JEE 2023

Relations & Functions

DPP-05

1. Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3),\ x \in R$. Then $f(2)$ equals:

(A) 8 (B) –4
(C) –2 (D) 30

2. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$, where the function f satisfies $f(x+y) = f(x)f(y)$ for all natural numbers *x*, *y* and $f(1) = 2$. Then the natural number *a* is:

(A) 2 (B) 3
(C) 16 (D) 4

3. The value of

$$6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}}\ldots}}\right)$$

is _____.

4. The range of the function $y = f(x)$ if $(34)^x + (34)^y = 34$ equals:

(A) $(-\infty, 1)$ (B) $(-\infty, 1)$
(C) $(-\infty, 1]$ (D) $(1, \infty)$

5. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{3x + x^3}{1 + 3x^2}\right)$ is equal to:

(A) $(f(x))^3$ (B) $(f(x))^2$
(C) $3f(x)$ (D) $2f(x)$

6. If $\log_4 5 = m$ and $\log_5 6 = n$ then $\log_3 2$ is equal to:

(A) $\frac{1}{2mn - 1}$ (B) $\frac{1}{2m + 1}$
(C) $\frac{1}{2n + 1}$ (D) $\frac{1}{2mn + 1}$

7. If $f(x)$ is a polynomial function satisfying $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right),\ \forall\ x \in R - \{0\}$ and $f(2) = -31$, then value of $f'(3)$ is:

(A) –324 (B) –405
(C) 405 (D) –108

8. $3^{((4\log_9 7) - 1)}$ is equal to:

(A) 49 (B) $\frac{49}{3}$
(C) 24 (D) 48

9. $\log_3 \log_2 \log_{\sqrt{5}}(5^4)$ is equal to:

(A) 2 (B) 1
(C) 3 (D) 0

10. If $p(x)$ is a polynomial function of degree four satisfying $p(1) = 3,\ p(2) = 5,\ p(3) = 7,\ p(4) = 9,\ p(5) = -13$, then value of $p(0)$ is:

(A) 1
(B) 23
(C) –23
(D) –24

Answer Key

Relations and Functions

1. (C)
2. (B)
3. 4
4. (B)
5. (C)
6. (A)
7. (B)
8. (B)
9. (B)
10. (C)

Hints & Solutions

Relations and Functions

1. **(C)**

$f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$

Differentiate w.r.t. x

$f'(x) = 3x^2 + 2xf'(1) + f''(2)$ …….(i)

Again, differentiate w.r.t x

$f''(x) = 6x + 2f'(1)$ …….(ii)

Again, differentiate w.r.t. x

$f'''(x) = 6$ ……..(iii)

Put $x = 1$ in equation (i)

$f'(1) = 3 + 2f'(1) + f''(2)$ …..(iv)

Put $x = 2$ in equation (ii)

$f''(2) = 12 + 2f'(1)$ …..(v)

Eliminate $f''(2)$ from equation (iv) and (v)

$-3 - f'(1) = 12 + 2f'(1)$

$\Rightarrow \quad f'(1) = -5$

Replace the value $f'(1)$ in equation (v)

$f''(2) = 2$

Put $x = 3$ in equation (iii)

$f'''(3) = 6$

$\because \quad f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$

$\Rightarrow \quad f(x) = x^3 - 5x^2 + 2x + 6$

$\Rightarrow \quad f(2) = 8 - 20 + 4 + 6 = -2$

2. **(B)**

$\because \ f(1) = 2$

And $f(x+y) = f(x) \cdot f(y)$

$f(x+y) = f(x) \cdot f(y)$

$\forall \ x, \ y \in N$

Now, $f(x) = f(n-1) \cdot f(1)$

$f(x) = f(n-1) \cdot f(1)$

……..

……..

$= (f(1))^n$

Hence, $f(x) = 2^x \ \forall x \in N$

Given, $f(a+1) + f(a+2) + ... + f(a+10)$

$= 16(2^{10} - 1)$

$\Rightarrow \quad 2^{a+1} + 2^{a+2} + ... + 2^{a+10}$

$= 16(2^{10} - 1)$

$\Rightarrow \quad 2^{a+1}(2^{10} - 1) = 16(2^{10} - 1)$

$\Rightarrow \quad a + 1 = 4$

$\Rightarrow \quad a = 3$

3. **4**

Let $\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}}.....} = y$

$\Rightarrow \quad \frac{1}{3\sqrt{2}}\sqrt{4 - y} = y$

$\Rightarrow \quad \sqrt{4 - y} = 18y^2$

$\Rightarrow \quad y = \frac{4}{9}$

Hence, $6 + \log_{\left(\frac{3}{2}\right)}\left(\frac{2}{3}\right)^2$

$= 6 - 2 = 4$

4. **(B)**

$(34)^x + (34)^y = 34$

$\Rightarrow \ (34)^x = 34 - 34^y$

$\Rightarrow \quad x = \log_{34}\left(34 - 34^y\right)$

$\Rightarrow 34 - 34^y > 0 \Rightarrow y < 1 \ \Rightarrow y \in (-\infty, 1)$

5. **(C)**

$f\left(\frac{3x + x^3}{1 + 3x^2}\right) = \log\left(\frac{1 + \frac{3x + x^3}{1 + 3x^2}}{1 - \frac{3x + x^3}{1 + 3x^2}}\right)$

$= \log\left(\frac{x^3 + 3x^2 + 3x + 1}{1 - x^3 + 3x^2 - 3x}\right) = \log\frac{(x+1)^3}{(1-x)^3}$

$= \log\left(\frac{1+x}{1-x}\right)^3 = 3f(x)$

6. **(A)**

Here $5 = 4^m$ and

$6 = 5^n \Rightarrow 6 = 5^n = (4^m)^n \Rightarrow 3 = 2^{2mn-1}$

$\therefore \quad \log_3 2 = \dfrac{1}{2mn-1}$

7. **(B)**

$f(x) = 1 \pm x^n$

$\because \quad f(2) = -31$

$\Rightarrow \quad 1 - 2^n = -31$

$\Rightarrow \quad 2^n = 32$

$\Rightarrow \quad n = 5$

$\therefore \quad f(x) = 1 - x^5$

$f'(x) = -5x^4$

$\therefore \quad f'(3) = -405$

8. **(B)**

$3^{((4\log_9 7)-1)}$

$= 3^{\left(\left(\frac{4}{2}\log_3 7\right)-\log_3 3\right)}$

$= 3^{((\log_3 49)-\log_3 3)}$

$= \dfrac{3^{\log_3 49}}{3^{\log_3 3}} = \dfrac{49}{3}$

9. **(B)**

$\log_3 \log_2 \log_{\sqrt{5}}(5^4)$

$= \log_3 \log_2 (8\log_5 5)$

$= \log_3 (3\log_2 2) = \log_3 3 = 1$

10. **(C)**

$p(x) = -(x-1)(x-2)(x-3)(x-4) + (2x+1)$

$\therefore \; p(0) = -(-1)(-2)(-3)(-4) + 1 = -23$