Chapter 12

Conic Sections-I

Solutions

SECTION - A

Objective Type Questions (One options is correct)

1. The point of intersection of the lines $x - y + 1 = 0$ and $x + y + 5 = 0$ is P. A circle with centre at (1, 0) passes through P. The tangent to the circle at P meets the x-axis at $(k, 0)$. The value of k is

(1) 2 (2) –3 (3) –2 (4) –4

Sol. Answer (4)

$L_1 : x - y + 1 = 0$

$L_2 : x + y + 5 = 0$

$\Rightarrow x = -3, y = -2$

$\Rightarrow P \equiv (-3, -2)$

Equation of circle is $(x - 1)^2 + y^2 = 20$

$\Rightarrow x^2 + y^2 - 2x - 19 = 0$

Equation of tangent at P is

$2x + y + 8 = 0$

Put $y = 0$

$\Rightarrow x = -4$

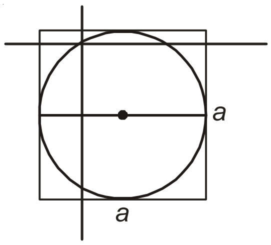
Point is (–4, 0)

$\Rightarrow k = -4$

2. The area of a square circumscribing the circle $3(x^2 + y^2) - 6x + 8y = 0$ is

(1) $\frac{50}{9}$ sq. units (2) $\frac{75}{9}$ sq. units (3) $\frac{25}{9}$ sq. units (4) $\frac{100}{9}$ sq. units

Sol. Answer (4)



$S : x^2 + y^2 - 2x + \frac{8}{3}y = 0$

$\Rightarrow (x-1)^2 + \left(y + \frac{4}{3}\right)^2 = \frac{16}{9} + 1 = \left(\frac{5}{3}\right)^2$

Let side of a square = a

Now diameter = a

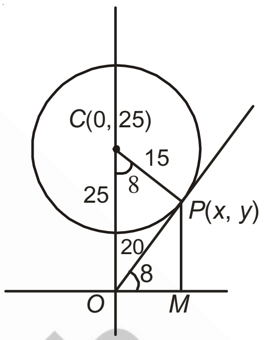
$\therefore \quad a = 2.\frac{5}{3} = \frac{10}{3}$

$\Rightarrow \quad a^2 = \frac{100}{9}$

3. Let a point P lie on the circle $x^2 + y^2 - 50y + 400 = 0$ such that $\angle POX$ is minimum. Then the co-ordinates of P are

(1) (12, 10) (2) (12, 16) (3) (12, 18) (4) (18, 12)

Sol. Answer (2)



Now, $S : x^2 + y^2 - 30y - 400 = 0$

$\Rightarrow \quad x^2 + (y - 15)^2 = 625 = (25)^2$

Now, $OP = \sqrt{25^2 - 15^2}$

$= \sqrt{625 - 225} = \sqrt{400} = 25$

Let $\angle POX = \theta$

$\therefore \quad \cos\theta = \frac{x}{20} \quad \Rightarrow \quad \frac{15}{25} = \frac{x}{20}$

$\Rightarrow \quad x = 12$

$\sin\theta = \frac{y}{20} \quad \Rightarrow \quad \frac{20}{25} = \frac{y}{20}$

$\Rightarrow \quad y = 16$

$\therefore \quad P \equiv (x, y) \equiv (12, 16)$

4. If the circle $O : x^2 + y^2 = 16$ intersects another circle C of radius 5 units in such a way that the common chord is of maximum length and has a slope equal to $\frac{3}{4}$, then the coordinates of the centre of C are

(1) $\left(\frac{9}{5}, \frac{-12}{5}\right)$ (2) $\left(\frac{-9}{5}, \frac{-12}{5}\right)$ (3) $\left(\frac{9}{5}, \frac{12}{5}\right)$ (4) $\left(\frac{-9}{15}, \frac{12}{15}\right)$

Sol. Answer (1)

Length of common chord is maximum and therefore common chord is diameter of small circle.

$\therefore$ Equation of common chord $y = \frac{3}{4}x$ ...(i)

Let centre of circle S_2 is C (a, b).

Equation of perpendicular line to (i) is $y = \frac{-4}{3}x$ centre (a, b) lie as this line.

$\therefore \quad b = -\frac{4}{3}a$ ...(ii)

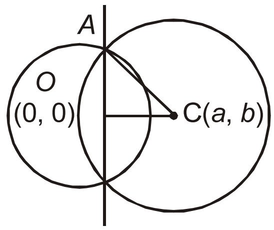
$AO = 4$

$AC = 5$

$\therefore \quad OC = 3$

$\sqrt{a^2 + b^2} = 3$

$a^2 + b^2 = 9$ ...(iii)



Solve equations (ii) and (iii),

$a = \frac{9}{5}$

$b = -\frac{12}{5}$

Centre $\left(\frac{9}{5}, \frac{-12}{5}\right)$

5. The equation of a circle which passes through $(2a, 0)$ and whose radical axis in relation to the circle $x^2 + y^2 = a^2$ is $x = \frac{a}{2}$, is

(1) $x^2 + y^2 - ax = 0$ (2) $x^2 + y^2 + 2ax = 0$

(3) $x^2 + y^2 - 2ax = 0$ (4) $x^2 + y^2 + ax = 0$

Sol. Answer (3)

The required circle is $s + \lambda L = 0$

$(x^2 + y^2 \;\; a^2) + \lambda\left(x - \frac{a}{2}\right) = 0$

This passes through $(2a, 0) \Rightarrow \lambda = -2a$

Hence, the required circle is

$(x^2 + y^2 \;\; a^2) - 2a\left(x - \frac{a}{2}\right) = 0$

$\Rightarrow \; x^2 + y^2 - 2ax = 0$

6. Two rods of lengths a and b slide along the axes in such a way that their ends are concyclic. The locus of the centre of the circle passing through these points is

(1) $4(x^2 + y^2) = a^2 + b^2$ (2) $x^2 - y^2 = a^2 - b^2$ (3) $4(x^2 - y^2) = a^2 - b^2$ (4) $x^2 + y^2 = a^2 + b^2$

Sol. Answer (3)

Let A_1A_2 and B_1B_2 be two rods of lengths a and b which slide along OX and OY respectively.

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be the circle passing through A_1, A_2, B_1, B_2. Then

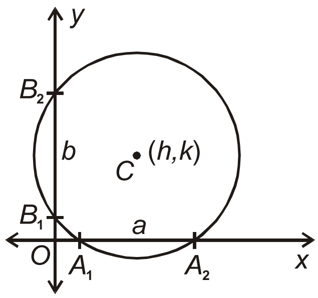
$A_1 A_2$ = Intercept of x-axis = $2\sqrt{g^2 - c}$

$\Rightarrow \; a = 2\sqrt{g^2 - c}$ …(i)

$B_1 B_2$ = Intercept of y-axis

$2\sqrt{f^2 - c}$

$\Rightarrow \; b = 2\sqrt{f^2 - c}$ …(ii)



Squaring (i) and (ii), we get

$a^2 = 4(g^2 \ \ c)$

$b^2 = 4(f^2 \ \ c)$

Subtracting $a^2 - b^2 = 4(g^2 - f^2)$

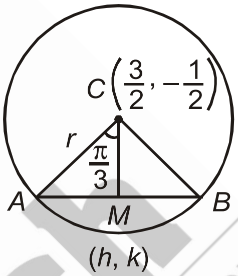
Hence, locus of centre is $a^2 - b^2 = 4(x^2 - y^2)$.

7. The equation of the locus of the mid-points of chords of the circle $4x^2 + 4y^2 - 12x + 4y + 1 = 0$ that subtend an angle of $\frac{2\pi}{3}$ at its centre, is

(1) $16x^2 + 16y^2 - 48x + 16y + 31 = 0$
(2) $16x^2 + 16y^2 + 48x + 48y + 31 = 0$
(3) $16x^2 - 16y^2 + 48x + 48y + 31 = 0$
(4) $16x^2 + 16y^2 - 48x - 16y - 31 = 0$

Sol. Answer (1)

$$r = \sqrt{\frac{9}{4} + \frac{1}{4} - \frac{1}{4}} = \frac{3}{2}$$



$$CM = r\cos\frac{\pi}{3} = \frac{r}{2}$$

$$\sqrt{\left(h - \frac{3}{2}\right)^2 + \left(k + \frac{1}{2}\right)^2} = \frac{3}{4}$$

Locus of $M(h, k)$

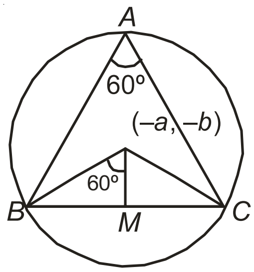
$$16x^2 + 16y^2 - 48x + 16y + 31 = 0$$

8. The area of an equilateral triangle inscribed in the circle $x^2 + y^2 + 2ax + 2by + c = 0$ is $\frac{m}{n}(a^2 + b^2 - c)$, then the value of $\frac{1}{\sqrt{3}}m + n$ is

(1) 5
(2) 7
(3) 6
(4) 8

Sol. Answer (2)

$$\therefore \quad \sin 60° = \frac{BM}{OB} = \frac{BM}{\sqrt{a^2 + b^2 - c}}$$



$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{BM}{\sqrt{a^2 + b^2 - c}}$$

$$\Rightarrow \quad BM = \frac{\sqrt{3}}{2} \times \sqrt{a^2 + b^2 - c}$$

$$\Rightarrow \quad BC = 2BM = \sqrt{3} \times \sqrt{a^2 + b^2 - c}$$

$$\therefore \quad \text{Area of } \Delta ABC = \frac{\sqrt{3}}{4} \times (BC)^2$$

$$= \frac{\sqrt{3}}{4} \times 3(a^2 + b^2 - c)$$

$$= \frac{3\sqrt{3}}{4} \times (a^2 + b^2 - c)$$

$\Rightarrow \quad \frac{m}{n}(a^2+b^2-c)=\frac{3\sqrt{3}}{4}\times(a^2+b^2-c)$

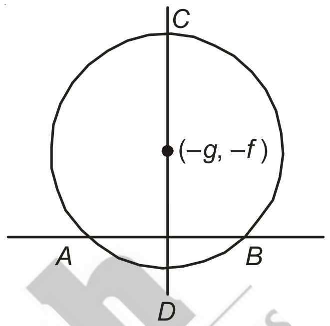
$\Rightarrow \quad \frac{m}{n}=\frac{3\sqrt{3}}{4}$

$\therefore \quad \frac{1}{\sqrt{3}}m+n=\frac{1}{3}\times 3\sqrt{3}+4=3+4=7$

9. The equation of the diameter of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which corresponds to the chord $ax + by + d = 0$ is $\lambda x - ay + \mu g + k = 0$, then $\lambda + \mu$ is

(1) $2a$ (2) $2b$ (3) $2c$ (4) $2d$

Sol. Answer (2)



$\therefore \quad AB : ax + by + d = 0$

Diameter = $CD : bx - ay + m = 0$

Which is passes through $(-g, -f)$

$\therefore \quad -bg + af + m = 0$

$\Rightarrow \quad m = bg - af$

Equation of diameter is

$bx - ay + bg - af = 0$

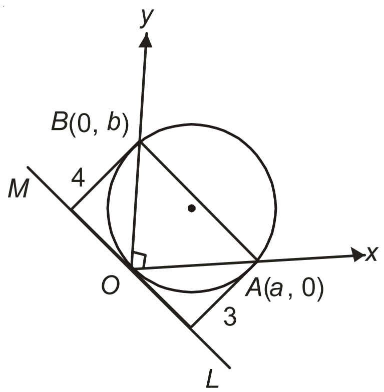
$\Rightarrow \quad \lambda x - ay + \mu g + k = 0$

$\therefore \quad \lambda + \mu = b + b = 2b$

10. A line meets the co-ordinate axes at $A(a, 0)$ and $B(0, b)$. A circle is circumscribed about the triangle OAB. If the distance of the points A and B from the tangent at origin to the circle are 3 and 4 respectively, then the value of $a^2 + b^2 + 1$ is

(1) 20 (2) 30 (3) 40 (4) 50

Sol. Answer (4)



$\therefore \quad \angle AOB = \frac{\pi}{2}$

$\Rightarrow \quad AB$ is the diameter

Equation of circle is $(x - a)(x - 0) + (y - 0)(y - b) = 0$

$\Rightarrow \quad x^2 + y^2 - ax - by = 0$

Tangent at O,

$x - O + y.O - \frac{a}{2}(x+O) - \frac{b}{2}(y+O) = 0$

$\Rightarrow \quad ax + by = 0$

$\therefore \quad AL = \frac{a.a+0}{\sqrt{a^2+b^2}} = 3$

$\Rightarrow \quad 3\sqrt{a^2+b^2} = a^2$

$\Rightarrow \quad 3 = \frac{a^2}{\sqrt{a^2+b^2}}$

Similarly, $4 = \frac{b^2}{\sqrt{a^2 + b^2}}$

$\Rightarrow \quad 3 + 4 = \frac{a^2 + b^2}{\sqrt{a^2 + b^2}} = \sqrt{a^2 + b^2}$

$\Rightarrow \quad a^2 + b^2 = 7^2 = 49$

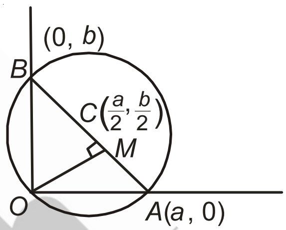
$\Rightarrow \quad a^2 + b^2 + 1 = 50$

11. A circle of constant radius r passes through the origin O, and cuts the axes at A and B. The locus of the foot of the perpendicular from O to AB is $(x^2 + y^2)^k = 4r^2x^2y^2$. Then the value of k is

(1) 2 (2) 1 (3) 3 (4) 4

Sol. Answer (3)

Let $A \equiv (a, 0)$ and $B \equiv (0, b)$ respectively



$\therefore \quad AB = \frac{x}{a} + \frac{y}{b} = 1$ ...(1)

Since AB is a diameter of a circle, center of a circle lie on AB

$\therefore \quad C \equiv \left(\frac{a}{2}, \frac{b}{2}\right)$

Radius = $\sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{\sqrt{a^2 + b^2}}{2}$

$\Rightarrow \quad a^2 + b^2 = 4r^2$ ...(*)

Now, $OM \perp AB$

$\Rightarrow \quad OM = ax - by = 0$ ...(2)

Now solving (1) and (2), we get

$$a = \frac{x^2 + y^2}{x}, \; b = \frac{y^2 + x^2}{y}$$

Put the values of a and b in (*), we get

$$\frac{(x^2 + y^2)^2}{x^2} + \frac{(x^2 + y^2)^2}{y^2} = 4r^2$$

$\Rightarrow \quad (x^2 + y^2)^2 \left(\frac{1}{x^2} + \frac{1}{y^2}\right) = 4r^2$

$\Rightarrow \quad (x^2 + y^2)^3 = 4r^2x^2y^2$

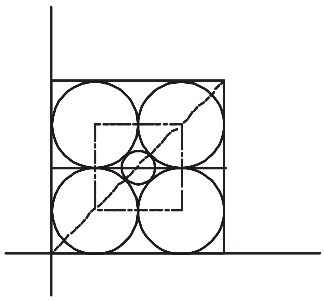
$\Rightarrow \quad k = 3$

12. Four circles are inscribed in a square of side 10 cm in such a way that the minimum radius is 2.5 cm. The radius of the smallest circle which touches all those four circles externally is

(1) $\frac{5}{2}(\sqrt{3} - 1)$ (2) $\frac{3}{2}(\sqrt{2} - 1)$ (3) $\frac{5}{2}(\sqrt{2} - 1)$ (4) $\frac{5}{2}(\sqrt{5} - 1)$

Sol. Answer (3)

Centre of the smallest circle = (5, 5)



Here, $QT = \frac{5}{2} = 2.5$

$PT = \frac{5}{2}$

$\therefore \quad PQ = \sqrt{\frac{50}{4}} = \sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}} = \frac{5}{2} \times \sqrt{2}$

13. Four distinct points (a, 0), (0, b), (c, 0) and (0, d) are lie on a plane in such a way that $ac = bd$, then they will

(1) Form a trapezium

(2) Form a triangle

(3) Lie on a circle

(4) Form a quadrilateral, whose area is zero

Sol. Answer (3)

As we know that, $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$ cuts the axes in four cyclic points, then $a_1a_2 = b_1b_2$

Here, $L_1 : \frac{x}{a} + \frac{y}{b} = 1$

$L_2 : \frac{x}{c} + \frac{y}{d} = 1$

$\Rightarrow \quad \frac{1}{a} \times \frac{1}{c} = \frac{1}{b} \times \frac{1}{d}$

$\Rightarrow \quad ac = bd$

Four distinct points are lie on a circle.

14. A circle of radius 2 units touches the co-ordinate axes in first quadrant. If the circle moves one complete roll on x-axis along the positive direction of x-axis and then centre is rotated about the origin at $\frac{\pi}{3}$ angle in anti-clockwise direction. Then the co-ordinates of the centre in the new position is

(1) $(2\pi + 1 - \sqrt{3},\ 1 + (1 + 2\pi)\sqrt{3})$

(2) $(2\pi - 1 - \sqrt{3},\ (1 + 2\pi)\sqrt{3})$

(3) $((1 + 2\pi)\sqrt{3}, (1 + 2\pi)\sqrt{3})$

(4) $((1 - 2\pi)\sqrt{3}, (1 - 2\pi)\sqrt{3})$

Sol. Answer (1)

Centre of a circle is (2, 2).

Circumference of the circle = $2\pi r$

$= 2\pi \times 2$

$= 4\pi$

Centre of the circle in the new position is $(2 + 4\pi, 2)$

Let $z_1 = (2 + 4\pi, 2)$

$P \equiv (z)$

Applying rotation, we have

$$\frac{z-0}{z_1-0}=e^{i\frac{\pi}{3}}$$

$$\Rightarrow \quad z=z_1\times\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$$

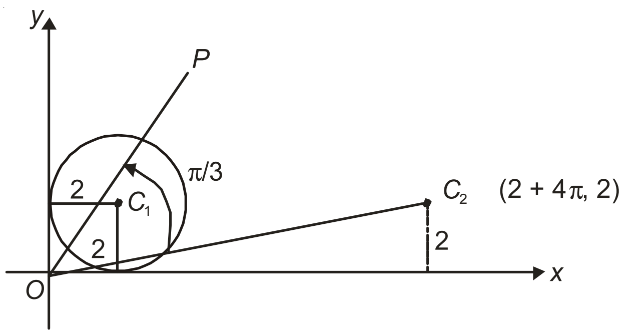
$$=\frac{1}{2}((2+4\pi)+2i)\ (1+i\sqrt{3})$$

$$=((1+2\pi)+i)\ (1+i\sqrt{3})$$

$$=\left((1+2\pi)+i+i(1+2\pi)\sqrt{3}-\sqrt{3}\right)$$

$$=\left((1-\sqrt{3}+2\pi)+i(1+(1+2\pi)\sqrt{3}\right)$$

$$=\left(2\pi+1-\sqrt{3},\ 1+(1+2\pi)\sqrt{3}\right)$$



15. If the circle $x^2 + y^2 - 4x - 8y + 16 = 0$ rolls up the tangent to it at $(2+\sqrt{3}, 3)$ by 2 units (assumes x-axis as horizontal), then the centre of the circle in the new position is

(1) $(3, 4)$ (2) $(3\sqrt{3}, 4+\sqrt{3})$ (3) $(3, 4+\sqrt{3})$ (4) $(3+\sqrt{3}, 4+\sqrt{3})$

Sol. Answer (3)

$S : x^2 + y^2 - 4x - 8y + 16 = 0$

$\Rightarrow (x-2)^2 + (y-4)^2 = 2^2$

$\therefore \quad P \equiv (2+\sqrt{3}, 3)$

Equation of tangent to the circle at P is

$$(2+\sqrt{3})x+3y-2\ (x+2+\sqrt{3})-4(y+3)+16=0$$

$\Rightarrow \quad \sqrt{3}x-y-2\sqrt{3}=0$

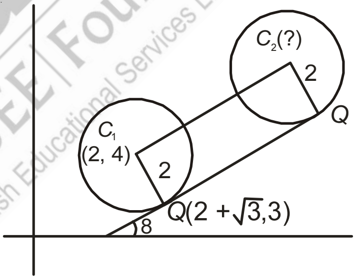
$\therefore \quad m=\sqrt{3}$

$\Rightarrow \quad \theta = 60^\circ$

$\therefore \quad B \equiv (2+2\cos 60^\circ,\ 4+2\sin 60^\circ)$

$= (2+1,\ 4+\sqrt{3})$

$= (3,\ 4+\sqrt{3})$



SECTION - B

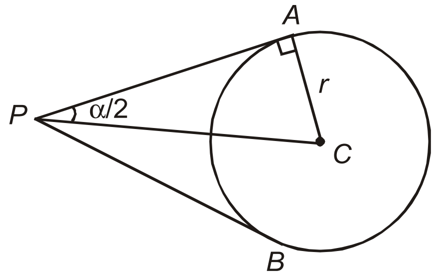
Objective Type Questions (More than one options are correct)

1. If α is the angle subtended at $P(x_1, y_1)$ by the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then

(1) $\cot\alpha=\dfrac{\sqrt{S_1}}{\sqrt{g^2+f^2-c}}$ (2) $\cot\dfrac{\alpha}{2}=\dfrac{\sqrt{S_1}}{\sqrt{g^2+f^2-c}}$

(3) $\tan\alpha=\dfrac{2\sqrt{g^2+f^2-c}}{\sqrt{S_1}}$ (4) $\tan\dfrac{\alpha}{2}=\left(\dfrac{\sqrt{g^2+f^2-c}}{\sqrt{S_1}}\right)$

Sol. Answer (2, 4)



$AP = \sqrt{S_1}$

$r = \sqrt{g^2 + f^2 - c}$

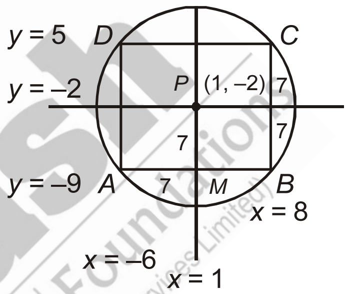
$\cot\frac{\alpha}{2} = \frac{AP}{r} = \frac{\sqrt{S_1}}{\sqrt{g^2 + f^2 - c}}$

$\tan\frac{\alpha}{2} = \frac{\sqrt{g^2 + f^2 - c}}{\sqrt{S_1}}$

2. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y - 93 = 0$ with its sides parallel to the axes of coordinates. The coordinates of the vertices are

(1) (–6, –9) (2) (–6, 5) (3) (8, –9) (4) (8, 5)

Sol. Answer (1, 2, 3, 4)



Centre $P(1, -2)$

$r = \sqrt{1+4+93} = 7 \cdot \sqrt{2}$

Side of square = $r\sqrt{2} = 14$

$A(-6, -9)$

$B(8, -9)$

$C(8, 5)$

$D(-6, 5)$

3. The equation of the tangent to the circle $x^2 + y^2 + 4x - 4y + 4 = 0$ which makes equal intercepts on the coordinate axes is given by

(1) $x - y = 2\sqrt{2}$ (2) $x + y = 2\sqrt{2}$ (3) $x - y + 2\sqrt{2} = 0$ (4) $x + y + 2\sqrt{2} = 0$

Sol. Answer (2, 4)

Equation of a line $x + y = a$ …(i)

Distance of line O from centre = radius

Centre (–2, 2),

radius = $\sqrt{4+4-4} = 2$

$\left|\frac{-2+2+a}{\sqrt{2}}\right| = (2)$

$|a| = 2\sqrt{2}$

Equation of tangent $x + y = \pm 2\sqrt{2}$

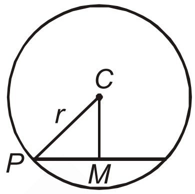
4. If a chord of the circle $x^2 + y^2 - 4x - 2y - k = 0$ is trisected at the points $\left(\frac{1}{3}, \frac{1}{3}\right)$ and $\left(\frac{8}{3}, \frac{8}{3}\right)$, then

(1) Length of the chord = $7\sqrt{2}$ (2) $k = 20$

(3) Radius of the circle = 5 (4) $k = 25$

Sol. Answer (1, 2, 3)

$A\left(\frac{1}{3}, \frac{1}{3}\right)$, $B\left(\frac{8}{3}, \frac{8}{3}\right)$



$$AB = \sqrt{\left(\frac{7}{3}\right)^2 + \left(\frac{7}{3}\right)^2} = \frac{7}{3} \cdot \sqrt{2}$$

Length of chord = $3 \cdot AB = 7 \cdot \sqrt{2}$

Equation of chord $\dfrac{y - \frac{1}{3}}{\frac{8}{3} - \frac{1}{3}} = \dfrac{x - \frac{1}{3}}{\frac{8}{3} - \frac{1}{3}}$

$x - y = 0$

Distance of chord from centre (2, 1)

$CM = \dfrac{2-1}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

Radius of circle $r = \sqrt{5+k}$

$r^2 = PM^2 + CM^2$

$5 + k = \left(\frac{7}{\sqrt{2}}\right)^2 + \frac{1}{2} = 25$

$k = 20$

Radius of circle, $r = \sqrt{5+20} = 5$

5. An equation of a circle through the origin, making an intercept of $\sqrt{10}$ on the line $y = 2x + \frac{5}{\sqrt{2}}$, which subtends an angle of 45° at the origin is

(1) $x^2 + y^2 - 4x - 2y = 0$ (2) $x^2 + y^2 - 2x - 4y = 0$

(3) $x^2 + y^2 + 4x + 2y = 0$ (4) $x^2 + y^2 + 2x + 8y = 0$

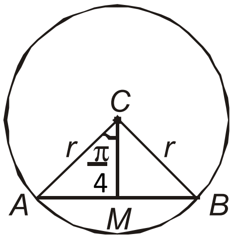
Sol. Answer (2, 4)

Chord subtend 90° angle at the centre of the circle.

$\therefore \quad AB^2 = r^2 + r^2 - 2r \cdot r \cos 90^\circ$

$10 = 2r^2$

$r = \sqrt{5}$



Let centre of circle be (a, b)

$\therefore \quad r = \sqrt{a^2 + b^2} = \sqrt{5}$

$a^2 + b^2 = 5$ ...(i)

$CM = r \cdot \cos 45° = \dfrac{r}{\sqrt{2}}$

$$\frac{2a - b + \frac{5}{\sqrt{2}}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{2}}$$

$$2a - b + \frac{5}{\sqrt{2}} = \frac{5}{\sqrt{2}} \Rightarrow 2a = b \quad \text{...(ii)}$$

Solve Equation (i) and (ii), $a = 1$ and $b = 2$

$\therefore$ Equation of circle $x^2 + y^2 - 2x - 4y = 0$

6. The equation of common tangent to the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$ is

(1) $x = 0$ (2) $y - 4 = 0$ (3) $3x + 4y = 10$ (4) $4x - 3y = 0$

Sol. Answer (1, 2, 3, 4)

$C_1(1, 3)$, $r_1 = 1$

If distance of line from centre = radius.

$\therefore$ Line is a tangent of the circle.

$\therefore$ All lines are the tangents of the circles.

7. If the circles $x^2 + y^2 - 2ax - 2by - c^2 = 0$ and $x^2 + y^2 = K^2$ touch each other, then

(1) $K = \sqrt{a^2+b^2} - \sqrt{a^2+b^2+c^2}$ (2) $K = \sqrt{a^2+b^2+c^2} - \sqrt{a^2+b^2}$

(3) $K = \sqrt{a^2+b^2+c^2} - \sqrt{b^2+c^2}$ (4) $K = \sqrt{a^2+b^2+c^2} - \sqrt{a^2+c^2}$

Sol. Answer (1, 2)

$C_1C_2 = r_1 + r_2,\ C_1(a,\ b)$

$r_1 = \sqrt{a^2 + b^2 + c^2}$

C_2 (0, 0)

$r_2 = k$

$\sqrt{a^2 + b^2} = \sqrt{a^2 + b^2 + c^2} + k$

$k = \sqrt{a^2 + b^2} - \sqrt{a^2 + b^2 + c^2}$

or $C_1C_2 = r_1 - r_2$

$\sqrt{a^2 + b^2} = \sqrt{a^2 + b^2 + c^2} - k$

$k = \sqrt{a^2 + b^2 + c^2} - \sqrt{a^2 + b^2}$

8. $C_1 : x^2 + y^2 = 25$, $C_2 : x^2 + y^2 - 2x - 4y - 7 = 0$ be two circles intersecting at A and B then

(1) Equation of common chord is $x + 2y - 9 = 0$

(2) Equation of common chord is $x + 2y + 7 = 0$

(3) Point of intersection of tangents at A and B to C_1 is $\left(\dfrac{25}{9}, \dfrac{50}{9}\right)$

(4) C_1, C_2 have four common tangents

Sol. Answer (1, 3)

Equation of common chord

$$C_1 - C_2 = 0 \Rightarrow 2x + 4y - 18 = 0$$

$$x + 2y - 9 = 0 \quad \ldots\text{(i)}$$

Let intersection point of tangents be $P(h, k)$.

$\therefore$ Equation of chord of contact $T = 0$

$$xh + yk = 25 \quad \ldots\text{(ii)}$$

Line (i) and (ii) are coincident lines

$$\frac{h}{1} = \frac{k}{2} = \frac{+25}{9}, \ h = \frac{25}{9}, \ k = \frac{50}{9}$$

$$P\left(\frac{25}{9}, \frac{50}{9}\right)$$

9. The positive integral value of λ, for which line $4x + 3y - 16\lambda = 0$ lies between the circles $x^2 + y^2 - 4x - 4y + 4 = 0$ and $x^2 + y^2 - 20x - 2y + 100 = 0$, and does not intersect either of the circles, may be

(1) 27 (2) 30 (3) 33 (4) 36

Sol. Answer (1, 2, 3, 4)

$x^2 + y^2 - 4x - 4y + 4 = 0$ …(1)

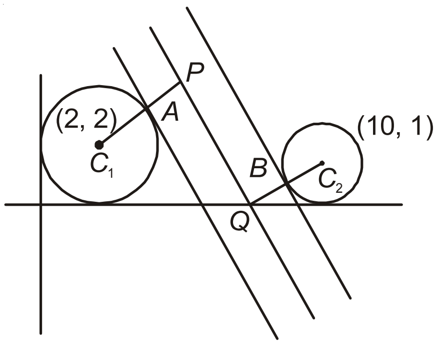
$x^2 + y^2 - 20x - 2y + 100 = 0$ ...(2)

$\because$ λ is positive

$\therefore$ Line intersects x and y axes on positive side i.e. intercepts lies in 1st quadrant.

Clearly from figure it will be between the tangents at A and B to the two circles.

Tangent parallel to $4x + 3y - \lambda = 0$ for circle (1) is



$$\left|\frac{8 + 6 - \lambda}{5}\right| = 2$$

$\Rightarrow$ 14 – = ± 10

$\Rightarrow$ λ = 24, 4

Tangent at A is $4x + 3y - 24 = 0$ …(3)

Tangent at B,

$$\left|\frac{40 + 3 - \lambda}{5}\right| = 1$$

$\Rightarrow$ 43 – = ± 5

$\Rightarrow$ λ = 48, 38

$\therefore$ Tangent at B is $4x + 3y - 38 = 0$

$\therefore$ $24 < \lambda < 38$

10. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts each of the circles $x^2 + y^2 - 4 = 0$, $x^2 + y^2 - 6x - 8y + 10 = 0$ and $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameters, then

(1) $c = -4$ (2) $g + f = c - 1$ (3) $g^2 + f^2 - c = 17$ (4) $gf = 6$

Sol. Answer (1, 2, 3, 4)

If circle S cuts S_1 at the extremities of diameter

∴ Centre of S_1 lie on common chord of the circles

Equation of common chord of S and S_1

$S - S_1 = 0$

$2gx + 2fy + c + 4 = 0$ centre of S_1 (0, 0) lie on this chord

∴ $c = -4$ …(i)

Equation of common chord of S and S_2

$S - S_2 = 0$

$(2g + 6)x + (2f + 8)y + c - 10 = 0$

Centre of S_2 (3, 4) lie on this chord

∴ $3g + 4f + 18 = 0$ …(ii)

Equation of common chord of S and S_3

$(2g - 2)x + (2f + 4)y + c + 2 = 0$

Centre (–1, 2) lie on this chord

$g - 2f - 4 = 0$ …(iii)

Solve Equation (ii) and (iii),

$g = -2$

$f = -3$

∴ All answers are correct.

11. If $(a\cos\theta_1, a\sin\theta_1)$, $(a\cos\theta_2, a\sin\theta_2)$, $(a\cos\theta_3, a\sin\theta_3)$ represents the vertices of an equilateral triangle inscribed in $x^2 + y^2 = a^2$, then

(1) $\Sigma\cos\theta_i = 0$ (2) $\Sigma\sin\theta_i = 0$

(3) $\Sigma\tan\theta_i = 0$ (4) $\Sigma\cot\theta_i = 0$

Sol. Answer (1, 2)

In equilateral Δ centroid and circumcentre are same.

$$\therefore \left(\frac{\Sigma a\cos\theta_i}{3}, \frac{\Sigma a\sin\theta_i}{3}\right) = (0, 0)$$

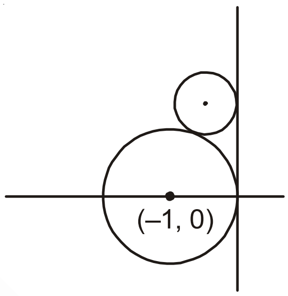
$\Sigma\cos\theta_i = 0, \ \Sigma\sin\theta_i = 0$

12. The locus of the centre of the circle which moves such that it touches the circle $(x + 1)^2 + y^2 = 1$ externally and also the y-axis is

(1) $y^2 = 4x, x \geq 0$ (2) $y^2 = -4x, x \leq 0$

(3) $y = 0, x > 0$ (4) $y = 0, \forall x \in R$

Sol. Answer (2, 3)



Let the centre of moving circle be (h, k).

$\because$ Circle touch externally

$\Rightarrow (h + 1)^2 + k^2 = (1 + |h|)^2$

$\Rightarrow h^2 + 2h + 1 + k^2 = 1 + h^2 + 2|h|$

$\Rightarrow k^2 = 2(|h| - h)$

$$= \begin{cases} -4h \ ; & h \leq 0 \\ 0 \ ; & h > 0 \end{cases}$$

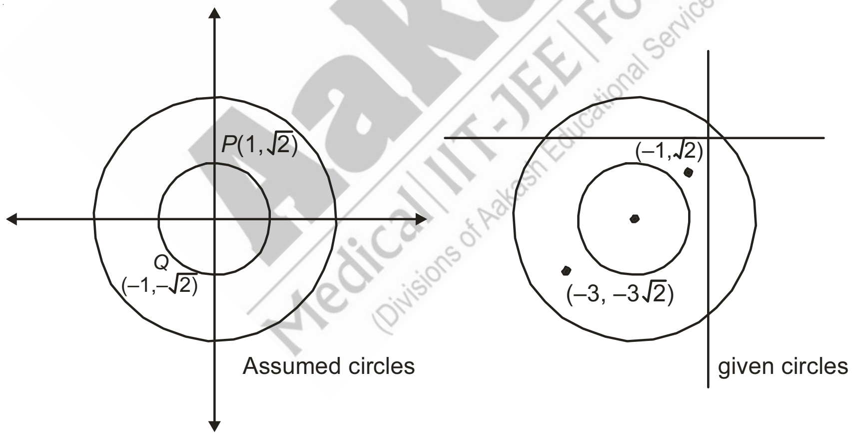
$\therefore$ Locus of (h, k) is $y^2 = -4x, x \leq 0$ and $y = 0$ if $x > 0$ or we can write as locus of (h, k) is the set S where $S = \{(x, y); y^2 = -4x, x \leq 0\} \cup \{(x, 0); x > 0\}$

13. If a point $(a, \sqrt{2}a)$ lies in region bounded between the circles $x^2 + y^2 + 4x + 4y + 7 = 0$ and $x^2 + y^2 + 4x + 4y - 1 = 0$, then the number of integral values of a exceeds

(1) 0 (2) 1 (3) 2 (4) 3

Sol. Answer (1, 2)

Given circles are $(x + 2)^2 + (y + 2)^2 = 1$ and $(x + 2)^2 + (y + 2)^2 = 9$



Our objective is to find number of points $(a, \sqrt{2}a)$ (numbers only not the co-ordinates of point), therefore lets find points $(a, \sqrt{2}a)$ for $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ (concentric and with radii 1 and 3)

$1 < a^2 + 2a^2 < 9$

$\Rightarrow \frac{1}{3} < a^2 < 3$

$\Rightarrow a \in \left(-\sqrt{3}, -\frac{1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}}, \sqrt{3}\right)$

Integral values of a are –1 and +1

$\therefore$ Two integral values of a exist.

14. Tangents and normal are drawn from a point (3, 1) to circle C whose equation is $x^2 + y^2 - 2x - 2y + 1 = 0$. Let the points of contact of tangents be $T_i\,(x_i, y_i)$, where $i = 1, 2$ and feet of normals be N_1 and N_2 (N_1 is near P). Tangents are drawn at N_1 and N_2 and normals are drawn at T_1 and T_2

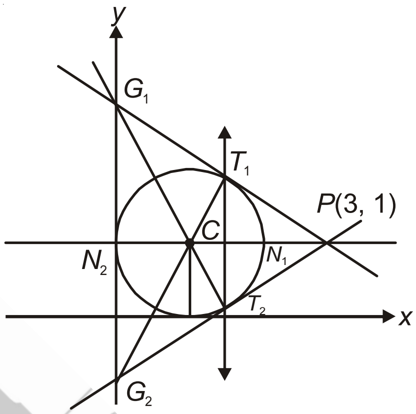
(1) $x_1 + x_2 + y_1 + y_2 = 5$

(2) $x_1x_2y_1y_2 = -\dfrac{9}{16}$

(3) Normal at T_1 and tangents at T_2 and N_2 are concurrent

(4) Circle is incircle of the triangle formed by tangents from P and tangent at N_2

Sol. Answer (1, 3, 4)



Chord of contact of tangents drawn from (3, 1) to the circle

$x^2 + y^2 - 2x - 2y + 1 = 0$ is $3x + y - (x + 3) - (y + 1) + 1 = 0$

$\Rightarrow\ 2x = 3$

$\Rightarrow\ \boxed{x = \dfrac{3}{2}}$ equation of T_1T_2

Points of contact T_1, T_2 are

$$\frac{9}{4} + y^2 - 3 - 2y + 1 = 0$$

$$y^2 - 2y + \frac{1}{4} = 0$$

$$y = \frac{2 \pm \sqrt{4-1}}{2} = 1 \pm \frac{\sqrt{3}}{2}$$

$$\therefore\ T_1 : \left(\frac{3}{2},\ \frac{\sqrt{3}}{2} + 1\right) = (x_1,\ y_1)$$

$$T_2 : \left(\frac{3}{2},\ 1 - \frac{\sqrt{3}}{2}\right) = (x_2, y_2)$$

Option (1) : $x_1 + x_2 + x_3 + x_4 = \dfrac{3}{2} + \dfrac{3}{2} + 1 + \dfrac{\sqrt{3}}{2} + 1 - \dfrac{\sqrt{3}}{2} = 5$ (given options is correct)

Option (2) : $x_1x_2y_1y_2 = \dfrac{9}{4}\left(1 - \dfrac{3}{4}\right) = \dfrac{9}{16}$ (gives options is wrong)

Option (3) : Equation of tangent at N_2 : $x = 0$ … (i)

Equation of tangent at T_2 : $y - 1 = \left(\dfrac{1 - \dfrac{\sqrt{3}}{2} - 1}{\dfrac{3}{2} - 3}\right)(x - 3)$

$$\Rightarrow\quad y - 1 = \frac{1}{\sqrt{3}}(x - 3) \qquad \text{… (ii)}$$

Equation of normal at T_1 : $y - 1 = \dfrac{1 + \dfrac{\sqrt{3}}{2} - 1}{\dfrac{3}{2} - 1}(x - 1)$

$$\Rightarrow\quad y - 1 = \sqrt{3}\,(x - 1) \qquad \text{… (iii)}$$

Clearly (i), (ii) and (iii) are concurrent at $(0, 1-\sqrt{3})$ (Option (3) is correct)

Option (4) : True clear from figure.

[Option (3) and (4) are true only if ΔPG_1G_2 is equilateral $\Delta \Rightarrow \angle$ between tangents from P is $\frac{\pi}{3}$]

Alter : Shift the centre of circle at (0, 0) and solve. Later change to given system. (For option (1) & (2) & (3) and (4) are verified by statement given above)

15. Let C_1 and C_2 be two concentric circles such that the radius of C_2 is double that of radius of C_1. Tangents PQ and PR are drawn from a point on C_2 to C_1.

(1) Centroid of ΔPQR lies on C_1

(2) Orthocentre of ΔPQR lies on C_1

(3) If radius of C_1 is $\sqrt{3}$ then area of ΔPQR is $\frac{9\sqrt{3}}{4}$ sq. units

(4) If radius of C_1 is $\sqrt{3}$ then area of ΔPQR is $\frac{27}{4}$ sq. units

Sol. Answer (1, 2, 3)

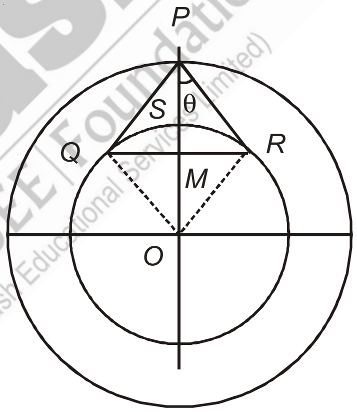
$OR = OQ = r$ (say)

$OP = 2r$

From ΔOPR,

$$PR = \sqrt{4r^2 - r^2} = \sqrt{3}\,r$$

Let $\angle OPR = Q \Rightarrow \tan\theta = \frac{OR}{PR} = \frac{r}{\sqrt{3}r} = \frac{1}{\sqrt{3}}$



$\Rightarrow \quad \theta = \frac{\pi}{6}$

$\therefore \quad \angle QPR = \frac{\pi}{3}$

Also $PQ = PR \Rightarrow \Delta PQR$ is equilateral

$$MR = \frac{1}{2}QR = \frac{1}{2}PR = \frac{\sqrt{3}}{2}r$$

$$\therefore \quad OM = \sqrt{r^2 - \frac{3}{4}r^2} = \frac{r}{2}$$

$\Rightarrow \quad SM = \frac{r}{2},\ PS = r$

S divides PM in the ratio 2 : 1

$\Rightarrow$ Centroid of ΔPQR

Also, since it is equilateral Δ

$\therefore$ Orthocenter is also same.

Area of ΔPQR = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(\sqrt{3}r)^2 = \frac{3\sqrt{3}}{4}r^2$

If $r = \sqrt{3} \Rightarrow$ Area = $\frac{9\sqrt{3}}{4}$ sq. units.

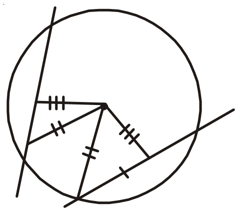
16. Chords are drawn to the circle $x^2 + y^2 - 2x - 2y - 8 = 0$ from the point (–1, –1). If circle cuts off equal intercept on these chords and the line $3x + 4y + 3 = 0$, then the equation of chord(s) may be

(1) $x + 1 = 0$ (2) $x + y + 2 = 0$ (3) $y + 1 = 0$ (4) $2x + y + 3 = 0$

Sol. Answer (1, 3)

Let the line be

$y + 1 = m(x + 1)$



$\because$ Circle cuts equal intercept on this line and the line $3x + 4y + 3 = 0$

$\Rightarrow$ Length of perpendicular from centre (1, 1) to these lines and equal (from figure)

$\Rightarrow \left|\dfrac{2m-2}{\sqrt{1+m^2}}\right| = \left|\dfrac{3+4+3}{\sqrt{9+16}}\right|$

$\Rightarrow 2(m-1) = 2\sqrt{1+m^2}$

$\Rightarrow m^2 + 1 - 2m = 1 + m^2$

$\Rightarrow m = 0$, also one line is vertical ($\because$ m^2 is cancelled)

Chords are $x + 1 = 0$ and $y + 1 = 0$

17. Let a circle cuts orthogonally each of the three circles $x^2 + y^2 + 3x + 4y + 11 = 0$, $x^2 + y^2 - 3x + 7y - 1 = 0$ and $x^2 + y^2 + 2x = 0$

(1) The centre of the circle is (–3, –2)

(2) Radius of the circle is 3

(3) Equation of chord of contact of tangents drawn from (2, 4) is $5x + 6y - 18 = 0$

(4) Length of tangent from (2, 4) to the circle is $\sqrt{13}$

Sol. Answer (1, 2)

(1) R.A. of (1) and (3) is

$x + 4y + 11 = 0$

R.A of (2) and (3) is

$5x - 7y + 1 = 0$

Solving these two equations we get radical centre, which is the cube of required circle.

$$\frac{x}{4+77} = \frac{y}{55-1} = \frac{1}{-7-20}$$

$x = -3,\ y = -2$

$\therefore$ Centre is (–3, –2)

(2) Radius of the circle = length of tangent from (–3, –2) to any of the three circles

$\sqrt{9+4-4} = 3$

(3) $\therefore$ Equation the circle is

$(x + 3)^2 + (y + 2)^2 = 9$

$x^2 + y^2 + 6x + 4y + 4 = 0$

Chord of contact of tangents drawn from (2, 4) is

$2x + 4y + 3(x + 2) + 2(y + 4) + 4 = 0$

$5x + 6y + 18 = 0$

(4) Length of tangent = $\sqrt{4+16+12+16+4} = 2\sqrt{13}$

18. The equation of a circle touching x-axis at (–4, 0) and cutting off an intercept of 6 units on y-axis can be

(1) $x^2 + y^2 + 8x + 12y + 16 = 0$
(2) $x^2 + y^2 + 8x - 12y + 16 = 0$
(3) $x^2 + y^2 + 8x + 10y + 16 = 0$
(4) $x^2 + y^2 + 8x - 10y + 16 = 0$

Sol. Answer (3, 4)

Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

$2\sqrt{g^2 - c} = 0 \Rightarrow g^2 = c$ ...(1)

$2\sqrt{f^2 - c} = 6 \Rightarrow f^2 - c = 9$ ...(2)

Passes through (–4, 0)

$\Rightarrow 16 + 0 - 8g + 0 + c = 0$

Putting $c = g^2$ from (1)

$\Rightarrow 16 - 8g + g^2 = 0$

$\Rightarrow (g - 4)^2 = 0$

$g = 4$

$\Rightarrow c = 16$

$\therefore f^2 = 9 + 16 = 25$

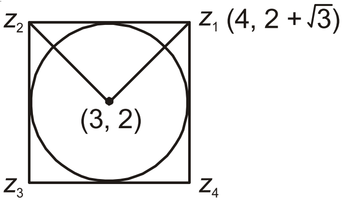
$\Rightarrow f = \pm 5$

$\therefore$ Equations of circles are $x^2 + y^2 + 8x \pm 10y + 16 = 0$.

19. Let one of the vertices of the square circumseribing the circle $x^2 + y^2 - 6x - 4y + 11 = 0$ be $(4,\ 2+\sqrt{3})$. The other vertices of the square may be

(1) $(3-\sqrt{3},\ 3)$
(2) $(2,\ 2-\sqrt{3})$
(3) $(3+\sqrt{3},\ +1)$
(4) (0, 0)

Sol. Answer (1, 2, 3)



$z_2(3+2i) = [4+(2+\sqrt{3})i-(3+2i)]e^{i\frac{\pi}{2}}$

$z_2 = (1+\sqrt{3}i)i + 3 + 2i$

$= -\sqrt{3} + i + 3 + 2i$

$= (3-\sqrt{3}) + 3i$

$z_2 = (3-\sqrt{3},\ 3)$

$$\frac{z_3+z_1}{2}=z_0 \Rightarrow z_3=(6+4i)-(4+(2+\sqrt{3})i)$$

$$= 2+(2-\sqrt{3})i$$

$$\Rightarrow \quad z_3=(2,\ 2-\sqrt{3})$$

$$\frac{z_4+z_2}{2}=z_0 \Rightarrow z_4=(6+4i)-(3-\sqrt{3}+3i)$$

$$= 3+\sqrt{3}+i$$

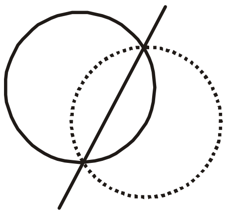
$$\therefore \quad z_4=(3+\sqrt{3},\ 1)$$

20. If $x^2 + y^2 - 2y - 15 + \lambda(2x + y - 9) = 0$ represents family of circles for different values of λ, then the equation of the circle(s) along these circles having minimum radius is/are

(1) $3x^2 + 3y^2 - 2x - 7y - 36 = 0$ (2) $x^2 + y^2 - 2y - 15 = 0$

(3) $5x^2 + 5y^2 - 32x - 26y + 69 = 0$ (4) $x^2 + y^2 - 10x - 7y + 30 = 0$

Sol. Answer (3)



The circle will be of minimum radius if chord $2x + y - 9 = 0$ is diameter

$$x^2 + y^2 + 2\lambda x + y(\lambda - 2) - (15 + 9\lambda) = 0$$

$\Rightarrow$ Centre $\left(-\lambda, \dfrac{2-\lambda}{2}\right)$ lies on $2x + y - 9 = 0$

$$\Rightarrow \quad -2\lambda+\frac{2-\lambda}{2}-9=0$$

$$-5\lambda - 16 = 0$$

$$\Rightarrow \quad \lambda=-\frac{16}{5}$$

$\therefore$ Required circle is

$$x^2+y^2-\frac{32}{5}x-\frac{26}{5}y+\frac{69}{5}=0$$

$$5x^2 + 5y^2 - 32x + 69 = 0$$

21. The straight line $\ell x + my = 1$ intersects $px^2 + 2qxy + ry^2 = s$ at AB. Chord AB subtends a right angle at the origin. If $\ell x + my = 1$ is a tangent to a circle $x^2 + y^2 = a^2$, then $a =$

(1) $\sqrt{\dfrac{s}{p+r}}$ (2) $\sqrt{\dfrac{p+r}{s}}$ (3) $\sqrt{\ell^2+m^2}$ (4) $\dfrac{1}{\sqrt{\ell^2+m^2}}$

Sol. Answer (1, 4)

Combined equation of OA and OB is

$$x^2 + 2qxy + ry^2 = s(lx + my)^2$$

$\because$ OA is perpendicular to OB

$\therefore$ Coefficient of x^2 + co-efficient of y^2 = 0

$$(p - sl^2) + (r - sm^2) = 0$$

$\Rightarrow$ $p + r = s(l^2 + m^2)$ … (1)

$lx + my = 1$ touches $x^2 + y^2 = a^2$

$\therefore \quad \left|\frac{1}{\sqrt{l^2+m^2}}\right| = a$

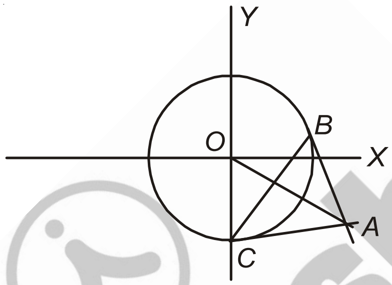
$\therefore \quad a = \frac{1}{\sqrt{l^2+m^2}}$

Also from (1),

$l^2 + m^2 = \frac{p+r}{s}$

$\therefore \quad a = \frac{1}{\sqrt{l^2+m^2}} = \sqrt{\frac{s}{p+r}}$

22. Let the midpoint of the chord of contact of tangents drawn from A to the circle $x^2 + y^2 = 4$ be $M(1, -1)$ and the points of contact be B and C



(1) The area of ΔABC is 2 sq. units
(2) The area of ΔABC is $\frac{1}{2}$ sq. units
(3) Co-ordinate of point A is (2, –2)
(4) ΔABC is right angled triangle

Sol. Answer (1, 3, 4)

Equation of chord BC is

$x - y = 2$ …(1)

Let A be (h, k), the equation BC i.e. chord of contact of tangents drawn from (h, k) is

$hx + ky = 4$ …(2)

From (1) & (2),

$\frac{h}{1} = \frac{k}{-1} = \frac{4}{2}$

$\Rightarrow \quad (h, k) = (2, -2)$

$AM = \left|\frac{2+2-2}{\sqrt{2}}\right| = \sqrt{2}$

$BM = \sqrt{OB^2 - OM^2} = \sqrt{4 - \left|\frac{2}{\sqrt{2}}\right|^2} = \sqrt{2}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}AM \times BC = \frac{1}{2} \times \sqrt{2} \times 2\sqrt{2} = 2$ sq. units

Point (2, –2) lies on director circle $x^2 + y^2 = 8$

$\therefore \quad \angle A = \frac{\pi}{2}$

SECTION - C

Linked Comprehension Type Questions

Comprehension-I

A circle C_1 of radius 2 units rolls on the outerside of the circle $C_2 : x^2 + y^2 + 4x = 0$, touching it externally.

1. The locus of the centre of C_1 is

(1) $x^2 + y^2 + 4y – 12 = 0$ (2) $x^2 + y^2 + 4x – 12 = 0$

(3) $x^2 + y^2 + 4x + 4y – 4 = 0$ (4) $x^2 + y^2 – 4x = 0$

Sol. Answer (2)

Let $C_1(h, k)$, $r_1 = 2$

$C_2(-2, 0)$, $r_2 = \sqrt{4} = 2$

$C_1C_2 = r_1 + r_2$

$\sqrt{(h+2)^2 + k^2} = 2 + 2$

$(h+2)^2 + k^2 = 16$

Locus of (h, k), $x^2 + y^2 + 4x - 12 = 0$

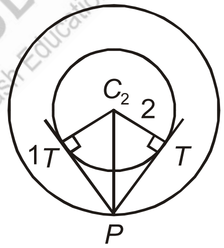
2. Area of a quadrilateral found by a pair of tangents from a point of $x^2 + y^2 + 4x – 12 = 0$ to C_2 with a pair of radii at the points of contact of the tangents is (in sq. units)

(1) $2\sqrt{3}$ (2) $4\sqrt{3}$ (3) $\sqrt{3}$ (4) $3\sqrt{3}$

Sol. Answer (2)

Let $P(x, y)$ be any point on $x^2 + y^2 + 4x - 12 = 0$

Length of tangent from $P(x, y)$ to $x^2 + y^2 + 4x = 0$



$PT = \sqrt{x^2 + y^2 + 4x} = \sqrt{12} = 2\sqrt{3}$

Area of $\Delta PTC_2 = \frac{1}{2} \cdot 2\sqrt{3} \cdot 2 = 2\sqrt{3}$

$\therefore$ Area of quadrilateral $= 2\,(2\sqrt{3}) = 4\sqrt{3}$ sq. units

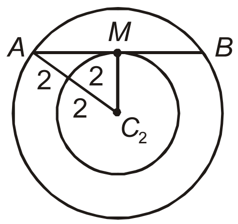
3. Square of the length of the intercept made by $x^2 + y^2 + 4x – 12 = 0$ on any tangent to C_2 is

(1) 12 (2) 24 (3) 16 (4) 48

Sol. Answer (4)

$AC_2 = 4$

$MC_2 = 2$



$\therefore$ $AM = \sqrt{16 - 4} = 2\sqrt{3}$

$AB = 4\sqrt{3}$

$AB^2 = 48$

Comprehension-II

$C : x^2 + y^2 - 2x - 2ay - 8 = 0$, a is a variable.

1. C represents a family of circles passing through two fixed points whose co-ordinates are

(1) (–2, 0), (4, 0) (2) (2, 0), (4, 0) (3) (–4, 0), (4, 0) (4) (2, 0), (–4, 0)

Sol. Answer (1)

$(x^2 + y^2 - 2x - 8) - 2ay = 0$

Represents family of circles which passes through the fixed point.

Point of intersection of $x^2 + y^2 - 2x - 8 = 0$ and $y = 0$.

Solve the equation $x^2 - 2x - 8 = 0 \Rightarrow x = 4, -2$ fixed points are (–2, 0) and (4, 0).

2. Equation of a circle C_1 of this family tangents to which at these fixed points intersects on the line $2y + x + 5 = 0$ is

(1) $x^2 + y^2 - 2x - 8y - 8 = 0$ (2) $x^2 + y^2 - 2x + 6y - 8 = 0$

(3) $x^2 + y^2 - 2x + 8y - 8 = 0$ (4) $x^2 + y^2 - 2x - 6y - 8 = 0$

Sol. Answer (4)

Fixed points are $A(-2, 0)$ and $B(4, 0)$.

Let line AB passes through a fixed point $P(h, k)$.

Equation of AB is $y = 0$

$P(h, k)$ lie on $y = 0$

$\therefore\ k = 0$

Tangents at A and B intersects on polar.

$\therefore$ Equation of polar $T = 0$

$$x \cdot h + y \cdot k - (x + h) - a(y + k) - 8 = 0$$

Put $k = 0$

$$\therefore\ (h-1)x - ay - (h+8) = 0 \quad \ldots(i)$$

Equation of polar

$$x + 2y + 5 = 0 \quad \ldots(ii)$$

These lines are coincident lines

$$\frac{h-1}{1} = \frac{-a}{2} = \frac{-(h+8)}{5}$$

$$5h - 5 = -h - 8$$

$$6h = -3 \qquad \frac{-a}{2} = -\frac{3}{2}$$

$$h = -\frac{1}{2} \qquad a = 3$$

$\therefore$ Equation of C_1

$$x^2 + y^2 - 2x - 6y - 8 = 0$$

3. If the chord joining the fixed points subtends an angle θ at the centre of the circle C_1 then θ equals

(1) 30° (2) 45° (3) 60° (4) 90°

Sol. Answer (4)

$A(-2, 0)$ and $B(4, 0)$

Centre $C_1(1, 3)$

Slope of $AC_1 = \dfrac{3-0}{1+2} = 1$

Slope of $BC_1 = \dfrac{3-0}{1-4} = \dfrac{3}{-3} = -1$

$\therefore$ Fixed points AB subtends 90° at the centre.

Comprehension-III

Let QR be the chord of contact of tangents drawn from $P(2, 0)$ to the circle $x^2 + y^2 = 1$. Let S be the orthocentre of the ΔPQR

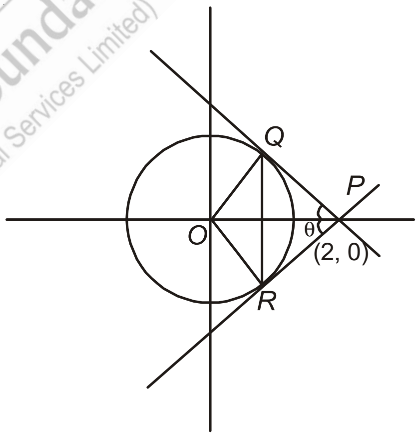
1. S lies

(1) Inside the circle (2) On the circle

(3) Outside the circle (4) Cannot be determined with given data

Sol. Answer (2)

Chord of contact of tangents from (2, 0) to $x^2 + y^2 = 1$ is $2x + 0 = 1$



$\Rightarrow$ $\boxed{x = \dfrac{1}{2}}$ $\rightarrow$ Equation of QR

$Q:\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$, $R:\left(\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$

Slope of $PR = \dfrac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow$ $\theta = \dfrac{\pi}{6}$ (from figure)

$\therefore$ $\angle QPR = \dfrac{\pi}{3}$

$\Rightarrow$ ΔPQR is equilateral

$\therefore$ Orthocenter = Centroid

$$= \left(\frac{\frac{1}{2}+\frac{1}{2}+2}{3}, \frac{\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}+0}{3}\right)$$

$= (1, 0)$

$\Rightarrow$ Lies on circle

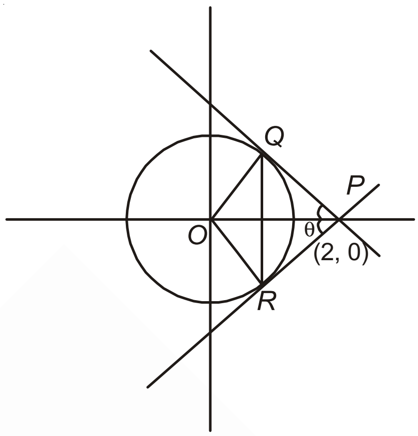
2. The orthocentre of the ΔPQS is

(1) Origin (2) R (3) Inside the ΔPQS (4) Inside the circle

Sol. Answer (2)

Chord of contact of tangents from (2, 0) to $x^2 + y^2 = 1$ is $2x + 0 = 1$

$\Rightarrow \boxed{x = \frac{1}{2}} \rightarrow$ Equation of QR



$Q:\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $R:\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Slope of $PR = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}$

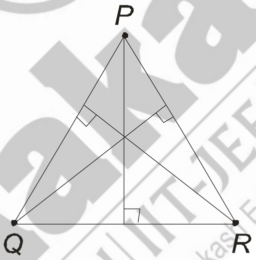
$\Rightarrow \theta = \frac{\pi}{6}$ (from figure)

$\therefore \angle QPR = \frac{\pi}{3}$

$\Rightarrow \Delta PQR$ is equilateral

In a ΔPQR if S is orthocenter then orthocenter of Δ formed by any two vertices of ΔPQR and S is the third vertex

$\Rightarrow$ Orthocenter of ΔPQS is R



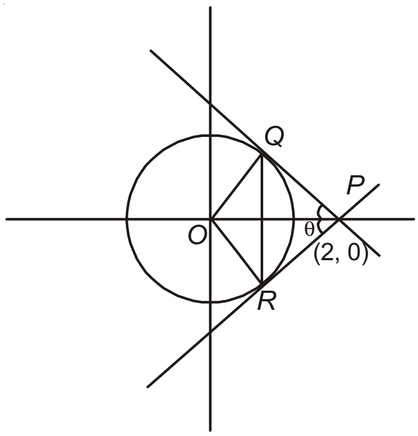
3. The angle subtended by the chord QR at the centre of the circle is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) $\frac{2\pi}{3}$

Sol. Answer (4)

Chord of contact of tangents from (2, 0) to $x^2 + y^2 = 1$ is $2x + 0 = 1$

$\Rightarrow \boxed{x = \frac{1}{2}} \rightarrow$ Equation of QR



$Q:\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $R:\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Slope of $PR = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \frac{\pi}{6}$ (from figure)

$\therefore \quad \angle QPR = \frac{\pi}{3}$

$\Rightarrow \quad \Delta PQR$ is equilateral

Angle subtended by chord QR is $\angle QOR = \pi - \angle QPR$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Comprehension-IV

Let the circle $x^2 + y^2 + 2ax + 2by + c = 0$ is orthogonal to each of the circles $x^2 + y^2 + 4x + 2y + 1 = 0$, $2x^2 + 2y^2 + 8x + 6y - 3 = 0$, $x^2 + y^2 + 6x - 2y - 3 = 0$

1. The value of $\frac{|abc|}{7}$ is

(1) 85 (2) 83 (3) 81 (4) None of these

Sol. Answer (1)

Radical centre of the 3 circle is the centres of the circle

$\therefore$ Radical axis of $x^2 + y^2 + 4x + 2y + 1 = 0$ and $x^2 + y^2 + 6x - 2y - 3 = 0$ is $2x - 4y - 4 = 0$

$\Rightarrow \quad x - 2y - 2 = 0 \qquad \ldots(1)$

Radical axis of $x^2 + y^2 + 4x + 2y + 1 = 0$ and $x^2 + y^2 + 4x + 3y - \frac{3}{2} = 0$ is

$\Rightarrow \quad y - \frac{5}{2} = 0 \qquad \ldots(2)$

From (1) and (2),

Radical centre is $\left(7, \frac{5}{2}\right)$

Radius = $\sqrt{49 + \frac{25}{4} + 28 + 5 + 1} = \sqrt{\frac{357}{4}}$

Circle is $(x - 7)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{357}{4}$

$x^2 + y^2 - 14x - 5y - 34 = 0$

$a = -7, \; b = -\frac{5}{2}, \; c = -34$

$$\frac{|abc|}{7} = \frac{7 \times \frac{5}{2} \times 34}{7} = 85$$

2. The centre of the circle

(1) Is $\left(7, \frac{5}{2}\right)$ (2) Lies on $x - 2y + 2 = 0$

(3) Is (7, 5) (4) Lies on $3x - 2y - 16 = 0$

Sol. Answer (1, 4)

Radical centre of the 3 circle is the centres of the circle

$\therefore$ Radical axis of $x^2 + y^2 + 4x + 2y + 1 = 0$ and $x^2 + y^2 + 6x - 2y - 3 = 0$ is $2x - 4y - 4 = 0$

$\Rightarrow$ $x - 2y - 2 = 0$ …(1)

Radical axis of $x^2 + y^2 + 4x + 2y + 1 = 0$ and $x^2 + y^2 + 4x + 3y - \frac{3}{2} = 0$ is

$\Rightarrow$ $y - \frac{5}{2} = 0$ …(2)

From (1) and (2),

Radical centre is $\left(7, \frac{5}{2}\right)$

Radius = $\sqrt{49 + \frac{25}{4} + 28 + 5 + 1} = \sqrt{\frac{357}{4}}$

Circle is $(x - 7)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{357}{4}$

$x^2 + y^2 - 14x - 5y - 34 = 0$

Center is $\left(7, \frac{5}{2}\right)$ it lies on $3x - 2y - 16 = 0$

$21 - 5 - 16 = 0$

3. Line $3x - 4y + 11 = 0$ is

(1) A tangent to the circle

(2) A chord of the circle

(3) A diameter of the circle

(4) Is no contact with the circle

Sol. Answer (3)

Radical centre of the 3 circle is the centres of the circle

$\therefore$ Radical axis of $x^2 + y^2 + 4x + 2y + 1 = 0$ and $x^2 + y^2 + 6x - 2y - 3 = 0$ is $2x - 4y - 4 = 0$

$\Rightarrow$ $x - 2y - 2 = 0$ …(1)

Radical axis of $x^2 + y^2 + 4x + 2y + 1 = 0$ and $x^2 + y^2 + 4x + 3y - \frac{3}{2} = 0$ is

$\Rightarrow$ $y - \frac{5}{2} = 0$ …(2)

From (1) and (2),

Radical centre is $\left(7, \frac{5}{2}\right)$

Radius = $\sqrt{49 + \frac{25}{4} + 28 + 5 + 1} = \sqrt{\frac{357}{4}}$

Circle is $(x-7)^2 + \left(y-\frac{5}{2}\right)^2 = \frac{357}{4}$

$x^2 + y^2 - 14x - 5y - 34 = 0$

Line $3x - 4y - 11 = 0$

Length of perpendicular from centre to the line = $\left|\frac{21-10-11}{5}\right| = 0$

$\therefore$ Line is a diameter.

Comprehension-V

L_1 is a line intersecting x and y-axis at $P(a, 0)$ and $Q(0, b)$. L_2 is a line perpendicular to L_1 intersecting x and y-axis at R and S respectively.

1. Locus of the point of intersection of the lines PS and QR is a circle passing through

(1) (0, 0) (2) (a, a) (3) (b, b) (4) (b, a)

Sol. Answer (1)

$P(a, 0)$, $Q(0, b)$

Slope of $PQ = -\frac{b}{a}$ $\therefore$ Slope of $L_2 = \frac{a}{b}$

Equation of L_2, $y = \frac{a}{b}x + \lambda$ $\therefore$ $R\left(\frac{-b\lambda}{a}, 0\right)$ and $S(0, \lambda)$

Equation of PS, $\frac{x}{a} + \frac{y}{\lambda} = 1$

$\frac{y}{\lambda} = 1 - \frac{x}{a}$ ...(i)

Equation of QR, $\frac{x \cdot a}{-b\lambda} + \frac{y}{b} = 1$

$\frac{xa}{b\lambda} = \frac{y}{b} - 1$ ...(ii)

Eliminate λ

$\therefore$ $\frac{yb}{xa} = \frac{(a-x)b}{(y-b)a}$

$y(y-b) = x(a-x)$

$x^2 + y^2 - ax - by = 0$

Circle passes through (0, 0) and (a, b).

2. Common chord of the circles on *QS* and *PR* as diameters passes through the point (*a*, *b*) if

(1) $a = 2b$ (2) $2a = b$ (3) $a = b$ (4) $2a = -b$

Sol. Answer (3)

$Q(0, b)$, $S(0, \lambda)$

Equation of circle $x \cdot x + (y - b)(y - \lambda) = 0$

$$x^2 + y^2 - (b + \lambda)y + b\lambda = 0 \quad \ldots\text{(i)}$$

$$P(a, 0),\ R\left(-\frac{b\lambda}{a}, 0\right)$$

$$(x - a)\left(x + \frac{b\lambda}{a}\right) + y^2 = 0$$

$$x^2 + y^2 + \left(\frac{b\lambda}{a} - a\right)x - b\lambda = 0 \quad \ldots\text{(ii)}$$

Equation of common chord $S_1 - S_2 = 0$

$$-(b + \lambda)y - \left(\frac{b\lambda}{a} - a\right)x + 2b\lambda = 0$$

$$\lambda\left(2b - y - \frac{b}{a}x\right) + (ax - by) = 0$$

Common chord passes through (*a*, *b*).

$$\lambda(2b - b - b) + (a^2 - b^2) = 0$$

$$a^2 = b^2$$

$$a = b$$

3. If the area of ΔORS in 4 times the area of ΔOPQ then equation of *PS* is, where *O* is the origin

(1) $x + 2y = 2b$ (2) $2x + y = 2a$ (3) $x - 2y = -2b$ (4) $2x - y + 2a = 0$

Sol. Answer (2)

$\Delta ORS = 4\ \Delta OPQ$

$$\frac{1}{2}\left(\frac{b\lambda}{a}\right)\lambda = 4 \cdot \frac{1}{2} a \cdot b$$

$$\lambda^2 = 4a^2 \Rightarrow \lambda = 2a$$

$$P(a, 0),\ S(0, \lambda)$$

$$P(a, 0),\ S(0, 2a)$$

Equation of *PS*, $\dfrac{x}{a} + \dfrac{y}{2a} = 1$

$$2x + y = 2a$$

SECTION - D

Matrix-Match Type Questions

1. If $S \equiv x^2 + y^2 + x - y - 2 = 0$, then

Column-I	Column-II
(A) (–2, 1) lies	(p) On S
(B) (2, –1) lies	(q) Outside S
(C) (0, 1) lies	(r) On the tangent at (1, 0) to S
(D) (2, 3) lies	(s) Inside the circle S

Sol. Answer : A(p), B(q), C(s), D(q, r)

(A) (–2, 1), $S_1 = 0$ lie on the circle

(B) (2, –1), $S_1 = 6$ lie outside the circle

(C) (0, 1), $S_1 = -2$ lie inside the circle

(D) Equation of tangent at (1, 0)

$3x - y - 3 = 0$

(2, 3) lie on this tangent.

2. $x^2 + y^2 - 14x - 10y + 24 = 0$, makes an

Column-I	Column-II
(A) Intercept on x-axis of length	(p) 0
(B) Intercept on y-axis of length	(q) 2
(C) Intercept on $y = x$ of length	(r) $8\sqrt{3}$
(D) Intercept on $7x + y - 4 = 0$ of length	(s) 10

Sol. Answer : A(s), B(q), C(r), D(p)

$C\,(7, 5),\ r = \sqrt{49+25-24} = 5\sqrt{2}$

(A) Intercept on x-axis = $2\sqrt{g^2 - C} = 2\sqrt{49-24} = 10$

(B) Intercept on y-axis = $2\sqrt{f^2 - C} = 2\sqrt{25-24} = 2$

(C) Put $y = x$ in the eq. of circle $x^2 - 12x + 12 = 0$

Let x_1, x_2 are the roots.

$x_1 + x_2 = 12$

$x_1 x_2 = 12$

Let $y = x$ intersect the circle at $A(x_1 y_1)$ and $B(x_2 y_2)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (x_2 - x_1)^2}$$

$$= \sqrt{(x_2 - x_1)^2} \cdot \sqrt{2}$$

$$= \sqrt{2} \cdot \sqrt{(x_1 + x_2)^2 - 4x_1x_2}$$

$$= \sqrt{2} \cdot \sqrt{144 - 48 = \sqrt{2}\sqrt{96}}$$

$$= \sqrt{2} \cdot 4\sqrt{6} = 8\sqrt{3} \quad \begin{bmatrix} \because & y = x \\ \therefore & y_1 - y_1 = x_2 - x_1 \end{bmatrix}$$

(D) $CM = \dfrac{49 + 5 - 4}{\sqrt{49 + 1}} = \sqrt{50} \; = 5\sqrt{2} = r$

$\therefore$ Line $7x + y - 4 = 0$ is a tangent.

$\therefore$ Intercept on this line is equal to zero.

3. If $S \equiv (x - 2)^2 + (y + 1)^2 = 1$ is a circle, then the equation of a

Column-I	**Column-II**
(A) A Tangent to S	(p) $3x + 4y - 2 = 0$
(B) A Diameter of S	(q) $x = 0$
(C) A Line perpendicular to a tangent to S	(r) $y = 0$
(D) A Chord of S	(s) $x - y - 2 = 0$

Sol. Answer : A(r), B(p), C(p, q, r, s), D(p, s)

$C(2, -1)$, $r = 1$

(A) Distance of $y = 0$ from $C(2, -1)$ is equal to radius.

$\therefore$ $y = 0$ is a tangent.

(B) Centre $(2, -1)$ lie on $3x + 4y - 2 = 0$.

$\therefore$ $3x + 4y - 2 = 0$ is a diameter.

(C) $y = 0$ is a tangent.

$\therefore$ Perpendicular line to this tangent is $x = 0$.

(D) Distance of line $x - y - 2 = 0$ from centre $= \dfrac{2 + 1 - 2}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} < 1$ (radius)

$\therefore$ $x - y - 2 = 0$ is a chord.

4. Match column-I to column-II according to given conditions.

Column-I	**Column-II**
(A) Length of common tangents of $x^2 + y^2 = 1$ and $x^2 + y^2 - 10x + 21 = 0$ is	(p) $2\sqrt{6}$
(B) Length of common tangents of $x^2 + y^2 = 1$ and $x^2 + y^2 - 4x + 3 = 0$ is	(q) 4
(C) Length of common tangents of $x^2 + y^2 = 1$ and $x^2 + y^2 - 2x - 3 = 0$ is always less than	(r) 2
(D) Length of common tangents of $x^2 + y^2 = 1$ and $x^2 + y^2 - 2x = 0$ is always less than	(s) 0
	(t) 1

Sol. Answer A(p, q), B(r, s), C(p, q, r, t), D(p, q, r, t)

(A) $C_1 = (0, 0)$, $C_2 = (5, 0)$, $r_1 = 1$, $r_2 = 2$

$C_1C_2 = 5$, $r_1 + r_2 = 3$, $|r_1 - r_2| = 1$

$C_1C_2 > r_1 + r_2$, Hence there exists four common tangents.

Length of direct common tangents = $\sqrt{(C_1C_2)^2 - (r_1 - r_2)^2} = \sqrt{24} = 2\sqrt{6}$

Length of transverse common tangent = $\sqrt{(C_1C_2)^2 - (r_1 + r_2)^2} = 4$

(B) $C_1 = (0, 0)$, $C_2 = (2, 0)$, $r_1 = 1$, $r_2 = 1$, $C_1C_2 = r_1 + r_2$

Hence circles externally touches

Length of direction common tangent $= \sqrt{(C_1C_2)^2 - (r_1 - r_2)^2} = \sqrt{2}$

Length of transverse common tangent is 0.

(C) $C_1 = (0, 0)$, $C_2 = (1, 0)$, $r_1 = 1$, $r_2 = 2$

$\Rightarrow$ $C_1C_2 = |r_2 - r_1|$

Hence circles internally touches. Hence the length of direct tangent is zero.

Transverse common tangent in this case does not exist.

(D) $C_1 = (0, 0)$, $C_2 = (0, 0)$

Circles are concentric hence no common tangent exist. Hence length is zero.

5. Match column-I to column-II according to the given conditions.

Column-I	Column-II
(A) The area of triangle formed by the tangents and the chord of contact drawn from (2, 3) to the circle $x^2 + y^2 = 1$, is	(p) $\frac{12\sqrt{12}}{13}$
(B) The minimum distance of (0, 0) from the circle $x^2 + y^2 - 10x + 24 = 0$ is	(q) 4
(C) The radius of smallest circle passing through (4, 0) and (0, 4) is	(r) 6
(D) The maximum number of normals of a circle passing through the centre is always greater than	(s) $2\sqrt{2}$
	(t) 1

Sol. Answer A(p), B(q), C(s), D(p, q, r, s, t)

(A) Equation of chord of contact $\Rightarrow$ $2x + 3y = 1$

Length of chord of contact = $\sqrt{\frac{12}{13}}$

Length of perpendicular from (2, 3) to chord of contact = $\frac{12}{\sqrt{13}}$

Area of triangle = $\frac{12\sqrt{12}}{13}$

(B) $C \equiv (5, 0)$

Let $P \equiv (0, 0)$

$CP = 5$

Radius $r = 1$

Minimum distance = 5 – 1 = 4

Maximum distance = 1 + 5 = 6

Hence the distance varies from 4 to 6.

(C) Radius = $\frac{4\sqrt{2}}{2} = 2\sqrt{2}$, for smallest circle (4, 0) and (0, 4) will be the end of diameter.A(p),

B(q, r), C(s), D(p, q, r, s, t)

(D) From centre infinite normals can be drawn.

6. Match the following columns

Column-I	**Column-II**
(A) A circle is inscribed in an equilateral triangle of side *a*, then the area of any square inscribed in the circle is	(p) $\frac{a^2}{6}$
(B) The equation of circle $x^2 + y^2 = 4a^2$ with origin as centre, passing through the vertices of an equilateral triangle whose median is of length	(q) a
(C) If a chord of the circle $x^2 + y^2 = \frac{9a^2}{2}$ makes equal intercept of length *k* on the co-ordinate axes, then the value of *k* can be	(r) $2a$
(D) If the line $hx + ky = 1$ touches $x^2 + y^2 = \frac{1}{4a^2}$, then the locus of the point (h, k) is a circle of radius	(s) $3a$

Sol. Answer A(p), B(s), C(p, q, r, s), D(r)

(A) $r = \frac{\Delta}{s} = \frac{a}{2\sqrt{3}}$

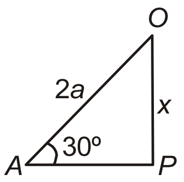
$\therefore$ Area of square inscribed = $\frac{2a^2}{12} = \frac{a^2}{6}$

(B) In ΔOAP,

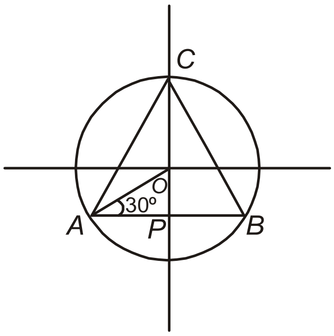
$$\frac{x}{2a} = \sin 30^\circ$$

$$x = 2a \times \frac{1}{2}$$

$x = a$



(Let)



Length of medium $CO + OP = 2a + a$

$= 3a$

(C) Let the equation of the chord $x \pm y = \pm k$ and the length of the perpendicular from the centre (0, 0) of the circle $x^2 + y^2 = \frac{9a^2}{2}$ must be less than the radius $\frac{3a}{\sqrt{2}}$ of the circle

$$\Rightarrow \frac{|\pm k|}{\sqrt{1+1}} \le \frac{\sqrt{a}}{\sqrt{2}}a$$

$$\frac{|k|}{\sqrt{2}} < \frac{3}{\sqrt{2}}a$$

$$k < 3a$$

(D) Since $hx + ky = 1$ touches $x^2 + y^2 = \frac{1}{4a^2}$

So, $\left|\frac{-1}{\sqrt{h^2+k^2}}\right| = \frac{1}{2a}$

$h^2 + k^2 = 4a^2$

So locus of (h, k) is $x^2 + y^2 = 4a^2$ which is a circle of radius $2a$.

SECTION - E

Assertion-Reason Type Questions

1. STATEMENT-1 : If two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other then $f'g = fg'$.

and

STATEMENT-2 : Two circles touch each other if distance between their centres is equal to difference of their radii.

Sol. Answer (1)

$C_1(-g, -f)$, $C_2(-g', -f')$

$r_1 = \sqrt{g^2+f^2}$, $r_2 = \sqrt{g'^2+f'^2}$

Circles touch each other

$$C_1C_2 = r_1 + r_2$$

$$\sqrt{(g-g')^2+(f-f')^2} = \sqrt{g^2+f^2} + \sqrt{g'^2+f'^2}$$

Squaring both sides, we get

$$-2gg' - 2ff' = 2\sqrt{g^2+f^2} \cdot \sqrt{g'^2+f'^2}$$

Again squaring both sides, we get

$$2gg'ff' = g^2f'^2 + f^2g'^2$$

$$(gf' - fg')^2 = 0$$

$$gf' = fg'$$

$\therefore$ Statement-2 is correct explanation for statement-1.

2. STATEMENT-1 : The farthest point on the circle $x^2 + y^2 - 2x - 4y + 4 = 0$ from (0, 0) is (1, 3).

and

STATEMENT-2 : The farthest and nearest points on a circle from a given point are the end points of the diameter through the point.

Sol. Answer (4)

Centre of circle (1, 2) and $r = \sqrt{1+4-4} = 1$

Farthest point on the circle from origin lie on a line joining O (0, 0) centre C (1, 2), i.e., $y = 2x$

But (1, 3) does not lie on $y = 2x$.

Statement-1 is false

3. STATEMENT-1 : Let $x^2 + y^2 = a^2$ and $x^2 + y^2 - 6x - 8y - 11 = 0$ be two circles. If $a = 5$, then two common tangents are possible.

and

STATEMENT-2 : If two circles are intersecting then they have two common tangents.

Sol. Answer (1)

$C_1(0, 0),\ r_1 = a$

$C_2(3, 4),\ r_2 = 6$

Condition of intersecting circles

$r_1 \sim r_2 < c_1c_2 < r_1 + r_2$

$|a - b| < 5 < a + 6 \quad \Rightarrow \quad -1 < a < 11$

$\therefore$ If $a = 5$, two common tangents are possible.

4. STATEMENT-1 : The shortest distance of the point (1, 1) form the circle $x^2 + y^2 - 4x - 6y + 4 = 0$ is $3 - \sqrt{5}$.

and

STATEMENT-2 : Shortest distance of a point = distance of the point from the centre – radius of the circle.

Sol. Answer (3)

Since point lies inside the circle.

5. STATEMENT-1 : Number of circle passing through (0, 3), (–1, 2) and (2, 5) is 1.

and

STATEMENT-2 : Through three non-collinear points, an unique circle can be drawn.

Sol. Answer (4)

Given points are collinear.

6. STATEMENT-1 : If n circles ($n \geq 3$), no two circles are con-centric and no three centre are collinear and number of radical centre is equal to number of radical axes, then $n = 5$.

and

STATEMENT-2 : If no three centres are collinear and no two circles are concentric, then number of radical centre is ${}^nC_3$ and number of radical axes is ${}^nC_2$.

Sol. Answer (1)

Statement-2 is true.

Now, ${}^nC_3 = {}^nC_2$

$n = 3 + 2 = 5$

7. STATEMENT-1 : Area of an equilateral triangle inscribed in the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $\frac{3\sqrt{3}}{4}(g^2 + f^2 - c)$.

and

STATEMENT-2 : Area of an equilateral triangle is $\frac{\sqrt{3}}{4}a^2$.

Sol. Answer (2)

Side of triangle = $2R \cos 30^\circ = \sqrt{3}\left(\sqrt{g^2 + f^2 - c}\right)$

8. STATEMENT-1 : If O is the origin and OP and OQ are tangents to the circle $x^2 + y^2 + 2x + 4y + 1 = 0$, the circumcentre of the triangle is $\left(\frac{-1}{2}, -1\right)$.

and

STATEMENT-2 : $OP.OQ = PQ^2$.

Sol. Answer (3)

Centre of circumcircle is mid point of O and centre of circle $x^2 + y^2 + 2x + 4y + 1 = 0$ *i.e.* $\left(-\frac{1}{2}, -1\right)$.

9. Let C_1 and C_2 be two given circles and C be a moving circle which touches both the circles.

STATEMENT-1 : The locus of centre of circle C must be an ellipse.

and

STATEMENT-2 : The locus of a moving point whose sum of distances from two given points is always constant, is called an ellipse.

Sol. Answer (4)

Let centres of circles C, C_1 and C_2 are C, C_1 and C_2 and radius r, r_1 and r_2 respectively

$C_1C = r + r_1$ …(i)

$C_2C = r + r_2$ …(ii)

r is variable

(i) – (ii),

$C_1C - C_2C = r_1 - r_2$

Locus of C is a hyperbola.

$\therefore$ Statement-1 is false, statement-2 is true.

10. STATEMENT-1 : A circle of smallest radius passing through A and B must be of radius $\frac{1}{2}$ AB.

and

STATEMENT-2 : A straight line is a shortest distance between two points.

Sol. Answer (2)

A circle of smallest radius passing through A and B.

Diameter = AB

$2r = AB$

$r = \frac{1}{2} \cdot AB$

$\therefore$ Statement-2 is not a correct explanation for statement-1.

SECTION - F

Integer Answer Type Questions

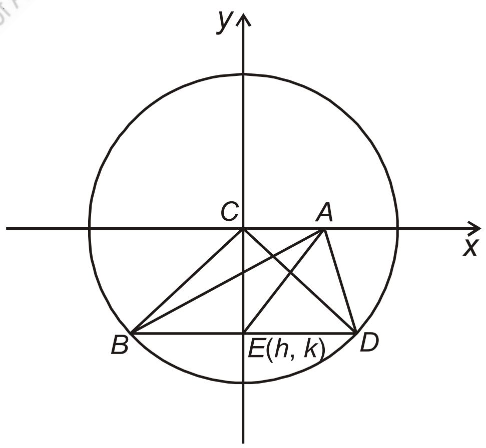
1. If the locus of middle points of chords of the circle $x^2 + y^2 = 4$, which subtend a right angle at the point $(a, 0)$ is $x^2 + y^2 - ax + \frac{a^2 - p}{2} = 0$, then the value of p is

Sol. Answer (4)

Let middle point $(h, k) \equiv E$

As we know that the middle point of hypotenuse in a right angle triangle is equidistant from all vertices.

$\Rightarrow$ $AE = BE = DE = (h - a)^2 + (k - 0)^2$



Now in triangle ECD,

$EC^2 + ED^2 = CD^2$

$\Rightarrow$ $(h^2 + k^2) + (h - a)^2 + k^2 = 4$

$\Rightarrow$ $h^2 + k^2 + h^2 + a^2 - 2ha + k^2 = 4$

$2(h^2 + k^2) - 2ah + a^2 - 4 = 0$

$$h^2 + k^2 - \frac{2ah}{2} + \frac{a^2 - 4}{2} = 0$$

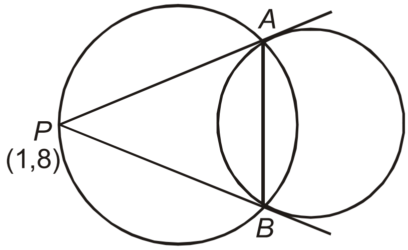
Locus of (h, k) is $x^2 + y^2 - ax + \frac{a^2 - 4}{2} = 0$

$\Rightarrow$ $p = 4$

2. Tangents are drawn from the point P (1, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. If the radius of circumcircle of triangle PAB is k then the value of $[k]$, where [ ] represents the greatest integer function.

Sol. Answer (3)

Equation of AB is given by



$$x.1 + y.8 - 6\left(\frac{x+1}{2}\right) - 4\left(\frac{y+8}{2}\right) - 11 = 0$$

$x + 8y - 3(x + 1) - 2(y + 8) - 11 = 0$

$x + 8y - 3x - 3 - 2y - 16 - 11 = 0$

$6y - 2x - 30 = 0$

$3y - x - 15 = 0$

$x - 3y + 15 = 0$ ...(i)

Equation of circumcircle of triangle PAB is

$x^2 + y^2 - 6x - 4y - 11 + \lambda(x - 3y + 15) = 0, \lambda \in R$ …(ii)

Putting $x = 1$, $y = 8$

$1 + 64 - 6 - 32 - 11 + \lambda(1 - 24 + 11) = 0$

$\Rightarrow$ $16 - 8\lambda = 0$ $\Rightarrow$ $\lambda = 2$

Putting $\lambda = 2$ in (ii),

$x^2 + y^2 - 6x - 4y + 2(x - 3y + 15) - 11 = 0$

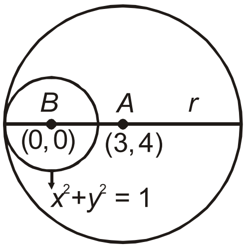
$x^2 + y^2 - 4x - 10y + 19 = 0$

Radius $k = \sqrt{4 + 25 - 19} = \sqrt{10}$ $\Rightarrow k = \sqrt{10}$

$[k] = [\sqrt{10}] = 3$

3. If the centre of a circle is (3, 4) and its size is just sufficient to contain to circle $x^2 + y^2 = 1$, then the radius of the required circle is

Sol. Answer (6)



In this case, the figure may be shown as following.

In this case, the radius of required circle is

$AB + 1 = 5 + 1 = 6$

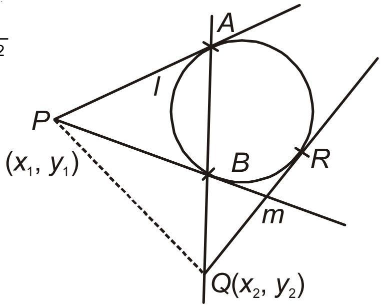
4. Two tangents are drawn from a point P to the given circle each of length l units. These tangents touch the circle at the points A and B. Two tangents are drawn again from at point Q on the chord of contact AB, each of length m units. If m and l are connected by the relation $ml - 2 = 0$, then the minimum distance between P and Q will be

Sol. Answer (2)

Let the equation of the circle is $x^2 + y^2 = a^2$ and P is any point whose coordinates are (x_1, y_1)

Then $l = \sqrt{x_1^2 + y_1^2 - a^2}$

$\Rightarrow$ $l^2 + a^2 = x_1^2 + y_1^2$



Now, equation of chord of contact AB is $xx_1 + yy_1 = a^2$

Let $Q(x_2, y_2)$ is any point on AB then $x_1x_2 + y_1y_2 = a^2$ and $m = \sqrt{x_2^2 + y_2^2 - a^2}$

$\Rightarrow$ $m^2 + a^2 = x_2^2 + y_2^2$ if d is the distance between P and Q, then

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{x_1^2 + y_1^2 + x_2^2 + y_2^2 - 2(x_1x_2 + y_1y_2)}$$

$$= \sqrt{l^2 + a^2 + m^2 + a^2 - 2a^2} \qquad [\because\ x_1x_2 + y_1y_2 = a^2]$$

$$d = \sqrt{l^2 + m^2}$$

Now, we know that $\dfrac{l^2 + m^2}{2} \geq (l^2m^2)^{1/2}$

$\Rightarrow$ $\ell^2 + m^2 \geq 2\ell m$

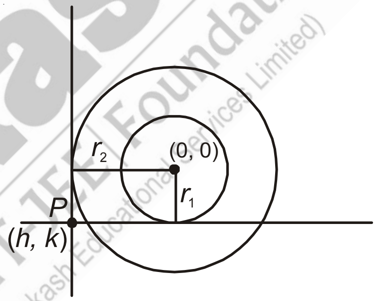
So, $d \geq \sqrt{2\ell m}$

But given that $ml - 2 = 0 \quad \Rightarrow \quad ml = 2$

So, minimum distance between P and Q will be 2 units.

5. There are two perpendicular lines, one touches to the circle $x^2 + y^2 = r_1^2$ and other touches to the circle $x^2 + y^2 = r_2^2$ if the locus of the point of intersection of these tangents is $x^2 + y^2 = 9$, then the value of $r_1^2 + r_2^2$ is

Sol. Answer (9)



From figure,

$h^2 + k^2 = r_1^2 + r_2^2$

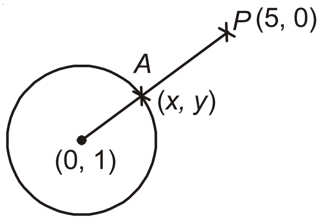
So locus of (h, k) is $x^2 + y^2 = r_1^2 + r_2^2$

But given locus is $x^2 + y^2 = 9$

So, $r_1^2 + r_2^2 = 9$

6. If the minimum value of $\sqrt{(1+\sqrt{4-x^2})^2 + (x-5)^2}$ for all $x \in R$ is $\sqrt{26} - 2k$, then the value of k is

Sol. Answer (1)



Let $r = \sqrt{(1+\sqrt{4-x^2})^2 + (x-5)^2}$

Suppose $y = 1 + \sqrt{4-x^2}$, then

$(y - 1) = 4 - x^2$

$\Rightarrow$ $x^2 + (y - 1)^2 = 4$ which is a circle

So, $r = \sqrt{y^2 + (x-5)^2}$

So, it is clear r is the distance between the points (x, y) and $(5, 0)$ as shown in the figure.

Now, we have to find the minimum value of PA.

From figure, minimum value of PA = $\sqrt{26} - 2$

But given value = $\sqrt{26} - 2k$

So, $k = 1$

7. Suppose the equation of circle is $x^2 + y^2 - 8x - 6y + 24 = 0$ and let (p, q) is any point on the circle, then the number of possible integral values of $|p + q|$ is

Sol. Answer (3)

The equation can be written as $(x - 4)^2 + (y - 3)^2 = 1$

and the coordinate $(p, q) \equiv (4 + \cos\theta, 3 + \sin\theta)$

So, $|p + q| = |7 + \sin\theta + \cos\theta|$ it is clear $7-\sqrt{2} \le |p+q| \le 7+\sqrt{2}$

and hence the integral values of $|p + q|$ = 6, 7, 8

and number of integral values = 3

8. Let the equation of the circle is $x^2 + y^2 = 4$. Find the total no. of points on $y = |x|$ from which perpendicular tangents can be drawn are

Sol. Answer (2)

We know that equation of director circle is $x^2 + y^2 = 8$ and the number of points of intersection of $y = |x|$ and $x^2 + y^2 = 8$ are 2.

So, total number of points from which perpendicular tangents are drawn = 2.

9. Let the equation of the circle is $x^2 + y^2 - 2x - 4y + 1 = 0$. A line through $P(\alpha, -1)$ is drawn which intersect the given circle at the point A and B. if PA. PB has the minimum value then the value of α is.

Sol. Answer (1)

We know that

$PA \cdot PB = (PT)^2$

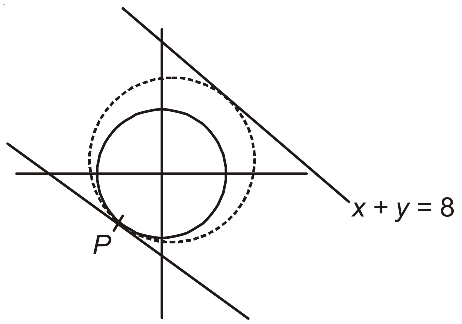
$\Rightarrow PA \cdot PB = \alpha^2 + 1 - 2\alpha + 4 + 1 = (\alpha - 1)^2 + 5$

$PA \cdot PB$ will be minimum at $\alpha = 1$

10. Let the equation of the circle is $x^2 + y^2 = 25$ and the equation of the line $x + y = 8$. If the radius of the circle of maximum area and also touches $x + y = 8$ and $x^2 + y^2 = 25$ is $\frac{(4\sqrt{2}+5)}{\lambda}$, then the value of λ is

Sol. Answer (2)

The possible figure is shown in the figure, it is clear the tangent at P will be $x+y+5\sqrt{2}=0$



So, radius of the required circle will be = $\frac{4\sqrt{2}+5}{2}$

So, the value of λ = 2.

11. If the circles $x^2 + y^2 + 2b_1y + 1 = 0$ and $x^2 + y^2 + 2b_2y + 1 = 0$ cuts orthogonally, then the value of b_1b_2 is

Sol. Answer (1)

We know that if two circle cuts orthogonally, then

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

So, $2b_1b_2 = 1 + 1$

$$b_1b_2 = 1$$

12. Let the equation of circle is $x^2 + y^2 = 9$ and the equation of the line is $x + y = a \ \forall \ a \in N$ and makes the intercepts AB by the circle $x^2 + y^2 = 9$. How many such intercepted portions are possible?

Sol. Answer (4)

Since AB is the intercept so its length will be $2\sqrt{9-\frac{a^2}{2}} = \sqrt{2}\ \sqrt{18-a^2}ac$

Since $a \in N$, so $a = 1, 2, 3, 4$, the number of such intercepted portions = 4

13. A circle of constant radius $2r$ passes through the origin and meets the axes in 'P' and 'Q'. Locus of the centroid of the ΔPOQ is $A(x^2 + y^2) = Br^2$, where $A, B \in N$, then $|A - B|$ is equal to

Sol. Answer (7)

Circle passes through origin.

$\therefore \quad c = 0$

Radius = $\sqrt{g^2+f^2}$

$$g^2 + f^2 = 4r^2 \qquad \text{...(1)}$$

Let circle intersect x axis at $P(a, 0)$ and y axis at $Q(0, b)$.

$\therefore \quad -g = \frac{a}{2}$ and $-f = \frac{b}{2}$

From (1),

$$\frac{a^2}{4} + \frac{b^2}{4} = 4r^2$$

$$a^2 + b^2 = 16r^2 \qquad \text{...(2)}$$

Let centroid of ΔPOQ be $C(h, k)$.

$\therefore \quad h = \frac{a}{3},\ k = \frac{b}{3} \qquad \therefore \quad 9h^2 + 9k^2 = 16r^2$

$\therefore$ Locus of centroid is $9(x^2+y^2) = 16r^2$.

14. The area of the triangle formed by joining the origin to the points of intersection of $\sqrt{5}x + 2y = 3\sqrt{5}$ and $x^2 + y^2 = 10$ is

Sol. Answer (5)

Combined equation of pair of lines joining origin and point of intersection of circle and line

$$x^2 + y^2 = 10\left(\frac{\sqrt{5}x+2y}{3\sqrt{5}}\right)^2$$

$$\frac{1}{9}x^2 - \frac{1}{9}y^2 + \frac{8}{9}\sqrt{5}xy = 0$$

$$a+b=\frac{1}{9}-\frac{1}{9}=0$$

$\therefore$ Δ is right-angled triangle at origin (0, 0).

$\therefore$ Area = $\frac{1}{2}r^2=\frac{1}{2}\left(\sqrt{10}\right)^2=5$

15. If $(1 + bx)^n = 1 + 8x + 24x^2 + ...$ and a line $P(b, n)$ cuts the circle $x^2 + y^2 = 9$ in C and D, then $PC \cdot PD = (\lambda^2 + 2)$, then λ is

Sol. Answer (3)

We have,

$(1 + bx)^n = 1 + 8x + 24x^2 + ...$

$\Rightarrow$ $1+nbx+\frac{n(n-1)}{2}\times b^2x^2+.......= 1 + 8x + 24x^2 + ...$

$\Rightarrow$ $nb = 8,\ \frac{n(n-1)}{2}\times b^2=24$

$\frac{n(n-1)}{2}\times b^2=24$

$\Rightarrow$ $n^2b^2 - nb.b = 48$

$\Rightarrow$ $(nb)^2 - (nb)b = 48$

$\Rightarrow$ $8^2 - 8b = 48$

$\Rightarrow$ $8b = 64 - 48 = 16$

$\Rightarrow$ $b = 2$

$\therefore$ Now, $nb = 8$

$\Rightarrow$ $n = 4$

$\therefore$ $P \equiv (2, 4)$

Now, $PC.PD = PT^2$

$= \sqrt{4+16-9}$

$= \sqrt{11}$

$\Rightarrow$ $\sqrt{\lambda^2+2}=\sqrt{11}$

$\Rightarrow$ $\lambda^2 = 9$

$\lambda = \pm 3$

❑ ❑ ❑