Constrained Motion

1. CONSTRAINED MOTION :

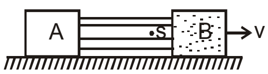
1.1 String constraint :

When the two object are connected through a string and if the string have the following properties :

- The length of the string remains constant i.e., it is inextensible string
- Always remains taut i.e., does not slacks.

Then the parameters of the motion of the objects along the length of the string have a definite relation between them.

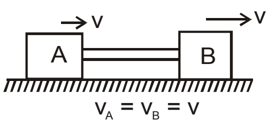
Ist format : - (when string is fixed)



The block B moves with velocity v. i.e. each particle of block B moves with velocity v.

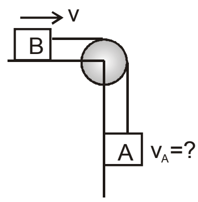
If string remain attached to block B it is necessary that velocity of each particle of string is same = v ($v_s = v$)

Now we can say that Block A also moves with velocity v.



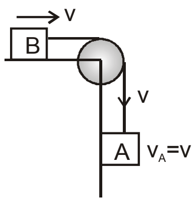
: If pulley is fixed then the velocity of all the particles of string is same along the string.

Ex.1

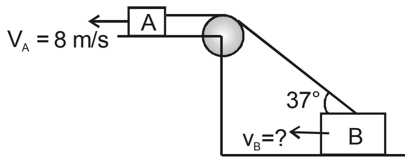


Sol. In the above situation block B is moving with velocity v. Then speed of each point of the string is v along the string.

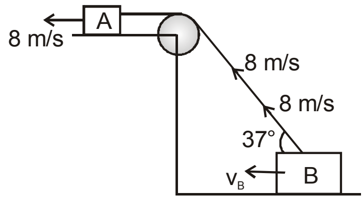
$\therefore$ speed of the block A is also v



Ex.2



Sol.



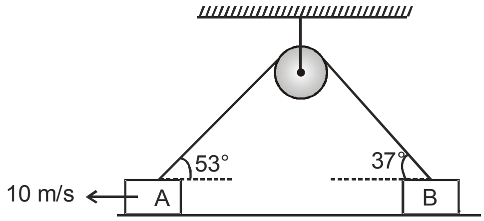
$\because$ Block A is moving with velocity 8 ms^{-1}.

$\therefore$ velocity of every point on the string must be 8m/s along the string.

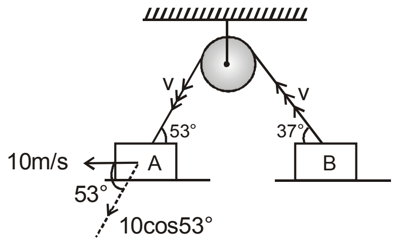
The real velocity of B is v_B. Then the string will not break only when the compoent of v_B along string is 8 m/s.

$$\Rightarrow \; v_B \cos 37° = 8 \;\Rightarrow\; v_B = \frac{8}{\cos 37°} = 10 \text{ m/sec}$$

Ex.3 ***Find out the velocity of block B in a pulley block system as shown in figure.***



Sol. In a given pulley block system the velocity of all the particle of string is let us assume v then.

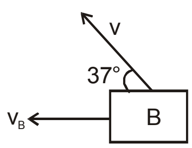


10 m/s is the real velocity of block A then its component along string is v.

$\Rightarrow$ $10 \cos 53° = v$ ...(1)

If v_B is the real velocity of block B then it component along string is v then

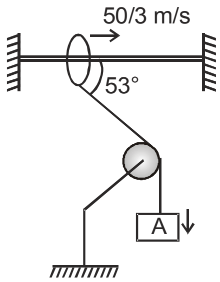
$v_B\cos37° = v$ ...(2)



from (1) & (2) $v_B \cos37° = 10 \cos53°$

$$\Rightarrow \quad v_B = \frac{10 \times 3/5}{4/5} = \frac{30}{4} = \frac{15}{2} \text{m/sec}$$

Ex.4



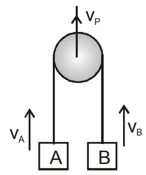
What is the velocity of block A in the figure as shown above.

Sol. The component of velocity of ring along string = velocity of A

$$= \frac{50}{3}\cos 53° = v_A \;\Rightarrow\; v_A = 10 \text{ m/s}$$

: In the first format only two points of string are attached or touched to moving bodies.

II[nd] **format (when pulley is also moving)**



To understand this format we consider the following example in which pulley is moving with velocity v_p and both block have velocity v_A & v_B respectively as shwon in figure.

If we observe the motion of A and B with respect to pulley. Then the pulley is at rest. Then from first format.

$$v_{AP} = -\, v_{BP}$$

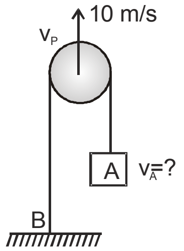
(–ve sign indicate the direction of each block is opposite with respect to Pulley)

$$v_A - v_p = -\, v_B + v_P$$

$$\Rightarrow \; v_P = \frac{v_A + v_B}{2}$$

: - To solve the problem put the values of v_A, v_B, & v_P with sign.

Ex.5



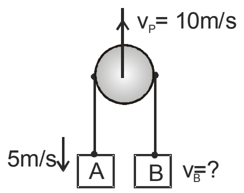
Sol. $v_P = \frac{v_A + v_B}{2}$

Putting $v_p = 10\ ms^{-1}$, $v_B = 0$,

we get

$v_A = 20\ ms^{-1}$ (upward direction)

Ex.6

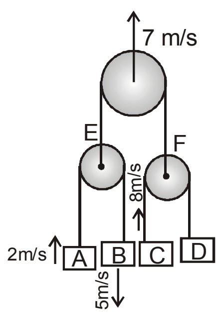


Sol. If we take upward direction as +ve then

$10 = \frac{-5 + v_B}{2}$

$v_B = 25$ m/sec (in upward direction)

Ex.7



Find out the velocity of Block D

Sol. From 2nd format of constrained motion

$v_E = \frac{v_A + v_B}{2}$

$v_E = \frac{2 - 5}{2} = -3/2$

(If upward direction is taken to be +ve)

$v_E = -3/2$ m/s

Now $\frac{v_E + v_F}{2} = 7$ m/s $\Rightarrow$ $7 = \frac{-3/2 + v_F}{2}$

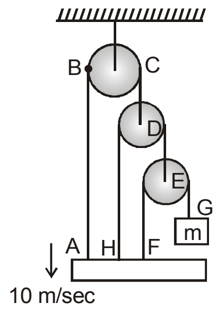
$\Rightarrow$ $14 + 3/2 = v_F \Rightarrow v_F = \frac{31}{2}$

Now $\frac{v_C + v_D}{2} = v_F$ $\Rightarrow$ $\frac{8 + v_D}{2} = \frac{31}{2} \Rightarrow v_D$

$= 31 - 8$

$v_D = 23$ m/s (upward direction)

Ex.8



Find the velocity of point G.

Sol. In string ABCD from first format of constrain

$V_D = 10$ m/s↑

Now $v_D = \frac{v_H + v_E}{2}$

$v_H = 10$ m/s ↓ if upward direction is taken to be positive

$+ 10 = \frac{-10 + v_E}{2}$ $\Rightarrow$ $v_E = 30$ m/s ↑

Now $\frac{v_F + v_G}{2} = v_E$ $\Rightarrow$ $30 = \frac{-10 + v_G}{2}$

$60 + 10 = v_G$

$v_G = 70$ m/s↑

: In IInd format three or four Points of the string is attached to the moving bodies.

III format :

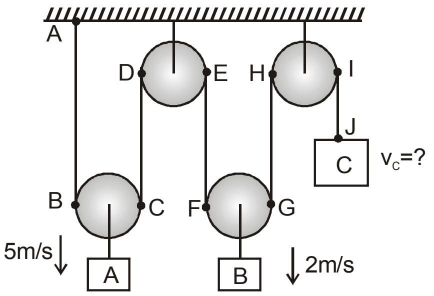
SOLVING STRATEGY :

1. First choose the longest string in the given problem which contains the point of which velocity/ acceleration to be find out.

2. Now mark a point on the string wherever it comes in contact or leaves the contact of real bodies.

3. If due to motion of a point, length of the part of a string with point is related, increases then its speed will be taken +ve otherwise –ve.

Ex.9



Sol.

Step 1. We choose a longest string ABCDEFGHIJ in which we have to find out velocity of point J (v_c)

Step 2. Mark all the point A, B ....

Step 3. Write equation

$v_A + v_B + v_C + v_D + v_E + v_F + v_{G+}v_H + v_I + v_J = 0$

$v_A = v_D = v_E = v_H = v_I = 0$

(No movement of that point because attached to fixed objects)

$\Rightarrow v_B + v_c + v_F + v_G + v_J = 0$ ...(1)

$v_B = v_C = 5$ m/s (increases the length)

$v_F = v_G = 2$m/s

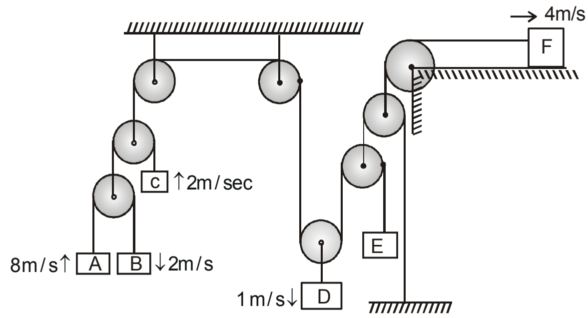
(It also increases the length)

Let us assume C is moving upward with velocity v_c so v_c negative because it decreasing the length

$\Rightarrow 5 + 5 + 2 + 2 - v_c = 0$

$v_C = 14$ m/sec (upward)

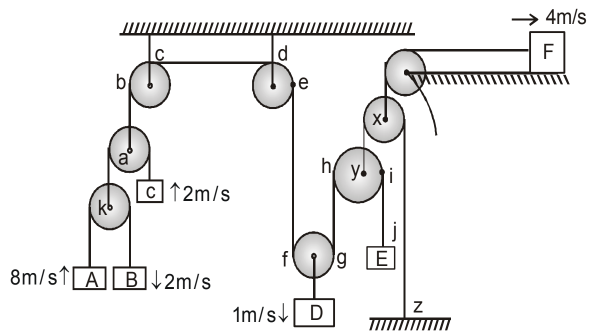
Ex.10



Find out the velocity of block E as shown in figure.

Sol.

Step-1 We first choose the longest string in which point j (block E) lie. (abcdefghij)



Step 2 : Now write equation according to the velocity of each point (either increase or decrease the length)

$v_a + v_b + v_c + v_d + v_e + v_f + v_g + v_h + v_i + v_j = 0$ ...(1)

Now find value of v_a, v_b ..... in a following way

$v_k = \dfrac{v_A + v_B}{2}$ (from second format)

$= \dfrac{8-2}{2} = 3$ m/sec. (upward)

$v_a = \dfrac{v_K + v_C}{2}$ (from 2nd format)

$= \frac{3+2}{2}$ = 5/2 m/sec. (upward)

v_x = 4m/s (from first format of constrain)

from 2nd format of constrain $v_x = \frac{v_y + v_z}{2}$

$\therefore v_z = 0$ (fixed)

$\Rightarrow v_y = 2\, v_x = 8$ m/s (upward)

$\Rightarrow$ Now $v_a = -\,5/2$ m/s (decreases the length)

$v_b = v_c = v_d = v_e = 0$ (attached to fixed object)

$v_f = v_g =$ 1m/s (increases the length)

$v_h = v_i = v_y = 8$ m/s (increase the length)

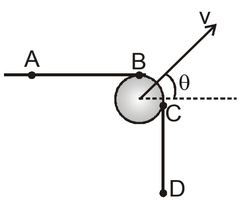
Let us assume block E move upward then $v_j = -\,v_E$ (decrease the length)

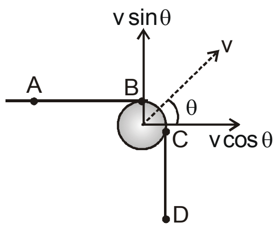
Puting the above values in eq. (1)

$\Rightarrow -5/2 + 1 + 1 + 8 + 8 - v_E = 0$

$v_E = 31/2$ m/s (upward)

: In the following figure pulley is moving with velocity v at an angle θ with the horizontal.

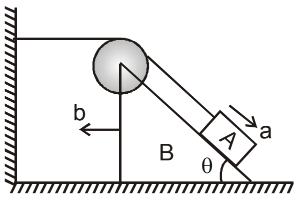




Only v cos θ is responsible to increase or decrease the length AB and v sin θ is responsible to either decrease or increase the length CD.

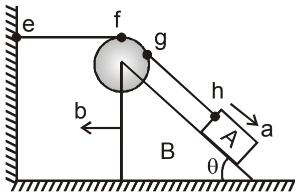
* Further solving strategy is same as 3rd format

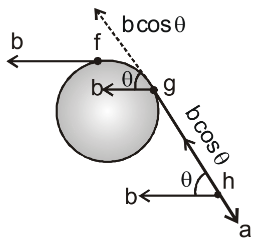
Ex.11 ***Find out the relation between acceleration a and b as shown in following figure.***



Sol.

Step 1. Mark the points on the string which is attached to the real object (e.f,g,h)





Step 2. Acceleration of each point which are responsible to effect the length of string

$a_e = 0$ (because it is attached to fixed object)

$a_f = -b$ (attach to pulley which is moving with wedge's acceleration & –ve because it decreases the length)

$a_g = b \cos \theta$ (only this component is responsible to effect the length of string)

$a_h = (a - b \cos \theta)$ (resultant velocity at point h along the string)

So now from 3rd format

$a_e + a_f + a_g + a_h = 0$

$\Rightarrow \quad 0 + (-b) + b \cos \theta + (a - b \cos \theta) = 0$

$a - b = 0$

$\Rightarrow \quad a = b$

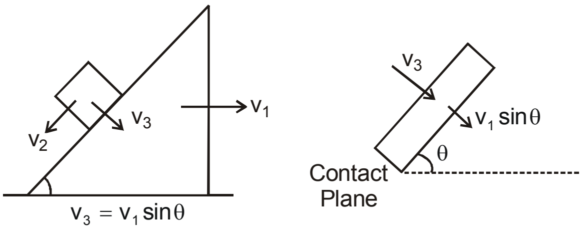
2. WEDGE CONSTRAINT :

Conditions :

(i) Contact must not be lost between two bodies.

(ii) Bodies are rigid.

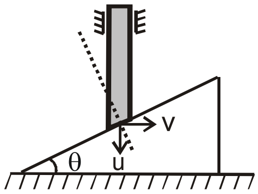
The relative velocity / acceleration perpendicular to the contact surface of the two rigid object is always zero. Wedge constraint is applicable for each contact.



In other words,

Components of velocity and acceleration perpendicular to the contact surface of the two objects is always equal if there is no deformation and they remain in contact.

Ex.12 Find the relation between velocity of rod and that of the wedge at any instant in the figure shown.



Sol. Using wedge constraint.

Component of velocity of rod along perpendicular to inclined surface is equal to velocity of wedge along that direction.

$u \cos\theta = v \sin\theta$

$$\frac{u}{v} = \tan\theta$$

$u = v \tan\theta$