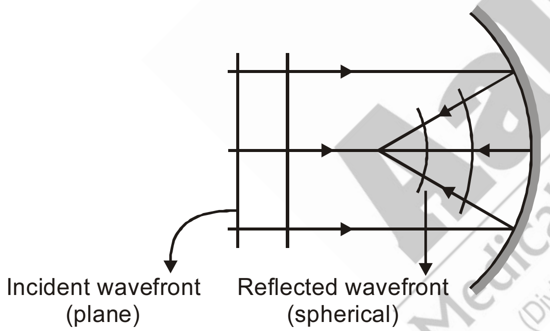
(b) Refraction through (i) Plane surface (ii) Curved surface

(c) Refraction through a glass prism

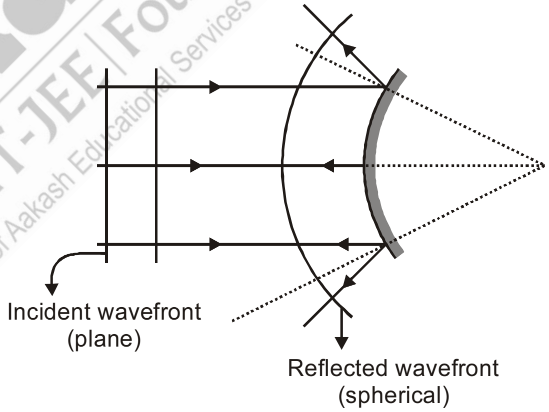
Sol. (a) (i) Reflection from plane mirror



(ii) Reflection from curved mirror

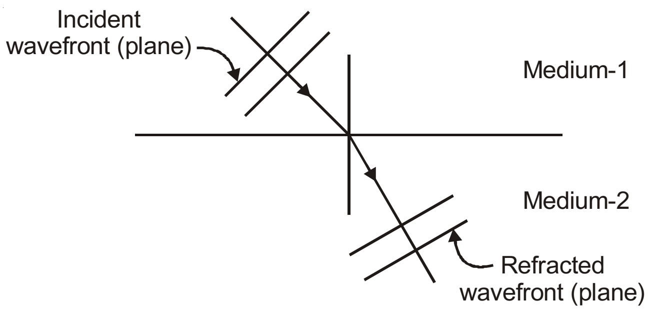


Reflection from concave mirror

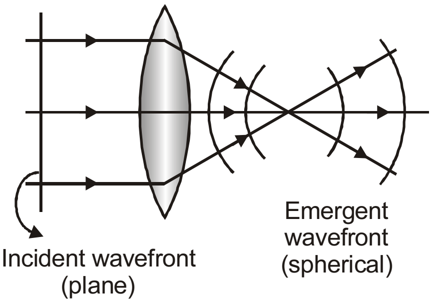


Reflection from convex mirror

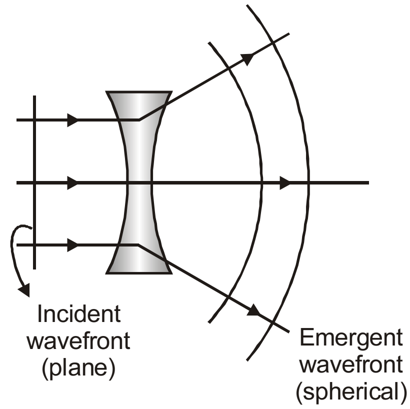
(b) (i) Refraction from plane surface



(ii) Refraction through lens

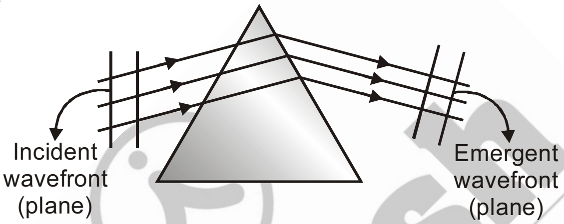


Refraction through convex lens



Refraction through concave lens

(c) Refraction through prism (Monochromatic beam)



33. Explain interference. Derive conditions for constructive and destructive interference. Also, find maximum and minimum intensity during constructive and destructive interference.

Sol. Interference : When two or more waves arrive at a particular point in space, they are said to superimpose with each other and a new wave is formed in which the resultant displacement at any instant is equal to the vector sum of the displacements produced due to individual waves at that instant.

i.e., $y = y_1 + y_2 + ...$

where y = resultant displacement

y_1, y_2 .... being individual displacements.

A special case of superposition when two or more waves of same frequency superimpose, there occurs redistribution of energy, this is called as Interference.

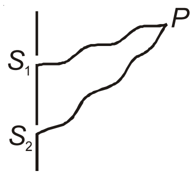
Let S_1 & S_2 be coherent sources emitting light waves reaching P.

Due to wave from S_1 source, instantaneous displacement of P is

$y_1 = a_1 \sin\omega t$ (a_1 = amplitude)

Due to wave from S_2 source, instantaneous displacement of P is

$y_2 = a_2 \sin(\omega t + \phi)$ (a_2 = amplitude; ϕ = phase difference)



From superposition principle,

$y = y_1 + y_2$

$= a_1\sin\omega t + a_2 [\sin\omega t \cos\phi + \cos\omega t \sin\phi]$

$= \sin\omega t[a_1 + a_2 \cos\phi] + \cos\omega t (a_2\sin\phi)$

$= A[\sin\omega t \cos\theta + \cos\omega t \sin\theta]$

$y = A[\sin(\omega t + \theta)]$

where ; $A\cos\theta = a_1 + a_2\cos\phi$

$A\sin\theta = a_2\sin\phi$ $\Rightarrow A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

Resultant intensity $I_R = A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$

$I_R = I_1 + I_2 + 2\sqrt{I_1I_2}\,(\cos\phi)$

For constructive interference

I_R = maximum

$\cos\phi = +1$

$\Rightarrow \phi = n(2\pi)$

where, $n = 0, 1, 2, 3 ...$

& path difference $\Delta = n(\lambda)$

$I_{max} = (a_1 + a_2)^2 = ((\sqrt{I_1} + \sqrt{I_2})^2$

For destructive interference :

I_R = min

$\cos\phi = -1$

$\Rightarrow \phi = (2n - 1)\pi$

where, $n = 1, 2, 3.....$

and path difference $\Delta = (2n-1)\dfrac{\lambda}{2}$

and $I_{min} = (a_1 - a_2)^2 = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

34. In a two slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the plane of slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 10^{-3} m, calculate the wavelength of the light used. Distance between the slits is 0.03 mm.

Sol. $\beta = \dfrac{\lambda D}{d}$

$\Delta\beta = \dfrac{\lambda \Delta D}{d}$

$\lambda = \dfrac{(\Delta\beta) \times d}{(\Delta D)}$

$\lambda = \dfrac{3 \times 10^{-5} \times 10^{-3}}{5 \times 10^{-2}} = 6 \times 10^{-7}\,\text{m}$

$\lambda = 6000$ Å

35. An unpolarised beam of light is incident on a group of four polarising sheets, which are arranged in such a way that the characteristic direction of each makes an angle of 30° with that of the preceding sheet. What fraction of incident unpolarized light is transmitted?

Sol. Let I_0 be the intensity of unpolarised light, then intensity of light from I^{st} polarising sheet is equal to $\frac{I_0}{2}$.

Intensity of light from 2nd polarizing sheet $I' = \frac{I_0}{2}\cos^2 30° = \frac{3I_0}{8}$

Intensity of light from 3rd polarising sheet $I'' = I'\cos^2 30° = \frac{9I_0}{32}$

Intensity of light from 4th polarising sheet $I''' = \frac{27}{128}I_0$

36. Derive expression for intensity of bright fringe and dark fringe in Young's double slit experiment.

Sol. See theory

37. Comment on the validity of Ray Optics. Also, discuss about Fresnel distance.

Sol. Whenever an aperture (hole or slit) of size a is illuminated by a parallel beam of light, the light after passing through the aperture, get diffracted into an angle approximately $\approx \frac{\lambda}{a}$. This is the angular size of the bright central maximum. Now as the beam progresses ahead a distance z, the diffracted beam acquires a width $\frac{z\lambda}{a}$.

Now, when we talk about ray optics we only take propagation of light to be rectilinear. But actually the light diffracts as it progresses. Now for value of z when the spreading becomes equal to size of aperture, the divergence becomes significant and the ray optics doesn't seem to work with accuracy, beyond this distance.

Hence, when $\frac{z\lambda}{a} \approx a$

or $\boxed{z \approx \frac{a^2}{\lambda}}$ Fresnel distance

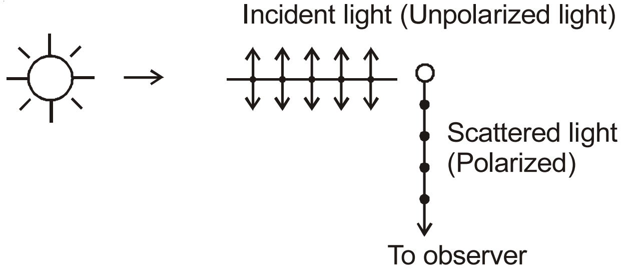
This distance is called Fresnel distance oftenly denoted as $Z_F \approx \frac{a^2}{\lambda}$. Now beyond this distance, the spreading becomes significant and we can no longer directly apply the laws of ray optics.

The equation also reveals that as $\lambda \to 0$, $Z_F \to \infty$. Hence, we can conclude that ray optics is actually an approximation in the limit of wavelength tending to zero.

But as wavelength of light is usually very small (in μm order) hence to a good distance ray optics seems to hold good.

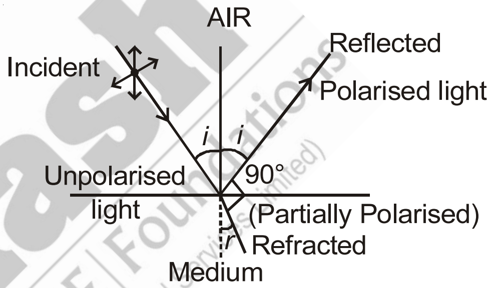
38. Explain by reflection and scattering how one can obtain plane polaried light. Explain what is Brewster's law and how is it related to refractive index of medium.

Sol. **Polarization by Scattering :** When light is incident on the small particles of atmosphere such as dust, air molecules it is obsorbed by the electrons in the molecules, hence electrons start vibrating. These vibrating electrons emit radiations in all directions except in its own line of vibration. The emitted radiations (light) scattered in a direction perpendicular to direction of incident light is plane polarised. The light in all other directions are partially polarised. The scattering of light by molecules was intensively investigated by C.V. Raman and his collaborates in Kolkata in the 1920s. For this work he was awarded with Nobel Prize for Physics in 1930.



Polarization by Reflection : When a light wave is incident on a boundary of a medium, a part of light wave is reflected back into the medium from which it is incident and a part of the wave is refracted into the other medium.

When unpolarized light is incident on the boundary between two transparent medium, for an angle of incidence in which reflected wave travels at right angle to the refracted wave, the reflected light is polarized while the refracted light is partially polarized.



By Snell's Law : $\frac{\sin i}{\sin r} = \mu$

Now, since Reflected ray ⊥ Refracted ray

$\therefore \quad i + 90° + r = 180°$

$\Rightarrow \quad i + r = 90°$

$\Rightarrow \quad r = 90° - i$

Now $\frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90 - i)} = \tan i = \mu$

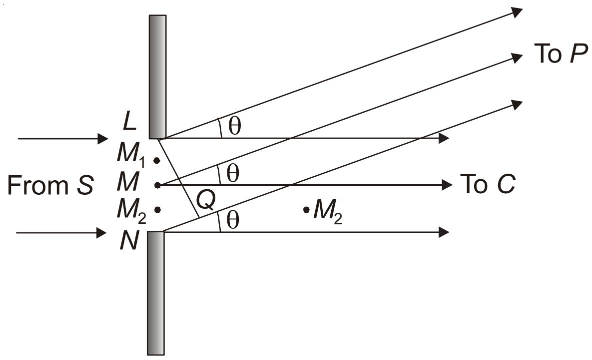
or $i = \tan^{-1}(\mu)$

This is called Brewster's law.

39. Explain in detail what do you mean by diffraction of light. Also discuss about diffraction of light at single slit. Also, determine positions of bright and dark fringes with respect to the path difference.

Sol. Spreading out of wave when it passes through a narrow opening is known as the phenomenon of diffraction.

The single slit diffraction : When the double slits in Young's experiment is replaced by a single narrow slit (illuminated by a monochromatic source), a broad pattern with a central bright region is seen. On both sides of this central bright fringe, there are alternate dark and bright regions, the intensity becoming weaker away from the centre. To understand this observation, let us mathematically study diffraction due to single slit. Consider a parallel beam of light falling normally on a single slit *LN* of width *a*. Mid point of slit is *M*.



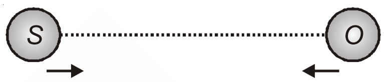
40. What do you mean by Doppler's effect in light? Explain how it differs from Doppler's effect on sound. Explain the concept of Red shift and Blue shift with context of Doppler's effect.

Sol. The Doppler's Effect

When light producing source moves away from the observer the frequency as measured by the observer will be smaller than that is actually generated by the source. Astronomers call the increase in wavelength due to Doppler effect as red shift. When observer moves towards the source or the source moves towards observer, then apparent wavelength decreases and visible spectrum appear to be shifted towards shorter wavelength. Hence, we call this as blue shift. However the formulae are different from that of sound. Here when either source or detector or both are in motion, only two cases are possible as speed of light is independent of relative motion between source and the observer.

Case-1 : Source and observer approach each other

$$f_A = f\sqrt{\frac{c+v}{c-v}}$$



Here, c is the speed of light

v is the relative speed of approach

f is the frequency of wave emitted by source

f_A is the observed frequency

Now, if $v << c$, then by binomial expansion, we have

$$f_A = f\left[1+\frac{v}{c}\right]^{1/2}\left[1-\frac{v}{c}\right]^{-1/2}$$

$$\Rightarrow \quad f_A = f\left[1+\frac{v}{2c}\right]\left[1+\frac{v}{2c}\right] \quad \left(\text{neglecting higher powers of } \frac{v}{c}\right)$$

$$\Rightarrow \quad \boxed{f_A = f\left[1+\frac{v}{c}\right]}$$

Case-2 : If the source or the observer recede from each other

$$f_R = f\sqrt{\frac{c-v}{c+v}}$$

which again reduces to $f_R = f\left[1-\frac{v}{c}\right]$ for $v << c$, here f_R represents frequency of recession.

The same relationship can be transformed in terms of the wavelength.

$$\lambda = \frac{c}{f}$$

$$\Rightarrow \quad \frac{c}{\lambda_A} = \frac{c}{\lambda}\left[1+\frac{v}{c}\right]$$

$$\Rightarrow \quad \lambda_A = \lambda\left[1+\frac{v}{c}\right]^{-1}$$

$$\Rightarrow \quad \lambda_A = \lambda\left[1-\frac{v}{c}\right]$$

$$\Rightarrow \quad \lambda_A - \lambda = -\lambda\frac{v}{c}$$

$$\boxed{-\frac{\Delta\lambda}{\lambda} = \frac{v}{c}}$$

Note that in the above formulae v is the component of the source velocity along the line joining the observer to the source relative to the observer.

v is positive when source moves relatively away from observer and v is negative when source moves towards the observer. The Doppler effect for the light is very important in astronomy. It is the basis for the measurements of the radial velocities of distant galaxies. Doppler effect in light is symmetric while in sound it is asymmetric.

41. Two sources S_1 and S_2 emitting light of wavelength 600 nm are placed at a distance of 1.0×10^{-2} cm apart. A detector can be moved on the line S_1P which is perpendicular to S_1S_2. (a) What would be the minimum and maximum path difference at the detector as it is moved along the line S_1P? (b) Locate the position of the farthest minimum detected.

Sol. (a) The situation is shown in Fig. (in question) The path difference is maximum when the detector is just at the position of S_1 and its value is equal to $d = 1.0 \times 10^{-2}$ cm.

The path difference is minimum when the detector is at a large distance from S_1. The path difference is then close to zero.

(b) The farthest minimum occurs at a point P, where path difference is equal to $\frac{\lambda}{2}$. If $S_1P = D$.

$$S_2P - S_1P = \frac{\lambda}{2}$$

$$\Rightarrow \quad \sqrt{D^2 + d^2} - D = \frac{\lambda}{2}$$

$$\Rightarrow \quad d^2 = D\lambda + \frac{\lambda^2}{4}$$

$$\text{or,} \quad D = \frac{d^2}{\lambda} - \frac{\lambda}{4}$$

$$D = \frac{\left(1.0\times10^{-4}\text{m}\right)^2}{\left(600\times10^{-9}\text{m}\right)} - 150 \times 10^{-9} \text{ m} = 1.7 \text{ cm}$$

42. Explain and discuss comparatively the corpuscular and wave theory of light. How wave theory was a step forward towards understanding of light?

Sol. 1. Corpuscular model : In 1637, Descartes gave the corpuscular model of light. According to this model, a luminous body emits a stream of particles in all directions. The particles are assumed to be very-very tiny. It explained the laws of reflection and refraction of light at an interface using concepts of elastic collisions and momentum conservation. Newton further developed this model and explained sensation of colours at the retina of the eye and spectrum formed due to prism. This model predicted that if the ray of light on refraction bends towards the normal, then the speed of light would be greater in the second medium. This prediction however proved wrong when Fermat formulated his famous principle of least time. Although this law could explain reflection and refraction, this law could not satisfactorily explain phenomenon like interference, polarization and diffraction.

2. Wave model : The wave theory of light was first put forward by Christian Huygens in 1678. During that period, everyone believed in Newton's corpuscular theory, which had satisfactorily explained the phenomenon of reflection, refraction, the rectilinear propagation of light and the fact that light could propagate through vacuum. So empowering was Newton's authority that scientist fraternity highly believed in Newton's theory. When Huygens put forward his wave theory, no one readily accepted it. Light as a wave model was not accepted as it was believed that a wave necessarily require a medium for propagation. On the basis of his wave theory. Huygen explained satisfactorily the phenomenon of reflection, refraction and total internal reflection. This model actually got acceptance when Thomas Young performed his famous interference experiments in 1801. Following Young's experiments many experiments were carried out involving the interference and diffraction of light waves, these experiments could only be satisfactorily explained by assuming a wave model of light. Thus around the middle of nineteenth century, the wave theory gained well acceptance.

The only phenomenon that wave theory failed to explain at that time was propagation of light in vacuum, as it was a firm belief that no wave could travel without a medium, till Maxwell propounded his famous electromagnetic theory. Maxwell had developed a set of equations describing the laws of electricity and magnetism and using these equations he derived what is known as the wave equation of electromagnetic waves. From these equations Maxwell calculated theoretically the speed of electromagnetic waves as $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$ which was later found quite close to the measured value of speed of light. According to Maxwell, light consists of varying electric and magnetic field.

43. Explain coherent and incoherent source of light.

Sol. Two sources which vibrate with a constant phase difference between them are said to be coherent. Two different light sources can never produce coherent waves. The reason of this lies in the origin of emission of light. In a conventional light source, light comes from a large number of independent atoms, each emitting a light for about 10^{-9} sec, *i.e.*, light emitted by an atom is essentially a pulse lasting for only 10^{-9} sec.

Even if different atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases. Hence we summarise from the above result that light beams from two different light sources can never have constant phase difference between them and thus will not be coherent. However if light from single monochromatic source is used, then even if the phases change as time passes but still the phase between the two or more sub-sources from this single source will remain constant and thus these two sources originating from a single source will be called coherent sources. The independent sources of light are called as incoherent sources.

44. Unpolarised light of Intensity 32 Wm^{-2} passes through three polarisers such that the transmission axis of the last polarizer is crossed with the first. If the intensity of emerging light is 3 Wm^{-2}, what is the angle between the transmission axis of first two polarisers? At what angle will the transmitted intensity be maximum?

Sol. Let θ = angle between transmission axis of P_1 and P_2

ϕ = angle between transmission axis of P_2 and P_3

$\therefore \quad \theta + \phi = 90°$ or $\phi = (90° - \theta)$

Here, $I_0 = 32\ Wm^{-2}$

$$I_1 = \frac{I_0}{2} = 16\ Wm^{-2}$$

$$I_2 = I_1\cos^2\theta \text{ and } I_3 = I_2 \cos^2\phi$$

$$I_3 = I_1\cos^2\theta\ \cos^2\theta$$

$$= I_1\cos^2\theta\ \sin^2\theta = 16\ \cos^2\theta\ \sin^2\theta$$

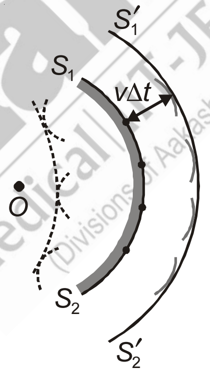
$$3 = 4\ (\sin 2\theta)^2$$

$$\theta = 30°$$

I_3 will be maximum, when $\sin 2\theta = 1 \Rightarrow \theta = 45°$

45. What is Huygen's principle? Explain the formation of spherical wavefronts. Write drawback of Huygen's principle.

Sol. According to Huygens' principle, each point on a wavefront is a source of secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of wave. The envelope of these wavelets gives the shape of the new wavefront. Let us suppose we want to determine the shape of the wavefront after a time interval of say Δt. Then, with each point on the wavefront as centre, we draw spheres of radius $v\Delta t$, where v is the speed of the wave in that medium. If we draw a common tangent to all these spheres, then we obtain an envelope which is again a sphere centered at point source.



S_1S_2 = Shape of the wavefront at time $t = t$

$S_1'S_2'$ = Shape of the wavefront at time $t = t + \Delta t$

Drawback with Huygens's Principle : Though Huygens' Principle could very well explain the various phenomenon unexplainable from corpuscular model, but it had one drawback. As we draw the wavefronts as suggested by Huygen we also obtain a backward wavefront which is not actually present. Huygen argued that presence of backwave is avoided by assuming that the amplitude of the secondary wavelets is not uniform in all directions, indeed it is maximum in the forward direction and zero in the backward direction. However this assumption was later explained by rigrous wave theory.

❑ ❑ ❑

Level-I

Chapter 10

Wave Optics

Solutions (Set-2)

[Huygen's Wave Theory]

1. By corpuscular theory of light, the phenomenon which can be explained is

   (1) Reflection (2) Interference (3) Diffraction (4) Polarisation

Sol. Answer (1)

2. The idea of secondary wavelets for the propagation of light wave was first given by

   (1) Newton (2) Huygens (3) Maxwell (4) Fresnel

Sol. Answer (2)

3. Wavefront means

   (1) All particles in it have same phase

   (2) All the particles have opposite phase of vibration

   (3) Few particles are in same phase, rest are in opposite phase

   (4) None of these

Sol. Answer (1)

4. Wavefront of a wave is _______to wave motion.

   (1) Parallel (2) Perpendicular (3) Opposite (4) At an angle

Sol. Answer (2)

5. The wavefront from a point source is

   (1) Cylindrical (2) Spherical (3) Plane (4) Cubical

Sol. Answer (2)

6. Huygens' wave theory allows, us to know

   (1) The wavelength of the wave (2) The velocity of the wave

   (3) The amplitude of the wave (4) The propagation of wavefronts

Sol. Answer (4)

[Young Double Slit Experiment]

7. Two coherent sources of light can be obtained by

(1) Two different lamps (2) Two different lamps but of same power

(3) Two different lamps of same power (4) By dividing a wavefront

Sol. Answer (4)

Two different sources can never be coherent.

8. Two sources of waves are called coherent if

(1) Both have same amplitude of vibration

(2) Both produce waves of same wavelength

(3) Both produce waves of the same wavelength having constant phase difference

(4) Both produce waves having same velocity

Sol. Answer (3)

Definition of coherent sources.

9. Two coherent monochromatic light beams of intensites I and $4I$ are superimposed. The maximum and minimum possible intensities in the resulting beam are

(1) $5I$ and I (2) $5I$ and $3I$ (3) $9I$ and I (4) $9I$ and $3I$

Sol. Answer (3)

$$I_{max} = \left(\sqrt{I} + \sqrt{4I}\right)^2 = 9I$$

$$I_{min} = \left(\sqrt{I} - \sqrt{4I}\right)^2 = I$$

10. Two identical light waves, propagating in the same direction, have a phase difference δ. After they superpose, the intensity of the resulting wave will be proportional to

(1) $\cos\delta$ (2) $\cos\left(\frac{\delta}{2}\right)$ (3) $\cos^2\left(\frac{\delta}{2}\right)$ (4) $\cos^2\delta$

Sol. Answer (3)

$$I_R = 4I_0\cos^2\left(\frac{\delta}{2}\right)$$

11. For destructive interference to take place between two monochromatic light waves of wavelength λ, the path difference should be

(1) $\frac{(2n-1)\lambda}{4}$ (2) $2n\lambda$ (3) $\frac{(2n+1)\lambda}{2}$ (4) $n\lambda$

Sol. Answer (3)

For destructive interference,

$$\Delta\phi = (2n + 1)\pi$$

$$\therefore \quad \Delta x = \frac{(2n+1)\lambda}{2}$$

12. If the amplitude ratio of two sources producing interference is 3 : 5, the ratio of intensities of maxima and minima is

(1) 25 : 16 (2) 5 : 3 (3) 16 : 1 (4) 25 : 9

Sol. Answer (3)

$$\frac{A_1}{A_2} = \frac{3}{5}, \frac{A_{max}}{A_{min}} = \frac{3+5}{5-3} = \frac{8}{2} = \frac{4}{1}$$

$$\frac{I_{max}}{I_{min}} \propto \left(\frac{A_{max}}{A_{min}}\right)^2 = \frac{16}{1}$$

13. Young's double slit experiment establishes that

(1) Light consists of waves
(2) Light consists of particles
(3) Light consists of neither particles nor waves
(4) Light consists of both particles and waves

Sol. Answer (1)

14. Monochromatic green light of wavelength 5×10^{-7} m illuminates a pair of slits 1 mm apart. The separation of bright lines on the interference pattern formed on a screen 2 m away is

(1) 0.25 mm (2) 0.1 mm (3) 1.0 mm (4) 0.01 mm

Sol. Answer (3)

Fringe width $\beta = \frac{\lambda D}{d}$

15. In Young's double slit interference experiment, the slit separation is made 3 folds. The fringe width becomes

(1) $\frac{1}{3}$ times (2) $\frac{1}{9}$ times (3) 3 times (4) 9 times

Sol. Answer (1)

$\beta = \frac{\lambda D}{d}$, $\beta \propto \frac{1}{d}$, so d made 3 folds

β becomes $\frac{1}{3}$ times.

16. In Young's double slit experiment, the distance between the slits is reduced to half and the distance between the slit and the screen is doubled, then fringe width

(1) Will not change
(2) Will become half
(3) Will be doubled
(4) Will become four times

Sol. Answer (4)

$\beta = \frac{\lambda D}{d}$

Now, new $D = 2D$

new $d = \frac{d}{2}$

$\beta' = 4\beta$

17. The maximum intensity of fringes in Young's experiment is I. If one of the slits is closed, then intensity at that place becomes I_0. Then relation between I and I_0 is

(1) $I = I_0$ (2) $I = 2I_0$ (3) $I = 4I_0$ (4) There is no relation

Sol. Answer (3)

Let individual intensity be I_0.

$I = 4I_0$

When one slit closed intensity is equal to I_0.

18. In Young's double slit experiment, the central point on the screen is

(1) Bright (2) Dark

(3) First bright then dark (4) First dark and then bright

Sol. Answer (1)

Centre point is central bright fringe.

19. In a Young's double slit experiment, the fringe width is found to be 0.4 mm. If the whole apparatus is immersed in water of refractive index $\frac{4}{3}$ without disturbing the geometrical arrangement, the new fringe width will be

(1) 0.30 mm (2) 0.40 mm (3) 0.53 mm (4) 450 μm

Sol. Answer (1)

$$\beta = \frac{\lambda D}{d}$$

$$\beta' = \frac{\lambda}{\mu}\frac{D}{d} = \frac{\beta}{\mu} = \frac{0.4\text{ mm}}{\frac{4}{3}} = 0.30\text{ mm}$$

20. In YDSE, d = 2 mm, D = 2 m and λ = 500 nm. If intensity of two slits are I_0 and $9I_0$, then find intensity at $y = \frac{1}{6}$ mm.

(1) $7I_0$ (2) $10I_0$ (3) $16I_0$ (4) $4I_0$

Sol. Answer (1)

$$\Delta x = \frac{dy}{\Delta} = \frac{2\times10^{-3}\times\frac{1}{6}\times10^{-3}}{2} = \frac{1}{6}\times10^{-6}\text{ m}$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{500\times10^{-9}}\times\frac{1}{6}\times10^{-6} = \frac{2\pi}{3}$$

$$I_{\text{net}} = I_0 + 9I_0 + 2\sqrt{I_0\times9I_0}\cos\Delta\phi = 10I_0 + 2\times3I_0\times\cos\frac{2\pi}{3} = 10I_0 + 6I_0\times\left(\frac{-1}{2}\right) = 7I_0$$

21. In Young's double slit experiment, the intensity at a point where path difference is $\frac{\lambda}{6}$ is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$.

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{2}$ (4) $\frac{3}{4}$

Sol. Answer (4)

$$I = I_{\text{max}}\cos^2\left(\frac{\phi}{2}\right)$$

$$I = I_0\cos^2\left(\frac{\phi}{2}\right)$$

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$$

$$\therefore \quad I = I_0 \cos^2\left(\frac{\pi}{6}\right)$$

$$I = \frac{3I_0}{4}$$

$$\therefore \quad \frac{I}{I_0} = \frac{3}{4}$$

22. Two slits, 4 mm apart, are illuminated by a light of wavelength 6000 Å. What will be the fringe width on a screen placed 2 m from the slits?

(1) 0.12 mm (2) 0.3 mm (3) 3.0 mm (4) 4.0 mm

Sol. Answer (2)

$\beta = \frac{\lambda D}{d}$, Put values in SI units.

23. In Young's double slit experiment, the central bright fringe can be identified

(1) By using white light instead of monochromatic light

(2) As it is narrower than other bright fringes

(3) As it is wider than other bright fringes

(4) As it has a greater intensity than other bright fringes

Sol. Answer (1)

24. In a Young's double slit experiment, the source illuminating the slits is changed from blue to violet. The width of the fringes

(1) Increases (2) Decreases (3) Becomes unequal (4) Remains same

Sol. Answer (2)

$$\beta = \frac{\lambda D}{d}, \beta \propto \lambda$$

$$\lambda_{violet} < \lambda_{blue}$$

25. If a torch is used in place of monochromatic light in Young's experiment what will happen

(1) Fringe will appear for a moment then it will disappear

(2) Fringes will appear as for monochromatic light

(3) Only bright fringes will appear

(4) No fringes will appear

Sol. Answer (4)

As torch light will have multiple wavelength. Overlap of fringe pattern will take place, hence we can say no fringes would appear.

26. In Young's double slit experiment, when two light waves form third minimum, they have

(1) Phase difference of 3π (2) Path difference of 3λ

(3) Phase difference of $\frac{5\pi}{2}$ (4) Path difference of $\frac{5\lambda}{2}$

Sol. Answer (4)

For third minima, $\Delta x = \left(n+\frac{1}{2}\right)\lambda$

$n = 2$,

$\Rightarrow \quad \Delta x = \frac{5\lambda}{2}$

Also, $\Delta\phi = \frac{2\pi}{\lambda}\times\frac{5\lambda}{2} = 5\pi$

27. For the sustained interference of light, the necessary condition is that the two sources should

(1) Have constant phase difference

(2) Be narrow

(3) Be close to each other

(4) Of same amplitude

Sol. Answer (1)

Coherence condition.

28. Which of the following is conserved when light waves interfere?

(1) Intensity (2) Energy (3) Amplitude (4) Momentum

Sol. Answer (2)

Interference is simply redistribution of energy.

29. In Young's double slit experiment, if one of the slits is closed fully, then in the interference pattern

(1) A bright slit will be observed, no interference pattern will exist

(2) The bright fringes will become more bright

(3) The bright fringes will become fainter

(4) Diffraction pattern due to single slit will be observed

Sol. Answer (4)

30. In Young's experiment, the separation between 5th maxima and 3rd minima is how many times as that of fringe width?

(1) 5 times (2) 3 times (3) 2.5 times (4) 2 times

Sol. Answer (3)

Position of 5^{th} maxima $\frac{5\lambda D}{d}$

Position of 3^{rd} minima $\frac{5\lambda D}{2d}$

Distance between them $\frac{5\lambda D}{d} - \frac{5\lambda D}{2d} = \frac{5\lambda D}{2d} = \frac{5}{2}\beta$

31. In a Young's double slit experiment, let β is the fringe width and I_0 is the intensity at the central bright fringe. At a distance x from the central bright fringe, intensity will be

(1) $I_0 \cos\left(\frac{x}{\beta}\right)$ (2) $I_0 \cos^2\left(\frac{x}{\beta}\right)$ (3) $I_0 \cos^2\left(\frac{\pi x}{\beta}\right)$ (4) $\frac{I_0}{4}\cos^2\left(\frac{\pi x}{\beta}\right)$

Sol. Answer (3)

$I = I_0 \cos^2 \frac{\varphi}{2}$

$\varphi = \frac{2\pi}{\lambda}$; Path difference = $\frac{xd}{D}$

using above relation

$I = I_0 \cos^2 \left[\frac{\pi}{\lambda} \times \frac{dx}{D}\right] = I_0 \cos^2\left[\frac{\pi x}{\beta}\right]$

32. In the Young's double slit experiment, when a glass plate (μ = 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength = λ), the intensity at the position where central maxima occurred previous remains unchanged. The minimum thickness of the glass plate is

(1) 2λ (2) λ (3) $\frac{2}{3}\lambda$ (4) $\frac{\lambda}{3}$

Sol. Answer (1)

For this the shift in fringe pattern shall be integral multiple of fringe width

$$\frac{Dt(\mu - 1)}{d} = n\frac{\lambda D}{d} \Rightarrow t(\mu - 1) = n\lambda \Rightarrow t = \frac{n\lambda}{0.5}$$

Minimum thickness = 2λ

33. It is found that when waves from two identical coherent sources superpose at a certain point, the resultant intensity is equal to intensity of one wave only. This means that the phase different between the two waves at that point is

(1) Zero (2) $\frac{\pi}{3}$ (3) $\frac{2\pi}{3}$ (4) π

Sol. Answer (3)

$I = 4I_1 \cos^2 \frac{\phi}{2}$

As $I = I_1$

So $\cos\frac{\phi}{2} = \frac{1}{2}$

$\frac{\phi}{2} = 60° \Rightarrow \phi = 120°$.

34. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by

(1) 18 (2) 12 (3) 24 (4) 30

Sol. Answer (1)

Let the width of the segment is y and fringe width is β

Number of fringes in segment = $\frac{y}{\beta} = \frac{yd}{\lambda D}$

Now for λ = 600 nm = $\frac{yd}{600D} = 12$

for λ = 400 nm = $\frac{yd}{400D} = n$

$\Rightarrow \frac{2}{3} = \frac{12}{n}$

$\Rightarrow n = 18$

35. In YDSE, $d = 5\lambda$, then the total no. of maxima observed upon screen will be

(1) 9 (2) 8 (3) 7 (4) 5

Sol. Answer (1)

[Diffraction]

36. In diffraction using single slit, a slit of width a is illuminated by white light. For red light (λ = 6500 Å), the first minima is obtained for θ = 30°. Then the value of a will be

(1) 3250 Å (2) 6.5×10^{-4} (3) 1.24 μm (4) 2.6×10^{-4}

Sol. Answer (3)

First minima $\theta = \frac{\lambda}{a}$

$\theta = \frac{\pi}{6}, \lambda = 6500 \times 10^{-10}\,\text{m}$

$\therefore \quad a = \frac{\lambda}{\theta} = 1.24\ \mu\text{m}$

37. The bending of beam of light around corners of obstacles is called

(1) Reflection (2) Diffraction (3) Refraction (4) Interference

Sol. Answer (2)

38. Angular width of central maximum of a diffraction pattern on a single slit does not depend upon

(1) Distance between slit and source

(2) Wavelength of the light used

(3) Width of the slit

(4) Frequency of light used

Sol. Answer (1)

Angular width of central maximum equals $2\sin^{-1}\left(\frac{\lambda}{d}\right)$

It is independent of D.

39. Red light is generally used to observe diffraction pattern from single slit. If blue light is used instead of red light, then diffraction pattern

(1) Will be more clear (2) Will contract

(3) Will expand (4) Will not be visualized

Sol. Answer (2)

$\lambda_{blue} < \lambda_{red}$ fringe pattern will contract because it is $\propto \lambda$

40. When a compact disk is illuminated by a source of white light, coloured 'lanes' are observed. This is due to

(1) Dispersion (2) Diffraction (3) Interference (4) Refraction

Sol. Answer (2)

41. For what distance is ray optics a good approximation when the aperture is 4 mm wide and the wavelength is 500 nm?

(1) 32 m (2) 69 m (3) 16 m (4) 8 m

Sol. Answer (1)

Fresnel distance $Z_F = \frac{a^2}{\lambda} = 32\text{ m}$

42. To observe diffraction the size of an obstacle

(1) Should be of the same order as wavelength (2) Should be much smaller than the wavelength

(3) Has no relation to wavelength (4) Should be exactly $\frac{\lambda}{2}$

Sol. Answer (1)

For diffraction $\lambda \approx a$ (aperture or obstacle size)

43. In a single slit diffraction pattern

(1) Central fringe has negligible width than others (2) All fringes are of same width

(3) Central fringe do not exist (4) Central fringe is twice as wide as other maxima

Sol. Answer (4)

See theory.

44. Diffraction and interference of light suggest

(1) Nature of light is electromagnetic (2) Wave nature of light

(3) Nature is quantum (4) Nature of light is transverse

Sol. Answer (2)

These are characteristics of a wave.

45. The limit of resolution of an optical instrument arises on account of

(1) Reflection (2) Diffraction (3) Polarisation (4) Interference

Sol. Answer (2)

See theory.

46. Choose the correct statement

(1) A telescope magnifies (2) A microscope resolves (3) A telescope resolves (4) Both (2) & (3)

Sol. Answer (3)

47. The distance upto which ray optics holds good is called

(1) Fresnel distance (2) Fraunhofer distance (3) Optical distance (4) Wave distance

Sol. Answer (1)

48. Diffraction effect can be observed in

(1) Only sound waves (2) Only light waves

(3) Only ultrasonic waves (4) Sound as well as light waves

Sol. Answer (4)

[Polarisation]

49. A polariser is used to

(1) Reduce intensity of light (2) Increase intensity of light

(3) Produce polarised light (4) Analyse polarised light

Sol. Answer (3)

50. Light waves can be polarised as they are

(1) Transverse (2) Longitudinal (3) Diffracted (4) Of high frequency

Sol. Answer (1)

Only transverse waves can be polarized.

51. Through which character we can differentiate between light waves and sound waves

(1) Interference (2) Reflection (3) Refraction (4) Polarisation

Sol. Answer (4)

Only light waves undergo polarization since they are transverse. Sound waves are longitudinal, hence do not undergo polarization.

52. The angle of polarisation for any medium is 60°. What will be critical angle for this?

(1) $\sin^{-1}\left(\sqrt{3}\right)$ (2) $\tan^{-1}\left(\sqrt{3}\right)$ (3) $\cos^{-1}\left(\sqrt{3}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Sol. Answer (4)

By Brewster's law,

$\tan i = \mu$

$\Rightarrow \mu = \tan 60° = \sqrt{3}$

Now, $c = \sin^{-1}\left(\frac{1}{\mu}\right)$

$\Rightarrow c = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

53. Which of the following waves cannot be polarised?

(1) Radiowaves (2) Ultraviolet rays (3) Infrared rays (4) Ultrasonic waves

Sol. Answer (4)

Only ultrasonic waves are longitudinal, rest three are transverse.

54. An unpolarised light of intensity I_0 is passed through a polariser. The intensity of the transmitted light will be

(1) I_0 (2) $\frac{I_0}{2}$ (3) $\frac{I_0}{4}$ (4) Zero

Sol. Answer (2)

55. Refractive index of material is equal to the tangent of polarising angle. It is called

(1) Brewster's law (2) Lambert's law (3) Malus' law (4) Bragg's law

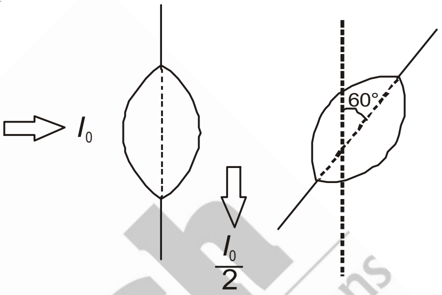
Sol. Answer (1)

56. Two polarizers are oriented with transmission planes making an angle of 60°. The percentage of incident unpolarized light that passes through the system is

(1) 50% (2) 100% (3) 12.5% (4) 37.5%

Sol. Answer (3)

$I = \frac{I_0}{2}\cos^2 60° = \frac{I_0}{8}$



% transmitted light $\frac{\frac{I_0}{8}}{I_0} \times 100 = \frac{100}{8} = 12.5\%$

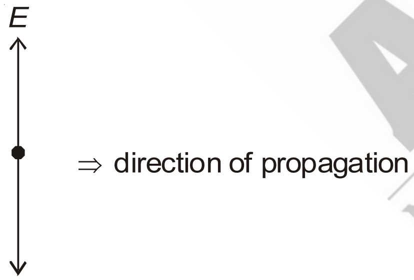
57. In case of linearly polarised light, the magnitude of electric field vector

(1) Doesn't change with time (2) Varies periodically with time

(3) Increases and decreases linearly with time (4) Is parallel to the direction of propagation

Sol. Answer (2)

Linearly polarized light



Direction of field vector is specified but the magnitude varies with time.

58. When angle of incidence of the material is 60°, the reflected light is completely polarized. The velocity of the refracted ray inside the material is (in m/s)

(1) 3×10^8 (2) $\frac{3}{\sqrt{2}} \times 10^8$ (3) $\sqrt{3} \times 10^8$ (4) 0.5×10^8

Sol. Answer (3)

Now, $\tan i = \mu$

$\Rightarrow \mu = \tan 60° = \sqrt{3}$

$v = \frac{c}{\mu} = \frac{3 \times 10^8}{\sqrt{3}} = \sqrt{3} \times 10^8$ m/s

[Doppler Effect]

59. It is believed that the Universe is expanding and hence the distant stars are receding from us. Light from such a star will show

(1) Shift in frequency towards longer wavelength

(2) Shift in frequency towards shorter wavelength

(3) No shift in frequency but a decrease in intensity

(4) A shift in frequency sometimes towards longer and sometimes towards shorter wavelength

Sol. Answer (1)

There will be a red shift.

60. The 6563 Å line emitted by hydrogen atom in a star is found to be red shifted by 5 Å. The speed with which the star is receding from the earth is

(1) 17.3×10^3 m/s (2) 4.29×10^7 m/s (3) 3.39×10^5 m/s (4) 2.29×10^5 m/s

Sol. Answer (4)

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

$\Delta\lambda = 5$ Å

$\lambda = 6563$ Å

$c = 3 \times 10^8$ m/s

$\therefore \quad v = 2.29 \times 10^5$ m/s

61. In context of Doppler's effect in light, the term'red shift' signifies

(1) Decrease in frequency (2) Increase in frequency

(3) Decrease in intensity (4) Increase in intensity

Sol. Answer (1)

62. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then,

(1) $\beta_G > \beta_B > \beta_R$ (2) $\beta_B > \beta_G > \beta_R$ (3) $\beta_R > \beta_B > \beta_G$ (4) $\beta_R > \beta_G > \beta_B$

Sol. Answer (4)

As $\lambda_R > \lambda_G > \lambda_B$

$\Rightarrow \beta_R > \beta_G > \beta_B$ as $\beta \propto \lambda$

63. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is

(1) $(2n+1)\frac{\lambda}{2}$ (2) $(2n+1)\frac{\lambda}{4}$ (3) $(2n+1)\frac{\lambda}{8}$ (4) $(2n+1)\frac{\lambda}{16}$

Sol. Answer (2)

$$I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{1}{2} = \cos^2\left(\frac{\phi}{2}\right)$$

$\Rightarrow \cos\phi = 0$

$\Rightarrow \phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\Rightarrow \Delta x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$

$\Rightarrow \Delta x = (2n+1)\frac{\lambda}{4}$

64. Using the expression $2d \sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0°,

(1) The absolute error in d remains constant
(2) The absolute error in d increases
(3) The fractional error in d remains constant
(4) The fractional error in d decreases

Sol. Answer (4)

$$d = \frac{\lambda}{2\sin\theta}$$

$$|d(d)| = \frac{\lambda}{2}|\text{cosec}\,\theta \cot\theta d\theta|$$

Absolute error in d decreases with increase in θ as cosec θ and cot θ decrease with increase in θ.

$$\left|\frac{d(d)}{d}\right| = |\cot\theta d\theta|$$

Fractional error decreases with increase in θ.

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