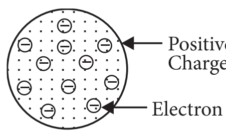
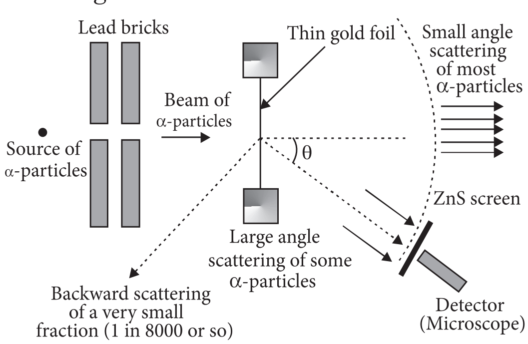
Atoms

Recap Notes

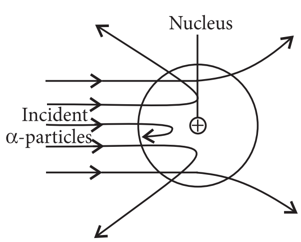
- **Thomson's model of atom :** It was proposed by J. J. Thomson in 1898. According to this model, the positive charge of the atom is uniformly distributed throughout the volume of the atom and the negatively charged electrons are embedded in it like seeds in a watermelon.



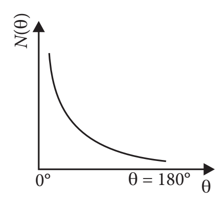
- **Rutherford's α-scattering experiment**
  - ▶ Rutherford and his two associates, Geiger and Marsden, studies the scattering of the α-particles from a thin gold foil in order to investigate the structure of the atom.



Schematic arrangement of the Geiger-Marsden experiment



  - ▶ **Rutherford's observations and results :**
    - – Most of the α-particles pass through the gold foil without any deflection. This shows that most of the space in an atom is empty.
    - – Few α-particles got scattered, deflecting at various angles from 0 to π. This shows that atom has a small positively charged core called 'nucleus' at centre of atom, which deflects the positively charged α-particles at different angles depending on their distance from centre of nucleus.
    - – Very few α-particles (1 in 8000) suffers deflection of 180°. This shows that size of nucleus is very small, nearly 1/8000 times the size of atom.



This graph shows deflection of number of particles with angle of deflection θ.

- **Rutherford's α-scattering formulae**
  - ▶ Number of α particles scattered per unit area, $N(\theta)$ at scattering angle θ varies inversely as $\sin^4(\theta/2)$,

    $$i.e., N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

  - ▶ **Impact parameter :** It is defined as the perpendicular distance of the initial velocity vector of the alpha particle from the centre of the nucleus, when the particle is far away from the nucleus of the atom.
    - – The scattering angle θ of the α particle and impact parameter b are related as

    $$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\varepsilon_0 K}$$

    where K is the kinetic energy of the α-particle and Z is the atomic number of the nucleus.

– Smaller the impact parameter, larger the angle of scattering θ.

▶ **Distance of closest approach :** At the distance of closest approach whole kinetic energy of the alpha particles is converted into potential energy.

– Distance of closest approach

$$r_0 = \frac{2Ze^2}{4\pi\varepsilon_0 K}$$

- **Rutherford's nuclear model of the atom :** According to this model the entire positive charge and most of the mass of the atom is concentrated in a small volume known as the nucleus with electrons revolving around it just as planets revolve around the sun.

- **Bohr's model :** Bohr combined classical and early quantum concepts and gave his theory of hydrogen and hydrogen-like atoms which have only one orbital electron. His postulates are

▶ An electron can revolve around the nucleus only in certain allowed circular orbits of definite energy and in these orbits it does not radiate. These orbits are known as stationary orbits.

▶ Angular momentum of the electron in a stationary orbit is an integral multiple of $h/2\pi$.

$$i.e.,\ L = \frac{nh}{2\pi} \quad \text{or} \quad mvr = \frac{nh}{2\pi}$$

This is known as Bohr's quantisation condition.

▶ The emission of radiation takes place when an electron makes a transition from a higher to a lower orbit. The frequency of the radiation is given by

$$\upsilon = \frac{E_2 - E_1}{h}$$

where E_2 and E_1 are the energies of the electron in the higher and lower orbits respectively.

- **Bohr's formulae**

▶ Radius of n^{th} orbit

$$r_n = \frac{4\pi\varepsilon_0 n^2 h^2}{4\pi^2 mZe^2};\quad r_n = \frac{0.53n^2}{Z}\ \text{Å}$$

▶ Velocity of the electron in the n^{th} orbit

$$v_n = \frac{1}{4\pi\varepsilon_0}\frac{2\pi Ze^2}{nh} = \frac{2.2\times10^6\ Z}{n}\ \text{m/s}$$

The kinetic energy of the electron in the n^{th} orbit

$$K_n = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r_n} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{2\pi^2 me^4 Z^2}{n^2h^2}$$

$$= \frac{13.6Z^2}{n^2}\ \text{eV}$$

The potential energy of the electron in the n^{th} orbit

$$U_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n} = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4\pi^2 me^4 Z^2}{n^2h^2}$$

$$= \frac{-27.2Z^2}{n^2}\ \text{eV}$$

▶ Total energy of electron in the n^{th} orbit

$$E_n = U_n + K_n = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{2\pi^2 me^4 Z^2}{n^2h^2}$$

$$= -\frac{13.6Z^2}{n^2}\ \text{eV}$$

$K_n = -E_n,\ U_n = 2E_n = -2K_n$

▶ Frequency of the electron in the n^{th} orbit

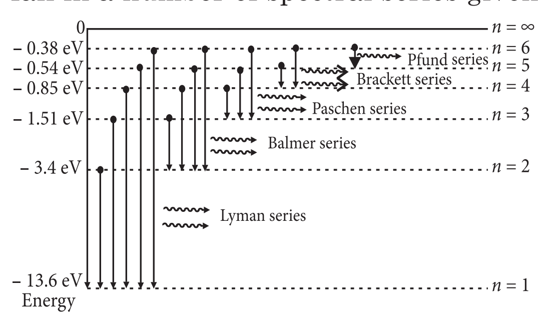
$$\upsilon_n = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4\pi^2 Z^2 e^4 m}{n^3h^3} = \frac{6.62\times10^{15} Z^2}{n^3}$$

▶ Wavelength of radiation in the transition from $n_2 \rightarrow n_1$ is given by $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

where R is called Rydberg's constant.

$$R = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2\pi^2 me^4}{ch^3} = 1.097\times10^7\ \text{m}^{-1}$$

- **Spectral series of hydrogen atom :** When the electron in a H-atom jumps from higher energy level to lower energy level, the difference of energies of the two energy levels is emitted as radiation of particular wavelength, known as spectral line. Spectral lines of different wavelengths are obtained for transition of electron between two different energy levels, which are found to fall in a number of spectral series given by



▶ **Lyman series**

– Emission spectral lines corresponding to the transition of electron from higher energy levels ($n_2 = 2, 3, ..., \infty$) to first energy level ($n_1 = 1$) constitute Lyman series.

$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$ where $n_2 = 2, 3, 4, ..., \infty$

- Series limit line (shortest wavelength) of Lyman series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = R \quad \text{or} \quad \lambda = \frac{1}{R}$$

- The first line (longest wavelength) of the Lyman series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4} \quad \text{or} \quad \lambda = \frac{4}{3R}$$

- Lyman series lie in the ultraviolet region of electromagnetic spectrum.
- Lyman series is obtained in emission as well as in absorption spectrum.

▶ **Balmer series**

- Emission spectral lines corresponding to the transition of electron from higher energy levels ($n_2 = 3, 4, ...\infty$) to second energy level ($n_1 = 2$) constitute Balmer series.

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right] \text{ where } n_2 = 3, 4, 5.........,\infty$$

- Series limit line (shortest wavelength) of Balmer series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{R}{4} \quad \text{or} \quad \lambda = \frac{4}{R}$$

- The first line (longest wavelength) of the Balmer series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36} \quad \text{or} \quad \lambda = \frac{36}{5R}$$

- Balmer series lie in the visible region of electromagnetic spectrum.
- This series is obtained only in emission spectrum.

▶ **Paschen series**

- Emission spectral lines corresponding to the transition of electron from higher energy levels ($n_2 = 4, 5, ...\infty$) to third energy level ($n_1 = 3$) constitute Paschen series.

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right] \text{ where } n_2 = 4, 5, 6.........,\infty$$

- Series limit line (shortest wavelength) of the Paschen series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{\infty^2}\right] = \frac{R}{9} \quad \text{or} \quad \lambda = \frac{9}{R}$$

The first line (longest wavelength) of the Paschen series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \frac{7R}{144} \quad \text{or} \quad \lambda = \frac{144}{7R}$$

- Paschen series lie in the infrared region of the electromagnetic spectrum.
- This series is obtained only in the emission spectrum.

▶ **Brackett series**

- Emission spectral lines corresponding to the transition of electron from higher energy levels ($n_2 = 5, 6, 7, ... ,\infty$) to fourth energy level ($n_1 = 4$) constitute Brackett series.

$$\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right] \text{ where } n_2 = 5, 6, 7..........,\infty$$

Series limit line (shortest wavelength) of Brackett series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right] = \frac{R}{16} \quad \text{or} \quad \lambda = \frac{16}{R}$$

- The first line (longest wavelength) of Brackett series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{5^2}\right] = \frac{9R}{400} \quad \text{or} \quad \lambda = \frac{400}{9R}$$

- Brackett series lie in the infrared region of the electromagnetic spectrum.
- This series is obtained only in the emission spectrum.

▶ **Pfund series**

- Emission spectral lines corresponding to the transition of electron from higher energy levels ($n_2 = 6, 7, 8,.......,\infty$) to fifth energy level ($n_1 = 5$) constitute Pfund series.

$$\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right] \text{ where } n_2 = 6, 7,...........,\infty$$

- Series limit line (shortest wavelength) of Pfund series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{\infty^2}\right] = \frac{R}{25} \quad \text{or} \quad \lambda = \frac{25}{R}$$

- The first line (longest wavelength) of the Pfund series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{6^2}\right] = \frac{11R}{900} \quad \text{or} \quad \lambda = \frac{900}{11R}$$

- Pfund series also lie in the infrared region of electromagnetic spectrum.
- This series is obtained only in the emission spectrum.

▶ Number of spectral lines due to transition of electron from n^{th} orbit to lower orbit is

$$N = \frac{n(n-1)}{2}$$

- **Ionization energy and ionization potential**

▶ Ionisation : The process of knocking an electron out of the atom is called ionisation. ionisation energy $= \frac{13.6}{n^2}$ eV

▶ Ionisation energy : The energy required, to knock an electron completely out of the atom.

▶ Ionisation potential $= \frac{13.6Z^2}{n^2}$ V

Practice Time

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. The first model of atom in 1898 was proposed by
(a) Ernest Rutherford (b) Albert Einstein
(c) J. J. Thomson (d) Niels Bohr

2. The transition from the state $n = 3$ to $n = 1$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from
(a) $2 \to 1$ (b) $3 \to 2$
(c) $4 \to 2$ (d) $4 \to 3$

3. In a hydrogen atom the total energy of electron is
(a) $\frac{e^2}{4\pi\varepsilon_0 r}$ (b) $\frac{-e^2}{4\pi\varepsilon_0 r}$
(c) $\frac{-e^2}{8\pi\varepsilon_0 r}$ (d) $\frac{e^2}{8\pi\varepsilon_0 r}$

4. The relation between the orbit radius and the electron velocity for a dynamically stable orbit in a hydrogen atom is (where, all notations have their usual meanings)
(a) $v = \sqrt{\frac{4\pi\varepsilon_0}{me^2 r}}$ (b) $r = \sqrt{\frac{e^2}{4\pi\varepsilon_0 v}}$
(c) $v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}}$ (d) $r = \sqrt{\frac{ve^2}{4\pi\varepsilon_0 m}}$

5. In the Geiger-Marsden scattering experiment the number of scattered particles detected are maximum and minimum at the scattering angles respectively at
(a) $0°$ and $180°$ (b) $180°$ and $0°$
(c) $90°$ and $180°$ (d) $45°$ and $90°$

6. Rutherford's experiments suggested that the size of the nucleus is about
(a) 10^{-14} m to 10^{-12} m (b) 10^{-15} m to 10^{-13} m
(c) 10^{-15} m to 10^{-14} m (d) 10^{-15} m to 10^{-12} m

7. A 10 kg satellite circles earth once every 2 h in an orbit having a radius of 8000 km. Assuming that Bohr's angular momentum postulate applies to a satellite just as it does to an electron in the hydrogen atom, then the quantum number of the orbit of satellite is
(a) 5.3×10^{40} (b) 5.3×10^{45}
(c) 7.8×10^{48} (d) 7.8×10^{50}

8. Which of the following is not correct about Bohrs model of the hydrogen atom?
(a) An electron in an atom could revolve in certain stable orbits without the emission of radiant energy.
(b) Electron revolves around the nucleus only in those orbits for which angular momentum $L_n = \frac{nh}{2\pi}$.
(c) When electron make a transition from one of its stable orbit to lower orbit then a photon emitted with energy $h\upsilon = E_f - E_i$.
(d) Bohr model is applicable to all atoms.

9. The shortest wavelength present in the Paschen series of spectral lines is
(a) 720 nm (b) 790 nm
(c) 800 nm (d) 820 nm

10. In a Geiger-Marsden experiment. Find the distance of closest approach to the nucleus of a 7.7 MeV α-particle before it comes momentarily to rest and reverses its direction.
(Z for gold nucleus = 79)
(a) 10 fm (b) 20 fm
(c) 30 fm (d) 40 fm

11. If an electron in hydrogen atom is revolving in a circular track of radius 5.3×10^{-11} m with a velocity of 2.2×10^6 m s^{-1} around the proton then the frequency of electron moving around the proton is
(a) 6.6×10^{12} Hz (b) 3.3×10^{15} Hz
(c) 3.3×10^{12} Hz (d) 6.6×10^{15} Hz

12. The Rydberg formula, for the spectrum of the hydrogen atom where all terms have their usual meaning is

(a) $h\upsilon_{if} = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_f} - \frac{1}{n_i}\right)$

(b) $h\upsilon_{if} = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

(c) $h\upsilon_{if} = \frac{8\varepsilon_0^2 h^2}{me^4}\left(\frac{1}{n_f} - \frac{1}{n_i}\right)$

(d) $h\upsilon_{if} = \frac{8\varepsilon_0^2 h^2}{me^4}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

13. In which of the following systems will the radius of the first orbit (n = 1) be minimum ?

(a) Doubly ionized lithium.
(b) Singly ionized helium.
(c) Deuterium atom.
(d) Hydrogen atom.

14. An electron in a hydrogen atom makes a transition from $n = n_1$ to $n = n_2$. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are

(a) $n_1 = 4, n_2 = 2$ (b) $n_1 = 8, n_2 = 2$
(c) $n_1 = 8, n_2 = 1$ (d) $n_1 = 6, n_2 = 2$

15. Energy is absorbed in the hydrogen atom giving absorption spectra when transition takes place from

(a) $n = 1 \rightarrow n'$ where $n' > 1$
(b) $n = 2 \rightarrow 1$
(c) $n' \rightarrow n$
(d) $n \rightarrow n' = \infty$

16. The value of ionisation energy of the hydrogen atom is

(a) 3.4 eV (b) 10.4 eV
(c) 12.09 eV (d) 13.6 eV

17. The moment of momentum for an electron in second orbit of hydrogen atom as per Bohr's model is

(a) $\frac{h}{\pi}$ (b) $2h$ (c) $\frac{2h}{\pi}$ (d) $\frac{\pi}{h}$

18. In the Geiger-Marsden scattering experiment, in case of head-on collision the impact parameter should be

(a) maximum (b) minimum
(c) infinite (d) zero

19. In an atom the ratio of radius of orbit of electron to the radius of nucleus is

(a) 10^3 (b) 10^4
(c) 10^5 (d) 10^6

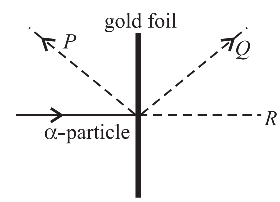
20. The radius of n^{th} orbit r_n in terms of Bohr radius (a_0) for a hydrogen atom is given by the relation

(a) na_0 (b) $\sqrt{n}\, a_0$
(c) $n^2 a_0$ (d) $n^3 a_0$

21. In the Bohr model of the hydrogen atom, the lowest orbit corresponds to

(a) infinite energy (b) maximum energy
(c) minimum energy (d) zero energy.

22. In an experiment on α-particle scattering, α-particles are directed towards a gold foil and detectors are placed in position P,Q and R. What is the distribution of α-particles as recorded at P, Q and R?



	P	Q	R
(a)	all	none	none
(b)	none	none	all
(c)	a few	some	most
(d)	most	some	a few

23. From quantisation of angular momentum, one gets for hydrogen atom, the radius of the n^{th} orbit as $r_n = \left(\frac{n^2}{m_e}\right)\left(\frac{h}{2\pi}\right)^2\left(\frac{4\pi^2\varepsilon_0}{e^2}\right)$

For a hydrogen like atom of atomic number Z,

(a) the radius of the first orbit will be the same
(b) r_n will be greater for larger Z values
(c) r_n will be smaller for larger Z values
(d) none of these.

24. Bohr's basic idea of discrete energy levels in atoms and the process of emission of photons from the higher levels to lower levels was experimentally confirmed by experiments performed by

(a) Michelson–Morley (b) Millikan
(c) Joule (d) Franck and Hertz

25. The binding energy of an electron in the ground state of He is equal to 24.6 eV. The energy required to remove both the electrons is

(a) 49.2 eV (b) 54.4 eV
(c) 79 eV (d) 108.8 eV

26. Out of the following which one is not a possible energy for a photon to be emitted by hydrogen atom according to Bohr's atomic model?

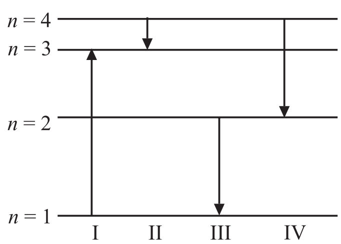
(a) 0.65 eV (b) 1.9 eV
(c) 11.1 eV (d) 13.6 eV

27. The ratio of the speed of the electron in the ground state of hydrogen atom to the speed of light in vacuum is

(a) $\frac{1}{2}$ (b) $\frac{2}{237}$ (c) $\frac{1}{137}$ (d) $\frac{1}{237}$

28. The diagram shows the energy levels for an electron in a certain atom.

Which transition shown represents the emission of a photon with the most energy?



(a) I (b) II (c) III (d) IV

29. If υ_1 is the frequency of the series limit of Lyman series, υ_2 is the frequency of the first line of Lyman series and υ_3 is the frequency of the series limit of the Balmer series, then

(a) $\upsilon_1 - \upsilon_2 = \upsilon_3$ (b) $\upsilon_1 = \upsilon_2 - \upsilon_3$

(c) $\frac{1}{\upsilon_2} = \frac{1}{\upsilon_1} + \frac{1}{\upsilon_3}$ (d) $\frac{1}{\upsilon_1} = \frac{1}{\upsilon_2} + \frac{1}{\upsilon_3}$

30. Suppose an electron is attracted towards the origin by a force k/r, where k is a constant and r is the distance of the electron from the origin. By applying Bohr model to this system, the radius of n^{th} orbit of the electron is found to be r_n and the kinetic energy of the electron is found to be T_n. Then which of the following is true?

(a) $T_n \propto \frac{1}{n^2}$

(b) T_n is independent of n; $r_n \propto n$

(c) $T_n \propto \frac{1}{n}; r_n \propto n$

(d) $T_n \propto \frac{1}{n}$ and $r_n \propto n^2$

31. The first line of the Lyman series in a hydrogen spectrum has a wavelength of 1210 Å. The corresponding line of a hydrogen-like atom of Z = 11 is equal to

(a) 4000 Å (b) 100 Å (c) 40 Å (d) 10 Å

32. The de-Broglie wavelength of an electron in the first Bohr orbit is

(a) equal to one-fourth the circumference of the first orbit
(b) equal to half the circumference of first orbit
(c) equal to twice the circumference of first orbit
(d) equal to the circumference of the first orbit.

33. The excitation energy of Lyman last line is

(a) the same as ionisation energy
(b) the same as the last absorption line in Lyman series
(c) both (a) and (b)
(c) different from (a) and (b)

34. If the wavelength of the first line of the Balmer series of hydrogen is 6561 Å, the wavelength of the second line of the series should be

(a) 13122 Å (b) 3280 Å
(c) 4860 Å (d) 2187 Å

35. The electric current I created by the electron in the ground state of H atom using Bohr model in terms of Bohr radius (a_0) and velocity of electron in first orbit v_0 is

(a) $\frac{ev_0}{2\pi a_0}$ (b) $\frac{2\pi a}{ev_0}$ (c) $\frac{2\pi a}{v_0}$ (d) $\frac{v_0}{2\pi a}$

36. If the radius of inner most electronic orbit of a hydrogen atom is 5.3×10^{-11} m, then the radii of n = 2 orbit is

(a) 1.12 Å (b) 2.12 Å
(c) 3.22 Å (d) 4.54 Å

37. The wavelength limit present in the Pfund series is ($R = 1.097 \times 10^7$ m^{-1})

(a) 1572 nm (b) 1898 nm
(c) 2278 nm (d) 2535 nm

38. An electron is revolving in the n^{th} orbit of radius 4.2 Å, then the value of n is (r_1 = 0.529 Å)

(a) 4 (b) 5 (c) 6 (d) 3

39. A hydrogen atom initially in the ground level absorbs a photon and is excited to n = 4 level then the wavelength of photon is

(a) 790 Å (b) 870 Å
(c) 970 Å (d) 1070 Å

40. If muonic hydrogen atom is an atom in which a negatively charged muon (μ) of mass about 207 m_e revolves around a proton, then first Bohr radius of this atom is ($r_e = 0.53 \times 10^{-10}$ m)

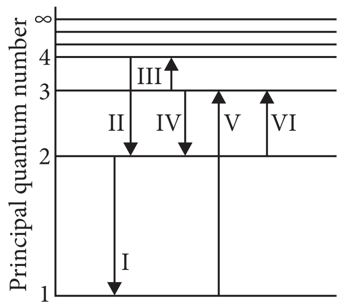
(a) 2.56×10^{-10} m (b) 2.56×10^{-11} m
(c) 2.56×10^{-12} m (d) 2.56×10^{-13} m

Case Based MCQs

Case I : Read the passage given below and answer the following questions from 41 to 43.

Electron Transitions for the Hydrogen Atom

Read the Bohr's model explains the spectral lines of hydrogen atomic emission spectrum. While the electron of the atom remains in the ground state, its energy is unchanged. When the atom absorbs one or more quanta of energy, the electrons moves from the ground state orbit to an excited state orbit that is farther away.



The given figure shows an energy level diagram of the hydrogen atom. Several transitions are marked as I, II, III and so on. The diagram is only indicative and not to scale.

41. In which transition is a Balmer series photon absorbed?

(a) II (b) III
(c) IV (d) VI

42. The wavelength of the radiation involved in transition II is

(a) 291 nm (b) 364 nm
(c) 487 nm (d) 652 nm

43. Which transition will occur when a hydrogen atom is irradiated with radiation of wavelength 1030 nm?

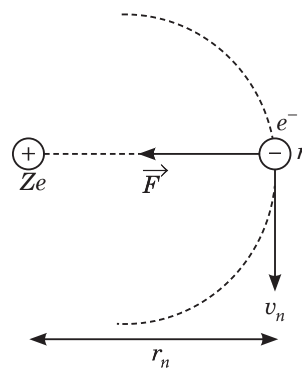
(a) I (b) II (c) IV (d) V

Case II : Read the passage given below and answer the following questions from 44 to 48.

Second Postulate of Bohr's Theory

Hydrogen is the simplest atom of nature. There is one proton in its nucleus and an electron moves around the nucleus in a circular orbit. According to Niels Bohr, this electron moves in a stationary orbit. When this electron is in the stationary orbit, it emits no electromagnetic radiation. The angular momentum of the electron is quantized, *i.e.*, $mvr = (nh/2\pi)$, where m = mass of the electron, v = velocity of the electron in the orbit, r = radius of the orbit and $n = 1, 2, 3, ...$ When transition takes place from K^{th} orbit to J^{th} orbit, energy photon is emitted. If the wavelength of the emitted photon is λ, we find that $\frac{1}{\lambda} = R\left[\frac{1}{J^2} - \frac{1}{K^2}\right]$, where R is Rydberg's constant.

On a different planet, the hydrogen atom's structure was somewhat different from ours. The angular momentum of electron was $P = 2n(h/2\pi)$, *i.e.*, an even multiple of $(h/2\pi)$.



44. The minimum permissible radius of the orbit will be

(a) $\frac{2\varepsilon_0 h^2}{m\pi e^2}$ (b) $\frac{4\varepsilon_0 h^2}{m\pi e^2}$ (c) $\frac{\varepsilon_0 h^2}{m\pi e^2}$ (d) $\frac{\varepsilon_0 h^2}{2m\pi e^2}$

45. In our world, the velocity of electron is v_0 when the hydrogen atom is in the ground state. The velocity of electron in this state on the other planet should be

(a) v_0 (b) $v_0/2$
(c) $v_0/4$ (d) $v_0/8$

46. In our world, the ionization potential energy of a hydrogen atom is 13.6 eV. On the other planet, this ionization potential energy will be

(a) 13.6 eV (b) 3.4 eV
(c) 1.5 eV (d) 0.85 eV

47. Check the correctness of the following statements about the Bohr model of hydrogen atom.

(i) The acceleration of the electron in $n = 2$ orbit is more than that in $n = 1$ orbit.
(ii) The angular momentum of the electron in $n = 2$ orbit is more than that in $n = 1$ orbit.
(iii) The kinetic energy of the electron in $n = 2$ orbit is less than that in $n = 1$ orbit.

(a) Only (iii) and (i) are correct.
(b) Only (i) and (ii) are correct.
(c) Only (ii) and (iii) are correct.
(d) All the statements are correct.

48. In Bohr's model of hydrogen atom, let PE represent potential energy and TE the total energy. In going to a higher orbit
(a) PE increases, TE decreases
(b) PE decreases, TE increases
(c) PE increases, TE increases
(d) PE decreases, TE decreases

Case III : Read the passage given below and answer the following questions from 49 to 52.

Hydrogen Emission Spectrum

Hydrogen spectrum consists of discrete bright lines in a dark background and it is specifically known as hydrogen emission spectrum. There is one more type of hydrogen spectrum that exists where we get dark lines on the bright background, it is known as absorption spectrum. Balmer found an empirical formula by the observation of a small part of this spectrum and it is represented by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \text{ where } n = 3, 4, 5, \ldots\ldots$$

For Lyman series, the emission is from first state to n^{th} state, for Paschen series, it is from third state to n^{th} state, for Brackett series, it is from fourth state to n^{th} state and for Pfund series, it is from fifth state to n^{th} state.

49. Number of spectral lines in hydrogen atom is
(a) 8 (b) 6 (c) 15 (d) ∞

50. Which series of hydrogen spectrum corresponds to ultraviolet region?
(a) Balmer series (b) Brackett series
(c) Paschen series (d) Lyman series

51. Which of the following lines of the H-atom spectrum belongs to the Balmer series?
(a) 1025 Å (b) 1218 Å
(c) 4861 Å (d) 18751 Å

52. Rydberg constant is
(a) a universal constant
(b) same for same elements
(c) different for different elements
(d) none of these.

Assertion & Reasoning Based MCQs

For question numbers 53-60, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false and R is also false

53. Assertion (A) : The force of repulsion between atomic nucleus and α-particle varies with distance according to inverse square law.
Reason (R) : Rutherford did α-particle scattering experiment.

54. Assertion (A) : According to classical theory, the proposed path of an electron in Rutherford atom model will be circular.
Reason (R) : According to electromagnetic theory an accelerated particle continuously emits radiation.

55. Assertion (A) : Between any two given energy levels, the number of absorption transitions is always less than the number of emission transitions.
Reason (R) : Absorption transitions start from the lowest energy level only and may end at any higher energy level. But emission transitions may start from any higher energy level and end at any energy level below it.

56. Assertion (A) : Electrons in the atom are held due to coulomb forces.
Reason (R) : The atom is stable only because the centripetal force due to Coulomb's law is balanced by the centrifugal force.

57. Assertion (A) : Total energy of revolving electron in any stationary orbit is negative.
Reason (R) : Energy is a scalar quantity. It can have positive or negative value.

58. Assertion (A) : Balmer series lies in the visible region of electromagnetic spectrum.

Reason (R) : $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{K^2}\right)$, where K = 3, 4, 5, ...

59. Assertion (A) : For the scattering of α-particles at large angles, only the nucleus of the atom is responsible.

Reason (R) : Nucleus is very heavy in comparison to electrons.

60. Assertion (A) : Fraunhofer lines are observed in the spectrum of the sun.

Reason (R) : The different elements have different spectra.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. Why is the classical (Rutherford) model for an atom of electron orbiting around the nucleus not able to explain the atomic structure?

2. When is H_α line of the Balmer series in the emission spectrum of hydrogen atom obtained?

3. What is the maximum number of spectral lines emitted by a hydrogen atom when it is in the third excited state?

4. Find the radius and energy of a He^+ ion in the states n = 2.

5. State Bohr's quantization condition of angular momentum.

6. Find the wavelength of the IInd line of Balmer series.

7. What is the wavelength of the electronic de Broglie wave in the 3^{rd} orbit of hydrogen?

8. Find the radius of the ground state orbit of hydrogen atom.

9. Find the speed in the ground state

10. Calculate the shortest wavelength of the Brackett series and state to which part of the electromagnetic spectrum does it belong.

Short Answer Type Questions (SA-I)

11. Find the number of unique radiations that can be emitted for a sample of hydrogen atoms excited to the n^{th} level.

12. An electron in the hydrogen atom makes a transition from n = 2 energy state to the ground state (corresponding to n = 1). Find the wavelength and frequency of the emitted photon.

13. Use the Bohr's model to estimate the wavelength of the K_α line in the X-ray spectrum of platinum (Z = 78).

14. The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the n = 2 and n = 3 orbits?

15. If Bohr's quantisation postulate (angular momentum = $nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

16. Would the Bohr formula for the H-atom remain unchanged if proton had a charge $(+4/3)e$ and electron a charge $(-3/4)e$, where $e = 1.6 \times 10^{-19}$C. Give reasons for your answer.

17. Consider a gas consisting Li^{++} (which is hydrogen like ion).

(i) Find the wavelength of radiation required to excite the electron in Li^{++} from n = 1 and n = 3.

(ii) How many spectral lines are observed in the emission spectrum of the above excited system?

18. Using Bohr model, calculate the electric current created by the electron when the H-atom is in the ground state.

19. The kinetic energy of the electron orbiting in the first excited state of hydrogen atom is 3.4 eV. Determine the de Broglie wavelength associated with it.

20. Calculate the shortest wavelength in the Balmer series of hydrogen atom. In which region (infrared), visible, ultraviolet of hydrogen spectrum does this wavelength lie?

Short Answer Type Questions (SA-II)

21. Assume that their is no repulsive force between the electrons in an atom but the force between positive and negative charges is given by Coulomb's law as usual. Under such circumstances, calculate the ground state energy of a He-atom.

22. Show that the first few frequencies of light that is emitted when electrons fall to the n^{th} level from levels higher than n, are approximate harmonics (*i.e.* in the ratio 1 : 2 : 3...) when $n >> 1$.

23. What is the minimum energy that must be given to a H atom in ground state so that it can emit an H_γ line in Balmer series. If the angular momentum of the system is conserved, what would be the angular momentum of such H_γ photon?

24. The electron in a given Bohr orbit has a total energy of –1.5 eV. Calculate its

(i) kinetic energy. (ii) potential energy.

(iii) wavelength of radiation emitted, when this electron makes a transition to the ground state.

[Given : Energy in the ground state = –13.6 eV and Rydberg's constant = 1.09×10^7 m^{-1}]

25. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. Upto which energy level the hydrogen atoms would be excited?

Calculate the wavelengths of the first member of Lyman and first member of Balmer series.

26. The value of ground state energy of hydrogen atom is –13.6 eV.

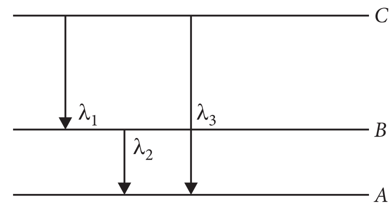
(i) Find the energy required to move an electron from the ground state to the first excited state of the atom.

(ii) Determine (a) the kinetic energy and (b) orbital radius in the first excited state of the atom.

(Given the value of Bohr radius = 0.53 Å).

27. (i) How does de-Broglie hypothesis explain Bohr's quantization condition for stationary orbits ?

(ii) Find the relation between the three wavelengths λ_1, λ_2 and λ_3 from the energy level diagram shown in the figure.



28. Show that the radius of the orbit in hydrogen atom varies as n^2, where n is the principal quantum number of the atom.

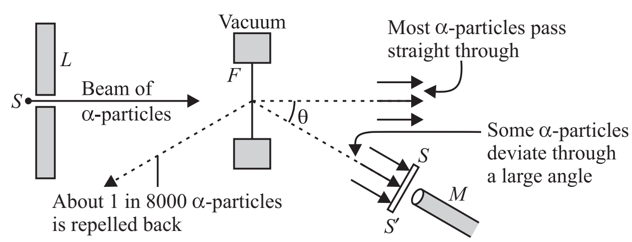
29. Direction : Read the following passage and answer the questions given below.

In 1911, Rutherford, along with his assistants, H. Geiger and E. Marsden, performed the alpha particle scattering experiment. H. Geiger and E. Marsden took radioactive source ($^{214}_{83}Bi$) for α-particles. A collimated beam of α-particles of energy 5.5 MeV was allowed to fall on 2.1×10^{-7} m thick gold foil. Observations of this experiment are as follows

(I) Most of the α-particles passed through the foil without deflection.

(II) Only about 0.14% of the incident α-particles scattered by more than 1 .

(III) Only about one α-particle in every 8000 α-particles deflected by more than 90°.



(i) Gold foil used in Geiger-Marsden experiment is about 10^{-8} m thick. What does it ensures?

(ii) On which factor, the trajectory traced by an α-particle depends?

(iii) In Rutherford scattering experiment the fact that only a small fraction of the number of incident particles rebound back. What it indicates?

30. Positronium is just like a H-atom with the proton replaced by the positively charged antiparticle of the electron (called the positron which is as massive as the electron). What would be the ground state energy of positronium?

31. The photons from Balmer series in hydrogen spectrum having wavelength between 450 nm to 700 nm are incident on a metal surface of work function 2 eV. Find the maximum kinetic energy of one photoelectron.

32. A small particle of mass m moves in such a way that the potential energy $U = ar^2$ where a is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantisation of angular momentum for circular orbits, find the radius of n^{th} allowed orbit.

33. A particle known as μ-meson, has a charge equal to that of an electron and mass 208 times the mass of the electron. It moves in a circular orbit around a nucleus of charge $+3e$. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to this system,

(i) derive an expression for the radius of the n^{th} Bohr orbit,

(ii) find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for a hydrogen atom.

34. A gas of hydrogen like atoms can absorb radiations of 68 eV. Consequently, the atoms emit radiations of only three different wavelength. All the wavelengths are equal or smaller than that of the absorbed photon.

(i) Determine the initial state of the gas atoms.

(ii) Identify the gas atoms.

(iii) Find the minimum wavelength of the emitted radiations.

Long Answer Type Questions (LA)

35. In the Auger process an atom makes a transition to a lower state without emitting a photon. The excess energy is transferred to an outer electron which may be ejected by the atom. (This is called an Auger electron). Assuming the nucleus to be massive, calculate the kinetic energy of an $n = 4$ Auger electron emitted by Chromium by absorbing the energy from a $n = 2$ to $n = 1$ transition.

36. (a) Write two important limitations of Rutherford model which could not explain the observed features of atomic spectra.

(b) How were these explained in Bohr's model of hydrogen atom?

37. Using Bohr's postulates, derive the expression for the frequency of radiation emitted when electron in hydrogen atom undergoes transition from higher energy state (quantum number n_i) to the lower state, (n_f). When electron in hydrogen atom jumps from energy state $n_i = 4$ to $n_f = 3, 2, 1$. Identify the spectral series to which the emission lines belong.

38. The first four spectral lines in the Lyman series of a H-atom are $\lambda = 1218$ Å, 1028 Å, 974.3 Å and 951.4 Å. If instead of Hydrogen, we consider Deuterium, calculate the shift in the wavelength of these lines.

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. (c) : Sir J. J. Thomson proposed the first model of atom called plum pudding model of atom. According to this model, the positive charge of the atom is uniformly distributed throughout the volume of the atom and the negatively charged electrons are embedded in it like seeds in a watermelon.

2. (d)

3. (c) : The kinetic energy of the electron in hydrogen atom are

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r} \qquad \left[\because \ v^2 = \frac{e^2}{4\pi\varepsilon_0 mr}\right]$$

⇒ Electrostatic potential energy,

and $U = \dfrac{-e^2}{4\pi\varepsilon_0 r}$

The total energy E of the electron in a hydrogen atom is

$E = K + U$

$$E = \frac{e^2}{8\pi\varepsilon_0 r} + \left(\frac{-e^2}{4\pi\varepsilon_0 r}\right) = -\frac{e^2}{8\pi\varepsilon_0 r}$$

Here negative sign shows that electron is bound to the nucleus.

4. (c) : In hydrogen atom electrostatic force of attraction (F_e) is acting between the revolving electrons and the nucleus provides the requisite centripetal force (F_c) to keep them in their orbits. Thus,

$$F_e = F_c$$

$$\therefore \quad \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$\text{or} \quad v^2 = \frac{e^2}{4\pi\varepsilon_0 mr} \quad \Rightarrow \quad v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}}$$

5. (a) : The number of scattered particles detected will be maximum at the angle of scattering $\theta = 0°$ and minimum at $\theta = 180°$.

6. (c)

7. (b) : Here, $m = 10$ kg, $r_n = 8 \times 10^6$ m

$T = 2 \times 60 \times 60 = 7200$ s

Velocity of n^{th} orbit, $v_n = \frac{2\pi r_n}{T}$ and from $m v_n r_n = \frac{nh}{2\pi}$

$$n = \frac{2\pi}{h} \times m \times \frac{2\pi r_n}{T} \times r_n$$

$$= (2\pi r_n)^2 \times \frac{m}{T \times h} = \frac{(2\pi \times 8 \times 10^6)^2 \times 10}{7200 \times 6.64 \times 10^{-34}} = 5.3 \times 10^{45}.$$

8. (d) : The first three options (a), (b) and (c) are Bohr's postulates of atomic model whereas option (d) is not correct as Bohr's model is applicable to hydrogen atom only.

9. (d) : The wavelength for Paschen series, $\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right]$

For shortest wavelength $n = \infty$

$$\therefore \quad \frac{1}{\lambda} = R\left[\frac{1}{9} - \frac{1}{\infty^2}\right] = \frac{R}{9}$$

$$\lambda = \frac{9}{R} = \frac{9}{1.097 \times 10^7}$$

$$= 8.20 \times 10^{-7} \text{ m} = 820 \text{ nm}.$$

10. (c) : Let d be the distance of closest approach then by the conservation of energy,

Initial kinetic energy of incoming α-particle, K

= Final electric potential energy U of the system

$$\text{As } K = \frac{1}{4\pi\varepsilon_0} \times \frac{(2e)(Ze)}{d}$$

$$\therefore \quad d = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{K} \qquad \text{...(i)}$$

Here, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$, $Z = 79$, $e = 1.6 \times 10^{-19}$ C.

$K = 7.7 \text{ MeV} = 7.7 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} = 1.2 \times 10^{-12}$ J

Substituting these values in (i)

$$d = \frac{2 \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2 \times 79}{1.2 \times 10^{-12}}$$

$$d = 3 \times 10^{-14} \text{ m}$$

$$= 30 \text{ fm} \qquad (\because 1 \text{ fm} = 10^{-15} \text{ m})$$

11. (d) : Frequency of the electron moving around the proton is

$$\upsilon = \frac{\text{velocity of electron } (v)}{\text{circumference } (2\pi r)}$$

Here, $v = 2.2 \times 10^6 \text{ m s}^{-1}$ and $r = 5.30 \times 10^{-11}$ m

$$\therefore \quad \upsilon = \frac{2.2 \times 10^6}{2 \times 3.14 \times 5.3 \times 10^{-11}}$$

$$\upsilon = 6.6 \times 10^{15} \text{ Hz}.$$

12. (b)

13. (a) : Radius of first orbit, $r \propto \frac{1}{Z}$,

For doubly ionized lithium, Z (= 3) will be maximum, hence for doubly ionized lithium, r will be minimum.

14. (a) : In the n^{th} orbit, let r_n = radius and v_n = speed of electron.

Time period, $T_n = \frac{2\pi r_n}{v_n} \propto \frac{r_n}{v_n}$.

Now, $r_n \propto n^2$ and $v_n \propto \frac{1}{n}$

$$\therefore \quad \frac{r_n}{v_n} \propto n^3 \quad \text{or} \quad T_n \propto n^3.$$

$$\text{Here, } 8 = \left(\frac{n_1}{n_2}\right)^3$$

$$\text{or} \quad \frac{n_1}{n_2} = 2 \quad \text{or} \quad n_1 = 2n_2$$

15. (a) : Absorption is from the ground state $n = 1$ to n' where $n' > 1$.

16. (d) : The minimum energy required to free the electron from the ground state of the hydrogen atom is called the ionisation energy of hydrogen atom and its value is 13.6 eV.

17. (a) : According to Bohr's second postulate

Angular momentum, $L = \frac{nh}{2\pi}$

Angular momentum is also called a moment of momentum.

For second orbit, $n = 2$

$$L = \frac{2h}{2\pi} = \frac{h}{\pi}$$

18. (b) : At minimum impact parameter α particles rebound back ($\theta \approx \pi$) and suffers large scattering.

19. (c) : The radius of orbit of electrons = 10^{-10} m

radius of nucleus = 10^{-15} m

$$\therefore \quad \text{Ratio} = \frac{10^{-10}}{10^{-15}} = 10^5$$

Hence the radius of electron orbit is 10^5 times larger than the radius of nucleus.

20. (c) : The radius of n^{th} orbit

$$r_n = n^2 \frac{\hbar^2 4\pi\varepsilon_0}{me^2}$$

where $\frac{\hbar^2 4\pi\varepsilon_0}{me^2} = a_0$ (Bohr radius)

Hence, $r_n = n^2 a_0$.

21. (c) : In hydrogen atom, the lowest orbit corresponds to minimum energy.

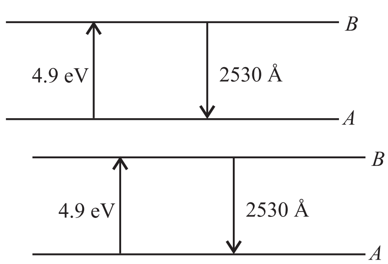
22. (c)

23. (c) : For an atom with a single electron, Bohr atom model in applicable.

As the value of attraction force between a proton and electron is proportional to e^2, for an ion with a single electron, $\frac{e^2}{4\pi\varepsilon_0}$ is replaced by $\frac{Ze^2}{4\pi\varepsilon_0}$

i.e. $r_n \propto \frac{n^2}{Z}$.

24. (d) : Franck and Hertz showed that exciting mercury vapour by electrons of energy 4.9 eV and more, mercury lines of energy 4.9 eV were obtained.



First they were excited to level *B* and then thec atoms emitted spectral lines of 2530 Å i.e. 4.9 eV, Bohr's concepts were verified.

25. (c) : The energy needed to remove one electron from the ground state of He = 24.6 eV.

As the He^+ is now hydrogen-like, ionisation energy

$$= |-13.6| \frac{2^2}{1^2} \text{ eV} \Rightarrow E = 54.4 \text{ eV}$$

∴ To remove both the electrons, energy needed

= (54.4 + 24.6) eV = 79 eV

26. (c) : The energy of n^{th} orbit of hydrogen atom is given as

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

$$\therefore \quad E_1 = -13.6 \text{ eV}$$

$$E_2 = -\frac{13.6}{2^2} = -3.4 \text{ eV}$$

$$E_3 = -\frac{13.6}{3^2} = -1.5 \text{ eV}$$

$$E_4 = -\frac{13.6}{4^2} = -0.85 \text{ eV}$$

$$\therefore \quad E_3 - E_2 = -1.5 - (-3.4) = 1.9 \text{ eV}$$

$$E_4 - E_3 = -0.85 - (-1.5) = 0.65 \text{ eV}$$

27. (c) : Speed of the electron in the ground state of hydrogen atom is

$$v = \frac{2\pi e^2}{4\pi\varepsilon_0 h} = \frac{c}{137} = c\alpha$$

where, c = speed of light in vacuum,

$\alpha = \frac{e^2}{2\varepsilon_0 hc}$ is the fine structure constant. It is a pure number whose value is $\frac{1}{137}$.

$$\therefore \quad \frac{v}{c} = \frac{1}{137}$$

28. (c) : I^{st} transition is showing absorption of a photon. From rest of three transitions, III is having maximum energy from level $n = 2$ to $n = 1$

$$\Delta E \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

29. (a) : For Lyman series

$$\upsilon = Rc\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$$

where n = 2, 3, 4,.......

For the series limit of Lyman series, $n = \infty$

$$\therefore \quad \upsilon_1 = Rc\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = Rc \quad \text{...(i)}$$

For the first line of Lyman series, $n = 2$

$$\therefore \quad \upsilon_2 = Rc\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}Rc \quad \text{...(ii)}$$

For Balmer series

$$\upsilon = Rc\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

where n = 3, 4, 5....

For the series limit of Balmer series, $n = \infty$

$$\therefore \quad \upsilon_3 = Rc\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{Rc}{4} \quad \text{...(iii)}$$

From equations (i), (ii) and (iii), we get

$$\upsilon_1 = \upsilon_2 + \upsilon_3 \quad \text{or} \quad \upsilon_1 - \upsilon_2 = \upsilon_3$$

30. (b) : Applying Bohr model to the given system,

$$\frac{mv^2}{r_n} = \frac{k}{r_n} \quad \text{...(i)}$$

and $mvr_n = \frac{nh}{2\pi}$ or $v = \frac{nh}{2\pi m r_n}$

Put in (i),

$$\frac{m}{r_n} \times \frac{n^2h^2}{4\pi^2 m^2 r_n^2} = \frac{k}{r_n}$$

$$r_n^2 = \frac{n^2h^2}{4\pi^2 mk} \quad \text{...(ii)}$$

$\therefore$ $r_n^2 \propto n^2$ or $r_n \propto n$

K.E. of the electron, $T_n = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{n^2h^2}{4\pi^2 m^2 r_n^2} = \frac{n^2h^2}{8\pi^2 m r_n^2}$

Using (ii), we get

$$T_n = \frac{n^2h^2 4\pi^2 mk}{8\pi^2 mn^2h^2} = \frac{k}{2}$$

$\therefore$ T_n is independent of n.

31. (d) : By Bohr's formula

$$\frac{1}{\lambda} = Z^2R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For first line of Lyman series $n_1 = 1$, $n_2 = 2$

$$\therefore \frac{1}{\lambda} = Z^2R\frac{3}{4}$$

In the case of hydrogen atom, $Z = 1$

$$\frac{1}{\lambda} = R\frac{3}{4}$$

For hydrogen-like atom, $Z = 11$

$$\frac{1}{\lambda'} = 121R\frac{3}{4} \quad \Rightarrow \quad \frac{\lambda'}{\lambda} = \frac{3R}{4} \times \frac{4}{121R \times 3} = \frac{1}{121}$$

$$\lambda' = \frac{\lambda}{121} = \frac{1210}{121} = 10 \text{ Å}$$

32. (d) : Angular momentum $= \frac{nh}{2\pi}$

$\Rightarrow$ Moment of momentum $= \frac{nh}{2\pi}$

$\Rightarrow$ $p \times r_n = \frac{nh}{2\pi}$

$$\frac{h}{\lambda}r_n = \frac{nh}{2\pi} \Rightarrow \lambda = \frac{2\pi r_n}{n}$$

For 1st orbit, $n = 1$, $\lambda = 2\pi r_1$

$\Rightarrow \lambda$ = circumference of 1st orbit.

33. (c)

34. (c) : For Balmer series, $n_1 = 2$, $n_2 = 3$ for 1st line and $n_2 = 4$ for second line.

$$\frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{1}{2^2} - \frac{1}{4^2}\right)}{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)} = \frac{3/16}{5/36} = \frac{3}{16} \times \frac{36}{5} = \frac{27}{20}$$

$$\lambda_2 = \frac{20}{27}\lambda_1 = \frac{20}{27} \times 6561 = 4860 \text{ Å}$$

35. (a) : In the ground state of hydrogen atom, suppose,

a_0 = Bohr radius

v_0 = velocity of electron in first orbit

$\therefore$ Time taken by electron to complete one revolution,

$$T = \frac{2\pi a_0}{v_0}$$

$\therefore$ Current created, $I = \frac{\text{charge}(e)}{\text{time}(T)} = \frac{ev_0}{2\pi a_0}$

36. (b) : As, $r_n = n^2 a_0$

Here, $a_0 = 5.3 \times 10^{-11}$ m

$n = 2$

$\therefore$ $r_2 = (2)^2 a_0$

$= 4a_0 = 4 \times 5.3 \times 10^{-11}$ m

$= 21.2 \times 10^{-11}$ m $= 2.12$ Å.

37. (c) : The wavelength for Pfund series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n^2}\right]$$

for series limit, $n = \infty$

$$\therefore \frac{1}{\lambda} = R\left[\frac{1}{25} - \frac{1}{\infty^2}\right] = \frac{R}{25}$$

$$\therefore \lambda = \frac{25}{R} = \frac{25}{1.097 \times 10^7} = 2278 \text{ nm.}$$

38. (d) : Since $r_n \propto n^2$

$$\frac{r_n}{r_1} = \frac{n^2}{(1)^2}$$

$\therefore$ $r_n = n^2 r_1$

or $n^2 = \frac{r_n}{r_1} \Rightarrow n = \sqrt{\frac{r_n}{r_1}} = \sqrt{\frac{4.2}{0.529}} = \sqrt{7.939}$

$= 2.81 \approx 3$

39. (c) : Here, $n_1 = 1$, and $n_2 = 4$

Energy of photon absorbed, $E = E_2 - E_1$

Since, $E_n = -\frac{13.6}{n^2}$ eV

Then, $E_2 - E_1 = -\frac{13.6}{(4)^2} - \left(-\frac{13.6}{(1)^2}\right)$

$= -\frac{13.6}{16} + 13.6 = 12.75$ eV

$= 12.75 \times 1.6 \times 10^{-19}$ J $= 20.4 \times 10^{-18}$ J

$$E_2 - E_1 = \frac{hc}{\lambda}$$

$$\therefore \lambda = \frac{hc}{E_2 - E_1} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{20.4 \times 10^{-18}}$$

$= 9.70 \times 10^{-8}$ m $= 970 \times 10^{-10} = 970$ Å.

40. (d) : According to Bohr's atomic model, $r \propto \frac{1}{m}$

$$\Rightarrow \frac{r_\mu}{r_e} = \frac{m_e}{m_\mu} \quad \text{...(i)}$$

Here, $r_e = 0.53 \times 10^{-10}$ m

$m_\mu = 207\, m_e$

$$\therefore \quad r_\mu = \frac{m_e}{207\, m_e} \times 0.53 \times 10^{-10} \quad \text{(using (i))}$$

$$= 2.56 \times 10^{-13} \text{ m.}$$

41. (d) : For Balmer series, $\underset{\text{(lower)}}{n_1 = 2}$; $\underset{\text{(higher)}}{n_2 = 3, 4, \ldots}$

Therefore, in transition (VI), photon of Balmer series is absorbed.

42. (c) : In transition II,

$E_2 = -3.4$ eV, $E_4 = -0.85$ eV,

$$\Delta E = 2.55 \text{ eV} \Rightarrow \Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} = 487 \text{ nm}$$

43. (d) : Wavelength of radiation = 1030 Å

$$\Delta E = \frac{12400}{1030 \text{ Å}} = 12.0 \text{ eV}$$

So, difference of energy should be 12.0 eV (approx.)

Hence for $n_1 = 1$ to $n_2 = 3$

$$E_{n_3} - E_{n_1} = -1.51 \text{ eV} - (-13.6 \text{ eV}) \approx 12 \text{ eV}$$

Therefore, transition V will occur.

44. (b) : On other planet : $mvr = 2n\frac{h}{2\pi} \Rightarrow v = \frac{nh}{\pi mr}$

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \Rightarrow \frac{mn^2h^2}{n^2m^2r^3} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

Putting $n = 1$, we get $r = \frac{4h^2\varepsilon_0}{m\pi e^2}$

45. (b) : On our planet : $v_0 = \frac{e^2}{2\varepsilon_0 nh}$

On other planet : $v = \frac{e^2}{2\varepsilon_0(2n)h} = \frac{v_0}{2}$

46. (b) : On our planet : $E_n = -\frac{13.6}{n^2}$

On other planet : $E'_n = -\frac{13.6}{(2n)^2}$

$$\Rightarrow \quad E'_n = \frac{E_n}{4} = -3.4 \text{ eV}$$

47. (c) : Centripetal acceleration = mv^2/r

Further, as n increases, r also increases. Therefore, centripetal acceleration for n = 2 is less than that for n = 1. So, statement (i) is wrong. Statement (ii) and (iii) are correct.

48. (c) : Potential energy = $-C/r^2$ and total energy = $-Rhc/n^2$. With higher orbit, both r and n increase. So, both become less negative; hence both increase.

49. (d) : Number of spectral lines in hydrogen atom is ∞.

50. (d) : Lyman series lies in the ultraviolet region.

51. (c) : The shortest Balmer line has energy

= |(3.4 – 1.51)| eV = 1.89 eV

and the highest energy = |(0 – 3.4)| = 3.4 eV

The corresponding wavelengths are

$$\frac{12400 \text{ eV Å}}{1.89 \text{ eV}} = 6561 \text{ Å} \quad \text{and} \quad \frac{12400 \text{ eV Å}}{3.4 \text{ eV}} = 3647 \text{ Å}$$

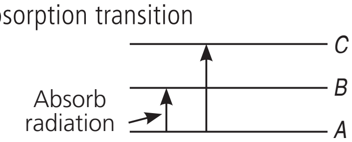
Only 4861 Å is between the first and last line of the Balmer series.

52. (a)

53. (b) : In Rutherford's α-particle scattering experiment, some of α-particles were found to be scattered at very large angles inspite of having very high kinetic energy. This shows that there are α-particles which will be passing very close to nucleus. Rutherford confirmed the repulsive force on α-particles due to nucleus varies with distance according to inverse square law and that the positive charges are concentrated at the centre and not distributed throughout the atom. This is the nuclear model of Rutherford.

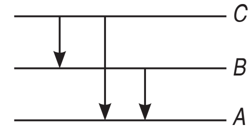
54. (b) : According to classical electromagnetic theory, an accelerated charge continuously emits radiation. As electrons revolving in circular paths are constantly experiencing centripetal acceleration, hence they will be losing their energy continuously and the orbital radius will go on decreasing and form spiral and finally the electron will fall into the nucleus.

55. (a) : Absorption transition



Two possibilities in absorption transition.

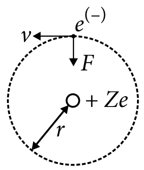
Emission transition



Three possibilities in emission transition. Therefore number of absorption transition < number of emission transition.

For any two states A and B such that $E_A < E_B$ we have absorption spectrum for $A \to B$ transition and emission $B \to A$. But most of the time atoms are in ground state, absorption is only from the ground state.

56. (c) : According to postulates of Bohr's atomic model, the electron revolve around the nucleus in fixed orbit of definite radii. As long as the electron is in a certain orbit it does not radiate any energy. Not only the centripetal force has to be the centrifugal force, even the stable orbits are fixed by Bohr's theory.



57. (b) : The reason is correct, but does not explain the assertion properly. Negative energy of revolving electron indicates that it is bound to the nucleus. The electron is not free to leave the nucleus.

58. (b) : When we put $R = 10^7$ m^{-1} and $K = 3, 4, 5$ in the given formula, values of λ calculated lie between 4000 Å and 8000 Å, which is the visible region. The reason is true, but does not explain the assertion properly.

59. (a) : We know that an electron is very light particle as compared to an α-particle. Hence electron cannot scatter the α-particle at large angles, according to law of conservation of momentum. On the other hand, mass of nucleus is comparable with the mass of α-particle, hence only the nucleus of atom is responsible for scattering of α-particles.

60. (b) : When white light from the photosphere (central portion of the sun) passes through vapours of various elements present in the outer chromosphere, then these elements absorb those wavelengths which they themselves emit to bring incandescent. Hence dark lines (absence of light) appear in the continuous solar spectrum, due to absorption of these lines. Absorption is possible in the sun, not only from the ground state but also in higher states because of the high temperature of the sun.

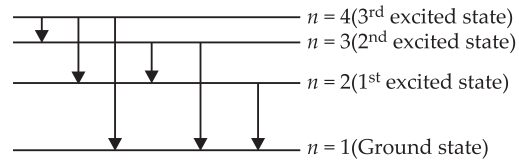
SUBJECTIVE TYPE QUESTIONS

1. According to electromagnetic theory, electron revolving around the nucleus are continuously accelerated. Since an accelerated charge emits energy, the radius of the circular path of a revolving electron should go on decreasing and ultimately it should fall into the nucleus. So, it could not explain the structure of the atom. As matter is stable, we cannot expect the atoms to collapse.

2. H_α line of the Balmer series in the emission spectrum of hydrogen atoms obtained when the transition occurs from $n = 3$ to $n = 2$ state.

3. Number of spectral lines obtained due to transition of electron from $n = 4$ (3rd excited state) to $n = 1$ (ground state) is

$$N = \frac{(4)(4-1)}{2} = 6$$



These lines correspond to Lyman series.

4. $r = 0.529\dfrac{n^2}{Z}$

$$r\,(n=2) = 0.529 \times \frac{2^2}{2} = 1.058 \text{ Å}$$

$$E\,(n=2) = -13.6 \times \frac{2^2}{2^2} = -13.6 \text{ eV}$$

5. Bohr's quantization condition : The electron can revolve round the nucleus only in those circular orbits in which angular momentum of an electron is an integral multiple of $\dfrac{h}{2\pi}$

i.e., $mvr = \dfrac{nh}{2\pi}$, $n = 1, 2, 3, \ldots$

6. $$\frac{1}{\lambda_{B_2}} = RZ^2\left[\frac{1}{2^2} - \frac{1}{(2+1)^2}\right] = RZ^2\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\Rightarrow \lambda_{B_2} = \frac{36}{5RZ^2}$$

7. $2\pi r = n\lambda$

Here, $n = 3 \Rightarrow 2\pi r = 3\lambda$ and $r = 0.53\dfrac{n^2}{Z}$ Å

or $r_3 = 0.53\left(\dfrac{9}{1}\right)$ Å

$$\Rightarrow \lambda = \frac{2\pi r_3}{3} = \frac{2\pi}{3}(0.53 \times 9) \text{ Å} \approx 9.99 \text{ Å}.$$

8. $$r_n = \frac{n^2h^2}{4\pi^2 m_e kZe^2} = \frac{n^2}{Z}(0.053 \text{ nm})$$

For our case, $n = 1$ and $Z = 2$, and the result is $r_1 = 0.027$ nm.

9. $$v = \frac{Ze^2}{2\varepsilon_0 nh} = \frac{c}{137}\left(\frac{Z}{n}\right)\frac{\text{m}}{\text{s}}$$

Here $n = 1$, $Z = 2$, $v_1 = \dfrac{2c}{137}$ m/s.

10. The shortest wavelength of Brackett series is given as

$$\frac{1}{\lambda} = 1.097 \times 10^7\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right] = \frac{1.097 \times 10^7}{16}$$

$\Rightarrow \quad \lambda = 1.4585 \times 10^{-6}$ m

This wavelength lies in the infrared region of electromagnetic spectrum.

11. The first excited level is 2nd line.

From the 2nd level electron can go to level 1 $\Rightarrow$ one radiation

3^{rd} level electron can go to levels 1, 2 $\Rightarrow$ three radiations
4^{th} level electron can go to levels 1, 2, 3 $\Rightarrow$ six radiations
n^{th} level electron can go to levels 1, 2, 3, ...$(n-1)$
$\therefore$ Total number of radiations
$= 1 + 2 + \ldots\ldots + (n-1) = \dfrac{(n-1)\cdot n}{2}$.

12. $$\lambda = \frac{hc}{(E_2 - E_1)}$$

$$\lambda = \frac{1242 \text{ eV-nm}}{(-3.4 \text{ eV}) - (-13.6 \text{ eV})} = 122 \text{ nm}.$$

This wavelength lies in the ultraviolet region.
Since, $c = \upsilon\lambda$, the frequency of the photon is

$$\upsilon = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{1.22 \times 10^{-7} \text{ m}} = 2.46 \times 10^{15} \text{ Hz}$$

13. $$\frac{1}{\lambda} = R(Z)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For characteristic X-rays, we replace Z by $Z - a$ or $(Z - 1)$ here. For K_α, the electron jumps to the K-shell, hence, $n_1 = 1$ and $n_2 = 2$.

$$\Rightarrow \frac{1}{\lambda} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1})(78-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$\frac{1}{\lambda} = 4.9 \times 10^{10} \text{ m}^{-1} \text{ or } \lambda = 2 \times 10^{-11} \text{ m} = 0.2 \text{ Å}.$$

14. Radius of innermost electron

$$r = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

For $n = 1$, $r_1 = \dfrac{h^2\varepsilon_0}{\pi me^2} = 5.3 \times 10^{-11}$ m

For $n = 2$, $r_2 = (2)^2 r_1 = 2.12 \times 10^{-10}$ m
For $n = 3$, $r_3 = (3)^2 r_1 = 4.77 \times 10^{-10}$ m.

15. Angular momentum $mvr = n\dfrac{h}{2\pi}$ associated with planetary motion are incomparably large relative to h.
For example angular momentum of earth in its orbital motion is of the order of $10^{70}\dfrac{h}{2\pi}$.
For such large value of n, the difference in successive energies and angular momenta of the quantised levels of the Bohr model are so small that one can predict the energy level continuous.

16. In Bohr's formula,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}(e)(-e)$$

Force $\propto (-e)(e) = -e^2$
If charge on proton is $\left(+\dfrac{4}{3}e\right)$ and charge on electron is $\left(-\dfrac{3}{4}e\right)$, then their product $\left(\dfrac{4}{3}e\right)\left(-\dfrac{3}{4}e\right) = -e^2$. Thus Bohr Formula remains the same.

17. (i) As, $\Delta E = 13.6 \times 3^2 \times \left(\dfrac{1}{1} - \dfrac{1}{3^2}\right) \text{eV} = 13.6 \times 8 \text{ eV}$

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{12400}{13.6 \times 8} \text{ Å} = 113.7 \text{ Å}$$

(ii) Number of spectral lines $= \dfrac{n(n-1)}{2} = \dfrac{3(3-1)}{2} = 3$

18. Let a_0 = Bohr radius.
v_0 = velocity of electron in first orbit.
$\therefore$ Time taken by electron to complete one revolution

$$T = \frac{2\pi a_0}{v_0}.$$

$\therefore$ Current created by electron, $I = \dfrac{e}{T}$

$$= \frac{e}{\left(\dfrac{2\pi a_0}{v_0}\right)} = \frac{ev_0}{2\pi a_0}.$$

19. Kinetic energy in the first excited state of hydrogen atom
$E_K = 3.4 \text{ eV} = 3.4 \times 1.6 \times 10^{-19}$ J

de-Broglie wavelength, $\lambda = \dfrac{h}{\sqrt{2m\,E_K}}$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}} = 0.67 \text{ nm}$$

20. Wavelength (λ) of Balmer series is given by

$$\frac{1}{\lambda} = R_H\left[\frac{1}{2^2} - \frac{1}{n_i^2}\right] \text{where } n_i = 3, 4, 5, \ldots$$

For shortest wavelength, when transition of electrons take place from $n_i = \infty$ to $n_f = 2$ orbit, wavelength of emitted photon is shortest.

$$\frac{1}{\lambda_{min}} = R_H\left[\frac{1}{2^2} - \frac{1}{\infty}\right] = \frac{1.097 \times 10^7}{4}$$

$\therefore$ $\lambda_{min} = 3.646 \times 10^{-7} \text{ m} = 3646$ Å

This wavelength lies in visible region of electromagnetic spectrum.

21. There are two protons and two neutrons in helium atom. Two electrons are revolving around the nucleus in first orbit. It is assumed that there is no interaction between the two electrons to He-atom.
so, replacing $Z = 1$ by $Z = 2$

$$E_n = \left(\frac{-13.6\,Z^2}{n^2}\right)\text{eV},$$

For ground state ($n = 1$) of helium atom, energy

$$E_1 = \left[\frac{-13.6}{1^2}(2)^2\right] \text{eV} = -54.4 \text{ eV}$$

Helium has two electrons in ground state, so total energy $E = 2E_1 = -108.8$ eV.

22. As $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$,

$$\upsilon = cRZ^2\left[\frac{1}{n^2} - \frac{1}{(n+p)^2}\right]$$

[as $c = \upsilon\lambda$, $n_f = n$, $n_i = (n + p)$, where $p = 1, 2, 3...$]

or $\upsilon = cRZ^2\left[\frac{(n+p)^2 - n^2}{n^2(n+p)^2}\right]$

$$= cRZ^2\left[\frac{(n^2 + p^2 + 2pn) - n^2}{n^2(n+p)^2}\right]$$

or $\upsilon \approx cRZ^2\left(\frac{2pn}{n^4}\right) \approx \left(\frac{2cRZ^2}{n^3}\right)p$ $\quad (\therefore n >> 1, p << n)$

obviously $\upsilon \propto p$, *i.e.*, the values of υ are approximately in the ratio 1 : 2 : 3.

23. In Balmer series, H_γ line corresponds to transition from state $n_i = 5$ to state $n_f = 2$.

Energy required, $E = E_5 - E_1$

$$= \left(\frac{-13.6}{5^2}\right) - \left(\frac{-13.6}{1^2}\right) = 13.06 \text{ eV}.$$

If angular momentum of system is conserved,

change in angular momentum of electron = change in angular momentum of photon

$$= 5\left(\frac{h}{2\pi}\right) - 2\left(\frac{h}{2\pi}\right) = \frac{3h}{2\pi}$$

$$= \frac{3 \times 6.6 \times 10^{-34}}{2 \times 3.14} = 3.17 \times 10^{-34} \text{ J s}$$

24. (i) The kinetic energy (E_k) of the electron in an orbit is equal to negative of its total energy (E).

$E_k = -E = -(-1.5) = 1.5$ eV

(ii) The potential energy (E_p) of the electron in an orbit is equal to twice of its total energy (E).

$E_p = 2E = -1.5 \times 2 = -3.0$ eV

(iii) Here, ground state energy of the H-atom = –13.6 eV

When the electron goes from the excited state to the ground state, energy emitted is given by

$E = -1.5 - (-13.6) = 12.1 \text{ eV} = 12.1 \times 1.6 \times 10^{-19}$ J

Now, $\quad E = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{12.1 \times 1.6 \times 10^{-19}}$$

$\lambda = 1.025 \times 10^{-7} = 1025$ Å

25. Here, $\Delta E = 12.5$ eV

Energy of an electron in n^{th} orbit of hydrogen atom is,

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

In ground state, $n = 1$

$E_1 = -13.6$ eV

Energy of an electron in the excited state after absorbing a photon of 12.5 eV energy will be

$E_n = -13.6 + 12.5 = -1.1$ eV

$$\therefore \quad n^2 = \frac{-13.6}{E_n} = \frac{-13.6}{-1.1} = 12.36 \quad \Rightarrow \quad n = 3.5$$

Here, state of electron cannot be in fraction.

So, $n = 3$ (2nd excited state).

The wavelength λ of the first member of Lyman series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}R$$

$$\Rightarrow \lambda = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7}$$

$\Rightarrow \quad \lambda = 1.215 \times 10^{-7}$ m

$\Rightarrow \quad \lambda = 121 \times 10^{-9}$ m $\Rightarrow \lambda = 121$ nm

The wavelength λ' of the first member of the Balmer series is given by

$$\frac{1}{\lambda'} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}R$$

$$\Rightarrow \lambda' = \frac{36}{5R} = \frac{36}{5 \times (1.097 \times 10^7)}$$

$= 6.56 \times 10^{-7} \text{ m} = 656 \times 10^{-9} \text{ m} = 656$ nm

26. (i) $\because \; E_n = \frac{-13.6}{n^2}$ eV

Energy of the photon emitted during a transition of the electron from the first excited state to its ground state is,

$\Delta E = E_2 - E_1$

$$= \frac{-13.6}{2^2} - \left(\frac{-13.6}{1^2}\right) = \frac{-13.6}{4} + \frac{13.6}{1} = -3.40 + 13.6$$

= 10.2 eV

This transition lies in the region of Lyman series.

(ii) (a) The energy levels of H-atom are given by

$$E_n = -\frac{Rhc}{n^2} = -\frac{13.6}{n^2} \text{ eV}$$

For first excited state $n = 2$

$$E_2 = -\frac{13.6}{(2)^2} \text{ eV} = -3.4 \text{ eV}$$

Kinetic energy of electron in ($n = 2$) state is

$K_2 = -E_2 = +3.4$ eV

(b) Radius in the first excited state

$r_1 = (2)^2\ (0.53)$ Å

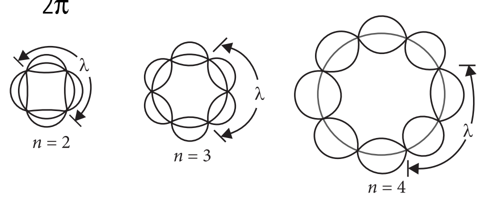
$r_1 = 2.12$ Å

27. (i) de-Broglie hypothesis may be used to derive Bohr's formula by considering the electron to be a wave spread over the entire orbit, rather than as a particle which at any instant is located at a point in its orbit. The stable orbits in an atom are those which are standing waves. Formation of standing waves require that the circumference of the orbit is equal in length to an integral multiple of the wavelength. Thus, if r is the radius of the orbit

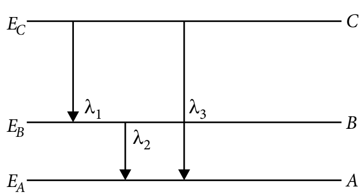
$$2\pi r = n\lambda = \frac{nh}{p}\text{ s} \qquad \left(\because\ \lambda = \frac{h}{p}\right)$$

which gives the angular momentum quantization.

$$L = pr = n\frac{h}{2\pi}$$



(ii)



Clearly, from energy level diagram,

$E_C - E_A = (E_C - E_B) + (E_B - E_A)$

(On the basis of energy of emitted photon).

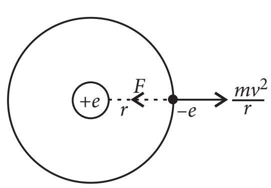
$$\therefore \quad \frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\Rightarrow \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \Rightarrow \lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$$

which is the required relation between the three given wavelengths.

28. Radius of n^{th} orbit of hydrogen atom : In H-atom, an electron having charge $-e$ revolves around the nucleus of charge $+e$ in a circular orbit of radius r, such that necessary centripetal force is provided by the electrostatic force of attraction between the electron and nucleus.

$$i.e., \quad \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e.e}{r^2} \quad \text{or} \quad mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \qquad \text{...(i)}$$



From Bohr's quantization condition

$$mvr = \frac{nh}{2\pi} \quad \text{or} \quad v = \frac{nh}{2\pi mr} \qquad \text{...(ii)}$$

Using equation (ii) in (i), we get

$$m\cdot\left(\frac{nh}{2\pi mr}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \quad \text{or} \quad \frac{m\cdot n^2h^2}{4\pi^2m^2r^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\text{or} \quad r = \frac{n^2h^2\varepsilon_0}{\pi me^2} \qquad \text{...(iii)}$$

where n = 1, 2, 3, ... is principal quantum number.

Equation (iii), gives the radius of n^{th} orbit of H-atom. So the radii of the orbits increase proportionally with n^2 *i.e.*, $[r \propto n^2]$. Radius of first orbit of H-atom is called Bohr radius a_0 and is given by

$$a_0 = \frac{h^2\varepsilon_0}{\pi me^2} \text{ for } n = 1 \text{ or } a_0 = 0.529 \text{ Å}$$

So, radius of n^{th} orbit of H-atom then becomes

$r = n^2 \times 0.529$ Å

29. (i) As the gold foil is very thin, it can be assumed that α-particles will suffer not more than one scattering during their passage through it. Therefore, computation of the trajectory of an α-particle scattered by a single nucleus is enough.

(ii) Trajectory of α-particles depends on impact parameter which is the perpendicular distance of the initial velocity vector of the α particles from the centre of the nucleus. For small impact parameter, α particle close to the nucleus suffers larger scattering.

(iii) In case of head-on-collision, the impact parameter is minimum and the α-particle rebounds back. So, the fact that only a small fraction of the number of incident particles rebound back indicates that the number of α-particles undergoing head-on collision is small. This in turn implies that the mass of the atom is concentrated in a small volume.

30. According to Bohr's formula,

$$E_n = \frac{-me^4}{8\varepsilon_0^2n^2h^2}$$

where m is called reduced mass.

In case of hydrogen, $m = m_e$ = mass of electron.

For positronium,

$$m = \frac{m_e \times m_e}{m_e + m_e} = \frac{m_e}{2}$$

Since for H-atom, $E_1 = \dfrac{m_e e^4}{8\varepsilon_0^2n^2h^2} = -13.6$ eV

So, for positronium $E_1' = \dfrac{-13.6}{2} = -6.8$ eV.

31. Wavelengths corresponding to minimum wavelength (λ_{min}) or maximum energy will emit photoelectrons having maximum kinetic energy.

Wavelengths belonging to Balmer series and lying in the given range (450 nm to 750 nm) corresponds to transition from (n = 4 to n = 2). Here,

$$E_4 = \frac{13.6}{(4)^2} = -0.85 \text{ eV and } E_2 = -\frac{13.6}{(2)^2} = -3.4 \text{ eV}$$

$\therefore \quad \Delta E = E_4 - E_2 = 2.55$ eV

K_{max} = Energy of photon – Work function
= 2.55 – 2.0 = 0.55 eV

32. The force at a distance r is

$$F = -\frac{dU}{dr} = -2ar$$

Suppose r be the radius of n^{th} orbit. The necessary centripetal force is provided by the above force. Thus,

$$\frac{mv^2}{r} = 2ar \qquad \text{...(i)}$$

Further, the quantisation of angular momentum gives,

$$mvr = \frac{nh}{2\pi} \qquad \text{...(ii)}$$

Solving, equations (i) and (ii) for r, we get $r = \left(\frac{n^2h^2}{8am\pi^2}\right)^{1/4}$

33. (i) We have

$$\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2} \quad \text{or} \quad v^2 r = \frac{Ze^2}{4\pi\varepsilon_0 m}$$

The quantisation rule is $vr = \frac{nh}{2\pi m}$

The radius is $r = \frac{(vr)^2}{v^2 r} = \left(\frac{nh}{2\pi m}\right)^2 \frac{4\pi\varepsilon_0 m}{Ze^2} = \frac{n^2h^2\varepsilon_0}{Z\pi me^2}$

For the given system, Z = 3 and m = 208 m_e

Thus, $r_\mu = \frac{n^2h^2\varepsilon_0}{624\pi m_e e^2}$

(ii) The radius of the first Bohr orbit for a hydrogen atom is

$$r_h = \frac{h^2\varepsilon_0}{\pi m_e e^2}$$

For $r_\mu = r_h$, $\frac{n^2h^2\varepsilon_0}{624\pi m_e e^2} = \frac{h^2\varepsilon_0}{\pi m_e e^2}$ or $n^2 = 624$

or $n \simeq 25$

34. (i) $\frac{n(n-1)}{2} = 3 \quad \therefore \quad n = 3$

i.e. after excitation atom jumps to second excited state.

Hence, $n_f = 3$. So n_i can be 1 or 2.

If $n_i = 1$ then energy emitted is either equal to, greater than or less than the energy absorbed. Hence, the emitted wavelength is either equal to, less than or greater than the absorbed wavelength.

Hence, $n_i \neq 1$

If $n_i = 2$, then $E_e \geq E_a$. Hence $\lambda_e \leq \lambda_0$

(ii) $E_3 - E_2 = 68$ eV

$\therefore \quad (13.6)(Z^2)\left(\frac{1}{4} - \frac{1}{9}\right) = 68$

$\therefore \quad Z = 6$

The gas atoms correspond to carbon.

(iii) $\lambda_{min} = \frac{12400}{E_3 - E_1} = \frac{12400}{(13.6)(6)^2\left(1 - \frac{1}{9}\right)}$

$= \frac{12400}{435.2} = 28.49$ Å

35. As the nucleus is massive, recoil momentum of the atom can be ignored. We can assume that the entire energy of transition is transferred to the Auger electron.

As there is a single valence electron in chromium (Z = 24), the energy states may be thought of as given by Bohr model. The energy of the n^{th} state is

$E_n = -\frac{RhcZ^2}{n^2}$ where R is Rydberg constant.

In the transition from n = 2 to n = 1, energy released,

$$\Delta E = -RhcZ^2\left(\frac{1}{4} - 1\right) = \frac{3}{4}RhcZ^2$$

The energy required to eject a n = 4 electron

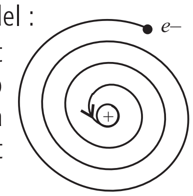
$$= RhcZ^2\left(\frac{1}{4}\right)^2 = \frac{RhcZ^2}{16}$$

$\therefore \quad KE$ of Auger electron $= \frac{3RhcZ^2}{4} - \frac{RhcZ^2}{16}$

$$\text{KE} = RhcZ^2\left(\frac{3}{4} - \frac{1}{16}\right) = \frac{11}{16}RhcZ^2$$

$= \frac{11}{16}(13.6 \text{ eV}) \times 24 \times 24 = 5385.6$ eV $\qquad [\because Rhc = 13.6 \text{ eV}]$

36. (a) (i) Limitation of Rutherford's model : Rutherford's atomic model is inconsistent with classical physics. According to electromagnetic theory, an electron is a charged particle moving in the circular orbit around the nucleus and is accelerated, so it should emit radiation continuously and thereby loose energy. Due to this, radius of the electron would decrease continuously and also the atom should then produce continuous spectrum, and ultimately electron will fall into the nucleus and atom will collapse in 10^{-8} s. But the atom is fairly stable and it emits line spectrum.



(ii) Rutherford's model is not able to explain the spectrum of even most simplest H-spectrum.

(b) Bohr's postulates to resolve observed features of atomic spectrum :

(i) Quantum condition: Of all the possible circular orbits allowed by the classical theory, the electrons are permitted to circulate only in those orbits in which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$, h being Planck's constant. Therefore, for any permitted orbit,

$$L = mvr = \frac{nh}{2\pi},\ n = 1, 2, 3\ldots..,$$

where n is called the principal quantum number, and this equation is called Bohr's quantisation condition.

(ii) Stationary orbits: While revolving in the permissible orbits, an electron does not radiate energy. These non-radiating orbits are called stationary orbits.

(iii) Frequency condition: An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit, respectively.

$$h\upsilon = E_i - E_f$$

where υ is frequency of radiation emitted, E_i and E_f are the energies associated with stationary orbits of principal quantum number n_i and n_f respectively (where $n_i > n_f$).

37. $\because \quad \frac{mv^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2}$ …(i)

and $mvr_n = \frac{nh}{2\pi}$ …(ii)

From eqn. (i) and (ii)

$$\therefore \quad r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2}$$

Total energy

$$E_n = \frac{1}{2}mv_n^2 - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = \frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_n} - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}$$

$$E_n = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_n} = -\frac{1}{8\varepsilon_0^2}\frac{me^4}{h^2 n^2}$$

$E_n = \frac{-Rhc}{n^2}$ where, Rydberg constant $R = \frac{me^4}{8\varepsilon_0^2 h^3 c}$

Energy emitted $\Delta E = E_i - E_f$

$$\Delta E = Rhc\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

But $\Delta E = h\upsilon$

$$\upsilon = Rc\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] \quad \text{or} \quad \upsilon = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

When electron in hydrogen atom jumps from energy state $n_i = 4$ to $n_f = 3, 2, 1$, the Paschen, Balmer and Lyman spectral series are found.

38. Let μ_H and μ_D are the reduced masses of electron for hydrogen and deuterium respectively.

We know that $\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

$\therefore \quad \lambda \propto \frac{1}{R}$ or $\frac{\lambda_D}{\lambda_H} = \frac{R_H}{R_D}$ …(i)

$$R_H = \frac{m_e e^4}{8\,\varepsilon_0 ch^3} = \frac{\mu_H e^4}{8\varepsilon_0 ch^3}$$

$R_D = \frac{m_e e^4}{8\,\varepsilon_0\, ch^3} = \frac{\mu_D\, e^4}{8\,\varepsilon_0 ch^3} \quad \therefore \quad \frac{R_H}{R_D} = \frac{\mu_H}{\mu_D}$ …(ii)

From equation (i) and (ii)

$\frac{\lambda_D}{\lambda_H} = \frac{\mu_H}{\mu_D}$ …(iii)

Reduced mass for hydrogen,

$$\mu_H = \frac{m_e}{1 + m_e / M} \simeq m_e\left(1 - \frac{m_e}{M}\right)$$

Reduced mass for deuterium,

$$\mu_D = \frac{2M \cdot m_e}{2M\left(1 + \frac{m_e}{2M}\right)} \simeq m_e\left(1 - \frac{m_e}{2M}\right)$$

where M is mass of proton.

$$\frac{\mu_H}{\mu_D} = \frac{m_e\left(1 - \frac{m_e}{M}\right)}{m_e\left(1 - \frac{m_e}{2M}\right)} = \left(1 - \frac{m_e}{M}\right)\left(1 - \frac{m_e}{2M}\right)^{-1}$$

$$= \left(1 - \frac{m_e}{M}\right)\left(1 + \frac{m_e}{2M}\right)$$

$$\Rightarrow \frac{\mu_H}{\mu_D} = \left(1 - \frac{m_e}{2M}\right)$$

or $\frac{\mu_H}{\mu_D} = \left(1 - \frac{1}{2 \times 1840}\right) = 0.99973$ …(iv)

($\therefore \quad M = 1840\ m_e$)

From (iii) and (iv)

$\frac{\lambda_D}{\lambda_H} = 0.99973, \quad \lambda_D = 0.99973\ \lambda_H.$

Using λ_H = 1218 Å, 1028 Å, 974.3 Å and 951.4 Å, we get λ_D = 1217.7 Å, 1027.7 Å, 974.04 Å, 951.1 Å

Shift in wavelength $(\lambda_H - \lambda_D) \approx 0.3$ Å.

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