JEE-MAIN 2025

Session - 2

Mock Practice Test - 6

PHYSICS

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. A man grows into a giant such that his linear dimension increase by a factor of 3. Assuming that his density remains same, the stress in the leg will change by factor of

1. 10 **2.** $\frac{1}{3}$ **3.** 3 **4.** $\frac{1}{81}$

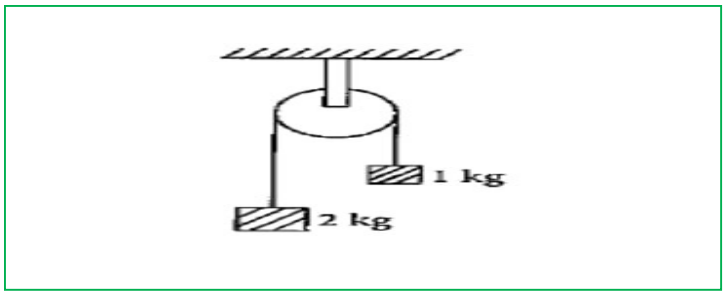
2. Two wires of the same material and length but diameter in the ratio 1:2 are stretched by the same load. The ratio of elastic potential energy per unit volume for the two wires is

1. 1:1 **2.** 2:1 **3.** 4:1 **4.** 16:1

3. A steel wire of length 1m has cross sectional area 1 cm^2. If Young's modulus of steel is $10^{11}\ {}^{N}\!/_{m^2}$, then force required to increase the length of wire by 1mm will be

1. 10^{11}N **2.** 10^{7}N **3.** 10^{4}N **4.** 10^{2}N

4. Two blocks of masses 1kg and 2kg are connected by a metal wire going over a smooth pulley as shown in figure. The breaking stress of metal is $\frac{40}{3\pi}\times 10^{6}\ {}^{N}\!/_{m^2}$. If $g = 10 m/s^2$ then the minimum radius of the wire used, if it is not to break, is



1. 0.5 mm **2.** 1mm **3.** 1.5 mm **4.** 2m

5. If 's' is stress and y is young's modulus of material of a wire, then the energy stored in the wire per unit volume is

1. $\frac{s}{2y}$ **2.** $\frac{2y}{s^2}$ **3.** $\frac{s^2}{2y}$ **4.** $2s^2y$

6. **ASSERTION (A):** Steel is more elastic than rubber.
REASON (R): Under given deforming force, steel is deforming less than rubber.
1. Both assertion and reason are true and the reason is the correct explanation of the assertion.
2. Both assertion and reason are true but reason is not the correct explanation of the assertion
3. Assertion is true but reason is false
4. Assertion and reason both are false

7. A pressure-pump has a horizontal tube of cross-sectional area $10^{-3}\,m^2$ for the outflow of water at a speed of $25\,{}^{m}\!/_{s}$. The force exerted on the vertical wall just in front of the tube which stops water horizontally flowing out of the tube is [density of water $=1000\,{}^{kg}\!/_{m^3}$]

1. 125 N **2.** 625 N **3.** 500 N **4.** 400 N

8. **Statement-I:-** Spraying of water causes cooling.
Statement-II:- For an isolated system, surface energy increases at the expense of internal energy.
1. Statement-I is true, statement-II is true, statement-II is correct explanation for statement-I
2. Statement-I is true, statement-II is true, statement-II is not correct explanation for statement-I
3. Statement-I is true, statement-II is false
4. Statement-I is false, statement-II is true

9. Match column- I with column- II and select the correct answer using the codes given below the lists.

Column I	Column II
A) Young's modulus of elasticity	P) $-\frac{\Delta r/r}{\Delta l/l}$
B) Bulk modulus	Q) $3K(1-2\sigma)$
C) Shear modulus	R) $\frac{Y}{2(1+\sigma)}$
D) Poisson's Ratio	S) $\frac{Y\eta}{9\eta-3Y}$

1. A-P,B-S,C-R,D-Q **2.** A-Q,B-S,C-R,D-P

3. A-Q,B-S,C-P,D-R **4)** A-Q,B-P,C-S,D-P

10. An ideal fluid flows through a pipe of circular cross-section made of two section with diameters 2.5cm and 3.75cm. The ratio of the velocities in the two pipes is

1. 9: 4 **2.** 3 :2 **3.** $\sqrt{3}:\sqrt{2}$ **4.** $\sqrt{2}:\sqrt{3}$

11. If the terminal speed of a sphere of gold (density= $19.5\,kg/m^3$) is $8\,m/s$ in a viscous liquid (density = $1.5\,kg/m^3$).Find the terminal speed of a sphere of silver (density = $10.5\,kg/m^3$) of the same size in the same liquid

1. $0.4\,m/s$ **2.** $4\,m/s$ **3.** $0.1\,m/s$ **4.** $0.2\,m/s$

12. Water is conveyed through a uniform tube of 8cm in diameter and 3140m in length at the rate of $2\times10^{-3}\,m^3$ per second. The pressure required to maintain the flow is (viscosity of water = 10^{-3} SI unit)

1. $6.25\,N/m^2$ **2.** $0.625\,N/m^2$ **3.** $6250\,N/m^2$ **4.** $0.00625\,N/m^2$

13. **Statement-I :-** The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Statement-II :- In any steady flow of an incompressible fluid , the volume flow rate of the fluid remains constant

1. Statement-I is true, statement-II is true, statement-II is correct explanation for statement-I

2. Statement-I is true, statement-II is true, statement-II is not correct explanation for statement-I

3. Statement-I is true, statement-II is false

4. Statement-I is false, statement-II is true

14. A tank full of water has a small hole at its bottom. If one -fourth of the tank is emptied in t_1 seconds and the remaining three fourth of the tank is emptied in t_2 second, then ratio $\left(\frac{t_1}{t_2}\right)=$

1. $\sqrt{3}$ **2.** $\sqrt{2}$ **3.** $\frac{2-\sqrt{2}}{\sqrt{2}}$ **4.** $\frac{2-\sqrt{3}}{\sqrt{3}}$

15. An aircraft of mass 4×10^5 kg with total wing area $500m^2$ in level flies at a speed of 720 kmh^{-1} . The density of air at that height is $1.2kg/m^3$. The fractional increase in the speed of the air on the upper surface of its wings relative to the lower surface is: (Take g=10m/s^2)

1. 0.04 **2.** 0.08 **3.** 0.17 **4.** 0.34

16. A stone of relative density 'k' is released from rest on the surface of a lake. If viscous effect are ignored, the stone in water has an acceleration of

1. $g(1-k)$ **2.** $g(1+k)$ **3.** $g\left(1-\frac{1}{k}\right)$ **4.** $g\left(1+\frac{1}{k}\right)$

17. The diameter of drop of water is 0.2 cm. The work done in breaking one drop into 1000 equal droplets will be (surface tension of water = 7×10^{-2} N/m)

1. $7.9\times10^{-6}J$ **2.** $5.92\times10^{-6}J$ **3.** $2.92\times10^{-6}J$ **4.** $1.92\times10^{-6}J$

18. **Statement-I:-** While blowing a soap bubble, to increase the size of soap bubble, we have to increase the air pressure within the soap bubble.

Statement-II:- To increase the size of soap bubble, more air has to be pushed into the bubble.

1) Both statement-I, and statement-2 are true and statement 2 is the correct explanation of statement-I.

2) Both statement-I and statement-2 are true but statement 2 is not the correct explanation of statement-I.

3) Statement-I is true but statement-2 is false**.**

4) Statement –I is false but statement-2 is true**.**

19. Water rises in a capillary up to a height of 4 cm. If it is tilted to 30^0 from the vertical, then the length of water column in it will be.

1. $\frac{8}{\sqrt{3}}$ cm **2.** $8\sqrt{3}cm$ **3.** 4 cm **4.** 2cm

20. A thin liquid film of thickness 2×10^{-4} m is formed between two glass plates of surface area $3\times10^{-2}\ m^2$. If surface tension of liquid is 60×10^{-2} N/m, then force required to separate the plate is

1. 60 N **2.** 120 N **3.** 180N **4.** 240 N

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

21. A body of mass m=10 kg is attached to one end of a wire of length 0.3m. The maximum angular speed (in rad/s) with which it can be rotated about its other end in space station is (Breaking stress of wire=$4.8\times10^7\ N/m^2$) and area of cross- section of the wire =$10^{-2}cm^2$) is:

22. A steel rod has a radius of 10mm and a length of 4m. A force of 94.2 kN stretches it along its length. Young's modulus of steel is $3\times10^{11}\ N/m^2$. The longitudinal strain produced in the wire is -$\times10^{-5}$. ($\pi = 3.14$)

23. A rope of rubber of density $1.5\times10^{3}\ kg/m^{3}$ and young's modulus $5\times10^{6}\ N/m^{2}$, 8m in length is hung from the ceiling of a room. Then the increase in length due to its own weight is -$\times10^{-3}$ m. $(g=10m/s^{2})$

24. A liquid of density 800 kg/m^{3} flows smoothly through a horizontal pipe that tapers in cross – sectional area from $A_1=1.2\times10^{-2}\ m^{2}$ to $A_2=\frac{A_1}{2}\ m^{2}$. The pressure difference between the wide and narrow sections of the pipe is 4800Pa. The rate of flow of liquid is -$\times10^{-3}\ m^{3}/s$.

25. The energy stored in a stretched spring is 36 J and spring constant 800 N/m, then the elongation in the spring is $\underline{n}\times10^{-1}$ (meters). n=

26. A soap bubble in vacuum has a radius of 3cm and another soap bubble in vacuum has a radius of 4cm. If the two bubbles coalesce under isothermal conditions, then the radius of the new bubble is-(cm)

27. A 20cm long capillary tube is dipped in water. The water rises up to 8cm. If the entire arrangement is put in a freely falling elevator, then the length of water column in capillary tube will be-(cm)

28. The pressure inside two soap bubbles is 1.01 and 1.02 atmosphere respectively. The ratio of their volumes is (atmospheric pressure = 1 atm) $\frac{v_1}{v_2}=2^{n}$. The value of n is------

29. The gauge pressure at the bottom of a lake, due to water is $4.9\times10^{6}\ N/m^{2}$. The depth of the lake is–(meters)

30. The reading of pressure meter attached with a closed water pipe is $3.5\times10^{5}\ N/m^{2}$. On opening the valve of the pipe, the reading of pressure meter is reduced to $3\times10^{5}\ N/m^{2}$. The speed of water (in m/s) flowing in the pipe is [density of water $\rho_w=1000\ kg/m^{3}$]

CHEMISTRY

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. A vessel has N_2 gas saturated with water vapor at a total pressure of 1 atm. The partial pressure of water vapor is 0.3atm. The contents of this vessel are completely transferred to another vessel having one third of the capacity of the original volume, at the same temperature. The total pressure of this system in the new vessel is

1. 3.0atm **2.** 1atm **3.** 3.3atm **4.** 2.4atm

32. When an open vessel is heated from 300K to 500K, What percentage of the gas will escape out?

1. 20 **2.** 60 **3.** 40 **4.** 80

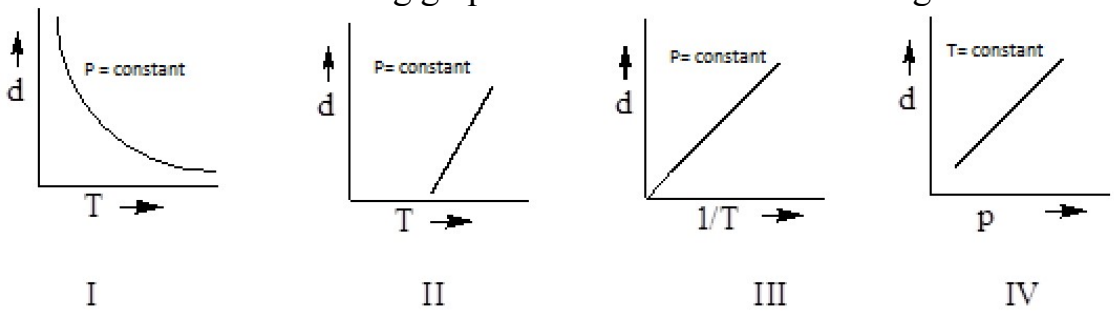
33. At 400K, the RMS speed of gas X (MW = 40) is equal to the most probable speed of gas Y at 60K. The molecular mass of the gas Y is

1. 40 **2.** 20 **3.** 30 **4.** 4

34 A uniform glass tube of 100cm length is connected to a bulb containing hydrogen at one end and another bulb containing oxygen at the other end at the same temperature and pressure. The two gases meet for the first time at the following distance from the oxygen end.

1. 80cm **2.** 50cm **3.** 20cm **4.** 6.66cm

35. Which one of the following graphs is not correct for an ideal gas ?



I II III IV

d = density; p = pressure, T = temperature

1. II **2.** III **3.** I **4.** IV

36. At very high pressure, the Vander Waals equation reduces to

1. $PV = RT - Pb$ **2.** $PV = \frac{aRT}{V^2}$ **3.** $P = \frac{RT}{(V-b)}$ **4.** $PV = RT - \frac{a}{V}$

37. Among the following gaseous elements with atomic numbers which will have the greatest rate of diffusion under similar conditions?

1. $Z = 7$ **2.** $Z = 8$ **3.** $Z = 17$ **4.** $Z = 10$

38. **Assertion (A):** Compressibility factor (Z) for non-ideal gases can be greater than or less than 1

Reason (R) : Non ideal gases always exert higher pressure than expected

1. Both (A) and (R) are correct but (R) is not the correct explanation of (A)

2. (A) is correct but (R) is not correct

3. (A) is not correct but (R) is correct

4. Both (A) and (R) are correct but (R) is the correct explanation of (A)

39. **Statement I:** SO_2 gas is easily liquifiable, while H_2 is not

Statement II: SO_2 has low critical temperature, while H_2 has high critical temperature.

1. Both Statement I and statement II are incorrect

2. Statement I is correct, and statement II is incorrect

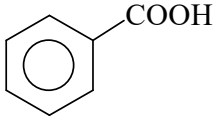
3. Statement I is incorrect, but statement II is correct

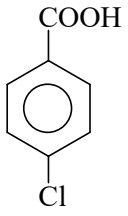
4. Both Statement I and Statement II are correct

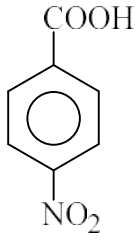
40. How much should be the decrease in pressure to increase the volume of a given mass of a gas by 10%?

1. 9.1% **2.** 1.9% **3.** 10% **4.** 6.5%

41. The decreasing order of acidity of the following carboxylic acids

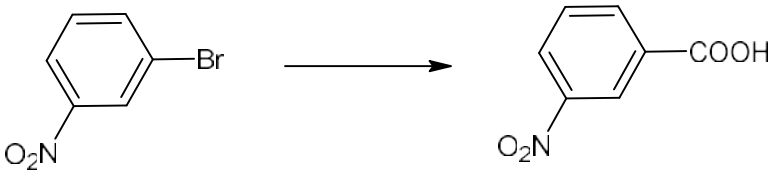
A. CH_3COOH **B.** 

C. 

D. 

1. A > B > C > D **2.** B > C > A > D **3.** D > C > B > A **4.** B > C > D > A

42.

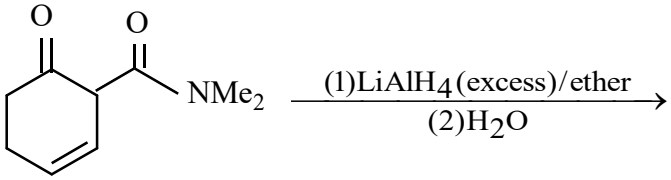


For the above conversion, the correct sequence of reagents used are :

1. $Mg / ether, H_3O^+, CO_2$ **2.** $Na / ether, H_3O^+, CO_2$

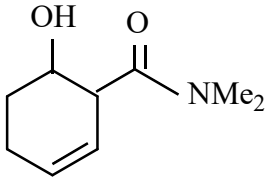
3. $Mg / ether, CO_2, H_3O^+$ **4.** $Li - Cu / ether, H_3O^+, CO_2$

43.

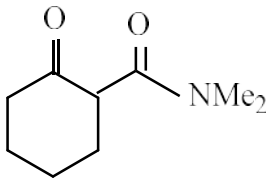


major product of the given reaction is.

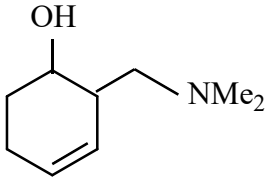
1.



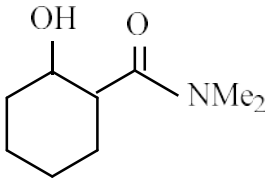
2.



3.



4.



44. **Statement 1: C**arboxylic acids are higher boiling liquids than aldehydes, ketones and even alcohols of comparable molecular mass

Statement 2: Alcohols have strong hydrogen bond than carboxylic acids.

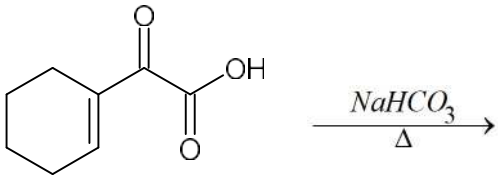
1. Both Statement 1 and Statement 2 are correct

2. Statement 1 correct and Statement 2 is false

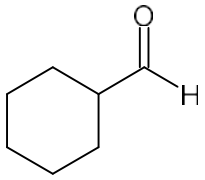
3. Statement 1 is incorrect and statement 2 is correct

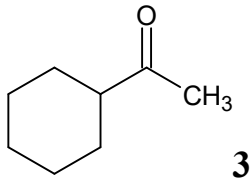
4. Both statements are false

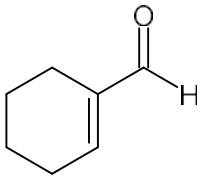
45.

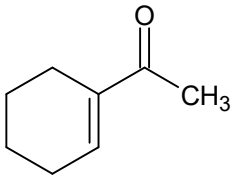


Find the major product of the reaction is :

1. 

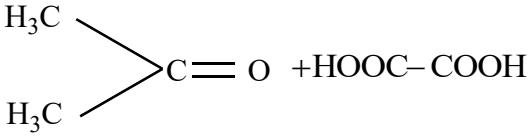
2. 

3. 

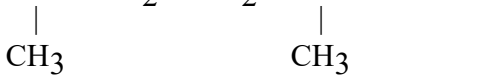
4. 

46. On vigorous oxidation of 2,5-dimethyl hexa -2,4-diene

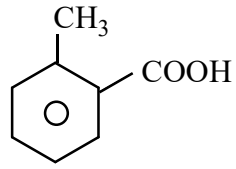
1. $CH_3CHO + CH_3COCH_3$

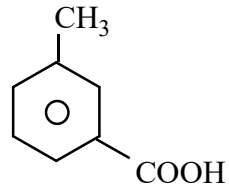
2. 

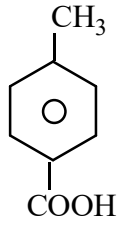
3. 

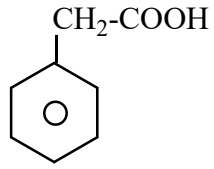
4. 

47. Photo catalyzed mono bromination of toluene gives ‘A’. the compound ‘A’ on treatment with aqueous NaOH gives ‘B’. The compound ‘B’ on treatment with $SOCl_2$ gives ‘C’. The compound ‘C’ is reacted with KCN followed by hydrolysis and heat gives ‘D’. The major product ‘D’ is.

1. 

2. 

3. 

4. 

48. Given below two statements

Statement 1: The esterification of carboxylic acid with alcohol is a nucleophilic acyl substitution.

Statement 2: Electron donating group in the carboxylic acid will increase the rate of esterification.

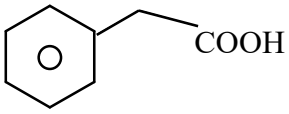
1. Both statement 1 and Statement 2 are correct

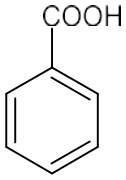
2. Both statement 1 and statement 2 are incorrect

3. Statement 1 is correct, but Statement 2 is incorrect

4. Statement 1 is incorrect but Statement 2 is correct

49. Which of the following does not give HVZ reaction.

A. 

B. HCOOH **C.** CH_3COOH **D.** 

The correct option is

1.A and C **2.** B and D **3.** C and D **4.** A and D

50. $RCOOH \xrightarrow[H_3O^+]{B_2H_6} X$ major product 'X' is

1. RCH_3 **2.** R – H **3.** RCOONa **4.** RCH_2OH

(NUMERICAL VALUE TYPE)

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Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

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51. For a gas obeying P(V−b)=RT a graph is plotted between Volume (V lit.) on Y-axis and Temperature (K) on X-axis at a constant pressure of 0.04105 atm. Find the slope of the line?

52. Two grams of hydrogen diffuse from a container in 10 minutes. How many grams of oxygen would diffuse through the same container in the same time under similar conditions

53. The temperature (in K) at which root mean square velocity of SO_2 gas is double of the root mean square velocity of O_2 gas at 27°C is

54. A mixture contains 16g oxygen, 28g of nitrogen and 8g of methane total pressure of the mixture is 740 mm. What is the partial pressure of nitrogen in mm?

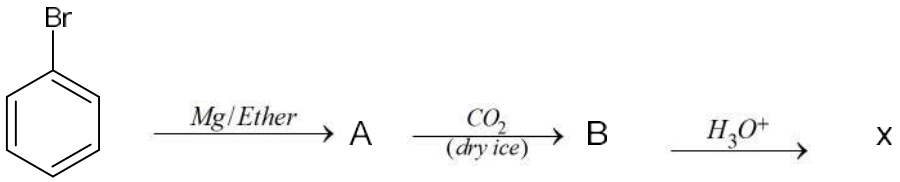
55. Under critical states for one mole of gas, Compressibility factor is $\frac{x}{y}$ then x+y is

56. How many of the following compounds are weakly acidic than $Cl_2CHCOOH$

1. CF_3COOH **2.** CCl_3COOH **3.** O_2N-CH_2COOH **4.** $NC-CH_2COOH$

5. FCH_2COOH **6.** $ClCH_2CH_2COOH$ **7.** $PhCOOH$

57.



Number of π bonds in one molecule of compound 'x' is ……

58. How many of the following statements are correct

a. Aldehydes have a higher boiling point than carboxylic acid.

b. Hydrolysis of methyl isocyanide gives acetic acid.

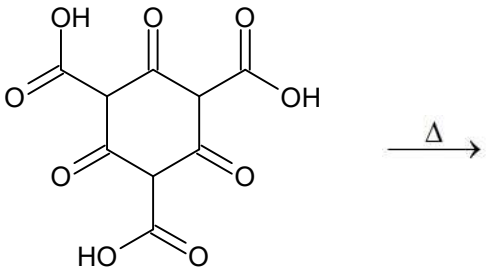
c. Aliphatic carboxylic acids up to nine carbon atoms are colorless liquids at room temperature

d. The higher acids are wax like solids (> 9 carbons)

e. Carboxylic acid reacts with Na_2CO_3 and liberated CO_2 gas.

f. Carboxylate ion is resonance stabilized.

59. Total number of moles of CO_2 released from 1 mole of compound in the following process



60. The volume of H_2 (in litre) liberated at STP when excess of sodium reacts with 108.92gm of benzoic acid

MATHEMATICS

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

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61. If $P=\begin{pmatrix}1 & 0\\ \frac{1}{2} & 1\end{pmatrix}$ then $P^{2022}=$

1. $\begin{pmatrix}1 & 0\\ 2022 & 1\end{pmatrix}$ **2.** $\begin{pmatrix}1 & 0\\ 2011 & 1\end{pmatrix}$ **3.** $\begin{pmatrix}1 & 0\\ 1011 & 1\end{pmatrix}$ **4.** $\begin{pmatrix}1 & 0\\ 1000 & 1\end{pmatrix}$

62. $P=\begin{pmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ 9 & 3 & 1\end{pmatrix}$ $Q=\left(q_{ij}\right)_{3\times 3}$ Matrices such that $Q-P^5=I_3$ then $\frac{q_{21}+q_{31}}{q_{32}}=$.....

1. 10 **2.** 15 **3.** 9 **4.** 135

63. If A and B two squares matrices of order $n\times n$ such that $A^2-B^2=(A-B)(A+B)$ then which of the following will be always true

1. $A=B$ **2.** $AB=BA$

3. Either $A(\text{or})B$ zero matrix **4.** Either A (or) B identity matrix

64. $A=\begin{pmatrix}1\\ 1\\ 1\end{pmatrix}$ $B=\begin{pmatrix}9^2 & -10^2 & 11^2\\ 12^2 & 13^2 & -14^2\\ -15^2 & 16^2 & 17^2\end{pmatrix}$ then value of $A'BA$

1. $[1224]$ **2.** $[1042]$ **3.** $[540]$ **4.** $[539]$

65. **Statement I:** A matrix $A=\left(a_{ij}\right)_{3\times3}$ such that $a_{ij}=\frac{i-j}{\hat{i}+2\hat{j}}$ cannot be expressed as sum of symmetric and skew symmetric matrix

Statement 2: A matrix = $\left(a_{ij}\right)_{3\times3}$ such that $a_{ij}=\frac{i-j}{\hat{i}+2\hat{j}}$ neither symmetric (nor) skew symmetric

1. Both Statements are true and statement 2 is correct explanation of statement I
2. Both statement are true but statement 2 is not a correct explanation of statement I
3. Statement I is true and statement 2 is false
4. Statement I is false Statement 2 is true

66. $A=\begin{pmatrix}1&0&1\\0&2&2\\1&1&3\end{pmatrix}$ then $t_r\left(A^3\,\text{adj}(A^2)\right)=.....$ Where $\text{tr}(A)$ denotes trace of A $\text{adj}A$ denotes adjoint of matrix A

1. 6 **2.** 24 **3.** 36 **4.** 12

67. $3A=\begin{pmatrix}1&2&2\\2&1&-2\\x&2&y\end{pmatrix}$ and $A.A^T=I$ then $x+y$

1. -1 **2.** 1 **3.** 2 **4.** -3

68. **If** $\theta=\frac{\pi}{5}$ and $A=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}$ and $B=A+A^4$ then $\det B$

1. 0 **2.** lies in $(2,3)$ **3.** 4 **4.** lies in $(1,2)$

69. $\begin{vmatrix}x^2+x&x+1&x-2\\2x^2+3x-1&3x&3x-3\\x^2+2x+3&2x-1&2x-1\end{vmatrix}=ax-12$ then $a=.....$

1. 24 **2.** -12 **3.** -24 **4.** 12

70. The solutions of the equation $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0 \ (0 < x < \pi)$ are

1. $\frac{\pi}{6}, \frac{5\pi}{6}$ **2.** $\frac{\pi}{12}, \frac{\pi}{6}$ **3.** $\frac{5\pi}{12}, \frac{7\pi}{12}$ **4.** $\frac{7\pi}{12}, \frac{11\pi}{12}$

71. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$ if $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$ then k

$= \ldots.$

1. 1 **2.** $-z$ **3.** z **4.** -1

72. **Statement 1:** $\left|\text{adj}\left(\text{adj}\left(\text{adj}A\right)\right)\right| = |A|^{(n-1)^3}$ where n is order of A

Statement 2: $|\text{adj}\, A| = |A|^n$

1. Both statement are true and statement 2 is correct explanation of statement 1

2. Both statement are true and statement 2 is incorrect explanation of statement 1

3. Statement 1 is true statement 2 is false

4. Statement 1 is false statement 2 is true

73. $\begin{vmatrix} x & 1 & 1 \\ 2 & x & 3 \\ x & -1 & 2 \end{vmatrix} = 4$ where x is a complex number then the value of $\begin{vmatrix} 2x+3 & -3 & 3-x \\ 3x-4 & x & 2-3x \\ -x^2-2 & 2x & x^2-2 \end{vmatrix} = ...$

1. 64 **2.** 16 **3.** 8 **4.** 10

74. for $n \in N$ $\Delta_n = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$ then $\lim\limits_{n\to\infty} \dfrac{\left(3n^3-5\right)\Delta_n}{\Delta_{n+1}} = ...$

1. zero **2.** $\frac{5}{2}$ **3.** Does not exists **4.** 3

75. Let A be 3×3 matrix such that $A\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ then $A^{-1} =$

1. $\begin{pmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{pmatrix}$ **2.** $\begin{pmatrix} 3 & 2 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix}$ **3.** $\begin{pmatrix} 0 & 1 & 3 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}$ **4.** $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 3 \end{pmatrix}$

76. Statement 1: If the matrices $A, B, (A+B)$ are non – singular then

$\left[A(A+B)^{-1}B\right]^{-1} = B^{-1} + A^{-1}$

Statement 2: $\left[A(A+B)^{-1}B\right]^{-1} = \left(A\left(A^{-1}+B^{-1}\right)B\right)^{-1} = \left(\left(I+AB^{-1}\right)B\right)^{-1} = (B+AI)^{-1} = B^{-1}+A^{-1}$

1. Both statement are true and statement 2 is correct explanation of statement 1

2. Both statement are true and statement 2 is incorrect explanation of statement 1

3. Statement 1 is true statement 2 is false

4. Statement 1 is false statement 2 is true

77. If the system of equations $x-2y+z=4;\ x+y+\lambda z=4;\ 2x-y+2z=2$ has no solution 'λ' lies in

1. $\left(0,\frac{3}{4}\right)$ **2.** $\left(\frac{3}{4},\frac{5}{4}\right)$ **3.** $\left(\frac{5}{4},2\right)$ **4.** $(2,4)$

78. A variable matrix $V=\begin{pmatrix} x & y & z \\ y & z & x \\ z & x & y \end{pmatrix}$ such that $VA=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ where $A=\begin{pmatrix} \log a \\ \log b \\ \log c \end{pmatrix}$ and a, b, c distinct +ve numbers in G.P. If (x, y, z) is solution of VA = 0 lie on line $\frac{x}{\alpha}=\frac{y}{\beta}=\frac{z}{\gamma}$ is $(x,y,z\neq 0)$ then $\frac{\alpha}{\beta}+\frac{\beta}{\gamma}+\frac{\gamma}{\alpha}$

1. 3 **2.** 4 **3.** 5 **4.** 7

79. The system of equations $2x+y-z=7;\ x-3y+2z=1,\ x+4y+\lambda z=k$ where $\lambda,k\in R$ has infinitely many solutions then $-\lambda+k=$.....

1. 9 **2.** 10 **3.** 4 **4.** –9

80. The system of equations $-kx+3y-14z=25;\ -15x+4y-kz=3;\ -4x+y+3z=4$ has unique solution then for all k in the set

1. R **2.** $R-\{-11,13\}$ **3.** $R-\{13\}$ **4.** $R-\{-11,11\}$

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**
Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

81. $A=\left(a_{ij}\right)_{4\times4}$ such that $\begin{matrix} a_{ij}=2 \text{ for } i=j \\ =0 \quad \text{for } i\neq j \end{matrix}$ then $\left\{\frac{\det\left(\text{adj}(\text{adj}A)\right)}{7}\right\}=\lambda$ then $7\lambda=.....\text{where}\{.\}$ represents fractional part function

82. If the equation $x+y+z=6;\ x+2y+3z=10,\ 2x+4y+\lambda z=24$ has no solution and product of roots of $f(y)=\begin{vmatrix} 2y^2 & -c \\ 1 & \lambda \end{vmatrix}=0$ is 12 then value of c = …..

83. If equations $(-\sin\theta)x+y=0\ ;\ x+\sin\theta=0,\ (\sin\theta)x+y+1=0$ are concurrents then no. of solutions for θ in $(0,2\pi)$

84. A is a square matrix of order 3 such that $|A|=2$ then $\left|\left(\text{adj}A^{-1}\right)^{-1}\right|=.....$

85. If $A=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $\det\left(A^n-I\right)=1-\lambda^n\ n\in N$ then value of λ is……

86. $\text{adj}A=\begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$ and matrix $A^{-2}=\left(x_{ij}\right)_{3\times3}$ where $1\le i,\ j\le3$ then $\frac{1}{4}\left(\sum_{j=1}^{3}\sum_{i=1}^{3}x_{ij}\right)=.....$

87. The sum of determinants $\begin{vmatrix} ax+xy & a^2+xb & ab+xc \\ bx+y^2 & ab+yb & b^2+yc \\ cx+yz & ac+zb & bc+cz \end{vmatrix}+\begin{vmatrix} 1 & -1 & 1 \\ \cos\theta & \cos\theta & \sin\theta \\ -\sin\theta & -\sin\theta & \cos\theta \end{vmatrix}=.....$

88. Let $A=\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{pmatrix}$ be zero of $f(x)=x^2-4x-5$. The sum of all elements of matrix A^3

89. If α be a root of the equation $(a-c)x^2+(b-a)x+c-b=0$ a, b, c are different real number such that matrix $\begin{pmatrix}\alpha^2 & \alpha & 1\\ 1 & 1 & 1\\ a & b & c\end{pmatrix}$ is singular then value of

$$\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)} \text{ is } k$$

Then $30k$ is

90. Let M be a square matrix of order 3 such that $MM^T=I$ and $M^2=I$. Also $M^{-1}+\text{adj}(M)=0$ If P is another matrix such that $P+2M=0$ then $\det\left(PP^TP^{-1}\right)=.....$

KEY SHEET

PHYSICS

1)	**3**	2)	**4**	3)	**3**	4)	**2**	5)	**3**
6)	**1**	7)	**2**	8)	**1**	9)	**2**	10)	**1**
11)	**2**	12)	**3**	13)	**1**	14)	**4**	15)	**3**
16)	**3**	17)	**1**	18)	**4**	19)	**1**	20)	**3**
21)	**4**	22)	**100**	23)	**96**	24)	**24**	25)	**3**
26)	**5**	27)	**20**	28)	**3**	29)	**500**	30)	**10**

CHEMISTRY

31)	**4**	32)	**3**	33)	**4**	34)	**3**	35)	**1**
36)	**3**	37)	**4**	38)	**2**	39)	**2**	40)	**1**
41)	**3**	42)	**3**	43)	**3**	44)	**2**	45)	**3**
46)	**2**	47)	**4**	48)	**3**	49)	**2**	50)	**4**
51)	**2**	52)	**8**	53)	**2400**	54)	**370**	55)	**11**
56)	**5**	57)	**4**	58)	**4**	59)	**3**	60)	**10**

MATHEMATICS

61)	**3**	62)	**1**	63)	**2**	64)	**4**	65)	**4**
66)	**2**	67)	**4**	68)	**4**	69)	**1**	70)	**4**
71)	**2**	72)	**3**	73)	**2**	74)	**4**	75)	**1**
76)	**3**	77)	**2**	78)	**1**	79)	**1**	80)	**4**
81)	**1**	82)	**144**	83)	**4**	84)	**4**	85)	**2**
86)	**3**	87)	**2**	88)	**375**	89)	**90**	90)	**8**

SOLUTIONS

PHYSICS

1. **Stress** $= \frac{force}{area} = \frac{f}{l \times b} = \frac{\rho v a}{l \times b}$

$$s^1 = \frac{\rho v a}{3l \times 3b} = \frac{3^3 \rho a}{3l \times 3b} = \frac{3\rho a}{Lb} \qquad \frac{s^1}{s} = \frac{\frac{3\rho a}{Lb}}{\frac{\rho a}{Lb}} = 3$$

Hence stress in leg will change by factor of '3'

2. PE$= \frac{1}{2}\frac{(stress)^2}{y}$

$U \alpha (stress)^2$

$$\frac{U_1}{U_2} = \frac{(F/A_1)^2}{(F/A_2)^2} = \left(\frac{A2}{A1}\right)^2 = \left(\frac{D_2}{D_1}\right)^4 = \left(\frac{2}{1}\right)^4 = \frac{16}{1}$$

3. L=1m, A= $10^{-4} m^2$, $\Delta L = 1 \times 10^{-3} m$

$$\Delta L = \frac{FL}{AY}$$

$$F = AY\frac{\Delta L}{L} = 10^{-4} \times 10^{11} \times \frac{10^{-3}}{1} = 10^4 N.$$

4. $$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g = \left(\frac{2 \times 1 \times 2}{1+2}\right)g = \frac{4}{3} \times g = \frac{40}{3}$$

Breaking stress $= \frac{T}{A} = \frac{40/3}{\pi r^2}$

$$\therefore \frac{40}{3\pi} \times 10^6 = \frac{40}{3\pi r^2}$$

$$r^2 = \frac{1}{10^6}$$

$$r = \frac{1}{10^3} = 10^{-3} = 1mm$$

5. $$\frac{E}{V} = \frac{1}{2} stress \times strain = \frac{1}{2} s \times \frac{S}{Y} = \frac{s^2}{2Y}$$

6. Conceptual

7. Momentum transformed by water per sec = $\rho AV \times V$

$F = \rho_w A v^2 = 10^3 \times 10^{-3} \times (25)^2 = 625N$

8. Conceptual

9. 1. Young's modulus $y = 3k(1-2\sigma)$

2. $y = \dfrac{9\eta k}{3k+\eta}$

Bulk modulus $k = \dfrac{y\eta}{9\eta - 3y}$

3. shear modulus $\eta = \dfrac{y}{2(1+\sigma)} \Rightarrow y = 2\eta(1+\sigma)$

4. poisson's ratio $\sigma = \dfrac{-\Delta r / r}{(\Delta l / l)}$

10. According to equation of continuity

$$A_1V_1 = A_2V_2 \Rightarrow \frac{V_1}{V_2} = \frac{A_2}{A_1} = \frac{\pi D2^2/4}{\pi D1^2/4} = \left(\frac{D_2}{D_1}\right)^2 \Rightarrow \frac{V_1}{V_2} = \left(\frac{3.75}{2.5}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

11. Terminal speed of spherical body in a viscous liquid is given by

$$V_T = \frac{2r^2(\rho-\sigma)}{9\eta}g$$

$$\therefore \frac{v_T(Ag)}{v_T(gold)} = \frac{\rho_{Ag}-\sigma_1}{\rho_{gold}-\sigma_1} = \left|\frac{10.5-1.5}{19.5-1.5}\right. \Rightarrow V_T(Ag) = \frac{9}{18}\times 8 = 4m/s$$

12. radius $r = 4cm = 0.04m$

$lengthL = 3140m$

$\eta = 10^{-3}$

Rate of water $Q = 2\times 10^{-3} m^3$

According to poisunllie's equation

$$Q = \frac{\pi \Pr^4}{8\eta L} \Rightarrow P = \frac{2\times 10^{-3}\times 8\times 10^{-3}\times 3140}{3.14\times(0.04)^4}$$

$$= 6250 \ \ N/m^2$$

13. As the water falls down words, its velocity increases there fore its area will decrease

According to equation of continuity Q=AV=constant

14. $T = \dfrac{2A}{a\sqrt{2g}}\left[\sqrt{H} - \sqrt{h}\right]$

$$t_1 = \frac{2A}{a\sqrt{2g}}\left[\sqrt{H} - \sqrt{\frac{3H}{4}}\right] \quad : t_2 = \frac{2A}{a\sqrt{2g}}\left[\sqrt{\frac{3H}{4}} - 0\right]$$

$$so: \ \frac{t_1}{t_2} = \frac{2-\sqrt{3}}{\sqrt{3}}$$

15. The weight of the air croft is balanced by the upward force due to the pressure difference

$$\Delta P = \frac{mg}{A} = \frac{4\times10^5\times10}{500} = \frac{4}{5}\times10^4 \quad N/m^2$$

By the beroulli's theorem

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\Delta p = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$\Delta p = \frac{1}{2}\rho\left(v_2 + v_1\right)\left(v_2 - v_1\right)$$

$$\Delta p = \rho v_{av}\left(v_2 - v_1\right) \Rightarrow v_2 - v_1 = \frac{\Delta p}{\rho v_{av}}$$

$$v_{av} = \frac{v_1 + v_2}{2} = 720km/n = 200m/s \Rightarrow \frac{v_2 - v_1}{v_{av}} = \frac{\Delta p}{\rho\left(v_{av}\right)^2} = \frac{\frac{4}{5}\times10^4}{1.2\times\left(200\right)^2} = 0.17$$

16. $k = \dfrac{\rho_s}{\rho_w}$

$$Fnet = mg - F_B \qquad v\rho_w g = \frac{v\rho_s g}{k} = \frac{mg}{k}$$

$$Fnet = mg - \frac{mg}{k}$$

$$Fnet = mg\left(1 - \frac{1}{k}\right)$$

$$ma = mg\left(1-\frac{1}{k}\right)$$

$$a = g\left(1-\frac{1}{k}\right)$$

17. $w = 4\pi r^2 T\left[n^{1/3}-1\right] : n = 1000$

$$w = 7.9\times10^{-6} J$$

18. Conceptual

19. $\cos\theta = {}^{h}/_{l}$

20. $$F = PA = \frac{2T}{\Delta x}A$$

$$F = \frac{2\times60\times10^{-2}}{2\times10^{-4}}\times3\times10^{-2} = 180N$$

21. $$Breakingstress = \sigma = \frac{F}{A}$$

$$\sigma A = \frac{mv^2}{L}$$

$$\sigma A = MLw^2$$

$$w = \sqrt{\frac{\sigma A}{Ml}} = \sqrt{\frac{4.8\times10^7\times10^{-4}\times10^{-2}}{10\times0.3}}$$

$$w = \sqrt{\frac{48}{3}} = \sqrt{16} = 4\,{}^{red}/_{s}$$

22. $$strain = \frac{stress}{y} = \frac{F}{AY} : r = 10\times10^{-3} m$$

$$= \frac{94.2\times10^3}{\pi r^2 y} = \frac{94.2\times10^3}{3.14\times\left(10\times10^{-3}\right)^2\times3\times10^{11}} = \frac{94.2\times10^3}{3.14\times100\times10^{-6}\times3\times10^{11}}$$

$$= \frac{94.2}{9.42\times10^4} = 10\times10^{-4} = 100\times10^{-5}$$

23. $$e = \frac{l^2\rho g}{2y} = \frac{64\times1.5\times10^3\times10}{2\times5\times10^6} = 96\times10^{-3} m$$

24. $p + \frac{1}{2}\rho v^2 = \text{constant}$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$A_1V_1 = A_2V_2$$
$$2A_2V_1 = A_2V_2$$
$$V_2 = 2V_1$$

$$\Rightarrow p_1 - p_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$\Delta p = \frac{1}{2}\rho 3v_1^2$$

$$v_1^2 = \frac{2}{3}\frac{\Delta p}{\rho} = \frac{2}{3}\times\frac{4800}{800} = 4$$

$$v_1 = 2\,m/s$$

$$\therefore roteof \quad flow = A_1V_1 = 1.2\times10^{-2}\times2$$

$$= 2.4\times10^{-2}$$

$$= 24\times10^{-3}$$

25. $u = \frac{1}{2}k\times^2$

$$36 = \frac{1}{2}\times800\times^2$$

$$X = 0.3 = 3\times10^{-1}$$

26. $p_{net} = p_a + p_b$

$$\frac{4T}{R}\times\frac{4}{3}\pi R^3 = \frac{4T}{r_1}\times\frac{4}{3}\pi r_1^3 + \frac{4T}{r_2}\quad\frac{4}{3}\pi r_2^3 \Rightarrow R^2 = r_1^2 + r_2^2 = 3^2 + 4^2$$

$$R = \sqrt{9+16} = \sqrt{25} = 5cm$$

27. $h = \frac{2T\cos\theta}{rg\rho}$

$$h\alpha\frac{1}{g}$$

For freely fall g=0

$$h\alpha\frac{1}{0} = \infty$$

$so, take L = h = 20cm$

28. $excess \quad pressure \quad \Delta p = \frac{4T}{R}$

$$0.01 = \frac{4T}{R_p} : 0.02 = \frac{4T}{R_2} \Rightarrow \frac{R_1}{R_2} = \frac{2}{1}$$

$$\Rightarrow v \infty R^3 \Rightarrow \frac{v_1}{v_2} = \left(\frac{R_1}{R_2}\right)^3 = \frac{8}{1} = 2^3$$

$n = 3$

29. $pressure \quad p = h\rho g \quad = 4.9 \times 10^6$

$$p_w = 1000 kg / m^3 : g = 9.8 m / s^2$$

$$\therefore h = \frac{p}{\rho g} = \frac{4.9 \times 10^6}{1000 \times 9.8} = 500 m$$

30. $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$

$$p_1 - p_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \qquad \left[\therefore v_1 = 0\right]$$

$$\left(3.5 - 3\right)10^5 = \frac{1}{2} \quad 10^3 v_2^2$$

$$0.5 \times 10^2 = \frac{1}{2} v_2^2$$

$$v_2 = \sqrt{100} = 10 \, m/s$$

CHEMISTRY

31. $P_{total} = P_{N_2} + P_{water\,vapour} \Rightarrow P_{N_2} = 1 - 0.3 = 0.7$ atm

$$P_1 V_1 = P_2 V_2 \ , 0.7 \times V = P_2 \times \frac{V}{3} \Rightarrow V = 2.1 atm$$

Total pressure in the new vessel $= 2.1 + 0.3 = 2.4 atm$

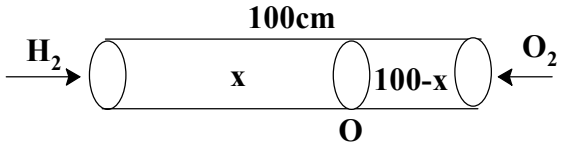
32. for open vessel $n_1T_1 = n_2T_2 \Rightarrow 1\times 300 = n_2 \times 500 \Rightarrow n_2 = \frac{3}{5}$

Number of moles escaped out = $1-\frac{3}{5} = \frac{2}{5} = 0.4$

% of gas escaped out = 40%

33. $\left(\sqrt{\frac{3RT}{M}}\right)_X = \left(\sqrt{\frac{2RT}{M}}\right)_y = \sqrt{\frac{3R\times 400}{40}} = \sqrt{\frac{2R\times 60}{M}} = \sqrt{30} = \sqrt{\frac{120}{M}} = M = 4$

34. $\frac{r_{H_2}}{r_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4$



$\frac{x}{(100-x)} = \frac{4}{1} \Rightarrow 400-4x = x \Rightarrow 5x = 400$

$x = 80cm$

Distance from oxygen end = 20cm

35. $PM = dRT \Rightarrow d\,\alpha\,\frac{1}{T}$

36. Vender waals equation for one mole gas $\left(P+\frac{a}{V^2}\right)(v-b) = RT$ at high pressure $P(v-b) = RT$

$P = \frac{RT}{(v-b)}$

37. $z = 7\ (N_2)\ Mw = 28$

$z = 8\ (O_2)\ Mw = 32$

$z = 17\ (Cl_2)\ Mw = 71$

$z = 10\ (Ne)\ Mw = 20$

$r\alpha\,\frac{1}{\sqrt{M}}$ (M=molar mass of gas)

38. Z can be greater than 1 (or) less than 1 non ideal gases exert less pressure than expected due to inter molecular attraction

39. Greater than T_C value more easily to liquefy

T_c of SO_2 : 630K

T_c of H_2 : 33.2K

40. $p_1 = 100,\ v_1 = 100, v_2 = 110$

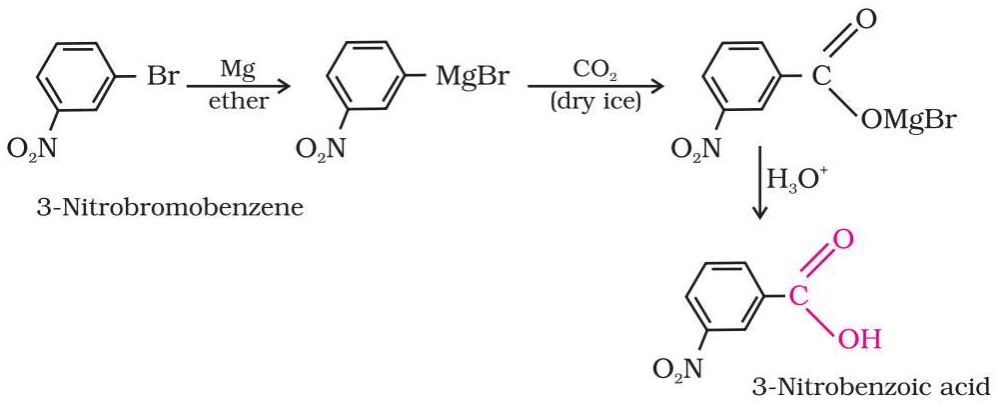
$p_1 v_1 = p_2 v_2$

$$p_2 = \frac{p_1 v_1}{v_2} = \frac{100 \times 100}{110} = 90.9$$

Decrease in pressure $100 - 90.9 = 9.1\%$

41. Aromatic acids are more acidic than CH_3COOH As – M or - I effect increases acidic nature increases

42.



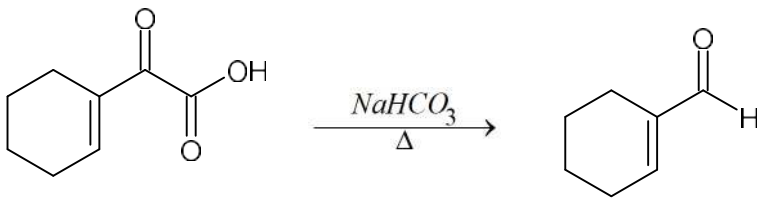
3-Nitrobromobenzene

3-Nitrobenzoic acid

43. $LiAlH_4$ can reduce keto to alcohol and amides to amines

44. Alcohol have weaker hydrogen bond than carboxylic acids

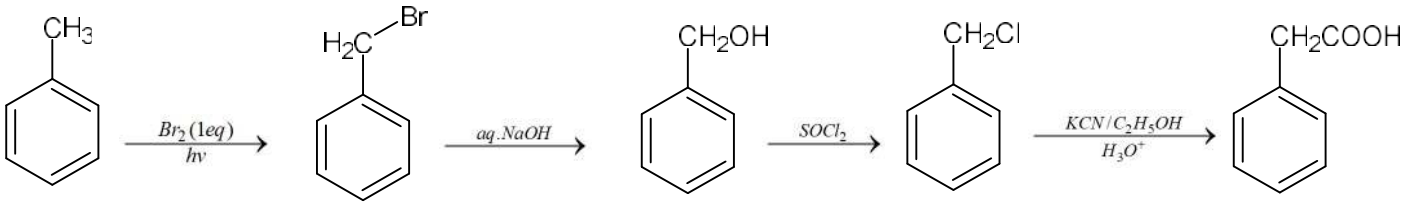
45.



This is a decarboxylation reaction.

46 Gives acetone and oxalic acid

47



48. Esterification is acyl nucleophilic substitution reaction

EDG on carboxylic acid decreases the rate of esterification

49. Carboxylic acids having α – hydrogen gives HVZ reactions

50. Diborane selectively reduce carboxylic acid to alcohols

51.

PV=RT+Pb

$$V = \frac{RT}{P} + b$$

$$\text{Slope} = \frac{R}{P} = \frac{0.0891}{0.04105} = 2$$

52. $$\frac{w_1}{w_2} = \sqrt{\frac{m_1}{m_2}}$$

$$\frac{2}{w_2} = \sqrt{\frac{2}{32}} \Rightarrow w_2 = 8$$

53. $$\left(\sqrt{\frac{3RT}{M}}\right)_{SO_2} = 2\left(\sqrt{\frac{3RT}{M}}\right)_{O_2}$$

$$\left(\frac{T}{M}\right)_{SO_2} = 4\left(\frac{T}{M}\right)_{O_2}$$

$$\frac{T_{SO_2}}{64} = 4 \times \frac{300}{32}$$

$$T_{SO_2} = 2400\,K$$

54. $n_{O_2} = \frac{16}{32} = \frac{1}{2}; n_{n_2} = \frac{28}{28} = 1$

$n_{CH_4} = \frac{8}{16} = \frac{1}{2} \qquad x_{N_2} = \frac{1}{2} = 0.5$

$p_{N_2} = x_{N_2}.P = 370 \text{ mm}$

55. Under critical state

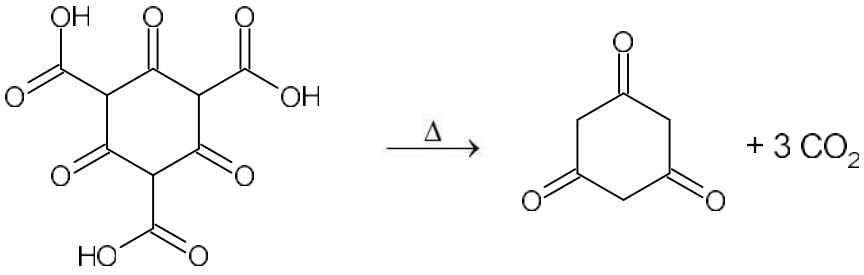
$z = \frac{3}{8} \Rightarrow x + y = 3 + 8 = 11$

56. 3, 4, 5, 6, 7 are less acidic than $Cl_2HC - COOH$

57. X = Benzoic acid

58. c, d, e, f are correct

59.



60. $PhCOOH + Na \rightarrow PhCOONa + \frac{1}{2}H_2$

122gm 11.2lits

$\Rightarrow 108.952\text{gr}$

Vol. of $H_2 = \frac{11.2 \times 108.92}{122} = 10\,\text{L}$

MATHEMATICS

61. $P = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix} \Rightarrow P^2 = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$P^3 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix}$

$P^4 = \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

Similarly $P^{2022} = \begin{pmatrix} 1 & 0 \\ 1011 & 1 \end{pmatrix}$

62. $P^2=\begin{pmatrix}1&0&0\\3&1&0\\9&3&1\end{pmatrix}\begin{pmatrix}1&0&0\\3&1&0\\9&3&1\end{pmatrix}=\begin{pmatrix}1&0&0\\6&1&0\\27&6&1\end{pmatrix}$

$P^4=\begin{pmatrix}1&0&0\\6&1&0\\27&6&1\end{pmatrix}\begin{pmatrix}1&0&0\\6&1&0\\27&6&1\end{pmatrix}=\begin{pmatrix}1&0&0\\12&1&0\\90&12&1\end{pmatrix}$

$P^5=\begin{pmatrix}1&0&0\\12&1&0\\90&12&1\end{pmatrix}\begin{pmatrix}1&0&0\\3&1&0\\9&3&1\end{pmatrix}=\begin{pmatrix}1&0&0\\15&1&0\\135&15&1\end{pmatrix}$

$Q-P^5=I_3\Rightarrow Q=I_3+P_5=\begin{pmatrix}2&0&0\\15&2&0\\135&15&2\end{pmatrix}\dfrac{q_{21}+q_{31}}{q_{32}}=\dfrac{15+135}{15}=10$

63. $(A-B)(A+B)=A^2+AB-BA-B^2=A^2-B^2\Rightarrow AB=BA$

64. $A'BA=\begin{pmatrix}1&1&1\end{pmatrix}\begin{pmatrix}9^2&-10^2&11^2\\12^2&13^2&-14^2\\-15^2&16^2&17^2\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}$

$=\left[9^2+12^2-15^2-10^2+13^2+16^2+11^2-14^2+17^2\right]=[539]$

65. $A=(a_{ij})=\dfrac{i-j}{i+2j}=\begin{pmatrix}0&-\frac{1}{5}&-\frac{2}{7}\\\frac{1}{4}&0&-\frac{1}{8}\\\frac{2}{5}&\frac{1}{7}&0\end{pmatrix}$ is neither symmetric nor skew symmetric but every square matrix can be expressed sum of symmetric and skew symmetric

66. $t_r\left(A^3(\text{adj}(A^2))\right)=t_r\left(A(A\,\text{adj}A)^2\right)=t_r\left(A|A|^2I\right)=|A|^2\,t_r(A)=24$

67. $AA^T=\dfrac{1}{4}\begin{pmatrix}1&2&2\\2&1&-2\\x&2&y\end{pmatrix}\begin{pmatrix}1&2&x\\2&1&2\\2&-2&y\end{pmatrix}=\dfrac{1}{9}\begin{pmatrix}9&0&x+4+2y\\0&9&2x+2-2y\\x+4+2y&2x+2-2y&x^2+4+y^2\end{pmatrix}$

$=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\Rightarrow x+y=-3$

68. $A=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}\Rightarrow A^n=\begin{pmatrix}\cos\theta&\sin n\theta\\\sin n\theta&\cos n\theta\end{pmatrix}$

$$B = A + A^4 = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} + \begin{pmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{pmatrix}$$

$$\det B = 4\sin^2\theta \begin{vmatrix} 0 & +1 \\ -1 & 0 \end{vmatrix} = 4\sin^2\left(\frac{\pi}{5}\right)$$

$$= \frac{10 - 2\sqrt{3}}{4} = 1.35 \in (1,2)$$

69. Put $x = -1$ we get $\begin{vmatrix} 0 & 0 & -3 \\ -2 & -3 & -5 \\ 2 & -3 & -3 \end{vmatrix} = -a - 12 \Rightarrow a = 24$

70. $c_1 \to c_1 - c_2$ $c_3 \to c_3 - c_2$

$$\begin{vmatrix} 1 & \sin^2 x & 0 \\ -1 & 1 + \cos^2 x & -1 \\ 0 & 4\sin 2x & 1 \end{vmatrix} = 0$$

$$\Rightarrow \sin 2x = -\frac{1}{2}$$

$$2x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right) \Rightarrow 2x = \frac{7\pi}{6}, \frac{11\pi}{6} \Rightarrow x = \frac{7\pi}{12}, \frac{11\pi}{12}$$

71. $2\omega + 1 = z \Rightarrow \omega = \frac{i\sqrt{3} - 1}{2} \Rightarrow \omega$ is a cube root of unity

Now $R_1 \to R_1 + R_2 + R_3$ $\begin{vmatrix} 1 & 0 & 0 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = -3(1 + 2\omega) \Rightarrow k = -z$

72. **We know that** $|\text{adj } A| = |A|^{n-1}$

Statement 2 is false

Now $|\text{adj}(\text{adj}A)| = |\text{adj}A|^{n-1} = |A|^{(n-1)^2}$

$\Rightarrow |\text{adj}(\text{adj}(\text{adj}A))| = |A|^{(n-1)^3}$

Statement 1 is true

73. **Given that** $|A| = \begin{vmatrix} x & 1 & 1 \\ 2 & x & 3 \\ x & -1 & 2 \end{vmatrix} = 4 \Rightarrow \begin{vmatrix} 2x+3 & -3 & 3-x \\ 3x-4 & x & 2-3x \\ -x^2-2 & 2x & x^2-2 \end{vmatrix} = |\text{adj } A| = |A|^2 = 16$

74. $\Delta_n = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$ $\Rightarrow \Delta_n = n!(n+1)!(n+2)! \begin{vmatrix} 1 & n+1 & (n+1) & (n+2) \\ 1 & n+2 & (n+2) & (n+3) \\ 1 & n+3 & (n+3) & (n+4) \end{vmatrix}$

$\Delta_n = 2n!(n+1)!(n+2)!$

$$\lim_{n\to\infty} \frac{\left(3n^3-5\right)\Delta_n}{\Delta_{n+1}} = \lim_{n\to\infty} \frac{\left(3n^3-5\right)}{(n+1)(n+2)(n+3)} = 3$$

75. $A\begin{pmatrix}1&2&3\\0&2&3\\0&1&1\end{pmatrix} = \begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix} \Rightarrow c_1 - c_3 \; A\begin{pmatrix}3&2&1\\3&2&0\\1&1&0\end{pmatrix} = \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$

$c_2 - c_3 \; A\begin{pmatrix}3&1&2\\3&0&2\\1&0&1\end{pmatrix} = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix}3&1&2\\3&0&2\\1&0&1\end{pmatrix}$

76. $\left[A(A+B)^{-1}B\right]^{-1} = B^{-1}(A+B)A^{-1} = \left(B^{-1}A+I\right)A^{-1} = B^{-1}+A^{-1}$

Statement I is true statement 2 is false as $(A+B)^{-1} = A^{-1}+B^{-1}$ is not true

77. $\begin{vmatrix}1&-2&1\\1&1&\lambda\\2&-1&\lambda\end{vmatrix} = 0 \Rightarrow 2+\lambda+2(2-2\lambda)+1(-3) = 0$

$\lambda = 1 \Rightarrow \lambda = 1 \in \left(\frac{3}{4}, \frac{5}{4}\right)$

78. $VA = 0$

$(\log a)x + (\log b)y + (\log c)z = 0 \Rightarrow (\log b)x + (\log c)y + (\log a)z = 0$

$(\log c)x + (\log a)y + (\log b)z = 0$ is homogeneous system of equation in x, y, z and $(x,y,z) \neq (0,0,0)$

$$\Rightarrow \Delta = \begin{vmatrix}\log a&\log b&\log c\\\log b&\log c&\log a\\\log c&\log a&\log b\end{vmatrix} = 0 \Rightarrow \log a + \log b + \log c = 0 \Rightarrow \log(abc) = 0$$

$abc = 1 \Rightarrow b^3 = 1 \; \left(b^2 = ac\right)$

$b = 1 \quad ac = 1 \quad c = \frac{1}{a}$

Equation be comes $\Rightarrow (\log a)x - (\log b)z = 0 \Rightarrow -(\log a)y + (\log a)z = 0$

$-(\log a)x - (\log a)y = 0$

$x - y = 0 \Rightarrow y - z = 0 \Rightarrow x - y = 0$

$x = y = z \Rightarrow \frac{x}{1} = \frac{y}{1} = \frac{z}{1} \Rightarrow \alpha : \beta : \gamma = 1:1:1 \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\gamma} + \frac{\gamma}{\alpha} = 3$

79. $\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \lambda \end{vmatrix} = 0\ \lambda = -3$

$\Delta_1 = \begin{vmatrix} 7 & -1 & -1 \\ 1 & -3 & 2 \\ k & +4 & -3 \end{vmatrix} = 0 \Rightarrow k = 6\ \ K - \lambda = 6 + 3 = 9$

80. $\Delta = \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -kl \\ -4 & 1 & 3 \end{vmatrix} = 121 - k^2 \neq 0$

For $k \in R - \{11, 11\}$

The system of equations has unique solution

81. $|A| = 2^4 \Rightarrow |adj(adjA))| = 2^{36} \Rightarrow \left\{\frac{2^{36}}{7}\right\} = \left\{\frac{(7+1)^{12}}{7}\right\} = \frac{1}{7} = \lambda$

$7\lambda = 1$

82. $\begin{vmatrix} 2 & 4 & \lambda \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \lambda = 6$

$f(y) = \begin{vmatrix} 2y^2 & -c \\ 1 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda 2y^2 + c = 0 \Rightarrow s_2 = \frac{c}{(2\lambda)} = 12$

$c = 144$

83. $\begin{vmatrix} -\sin\theta & 1 & 0 \\ 1 & 0 & \sin\theta \\ \sin\theta & 1 & 1 \end{vmatrix} = 0 \Rightarrow \sin^2\theta = \frac{1}{\sqrt{2}} \Rightarrow \sin^2\theta = \sin^2\frac{\pi}{4}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

84. $\left|\left(adjA^{-1}\right)^{-1}\right| = \frac{1}{\left|adj\,A^{-1}\right|} = \frac{1}{\left|(adjA)^{-1}\right|} = |adjA| = |A|^2 = 2^2 = 4$

85. $\left|A^n - 1\right| = \begin{vmatrix} 2^{n-1} - 1 & 2^{n-1} \\ 2^{n-1} & 2^{n-1} - 1 \end{vmatrix} = \left(2^{n-1} - 1\right)^2 - \left(2^{n-1}\right)^2 = \left|A^n - 1\right| = 1 - 2^n\ 1 - \lambda^n \Rightarrow \lambda = 2$

86. $|adjA| = 4 \Rightarrow |A|^2 = 4$

Now $A^{-1} = \frac{adjA}{\det A}$

$$A^{-2}=\frac{(\text{adj}A)^2}{|A|^2}=\frac{1}{4}\begin{bmatrix}1&1&2\\2&1&1\\1&2&1\end{bmatrix}^2$$

$$=\frac{1}{4}\begin{bmatrix}5&6&5\\5&5&6\\6&5&5\end{bmatrix}\Rightarrow\frac{1}{4}\sum_{j=1}^{3}\sum_{i=1}^{3}\ =\frac{1}{4}(\text{sum of all elements})=\frac{48}{4\times4}=\frac{12}{4}=3$$

87. $$\begin{vmatrix}a&x&0\\b&y&0\\c&z&0\end{vmatrix}\times\begin{vmatrix}x&a&b\\y&b&c\\z&c&a\end{vmatrix}+\begin{vmatrix}1&-1&1\\\cos\theta&\cos\theta&\sin\theta\\-\sin\theta&-\sin\theta&\cos\theta\end{vmatrix}=2$$

88. G.T $A^2-4A-5I=0$

$$A^3=A.A^2=A(4A+5I)=4A^2+A5=4(4A+5I)+5A\ =\ 21BA+20I$$

$$\begin{pmatrix}21&42&42\\42&21&42\\42&42&21\end{pmatrix}+\begin{pmatrix}20&0&0\\0&20&0\\0&0&20\end{pmatrix}=375$$

89. $$\Delta=\begin{vmatrix}\alpha^2&\alpha&1\\1&1&1\\a&b&c\end{vmatrix}=0\Rightarrow\alpha^2(c-b)-\alpha(c-a)+b-a=0\text{ if }\alpha=1$$

$$\Rightarrow c-b+a-c+b-a=0\Rightarrow\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$$

$$=3\frac{(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)}=3$$

$30k=90$

90. $MM^T=I\ M^2=I$

$M^{-1}+\text{adj}M=0$

$M^{-1}=-\text{adj}(M)$

$$\left|M^{-1}\right|=(-1)^3|\text{ad}\,M|\Rightarrow\frac{1}{|M|}=-|M|^2\Rightarrow|M|=-1$$

$P+2M=0$

$P=-2M$

$$|P|=-8|M|=8\Rightarrow\left|PP^TP^{-1}\right|=8$$