10

POLYNOMIALS

Important Point

- If a polynomial $p(x)$ can be expressed as $p(x) = q(x)\ r(x)$ then $q(x)$ and $r(x)$ are factors of $p(x)$
- The remainder got by dividing a polynomial $p(x)$ by $(x - a)$ is $p(a)$ and $p(-a)$ is the remainder when it is divided by $(x+a)$.
- If $p(a) = 0$, then $(x - a)$ is factors of $p(x)$ and if $p(-a) = 0$, $(x+a)$ is a factor of $p(x)$.
- If $p(a) \neq 0$, then $(x - a)$ is not a factor of $p(x)$ and if $p(-a) \neq 0$ then $(x + a)$ is not a factor of $p(x)$.
- $p(x) = (x - a)\ q(x) + r$, then the quotient and remainder got by dividing $p(x)$ by $(x - a)$ are $q(x)$ and 'r' respectively.
- The remainder got by dividing the polynomial $p(x)$ by $(ax + b)$ is $p(-b/a)$.
- If p and q are the solutions of the second degree equation $ax^2 + bx + c = 0$, then the polynomial $ax^2 + bx + c$ can be expressed as $a(x-p)\ (x-q)$.
- In the polynomial $p(x)$, $p(1)$ will be the sum of coefficients of the variables and $p(0)$ will be constant term.
- If $p(x)$ is a polynomial and 'a' is any number, then $x-a$ will always be a factor of $p(x) - p(a)$.

PART - A (Score : 1)

1. If $p(x) = 7x^3 - 3x^2 - 9x + 18$ then what is $p(0)$?
2. If $p(x) = 2x^4 + 5x^3 + 3x^2$ then what is $p(1)$?
3. $p(x) = x^2 + 9x$ then what is $p(1)$?
4. In the polynomial $p(x) = 5x^3 - 3x^2 + 2x + k$. If $p(0) = 10$ find the value of k ?
5. In the polynomial $p(x) = 2x^2 + 3x + k$. If $p(1) = 12$ find the value of k ?

PART - B (Score : 2)

1. If $p(x) = x^3 - 5x^2 + 6x - 2$, find $p(-2)$
2. What is the remainder got by dividing $x^3 - 2x^2 + 5x + 1$ by $(x - 2)$
3. If $(x - 1)$ is a factor of the polynomial $x^2 + 6x + k$ find the value of k.
4. Check whether $(x - 3)$, a factor of the polynomial $x^2 + 2x - 15$
5. Which number should be added to the polynomial $2x^2 - 5x + 1$ so that $(x - 1)$ is a factor of the new polynomial.

PART - C (Score : 3)

1. (i) Express $x^2 + 5x + 6$ as the product of two first degree polynomials.
   (ii) Find the solutions of $x^2 + 5x + 6 = 0$
2. (i) If $p(x) = x^2 - 3x + 2$. Find $p(2)$.
   (ii) Is $(x - 2)$ a factor of $p(x)$? Why?
3. Show that $2x^2 + 3x + 5$ cannot be expressed as the product of two first degree polynomials.
4. (i) If $p(x) = (x - 2)(x - 4)$. Find $p(2)$
   (ii) Which number should be added to $p(x)$ to obtain a perfect square.
5. $p(x) = x^2 - 12x + 20$
   (i) Express $p(x)$ as the product of two first degree polynomials
   (ii) Find solutions of the equation $p(x) = 0$
6. If $-3, 5$ are the solutions of the equation $x^2 + ax + b = 0$
   (i) Express $x^2 + ax + b$ as the product of two first degree polynomials
   (ii) Find the value of a, b

PART - D (Score : 4)

1. If $p(x) = x^2 - 5x + 10$
   (i) Find $p(2)$

(ii) Which number should be subtracted from $p(x)$ so that $(x - 2)$ is a factor of the resulting polynomial.

(iii) If $q(x) = x^2 - 5x + 6$, express it as the product of two first degree polynomials.

2. (i) If $p(x) = 2x^2 - 4x - 16$. Find $p(3)$.

   (ii) $q(x) = p(x) - p(3)$. Express $q(x)$ as the product of two first degree polynomials.

3. (i) Which number should be taken as 'k' so that the polynomial $kx^2 + 12x - 14$ has $(x + 7)$ as a factor of it.

   (ii) Find the second factor of this polynomial.

4. (i) If $p(0) = -5$ in $p(x) = ax^2 + bx + c$ then find the value of c.

   (ii) If $(x - 1)$ is a factor of $p(x)$. Prove that $a + b = 5$

5. (i) If 3 is the remainder got by dividing the polynomial $p(x) = 2x^3 - x^2 + kx + 24$ by $(x-3)$ then find the value of k.

   (ii) What is the remainder got by dividing $p(x)$ by $(x - 1)$

PART - E (Score : 5)

1. Express the polynomial $p(x) = 4x^2 + 20x + 25$ as the product of two first degree polynomials.

2. If $p(-5) = 0$, $p(6) = 0$ in the polynomial $p(x)$.

   (i) Write the first degree polynomials, which are the factors of $p(x)$.

   (ii) Find the polynomial $p(x)$

3. (i) If $(x - 2)$ is a factor of the polynomial $p(x) = x^2 - kx + 8$. Find the value of k.

   (ii) Express the polynomial $p(x)$ as the product of two first degree polynomials.

4. $p(x) = 2x^2 - 5x + 8$

   (i) Which number should be subtracted from $p(x)$ so that $(x - 2)$ is a factor of the resulting polynomial.

   (ii) Find the second factor of the resulting polynomial.

Answer : PART - A

1. $p(x) = 7x^3 + 3x^2 - 9x + 18$

   $\therefore p(0) = 18$

2. $p(x) = 2x^4 + 5x^3 + 3x^2$

   $p(1) = 2 + 5 + 3$

   $= 10$

3. $p(x) = x^2 + 9x$

   $p(1) = 1 + 9$

   $= 10$

4. $p(x) = 5x^3 - 3x^2 + 2x + k$

   $\therefore p(0) = k = 10$

5. $p(x) = 2x^2 + 3x + k$

   $p(1) = 2 + 3 + k = 12$

   $k = 12 - 5 = 7$

Answer : PART - B

1. $p(x) = x^3 - 5x^2 + 6x - 2$

   $p(-2) = (-2)^3 - 5(-2)^2 + 6(-2) - 2$

   $= 8 - 5 \text{ x } 4 - 12 - 2$

   $= -8 - 20 - 12 - 2$

   $= -42$

2. $p(x) = x^3 - 2x^2 + 5x + 1$

   remainder = p(2)

   $= 2^3 - 2 \text{ x } 2^2 + 5 \text{ x } 2 + 1$

   $= 8 - 8 + 10 + 1 = 11$

3. $p(x) = x^2 + 6x + k$

   Since $(x-1)$ is a factor of $p(x)$, $p(1) = 0$

   ie, $p(1) = 0 \Rightarrow 1^2 + 6 \text{ x } 1 + k = 0$

   $1 + 6 + k = 0$

   $7 + k = 0$

   $k = -7$

4. $p(x) = x^2 + 2x - 15$

   for $(x - 3)$ to be a factor of $p(x)$ then $p(3)$ must be zero

   $p(3) = 3^2 + 2 \text{ x } 3 - 15$

   $= 9 + 6 - 15 = 0$

   $\therefore (x - 3)$ is a factor of $p(x)$

5. $p(x) = 2x^2 - 5x + 1$

$p(1) = 2 \times 1^2 - 5 \times 1 + 1$

$= 2 - 5 + 1$

$= -2$

$\therefore$ The number should be added to $p(x)$ is $+ 2$

Answer : PART - C

1. $p(x) = x^2 + 5x + 6$

(i) $x^2 + 5x + 6 = (x + a)(x + b)$, say

ie, $x^2 + 5x + 6 = x^2 + (a+b)x + ab$

$a+b = 5$ $\quad\quad$ $ab = 6$

$\therefore a = 3, b = 2$

$\therefore x^2 + 5x + 6 = (x + 3)(x + 2)$

(ii) Solutions of the equation $x^2 + 5x + 6 = 0$ are -3 and -2

2. (i) $p(x) = x^2 - 3x + 2$

$p(2) = 2^2 - 3 \times 2 + 2$

$= 4 - 6 + 2 = 0$

(ii) Since $p(2) = 0$, $(x - 2)$ is a factor of $p(x)$

3. $p(x) = 2x^2 + 3x + 5$

consider the equation $2x^2 + 3x + 5 = 0$,

$a = 2$, b = -5, c = 8

$\therefore \sqrt{b^2 - 4ac} = \sqrt{(-5)^2 - 4 \times 2 \times 8}$

$= \sqrt{25 - 64}$

$= \sqrt{-39}$

Negative numbers has no square root.

$\therefore$ The polynomial $2x^2 + 3x + 5$ cannot be written as the product of two first degree polynomials.

4. $p(x) = (x - 2)(x - 4)$

(i) $p(2) = (2 - 2)(2 - 4) = 0 \times -2 = 0$

(ii) $p(x) = x^2 - 4x - 2x + 8$

1 should be added for this polynomial to be a perfect square

$\therefore p(x) = x^2 - 6x + 8 + 1$

$= x^2 - 6x + 9$

$= (x - 3)^2$

5. $p(x) = x^2 - 12x + 20$

(i) $x^2 - 12x + 20 = (x - a)(x - b)$

$x^2 - 12x + 20 = x^2 - (a + b)x + ab$

$\therefore a + b = 12 \qquad ab = 20$

$\therefore a = 10, b = 2$

$\therefore x^2 - 12x + 20 = (x - 10)(x - 2)$

(ii) The solutions of the equation $x^2 - 12x + 20 = 0$ are 10 and 2

6. Since - 3, 5 are the solutions of the equation $x^2 + ax + b = 0$

(i) $x^2 + ax + b = (x + 3)(x - 5)$

$x^2 + ax + b = x^2 - 5x + 3x - 15$

$x^2 + ax + b = x^2 - 2x - 15$

(ii) $\therefore a = -2, b = -15$

Answer : PART - D

1. $p(x) = x^2 - 5x + 10$

(i) $p(2) = 2^2 - 5 \times 2 + 10$

$= 4 - 10 + 10$

$= 4$

(ii) When p(2) subtracted from $p(x)$, we get a polynomial with $(x - 2)$ as a factor.

$p(x) - p(2) = x^2 - 5x + 10 - 4$

$= x^2 - 5x + 6$

(iii) $x^2 - 5x + 6$ has a factor as $(x-2)$

If the second factor is $(x - b)$, then

$x^2 - 5x + 6 = (x-2)(x - b)$

$6 = -2 \times -b$

$2b = 6$

$b = 3$

$\therefore$ second factor $= x - 3$

$\therefore x^2 - 5x + 6 = (x - 2)(x - 3)$

2. (i) $p(x) = 2x^2 - 4x - 16$

$p(3) = 2 \times 3^2 - 4 \times 3 - 16$

$= 2 \times 9 - 12 - 16$

$= 18 - 12 - 16$

$= 6 - 16 = -10$

(ii) $q(x) = p(x) - p(3)$

$= 2x^2 - 4x - 16 - (-10)$

$= 2x^2 - 4x - 16 + 10$

$= 2x^2 - 4x - 6$

$= 2(x^2 - 2x - 3)$

$(x-3)$ is a factor of $q(x)$

If $(x-b)$ is the second factor, then

$x^2 - 2x - 3 = (x-3)(x-b)$

$\therefore -3 = (-3) \times (-b)$

$-3 = 3b$

$b = -1$

$\therefore q(x) = 2(x-3)(x-b)$

$= 2(x-3)(x- -1)$

$= 2(x-3)(x+1)$

$= (x-3)(2x+2)$

3. (i) $p(x) = kx^2 + 12x - 14$

since $(x+7)$ is a factor of $p(x)$, $p(-7) = 0$

ie, $p(-7) = 0 \Rightarrow k \times (-7)^2 + 12 \times -7 - 14 = 0$

$k \times 49 - 84 - 14 = 0$

$49k - 98 = 0$

$k = 98/49 = 2$

(ii) $p(x) = 2x^2 + 12x - 14$

$= 2(x^2 + 6x - 7)$

$(x+7)$ is a factor of $p(x)$

If $(x+b)$ is the second factor

$x^2 + 6x - 7 = (x+7)(x+b)$

$-7 = 7 \times b$

$7b = -7$

$b = -7/7 = -1$

$\therefore$ second factor $= x - 1$

4. (i) $p(x) = ax^2 + bx + c$

$p(0) = -5$

$\therefore a \times o^2 + b \times o + c = -5$

$c = -5$

(ii) $p(x) = ax^2 + bx - 5$

since $(x-1)$ is a factor of $p(x)$, $p(1)$

$\therefore a \times 1^2 + b \times 1 - 5 = 0$

$a + b = 5$

5. $p(x) = 2x^3 - x^2 + kx + 24$

(i) The remainder got by dividing $p(x)$ by $(x - 3) = p(3)$

$\therefore p(3) \Rightarrow 2 \times 3^3 - 3^2 + k \times 3 + 24 = 3$

$2 \times 27 - 9 + 3k + 24 = 3$

$54 - 9 + 3k + 24 = 3$

$69 + 3k = 3$

$3k = 3 - 69$

$k = -66 / 3 = -22$

$\therefore p(x) = 2x^3 - x^2 - 22x + 24$

(ii) $p(1)$ is the remainder got by dividing $p(x)$ by $(x - 1)$

$p(1) = 2 \times 1^3 - 1^2 - 22 \times 1 + 24$

$= 2 - 1 - 22 + 24$

$= 26 - 23 = 3$

Answer : PART - E

1. $4x^2 + 20x + 25 = 0$

then, $a = 4$, $b = 20$, $c = 25$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-20 \pm \sqrt{20^2 - 4 \times 4 \times 25}}{2 \times 4}$$

$$x = \frac{-20 \pm \sqrt{400 - 400}}{8}$$

$$x = \frac{-20 \pm 0}{8}$$

$$x = \frac{-20}{8}$$

$$x = \frac{-5}{2} \quad \Rightarrow 2x + 5 = 0$$

$\therefore 4x^2 + 20x + 25 = (2x + 5)(2x + 5)$

2. (i) Since $p(-5) = 0$, $p(6) = 0$ in the polynomial $p(x)$, $(x + 5)$ and $(x - 6)$ are factors of $p(x)$

(ii) $p(x) = (x + 5)(x - 6)$

$= x^2 - 6x + 5x - 30$

$= x^2 - x - 30$

3. (i) $p(x) = x^2 - kx + 8$.

Since $(x - 2)$ is a factor of $p(x)$, $p(2)$ must be zero.

$\therefore (2)^2 - k \times 2 + 8 = 0$

$4 - 2k + 8 = 0$

$12 - 2k = 0$

$2k = 12$

$k = 12/2 = 6$

(ii) $p(x) = x^2 - 6x + 8$

one factor of $p(x)$ is $(x - 2)$, Let the other factor be $(x - b)$

$\therefore x^2 - 6x + 8 = (x - 2)(x - b)$

$\therefore -2 \times -b = 8$

$2b = 8$

$\therefore b = 8/2 = 4$

$\therefore x^2 - 6x + 8 = (x - 2)(x - 4)$

4. (i) $p(x) = 2x^2 - 5x + 8$

p(2) should be subtracted from $p(x)$

$p(2) = 2 \times 2^2 - 5 \times 2 + 8$

$= 8 - 10 + 8 = 16 - 10$

$= 6$

(ii) $p(x) - p(2) = 2x^2 - 5x + 8 - 6$

$= 2x^2 - 5x + 2$

$= 2\left(x^2 - \frac{5}{2}x + 1\right)$

$p(x) - p(2)$ has a factor as $(x - 2)$

If second factor is $(x - b)$, then

$x^2 - \frac{5}{2}x + 1 = (x - 2)(x - b)$

$\therefore 1 = -2 \times -b$

$2b = 1$

$b = \frac{1}{2}$

$\therefore$ Second factor $= \left(x - \frac{1}{2}\right)$

$\therefore 2x^2 - 5x + 2 = 2(x - 2)(x - b)$

$= 2(x - 2)\left(x - \frac{1}{2}\right)$

$= \cancel{2}(x - 2)\frac{(2x - 1)}{\cancel{2}}$

$= (x - 2)(2x - 1)$