Quadratic Equation and Inequalities

GENERAL POLYNOMIAL

An expression written as, $a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$, where $a_0, a_1, a_2, ..., a_n \in R$ is called a **polynomial** of degree n where ($a_n \neq 0, n \in W$).

If $a_0, a_1, a_2, ..., a_n \in C$, then it is called **complex cofficient polynomial**.

QUADRATIC POLYNOMIAL

$ax^2 + bx + c$, is a polynomial of degree two in one variable, where $a \neq 0$ & $a, b, c \in R$. ($a \rightarrow$ leading coefficient, $c \rightarrow$ absolute term / constant term)

If $a = 0$ then above expression redues to, $bx + c$, which kown as **linear polynomial** ($b \neq 0$). If $c = 0$ in linear polynomial it reduces to, bx which is **odd linear polynomial**.

QUADRATIC EQUATIONS WITH REAL COEFFICIENTS

The expression, $ax^2 + bx + c$, where $a, b, c \in R$ and $a \neq 0$ is called as a quadratic expression in x. The quadratic expression when equated to zero, $ax^2 + bx + c = 0$, is called as a quadratic equation in x. Where the numbers a, b, c are called the coefficients of the equation.

1. The values of x which satisfy the quadratic equation is called roots (also called solutions or zeros) of the quadratic equation.

2. This equation has two roots which are given by

$$x = \frac{-b \pm \sqrt{D}}{2a},$$

where D (or Δ) $= b^2 - 4ac$ is called as discriminant of the quadratic equation.

3. If α and β denote the roots of $ax^2 + bx + c = 0$, then,

(i) $\alpha + \beta = -\frac{b}{a}$ (ii) $\alpha\beta = \frac{c}{a}$ (iii) $|\alpha - \beta| = \frac{\sqrt{D}}{|a|}$

SOLVED EXAMPLE

Example-1

If α and β are the roots of $ax^2 + bx + c = 0$ and γ, δ are those of $lx^2 + mx + n = 0$, find the equation whose roots are $\alpha\gamma + \beta\delta$ and $\alpha\delta + \beta\gamma$.

Sol.: $ax^2 + bx + c = 0$ has roots a and b

$\therefore\ \alpha + \beta = -\frac{b}{a}$; $\alpha\beta = \frac{c}{a}$

$lx^2 + mx + n = 0$ has roots γ and δ

$\therefore\ \gamma + \delta = -\frac{m}{l}$; $\gamma\delta = \frac{n}{l}$

The required quadratic equation is

$$x^2 - (\alpha\gamma + \beta\delta + \alpha\delta + \beta\gamma)x + (\alpha\gamma + \beta\delta)(\alpha\delta + \beta\gamma) = 0$$

Where $\alpha\gamma + \beta\delta + \alpha\delta + \beta\gamma = (\alpha + \beta)(\gamma + \delta) = \frac{b}{a}.\frac{m}{l}$

$(\alpha\gamma + \beta\delta)(\alpha\delta + \beta\gamma)$

$$= \frac{n}{l}\left(\frac{b^2 - 2ac}{a^2}\right) + \frac{c}{a}\left(\frac{m^2 - 2nl}{l^2}\right)$$

$$= \frac{ln(b^2 - 2ac) + ac(m^2 - 2nl)}{a^2 l^2}$$

$$\therefore\ a^2 l^2 \cdot x^2 - ablm\ x + ln(b^2 - 2ac) + ac(m^2 - 2nl) = 0$$

NATURE OF ROOTS

- Consider the quadratic equation $ax^2 + bx + c = 0$ where $a, b, c \in R$ & $a \neq 0$ then ;

(i) $D > 0 \Leftrightarrow$ roots are real & distinct (unequal)

(ii) $D = 0 \Leftrightarrow$ roots are real & coincident (equal)

(iii) $D < 0 \Leftrightarrow$ roots are imaginary

(iv) If $p + iq$ is one root of a quadratic equation, then the other must be it's conjugate $p - iq$ & vice versa. ($p, q \in R$ & $i = \sqrt{-1}$)

- Consider the quadratic equation $ax^2 + bx + c = 0$ where $a, b, c \in Q$ & $a \neq 0$ then ;

(i) If $D > 0$ & is a perfect square, then roots are rational & unequal.

(ii) If $\alpha = p + \sqrt{q}$ is one root in this case, (where p is rational & $\sqrt{q}$ is a surd) then the other root must be the conjugate of it i.e. $\beta = p - \sqrt{q}$ & vice versa.

SOLVED EXAMPLE

Example-2

Determine the values of m for which the equation $5x^2 - 4x + 2 + m(4x^2 - 2x - 1) = 0$ will have

(a) equal roots (b) product of the roots as 2

Sol. : $5x^2 - 4x + 2 + m(4x^2 - 2x - 1) = 0$

$(5 + 4m)x^2 - 2(m + 2)x + (2 - m) = 0$ ...(1)

(a) For equal roots, the discriminant of (1) should be equal to zero.

$\therefore\ 4(m+2)^2 - 4.(5+4m)(2-m) = 0$

$\Rightarrow m^2 + 4m + 4 - (10 + 8m - 5m - 4m^2) = 0$

$\Rightarrow 5m^2 + m - 6 = 0 \qquad \Rightarrow (5m + 6)(m - 1) = 0$

$\Rightarrow m = -\dfrac{6}{5}$ or $m = 1$

(b) $\dfrac{2 - m}{5 + 4m} = 2$

$\therefore\ 2 - m = 10 + 8m \Rightarrow m = -\dfrac{8}{9}$

RELATION BETWEEN ROOTS AND CO-EFFICIENTS

(i) The solutions of quadratic equation, $ax^2 + bx + c = 0$, $(a \neq 0)$ are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(ii) If α, β are the roots of quadratic equation,

$ax^2 + bx + c = 0$ ....(i)

then equation (i) can be written as

$a(x - \alpha)(x - \beta) = 0$

or $ax^2 - a(\alpha + \beta)x + a\,\alpha\beta = 0$ ......(ii)

equations (i) and (ii) are identical,

$\therefore$ by comparing the coefficients

sum of the roots,

$$\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and product of the roots,

$$\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

(iii) Dividing the equation (i) by a,

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$\Rightarrow\ x^2 - \left(\dfrac{-b}{a}\right)x + \dfrac{c}{a} = 0$

$\Rightarrow\ x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow\ x^2 -$ (sum of the roots) x + (product of the roots) = 0

Hence we conclude that the quadratic equation whose roots are α & β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

If $\Delta = b^2 - 4ac > 0$.

Now consider the following cases.

Case		**Nature of roots**
Case-I	$a > 0,\ b > 0,\ c > 0$ $\Rightarrow \alpha + \beta < 0, \alpha\beta > 0$	Both roots are negative.
Case-II	$a > 0,\ b > 0,\ c < 0$ $\Rightarrow \alpha + \beta < 0, \alpha\beta < 0$	Both roots are opposite in sign; Magnitude of negative root is more than the magnitude of positive root.
Case-III	$a > 0,\ b < 0,\ c > 0$ $\Rightarrow \alpha + \beta > 0,\ \alpha\beta > 0$	Both roots are positive.
Case-IV	$a > 0,\ b < 0,\ c < 0$ $\Rightarrow \alpha + \beta > 0, \alpha\beta < 0$	Roots are opposite in sign. Magnitude of positive root is more than magnitude of negative root.

Remember :

(i) Roots are rational $\Leftrightarrow$ D is a perfect square.

(ii) Roots are irrational $\Leftrightarrow$ D is positive but not a perfect square.

(iii) If $a + b + c = 0$, then 1 is a root of the equation $ax^2 + bx + c = 0$.

(iv) If a and c are of opposite sign, the roots must be of opposite sign.

(v) If the roots are equal in magnitude but opposite in sign, then $b = 0$, $ac < 0$.

(vi) If the roots are reciprocal of each other, then $c = a$.

(vii) The quadratic equation whose roots are reciprocals of the roots of $ax^2 + bx + c = 0$ is $cx^2 + bx + a = 0$ (*i.e.*, the coefficients are written in reverse order).

(viii) If $a = 1$, b, $c \in Z$ and the roots are rational numbers, then these roots must be integers.

(ix) If $ax^2 + bx + c = 0$ is satisfied by more than two values, it is an identity and $a = b = c = 0$ and vice-versa.

(x) If $P(x) = a_1x^2 + b_1x + c_1$ and $Q(x) = a_2x^2 + b_2x + c_2$, then P(x). Q(x) = 0 have atleast two real roots if $D_1 + D_2 > O$.

Condition for a root of a quadratic equation $ax^2 + bx + c = 0$ to be reciprocal of the root of $a'x^2 + b'x + c' = 0$ is

$$(cc' - aa')^2 = (ba' - b'c)(ab' - bc')$$

SYMMETRIC FUNCTION OF THE ROOTS

A function of α and β is said to be a symmetric function if it remains unchanged when α and β are interchanged.

For example, $\alpha^2 + \beta^2 + 2\alpha\beta$ is a symmetric function of α and β whereas $\alpha^2 - \beta^2 + 3\alpha\beta$ is not a symmetric function of α and β.

In order to find the value of a symmetric function of α and β, express the given function in terms of $\alpha + \beta$ and $\alpha\beta$. The following results may be useful.

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

(ii) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

(iii) $\alpha^4 + \beta^4 = (\alpha^3 + \beta^3)(\alpha + \beta) - \alpha\beta(\alpha^2 + \beta^2)$

(iv) $\alpha^5 + \beta^5 = (\alpha^3 + \beta^3)(\alpha^2 + \beta^2) - \alpha^2\beta^2(\alpha + \beta)$

(v) $|\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

(vi) $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta)$

(vii) $\alpha^3 - \beta^3 = (\alpha - \beta)[(\alpha + \beta)^2 - \alpha\beta]$

(viii) $\alpha^4 - \beta^4 = (\alpha + \beta)(\alpha - \beta)(\alpha^2 + \beta^2)$

QUADRATIC EXPRESSION

Let $y = ax^2 + bx + c$

$$= a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right) \quad \ldots(1)$$

(1) represents a parabola with vertex $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

and axis of the parabola is $x = \frac{-b}{2a}$

If $a > 0$, the parabola opens upward while if $a < 0$, the parabola opens downward. The parabola cuts the x-axis at points corresponding to roots of $ax^2 + bx + c = 0$. If this equation has

(i) $D > 0$, the parabola cuts x-axis at two real and distinct points.

(ii) $D = 0$, the parabola touches x-axis at $x = \frac{-b}{2a}$.

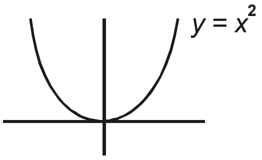
(iii) $D < 0$, then;

if $a > 0$, parabola lies above x-axis.

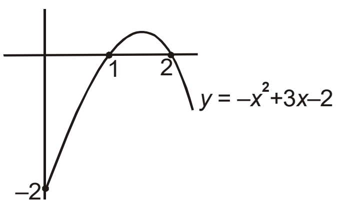
if $a < 0$, parabola lies below x-axis.

e.g.

1. $y = x^2$



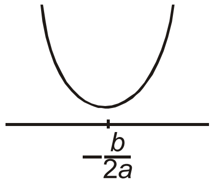
2. $y = -x^2 + 3x - 2$



GRAPHS OF QUADRATIC EXPRESSIONS

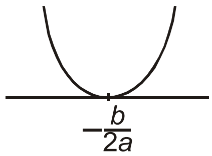
Let $f(x) = ax^2 + bx + c$ and $\alpha, \beta : \alpha < \beta$, be its roots

(i) $a > 0$ and $D < 0 \Leftrightarrow f(x) > 0 \quad \forall \quad x \in R$



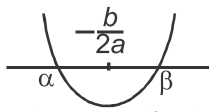
α and β are complex conjugates

(ii) $a > 0$ and $D = 0 \Leftrightarrow f(x) \geq 0 \quad \forall \quad x \in R$



$f(x) = 0$ at $x = \frac{-b}{2a}$

(iii) $a > 0$ and $D > 0$; then,

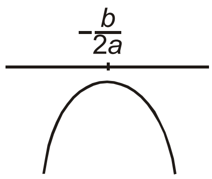


$f(x) > 0 \ \forall \ x \in (-\infty, \alpha) \cup (\beta, \infty)$

$f(x) < 0 \ \forall \ x \in (\alpha, \beta)$

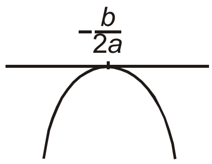
$f(x)_{min} = -\frac{D}{4a}$ at $x = -\frac{b}{2a}$

(iv) $a < 0$ and $D < 0 \Leftrightarrow f(x) < 0 \quad \forall \quad x \in R$



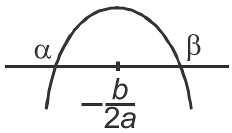
α and β are complex conjugates

(v) $a < 0$ and $D = 0 \Leftrightarrow f(x) \leq 0 \quad \forall \quad x \in R$



$f(x) = 0$ at $x = \frac{-b}{2a}$

(vi) $a < 0$ and $D > 0$; then



$f(x) < 0 \quad \forall \quad x \in (-\infty, \alpha) \cup (\beta, \infty)$

$f(x) > 0 \quad \forall \quad x \in (\alpha, \beta)$

$f(x)_{max} = -\frac{D}{4a}$ *at* $x = -\frac{b}{2a}$

SOLVED EXAMPLE

Example-3

A quadratic equation p(x) = 0 has $1+\sqrt{5}$ and $1-\sqrt{5}$ as roots and it satisfies p(1) = 2. Find the quadratic polynomial.

Sol. sum of the roots = 2, product of the roots = – 4

$\therefore$ let $p(x) = a(x^2 - 2x - 4) \Rightarrow p(1) = 2$

$\Rightarrow$ $2 = a(1^2 - 2\cdot 1 - 4) \Rightarrow a = -2/5$

$\therefore$ $p(x) = -2/5\,(x^2 - 2x - 4)$

Example-4

The quadratic equation $x^2 + mx + n = 0$ has roots which are as twice as those of $x^2 + px + m = 0$ and m, n and $p \neq 0$. Find the value of $\frac{n}{p}$.

Sol. $x^2 + mx + n = 0$ with roots 2α, 2β

and $x^2 + px + m = 0$ with roots α, β

$2(\alpha+\beta) = -m$ ....(1)

$4\alpha\beta = n$ ....(2)

and $\alpha+\beta = -p$ ....(3)

$\alpha\beta = m$ ....(4)

$\therefore$ from (1) and (3)

$\Rightarrow$ $2p = m$

and from (2) and (4)

$\Rightarrow$ $4m = n \Rightarrow \frac{n}{p} = \frac{4m}{m/2} = 8$

Example-5

For what values of m the equation $(1+m)x^2 - 2(1+3m)x + (1+8m) = 0$ has equal roots.

Sol. Given equation is

$(1+m)x^2 - 2(1+3m)x + (1+8m) = 0$

Roots of equation will be equal if, D = 0.

$\Rightarrow$ $4(1+3m)^2 - 4(1+m)(1+8m) = 0$

$\Rightarrow$ $4(1+9m^2+6m-1-9m-8m^2) = 0$

$\Rightarrow$ $m^2 - 3m = 0$ or, $m(m-3) = 0$

$\therefore$ $m = 0, 3$.

Example-6

Find all the integral values of a for which the quadratic equation $(x-a)(x-10) + 1 = 0$ has integral roots.

Sol. Here the equation is $x^2 - (a+10)x + 10a + 1 = 0$. For integral roots D should be a perfect square.

$D = a^2 - 20a + 96$.

$\Rightarrow$ $D = (a-10)^2 - 4 \Rightarrow 4 = (a-10)^2 - D$

If D is a perfect square means difference of two perfect square as 4 which is possible only when $(a-10)^2 = 4$ and D = 0.

$\Rightarrow$ $(a-10) = \pm 2; \Rightarrow a = 12, 8$

EQUATION V/S IDENTITY

A quadratic equation is satisfied by exactly two values of 'x' which may be real or imaginary. The equation, $ax^2 + bx + c = 0$ is :

A **quadratic equation** if $a \neq 0$ and An **identity** if $a = b = c = 0$ Infinite Roots

If a equation is satisfied by three distinct values of 'x', then it is an identity.

$(x+1)^2 = x^2 + 2x + 1$ is an identity in x.

Here highest power of x in the given relation is 2 and this relation is satisfied by three different values x = 0, x = 1 and x = –1 and hence it is an identity because a polynomial equation of n^{th} degree cannot have more than n distinct roots.

SOLVED EXAMPLE

Example-7

If $a+b+c = 0$; $an^2 + bn + c = 0$ and $a + bn + cn^2 = 0$ where $n \neq 0, 1$, then prove that $a = b = c = 0$.

Sol. Note that $ax^2 + bx + c = 0$ is satisfied by $x = 1$; $x = n$ & $x = \frac{1}{n}$ where $n \neq \frac{1}{n}$

$\Rightarrow$ Q.E. has 3 distinct real roots which implies that it must be an identity.

SOLUTION OF QUADRATIC INEQUALITIES

The values of 'x' satisfying the inequality, $ax^2 + bx + c > 0$ $(a \neq 0)$ are :

(i) If D > 0, i.e. the equation $ax^2 + bx + c = 0$ has two different roots $\alpha < \beta$.

Then $a > 0 \Rightarrow x \in (\infty, \alpha) \cup (\beta, \infty)$

$a < 0 \Rightarrow x \in (\alpha, \beta)$

(ii) If D = 0, i.e. roots are equal, i.e. $\alpha = \beta$.

Then $a > 0 \Rightarrow x \in (-\infty, \alpha) \cup (\alpha, \infty)$

$a < 0 \Rightarrow x \in \phi$

(iii) If D < 0, i.e. the equation $ax^2 + bx + c = 0$ has no real roots. Then $a > 0 \Rightarrow x \in R$

$a < 0 \Rightarrow x \in \phi$

(iv) Inequalities of the form $\frac{P(x)}{Q(x)} \lesseqgtr 0$ can be solved using the method of intervals.

Example-8

Find the solution set of k so that y = kx is secant to the curve $y = x^2 + k$.

Sol. put $y = kx$ in $y = x^2 + k$

$\Rightarrow kx = x^2 + k = 0$

$\Rightarrow$ $x^2 - kx + k = 0$ for line to be secant, $D > 0$

$\Rightarrow \quad k^2 - 4k > 0 \; k(k-4) > 0$

hence $k > 4$ or $k < 0$

$\Rightarrow k \in (-\infty, 0) \cup (4, \infty)$

Example-9

Solve inequatility, $\dfrac{x^2+x+1}{|x+1|} > 0.$

Sol. $\dfrac{x^2+x+1}{|x+1|} > 0$ ....(i)

$\therefore \quad |x+1| > 0, \; \forall x \in R - \{-1\}$

$\therefore \quad$ (i) becomes $\qquad x^2 + x + 1 > 0 \; x \neq -1$

$\because \quad x^2 + x + 1 > 0, \forall x \in R$ (as its $a > 0$ and $D < 0$)

$\therefore \quad$ Solution of (i) is $x \in R - \{-1\}$

Example-10

Solve $\left|\dfrac{x^2-3x-1}{x^2+x+1}\right| < 3.$

Sol. $\dfrac{|x^2-3x-1|}{x^2+x+1} < 3.$

$\because \quad (x^2+x+1) > 0, \forall x \in R$

$\therefore \quad |x^2-3x-1| < 3(x^2+x+1)$

$\Rightarrow \quad (x^2-3x-1)^2 - \{3(x^2+x+1)\}^2 < 0$

$\Rightarrow \quad (4x^2+2)(-2x^2-6x-4) < 0$

$\Rightarrow \quad (2x^2+1)(x+2)(x+1) > 0$

$\Rightarrow \quad x \in (-\infty, -2) \cup (-1, \infty)$

COMMON ROOTS:

One root common :

If $\alpha \neq 0$ is a common root of the equation

$a_1x^2 + b_1x + c_1 = 0$ ...(i)

and $a_2x^2 + b_2x + c_2 = 0$ ...(ii)

then we have

$a_1\alpha^2 + b_1\alpha + c_1 = 0$ and

$a_2\alpha^2 + b_2\alpha + c_2 = 0$

These give $\dfrac{\alpha^2}{b_1c_2 - b_2c_1} = \dfrac{\alpha}{c_1a_2 - c_2a_1}$

$$= \frac{1}{a_1b_2 - a_2b_1} (a_1b_2 - a_2b_1 \neq 0).$$

Thus, the required condition for one common root is $(a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (c_1a_2 - c_2a_1)^2$ and the value of the common root is

$$\alpha = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \text{ or } \frac{b_1c_2 - b_2c_1}{c_1a_2 - c_2a_1}.$$

Both roots common :

If the equations (i) and (ii) have both roots common, then these equations will be identical. Thus the required condition for both roots common is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \text{(If no root is equal to zero)}$$

Remember :

(i) To find the common root of two equations, make the coefficient of second degree terms in two equations equal and subtract. The value of x so obtained is the required common root.

(ii) If two quadratic equations with real and rational coefficients have a common imaginary or irrational root, then both roots will be common and the two equations will be identical. The required condition is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

(iii) If α is a repeated root of the quadratic equation $f(x) = ax^2 + bx + c = 0$, then α is also a root of the equation $f'(x) = 0$.

Note

If $f(x) = 0$ & $g(x) = 0$ are two polynomial equation having some common root(s) then those common root(s) is/are also the root(s) of $h(x) \equiv a\,f(x) + b\,g(x) = 0$.

SOLVED EXAMPLE

Example-11

Find m and n in order that the equations $mx^2 + 5x + 2 = 0$ and $3x^2 + 10x + n = 0$ may have both the roots common.

Sol. : The equations are $mx^2 + 5x + 2 = 0$ and $3x^2 + 10x + n = 0$. Since they have both the roots common,

$\dfrac{m}{3} = \dfrac{5}{10} = \dfrac{2}{n} \qquad \because \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_1}$

From the first-relation, $m = \dfrac{15}{10} = \dfrac{3}{2}$.

From the last-relation, $n = 4$.

Example-12

If the quadratic equations, $x^2 + bx + c = 0$ and $bx^2 + cx + 1 = 0$ have a common root then prove that either $b + c + 1 = 0$ or $b^2 + c^2 + 1 = bc + b + c$.

Sol. $\alpha^2 + b\alpha + c = 0$ ....(1)

$b\alpha^2 + c\alpha + 1 = 0$ ....(2)

$$\therefore \quad \frac{\alpha^2}{b - c^2} = \frac{\alpha}{bc - 1} = \frac{1}{c - b^2}$$

$$\Rightarrow \quad \alpha^2 = \frac{b - c^2}{c - b^2} \text{ or } \alpha = \frac{bc - 1}{c - b^2}$$

$$\Rightarrow \quad (bc-1)^2 = (b - c^2)(c - b^2)$$

or $b^3 + c^3 + 1 - 3bc = 0$

$\Rightarrow (b + c + 1)(b^2 + c^2 + 1 - bc - c - b) = 0$

$\Rightarrow b + c + 1 = 0$ or $b^2 + c^2 + 1 = bc + b + c$

RANGE OF QUADRATIC EXPRESSION

$f(x) = ax^2 + bx + c$

(i) Range when $x \in R$: If $a > 0 \Rightarrow f(x) \in \left[-\frac{D}{4a}, \infty\right)$

$a < 0 \Rightarrow f(x) \in \left(-\infty, \frac{D}{4a}\right]$

Maximum & Minimum Value of $y = ax^2 + bx + c$ occurs at $x = -(b/2a)$ according as $a < 0$ or $a > 0$ respectively

(ii) Range in restricted domain : Given $x \in [x_1, x_2]$

(a) If $-\frac{b}{2a} \notin [x_1, x_2]$ then,

$f(x) \in \left[\min\{f(x_1), f(x_2)\},\ \max\{f(x_1), f(x_2)\}\right]$

(b) If $-\frac{b}{2a} \in [x_1, x_2]$ then,

$f(x) \in \left[\min\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\},\ \max\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\}\right]$

SOLVED EXAMPLE

Example-13

Find the minimum value of $f(x) = x^2 - 5x + 6$.

Sol. Minimum value of $f(x) = -\frac{D}{4a} = -\left(\frac{25-24}{4}\right)$

$= -\frac{1}{4}\ \left(\text{at } x = -\frac{b}{2a} = \frac{5}{2}\right)$. Hence range is $\left[-\frac{1}{4}, \infty\right)$.

Example-14

Find the range of $y = \frac{x+2}{2x^2+3x+6}$, if x is real.

Sol. $y = \frac{x+2}{2x^2+3x+6}$

$\Rightarrow 2yx^2 + 3yx + 6y = x + 2$

$\Rightarrow 2yx^2 + (3y - 1)x + 6y - 2 = 0$ ....(i)

<u>case-I</u> :

if $y \neq 0$, then equation (i) is quadratic in x

$\because$ x is real

$\therefore D \geq 0$

$\Rightarrow (3y - 1)^2 - 8y(6y - 2) \geq 0$

$\Rightarrow (3y - 1)(13y + 1) \leq 0$

$y \in \left[-\frac{1}{13}, \frac{1}{3}\right] - \{0\}$

<u>case-II</u> : if $y = 0$, then equation becomes

$x = -2$

which is possible as x is real

$\therefore$ Range $y \in \left[-\frac{1}{13}, \frac{1}{3}\right]$

THEORY OF EQUATIONS

If $\alpha_1, \alpha_2, \alpha_3, ...\alpha_n$ are the roots of the equation ; $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$ where $a_0, a_1, ... a_n$ are all real & $a_0 \neq 0$ then,

$\Sigma\alpha_1 = -\frac{a_1}{a_0}$, $\Sigma\alpha_1\alpha_2 = +\frac{a_2}{a_0}$, $\Sigma\alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0}$, ...,

$\alpha_1\alpha_2\alpha_3........\alpha_n = (-1)^n\frac{a_n}{a_0}$

Note

(i) If α is a root of the equation $f(x) = 0$, then the polynomial f(x) is exactly divisible by $(x - \alpha)$ or $(x - \alpha)$ is a factor of f(x) and conversely.

(ii) Every equation of nth degree $(n \geq 1)$ has exactly n roots & if the equation has more than n roots, it is an identity.

(iii) If the coefficients of the equation $f(x) = 0$ are all real and $\alpha + i\beta$ is its root, then $\alpha - i\beta$ is also a root. i.e. **imaginary roots occur in conjugate pairs**.

(iv) If the coefficients in the equation are all rational & $\alpha + \sqrt{\beta}$ is one of its roots, then $\alpha - \sqrt{\beta}$ is also a root where $\alpha, \beta \in Q$ & β is not a perfect square.

(v) If there be any two real numbers 'a' & 'b' such that f(a) & f(b) are of opposite signs, then $f(x) = 0$ must have atleast one real root between 'a' and 'b'.

(vi) Every equation $f(x) = 0$ of degree odd has atleast one real root of a sign opposite to that of its last term.

SOLVED EXAMPLE

Example-15

If $x = 1$ and $x = 2$ are solutions of the equation $x^3 + ax^2 + bx + c = 0$ and $a + b = 1$, then find the value of b.

Sol. $a + b + c = -1 \Rightarrow c = -2$ & $8 + 4a + 2b + c = 0$

$\Rightarrow 4a + 2b = -6 \Rightarrow 2a + b = -3$

$\Rightarrow a = -4,\ b = 5$

Example-16

A polynomial in x of degree greater than 3 leaves the remainder 2, 1 and –1 when divided by $(x - 1)$; $(x + 2)$ & $(x + 1)$ respectively. Find the remainder, if the polynomial is divided by, $(x^2 - 1)(x + 2)$.

Sol. $f(x) = Q_1(x - 1) + 2 = Q_2(x + 2) + 1 = Q_3(x + 1) - 1$

$\Rightarrow f(1) = 2$; $f(-2) = 1$; $f(-1) = -1$

Let $f(x) = Q_r(x^2 - 1)(x + 2) + ax^2 + bx + c$

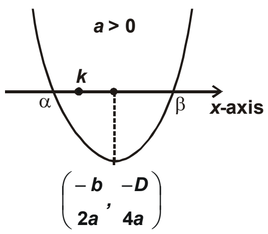
Hence $a + b + c = 2$; $4a - 2b + c = 1$

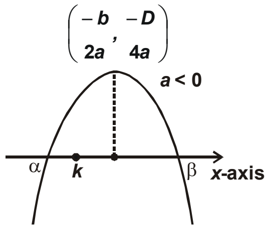
and $a - b + c = -1$

LOCATION OF ROOTS

Let $f(x) = ax^2 + bx + c,\ a, b, c \in R,\ a \neq 0$ and α, β be roots of $f(x) = 0$

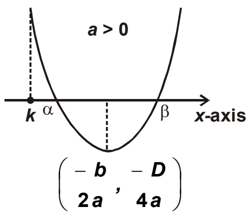
(1) A real number k lies between the roots of $f(x) = 0$

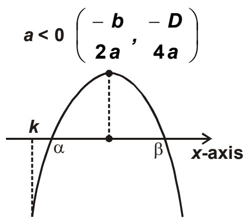




(i) $D > 0$, (ii) $af(k) < 0$, where $\alpha < \beta$

(2) If both roots of quadratic equation $f(x) = 0$ are greater than k, then

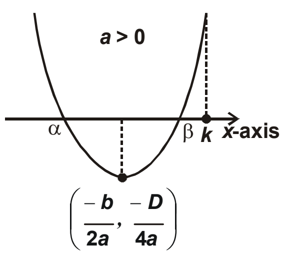


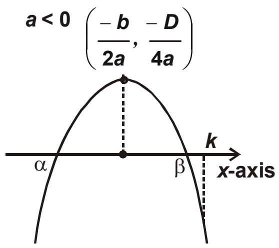


(i) $D \geq 0$, (ii) $af(k) > 0$

(iii) $k < \dfrac{-b}{2a}$, where $\alpha \leq \beta$

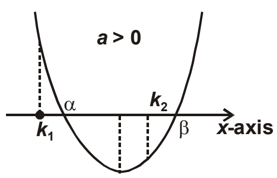
(3) If both roots are less than real number k, then

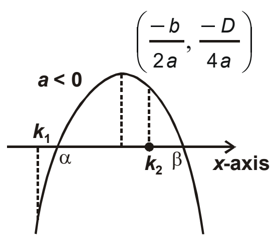




(i) $D \geq 0$, (ii) $af(k) > 0$ (iii) $k > \dfrac{-b}{2a}$, where $\alpha \leq \beta$

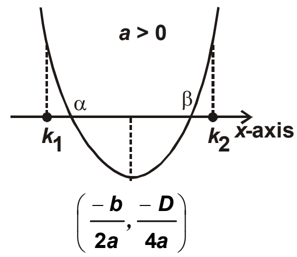
(4) Exactly one root lies between real numbers k_1 and k_2, where $k_1 < k_2$.

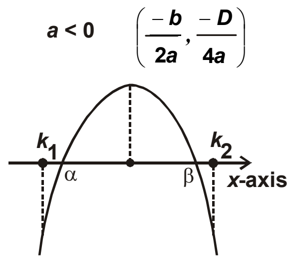




(i) $D > 0$, (ii) $f(k_1).f(k_2) < 0$, where $\alpha < \beta$

(5) If both roots of $f(x) = 0$ are confined between real numbers k_1 and k_2, where $k_1 < k_2$.

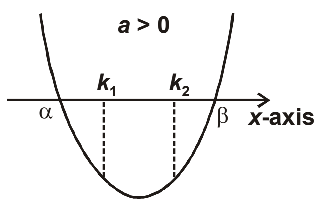


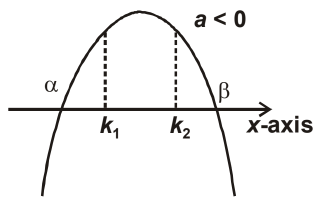


(i) $D \geq 0$, (ii) $af(k_1) > 0$

(iii) $af(k_2) > 0$, (iv) $k_1 < \dfrac{-b}{2a} < k_2$, where $\alpha \leq \beta$

(6) If real numbers k_1 and k_2 lie between roots of $f(x) = 0$, where $k_1 < k_2$.





(i) $D > 0$, (ii) $af(k_1) < 0$

(iii) $af(k_2) < 0$, where $\alpha < \beta$

SOLVED EXAMPLE

Example-17

If α & β are the roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always (p, q, r, s $\in$ R)

Sol. $\alpha + \beta = -p$; $\alpha\beta = q$; $\alpha^4 + \beta^4 = r$; $\alpha^4\beta^4 = s$

Now for equation $x^2 - 4q^2 + 2q^2 - r = 0$

$D = 16q^2 - 4(2q^2 - r)$

$= 8q^2 + 4r = 4(2q^2 + r) = 4[2\alpha^2\beta^2 + \alpha^4 + \beta^4]$

$= 4[(\alpha^2 + \beta^2)^2] = 4[(\alpha + \beta)^2 - 2\alpha\beta]^2 = 4(p^2 - 2q)^2 > 0$

Again consider the product of the roots $= 2q^2 - r$ which can be either positive or negative.

Example-18

Find the range of values of a for which the equation $x^2 - (a - 5)x + \left(a - \dfrac{15}{4}\right) = 0$ has at least one positive root.

Sol. $D \ge 0 \Rightarrow (a-5)^2 - 4\left(a - \dfrac{15}{4}\right) \ge 0$

$\Rightarrow a^2 - 10a + 25 - 4a + 15 \ge 0$

$\Rightarrow a^2 - 14a + 40 \ge 0$

$\Rightarrow (a-4)(a-10) \ge 0$

$\Rightarrow a \in (-\infty, 4] \cup [10, \infty)$

Case I : When both roots are positive

$D \ge 0,\ a - 5 > 0,\ a - \dfrac{15}{4} > 0$

$\Rightarrow D \ge 0,\ a > 5,\ a > \dfrac{15}{4} \Rightarrow a \in [10, \infty)$

Case II : when exactly one root is positive

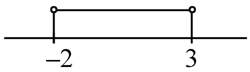
$\Rightarrow a - \dfrac{15}{4} \le 0 \Rightarrow a \le \dfrac{15}{4}$

then finally $a \in (-\infty, 15/4] \cup [10, \infty)$

Example-19

Find the value of k for which one root of the equation of $x^2 - (k+1)x + k^2 + k - 8 = 0$ exceed 2 and other is smaller than 2.

Sol. Since $a > 0,\ f(0) < 0$ $\quad \therefore f(2) < 0$



$\therefore \quad 4 - 2(k+1) + k^2 + k - 8 < 0$

$\Rightarrow k^2 - K + 6 < 0$

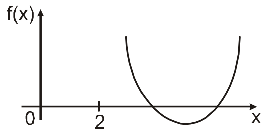
$\Rightarrow (k+2)(k-3) < 0 \quad \therefore k \in (-2, 3)$

Example-20

Let $x^2 - (m-3)x + m = 0$ $(m \in R)$ be a quadratic equation, then find the values of 'm' for which

(a) both the roots are greater than 2.
(b) both roots are positive.
(c) one root is positive and other is negative.
(d) One root is greater than 2 and other smaller than 1
(e) Roots are equal in magnitude and opposite in sign.
(f) both roots lie in the interval (1, 2)

Sol. (a)



Condition - I : $D \ge 0 \Rightarrow (m-3)^2 - 4m \ge 0$

$\Rightarrow \quad m^2 - 10m + 9 \ge 0$

$\Rightarrow \quad (m-1)(m-9) \ge 0$

$\Rightarrow \quad m \in (-\infty, 1] \cup [9, \infty)$ ......(i)

Condition - II : $f(2) > 0$

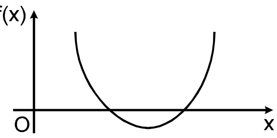
$\Rightarrow \quad 4 - (m-3)2 + m > 0$

$\Rightarrow \quad m < 10$ ......(ii)

Condition - III : $-\dfrac{b}{2a} > 2 \Rightarrow m > 7$ .....(iii)

Intersection of (i), (ii) and (iii) gives $m \in [9, 10)$

(b)



Condition - I $D \ge 0$

$\Rightarrow \quad m \in (-\infty, 1] \cup [9, \infty)$

Condition - II $f(0) > 0$

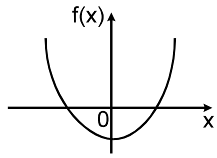
$\Rightarrow \quad m > 0$

Condition - III $-\dfrac{b}{2a} > 0 \Rightarrow \dfrac{m-3}{2} > 0$

$\Rightarrow \quad m > 3$

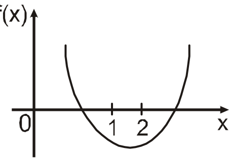
intersection gives $m \in [9, \infty)$ Ans.

(c)



Condition - I $f(0) < 0 \Rightarrow m < 0$ Ans.

(d)



Condition - I $f(1) < 0 \Rightarrow 4 < 0 \Rightarrow m \in \phi$

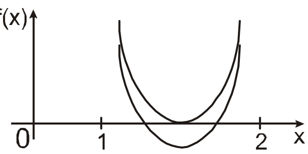
Condition - II $f(2) < 0 \Rightarrow m > 10$

Intersection gives $m \in \phi$ Ans.

(e) sum of roots $= 0 \Rightarrow m = 3$
and $f(0) < 0 \Rightarrow m < 0$

$\therefore \quad m \in \phi$ **Ans.**

(f)



Condition - I $D \ge 0 \Rightarrow m \in (-\infty, 1] \cup [9, \infty)$

Condition - II $f(1) > 0 \Rightarrow 1 - (m-3) + m > 0 \Rightarrow 4 > 0$

which is true $\forall\ m \in R$

Condition - III $f(2) > 0 \Rightarrow m < 10$

Condition - IV $1 < -\dfrac{b}{2a} < 2$

$\Rightarrow 1 < \dfrac{m-3}{2} < 2$

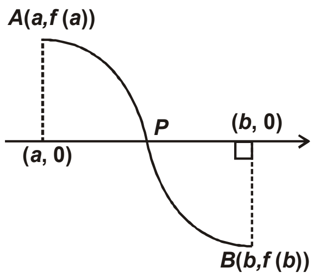
$\Rightarrow 5 < m < 7$

intersection gives $m \in \phi$ **Ans.**

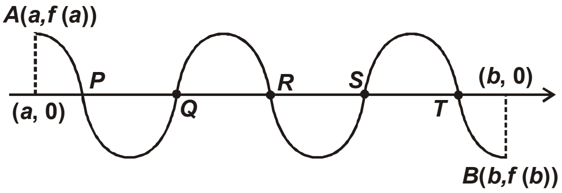
NUMBER OF ROOTS OF A POLYNOMIAL EQUATION

1. If $f(x)$ is an increasing function in $[a, b]$, then $f(x) = 0$ will have atmost one root in $[a, b]$.

2. Let $f(x) = 0$ be a polynomial equation and a, b are two real numbers. Then $f(x) = 0$ will have at least one real root or odd number of real roots in (a, b) if $f(a)$ and $f(b)$ $(a < b)$ are of opposite signs.

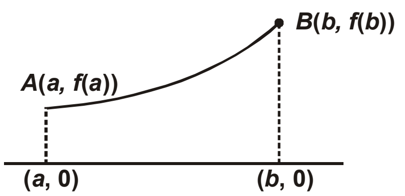


One real root

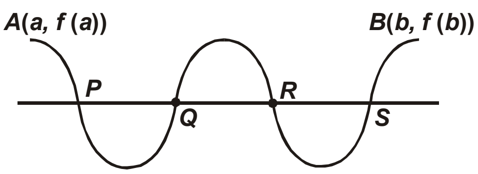


Odd number of real roots

But if $f(a)$ and $f(b)$ are of same signs, then either $f(x) = 0$ has no real root or even number of real roots in (a, b).



No real root



Even number of real roots

3. If the equation $f(x) = 0$ has two real roots a and b then $f'(x) = 0$ will have atleast one real root lying between a and b (using Rolle's Theorem).

EQUATIONS REDUCIBLE TO QUADRATIC EQUATIONS

1. **Reciprocal equation** of the standard form can be reduced to an equation of half its dimension.

2. **Equations of the form**

(a) $(x - a)(x - b)(x - c)(x - d) = A$ where $a < b < c < d$ can be solved by change of variable.

$$y = x - \left(\frac{a+b+c+d}{4}\right)$$

(b) $(x - a)(x - b)(x - c)(x - d) = Ax^2$ where

$ab = cd$ can be solved by assumption $y = x + \frac{ab}{x}$.

3. **For the equation of the type $(x - a)^4 + (x - b)^4 = A$**

Substitute $y = \frac{(x-a)+(x-b)}{2}$

SOLVED EXAMPLE

Example-21

Solve $2x^4 + x^3 - 11x^2 + x + 2 = 0$

Sol. Since $x = 0$ is not a solution hence, divide by x^2

We get,

$$2x^2 + x - 11 + \frac{1}{x} + \frac{2}{x^2} = 0$$

or $2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 11 = 0$

Let $x + \frac{1}{x} = y \Rightarrow 2(y^2 - 2) + y - 11 = 0$

or $2y^2 + y - 15 = 0 \Rightarrow y = -3, \frac{5}{2}$

Corresponding values of x are $\frac{1}{2}, 2, \frac{-3 \pm \sqrt{5}}{2}$

Example-22

$(x + 4)(x + 6)(x + 8)(x + 12) = 1680x^2$

Sol. Here $(-4 \times -12) = 48 = (-6) \times (-8)$

$(x + 4)(x + 12)(x + 6)(x + 8) = 1680x^2$

$\Rightarrow (x^2 + 16x + 48)(x^2 + 14x + 48) = 1680x^2$

$$\Rightarrow \left(x + 16 + \frac{48}{x}\right)\left(x + 14 + \frac{48}{x}\right) = 1680 \quad (\because x \neq 0)$$

Let $y = x + \frac{48}{x} \Rightarrow (y + 16)(y + 14) = 1680$

$\Rightarrow y^2 + 30y - 1680 + 224 = 0$

$\Rightarrow y^2 + 30y - 1456 = 0 \Rightarrow y^2 + 56y - 26y - 1456 = 0$

$\Rightarrow (y + 56)(y - 26) = 0 \quad \Rightarrow y = 26, -56$

$\therefore\ x + \frac{48}{x} = 26$ and $x + \frac{48}{x} = -56$

$\Rightarrow x^2 - 26x + 48 = 0 \Rightarrow x^2 + 56x + 48 = 0$

$\Rightarrow x = 2, 24 \Rightarrow x = \frac{-56 \pm \sqrt{2944}}{2} = -28 \pm \sqrt{736}$

$= -28 \pm 4\sqrt{46}$

$\therefore\ x = \left\{2, 24, -28 - 4\sqrt{46}, -28 + 4\sqrt{46}\right\}$

IMPORTANT THEOREMS AND RESULTS

1. If α is a root of the equation $f(x) = 0$, then $f(x)$ is exactly divisible by $(x - \alpha)$ and conversely, if $f(x)$ is exactly divisible by $(x - \alpha)$ then α is a root of the equation $f(x) = 0$.

2. Every equation of an odd degree has atleast one real root, whose sign is opposite to that of its last term, provided that the coefficient of the first term is positive.

3. Every equation of an even degree whose last term is negative has at least two real roots, one positive and one negative, provided that the coefficient of the first term is positive.

4. If an equation has no odd powers of x, then all roots of the equation are complex provided all the coefficients of the equation are having positive sign.

5. If $x = \alpha$ is root repeated m times in $f(x) = 0$, ($f(x) = 0$ is an nth degree equation in x) then

$$f(x) = (x - \alpha)^m g(x)$$

when $g(x)$ is a polynomial of degree $(n - m)$ and the root $x = \alpha$ is repeated $(m - 1)$ times in $f'(x) = 0$, $(m - 2)$ times in $f''(x) = 0$,...$(m - (m - 1))$ times in $f^{m-1}(x) = 0$.

6. The condition that a quadratic function $f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ may be resolved into two linear factor if

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

or $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ is 0.

SOLVED EXAMPLE

Example-23

If the equations $4x^2 - 11x + 2k = 0$ and $x^2 - 3x - k = 0$ have a common root, then the value of k and common root is

(1) $0, \frac{17}{6}$ (2) $\frac{-17}{36}, \frac{17}{6}$ (3) $0, \frac{-17}{6}$ (4) $\frac{-17}{6}, 0$

Sol. The condition for the equations $ax^2 + bx + c = 0$ and $a'x^2 + b'x + c' = 0$ to have a common root α is

$(ca' - c'a)^2 = (bc' - b'c)(ab' - a'b)$ and $\alpha = \frac{ca' - c'a}{ab' - a'b}$

$4x^2 - 11x + 2k = 0 \Rightarrow x^2 - 3x - k = 0$

$\therefore\ (2k + 4k)^2 = (11k + 6k)(-12 + 11)$

$\therefore\ (6k)^2 = -17k \Rightarrow k = 0$ or $k = -\frac{17}{36}$

The common root α is given by $\alpha = \frac{6k}{-1}$

$\therefore$ If $k = 0$, then $\alpha = 0$ and if $k = -\frac{17}{36}$

then $\alpha = \frac{17}{6}$. Hence answer is **(2)**

Example-24

The value of $\sqrt{7 + \sqrt{7 - \sqrt{7 + \sqrt{7 ... \infty}}}}$ is

(1) *5* (2) *4*
(3) *3* (4) *2*

Sol. : Let $y = \sqrt{7 + \sqrt{7 - \sqrt{7 + \sqrt{7 -}}}}$

or $y = \sqrt{7 + \sqrt{7 - y}}$

or $y^2 - 7 = \sqrt{7 - y}$ ...(i)

Clearly $y^2 - 7 \geq 0 \Rightarrow y^2 \geq 7$

$\Rightarrow$ $y \geq \sqrt{7}$ or $y \leq -\sqrt{7}$

$y \leq -\sqrt{7}$ is not acceptable as $y \geq 0$ so only solution may be in the interval $y \geq \sqrt{7}$

Since $\sqrt{7} > 2$ so $y = 2$ is not possible

Keeping $y = 3$, 4 and 5 in equation (i) we see

That only $y = 3$ satisfies it and not $y = 4$ or $y = 5$.

So only solution is $y = 3$.

Answer is **(3)**

Example-25

If $0 \leq x \leq \pi$, then the solution of the equation $16^{\sin^2 x} + 16^{\cos^2 x} = 10$ is given by x equal to

(1) $\frac{\pi}{6}, \frac{\pi}{3}$ (2) $\frac{\pi}{3}, \frac{\pi}{2}$

(3) $\frac{\pi}{6}, \frac{\pi}{2}$ (4) None of these

Sol. Let $16^{\sin^2 x} = y$, then $16^{\cos^2 x} = 16^{1 - \sin^2 x} = \frac{16}{y}$

Hence, $y + \frac{16}{y} = 10$

$\Rightarrow y^2 - 10y + 16 = 0$ or $y = 2, 8$

Now $16^{\sin^2 x} = 2 \Rightarrow 4\sin^2 x = 1$

$\therefore\ \sin x = \pm\frac{1}{2} \Rightarrow x = \frac{\pi}{6}$

Also $16^{\sin^2 x} = 8 \Rightarrow 4\sin^2 x = 3$

$\therefore \sin x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}$

Hence **(1)** is the correct answer

EXERCISE-I

Quadratic Equation & Nature of roots

Q.1 The roots of the equation $(b-c)\,x^2+(c-a)\,x+(a-b)=0$ are

(1) $\dfrac{c-a}{b-c}, 1$ (2) $\dfrac{a-b}{b-c}, 1$

(3) $\dfrac{b-c}{a-b}, 1$ (4) $\dfrac{c-a}{a-b}, 1$

Q.2 If a, b, c are integers and $b^2 = 4(ac + 5d^2)$, $d \in N$, then roots of the equation $ax^2 + bx + c = 0$ are

(1) Irrational (2) Rational & different

(3) Complex conjugate (4) Rational & equal

Q.3 Let a, b and c be real numbers such that $4a + 2b + c = 0$ and $ab > 0$. Then the equation $ax^2 + bx + c = 0$ has

(1) real roots (2) imaginary roots

(3) exactly one root (4) none of these

Q.4 Consider the equation $x^2 + 2x - n = 0$, where $n \in N$ and $n \in [5, 100]$. Total number of different values of 'n' so that the given equation has integral roots, is

(1) 4 (2) 6 (3) 8 (4) 3

Q.5 The entire graph of the expression $y = x^2 + kx - x + 9$ is strictly above the x-axis if and only if

(1) $k < 7$ (2) $-5 < k < 7$

(3) $k > -5$ (4) none

Q.6 If $a, b \in R$, $a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots then $a + b + 1$ is:

(1) positive (2) negative

(3) zero (4) depends on the sign of b

Q.7 If a and b are the non-zero distinct roots of $x^2 + ax + b = 0$, then the least value of $x^2 + ax + b$ is

(1) $\dfrac{3}{2}$ (2) $\dfrac{9}{4}$ (3) $-\dfrac{9}{4}$ (4) 1

Q.8 If both roots of the quadratic equation $(2-x)(x+1) = p$ are distinct & positive, then p must lie in the interval

(1) $(2, \infty)$ (2) $(2, 9/4)$

(3) $(-\infty, -2)$ (4) $(-\infty, \infty)$

Q.9 If $(1 - p)$ is root of quadratic equation $x^2 + px + (1-p) = 0$, then its roots are

(1) 0, 1 (2) –1, 1 (3) 0, –1 (4) –1, 2

Sum and Product of roots

Q.10 If a, b are the roots of quadratic equation $x^2 + px + q = 0$ and g, d are the roots of $x^2 + px - r = 0$, then $(a-g)\,.\,(a-d)$ is equal to :

(1) $q + r$ (2) $q - r$

(3) $-(q + r)$ (4) $-(p + q + r)$

Q.11 Two real numbers a & b are such that $a+b = 3$ & $|a-b| = 4$, then a & b are the roots of the quadratic equation:

(1) $4x^2 - 12x - 7 = 0$ (2) $4x^2 - 12x + 7 = 0$

(3) $4x^2 - 12x + 25 = 0$ (4) none of these

Q.12 Let conditions C_1 and C_2 be defined as follows : $C_1 : b^2 - 4ac \geq 0$, C_2 : a, – b, c are of same sign. The roots of $ax^2 + bx + c = 0$ are real and positive, if

(1) both C_1 and C_2 are satisfied

(2) only C_2 is satisfied

(3) only C_1 is satisfied

(4) None of these

Q.13 If α, β are roots of the equation $ax^2 + bx + c = 0$, then the value of $\alpha^3 + \beta^3$ is

(1) $\dfrac{3abc + b^3}{a}$ (2) $\dfrac{a^3 + b^3}{3abc}$

(3) $\dfrac{3abc - b^3}{a^3}$ (4) $\dfrac{-(3abc + b^3)}{a^3}$

Common Roots

Q.14 If a, b, p, q are non-zero real numbers, then the equations $2a^2x^2 - 2\,ab\,x + b^2 = 0$ and $p^2x^2 + 2\,pq\,x + q^2 = 0$ have:

(1) no common root

(2) one common root if $2\,a^2 + b^2 = p^2 + q^2$

(3) two common roots if $3\,pq = 2\,ab$

(4) two common roots if $3\,qb = 2\,ap$

Theory of Equation and Identity, Inequalities

Q.15 Number of values of 'p' for which the equation $(p^2 - 3p + 2)\,x^2 - (p^2 - 5p + 4)\,x + p - p^2 = 0$ possess more than two roots, is :

(1) 0 (2) 1 (3) 2 (4) none

Q.16 The number of the integer solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is

(1) 1 (2) 2 (3) 3 (4) None of these

Q.17 The complete set of values of 'x' which satisfy the inequations : $5x+2<3x+8$ and $\frac{x+2}{x-1}<4$ is
(1) $(-\infty, 1)$ (2) $(2, 3)$
(3) $(-\infty, 3)$ (4) $(-\infty, 1)\cup(2, 3)$

Q.18 The complete solution set of the inequality $\frac{x^4-3x^3+2x^2}{x^2-x-30}\ge 0$ is:
(1) $(-\infty,-5)\cup(1,2)\cup(6,\infty)\cup\{0\}$
(2) $(-\infty,-5)\cup[1,2]\cup(6,\infty)\cup\{0\}$
(3) $(-\infty,-5]\cup[1,2]\cup[6,\infty)\cup\{0\}$
(4) none of these

Q.19 If the inequality $(m-2)x^2+8x+m+4>0$ is satisfied for all $x\in R$, then the least integral 'm' is:
(1) 4 (2) 5 (3) 6 (4) none

Q.20 The complete set of real 'x' satisfying $||x-1|-1|\le 1$ is :
(1) $[0, 2]$ (2) $[-1, 3]$ (3) $[-1, 1]$ (4) $[1, 3]$

Q.21 If $\log_{1/3}\frac{3x-1}{x+2}$ is less than unity, then 'x' must lie in the interval :
(1) $(-\infty,-2)\cup(5/8,\infty)$
(2) $(-2, 5/8)$
(3) $(-\infty,-2)\cup(1/3, 5/8)$
(4) $(-2, 1/3)$

Q.22 Solution set of the inequality $2-\log_2(x^2+3x)\ge 0$ is :
(1) $[-4, 1]$
(2) $[-4,-3)\cup(0, 1]$
(3) $(-\infty,-3)\cup(1,\infty)$
(4) $(-\infty,-4)\cup[1,\infty)$

Q.23 The set of all the solutions of the inequality $\log_{1-x}(x-2)\ge 0$ is
(1) $(-\infty, 0)$ (2) $(2,\infty)$
(3) $(-\infty, 1)$ (4) ϕ

Q.24 If $\log_{0.3}(x-1)<\log_{0.09}(x-1)$, then x lies in the interval
(1) $(2,\infty)$ (2) $(1, 2)$
(3) $(-2,-1)$ (4) none of these

Q.25 If $\log_{0.5}\log_5(x^2-4)>\log_{0.5}1$, then 'x' lies in the interval
(1) $(-3,-\sqrt{5})\cup(\sqrt{5}, 3)$
(2) $(-3,-\sqrt{5})\cup(\sqrt{5}, 3\sqrt{5})$
(3) $(\sqrt{5}, 3\sqrt{5})$
(4) ϕ

Q.26 The set of all solutions of the inequality $(1/2)^{x^2-2x}<1/4$ contains the set
(1) $(-\infty, 0)$ (2) $(-\infty, 1)$
(3) $(1,\infty)$ (4) $(3,\infty)$

Q.27 If $\frac{6x^2-5x-3}{x^2-2x+6}\le 4$, then least and the highest values of $4x^2$ are
(1) 0 & 81 (2) 9 & 81
(3) 36 & 81 (4) None of these

Q.28 If two roots of the equation $x^3-px^2+qx-r=0$ are equal in magnitude but opposite in sign, then:
(1) $pr=q$ (2) $qr=p$
(3) $pq=r$ (4) none

Q.29 If α, β & γ are the roots of the equation $x^3-x-1=0$ then, $\frac{1+\alpha}{1-\alpha}+\frac{1+\beta}{1-\beta}+\frac{1+\gamma}{1-\gamma}$ has the value equal to:
(1) zero (2) -1 (3) -7 (4) 1

Q.30 For what value of a and b the equation $x^4-4x^3+ax^2+bx+1=0$ has four real positive roots ?
(1) $(-6,-4)$ (2) $(-6, 5)$
(3) $(-6, 4)$ (4) $(6,-4)$

Q.31 If α, β are roots of the equation $ax^2+bx+c=0$ then the equation whose roots are $2\alpha+3\beta$ and $3\alpha+2\beta$ is
(1) $ab\,x^2-(a+b)\,cx+(a+b)^2=0$
(2) $ac\,x^2-(a+c)\,bx+(a+c)^2=0$
(3) $ac\,x^2\,(a+c)\,bx-(a+c)^2=0$
(4) None of these

Q.32 If S is the set of all real x such that $\frac{2x-1}{2x^3+3x^2+x}$ is positive, then S contains
(1) $(-\infty,-3/2)$ (2) $(-3/2, 1/4)$
(3) $(-1/4, 1/2)$ (4) $(-1/2, 3)$

Max and Min Value, Factorization

Q.33 If $y=-2x^2-6x+9$, (for $x\in R$) then
(1) maximum value of y is -11 and it occurs at $x=2$
(2) minimum value of y is -11 and it occurs at $x=2$
(3) maximum value of y is 13.5 and it occurs at $x=-1.5$
(4) minimum value of y is 13.5 and it occurs at $x=-1.5$

Q.34 If 'x' is real and $k=\frac{x^2-x+1}{x^2+x+1}$, then :
(1) $\frac{1}{3}\le k\le 3$ (2) $k\ge 5$
(3) $k\le 0$ (4) none

Q.35 Consider $y=\frac{2x}{1+x^2}$, where x is real, then the range of expression y^2+y-2 is
(1) $[-1, 1]$ (2) $[0, 1]$
(3) $[-9/4, 0]$ (4) $[-9/4, 1]$

Q.36 The values of x and y besides y can satisfy the equation (x, y $\in$ real numbers) $x^2 - xy + y^2 - 4x - 4y + 16 = 0$

(1) 2, 2 (2) 4, 4
(3) 3, 3 (4) None of these

Q.37 If x is real, then $\frac{x^2 - x + c}{x^2 + x + 2c}$ can take all real values if

(1) $c \in [0, 6]$
(2) $c \in [-6, 0]$
(3) $c \in (-\infty, -6) \cup (0, \infty)$
(4) $c \in (-6, 0)$

Location of roots

Q.38 The real values of 'a' for which the quadratic equation $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite sign is given by:

(1) $a > 5$ (2) $0 < a < 4$
(3) $a > 0$ (4) $a > 7$

Q.39 If a, b are the roots of the quadratic equation $x^2 - 2p(x - 4) - 15 = 0$, then the set of values of 'p' for which one root is less than 1 & the other root is greater than 2 is:

(1) $(7/3, \infty)$ (2) $(-\infty, 7/3)$
(3) $x \in R$ (4) none

Q.40 The values of k, for which the equation $x^2 + 2(k - 1)x + k + 5 = 0$ possess atleast one positive root, are

(1) $[4, \infty)$ (2) $(-\infty, -1] \cup [4, \infty)$
(3) $[-1, 4]$ (4) $(-\infty, -1]$

EXERCISE-II

Q.1 The roots of the given equation $(p - q)x^2 + (q - r)x + (r - p) = 0$ are

(1) $\frac{p-q}{r-p}, 1$ (2) $\frac{q-r}{p-q}, 1$

(3) $\frac{r-p}{p-q}, 1$ (4) $1, \frac{q-r}{p-q}$

Q.2 If a + b + c = 0, and a, b, c $\in$ R, then the roots of the equation $4ax^2 + 3bx + 2c = 0$ are

(1) Equal (2) Imaginary
(3) Real (4) Both (1) and (2)

Q.3 The roots of the given equation $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$ are
[Where $a \neq b$]

(1) Rational (2) Irrational
(3) Real (4) Imaginary

Q.4 If $a > 0$, $b > 0$, $c > 0$ then both the roots of the equation $ax^2 + bx + c = 0$

(1) Are real and negative
(2) Have negative real parts
(3) Are rational numbers
(4) Both (1) and (3)

Q.5 If a, b, c $\in$ Q, then roots of the equation $(b + c - 2a)x^2 + (c + a - 2b)x + (a + b - 2c) = 0$ are

(1) Rational (2) Non-real
(3) Irrational (4) Equal

Q.6 The value of k for which the quadratic equation, $kx^2 + 1 = kx + 3x - 11x^2$ has real and equal roots are

(1) –11, –3 (2) 5, 7
(3) 5, –7 (4) –7, 25

Q.7 The value of k for which the equation $(k - 2)x^2 + 8x + k + 4 = 0$ has both real, distinct and negative is -

(1) 0 (2) 2 (3) 3 (4) – 4

Q.8 $x^2 + x + 1 + 2k(x^2 - x - 1) = 0$ is a perfect square for how many values of k

(1) 2 (2) 0 (3) 1 (4) 3

Q.9 If one root of $5x^2 + 13x + k = 0$ is reciprocal of the other, then k =

(1) 0 (2) 5 (3) 1/6 (4) 6

Q.10 If α and β are the roots of the equation $4x^2 + 3x + 7 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

(1) $-\frac{3}{7}$ (2) $\frac{3}{7}$ (3) $-\frac{3}{5}$ (4) $\frac{3}{5}$

Q.11 If α, β are the roots of the equation $ax^2 + bx + c = 0$ then the equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is

(1) $acx^2 + (a + c)bx + (a + c)^2 = 0$
(2) $abx^2 + (a + c)bx + (a + c)^2 = 0$
(3) $acx^2 + (a + b)cx + (a + c)^2 = 0$
(4) $acx^3 + (a + c)bx + (a + c)^3$

Q.12 If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} =$

(1) $\frac{2}{a}$ (2) $\frac{2}{b}$ (3) $\frac{2}{c}$ (4) $-\frac{2}{a}$

Q.13 If the sum of the roots of the equation $ax^2 + bx + c = 0$ be equal to the sum of their squares, then

(1) $a(a+b) = 2bc$ (2) $c(a+c) = 2ab$
(3) $b(a+b) = 2ac$ (4) $b(a+b) = ac$

Q.14 If α, β be the roots of the equation $x^2 - 2x + 3 = 0$, then the equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ is

(1) $x^2+2x+1=0$ (2) $9x^2+2x+1=0$
(3) $9x^2-2x+1=0$ (4) $9x^2+2x-1=0$

Q.15 If the product of roots of the equation, $mx^2 + 6x + (2m - 1) = 0$ is -1, then the value of m will be

(1) 1 (2) -1 (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

Q.16 If α and β are the roots of the equation $x^2 - 4x + 1 = 0$ the value of $\alpha^3 + \beta^3$ is

(1) 76 (2) 52 (3) -52 (4) -76

Q.17 If the roots of the equation $12x^2 - mx + 5 = 0$ are in the ratio 2 : 3, then m =

(1) $5\sqrt{10}$ (2) $3\sqrt{10}$
(3) $2\sqrt{10}$ (4) $10\sqrt{5}$

Q.18 If both the roots of $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are common, then $2r - p$ is equal to

(1) -1 (2) 0 (3) 1 (4) 2

Q.19 If the equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$, have a common root,(Where $p \neq q$) then $p + q + 1 =$

(1) 0 (2) 1 (3) 2 (4) -1

Q.20 $x^2 - 11x + a$ and $x^2 - 14x + 2a$ will have a common factor, if $a =$

(1) 24 (2) 0, 24 (3) 3, 24 (4) 0, 3

Q.21 If x is real and satisfies $x + 2 > \sqrt{x+4}$, then

(1) $x < -2$ (2) $x > 0$
(3) $-3 < x < 0$ (4) $-3 < x < 4$

Q.22 The set of all real numbers x for which $x^2 - |x + 2| + x > 0$ is

(1) $(-\infty, -2) \cup (2, \infty)$ (2) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ (3) $(-\infty, -1) \cup (1, \infty)$ (4) $(\sqrt{2}, \infty)$

Q.23 If α, β, γ are the roots of the equation $x^3 + 4x + 1 = 0$, then $(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1}$

(1) 2 (2) 3 (3) 4 (4) 5

Q.24 If α, β, γ are the roots of the equation $x^3 + x + 1 = 0$, then the value of $\alpha^3 \beta^3 \gamma^3$

(1) 0 (2) -3 (3) 3 (4) -1

Q.25 If x is real, then the maximum and minimum values of the expression $\frac{x^2 - 3x + 4}{x^2 + 3x + 4}$ will be

(1) 2, 1 (2) $5, \frac{1}{5}$ (3) $7, \frac{1}{7}$ (4) 2, 7

Q.26 The smallest value of $x^2 - 3x + 3$ in the interval $(-3, 3/2)$ is

(1) 3/4 (2) 5 (3) -15 (4) -20

Q.27 If $x^2 - 3x + 2$ be a factor of $x^4 - px^2 + q$, then (pq) =

(1) (3, 4) (2) (4, 5)
(3) (4, 3) (4) (5, 4)

Q.28 If the roots of $x^2 + x + a = 0$ exceed a , then

(1) $2 < a < 3$ (2) $a > 3$
(3) $-3 < a < 3$ (4) $a < -2$

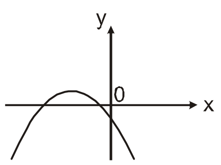
Q.29 The value of p for which both the roots of the equation $4x^2 - 20px + (25p^2 + 15p - 66) = 0$ are less than 2, lies in

(1) (4/5, 2) (2) $(2, \infty)$
(3) $(-1, -4/5)$ (4) $(-\infty, -1)$

EXERCISE-III

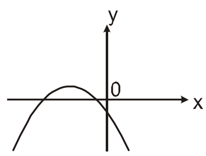
MCQ/COMPREHENSION/MATCHING/NUMERICAL

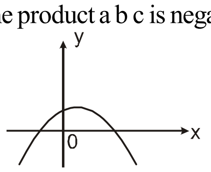
Q.1 The graph of the quadratic polynomial $y = ax^2 + bx + c$ is as shown in the figure. Then :

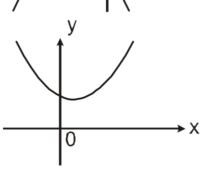


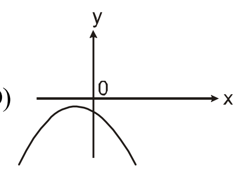
(A) $b^2 - 4ac > 0$ (B) $b < 0$
(C) $a > 0$ (D) $c < 0$

Q.2 For which of the following graphs of the quadratic expression $y = ax^2 + bx + c$, the product a b c is negative ?

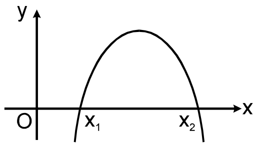
(A) 

(B) 

(C) 

(D) 

Q.3 The adjoining figure shows the graph of $y = ax^2 + bx + c$. Then



(A) $a < 0$
(B) $b^2 < 4ac$
(C) $c > 0$
(D) a and b are of opposite sign

Q.4 For the equation $|x|^2 + |x| - 6 = 0$, the correct statement (s) is (are) :
(A) sum of roots is 0 (B) product of roots is – 4
(C) there are 4 roots (D) there are only 2 roots

Q.5 If a, b are the roots of $ax^2 + bx + c = 0$, and a + h, b + h are the roots of $px^2 + qx + r = 0$, (where h ¹ 0), then

(A) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$ (B) $h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

(C) $h = \frac{1}{2}\left(\frac{b}{a} + \frac{q}{p}\right)$ (D) $\frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$

Q.6 If a, b are non-zero real numbers and a, b the roots of $x^2 + ax + b = 0$, then
(A) α^2, β^2 are the roots of $x^2 - (2b - a^2)x + a^2 = 0$
(B) $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of $bx^2 + ax + 1 = 0$
(C) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $bx^2 + (2b - a^2)x + b = 0$
(D) (a – 1), (b – 1) are the roots of the equation $x^2 + x(a + 2) + 1 + a + b = 0$

Q.7 $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d = 0$, then the real root of above equation is
($a, b, c, d \in R$)
(A) – d/a (B) d/a
(C) (b – a)/a (D) (a – b)/a

Q.8 If $\frac{1}{2} \le \log_{0.1} x \le 2$, then

(A) maximum value of x is $\frac{1}{\sqrt{10}}$

(B) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$

(C) minimum value of x is $\frac{1}{\sqrt{10}}$

(D) minimum value of x is $\frac{1}{100}$

Q.9 If the quadratic equations $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root, then the equation containing their other roots is/are :
(A) $x^2 + a(b + c)x - a^2bc = 0$
(B) $x^2 - a(b + c)x + a^2bc = 0$
(C) $a(b + c)x^2 - (b + c)x + abc = 0$
(D) $a(b + c)x^2 + (b + c)x - abc = 0$

Q.10 If the quadratic equations $ax^2 + bx + c = 0$ ($a, b, c \in R$, $a \neq 0$) and $x^2 + 4x + 5 = 0$ have a common root, then a, b, c must satisfy the relations:
(A) $a > b > c$
(B) $a < b < c$
(C) $a = k$; $b = 4k$; $c = 5k$ ($k \in R, k \neq 0$)
(D) $b^2 - 4ac$ is negative.

Q.11 If α, β are the real and distinct roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always
(A) two real roots
(B) two negative roots
(C) two positive roots
(D) one positive root and one negative root

Q.12 If $(x^2 + x + 1) + (x^2 + 2x + 3) + (x^2 + 3x + 5) + \ldots\ldots + (x^2 + 20x + 39) = 4500$, then x is equal to
(A) 10 (B) – 10 (C) 20.5 (D) –20.5

Q.13 If roots of equation, $x^3 + bx^2 + cx - 1 = 0$ forms an increasing G.P., then
(A) $b + c = 0$
(B) $b \in (-\infty, -3)$
(C) one of the roots = 1
(D) one root is smaller than 1 & other > 1

Q.14 Let $f(x) = \frac{3}{x-2} + \frac{4}{x-3} + \frac{5}{x-4}$, then f(x) = 0 has
(A) exactly one real root in (2, 3)
(B) exactly one real root in (3, 4)
(C) at least one real root in (2, 3)
(D) None of these

Q.15 If $p = -1$, $x_1 = -1$, $x_2 = -5$ then the vertex V is
(A) (–3, –4)(B) (–3, 1)
(C) (–4, 3) (D) (–3, 4)

Q.16 $r(p - q - r)$ is
(A) positive (B) negative
(C) zero (D) $> f(x_3)$

Comprehension # 3 (Q. No. 17 to 19)

Consider the equation $x^4 - \lambda x^2 + 9 = 0$. This can be solved by substituting $x^2 = t$ such equations are called as pseudo quadratic equations.

Q.17 If the equation has four real and distinct roots, then λ lies in the interval
(A) $(-\infty, -6) \cup (6, \infty)$ (B) $(0, \infty)$
(C) $(6, \infty)$ (D) $(-\infty, -6)$

Q.18 If the equation has no real root, then λ lies in the interval

(A) $(-\infty, 0)$ (B) $(-\infty, 6)$
(C) $(6, \infty)$ (D) $(0, \infty)$

Q.19 If the equation has only two real roots, then set of values of λ is

(A) $(-\infty, -6)$ (B) $(-6, 6)$
(C) $\{6\}$ (D) ϕ

Q.20 **MATCH THE COLUMN:**

Column – I	Column – II
(A) If the roots of $x^2 - bx + c = 0$ are two consecutive integers, then value of $b^2 - 4c$	(P) 1
(B) If $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ $(a \neq 0)$ have a common root, then value of $a + b$	(Q) 7
(C) If α, β are roots of $x^2 - x + 3 = 0$ then value of $\alpha^4 + \beta^4$	(R) 17
(D) If α, β, γ are the roots of $x^3 - 7x^2 + 16x - 12 = 0$	(S) – 1

Q.21

Column – I	Column – II
(A) If $\alpha, \alpha + 4$ are two roots of $x^2 - 8x + k = 0$, then possible value of k is	(P) 2
(B) Number of real roots of equation $x^2 - 5\|x\| + 6 = 0$ are 'n', then value of $\frac{n}{2}$ is	(Q) 3
(C) If $3 - i$ is a root of $x^2 + ax + b = 0$ $(a, b \in R)$, then b is	(R) 12
(D) If both roots of $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then 'k' may be equal to	(S) 10

NUMERICAL BASED QUESTIONS

Q.22 Find number of integer roots of equation $x(x + 1)(x + 2)(x + 3) = 120$.

Q.23 Find product of all real values of x satisfying $(5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{6})^{x^2-3} = 10$

Q.24 The least prime integral value of '2a' such that the roots α, β of the equation $2x^2 + 6x + a = 0$ satisfy the inequality $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} < 2$ is

Q.25 If a, b are the roots of $x^2 + px + 1 = 0$ and c, d are the roots of $x^2 + qx + 1 = 0$. Then find the value of $(a - c)(b - c)(a + d)(b + d)/(q^2 - p^2)$.

Q.26 α, β are roots of the equation $\lambda(x^2 - x) + x + 5 = 0$. If λ_1 and λ_2 are the two values of λ for which the roots α, β are connected by the relation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 4$, then the value of $\left(\frac{\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1}}{14}\right)$ is

Q.27 Let α, β be the roots of the equation $x^2 + ax + b = 0$ and γ, δ be the roots of $x^2 - ax + b - 2 = 0$. If $\alpha\beta\gamma\delta = 24$ and $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{5}{6}$, then find the value of a.

Q.28 The least value of expression $x^2 + 2xy + 2y^2 + 4y + 7$ is:

Q.29 If $a > b > 0$ and $a^3 + b^3 + 27ab = 729$ then the quadratic equation $ax^2 + bx - 9 = 0$ has roots α, β $(\alpha < \beta)$. Find the value of $4\beta - a\alpha$.

Q.30 Let α and β be roots of $x^2 - 6(t^2 - 2t + 2)x - 2 = 0$ with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then find the minimum value of $\frac{a_{100} - 2a_{98}}{a_{99}}$ (where $t \in R$)

Q.31 If roots of the equation $x^2 - 10ax - 11b = 0$ are c and d and those of $x^2 - 10cx - 11d = 0$ are a and b, then find the value of $\frac{a+b+c+d}{110}$. (where a, b, c, d are all distinct numbers)

EXERCISE-IV

JEE-MAIN
PREVIOUS YEAR'S

Q.1 The sum of all real values of x satisfying the equation **[JEE Main-2016]**
$(x^2-5x+5)^{x^2+4x-60}=1$ is:
(1) 3 (2) –4 (3) 6 (4) 5

Q.2 If, for a positive integer n, the quadratic equation, $x(x+1)+(x+1)(x+2)+\ldots\ldots+(x+\overline{n-1})(x+n)=10n$ has two consecutive integral solutions, then n is equal to : **[JEE Main-2017]**
(1) 11 (2) 12 (3) 9 (4) 10

Q.3 If $\alpha, \beta \in C$ are the distinct roots of the equation $x^2-x+1=0$, then $\alpha^{101}+\beta^{107}$ is equal to- **[JEE Main-2018]**
(1) 0 (2) 1 (3) 2 (4) –1

Q.4 Let α and β be two roots of the equation $x^2+2x+2=0$, then $\alpha^{15}+\beta^{15}$ is equal to :- **[JEE Main - 2019 (January)]**
(1) –256 (2) 512
(3) –512 (4) 256

Q.5 If both the roots of the quadratic equation $x^2-mx+4=0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval **[JEE Main - 2019 (January)]**
(1) (4, 5) (2) (3, 4)
(3) (5, 6) (4) (–5, –4)

Q.6 The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2-11x+\alpha=0$ are rational numbers is : **[JEE Main - 2019 (January)]**
(1) 2 (2) 5
(3) 3 (4) 4

Q.7 Consider the quadratic equation $(c-5)x^2-2cx+(c-4)=0$, $c\neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is : **[JEE Main - 2019 (January)]**
(1) 18 (2) 12
(3) 10 (4) 11

Q.8 If one real root of the quadratic equation $81x^2+kx+256=0$ is cube of the other root, then a value of k is : **[JEE Main - 2019 (January)]**
(1) –81 (2) 100
(3) 144 (4) –300

Q.9 The number of integral values of m for which the quadratic expression, $(1+2m)x^2-2(1+3m)x+4(1+m)$, $x\in R$, is always positive, is :- **[JEE Main - 2019 (January)]**
(1) 3 (2) 8 (3) 7 (4) 6

Q.10 If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2+m(m-4)x+2=0$, then the least value of m for which $\lambda+\dfrac{1}{\lambda}=1$, is **[JEE Main - 2019 (January)]**
(1) $2-\sqrt{3}$ (2) $4-3\sqrt{2}$
(3) $-2+\sqrt{2}$ (4) $4-2\sqrt{3}$

Q.11 If α and β be the roots of the equation $x^2-2x+2=0$, then the least value of n for which $\left(\dfrac{\alpha}{\beta}\right)^n=1$ is : **[JEE Main - 2019(April)]**
(1) 2 (2) 3 (3) 4 (4) 5

Q.12 If three distinct numbers a,b,c are in G.P. and the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root, then which one of the following statements is correct ? **[JEE Main - 2019 (April)]**
(1) d, e, f are in A.P. (2) $\dfrac{d}{a},\dfrac{e}{b},\dfrac{f}{c}$ are in G.P.
(3) $\dfrac{d}{a},\dfrac{e}{b},\dfrac{f}{c}$ are in A.P. (4) d, e, f are in G.P.

Q.13 The number of integral values of m for which the equation $(1+m^2)x^2-2(1+3m)x+(1+8m)=0$ has no real root is : **[JEE Main-2019(April)]**
(1) infinitely many (2) 2
(3) 3 (4) 1

Q.14 Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2+px+q=0$, then : **[JEE Main-2019(April)]**
(1) $q^2+4p+14=0$ (2) $p^2-4q-12=0$
(3) $q^2-4p-16=0$ (4) $p^2-4q+12=0$

Q.15 If m is chosen in the quadratic equation $(m^2+1)x^2-3x+(m^2+1)^2=0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is :- **[JEE Main - 2019(April)]**
(1) $8\sqrt{3}$ (2) $4\sqrt{3}$ (3) $10\sqrt{5}$ (4) $8\sqrt{5}$

Q.16 If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then

$$\frac{\alpha^{12}+\beta^{12}}{\left(\alpha^{-12}+\beta^{-12}\right)(\alpha-\beta)^{24}}$$ is equal to :

[JEE Main - 2019(April)]

(1) $\frac{2^6}{(\sin\theta+8)^{12}}$ (2) $\frac{2^{12}}{(\sin\theta-8)^6}$

(3) $\frac{2^{12}}{(\sin\theta-4)^{12}}$ (4) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

Q.17 The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is :**[JEE Main - 2019(April)]**

(1) 2 (2) 3 (3) 4 (4) 1

Q.18 If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\alpha^r + \lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\beta^r$ is equal to :

[JEE Main-2019(April)]

(1) $\frac{21}{346}$ (2) $\frac{29}{358}$ (3) $\frac{1}{12}$ (4) $\frac{7}{116}$

Q.19 If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to : **[JEE Main - 2019(April)]**

(1) $\beta\gamma$ (2) 0 (3) $\alpha\gamma$ (4) $\alpha\beta$

Q.20 Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which of the following statements is not true? **[JEE Main-2020 (January)]**

(1) $p_5 = p_2 \cdot p_3$ (2) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(3) $p_3 = p_5 - p_4$ (4) $p_5 = 11$

Q.21 The least positive value of 'a' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is _______.

[JEE Main-2020 (January)]

Q.22 Let S be the set of all real roots of the equation, **[JEE Main-2020 (January)]**

$3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S :

(1) contains exactly two elements

(2) is a singleton

(3) contains at least four elements

(4) is an empty set

Q.23 The number of real roots of the equation, **[JEE Main-2020 (January)]**

$e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$

(1) 3 (2) 1 (3) 4 (4) 2

Q.24 Let a, b $\in$ R, a $\neq$ 0 be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, $\alpha^2 + \beta^2$ is equal to :

[JEE Main-2020 (January)]

(1) 25 (2) 26 (3) 24 (4) 28

Q.25 Let f(x) be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in : **[JEE Main-2020 (September)]**

(1) (–1, 0) (2) (–3, –1)

(3) (0, 1) (4) (1, 3)

Q.26 Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, , n = 1, 2, 3, ..., then :

[JEE Main-2020 (September)]

(1) $5S_6 + 6S_5 = 2S_4$ (2) $6S_6 + 5S_5 + 2S_4 = 0$

(3) $6S_6 + 5S_5 = 2S_4$ (4) $5S_6 + 6S_5 + 2S_4 = 0$

Q.27 The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0,1) is :

[JEE Main-2020 (September)]

(1) (–3, –1) (2) (2, 4]

(3) (0, 2) (4) (1, 3]

Q.28 If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to : **[JEE Main-2020 (September)]**

(1) $\frac{9}{4}(9 - q^2)$ (2) $\frac{9}{4}(9 + p^2)$

(3) $\frac{9}{4}(9 + q^2)$ (4) $\frac{9}{4}(9 - p^2)$

Q.29 Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to:

[JEE Main-2020 (September)]

(1) 18 (2) 9

(3) 27 (4) 36

Q.30 Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is:

[JEE Main-2020 (September)]

(1) 3 : 1 (2) 5 : 3

(3) 9 : 7 (4) 33 : 31

Q.31 If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2}+\frac{\beta}{1-\beta^2}$ is equal to :

[JEE Main-2020 (September)]

(1) $\frac{1}{24}$ (2) $\frac{27}{32}$ (3) $\frac{3}{8}$ (4) $\frac{27}{16}$

Q.32 The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is : **[JEE Main-2020 (September)]**

(1) $\frac{25}{9}$ (2) $\frac{25}{81}$ (3) $\frac{5}{9}$ (4) $\frac{5}{27}$

Q.33 If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to : **[JEE Main-2020 (September)]**

(1) $2\alpha^2$ (2) $-2\alpha(\alpha+1)$
(3) $2\alpha(\alpha-1)$ (4) $2\alpha(\alpha+1)$

Q.34 If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8}+\left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is :

[JEE Main-2020 (September)]

(1) 3 (2) 2 (3) 4 (4) 1

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is **[IIT JEE-2009]**

Q.2 Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

[IIT JEE-2010]

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

Q.3 Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is

[IIT JEE-2011]

(A) 1 (B) 2
(C) 3 (D) 4

Q.4 A value of b for which the equations **[IIT JEE-2011]**
$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$
have one root in common is
$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

Q.5 The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has **[JEE Advanced-2014]**
(A) only purely imaginary roots
(B) all real roots
(C) two real and two purely imaginary roots
(D) neither real nor purely imaginary roots

Q.6 Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S ? **[JEE Advanced-2015]**

(A) $\left(-\frac{1}{2},-\frac{1}{\sqrt{5}}\right)$ (B) $\left(-\frac{1}{\sqrt{5}},0\right)$

(C) $\left(0,\frac{1}{\sqrt{5}}\right)$ (D) $\left(\frac{1}{\sqrt{5}},\frac{1}{2}\right)$

Comprehension #1 (7 and 8)

Let p, q_n be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$ where $\alpha \neq \beta$. For n = 0, 1, 2,........let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$. **[JEE Advanced-2017]**

Q.7 $a_{12} =$
(A) $a_{11} + 2a_{10}$ (B) $2a_{11} + a_{10}$
(C) $a_{11} - a_{10}$ (D) $a_{11} + a_{10}$

Q.8 If $a_4 = 28$, then $p + 2q =$
(A) 14 (B) 7 (C) 21 (D) 12

Q.9 Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define

$a^n = \frac{\alpha^n-\beta^n}{\alpha-\beta}$, $n \geq 1$ **[JEE Advanced-2019]**

$b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \geq 2$.
Then which of the following options is/are correct ?
(1) $a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$ for all $n \geq 1$
(2) $\sum_{n=1}^{\infty}\frac{a_n}{10^n}=\frac{10}{89}$
(3) $\sum_{n=1}^{\infty}\frac{b_n}{10^n}=\frac{8}{89}$
(4) $b_n = \gamma^n + \beta^n$ for all $n \geq 1$

Q.10 Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define

$$a^n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 1, b_1 = 1 \text{ and } b_n = a_{n-1} + a_{n+1}, n \geq 2.$$

Then which of the following options is/are correct ?

[JEE Advanced-2019]

(1) $a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$ for all $n \geq 1$

(2) $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(3) $\displaystyle\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

(4) $b_n = \gamma^n + \beta^n$ for all $n \geq 1$

Q.11 Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c,d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of $ac(a - c) + ad(a - d) + bc(b - c) + bd(b - d)$ is

[JEE(Advanced) - 2020]

(A) 0

(B) 8000

(C) 8080

(D) 16000

Answer Key

EXERCISE-I

Q.1 (2)	**Q.2** (1)	**Q.3** (1)	**Q.4** (3)	**Q.5** (2)	**Q.6** (1)	**Q.7** (3)	**Q.8** (2)	**Q.9** (3)	**Q.10** (3)
Q.11 (1)	**Q.12** (1)	**Q.13** (3)	**Q.14** (1)	**Q.15** (2)	**Q.16** (4)	**Q.17** (4)	**Q.18** (2)	**Q.19** (2)	**Q.20** (2)
Q.21 (1)	**Q.22** (2)	**Q.23** (4)	**Q.24** (1)	**Q.25** (1)	**Q.26** (4)	**Q.27** (1)	**Q.28** (3)	**Q.29** (3)	**Q.30** (4)
Q.31 (4)	**Q.32** (1)	**Q.33** (3)	**Q.34** (1)	**Q.35** (3)	**Q.36** (2)	**Q.37** (4)	**Q.38** (2)	**Q.39** (2)	**Q.40** (4)

EXERCISE-II

Q.1 (3)	**Q.2** (3)	**Q.3** (4)	**Q.4** (2)	**Q.5** (1)	**Q.6** (3)	**Q.7** (3)	**Q.8** (1)	**Q.9** (2)	**Q.10** (1)
Q.11 (1)	**Q.12** (4)	**Q.13** (3)	**Q.14** (2)	**Q.15** (3)	**Q.16** (2)	**Q.17** (1)	**Q.18** (2)	**Q.19** (1)	**Q.20** (2)
Q.21 (2)	**Q.22** (2)	**Q.23** (3)	**Q.24** (4)	**Q.25** (3)	**Q.26** (1)	**Q.27** (4)	**Q.28** (4)	**Q.29** (4)	

EXERCISE-III

MCQ/COMPREHENSION/MATCHING/NUMERICAL

Q.1 (A, B, D)	**Q.2** (A,B,C,D)	**Q.3** (A,D)	**Q.4** (A, B, D)	**Q.5** (B,D)
Q.6 (B,C,D)	**Q.7** (A,D)	**Q.8** (A,B,D)	**Q.9** (B, D)	**Q.10** (C,D)
Q.11 (A, D)	**Q.12** (A,D)	**Q.13** (A,B,C,D)	**Q.14** (A,B)	**Q.15** (D)
Q.16 (B)	**Q.17** (C)	**Q.18** (B)	**Q.19** (D)	

Q.20 (A) $\rightarrow$ (P); (B) $\rightarrow$ (S); (C) $\rightarrow$ (Q); (D) $\rightarrow$ (R)

Q.21 (A) $\rightarrow$ (r),(B) $\rightarrow$ (p),(C) $\rightarrow$ (s), (D) $\rightarrow$ (p, q)

Q.22 [2]	**Q.23** [8]	**Q.24** [11]	**Q.25** [1]	**Q.26** [73]
Q.27 [10]	**Q.28** [3]	**Q.29** [13]	**Q.30** [6]	**Q.31** [11]

EXERCISE-IV

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (1)	**Q.2** (1)	**Q.3** (2)	**Q.4** (1)	**Q.5** (Bonus)	**Q.6** (3)	**Q.7** (4)	**Q.8** (4)	**Q.9** (3)	**Q.10** (2)
Q.11 (3)	**Q.12** (3)	**Q.13** (1)	**Q.14** (2)	**Q.15 (4)**	**Q.16 (4)**	**Q.17 (4)**	**Q.18 (3)**	**Q.19** (1)	**Q.20** (1)
Q.21 8	**Q.22** (2)	**Q.23** (2)	**Q.24** (1)	**Q.25** (1)	**Q.26** (1)	**Q.27** (4)	**Q.28** (4)	**Q.29** (1)	**Q.30** (3)
Q.31 (4)	**Q.32** (2)	**Q.33** (2)	**Q.34** (2)						

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Q.1 (2)	**Q.2** (B)	**Q.3** (C)	**Q.4** (B)	**Q.5** (D)	**Q.6** (A, D)	**Q.7** (D)	**Q.8** (D)	**Q.9** (1,2,4)	**Q.10** (1,2,4)
Q.11 (D)									

EXERCISE (Solution)

EXERCISE-I

Q.1 (2)

check by options

$x = 1$ is root

Let other root = a

$\therefore$ Product of the roots $= (1)(1) = \dfrac{a-b}{b-c}$

$\Rightarrow$ roots are $1, \dfrac{a-b}{b-c}$

Q.2 (1)

$D = b^2 - 4ac = 20d^2$

$\sqrt{D} = 2\sqrt{5}d$ here $\sqrt{5}$ is irrational

So roots are irrational.

Q.3 (1)

$D = b^2 - 4ac = b^2 - 4a(-4a - 2b)$

$= b^2 + 16a^2 + 8ab$

Since $ab > 0$

$\therefore D > 0$

So equation has real roots.

Q.4 (3)

For integral roots, D of equation should be perfect sq.

$\because \quad D = 4(1+n)$

By observation, for $n \in N$, D should be perfect sq. of even integer.

So $D = 4(1+n) = 6^2, 8^2, 10^2, 12^2, 14^2, 16^2, 18^2, 20^2$

No. of values of $n = 8$.

Q.5 (2)

Here for $D < 0$, entire graph will be above x-axis

$(\because a > 0)$

$\Rightarrow \quad (k-1)^2 - 36 < 0$

$\Rightarrow \quad (k-7)(k+5) < 0$

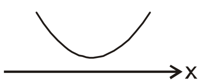
$\Rightarrow \quad -5 < k < 7$

Q.6 (1)

Let $f(x) = ax^2 - bx + 1$

Given $D < 0$ & $f(0) = 1 > 0$

$\therefore$ possible graph is as shown



i.e. $f(x) > 0 \ \forall \ x \in R$

or $f(-1) > 0$

$f(-1) = a + b + 1 > 0$

Q.7 (3)

$x^2 + ax + b = 0 \begin{matrix} a \\ b \end{matrix}$

$a + b = -a$

$\Rightarrow 2a + b = 0$

and $\quad ab = b$

$ab - b = 0$

$b(a-1) = 0$

$\Rightarrow \quad$ Either $b = 0$ or $a = 1$

But $\quad b \neq 0$(given)

$\therefore \quad a = 1$

$\therefore \quad b = -2$

$\therefore \quad f(x) = x^2 + x - 2$

Least value occurs at $x = -\dfrac{1}{2}$

Least value $= \dfrac{1}{4} - \dfrac{1}{2} - 2 = -\dfrac{9}{4}$

Q.8 (2)

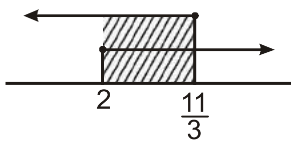
$(2-x)(x+1) = p$

$(x-2)(x+1) + p = 0$

$\Rightarrow x^2 - x - 2 + p = 0$

$\dfrac{c}{a} > 0 \Rightarrow p - 2 > 0$

& $D > 0 \Rightarrow 1 - 4(p-2) > 0 \Rightarrow p < \dfrac{9}{4}$



$\dfrac{-b}{2a} > 0, \dfrac{-1}{2(2-p)} > 0, \qquad P \in (2, \infty)$

Taking intersection of all $p \in \left(2, \dfrac{9}{4}\right)$

Q.9 (3)

$x^2 + px + (1-p) = 0$

$(1-p)^2 + p(1-p) + (1-p) = 0$

$(1-p)[1 - p + p + 1] = 0 \Rightarrow p = 1$

Q.E. will be $\Rightarrow x^2 + x = 0 \Rightarrow x(x+1) = 0$

$\Rightarrow \quad x = 0, -1$

Aliter
$\alpha + 1 - p = -p \Rightarrow \alpha = -1$
Satisfies
$1 - p + 1 - p = 0 \Rightarrow p = 1$
$\beta = 1 - p = 0 \Rightarrow \beta = 0$

Q.10 (3)
$a + b = -p$
$ab = q$
$g + d = -p$
$gd = -r$
$(a - g)(a - d) = a^2 - a(g + d) + gd$
$= a^2 + pa - r$
$= a(a + p) - r$
$= -ab - r$
$= -q - r$
$= -(q + r)$

Q.11 (1)
$|a - b| = 4 \Rightarrow (a - b)^2 = 16 \quad \Rightarrow (a + b)^2 - 4ab = 16$

$\Rightarrow 9 - 4ab = 16 \Rightarrow ab = -\dfrac{7}{4}$

$\Rightarrow$ equation is $x^2 - 3x - \dfrac{7}{4} = 0$

Q.12 (1)
$C_1 : b^2 - 4ac \geqslant 0,$
$ax^2 + bx + c = 0$ real roots C_1 satisfied
C_2 : a, –b, c are same sign

$\alpha + \beta > 0 \Rightarrow \dfrac{-b}{a} > 0$

$\alpha\beta > 0 \Rightarrow \dfrac{c}{a} > 0$

C_2 satisfied
C_1 & C_2 are satisfied

Q.13 (3)

$ax^2 + bx + c = 0, \alpha + \beta = \dfrac{-b}{a}, \alpha\beta = \dfrac{c}{a}$

$\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$$\alpha^3 + \beta^3 = \left(\frac{-b}{a}\right)\left[\left(\frac{-b}{a}\right)^2 - 3\frac{c}{a}\right]$$

$$= \frac{-b}{a}\left[\frac{b^2}{a^2} - \frac{3c}{a}\right] = \frac{-b}{a}\frac{(b^2 - 3ac)}{a^2} = \frac{3abc - b^3}{a^3}$$

Q.14 (1)
$D_1 = 4a^2b^2 - 8a^2b^2 = -4a^2b^2 < 0$ img. root
$D_2 = 4p^2q^2 - 4p^2q^2 = 0$ equal, real roots
So no common roots.

Q.15 (2)
For $(p^2 - 3p + 2)x^2 - (p^2 - 5p + 4)x + p - p^2 = 0$ to be an identity
$p^2 - 3p + 2 = 0 \Rightarrow p = 1, 2$ ...(1)
$p^2 - 5p + 4 = 0 \Rightarrow p = 1, 4$ ...(2)
$p - p^2 = 0 \Rightarrow p = 0, 1$ ...(3)
For (1), (2) & (3) to hold simultaneously
$p = 1.$

Q.16 (4)
$x^2 + 9 < (x + 3)^2 < 8x + 25$
$x^2 + 9 < x^2 + 6x + 9 \Rightarrow x > 0$
& $(x + 3)^2 < 8x + 25$
$x^2 + 6x + 9 - 8x - 25 < 0$
$x^2 - 2x - 16 < 0$
$1 - \sqrt{17} < x < 1 + \sqrt{17}$ & $x > 0$
$\Rightarrow \quad x \in (0, 1 + \sqrt{17})$
Integer x = 1, 2, 3, 4, 5
No. of integer are = 5

Q.17 (4)
$5x + 2 < 3x + 8 \Rightarrow 2x < 6 \Rightarrow x < 3$...(i)

$\dfrac{x+2}{x-1} < 4 \Rightarrow \dfrac{x+2}{x-1} - 4 < 0 \Rightarrow \dfrac{-3x+6}{x-1} < 0$

$\Rightarrow \dfrac{x-2}{x-1} > 0 \Rightarrow x \in (-\infty, 1) \cup (2, \infty)$...(ii)

Taking intersection of (i) and (ii) $x \in (-\infty, 1) \cup (2, 3)$

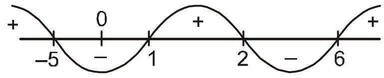
Q.18 (2)

$$\frac{x^2(x^2 - 3x + 2)}{x^2 - x - 30} \geq 0$$

$$\Rightarrow \quad \frac{x^2(x-1)(x-2)}{(x+5)(x-6)} \geq 0$$

$x \neq -5, 6$

$x \in (-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$



Q.19 (2)
$\because \quad (m - 2)x^2 + 8x + m + 4 > 0 \ \forall x \in R$
$\Rightarrow \quad m > 2 \ \& \ D < 0$
$64 - 4(m - 2)(m + 4) < 0$
$16 - [m^2 + 2m - 8] < 0$
$\Rightarrow \quad m^2 + 2m - 24 > 0$
$\Rightarrow \quad (m + 6)(m - 4) > 0$

$m \in (-\infty, -6) \cup (4, \infty)$

But $\quad m > 2$

$\Rightarrow \quad m \in (4, \infty)$

Then least integral m is $m = 5$.

Q.20 (2)

$-1 \le |x-1| - 1 \le 1$

$\Rightarrow \quad 0 \le |x-1| \le 2$

$\Rightarrow \quad 0 \le |x-1|$

$\Rightarrow \quad x \in R \quad$ ...(1)

and $\quad |x-1| \le 2$

$\Rightarrow \quad -2 \le x - 1 \le 2$

$\Rightarrow \quad -1 \le x \le 3 \quad$ ...(2)

$(1) \cap (2)$

$\Rightarrow \quad x \in [-1, 3]$.

Q.21 (1)

$$\log_{1/3} \frac{3x-1}{x+2} < 1$$

$$\Rightarrow \frac{3x-1}{x+2} > 0 \Rightarrow x \in (-\infty, -2) \cup \left(\frac{1}{3}, \infty\right) \quad \text{....(i)}$$

and $\quad \dfrac{3x-1}{x+2} > \dfrac{1}{3}$

$\Rightarrow \quad \dfrac{8x-5}{x+2} > 0$

$\Rightarrow \quad x \in (-\infty, -2) \cup \left(\dfrac{5}{8}, \infty\right) \quad$ .....(ii)

$(i) \cap (ii) \Rightarrow x \in (-\infty, -2) \cup \left(\dfrac{5}{8}, \infty\right)$

Q.22 (2)

$2 - \log_2 (x^2 + 3x) \ge 0$

$\Rightarrow \quad \log_2 (x^2 + 3x) \le 2$

$\quad x^2 + 3x > 0$

$\Rightarrow \quad x \in (-\infty, -3) \cup (0, \infty) \quad$ .....(i)

and $\quad x^2 + 3x \le 4$

$\Rightarrow \quad (x-1)(x+4) \le 0$

$\Rightarrow \quad x \in [-4, 1] \quad$ .....(ii)

$(i) \cap (ii) \Rightarrow x \in [-4, -3) \cup (0, 1]$

Q.23 (4)

$\log_{1-x} (x-2) \ge 0$

$x > 2 \quad$ ....(1)

(i) When $0 < 1 - x < 1 \quad \Rightarrow \quad 0 < x < 1$

So no common range comes out.

(ii) When $1 - x > 1 \quad \Rightarrow \quad x < 0$ but $x > 2$

here, also no common range comes out. , hence no solution.

Finally, no solution

Q.24 (1)

$\log_{0.3} (x-1) < \log_{0.09} (x-1)$

$$\log_{0.3} (x-1) < \frac{\log_{0.3} (x-1)}{2}$$

$\Rightarrow \quad \log_{0.3} (x-1) < 0 \quad \Rightarrow \quad x - 1 > 1 \quad \Rightarrow x > 2$

Q.25 (1)

$\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$

$\log_{0.5} \log_5 (x^2 - 4) > 0$

$\Rightarrow \quad x^2 - 4 > 0$

$\Rightarrow x \in (-\infty, -2) \cup (2, \infty) \quad$ ....(i)

$\log_5 (x^2 - 4) > 0 \quad \Rightarrow x^2 - 5 > 0$

$\Rightarrow x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty) \quad$ ....(ii)

$\log_5 (x^2 - 4) < 1$

$\Rightarrow x^2 - 9 < 0 \quad \Rightarrow x \in (-3, 3) \quad$ ....(iii)

$(i) \cap (ii) \cap (iii) \Rightarrow x \in (-3, \sqrt{5}) \cup (\sqrt{5}, 3)$

Q.26 (4)

$\left(\dfrac{1}{2}\right)^{x^2 - 2x} < \left(\dfrac{1}{2}\right)^2$ here base is less than zero so inequality change

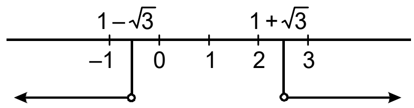
$\Rightarrow x^2 - 2x > 2 \Rightarrow x^2 - 2x - 2 > 0$

$$\alpha, \beta = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$$

$a = 1 - \sqrt{3}$, $b = 1 + \sqrt{3}$

$(x-a)(x-b) > 0$

$x \in \left(-\infty, \ 1 - \sqrt{3}\right) \cup \left(1 + \sqrt{3}, \ \infty\right)$, x can be in $(3, \infty)$



Q.27 (1)

$$\frac{6x^2 - 5x - 3}{x^2 - 2x + 6} \le 4$$

D^r is always > 0

$6x^2 - 5x - 3 - 4x^2 + 8x - 24 \le 0$

$\Rightarrow \quad 2x^2 + 3x - 27 \le 0$

$\Rightarrow \quad (2x + 9)(x - 3) \le 0 \Rightarrow x \in \left[\dfrac{-9}{2}, 3\right]$

least value of $4x^2 = 4.0^2 = 0$

Highest value of $4x^2$ is $= \max\left(4\cdot\left(-\frac{9}{2}\right)^2,\ 4\cdot 3^2\right)$

$= \max(81, 36) = 81$

Q.28 (3)

Let the roots be $\alpha, \beta, -\beta$

then $\alpha + \beta - \beta = p$

$\Rightarrow \quad \alpha = p \qquad ...(1)$

and $\quad \alpha\beta - \alpha\beta - \beta^2 = q$

$\Rightarrow \quad \beta^2 = -q \qquad ...(2)$

also $-\alpha\beta^2 = r$

$\Rightarrow \quad pq = r$ [using (1)].

Q.29 (3)

$x^3 - x - 1 = 0 \; \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}$

then $\alpha^3 - \alpha - 1 = 0.$ ....(1)

Let $\frac{1+\alpha}{1-\alpha} = y \Rightarrow \quad \alpha = \frac{y-1}{y+1}$

from equation (1) $\left(\frac{y-1}{y+1}\right)^3 - \left(\frac{y-1}{y+1}\right) - 1 = 0$

$\Rightarrow \; y^3 + 7y^2 - y + 1 = 0 \; \begin{matrix} \frac{1+\alpha}{1-\alpha} \\ \frac{1+\beta}{1-\beta} \\ \frac{1+\gamma}{1-\gamma} \end{matrix}$

then $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma} = -7$ Ans.

Q.30 (4)

$x^4 - 4x^3 + ax^2 + bx + 1 = 0$

real & positive roots

$\alpha + \beta + r + \delta = 4$ & $\alpha\beta r\delta = 1$

$\Rightarrow \quad \alpha = \beta = r = \delta = 1$

$\Sigma\alpha\beta = a \Rightarrow a = 6$

$\Sigma\alpha\beta r = -b \Rightarrow b = -4$

or $\quad (x-1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$

Q.31 (4)

$ax^2 + bx + c = 0 \; \begin{matrix} \alpha \\ \beta \end{matrix}$

sum of roots $= (2\alpha + 3\beta) + (3\alpha + 2\beta)$

$= 5(\alpha + \beta) = 5\left(-\frac{b}{a}\right)$

Product of roots $= 6\alpha^2 + 6\beta^2 + 13\alpha\beta = 6(\alpha+\beta)^2 + \alpha\beta$

$= 6\left(\frac{-b}{a}\right)^2 + \frac{c}{a} \quad = \frac{6b^2}{a^2} + \frac{c}{a}$

Q. E. $\quad x^2 + \frac{5b}{a}x + \frac{6b^2}{a^2} + \frac{c}{a} = 0$

$a^2x^2 + 5abx + 6b^2 + ac = 0$

Q.32 (1)

$\Rightarrow \frac{(2x-1)}{x(2x^2+3x+1)} > 0$

$\Rightarrow \frac{(2x-1)}{x(x+1)(2x+1)} > 0$

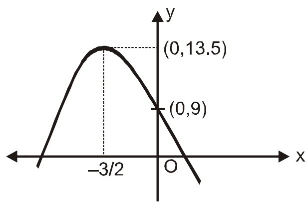
$$\begin{array}{ccccccccc} + & & - & & + & & - & & + \\ & -1 & & -1/2 & & 0 & & 1/2 & \end{array}$$

consontains $\left(-\infty, \frac{-3}{2}\right)$

Q.33 (3)

$y = -2x^2 - 6x + 9$

$\therefore \quad \frac{-b}{2a} = \frac{6}{2(-2)} = -\frac{3}{2} = -1.5$



& $\quad D = 36 - 4(-2)(9) = 36 + 72 = 108$

$\therefore \quad -\frac{D}{4a} = -\frac{108}{4(-2)} = +\frac{108}{8} = 13.5$

$\Rightarrow y \in (-\infty, 13.5]$

Q.34 (1)

$k = \frac{x^2 - x + 1}{x^2 + x + 1}$

$\Rightarrow \quad (k-1)x^2 + (k+1)x + (k-1) = 0$

Q $\quad$ x is real

$\therefore \quad D \geq 0$

$\Rightarrow (k+1)^2 - 4(k-1)^2 \geq 0$

$\Rightarrow (3k-1)(k-3) \leq 0$

$\Rightarrow \quad k \in \left[\frac{1}{3}, 3\right]$

Q.35 (3)

$y = \frac{2x}{1+x^2}, x \in R$

$\Rightarrow yx^2 - 2x + y = 0$

$\Rightarrow \quad D \geq 0 \Rightarrow 4 - 4y^2 \geq 0$

$\Rightarrow \quad (y^2 - 1) \leq 0 \Rightarrow y \in [-1, 1]$

$\therefore$ Range of $f(y) = y^2 + y - 2$

Min value $= \frac{-D}{4a} = \frac{-9}{4}$ at $y = \frac{-b}{2a} = \frac{-1}{2}$

$y = \frac{-1}{2} \in [-1, 1]$

$f(-1) = 1 - 1 - 2 = -2$
$f(1) = 1 + 1 - 2 = 0$
max value is $= 0$

Range $\left[\frac{-9}{4},\ 0\right]$

Q.36 (2)

$x^2 - xy + y^2 - 4x - 4y + 16 = 0, x, y \in R$

$x^2 - x(y + 4) + (y^2 - 4y + 16) = 0 \quad \ldots(1)$

$x \in R \Rightarrow D \geqslant 0$

$(y + 4)^2 - 4(y^2 - 4y + 16) \geqslant 0$

$\Rightarrow \quad y^2 + 8y + 16 - 4y^2 + 16y - 64 \geqslant 0 \quad \Rightarrow y^2 - 8y + 16 \leqslant 0 \quad \Rightarrow (y - 4)^2 \leqslant 0 \Rightarrow y = 4$

Put is given equation (i)

$x^2 - 8x + 16 = 0$

$\Rightarrow \quad (x - 4)^2 = 0 \Rightarrow x = 4$

Q.37 (4)

$(y - 1)x^2 + (y + 1)x + (2cy - c) = 0$

$D \geqslant 0 \therefore\ x \in R$

$\Rightarrow \quad (y + 1)^2 - 4(y - 1)(2cy - c) \geqslant 0$

$y^2 + 2y + 1 - 8cy^2 + 12cy - 4c \geqslant 0$

$(1 - 8c)y^2 + (2 + 12c)y + (1 - 4c) \geqslant 0$

$\forall\ y \in R, D \leqslant 0$

$(2 + 12c)^2 - 4(1 - 8c)(1 - 4c) \leqslant 0$

$(1 + 6c)^2 - (1 - 8c)(1 - 4c) \leqslant 0$

$4c^2 + 24c \leqslant 0 \Rightarrow c \in [-6, 0]$

& N^r & D^r have no any common root

(i) both common factor (root) (not possible)

$\frac{1}{1} = \frac{-1}{+1} = \frac{c}{2c}$

(ii) If one common root is α

$(\alpha^2 - \alpha + c = 0) \times 2$

& $\quad \alpha^2 + \alpha + 2c = 0$

$\alpha^2 - 3\alpha = 0$

$\alpha = 0 \Rightarrow c = 0$

or $\quad \alpha = 3 \Rightarrow c = -6$

$\therefore \quad c \neq 0$ & $c \neq -6$

$\therefore \quad c \in (-6, 0)$

Q.38 (2)

$2x^2 - (a^3 + 8a - 1)x + (a^2 - 4a) = 0$

since the roots are of opposite sign,

$f(0) < 0$

$\Rightarrow \quad a^2 - 4a < 0$

$\Rightarrow \quad a(a - 4) < 0$

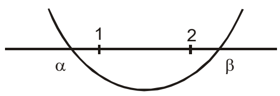
$\Rightarrow \quad a \in (0, 4)$

Q.39 (2)

$x^2 - 2px + (8p - 15) = 0$

$f(1) < 0$ and $f(2) < 0$

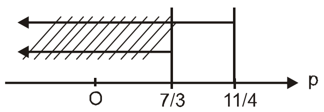
$\Rightarrow f(1) = 1 - 2p + 8p - 15 < 0$



$\Rightarrow p < 7/3$

and $f(2) = 4 - 4p + 8p - 15 < 0$

$\Rightarrow 4p - 11 < 0 \Rightarrow p < \frac{11}{4}$



Hence $p \in (-\infty, 7/3)$ Ans.

Q.40 (4)

$x^2 + 2(k - 1)x + k + 5 = 0$

Case - I

(i) $\quad D \ldots 0$

$\Rightarrow \quad 4(k - 1)^2 - 4(k + 5) \ldots 0$

$\Rightarrow \quad k^2 - 3k - 4 \ldots 0 \Rightarrow (k + 1)(k - 4) \ldots 0$

$\Rightarrow \quad k \in (-\infty, -1] \cup [4, \infty)$

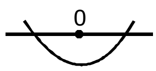
& (ii) $f(0) > 0 \Rightarrow k + 5 > 0 \Rightarrow k \in (-5, \infty)$

& (iii) $\frac{-b}{2a} > 0 \Rightarrow \frac{-2(k-1)}{2} > 0$

$\Rightarrow \quad k \in (-\infty, 1) \therefore k \in [-5, -1]$

Case - II $\quad f(0) \leq 0 \Rightarrow k + 5 \leq 0$

$\Rightarrow k \in (-\infty, -5]$



Finally $k \in$ (Case - I) $\cup$ (Case - II)

$k \in (-\infty, -1]$

EXERCISE-II

Q.1 (3)

(3) Given equation is

$(p-q)x^2 + (q-r)x + (r-p) = 0$

$$x = \frac{(r-q) \pm \sqrt{(q-r)^2 - 4(r-p(p-q)}}{2(p-q)}$$

$$\Rightarrow x = \frac{(r-q) \pm \sqrt{(q+r-2p)}}{2(p-q)} \Rightarrow x = \frac{r-p}{p-q}, 1$$

Q.2 (3)

We have $4ax^2 + 3bx + 2c = 0$ Let roots are α and β

Let $D = B^2 - 4AC = 9b^2 - 4(4a)(2c) = 9b^2 - 32\,ac$

Given that, $(a + b + c) = 0 \Rightarrow b = -(a + c)$

Putting this value, we get

$= 9(a+c)^2 - 32\,ac = 9(a-c)^2 + 4ac$

Hence roots are real.

Q.3 (4)

Given equation

$2(a^2 + b^2)x^2 + 2(a+b)x + 1 = 0$

Let $A = 2(a^2 + b^2)$, $B = 2(a + b)$ and $C = 1$

$B^2 - 4AC = 4(a^2 + b^2 + 2ab) - 4.2\,(a^2 + b^2)1$

$\Rightarrow B^2 - 4AC = -4(a-b)^2 < 0$

Thus given equation has imaginary roots.

Q.4 (2)

The roots of the equations are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(i) Let $b^2 - 4ac > 0$, $b > 0$

Now if $a > 0$, $c > 0$, $b^2 - 4ac < b^2$

$\Rightarrow$ the roots are negative.

(ii) Let $b^2 - 4ac < 0$, then the roots are given by

$$x = \frac{-b \pm i\sqrt{(4ac - b^2)}}{2a}, \ (i = \sqrt{-1})$$

Which are imaginary and have negative real part

$(\because b > 0)$

$\therefore$ In each case, the roots have negative real part.

Q.5 (1)

Here $(b + c - 2a) + (c + a - 2b) + (a + b - 2c) = 0$

Therefore the roots are rational.

Q.6 (3)

The quadratic is $(k + 11)x^2 - (k + 3)x + 1 = 0$

Accordingly, $(k + 3)^2 - 4(k + 11)(1) = 0 \Rightarrow k = -7, 5$

Q.7 (3)

From options put $k = 3 \Rightarrow x^2 + 8x + 7 = 0$

$\Rightarrow (x + 1)(x + 7) = 0 \Rightarrow x = -1, -7$

means for $k = 3$ roots are negative.

Q.8 (1)

Given equation $(1 + 2k)x^2 + (1 - 2k)x + (1 - 2k) = 0$

If equation is a perfect square then root are equal

i.e., $(1 - 2k)^2 - 4(1 + 2k)(1 - 2k) = 0$

i.e., $k = \frac{1}{2}, \frac{-3}{10}$. Hence total number of values $= 2$.

Q.9 (2)

Let first root $= \alpha$ and second root $= \frac{1}{\alpha}$

Then $\alpha, \frac{1}{\alpha} = \frac{k}{5} \Rightarrow k = 5$

Q.10 (1)

Given equation $4x^2 + 3x + 7 = 0$, therefore

$\alpha + \beta = -\frac{3}{4}$ and $\alpha\beta = \frac{7}{4}$

Now $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-3/7}{7/4} = \frac{-3}{4} \times \frac{4}{7} = -\frac{3}{7}$

Q.11 (1)

Here $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

If roots are $\alpha + \frac{1}{\beta}$, $\beta + \frac{1}{\alpha}$ then sum of roots are

$$= \left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = (\alpha + \beta) + \frac{(\alpha + \beta)}{\alpha\beta} = \frac{b}{ac}(a + c)$$

and product $= \left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$

$$= \alpha\beta + 1 + 1 + \frac{1}{\alpha\beta} = 2 + \frac{c}{a} + \frac{a}{c}$$

$$= \frac{2ac + c^2 + a^2}{ac} = \frac{(a + c)^2}{ac}$$

Hence required equation is given by

$$x^2 + \frac{b}{ac}(a + c)x + \frac{(a + c)^2}{ac} = 0$$

$\Rightarrow acx^2 + (a + c)bx + (a + c)^2 = 0$

Trick : Let $a = 1$, $b = -3$, $c = 2$, then $\alpha = 1$, $\beta = 2b$ $= -3$, $c = 2$, then $\alpha = 1$, $\beta = 2$

$\therefore \alpha + \frac{1}{\beta} = \frac{3}{2}$ and $\beta + \frac{1}{\alpha} = 3$

Therefore, required equation must be

$(x - 3)(2x - 3) = 0$ i.e. $2x^2 - 9x + 9 = 0$

Here (1) gives this equation on putting

$a = 1$, $b = -3$, $c = 2$

Q.12 (4)

$\alpha+\beta=-\frac{b}{a}$, $\alpha\beta=\frac{c}{a}$

and $\alpha^2+\beta^2=\frac{(b^2-2ac)}{a^2}$

Now $\frac{\alpha}{a\beta+b}+\frac{\beta}{a\alpha+b}=\frac{\alpha(a\alpha+b)+\beta(a\beta+b)}{(a\beta+b)(a\alpha+b)}$

$$=\frac{a(\alpha^2+\beta^2)+b(\alpha+\beta)}{\alpha\beta a^2+ab(\alpha+\beta)+b^2}=\frac{a\frac{(b^2-2ac)}{a^2}+b\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)a^2+ab\left(-\frac{b}{a}\right)+b^2}$$

$$=\frac{b^2-ac-b^2}{a^2c-ab^2+ab^2}=\frac{-ac}{a^2c}=-\frac{2}{a}$$

Q.13 (3)

Let α and β be two roots of $ax^2+bx+c=0$

Then $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

$\Rightarrow \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{b^2}{a^2}-2\frac{c}{a}$

So under condition $\alpha+\beta=\alpha^2+\beta^2$ $\alpha+\beta=a^2+\beta^2$

$\Rightarrow -\frac{b}{a}=\frac{b^2-2ac}{a^2}\Rightarrow b(a+b)=2ac$

Q.14 (2)

α,β be the roots of $x^2-2x+3=0$, then $\alpha+\beta=2$ and $\alpha\beta=3$. Now required equation whose roots are $\frac{1}{\alpha^2},\frac{1}{\beta^2}$ is

$$x^2-\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)x+\frac{1}{\alpha^2\beta^2}=0$$

$$\Rightarrow x^2-\left(-\frac{2}{9}\right)x+\frac{1}{9}=0\Rightarrow 9x^2+2x+1=0$$

Q.15 (3)

According to condition

$$\frac{2m-1}{m}=-1\Rightarrow 3m=1\Rightarrow m=\frac{1}{3}$$

Q.16 (2)

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$=(4)^3-3\times 1(4)=52$

Q.17 (1)

Let roots are α,β so, $\frac{\alpha}{\beta}=\frac{2}{3}\Rightarrow\alpha=\frac{2\beta}{3}$

$\therefore\ \alpha+\beta=\frac{m}{12}$

$\Rightarrow\frac{2\beta}{3}+\beta=\frac{m}{12}\Rightarrow\frac{5\beta}{3}=\frac{m}{12}$ .....(i)

and

$\alpha\beta=\frac{5}{12}\Rightarrow\frac{2\beta}{3}.\beta=\frac{5}{12}\Rightarrow\beta^2=\frac{5}{8}$

$\Rightarrow\beta=\sqrt{5/8}$

Put the value of β in (i), $\frac{5}{3}.\sqrt{\frac{5}{8}}=\frac{m}{12}\Rightarrow m=5\sqrt{10}$.

Q.18 (2)

Given equation can be written as

$(6k+2)x^2+rx+3k-1=0$ .....(i)

and $2(6k+2)x^2+px+2(3k-1)=0$ .....(ii)

Condition for common roots is

$$\frac{12k+4}{6k+2}=\frac{p}{r}=\frac{6k-2}{3k-1}=2 \text{ or } 2r-p=0$$

Q.19 (1)

Let α is the common root,

so $\alpha^2+p\alpha+q=0$ .....(i)

and $\alpha^2+q\alpha+p=0$ ....(ii)

from (i) – (ii),

$\Rightarrow(p-q)\alpha+(q-p)=0\Rightarrow\alpha=1$

Put the value of α in (i), $p+q+1=0$.

Q.20 (2)

Expressions are $x^2-11x+a$ and $x^2-14x+2a$ will have a common factor, then

$$\Rightarrow\frac{x^2}{-22a+14a}=\frac{x}{a-2a}=\frac{1}{-14+11}$$

$$\Rightarrow\frac{x^2}{-8a}=\frac{x}{-a}=\frac{1}{-3}\Rightarrow x^2=\frac{8a}{3}\text{ and }x=\frac{a}{3}$$

$$\Rightarrow\left(\frac{a}{3}\right)^2=\frac{8a}{3}\Rightarrow\frac{a^2}{9}=\frac{8a}{3}\Rightarrow a=0,24.$$

Trick : We can check by putting the values of *a* from the options.

Q.21 (2)

Given, $x+2 > \sqrt{x+4} \Rightarrow (x+2)^2 > (x+4)$

$\Rightarrow x+4x+4 > x+4 \Rightarrow x^2+3x > 0$

$\Rightarrow x(x+3) > 0 \Rightarrow x < -3$ or $x > 0 \Rightarrow x > 0$

Q.22 (2)

Case I: When $x+2 \geq 0$ i.e. $x \geq -2$

Then given inequality becomes

$x^2-(x+2)+x > 0 \Rightarrow x^2-2 > 0 \Rightarrow |x| > \sqrt{2}$

$\Rightarrow x < -\sqrt{2}$ or $x > \sqrt{2}$

As $x \geq -2$, therefore, in this case the part of the solution set is $[-2,-\sqrt{2}) \cup (\sqrt{2},\infty)$.

Case II: When $x+2 \leq 0$ *i.e.* $x \leq -2$,

Then given inequality becomes $x^2+(x+2)+x > 0$

$\Rightarrow x^2+2x+2 > 0 \Rightarrow (x+1)^2+1 > 0$, which is true for all real x

Hence, the part of the solution set in this case is $(-\infty, -2]$. Combining the two cases, the solution set is

$(-\infty,-2) \cup ([-2,-\sqrt{2}] \cup (\sqrt{2},\infty) = (-\infty,-\sqrt{2}) \cup (\sqrt{2},\infty)$

Q.23 (3)

If α, β, γ are the roots of the equation.

$x^3-px^2+qx-r=0$

$\therefore (\alpha+\beta)^{-1}+(\beta+\gamma)^{-1}+(\gamma+\alpha)^{-1} = \dfrac{p^2+q}{pq-r}$

Given, $p=0, q=4, r=-1$

$\Rightarrow \dfrac{p^2+q}{pq-r} = \dfrac{0+4}{0+1} = 4$

Q.24 (4)

We know that the roots of the equation

$ax^3+bx^2+cx+d=0$ follows $\alpha\beta\gamma = -d/a$

Comparing above equation with given equation

we get $d=1, a=1$

So, $\alpha\beta\gamma = -1$ or $\alpha^3\beta^3\gamma^3 = -1$.

Q.25 (3)

Let $y = \dfrac{x^2-3x+4}{x^2+3x+4} \Rightarrow (y-1)x^2+3(y+1)x+4(y-1)=0$

For x is real $D \geq 0$

$\Rightarrow 9(y+1)^2-16(y-1)^2 \geq 0$ $9(y+1)^2-16(y-1)^2 \geq 0$

$\Rightarrow -7y^2+50y-7 \geq 0 \Rightarrow 7y^2-50y+7 \leq 0$

$\Rightarrow (y-7)(7y-1) \leq 0$

Now, the product of two factors is negative if one in –ve and one in +ve.

Case I : $(y-7) \geq 0$ and $(7y-1) \leq 0$

$\Rightarrow y \geq 7$ and $y \geq \dfrac{1}{7}$. But it is impossible

Case II : $(y-7) \leq 0$ and $(7y-1) \geq 0$

$\Rightarrow y \leq 7$ and $y \geq \dfrac{1}{7} \Rightarrow \dfrac{1}{7} \leq y \leq 7$

Hence maximum value is 7 and minimum value is $\dfrac{1}{7}$

Q.26 (1)

$x^2-3x+3 = \left(x-\dfrac{3}{2}\right)^2+\dfrac{3}{4}$

Therefore, smallest value is $\dfrac{3}{4}$, which lie in $\left(-3,\dfrac{3}{2}\right)$

Q.27 (4)

x^2-3x+2 be factor of $x^4-px^2+q=0$

Hence $(x^2-3x+2)=0 \Rightarrow (x-2)(x-1)=0$

$\Rightarrow x=2, 1$ putting these values in given equation

so $4p-q-16=0$ .....(i)

and $p-q-1=0$ .....(ii)

Solving (i) and (ii), we get $(p, q) = (5, 4)$

Q.28 (4)

If the roots of the quadratic equation $ax^2+bx+c=0$ exceed a number k, then $ak^2+bk+c > 0$ if $a > 0$, $b^2-4ac \geq 0$ and sum of the roots $> 2k$. Therefore, if the roots of $x^2+x+a=0$ exceed a number a, then

$a^2+a+a > 0$, $1-4a \geq 0$ and $-1 > 2a$

$\Rightarrow a(a+2) > 0, a \leq \dfrac{1}{4}$ and $a < -\dfrac{1}{2}$ $\Rightarrow$ $a > 0$ or

$a < -2, a < \dfrac{1}{4}$ and $a < -\dfrac{1}{2}$

Hence $a < -2$.

Q.29 (4)

Let

$f(x) = 4x^2-20px+(25p^2+15p-66) = 0$ .....(i)

The roots of (i) are real if

$b^2-4ac = 400p^2-16(25p^2+15p-66)$

$= 16(66-15p) \geq 0$

$\Rightarrow p \leq 22/5$ .....(ii)

Both roots of (i) are less than 2. Therefore $f(2) > 0$ and sum of roots < 4.

$\Rightarrow 4.2^2-20p.2+(24p^2+15p-66) > 0$ and $\dfrac{20p}{4} < 4$

$\Rightarrow p^2 - p - 2 > 0$ and $p < \dfrac{4}{5}$

$\Rightarrow (p+1)(p-2) > 0$ and $p < \dfrac{4}{5}$

$\Rightarrow p < -1$ or $p > 2$ and $p < \dfrac{4}{5} \Rightarrow p < -1$ .....(iii)

From (ii) and (iii), we get $p < -1$ i.e. $p \in (-\infty, -1)$.

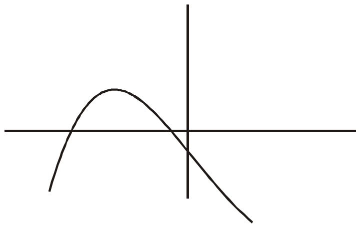
EXERCISE-III

Q.1 (A, B, D)

$y = ax^2 + bx + c$

Clearly $a < 0$

and $\dfrac{-b}{2a} < 0$



$\Rightarrow b < 0$

also $f(0) < 0 \Rightarrow c < 0$

and $D > 0$

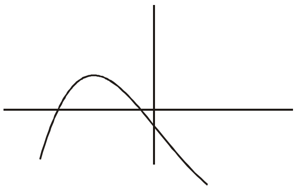
$\therefore$ (A), (B) and (D).

Q.2 (A,B,C,D)

(A) $a < 0$,

$-\dfrac{-b}{2a} < 0 \Rightarrow b < 0$

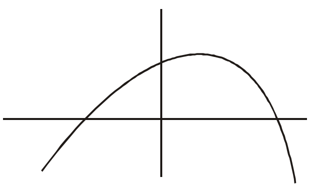
& $f(0) < 0 \Rightarrow c < 0$



$\therefore$ $abc < 0$

(B) $a < 0$,

$\dfrac{-b}{2a} > 0 \Rightarrow b > 0$

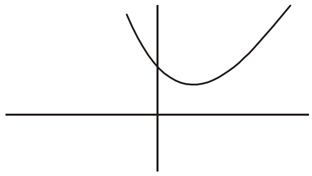


$f(0) > 0 \Rightarrow c > 0$

$\Rightarrow$ $abc < 0$

(C) $a > 0$

$\dfrac{-b}{2a} > 0 \Rightarrow b < 0$

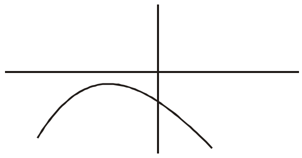


$f(0) > 0$ $c > 0$

$\Rightarrow$ $abc < 0$

(D) $a < 0$

$\dfrac{-b}{2a} < 0 \Rightarrow b < 0$



$f(0) < 0 \Rightarrow c < 0$

$\therefore$ (A), (B), (C), (D)

Q.3 (A,D)

Clearly $a < 0$

$\dfrac{-b}{2a} > 0 \Rightarrow b > 0$

$\therefore$ (A), (D)

Q.4 (A, B, D)

$|x|^2 + |x| - 6 = 0 \Rightarrow |x| = -3, 2 \Rightarrow |x| = 2$

$\Rightarrow x = \pm 2$

Q.5 (B,D)

$ax^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

$a + b = -b/a$, $ab = c/a$

$px^2 + qx + r = 0 \begin{matrix} \nearrow \alpha + h \\ \searrow \beta + h \end{matrix}$

$(a + b) + 2h = \dfrac{-q}{p}$

$h = \dfrac{\dfrac{-q}{p} + \dfrac{b}{a}}{2} = \dfrac{1}{2}\left(\dfrac{b}{a} - \dfrac{q}{p}\right)$ Ans.

$|\alpha - \beta| = |(\alpha + h) - (\beta + h)|$

$= \sqrt{[(\alpha + h) + (\beta + h)]^2 - 4(\alpha + h)(\beta + h)}$

$= \dfrac{b^2}{a^2} - \dfrac{4c}{a} = \dfrac{q^2}{p^2} - \dfrac{4r}{p} \Rightarrow \dfrac{b^2 - 4ac}{a^2} = \dfrac{q^2 - 4pr}{p^2}$

Ans.

Q.6 (B,C,D)

(A) $S = a^2 + b^2 = a^2 - 2b$

$P = a^2 b^2 = b^2$

$\therefore$ equation is $x^2 - (a^2 - 2b)\,x + b^2 = 0$

(B) $S = \dfrac{1}{\alpha} + \dfrac{1}{\beta} = -\dfrac{a}{b}$, $P = \dfrac{1}{\alpha} \cdot \dfrac{1}{\beta} = \dfrac{1}{b}$

$\therefore x^2 + \dfrac{a}{b}x + \dfrac{1}{b} = 0$

$\Rightarrow bx^2 + ax + 1 = 0$

(C) $S = \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{a^2 - 2b}{b}$

$P = \dfrac{\alpha}{\beta} \cdot \dfrac{\beta}{\alpha} = 1$

$x^2 - \dfrac{a^2 - 2b}{b}\,x + 1 = 0 \Rightarrow bx^2 - (a^2 - 2b)\,x + b = 0$

(D) $S = a + b - 2 = -a - 2$

$P = (a - 1)(b - 1)$

$= ab - (a + b) + 1$

$= b + a + 1$

$\therefore$ equation is

$x^2 + (a + 2)x + (a + b + 1) = 0.$

Q.7 (A,D)

$ax^3 + bx^2 + cx + d = 0 \begin{smallmatrix}\alpha\\ \beta\\ \gamma\end{smallmatrix}$

Let $ax^3 + bx^2 + cx + d \equiv (x^2 + x + 1)(Ax + B)$

Roots of $x^2 + x + 1 = 0$ are imaginary, Let these are α, β

So the third root 'γ' will be real.

$\alpha + \beta + \gamma = \dfrac{-b}{a}$

$-1 + \gamma = \dfrac{-b}{a} \quad \Rightarrow \gamma = \dfrac{a - b}{a}$

Also $\alpha\beta\gamma = \dfrac{-d}{a}$

But $\alpha\beta = 1$

$\therefore\ \gamma = \dfrac{-d}{a}$

$\therefore$ Ans are (A) & (D).

Q.8 (A,B,D)

$\dfrac{1}{2} \le \log_{1/10} x \le 2$

$\Rightarrow \dfrac{1}{100} \le x \le \dfrac{1}{\sqrt{10}}$

Q.9 (B, D)

$x^2 + abx + c = 0 \begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}$ ...(1)

$\alpha + \beta = -ab,\ \alpha\beta = c$

$x^2 + acx + b = 0 \begin{smallmatrix}\alpha\\ \delta\end{smallmatrix}$ ...(2)

$\alpha + \delta = -ac,\ \alpha\delta = b$

$\alpha^2 + ab\,\alpha + c = 0$

$\alpha^2 + ac\,\alpha + b = 0$

$\dfrac{\alpha^2}{ab^2 - ac^2} = \dfrac{\alpha}{c - b} = \dfrac{1}{ac - ab} \Rightarrow \alpha^2 = \dfrac{a(b^2 - c^2)}{a(c - b)} = -(b + c)$

& $\alpha = \dfrac{c - b}{a(c - b)} = \dfrac{1}{a}$ $\quad\therefore$ common root, $\alpha = \dfrac{1}{a}$

$\therefore\ -(b + c) = \dfrac{1}{a^2} \Rightarrow \quad a^2(b + c) = -1$

Product of the roots of equation (1) & (2) gives

$\beta \times \dfrac{1}{a} = c \Rightarrow \beta = ac$

& $\delta \times \dfrac{1}{a} = b \Rightarrow \delta = ab.$

$\therefore$ equation having roots β, δ is

$x^2 - a(b + c)\,x + a^2bc = 0$

$a(b + c)\,x^2 - a^2(b + c)^2\,x + a.(b + c)\,a^2bc = 0$

$a(b + c)\,x^2 + (b + c)\,x - abc = 0.$

Q.10 (C,D)

$\because$ D of $x^2 + 4x + 5 = 0$ is less than zero

$\Rightarrow$ both the roots are imaginary $\Rightarrow$ both the roots of quadratic are same

$\Rightarrow$ $b^2 - 4ac < 0$ & $\dfrac{a}{1} = \dfrac{b}{4} = \dfrac{c}{5} = k$

$\Rightarrow$ $a = k,\ b = 4k,\ c = 5k.$

Q.11 (A, D)

$x^2 + px + q = 0 \begin{array}{l} \alpha \\ \beta \end{array}$

$\alpha + \beta = -p,\ \alpha\beta = q$ and $p^2 - 4q > 0$

$x^2 - rx + s = 0 \begin{array}{l} \alpha^4 \\ \beta^4 \end{array}$ .....(1)

Now $\alpha^4 + \beta^4 = r$

$\Rightarrow \quad \alpha^4 + \beta^4 = r,\ (\alpha\beta)^4 = s = q^4$

$\therefore\ (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = r \Rightarrow [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2\alpha^2\beta^2 = r$

$\Rightarrow (p^2 - 2q)^2 - 2q^2 = r$

$\Rightarrow (p^2 - 2q)^2 = 2q^2 + r > 0$ .....(2)

Now, for $x^2 - 4qx + 2q^2 - r = 0$

$D = 16q^2 - 4(2q^2 - r)$ by equation (2)

$= 8q^2 + 4r = 4(2q^2 + r) > 0$

$\Rightarrow \quad D > 0$ two real and distinct roots

Product of roots $= 2q^2 - r$

$= 2q^2 - [(p^2 - 2q)^2 - 2q^2]$

$= 4q^2 - (p^2 - 2q)^2$

$= -p^2(p^2 - 4q) < 0$ from (1)

So product of roots is – ve

hence roots are opposite in sign

Q.12 (A,D)

$20x^2 + 210x + 400 = 4500 \Rightarrow 2x^2 + 21x - 410 = 0$

$\Rightarrow (2x + 41)(x - 10) = 0$

$\Rightarrow x = \frac{-41}{2},\ x = 10 \quad \Rightarrow x = -20.5,\ x = 10$

Q.13 (A,B,C,D)

$x^3 + bx^2 + cx - 1 = 0 \begin{array}{l} \alpha = \frac{a}{r} \\ \beta = a \\ r = ar \end{array}$

$\frac{a}{r} + a + ar = -b \quad \Rightarrow \quad a\left(\frac{1}{r} + 1 + r\right) = -b$

& $\quad \frac{a}{r} \times a \times ar = 1$

$a^3 = 1 \Rightarrow a = 1$

& $\quad \frac{a}{r}a + a \cdot ar + \frac{a}{r} \cdot ar = c$

$a^2\left(\frac{1}{r} + r + 1\right) = c$

$\frac{1}{r} + r + 1 = -b$ & $\frac{1}{r} + r + 1 = c \Rightarrow b + c = 0$

we know $\frac{1}{r} + r > 2 \Rightarrow \left(\frac{1}{r} + r + 1\right) > 3$

$-b > 3 \Rightarrow b < -3 \quad \Rightarrow b \in (-\infty, -3)$

& other two roots are $\frac{1}{r}$ & r

if $\frac{1}{r} > 1 \Rightarrow r < 1$ if $r > 1 \Rightarrow r < 1$

Q.14 (A,B)

$f(x) = \frac{3}{(x-2)} + \frac{4}{(x-3)} + \frac{5}{(x-4)} = 0$

$6x^2 - 14x - 21x + 49 = 0$

$(3x - 7)(2x - 7) = 0$

$x = \frac{7}{2}, x = \frac{7}{2}$

$2 < \frac{7}{2} < 3 3 < \frac{7}{2} < 4$

2nd Method

$g(x) = 3(x-3)(x-4) + 4(x-2)(x-4) + 5(x-2)(x-3)$
$= 0$

$g(2) > 0; g(3) < 0, g(4) > 0$

one root lie b/w (2, 3)

& other root lie b/w (3, 4)

Q.15 (D)

$f(-1) = 0 \Rightarrow -1 + q - r = 0$

$f(-5) = 0 \Rightarrow -25 + 5q - r = 0$

$q = 6 \qquad r = 5$

$f(x) = -x^2 - 6x + 5$ vertex is $(-3, 4)$

Q.16 (B)

$f(x) = px^2 - qx - r$ Since $f(0)\,f(1) > 0$

$\Rightarrow (-r)(p - q - r) > 0 \Rightarrow r(p - q - r) < 0$

Q.17 (C)

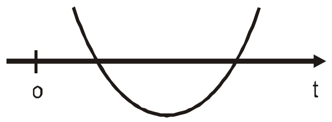
Q.18 (B)

Q.19 (D)

Sol. **(17 to 18)**

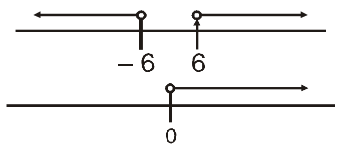
$x^4 - \lambda x^2 + 9 = 0 \Rightarrow x^2 = t \geq 0 \Rightarrow f(t) = t^2 - \lambda t + 9 = 0$

(17) given equation has four real & distinct roots



$D > 0$

$\Rightarrow \lambda^2 - 36 > 0$



$\frac{-b}{2a} > 0$

$\Rightarrow \frac{\lambda}{2} > 0$

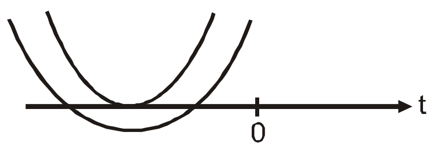
$\Rightarrow \lambda > 0$

$f(0) > 0$

$\Rightarrow 9 > 0$

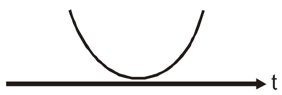
$\therefore \lambda \in (6, \infty)$

(18) Equation has no real roots.



case-I $D \geq 0 \Rightarrow \lambda^2 - 36 \geq 0$

$\frac{-b}{2a} < 0 \Rightarrow \lambda < 0$



$f(0) > 0 \Rightarrow 9 > 0.$

$\therefore \lambda \in (-\infty, -6]$

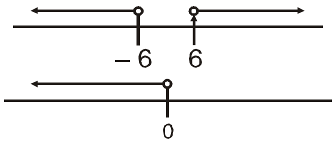
case-II $D < 0$

$\Rightarrow \lambda^2 - 36 < 0$

$\Rightarrow \lambda \in (-6, 6)$

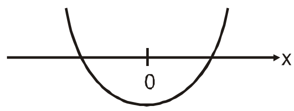
union of both cases gives

$\lambda \in (-\infty, 6)$



(19) Equation has only two real roots

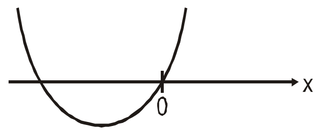
case-I $f(0) < 0$ $9 < 0$



which is false

case-II $f(0) = 0$

and $\frac{-b}{2a} < 0$



$\therefore$ No solution

$\therefore$ Final answer is ϕ

Q.20 (A) $\rightarrow$ (P); (B) $\rightarrow$ (S); (C) $\rightarrow$ (Q); (D) $\rightarrow$ (R)

(A) $x^2 - bx + c = 0 \; \begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}$

$\because |\alpha - \beta| = 1$

$\Rightarrow (\alpha - \beta)^2 = 1$

$b^2 - 4c = 1.$

(B) Let α be common root then

$\alpha^2 + a\alpha + b = 0$

$\alpha^2 + b\alpha + a = 0$

$$\Rightarrow \frac{\alpha^2}{a^2 - b^2} = \frac{\alpha}{b - a} = \frac{1}{b - a}$$

$$\Rightarrow \frac{\alpha^2}{a^2 - b^2} = \frac{\alpha}{b - a} = \frac{1}{b - a}$$

$\Rightarrow \alpha = 1$ and $\alpha = -(a + b)$

$\therefore 1 = -(a + b).$

(C) $\because \alpha + \beta = 1$ and $\alpha\beta = 3$

$\because \alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= [(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

$= (1-6)^2 - 2(9)$

$= 25 - 18 = 7$

(D) $\because \Sigma\,\alpha = 7$

$\Sigma\,\alpha\beta = 16$

$\Sigma\,\alpha = 12$

$\therefore \Sigma\alpha^2 = (\Sigma\,\alpha)^2 - 2\,(\Sigma\,\alpha\,\beta)$

$= 49 - 32$

$\therefore \alpha^2 + \beta^2 + \gamma^2 = 17$

Q.21 (A) $\rightarrow$ (r), (B) $\rightarrow$ (p), (C) $\rightarrow$ (s), (D) $\rightarrow$ (p, q)

(A) $x^2 - 8x + k = 0 \begin{cases} \alpha \\ \alpha + 4 = \beta \end{cases}$

$\because$ $(\beta-\alpha)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$\Rightarrow$ $16 = 64 - 4k \Rightarrow 4k = 48 \Rightarrow k = 12$

(B) $\because$ $(|x|-2)\,(|x|-3) = 0$

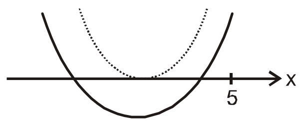
$\Rightarrow$ $x = \pm 2\,;\ x = \pm 3$

$\therefore$ $n = 4 \therefore\ \frac{n}{2} = 2$

(C) $\because$ $b = (3-i)\,(3+i)$

$b = 10$

(D) $x^2 - 2kx + (k^2 + k - 5) = 0$



(i) $D \geq 0$

$\Rightarrow 4k^2 - 4(k^2 + k - 5) \geq 0$

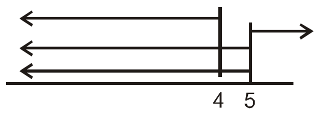
$\Rightarrow k - 5 \leq 0$

(ii) $f(5) > 0$

$\Rightarrow$ $25 - 10\,k + k^2 + k - 5 > 0$

$\Rightarrow$ $k^2 - 9k + 20 > 0 \Rightarrow (k-5)\,(k-4) > 0$

(iii) $-\frac{b}{2a} < 5 \Rightarrow k < 5$



$\Rightarrow$ $k \in (-\infty, 4)$

So k may be 2, 3.

NUMERICAL VALUE BASED

Q.22 2

$(x^2 + 3x + 2)\ (x^2 + 3x) = 120$

Let $x^2 + 3x = y$

$\Rightarrow$ $y^2 + 2y - 120 = 0$

$\Rightarrow$ $(y+12)\,(y-10) = 0$

$\Rightarrow$ $y = -12 \Rightarrow x^2 + 3x + 12 = 0$

$\Rightarrow$ $x \in \phi$

$y = 10 \quad \Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow$ $(x+5)\,(x-2) = 0 \quad \Rightarrow \quad x = \{-5, 2\}$

x = 2, – 5 are only two integer roots.

Q.23 8

$\left(5+2\sqrt{6}\right)^{x^2-3} + \frac{1}{\left(5+2\sqrt{6}\right)^{x^2-3}} = 10$

$\Rightarrow$ $t + \frac{1}{t} = 10$

$\Rightarrow$ $t^2 - 10t + 1 = 0 \quad t = \frac{10 \pm \sqrt{96}}{2} = 5 \pm 2\sqrt{6}$

$\Rightarrow$ $(5+2\sqrt{6})^{x^2-3} = (5+2\sqrt{6})$ or $\frac{1}{5+2\sqrt{6}}$

$\Rightarrow$ $x^2 - 3 = 1$ or $x^2 - 3 = -1$

$\Rightarrow$ $x = 2$ or -2 or $-\sqrt{2}$ or $\sqrt{2}$

Product 8

Q.24 11

$2x^2 + 6x + a = 0$

Its roots are α, β

$\Rightarrow$ $\alpha + \beta = -3$ & $\alpha\beta = \frac{a}{2} \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} < 2$

$\Rightarrow$ $\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} < 2$

$\Rightarrow$ $\frac{9-a}{a} < 1$

$\Rightarrow$ $\frac{2a-9}{a} > 0$

$\Rightarrow$ $a \in (-\infty, 0) \cup \left(\frac{9}{2}, \infty\right)$

$\Rightarrow$ 2a = 11 is least prime.

Q.25 [1]

$x^2 + px + 1 = 0 \begin{cases} a \\ b \end{cases}$ $a + b = -p,\ ab = 1\,;$

$x^2 + qx + 1 = 0 \to c, d \qquad c + d = -q,\ cd = 1$

$a + b = -p,\ ab = 1 \Rightarrow c + d = -q,\ cd = 1$

$RHS = (a - c)(b - c)(a + d)(b + d) = (ab - ac - bc + c^2)(ab + ad + bd + d^2)$

$= (1 - ac - bc + c^2)(1 + ad + bd + d^2)$

$= 1 + ad + bd + d^2 - ac - a^2cd - abcd - acd^2 - bc - abcd - b^2cd - bcd^2 + c^2 + adc^2 + bdc^2 + c^2d^2$

$= 1 + ad + bd + d^2 - ac - a^2 - 1 - ad - bc - 1 - b^2 - bd + c^2 + ac + bc + 1$

$[\because\ ab = cd = 1]$

$= c^2 + d^2 - a^2 - b^2 = (c + d)^2 - 2cd - (a + b)^2 + 2ab$

$= q^2 - 2 - p^2 + 2 = q^2 - p^2 = LHS.$ **Proved.**

2nd Method:

$RHS = (ab - c(a + b) + c^2)(ab + d(ab + d(a + b) + d^2)$

$= (c^2 + pc + 1)(1 - pd + d^2)$ ...(1)

Since c & d are the roots of the equation $x^2 + qx + 1 = 0$

$\therefore\ c^2 + qc + 1 = 0 \Rightarrow c^2 + 1 = -qc$ & $d^2 + qd + 1 = 0$

$\Rightarrow d^2 + 1 = -qd.$

$\therefore$ (i) Becomes $= (pc - qc)(-pd - qd) = c(p - q)(-d)(p + q) = -cd(p^2 - q^2)$

$= cd(q^2 - p^2) = q^2 - p^2 = LHS.$ **Proved.**

Q.26 **73**

$\because\ \alpha, \beta$ are roots of $\lambda x^2 - (\lambda - 1)x + 5 = 0$

$\therefore\ \alpha + \beta = \dfrac{\lambda - 1}{\lambda}$ and $\alpha\beta = \dfrac{5}{\lambda}$

$\because\ \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = 4 \qquad \Rightarrow \qquad \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = 4$

$\Rightarrow (\alpha + \beta)^2 = 6\alpha\beta \qquad \Rightarrow \dfrac{(\lambda - 1)^2}{\lambda^2} = \dfrac{30}{\lambda}$

$\Rightarrow \lambda^2 - 32\lambda + 1 = 0$ ....(1)

$\because\ \lambda_1, \lambda_2$ are roots of (1)

$\therefore\ \lambda_1 + \lambda_2 = 32$ and $\lambda_1\lambda_2 = 1$

$\therefore\ \dfrac{\lambda_1}{\lambda_2} + \dfrac{\lambda_2}{\lambda_1} = \dfrac{(\lambda_1 + \lambda_2)^2 - 2\lambda_1\lambda_2}{\lambda_1\lambda_2} = \dfrac{(32)^2 - 2}{1}$

$= 1022 \Rightarrow \left(\dfrac{\dfrac{\lambda_1}{\lambda_2} + \dfrac{\lambda_2}{\lambda_1}}{14}\right) = 73$

Q.27 10

$\alpha.\beta = b\ ;\ \gamma\delta = b - 2$

$\Rightarrow \qquad \alpha\beta\gamma\delta = b(b - 2) = 24$

$\therefore\ bx^2 + ax + 1 = 0$ has roots $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$

$\Rightarrow \dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{-a}{b}$

$(b - 2)x^2 - ax + 1 = 0$ has root $\dfrac{1}{\gamma}, \dfrac{1}{\delta} \Rightarrow \dfrac{1}{\gamma} + \dfrac{1}{\delta} = \dfrac{a}{b - 2}$

$\dfrac{1}{\gamma} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta} = \dfrac{-a}{b} + \dfrac{a}{b - 2} = \dfrac{5}{6}\ ;\ \dfrac{+2a}{b(b - 2)} = \dfrac{5}{6}\ ;$

$\dfrac{+2a}{24} = \dfrac{5}{6} \qquad ; \quad a = 10.$

Q.28 **3**

$x^2 + 2xy + 2y^2 + 4y + 7$

$= (x + y)^2 + (y + 2)^2 + 3 \ge 0 + 0 + 3$

$\therefore$ Least value = 3.

Q.29 13

$a^3 + b^3 + (-9)^3 = 3 \cdot a \cdot b(-9)$

$\Rightarrow a + b - 9 = 0$ or

$a = b = -9$. Which is rejected.

As $\quad a > b > -9$

$\Rightarrow a + b - 9 = 0 \ \Rightarrow x = 1$ is a root

other root $= \dfrac{-9}{a}$. $\quad \therefore \quad \alpha = \dfrac{-9}{a}, \beta = 1$

$\Rightarrow 4\beta - a\alpha = 4 - a\left(\dfrac{-9}{a}\right) = 4 + 9 = 13.$

Q.30 **6**

Let $t^2 - 2t + 2 = k$

$\Rightarrow \qquad \alpha^2 - 6k\alpha - 2 = 0$

$\Rightarrow \qquad \alpha^2 - 2 = 6k\alpha$

$a_{100} - 2a_{98} = \alpha^{100} - 2.\alpha^{98} - \beta^{100} + 2.\beta^{98}$

$= \alpha^{98}(\alpha^2 - 2) - \beta^{98}(\beta^2 - 2) = 6k(\alpha^{99} - \beta^{99})$

$a_{100} - 2a_{98} = 6k.a_{99}$

$\dfrac{a_{100} - 2a_{98}}{a_{99}} = 6k = 6(t^2 - 2t + 2) = 6[(t - 1)^2 + 1]$

$\therefore \qquad$ min. value of $\dfrac{a_{100} - 2a_{98}}{a_{99}}$ is 6.

Q.31 **11**

Given that, roots of equation $x^2 - 10ax - 11b = 0$ are c, d

So $c + d = 10a$ and $cd = -11b$ and a, b are the roots of equation $x^2 - 10cx - 11d = 0$

$\therefore\ a + b = 10c,\ ab = -11d$

So $a+b+c+d=10(a+c)$ and $(c+d)-(a+b)=10(a-c)$

$(c-a)-(b-d)+10(c-a)=0$

$\Rightarrow \quad b+d=9(a+c) \quad$ ....(i)

$abcd=121\,bd$

$\Rightarrow \quad ac=121 \quad$ ....(ii)

$b-d=11(c-a) \quad$ ....(iii)

c & a satisfies the equation $x^2-10ax-11b=0$ and $x^2-10cx-11d=0$ respectively

$\therefore \quad c^2-10ac-11b=0$

$a^2-10ca-11d=0$

$(c^2-a^2)-11(b-d)=0$

$(c-a)(c+a)=11(b-d)=11.\,11(c-a)$

(by equation (iii))

$c+a=121$

$\Rightarrow \quad a+b+c+d=10(c+a)$

$$\Rightarrow \quad 10.121 \quad \Rightarrow \frac{a+b+c+d}{110}=11.$$

EXERCISE-IV

JEE-MAIN

PREVIOUS YEAR'S

Q.1 (1)

$$(x^2-5x+5)^{x^2+4x-60}=1$$

$x^2-5x+5=1 \Rightarrow x=1,4$

or, $x^2+4x-60=0$ $x=-10, 6$

or, $x^2-5x+5=-1$ Þ $x=2,3$

But for $x=3$, $x^2+4x-60$ is odd

So Required values of x are $1,4,-10,6,2$ Sum $=3$

Q.2 (1)

We have $\displaystyle\sum_{r=1}^{n}(x+r-1)(x+r)=10n$

$$\Rightarrow \sum_{r=1}^{n}\left(x^2+(2r-1)x+(r^2-r)\right)=10n$$

$\therefore$ On solving, we get

$$x^2+nx+\left(\frac{n^2-31}{3}\right)=0 \qquad \therefore (2\alpha+1)=-n$$

/ \

$\alpha \quad \alpha+1$

$$\Rightarrow \alpha=\frac{-(n+1)}{2} \qquad .....(1)$$

$$\text{and } \alpha(\alpha+1)=\frac{n^2-31}{3} \qquad .....(2)$$

$\Rightarrow \quad n^2=121 \quad$ (using (1) in (2))

or $\quad n=11$

Q.3 (2)

α, β are roots of $x^2-x+1=0$

$\therefore\ \alpha=-\omega$ and $\beta=-\omega^2$

where ω is non-real cube root of unity

so, $a^{101}+\alpha^{107}$

$\Rightarrow (-\omega)^{101}+(-\omega^2)^{107}$

$\Rightarrow -[\omega^2+\omega]$

$\Rightarrow -[-1]=1$

(As $1+\omega+\omega^2=0$ & $\omega^3=1$)

Q.4 (1)

$x^2+2x+2=0 \Rightarrow (x+1)^2=-1$

$$x=-1\pm i=\sqrt{2}e^{i\left(\pm\frac{3\pi}{4}\right)}$$

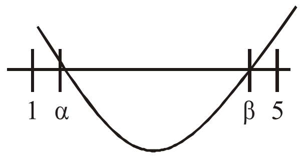
$$\therefore\ \alpha^{15}, \beta^{15}=\left(\sqrt{2}\right)^{15}\times 2\cos\left(15.\frac{3\pi}{4}\right)$$

$$=2^8\sqrt{2}\times\left(-\frac{1}{\sqrt{2}}\right)=-256\,]$$

Q.5 (Bonus)

$x^2-mx+4=0$

$\alpha, \beta \in [1, 5]$



(1) $\quad D>0 \Rightarrow m^2-16>0$

$\Rightarrow m \in (-\infty,-4)\cup(4,\infty)$

(2) $\quad f(1)\geq 0 \Rightarrow 5-m\geq 0 \Rightarrow m\in(-\infty,5)$

(3) $\quad f(5)\geq 0 \Rightarrow 29-5m\geq 0 \Rightarrow m \in \left(-\infty,\frac{29}{5}\right]$

(4) $\quad 1<\frac{-b}{2a}<5 \Rightarrow 1<\frac{m}{2}<5 \Rightarrow m\in(2,10)$

$\Rightarrow m\in(4,5)$

No option correct : Bonus

* If we consider $\alpha, \beta \in (1,5)$ then option (1) is correct.

Q.6 (3)

D must be perfect square

$\Rightarrow 121-24\alpha=\lambda^2$

$\Rightarrow$ maximum value of α is 5

$\alpha=1 \Rightarrow \lambda \notin 1$

$\alpha = 2 \Rightarrow \lambda \notin 1$

$\alpha = 3 \Rightarrow \lambda \in 1$

$\Rightarrow$ 3 integral values

$\alpha = 4 \Rightarrow \lambda \in 1$

$\alpha = 5 \Rightarrow \lambda \in 1$

Q.7 (4)

Case - 1

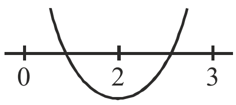
$c - 5 > 0$ ....(i)

$f(0) > 0$

$c - 4 > 0$ ...(ii)

$f(2) < 0$

$4(c-5) - 4c + c - 4 < 0$



$c < 24$ ...(iii)

$f(2) > 0$

$9(c-5) - 6c + c - 4 > 0$

$4c - 49 > 0 \Rightarrow c > \frac{49}{4}$ ...(iv)

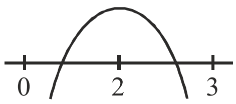
Here (i) $\cap$ (ii) $\cap$ (iii) $\cap$ (iv)

$c \in \left(\frac{49}{4}, 24\right)$

Case - II

$c - 5 < 0$ ...(i)

$f(0) < 0$



$c < 4$ ...(ii)

$f(2) > 0 \Rightarrow c > 24$ ...(iii)

$f(3) < 0 \Rightarrow c > 49$ ...(iv)

$4 \Rightarrow c \in \phi$

$c \in \left(\frac{49}{4}, 24\right)$.

Q.8 (4)

$\alpha + \alpha^3 = -\frac{K}{81}$ ....(1)

$\alpha^4 = \frac{256}{81}$

$\alpha = \pm\frac{4}{3}$ ....(2)

From (1) and (2)

$\frac{4}{3} + \frac{64}{27} = \frac{-K}{81}$

$K = -300$

Q.9 (3)

Expression is always positive if $2m + 1 > 0 \Rightarrow m > -\frac{1}{2}$

and $D < 0 \Rightarrow m^2 - 6m - 3 < 0$

$3\sqrt{12} < m < 3 + \sqrt{12}$

$\therefore$ common interval is $3 - \sqrt{12} < m < 3 + \sqrt{12}$

$\therefore$ Integral value of $m\{0,1,2,3,4,5,6\}$.

Q.10 (2)

Let roots are α and β now

$\lambda + \frac{1}{\lambda} = 1 \quad \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1 \Rightarrow \alpha^2 + \beta^2 = \alpha\beta$

$(\alpha + \beta)^2 = 2\alpha\beta$

$\left(\frac{-m(m-4)}{3m^2}\right)^2 = 3\cdot\frac{2}{3m^2}$

$m^2 - 8m - 2 = 0$

$m = 4 \pm 3\sqrt{2}$

So least value of $m = 4 - 3\sqrt{2}$

Q.11 (3)

$(x-1)^2 + 1 = 0 \quad \Rightarrow x = 1 + i, 1 - i$

$\therefore \quad \left(\frac{\alpha}{\beta}\right)^n = 1 \Rightarrow (\pm 1)^n = 1$

$\therefore$ n (least natural number) = 4

Q.12 **(3)**

a, b, c, in G.P.

say a, ar, ar^2

satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$

x = –r is the common root, satisfies second equation $d(-r)^2 + 2e(-r) + f = 0$

$\Rightarrow d.\frac{c}{a} - \frac{2ce}{b} + f = 0 \qquad \Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$

Q.13 (1)

$D < 0$

$4(1+3m)^2 - 4(Hm^2)(1+8m) < 0$

$\Rightarrow m(2m-1)^2 > 0 \Rightarrow m > 0$

Q.14 (2)

In given question p, q $\in$ R. If we take other root as any real number α,

then quadratic equation will be

$x^2 - (\alpha + 2 - \sqrt{3})\,x + \alpha.(2 - \sqrt{3}) = 0$

Now, we can have none or any of the options can be correct depending upon 'α' Instead of p, q $\in$ R it should be p, q $\in$ Q then other root will be $2 + \sqrt{3}$

$\Rightarrow p = -(2 + \sqrt{3} - 2 - \sqrt{3}) = -4$

and $q = (2 + \sqrt{3})(2 - \sqrt{3}) = 1$

$\Rightarrow p^2 - 4q - 12 = (-4)^2 - 4 - 12$

$= 16 - 16 = 0$

Option (2) is correct

Q.15 (4)

$SOR = \frac{3}{m^2+1} \Rightarrow (S.O.R)_{max} = 3$

when m = 0

$x^2 - 3x + 1 = 0 \rightarrow \alpha, \beta$

$\alpha + \beta = 3$

$\alpha\beta = 1$

$|\alpha^3 - \beta^2| = ||\alpha - \beta|(\alpha^2 + \beta^2 + \alpha\beta)|$

$= \left|\sqrt{(\alpha-\beta)^2 - \alpha\beta}\,((\alpha+\beta)^2 - \alpha\beta\right|$

$= \left|\sqrt{9-4}\,(9-1)\right|$

$= \sqrt{5} \times 8$

Q.16 (4)

$$\frac{\alpha^{12} + \beta^{12}}{\left(\frac{1}{\alpha^{12}} + \frac{1}{\beta^{12}}\right)(\alpha-\beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$$

$$= \frac{(\alpha\beta)^{12}}{\left[(\alpha+\beta)^2 - 4\alpha\beta\right]^{12}} = \left[\frac{\alpha\beta}{(\alpha+\beta)^2 - 4\alpha\beta}\right]^{12}$$

$$= \left(\frac{-2\sin\theta}{\sin^2\theta + 8\sin\theta}\right)^{12} = \frac{2^{12}}{(\sin\theta + 8)^{12}}$$

Q.17 (4)

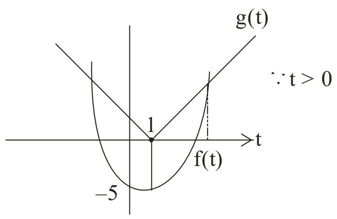
Let $2^x = t$

$5 + |t-1| = t^2 - 2t$

$\Rightarrow |t - 1| = (t^2 - 2t - 5)$

g(t) f(t)

From the graph



So, number of real root is 1.

Q.18 (3)

$375x^2 - 25x - 2 = 0$

$\alpha + \beta = \frac{25}{375}, \alpha\beta = \frac{-2}{375}$

$\Rightarrow (\alpha + \alpha^2 + ...$ upto infinite terms$) + (\beta + \beta^2 + ...$ upto infinite terms$) = \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} = \frac{1}{12}$

Q.19 (1)

$\alpha x^2 + 2\beta x + \gamma = 0$

Let $\beta = \alpha t, \gamma = \alpha t^2$

$\therefore \alpha x^2 + 2\alpha tx + \alpha t^2 = 0$

$\Rightarrow x^2 + 2tx + t^2 = 0$

$\Rightarrow (x + t)^2 = 0$

$\Rightarrow x = -t$

it must be root of equation $x^2 + x - 1 = 0$

$\therefore t^2 - t - 1 = 0$ ....(1)

Now

$\alpha(\beta + \gamma) = \alpha^2(t + t^2)$

Option 1 $\beta\gamma = \alpha t . \alpha t^2 = \alpha^2 t^3 = a^2(t^2 + t)$

from equation 1

Q.20 (1)

$\alpha^5 = 5\alpha + 3$

$\beta^5 = 5\beta + 3$

$p_5 = 5(\alpha + \beta) + 6$

$= 5(1) + 6$

$P_5 = 11$ and $p_5 = \alpha^2 + \beta^2 = \alpha + 1 + \beta + 1$

$P_2 = 3$ and $p_3 = \alpha^3 + \beta^3 = 2\alpha + 1 + 2\beta + 1 = 2(1) + 2 = 4$

$P_2 \times P_3 = 12$ and $P_5 = 11 \quad \Rightarrow P_5 \neq P_2 \times P_3$

Q.21 **8**

$D \geq 0$

$(a - 10)^2 - 4(2)\left(\frac{33}{2} - 2a\right) \geq 0$

$(a - 10)^2 - 4(33 - 4a) \geq 0$

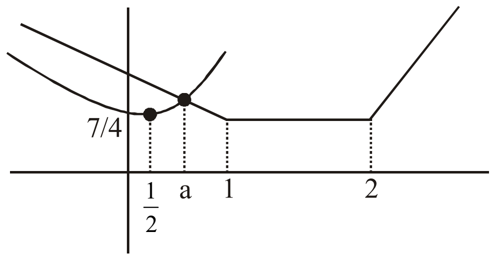
$a^2 - 4a - 32 \geq 0 \Rightarrow a \in (-\infty, -4] \cup [8, \infty)$.

Q.22 (2)

Let $3^x = t$

$t(t - 1) + 2 = |t - 1| + |t - 2|$

$t^2 - t + 2 = |t - 1| + |t - 2|$



are positive solution

$t = a$

$3^x = a$

$x = \log_3 a$ so singleton set.

Q.23 (2)

Let $e^x = t \in (0, \infty)$

Given equation

$t^4 + t^3 - 4t^2 + t + 1 = 0$

$t^2 + t - 4 + \frac{1}{t} + \frac{1}{t^2} = 0$

$\left(t^2 + \frac{1}{t^2}\right) + \left(t + \frac{1}{t}\right) - 4 = 0$

Let $t + \frac{1}{t} = \alpha$

$(\alpha^2 - 2) + \alpha - 4 = 0$

$\alpha^2 + \alpha - 6 = 0$

$\alpha^2 + \alpha - 6 = 0$

$\alpha = -3, 2 \quad \Rightarrow \alpha = 2 \quad \Rightarrow e^x + e^{-x} = 2$

$x = 0$ only solution

Q.24 (1)

$2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$ and $\alpha^2 = \frac{5}{a} \quad \Rightarrow \frac{b^2}{a^2} = \frac{5}{a}$

$\Rightarrow b^2 = 5a$ .....(i) $(a \neq 0)$

$\alpha + \beta = 2b$ ....(ii)

$\alpha\beta = -10$ ....(iii)

$\alpha = \frac{b}{a}$ is also root of $x^2 - 2bx - 10 = 0$

$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$

by (i) $\Rightarrow 5a - 10a^2 - 10a^2 = 0$

$\Rightarrow 20a^2 = 5a$

$\Rightarrow a = \frac{1}{4}$ and $b^2 = \frac{5}{4}$

$\alpha^2 = 20$ and $\beta^2 = 5$

Now $\alpha^2 + \beta^2$

$= 5 + 20$

$= 25$

Q.25 (1)

Let $f(x) = ax^2 + bx + c$

Let roots are 3 and α

and $f(-1) + f(2) = 0$

$4a + 2b + c + a - b + c = 0$

$5a + b + 2c = 0$ ...(i)

$\because f(3) = 0 \Rightarrow 9a + 3b + c = 0$ ...(ii)

From equation (i) and (ii)

$\frac{a}{1-6} = \frac{b}{18-5} = \frac{c}{15-9} \Rightarrow \frac{a}{-5} = \frac{b}{13} = \frac{c}{6}$

$\therefore f(x) = k(-5x^2 + 13x + 6)$

$= -k(5x + 2)(x - 3)$

= Root are 3 and $-\frac{2}{5}$

$\therefore -\frac{2}{5}$ lies in interval $(-1, 0)$

Q.26 (1)

$\because \alpha$ is a root of given equation, then

$5\alpha^2 + 6\alpha = 2$

$\Rightarrow 5\alpha^6 + 6\alpha^5 = 2\alpha^4$ ...(1)

Similarly $5\beta^6 + 6\beta^5 = 2\beta^4$ ...(2)

Adding (1) and (2), we get

$5S_6 + 6S_5 = 2S_4$

Q.27 (4)

$\because$ Equation is : $(\lambda^2+1)x^2-4\lambda x+2=0$

$\because$ One root in interval $(0,1)$

$\therefore$ f(0). f(1) < 0

$2.(\lambda^2+1-4\lambda+2)<0$

$(\lambda-3)(\lambda-1)<0$

$\therefore \lambda \in (1,3)$

If $\lambda = 3$, then roots are 1 and $\frac{1}{5}$

$\therefore \lambda \in (1,3]$

Q.28 (4)

$\alpha.\beta = 2$ and $\alpha+\beta=-p$ also $\frac{1}{\alpha}+\frac{1}{\beta}=-q$

$\Rightarrow p = 2q$

Now $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

$=\left[\alpha\beta+\frac{1}{\alpha\beta}-\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right]\left[\alpha\beta+\frac{1}{\alpha\beta}+1+1\right]$

$=\frac{9}{2}\left[\frac{5}{2}-\frac{\alpha^2+\beta^2}{2}\right]=\frac{9}{4}\left[5-(p^2-4)\right]$

$=\frac{9}{4}(9-p^2)$

Q.29 (1)

Roots of $x^2-x+2\lambda=0$ are α and β

and roots of $3x^2-10x+27\lambda=0$ and α and γ

Here,

$3\alpha^2-10\alpha+27\lambda=0$ ....(i)

$3\alpha^2-3\alpha+6\lambda=0$ ....(ii)

$\therefore \alpha = 3\lambda$

Now,

$3\lambda+\beta=1$ and $3\lambda\cdot\beta=2\lambda$

and, $3\lambda+\gamma=\frac{10}{3}$ and $3\lambda.\gamma=9\lambda$

$\therefore\ \gamma=3, \alpha=\frac{1}{3}$ and $\beta=\frac{2}{3}, \lambda=\frac{1}{9}$

$\frac{\beta\gamma}{\lambda}=18$

Q.30 (3)

$\because\ \alpha,\beta,\gamma,\delta$ are in G.P, so $\alpha\delta=\beta\gamma$

$\Rightarrow \frac{\alpha}{\beta}=\frac{\gamma}{\delta} \Rightarrow \left|\frac{\alpha-\beta}{\alpha+\beta}\right|=\left|\frac{\gamma-\delta}{\gamma+\delta}\right|$

$\Rightarrow \sqrt{\frac{9-4p}{3}}=\sqrt{\frac{36-4p}{6}}$

$\Rightarrow 36-16p=36-4q$

$\Rightarrow q=4p$

So, $\frac{2q+p}{2q-p}=\frac{9p}{7p}=\frac{9}{7}$

Q.31 (4)

$7x^2-3x-2=0 \Rightarrow \alpha+\beta=\frac{3}{7}, \alpha\beta=\frac{-2}{7}$

Now $\frac{\alpha}{1-\alpha^2}+\frac{\beta}{1-\beta^2}$

$=\frac{\alpha-\alpha\beta(\alpha+\beta)+\beta}{1-(\alpha^2+\beta^2)+(\alpha\beta)^2}=\frac{(\alpha+\beta)-\alpha\beta(\alpha+\beta)}{1-(\alpha^2+\beta^2)+2\alpha\beta+(\alpha\beta)^2}$

$=\frac{\frac{3}{7}+\frac{2}{7}\times\frac{3}{7}}{1-\frac{9}{49}+2\times\frac{-2}{7}+\frac{4}{49}}=\frac{21+6}{49-9-28+4}=\frac{27}{16}$

Q.32 (2)

Let $|x|=t$ we have

$9t^2-18t+5=0$

$9t^2-15t-3t+5=0$

$(3t-1)(3t-5)=0$

$\Rightarrow\ t=\frac{1}{3}$ or $\frac{5}{3} \Rightarrow |x|=\frac{1}{3}$ or $\frac{5}{3}$

Roots are $\pm\frac{1}{3}$ and $\pm\frac{5}{3}$

Product $=\frac{25}{81}$

Q.33 (2)

$\alpha+\beta=-\frac{1}{2} \Rightarrow -1=2\alpha+2\beta$

and $4\alpha^2+2\alpha-1=0$

$\Rightarrow 4\alpha^2+2\alpha+2\alpha+2\beta=0$

$\Rightarrow \beta=-2\alpha(\alpha+1)$

Q.34 (2)

$$\frac{\alpha^{3/8}}{\beta^{5/8}}+\frac{\beta^{3/8}}{\alpha^{5/8}}=\frac{\alpha+\beta}{(\alpha\beta)^{5/8}}$$

For $x^2-64x+256=0$

$\alpha+\beta=64$

$\alpha\beta=256$

$$\therefore\ \frac{\alpha+\beta}{(\alpha\beta)^{5/8}}=\frac{64}{\left(2^8\right)^{5/8}}=\frac{64}{32}=2$$

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Q.1 (2)

(i) $x^2-8kx+16(k^2-k+1)=0$

$\therefore$ $D=64\,(k^2-(k^2-k+1))=64\,(k-1)>0$

$\Rightarrow$ $k>1$ ....(1)

(ii) $-\dfrac{b}{2a}>4\Rightarrow\dfrac{8k}{2}>4$ $\Rightarrow k>1$.......(2)

(iii) $f(4)\geq 0$

$\Rightarrow 16-32k+16\,(k^2-k+1)\geq 0\ \Rightarrow k^2-3k+2\geq 0$

$\Rightarrow (k-2)\,(k-1)\geq 0\ \Rightarrow k\leq 1$ or $k\geq 2$.......(3)

$(1)\cap(2)\cap(3)$. Hence $k=2$

Q.2 (B)

Product $=1$

Sum $=\dfrac{\alpha^2+\beta^2}{\alpha\beta}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

Since $\alpha^3+\beta^3=q\Rightarrow -p\,(\alpha^2+\beta^2-\alpha\beta)=q$

$((\alpha+\beta)^2-3\alpha\beta\,)=-\dfrac{q}{p}\ \Rightarrow p^2+\dfrac{q}{p}=3\alpha\beta$

Hence sum $=\dfrac{\left\{p^2-\dfrac{2}{3}\left(\dfrac{p^3+q}{p}\right)\right\}3p}{(p^3+q)}=\dfrac{p^3-2q}{p^3+q}$

so the equation is $x^2-\left(\dfrac{p^3-2q}{p^3+q}\right)x+1=0$

$\Rightarrow$ $(p^3+q)\,x^2-(p^3-2q)x+(p^3+q)=0$

Q.3 (C)

$x^2-6x-2=0$ having roots α and β $\Rightarrow \alpha^2-6\alpha-2=0$

$\Rightarrow$ $\alpha^{10}-6\alpha^9-2\alpha^8=0$

$\Rightarrow$ $\alpha^{10}-2\alpha^8=6\alpha^9$ .... (i)

similarly $\beta^{10}-2\beta^8=6\beta^9$ .... (ii)

by (i) and (ii)

$(\alpha^{10}-\beta^{10})-2(\alpha^8-\beta^8)=6\,(\alpha^9-\beta^9)\Rightarrow a_{10}-2a_8=6a_9$

$\Rightarrow\ \dfrac{a_{10}-2a_8}{2a_9}=3$

Aliter

$$\frac{\alpha^{10}-\beta^{10}-2(\alpha^8-\beta^8)}{2(\alpha^9-\beta^9)}=\frac{\alpha^{10}-\beta^{10}+\alpha\beta(\alpha^8-\beta^8)}{2(\alpha^9-\beta^9)}=$$

$$\frac{\alpha^9(\alpha+\beta)-\beta^9(\alpha+\beta)}{2(\alpha^9-\beta^9)}=\frac{\alpha+\beta}{2}=\frac{6}{2}=3$$

Q.4 (B)

$$\begin{array}{c} x^2+bx-1=0\\ x^2+x+b=0\\ \hline \dfrac{x^2}{b^2+1}=\dfrac{x}{-1-b}=\dfrac{1}{1-b}\end{array}\ \Rightarrow\ x=\frac{b^2+1}{-(b+1)}=\frac{-(b+1)}{1-b}$$

$\Rightarrow (b^2+1)(1-b)=(b+1)^2$

$\Rightarrow$ $b^2-b^3+1-b=b^2+2b+1\ \Rightarrow\ b^3+3b=0\ \Rightarrow b=0$; $b^2=-3\Rightarrow b=0,\ \pm\sqrt{3}\,i$

Q.5 (D)

p(x) will be of the form ax^2+c. Since it has purely imaginary roots only.

Since p(x) is zero at imaginary values while ax^2+c takes real value only at real 'x', no root is real.

Also $p(p(x))=0\Rightarrow p(x)$ is purely imaginary

$\Rightarrow$ $ax^2+c=$ purely imaginary

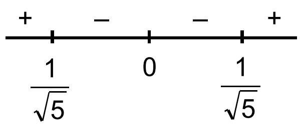
Hence x can not be purely imaginary since x^2 will be negative in that case and ax^2+c will be real.

Thus .(D) is correct.

Q.6 (A, D)

$(x_1+x_2)^2-4x_1x_2<1$

$\dfrac{1}{\alpha^2}-4<1$ $\Rightarrow$ $5-\dfrac{1}{\alpha^2}>0\ \Rightarrow\dfrac{5\alpha^2-1}{\alpha^2}>0$



$$\alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right) \quad ...(1)$$

$D > 0$

$1 - 4\alpha^2 > 0$

$$\alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right) \quad ...(2)$$

(1) & (2)

$$\alpha \in \left(-\frac{1}{2}, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{2}}\right)$$

Q.7 (D)

As α and β are roots of equation $x^2 - x - 1 = 0$, we get :

$\alpha^2 - \alpha - 1 = 0 \quad \Rightarrow \alpha^2 = \alpha + 1$

$\beta^2 - \beta - 1 = 0 \quad \Rightarrow \beta^2 = \beta + 1$

$\therefore a_{11} + a_{10} = p\alpha^{11} + q\beta^{11} + p\alpha^{10} + q\beta^{10}$

$= p\alpha^{10}(\alpha + 1) + q\beta^{10}(\beta + 1)$

$= p\alpha^{10} \times \alpha^2 + q\beta^{10} \times \beta^2$

$= p\alpha^{12} + q\beta^{12} = \alpha_{12}$

Q.8 (D)

$a_{n+2} = a_{n+1} + a_n$

$a_4 = a_3 + a_2 = 3a_1 + 2a_0 = 3p\alpha + 3q\beta + 2(p + q)$

As $\alpha = \frac{1+\sqrt{5}}{2}, \beta = \frac{1-\sqrt{5}}{2}$, we get

$$a_4 = 3p\left(\frac{1+\sqrt{5}}{2}\right) + 3q\left(\frac{1-\sqrt{5}}{2}\right) + 2p + 2q = 28$$

$$\Rightarrow \left(\frac{3p}{2} + \frac{3q}{2} + 2p + 2q - 28\right) = 0 \quad ....(i)$$

and $\Rightarrow \frac{3p}{2} - \frac{3q}{2} = 0$ . ...(ii)

$\Rightarrow$ $p = q$ (from (ii))

$\Rightarrow$ $7p = 28$ (from (i) and (ii))

$\Rightarrow$ $p = 4$

$\Rightarrow$ $q = 4$

$\Rightarrow$ $p + 2q = 12$

Q.9 (1,2,4)

α, β are roots of $x^2 - x - 1$

$$a_{r+2} - a_r = \frac{(\alpha^{r+2} - \beta^{r+2}) - (\alpha^r - \beta^r)}{\alpha - \beta}$$

$$= \frac{(\alpha^{r+2} - \alpha^r) - (\beta^{r+2} - \beta^r)}{\alpha - \beta}$$

$$= \frac{\alpha^r(\alpha^2 - 1) - \beta^r(\beta^2 - 1)}{\alpha - \beta} = \frac{\alpha^r\alpha - \beta^r\beta}{\alpha - \beta}$$

$$= \frac{\alpha^{r+1} - \beta^{r+1}}{\alpha - \beta} = a_{r+1}$$

$\Rightarrow a_{r+2} - a_{r+1} = a_r$

$$\Rightarrow \sum_{r=1}^{n} a_r = a_{n+2} - a_2 = a_{n+2} - \frac{\alpha^2 - \beta^2}{\alpha - \beta}$$

$= a_{n+2} - (\alpha + \beta) = a_{n+2} - 1$

$$\text{Now } \sum_{r=1}^{\infty} \frac{a_n}{10^n} = \frac{\sum_{r=1}^{\infty}\left(\frac{\alpha}{10}\right)^n - \sum_{r=1}^{\infty}\left(\frac{\beta}{10}\right)^n}{\alpha - \beta}$$

$$\frac{\frac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} - \frac{\frac{\beta}{10}}{1 - \frac{\beta}{10}}}{\alpha - \beta} = \frac{\frac{\alpha}{10 - \alpha} - \frac{\beta}{10 - \beta}}{(\alpha - \beta)} = \frac{10}{(10 - \alpha)(10 - \beta)} = \frac{10}{89}$$

$$\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \frac{a_{n-1} + a_{n+1}}{10^n} = \frac{\frac{\alpha}{10} +}{1 - \frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1 - \frac{\beta}{10}} = \frac{12}{89}$$

Further, $b_n = a_{n-1} + a_{n+1}$

$$= \frac{(\alpha^{n-1} - \beta^{n-1}) + (\alpha^{n+1} - \beta^{n+1})}{\alpha - \beta}$$

(as $\alpha\beta = -1 \Rightarrow \alpha^{n-1} = -\alpha^n\beta$ & $\beta^{n-1} = -\alpha\beta^n$)

$$= \frac{\alpha^n(\alpha - \beta) + (\alpha - \beta)\beta^n}{\alpha - \beta} = \alpha^n + \beta^n$$

Q.10 (1,2,4)

α, β are roots of $x^2 - x - 1$

$$a_{r+2} - a_r = \frac{(\alpha^{r+2} - \beta^{r+2}) - (\alpha^r - \beta^r)}{\alpha - \beta}$$

$$= \frac{(\alpha^{r+2} - \alpha^r) - (\beta^{r+2} - \beta^r)}{\alpha - \beta}$$

$$= \frac{\alpha^r(\alpha^2 - 1) - \beta^r(\beta^2 - 1)}{\alpha - \beta} = \frac{\alpha^r\alpha - \beta^r\beta}{\alpha - \beta}$$

$$= \frac{\alpha^{r+1} - \beta^{r+1}}{\alpha - \beta} = a_{r+1}$$

$\Rightarrow a_{r+2} - a_{r+1} = a_r$

$$\Rightarrow \sum_{r=1}^{n} a_r = a_{n+2} - a_2 = a_{n+2} - \frac{\alpha^2 - \beta^2}{\alpha - \beta}$$

$$= a_{n+2} - (\alpha + \beta) = a_{n+2} - 1$$

$$\text{Now } \sum_{r=1}^{\infty} \frac{a_n}{10^n} = \frac{\sum_{r=1}^{\infty}\left(\frac{\alpha}{10}\right)^n - \sum_{r=1}^{\infty}\left(\frac{\beta}{10}\right)^n}{\alpha - \beta}$$

$$\frac{\frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} - \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}}{\alpha - \beta} = \frac{\frac{\alpha}{10-\alpha} - \frac{\beta}{10-\beta}}{(\alpha - \beta)} = \frac{10}{(10-\alpha)(10-\beta)} = \frac{10}{89}$$

$$\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \frac{a_{n-1} + a_{n+1}}{10^n} = \frac{\frac{\alpha}{10} +}{1-\frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}} = \frac{12}{89}$$

Further, $b_n = a_{n-1} + a_{n+1}$

$$= \frac{(\alpha^{n-1} - \beta^{n-1}) + (\alpha^{n+1} - \beta^{n+1})}{\alpha - \beta}$$

(as $\alpha\beta = -1 \Rightarrow \alpha^{n-1} = -\alpha^n\beta$ & $\beta^{n-1} = -\alpha\beta^n$)

$$= \frac{\alpha^n(\alpha - \beta) + (\alpha - \beta)\beta^n}{\alpha - \beta} = \alpha^n + \beta^n$$

Q.11 (D)

$x^2 + 20x - 2020 = 0$ has two roots $a, b \in R$

$x^2 - 20x + 2020 = 0$ has two roots $c, d \in$ complex

$ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$

$= a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$

$= a^2(c+d) + b^2(c+d) - c^2(a+b) - d^2(a+b)$

$= (c+d)(a^2+b^2) - (a+b)(c^2+d^2)$

$= (c+d)((a+b)^2 - 2ab) - (a+b)((c+d)^2 - 2cd)$

$= 20[(20)^2 + 4040] + 20[(20)^2 - 4040]$

$= 20[(20)^2 + 4040 + (20)^2 - 4040]$

$= 20 \times 800 = 16000$