CHAPTER 2

Motion in a Straight Line

1. **(d)** Given $x = ae^{-\alpha t} + be^{\beta t}$

Velocity, $v = \frac{dx}{dt} = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$

$$= -\frac{a\alpha}{e^{\alpha t}} + b\beta e^{\beta t}$$

i.e., go on increasing with time.

2. **(d)** In (a), at the same time particle has two positions which is not possible. In (b), particle has two velocities at the same time. In (c), speed is negative which is not possible.

3. **(c)** We have, $S_n = u + \frac{a}{2}(2n-1)$

or $65 = u + \frac{a}{2}(2 \times 5 - 1)$

or $65 = u + \frac{9}{2}a$ ..... (1)

Also, $105 = u + \frac{a}{2}(2 \times 9 - 1)$

or $105 = u + \frac{17}{2}a$ ..... (2)

Equation (2) – (1) gives,

$$40 = \frac{17}{2}a - \frac{9}{2}a = 4a \text{ or } a = 10 \text{ m/s}^2.$$

Substitute this value in (1) we get,

$$u = 65 - \frac{9}{2} \times 10 = 65 - 45 = 20 \text{ m/s}$$

$\therefore$ The distance travelled by the body in 20 s is,

$$s = ut + \frac{1}{2}at^2 = 20 \times 20 + \frac{1}{2} \times 10 \times (20)^2$$
$$= 400 + 2000 = 2400 \text{ m}.$$

4. **(c)** Let v_A and v_B are the velocities of two bodies.

In first case, $v_A + v_B = 6\text{m/s}$ .....(1)

In second case, $v_A - v_B = 4\text{m/s}$ .....(2)

From (1) & (2) we get, $v_A = 5$ m/s and $v_B = 1$ m/s.

5. **(b)** Distance in last two second

$$= \frac{1}{2} \times 10 \times 2 = 10 \text{ m}.$$

Total distance $= \frac{1}{2} \times 10 \times (6+2) = 40$ m.

6. **(b)** Average velocity

$$= \frac{v_1 + v_2 + v_3}{3} = \frac{3+4+5}{3} = 4 \text{ m/s}$$

7. **(b)** Distance along a line i.e., displacement (s) $= t^3$ ($\because s \propto t^3$ given)

By double differentiation of displacement, we get acceleration.

$$V = \frac{ds}{dt} = \frac{dt^3}{dt} = 3t^2 \text{ and } a = \frac{dv}{dt} = \frac{d3t^2}{dt} = 6t$$

$a = 6t$ or $a \propto t$

8. **(d)** Let 'S' be the distance between two ends 'a' be the constant acceleration

As we know $v^2 - u^2 = 2aS$

or, $aS = \frac{v^2 - u^2}{2}$

Let v be velocity at mid point.

Therefore, $v_c^2 - u^2 = 2a\frac{S}{2}$

$$v_c^2 = u^2 + \frac{v^2 - u^2}{2} \Rightarrow v_c = \sqrt{\frac{u^2 + v^2}{2}}$$

9. **(b)** Time taken by same ball to return to the hands of juggler $= \frac{2u}{g} = \frac{2 \times 20}{10} = 4$ s. So he is throwing the balls after each 1 s. Let at some instant he is throwing ball number 4. Before 1 s of it he throws ball. So height of ball 3 :

$$h_3 = 20 \times 1 - \frac{1}{2}10(1)^2 = 15 \text{ m}$$

Before 2s, he throws ball 2. So height of ball 2 :

$$h_2 = 20 \times 2 - \frac{1}{2}10(2)^2 = 20 \text{ m}$$

Before 3 s, he throws ball 1. So height of ball 1 :

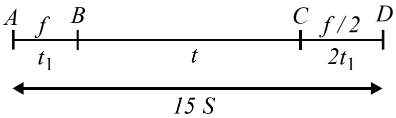
$$h_1 = 20 \times 3 - \frac{1}{2}10(3)^2 = 15 \text{ m}$$

10. **(d)** Distance from A to $B = S = \frac{1}{2}ft_1^2$

Distance from B to $C = (ft_1)t$

Distance from C to $D = \frac{u^2}{2a} = \frac{(ft_1)^2}{2(f/2)}$

$$= ft_1^2 = 2S$$



$\Rightarrow \quad S + f\,t_1 t + 2S = 15\,S$

$\Rightarrow \quad f\,t_1 t = 12\,S$ .... (i)

$\frac{1}{2} f\,t_1^2 = S$ .... (ii)

Dividing (i) by (ii), we get $t_1 = \frac{t}{6}$

$\Rightarrow \quad S = \frac{1}{2} f \left(\frac{t}{6}\right)^2 = \frac{f\,t^2}{72}$

11. **(a)** $v = \alpha\sqrt{x}$,

$\Rightarrow \quad \frac{dx}{dt} = \alpha\sqrt{x} \;\Rightarrow\; \frac{dx}{\sqrt{x}} = \alpha\,dt$

Integrating both sides,

$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t dt \;;\; \left[\frac{2\sqrt{x}}{1}\right]_0^x = \alpha[t]_0^t$

$\Rightarrow 2\sqrt{x} = \alpha t \;\Rightarrow x = \frac{\alpha^2}{4} t^2$

12. **(c)** Person's speed walking only is

$\frac{1}{60}\,\frac{\text{"escalator"}}{\text{second}}$

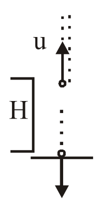
Standing the escalator without walking the speed is $\frac{1}{40}\,\frac{\text{"escalator"}}{\text{second}}$

Walking with the escalator going, the speed add.

So, the person's speed is $\frac{1}{60} + \frac{1}{40} = \frac{15}{120}$ $\frac{\text{"escalator"}}{\text{second}}$

So, the time to go up the escalator $\text{t} = \frac{120}{5} = 24$ second.

13. **(c)** Speed on reaching ground



$v = \sqrt{u^2 + 2gh}$

Now, $v = u + at$

$\Rightarrow \quad \sqrt{u^2 + 2gh} = -u + gt$

Time taken to reach highest point is $t = \frac{u}{g}$,

$\Rightarrow t = \frac{u + \sqrt{u^2 + 2gH}}{g} = \frac{nu}{g}$ (from question)

$\Rightarrow \quad 2gH = n(n-2)u^2$

14. **(a)** In first case

$u_1 = u\,;\, v_1 = \frac{u}{2}\,,\, s_1 = 3\text{ cm},\, a_1 = ?$

Using, $v_1^2 - u_1^2 = 2a_1 s_1$ ...(i)

$\left(\frac{u}{2}\right)^2 - u^2 = 2 \times a \times 3$

$\Rightarrow \quad a = \frac{-u^2}{8}$

In second case: Assuming the same retardation

$u_2 = u/2\,;\, v_2 = 0\,;\, s_2 = ?;\; a_2 = \frac{-u^2}{8}$

$v_2^2 - u_2^2 = 2a_2 \times s_2$ ...(ii)

$\therefore \; 0 - \frac{u^2}{4} = 2\left(\frac{-u^2}{8}\right) \times s_2$

$\Rightarrow \; s_2 = 1\text{ cm}$

15. **(b)** For downward motion $v = -gt$

The velocity of the rubber ball increases in downward direction and we get a straight line between v and t with a negative slope.

Also applying $y - y_0 = ut + \frac{1}{2}at^2$

We get $y - h = -\frac{1}{2}gt^2 \Rightarrow y = h - \frac{1}{2}gt^2$

The graph between y and t is a parabola with $y = h$ at $t = 0$. As time increases y decreases.

For upward motion.

The ball suffer elastic collision with the horizontal elastic plate therefore the direction of velocity is reversed and the magnitude remains the same.

Here $v = u - gt$ where u is the velocity just after collision.

As t increases, v decreases. We get a straight line between v and t with negative slope.

Also $y = ut - \frac{1}{2}gt^2$

All these characteristics are represented by graph (b).

16. **(b)** $x = at + bt^2 - ct^3$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(at + bt^2 + ct^3)$

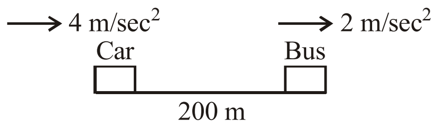
$= a + 2bt - 3ct^2$

Acceleration, $\frac{dv}{dt} = \frac{d}{dt}(a + 2bt - 3ct^2)$

or $0 = 2b - 3c \times 2t \quad \therefore \; t = \left(\frac{b}{3c}\right)$

and $v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2 = \left(a + \frac{b^2}{3c}\right)$

17. **(c)**



Given, $u_C = u_B = 0$, $a_C = 4\ m/s^2$, $a_B = 2\ m/s^2$
hence relative acceleration, $a_{CB} = 2\ m/sec^2$

Now, we know, $s = ut + \frac{1}{2}at^2$

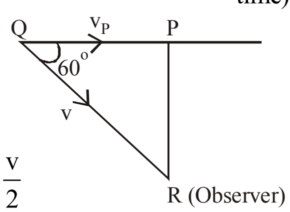
$200 = \frac{1}{2} \times 2t^2 \quad \because u = 0$

Hence, the car will catch up with the bus after time

$t = 10\sqrt{2}$ second

18. **(d)** Distance, $PQ = v_p \times t$ (Distance = speed × time)

Distance, $QR = V.t$



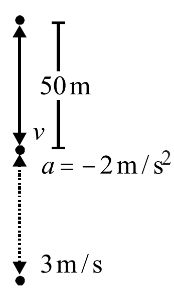
$\cos 60° = \frac{PQ}{QR}$

$\frac{1}{2} = \frac{v_p \times t}{V.t} \Rightarrow v_p = \frac{v}{2}$

19. **(a)**

20. **(b)** The slope of v-t graph is constant and velocity decreasing for first half. It is positive and constant over next half.

21. **(293)** Initial velocity of parachute after bailing out,



$u = \sqrt{2gh}$

$u = \sqrt{2 \times 9.8 \times 50} = 14\sqrt{5}$

The velocity at ground,

$v = 3m/s$

$S = \frac{v^2 - u^2}{2 \times 2} = \frac{3^2 - 980}{4}$

≈ 243 m

Initially he has fallen 50 m.

$\therefore$ Total height from where he bailed out = 243 + 50 = 293 m

22. **(80)** In first case speed,

$u = 60 \times \frac{5}{18} m/s = \frac{50}{3} m/s$

$d = 20m$,

Let retardation be a then

$(0)^2 - u^2 = -2ad$

or $\quad u^2 = 2ad \qquad \ldots(i)$

In second case speed, $u' = 120 \times \frac{5}{18}$

$= \frac{100}{3} m/s$

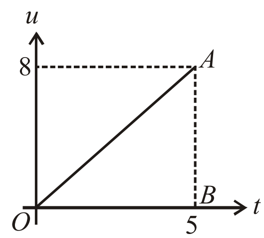
and $(0)^2 - u'^2 = -2ad'$

or $\quad u'^2 = 2ad' \qquad \ldots(ii)$

(ii) divided by (i) gives,

$4 = \frac{d'}{d} \Rightarrow d' = 4 \times 20 = 80m$

23. **(20)**



Distance travelled = Area of speed-time graph

$= \frac{1}{2} \times 5 \times 8 = 20\ m$

24. **(3)** Distance X varies with time t as

$x^2 = at^2 + 2bt + c$

$\Rightarrow 2x\frac{dx}{dt} = 2at + 2b$

$\Rightarrow x\frac{dx}{dt} = at + b \Rightarrow \frac{dx}{dt} = \frac{(at+b)}{x}$

$\Rightarrow x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 = a$

$\Rightarrow \frac{d^2x}{dt^2} = \frac{a - \left(\frac{dx}{dt}\right)^2}{x} = \frac{a - \left(\frac{at+b}{x}\right)^2}{x}$

$= \frac{ax^2 - (at+b)^2}{x^3} = \frac{ac - b^2}{x^3}$

$\Rightarrow \quad a \propto x^{-3} \qquad$ Hence, n = 3

25. **(08.00)** Let the ball takes time t to reach the ground

Using, $S = ut + \frac{1}{2}gt^2$

$\Rightarrow S = 0 \times t + \frac{1}{2}gt^2$

$\Rightarrow \quad 200 = gt^2 \qquad [\because 2S = 100m]$

$\Rightarrow t = \sqrt{\frac{200}{g}} \qquad \ldots(i)$

In last $\frac{1}{2}s$, body travels a distance of 19 m, so in

$\left(t - \frac{1}{2}\right)$ distance travelled = 81

Now, $\frac{1}{2}g\left(t - \frac{1}{2}\right)^2 = 81$

$\therefore \; g\left(t - \frac{1}{2}\right)^2 = 81 \times 2$

$\Rightarrow \left(t - \frac{1}{2}\right) = \sqrt{\frac{81 \times 2}{g}}$

$\therefore \; \frac{1}{2} = \frac{1}{\sqrt{g}}(\sqrt{200} - \sqrt{81 \times 2})$ using (i)

$\Rightarrow \sqrt{g} = 2(10\sqrt{2} - 9\sqrt{2})$

$\Rightarrow \sqrt{g} = 2\sqrt{2}$

$\therefore$ g = 8 m/s^2

26. **(2)** Given, $\frac{dv}{dt} = -2.5\sqrt{v}$

$\Rightarrow \frac{dv}{\sqrt{v}} = -2.5\,dt$

Integrating,

$\int_{6.25}^{0} v^{-1/2} dv = -2.5\int_0^t dt$

$\Rightarrow \left[\frac{v^{+1/2}}{(1/2)}\right]_{6.25}^{0} = -2.5[t]_0^t$

$\Rightarrow -2(6.25)^{1/2} = -2.5t$

$\Rightarrow -2 \times 2.5 = -2.5t$

$\Rightarrow t = 2$ s

27. **(5)** For rat $s = \frac{1}{2}\beta t^2 \qquad \ldots.(i)$

for cat $s = d = ut + \frac{1}{2}\alpha t^2 \qquad \ldots.(ii)$

Solving (i) and (ii) fot t to be real

$\beta = \alpha + \frac{u^2}{2d}$,

$\Rightarrow \beta = 2.5 + \frac{5^2}{2 \times 5} = 5 \text{ ms}^{-2}$

28. **(50)** The distance travel in n^{th} second is

$S_n = u + ½(2n–1)a \qquad \ldots.(1)$

so distance travel in t^{th} & $(t+1)^{th}$ second are

$S_t = u + ½(2t–1)a \qquad \ldots.(2)$

$S_{t+1} = u + ½(2t+1)a \qquad \ldots.(3)$

As per question,

$S_t + S_{t+1} = 100 = 2(u + at) \qquad \ldots.(4)$

Now from first equation of motion the velocity, of particle after time t, if it moves with an accleration a is

$v = u + a\,t \qquad \ldots.(5)$

where u is initial velocity

So from eq(4) and (5), we get v = 50cm./sec.

29. **(49)** S_n = Distance covered in n^{th} sec

$\Rightarrow \; S_n = u + \frac{a}{2}(2n - 1)$

Putting a = – g and n = 5, we get

$\Rightarrow S_5 = u - \frac{g}{2} \times 9 = u - \frac{9g}{2} \qquad \ldots(1)$

Distance covered in two continuous seconds can only be equal when the body reaches the highest point after the fifth second and comes down in the sixth second for which u = 0 and n = 1.

$\Rightarrow S_6 = 0 + \frac{g}{2}(2 \times 1 - 1) = \frac{g}{2} \qquad \ldots(2)$

Equating (1) and (2)

or, $u - \frac{9g}{2} = \frac{g}{2} \Rightarrow u = \frac{10g}{2} = 49\,m/s$

30. **(10)** The only force acting on the ball is the force of gravity. The ball will ascend until gravity reduces its velocity to zero and then it will descend. Find the time it takes for the ball to reach its maximum height and then double the time to cover the round trip.

Using $v_{\text{at maximum height}} = v_0 + at = v_0 - gt$, we get:

0 m/s = 50 m/s – (9.8 m/s^2) t

Therefore,

t = (50 m/s)/(9.8 m/s^2) ~ (50 m/s)/ (10 m/s^2) ~ 5s

This is the time it takes the ball to reach its maximum height. The total round trip time is 2t ~ 10s.