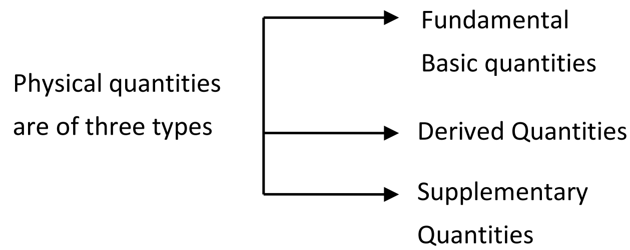
Unit, Dimension and Measurement

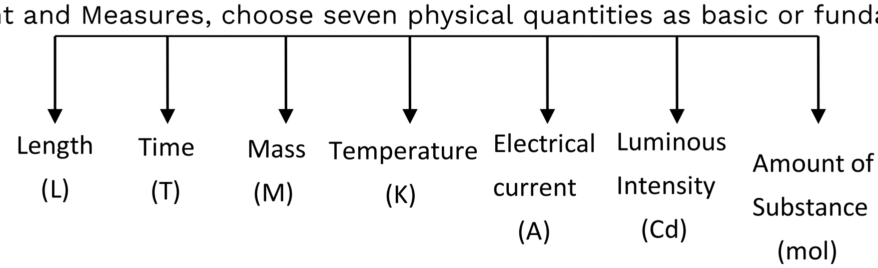
1. Physical Quantities

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called **physical quantities**.
e.g. length, velocity, acceleration, force, time, pressure, mass, density etc.



1.1 Fundamental Quantities

- These are the elementary quantities which covers the entire span of physics.
- Any other quantities can be derived from these.
- All the basic quantities are chosen such that they should be different, that means independent of each other. (i.e., distance, time and velocity cannot be chosen as basic quantities as $V = \frac{d}{t}$). An International Organization named CGPM : General Conference on weight and Measures, choose seven physical quantities as basic or fundamental.



Example 1:

Which of the following sets cannot enter into the list of fundamental quantities in any newly proposed system of units?

(A) length, mass and velocity
(B) pressure, density and velocity
(C) force, velocity and time
(D) force, momentum and time

Ans. **(B, D)**

Solution:

For (A): Length [L], mass [M] and velocity $[LT^{-1}]$ are independent.

For (B): Pressure $[M^1L^{-1}T^{-2}]$, density $[M^1L^{-3}T^0]$ and velocity $[M^0LT^{-1}]$ are dependent

For (C): Force $[MLT^{-2}]$, velocity $[LT^{-1}]$ and time [T] are independent

For (D): Force = $\frac{\text{Momentum}}{\text{Time}}$

$\Rightarrow$ force, momentum and time are dependent.

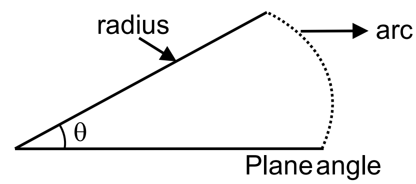
1.2 Derived Quantities

Physical quantities which can be expressed in terms of basic quantities (M,L,T....) are called derived quantities.
i.e., Momentum $\quad P = mV$

1.3 Supplementary Quantities

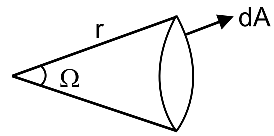
Besides seven fundamental quantities two supplementary quantities are also defined. They are

- **Plane angle** (The angle between two lines)
  Plane angle is the radius measurement of rotation of line along a plane.

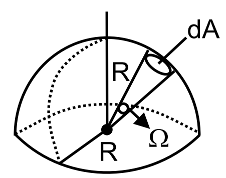


  Plane angle $\theta = \dfrac{\text{arc length}}{\text{radius}} = \dfrac{s}{r}$

- **Solid Angle**
  It is the 3D analogue of an angle. Such as that subtended by cone or formed by planes meeting at a point



  Solid angle $\Omega = \dfrac{dA}{r^2}$



2. The International System of Units

In earlier time scientists of different countries were using different system of units for measurement. There such systems, the CGS, the FPS (or British) system and the MKS system were in use extensively till recently.

The base units for length, mass and time in these systems were as follows :

- In CGS system they were centimetre, gram and second respectively.
- In FPs system they were foot, pound and second respectively.
- IN MKS system they were metre, kilogram and second respectively.

The system of units which is at present internationally accepted for measurement is the system international d' Unites (French for International system of Units), units and abbreviations, was developed and recommended by General Conference on Weights and Measures in 1971 for international usage in scientific, technical, industrial and commercial work. Because SI units used decimal system, conversions within the system are quite simple and convenient.

3. SI Base Quantities and Units

Length (meter): The meter is the length of the path traveled by light in vacuum during a time interval of 1/299,792,458 of a second (1983). It is denoted by m.

Mass (kilogram): The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1889). It is denoted by kg.

Time (second): The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967). It is denoted by s.

Electric Current (ampere): The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2 × 10^{-7} Newton per metre of length. (1948). It is denoted by A.

Thermodynamic Temperature (kelvin): The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967). It is denoted by K.

Amount of Substance (mole): The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971). It is denoted by mol.

Luminous Intensity (candela): The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 × 10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian (1979). It is denoted by Cd.

- **SI units of derived Quantities**
  - Velocity = $\frac{\text{displacement} \rightarrow \text{meter}}{\text{time} \rightarrow \text{second}}$

    So unit of velocity will be m/s
  - Acceleration = $\frac{\text{change in velocity}}{\text{time}} = \frac{m/s}{s} = \frac{m}{s^2}$
  - Momentum = mV

    so unit of momentum will be

    = (kg) (m/s) = kg m/s
  - Force = ma

    Unit will be = (kg) × (m/s^2) = kg m/s^2 called newton (N)
  - Work = FS

    unit = (N) × (m) = N m called joule (J)
  - Power = $\frac{\text{work}}{\text{time}}$

    Unit = J / s called watt (W)

4. Supplementary Units, SI Prefixes

- **Two supplementary units were also defined**
  - Plane angle – Unit = radian (rad)
  - Solid angle – Unit = Steradian (sr)

Power of 10	prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deca	da

Power of 10	prefix	Symbol
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

5. Principal of Measurement, Numerical Value and Unit

The result of measurement of a physical quantity is expressed by a number 'n' (numerical measure) accompanied by a unit 'u'.

Measurement = nu

Example 2:

Convert all in meters (m) :

(i) 5 μm (ii) 3 km (iii) 20 mm (iv) 73 pm (v) 7.5 nm

Solution:

(i) 5 μm = 5 × 10^{-6}m (ii) 3 km = 3 × 10^{3} m (iii) 20 mm = 20 × 10^{-3}m

(iv) 73 pm = 73 ×10^{-12} m (v) 7.5 nm = 7.5 × 10^{-9} m

Example 3:

F = 5 N convert it into CGS system

Solution:

$$F = 5\ \frac{kg \times m}{s^2} = (5)\ \frac{(10^3 g)(100cm)}{s^2} = 5 \times 10^5\ \frac{g\,cm}{s^2}\text{ (in CGS system).}$$

This unit ($\frac{g\,cm}{s^2}$) is also called dyne

Example 4:

$G = 6.67 \times 10^{-11}\ \frac{kg^{-1}\ m^3}{s^2}$ convert it into CGS system.

Solution:

$$G = 6.67 \times 10^{-11}\ \frac{kg^{-1}\ m^3}{s^2}$$

$$= (6.67\times10^{-11})\left(\frac{1}{1000\,g}\right)\ \frac{(100cm)^3}{s^2}$$

$$= 6.67 \times 10^{-11} \times \frac{10^{+6}}{10^3} = 6.67\times10^{-8}\ g^{-1}\frac{cm^3}{s^2}$$

Concept Builder-1

Q.1 Can Pressure (P), density (ρ) and velocity (v) be taken as fundamental quantities ?

Q.2 A cyclist moves 20 m on a curved path. By what angle the position vector of cyclist will rotate if centre (origin) is assumed to be 0.2 km away.

Q.3 Convert 7 pm into μm

Q.4 v = 90 km / hour convert it into m/s

Q.5 If unit of length is doubled, the numerical value of Area will be ....

Q.6 Force acting on a particle is 5N. If unit of length and time are doubled and unit of mass is halved than the numerical value of the force in the new unit will be.... .

6. Dimensions

The nature of a physical quantity is described by its dimensions. All the physical quantities represented by derived units and be expressed in terms of some combination of seven fundamental or base quantities. We shall call these base quantities as the seven dimensions of the physical world, which are denoted with square brackets [ ]. Thus, length has the dimension [L], mass [M], time [T], electric current [A], thermodynamic temperature [K], luminous intensity [cd], and amount of substance [mol]. The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.

7. Dimensional Formula, Dimensional Equation & Characteristics

An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity. Thus, the dimensional equations are the equations, which represent the dimensions of a physical quantity in terms of the base quantities. For example, the dimensional equations of volume [V], speed [v], force [F] and mass density [ρ] may be expressed as

$[V] = [M^0L^3T^0]$ $\quad$ $[v] = [M^0LT^{-1}]$

$[F] = [M\ L\ T^{-2}]$ $\quad$ $[\rho] = [M\ L^{-3}T^0]$

- Pure numbers are dimensionless.
- All trigonometric ratios, powers, exponential and logarithmic functions are dimensionless.
- All ratio of physical quantities having same dimensional formula are dimensionless. e.g. relative density, relative permeability, dielectric constant, angles, refractive index etc.
- Dimensions do not depend upon magnitude.
- Dimension of a physical quantity will be same, it doesn't depend on which formula we are using for that quantity.

Principle of Homogeneity of dimensions : It states that in a correct equation, the dimensions of each term added or subtracted must be same. Every correct equation must have same dimensions on both sides of the equation.

Example 5:

Find out the dimensional formula of the following

(i) Density (ii) Velocity (iii) Acceleration (iv) Momentum

(v) Angle (vi) Torque

Solution:

(i) $[\text{Density}] = \dfrac{[\text{mass}]}{[\text{volume}]} = \dfrac{M}{L^3} = [M^1L^{-3}]$

(ii) Velocity $[v] = \dfrac{[\text{Displacement}]}{[\text{time}]} = \dfrac{L}{T} = [M^0L^1T^{-1}]$

(iii) Acceleration $(a) = \dfrac{dv}{dt} = \dfrac{LT^{-1}}{T} = LT^{-2}$

(iv) Momentum $(P) = mV = [M^1L^1T^{-1}]$

(v) Angle $(\theta) = \dfrac{[\text{Arc}]}{[\text{radius}]} = \dfrac{L}{L} = [M^0L^0T^0]$ (Dimensionless)

(vi) Torque = Force × Arm length = $[M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$

Example 6:

To increase the temperature of a body by ΔT, Heat required is $Q = ms\,\Delta T$

Here 's' is specific heat capacity. Then find out the dimensional formula of 's'

Solution:

$[Q] = [m]\,[s]\,[\Delta T]$

Here Q is heat : A kind of energy so

$[Q] = M^1L^2T^{-2}$

$[M^1L^2T^{-2}] = [M]\,[S]\,[K]$

$[S] = [M^0L^2T^{-2}K^{-1}]$

Example 7:

If any spherical ball of radius r moves with velocity v in a viscous liquid, then viscous force acting on it is given by

$F_v = 6\pi\eta rv$

Here η is coefficient of viscosity. Find out the dimensional formula of 'η'

Solution:

$[F_v] = [6\pi]\,[\eta]\,[r]\,[v]$

$M^1L^1T^{-2} = (1)\,[\eta]\,[L]\,[LT^{-1}]$

$[\eta] = M^1L^{-1}T^{-1}$

Example 8:

If $\dfrac{\alpha}{t^2} = Fv + \dfrac{\beta}{x^2}$

Find dimension formula for $[\alpha]$ and $[\beta]$

(here t = time, F = force, v = velocity, x = distance)

Solution:

Since $[Fv] = M^1L^2T^{-3}$, so $\left[\dfrac{\beta}{x^2}\right]$ should also be $M^1L^2T^{-3}$

$\dfrac{[\beta]}{[x^2]} = [M^1 L^2T^{-3}]$ $\qquad [\beta] = [M^1L^4T^{-3}]$

and $\left[Fv + \dfrac{\beta}{x^2}\right]$ will also have dimension $[M^1L^2T^{-3}]$

so $\dfrac{[\alpha]}{[t^2]} = [M^1L^2T^{-3}] \Rightarrow [\alpha] = [M^1L^2T^{-1}]$

Example 9:

$\alpha = \dfrac{FV^2}{\beta^2}\log_e\left(\dfrac{2\pi\beta}{V^2}\right)$ where F = force, V = velocity. Find the dimensions of α and β.

Solution:

$$\alpha = \frac{FV^2}{\beta^2}\log_e\frac{2\pi\beta}{V^2}$$

dimensionless

$\dfrac{[2\pi][\beta]}{[V^2]} = M^0L^0T^0$

$\Rightarrow \dfrac{[1][\beta]}{L^2T^{-2}} = M^0L^0T^0 \Rightarrow [\beta] = L^2T^{-2}$

$[\alpha] = \dfrac{[F][V^2]}{[\beta^2]} \Rightarrow [\alpha] = \dfrac{[M^1L^1T^{-2}][L^2T^{-2}]}{[L^2T^{-2}]^2}$

$\Rightarrow [\alpha] = M^1L^{-1}T^0$

Example 10:

The dimensional formula of product and quotient of two physical quantities A and B are given by $[AB] = [ML^2T^{-2}]$; $\left[\dfrac{A}{B}\right] = [MT^{-2}]$. The quantities A and B respectively are

(A) Force and velocity
(B) Force and displacement
(C) Momentum and displacement
(D) Work and velocity

Ans. **(B)**

Solution:

$[A]^2 = [AB].\left[\dfrac{A}{B}\right] = [M^2L^2T^{-4}]$

$\Rightarrow [A] = [MLT^{-2}] \equiv$ Force

$[B] = \dfrac{[AB]}{[A]} = [L] \Rightarrow [B] = [L] \equiv$ Displacement

Example 11:

The Vanderwall's equation for n moles of a real gas is given by $\left(P+\frac{n^2a}{V^2}\right)$ (V – nb) = nRT, where

P = pressure of gas, V = volume of gas, T = temperature of gas, R = molar gas constant,
a & b = Vander wall's constants
Which of the following have the same dimensions as those of nRT.

(A) PV (B) $\frac{aV}{b^2}$ (C) $\frac{PV^2}{nb}$ (D) $\frac{na}{b}$

Ans. **(A, B, C, D)**

Solution:

Here $[P] = \left[\frac{n^2a}{V^2}\right]$, [V] = [nb]

so [PV] = [nRT]

Also $\left[\frac{aV}{b^2}\right] = \left[\left(\frac{PV^2}{n^2}\right)\frac{V}{(V/n)^2}\right]$ = [PV] = [nRT]

$\Rightarrow \left[\frac{PV^2}{nb}\right] = n^2a$

Concept Builder-2

Q.1 Find out dimensional formula of
(i) Force (ii) Work (iii) Energy (iv) Power
(v) Pressure (vi) Angular velocity (vii) Angular acceleration

Q.2 If two bodies of mass m_1 and m_2 are placed at r distance, both feel gravitational attraction force, whose value is, Gravitational force $F_g = \frac{Gm_1m_2}{r^2}$ where G is a constant called Gravitational constant. Find out dimensional formula of 'G'

Q.3 If light of frequency υ is falling on a surface & the energy of a photon is given by
$E = h\upsilon$ Here h = Planck's constant
then find out the dimensional formula of 'h'

Q.4 If $\alpha = \frac{F}{V^2}\sin(\beta t)$ (here V = velocity, F = force, t = time). Find the dimension of α and β

Q.5 If the dimensions of a physical quantity are given by $M^xL^yT^z$, then physical quantity may be
(A) acceleration due to gravity, if x =0, y=1, z=–2
(B) atmospheric pressure, if x=1, y=1, z=–2
(C) linear momentum, if x=1, y=1, z=–1
(D) potential energy, if x=1, y=2, z=–2

8. Application of Dimensions

8.1 To Check the Correctness of the Formula

If the dimensions of the L.H.S and R.H.S are same, then we can say that this eqn. is atleast dimensionally correct. So this equation may be correct.

But if dimensions of L.H.S and R.H.S is not same then the equation is not even dimensionally correct. So it cannot be correct.

Formula for centrifugal force F_e is given by

$$F_e = \frac{mv^2}{r}$$

(where m = mass, v = velocity, r = radius)

we have to check whether it is correct or not.

Dimension of L.H.S is

$[F] = [M^1L^1T^{-2}]$

Dimension of R.H.S is

$$\frac{[m][v^2]}{[r]} = \frac{[M][LT^{-1}]^2}{[L]} = [M^1L^1T^{-2}]$$

So this eqn. is at least dimensionally correct.

$\Rightarrow$ we can say that this equation may be correct.

Example 12:

Check whether this equation may be correct or not Pressure $P_r = \frac{3FV^2}{\pi^2 t^2 x}$ (where F = force, V = velocity, t = time, x = distance)

Solution:

Dimension of L.H.S $= [P_r] = M^1L^{-1}T^{-2}$

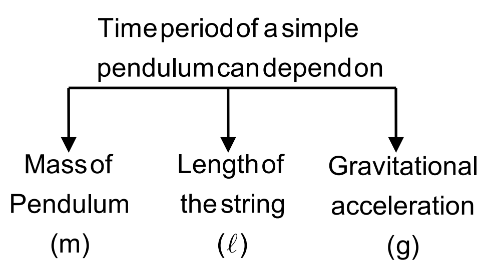
Dimension of R.H.S = $\frac{[3][F][v^2]}{[\pi][t^2][x]} = \frac{[M^1L^1T^{-2}][L^2T^{-2}]}{[T^2][L]} = M^1L^2T^{-6}$

Dimension of L.H.S and R.H.S are not same. So the relation cannot be correct.

8.2 Designing an Expression of Relationship Between Variables

If a quantity depends on many parameters, we can estimate, to what extent, the quantity depends on the given parameters.

Example :



So we can say that expression of T should be in this form

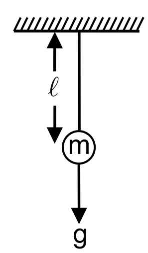
$T \propto (m)^a(\ell)^b(g)^c$

$M^0L^0T^1 = (1)\,[M^1]^a\,[L^1]^b\,[L^1T^{-2}]^c$

$M^0L^0T^1 = M^a\,L^{b+c}\,T^{-2c}$

Comparing the powers of M, L and T,

get a = 0, b + c = 0, – 2c = 1



so a = 0, b = $\frac{1}{2}$, c = $-\frac{1}{2}$

so $T \propto M^0 L^{1/2} g^{-1/2}$

$T = (\text{Constant}) \sqrt{\frac{\ell}{g}}$

The "Constant" can be found experimentally.

Example 13:

If velocity (V), force (F) and time (T) are chosen as fundamental quantities, express (i) mass and (ii) energy in terms of V,F and T.

Solution:

Let $M \propto (V)^a (F)^b (T)^c$

Equating dimensions of both the sides

$M^1L^0T^0 = (1)\,[L^1T^{-1}]^a\,[M^1L^1T^{-2}]^b\,[T^1]^c$

$M^1L^0T^0 = M^b\,L^{a+b}\,T^{-a-2b+c}$

get a = – 1, b = 1, c = 1

$M = (V^{-1} F^1 T^1) \Rightarrow [M] = [V^{-1} F^1 T^1]$

Similarly we can also express energy in terms of V, F, T

Let $[E] \propto [V]^a\,[F]^b\,[T]^c$

$\Rightarrow [ML^2T^{-2}] = [LT^{-1}]^a\,[MLT^{-2}]^b\,[T]^c$

$\Rightarrow [M^1L^2T^{-2}] = [M^b\,L^{a+b}\,T^{-a-2b+c}]$

$\Rightarrow b = 1;\ a + b = 2;\ -a-2b + c = -2$

a = 1 c = 1

$E \propto V^1\,F^1\,T^1$

8.3 Conversion of Units

$n[u]$ = constant i.e., $n_1[u_1] = n_2[u_2]$ where n is the numerical value and u is the unit.

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

where the dimensional formula of the physical quantity is $[M^aL^bT^c]$.

Example 14:

The density of a material in CGS system is 2g /cm^3. In a system of units in which unit of length is 2 cm and unit of mass is 4 g, what is the numerical value of the density of the material?

Solution:

$n_1u_1 = n_2u_2$

$$\Rightarrow n_2 = n_1\left[\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^{-3}\right] = 2\left[\left(\frac{1g}{4g}\right)\left(\frac{1cm}{2cm}\right)^{-3}\right] = 4$$

Concept Builder-3

Q.1 If A and B are two physical quantities having different dimensions then which of the following can denote a new physical quantity?

(A) $A + \frac{A^3}{B}$ (B) $\exp\left(-\frac{A}{B}\right)$ (C) AB^2 (D) $\frac{A}{B^4}$

Q.2 "A particle of mass m is located in a region where its potential energy [U(x)] depends on the position x as Potential Energy $[U(x)] = \frac{a}{x^2} - \frac{b}{x}$ here a & b are positive constants"

(i) Write dimensional formula of a & b.

(ii) If the time period of oscillation which is calculated from above formula is stated by a student as $T = 4\pi a \sqrt{\frac{ma}{b^2}}$, check whether this answer is dimensionally correct.

Q.3 Natural frequency (f) of a closed pipe depends upon

(i) length of tube ℓ (ii) density of air ρ (iii) pressure of air P

Then find out the relation of f with all these variables

Q.4 If the momentum is P and mass M are chosen as fundamental quantities, then derive K.E. in terms of P and M.

Q.5 $\rho = 2\ g/cm^3$ convert it into MKS system

9. Limitations of Dimensional Analysis

- Dimensional analysis doesn't give information about the "constant" (The dimensional constant).
- This method is useful only when a physical quantity depends on other quantities by multiplication and power relations.
  (i.e., $f = x^a y^b z^c$)
  It fails if a physical quantity depends on sum or difference of two quantities
  (i.e.$f = x + y - z$)
  i.e., we cannot get the relation
  $S = ut + \frac{1}{2}at^2$ from dimensional analysis.
- This method will not work if a quantity depends on another quantity as sine or cosine, logarithmic or exponential relation. The method works only if the dependence is by power functions.
- We equate the powers of M, L and T hence we get only three equations. So we can have only three variable (only three dependent quantities)
  So dimensional analysis will work only if the quantity depends only on three parameters, not more than that.

10. Measurement

(i) Measurement of Length

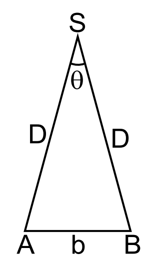
You are already familiar with some direct methods for the measurement of length. For example, a metre scale is used for lengths from 10^{-3} m to 10^{2} m. A Vernier calipers is used for lengths to an accuracy of 10^{-4} m. A screw gauge and a spherometer can be used to measure lengths as less as to 10^{-5} m. To measure lengths beyond these ranges, we make use of some special indirect methods.

(ii) Measurement of Large Distances

Large distances such as the distance of a planet or a star from the earth cannot be measured directly with a metre scale. An important method in such cases is the parallax method.

When you hold a pencil in front of you against some specific point on the background (a wall) and look at the pencil first through your left eye A (closing the right eye) and then look at the pencil through your right eye B (closing the left eye), you would notice that the position of the pencil seems to change with respect to the point on the wall. This is called parallax. The distance between the two points of observation is called the basis. In this example, the basis is the distance between the eyes.

To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions (observatories) A and B on the earth, separated by distance AB = b at the same time as shown in figure. We measure the angle between the two directions along which the planet is viewed at these two points. The $\angle ASB$ is represented by symbol θ is called the parallax angle or parallactic angle.



As the planet is very far away, $\frac{b}{D} << 1$, and therefore, θ is very small. Then we approximately take AB as an arc of length b of a circle with centre at S and the distance D as the radius AS = BS so that AB = b = D θ where θ is in radians.

$$D = \frac{b}{\theta}$$

11. Range of Lengths

The sizes of the objects we come across in the universe vary over a very wide range. These may vary from the size of the order of 10^{-14} m of the tiny nucleus of an atom to the size of the order of 10^{26} m of the extent of the observable universe.

12. Accuracy, Precision of Instrument

- **Accuracy**

  The closeness of the measured value to the true value of the physical quantity is known as the accuracy of the measurement.

- **Precision**

  It is the measure of the extent to which successive measurements of a physical quantity differ from one another.

The accuracy in measurement may depend on several factors, including the limit of the resolution of the measuring instrument. For example, suppose the true value of a certain length is near 3.678 cm. In one experiment, using a measuring instrument of resolution 0.1 cm, the measured value is found to be 3.5 cm, while in another experiment using a measuring device of greater resolution, say 0.01 cm, the length is determined to be 3.88 cm. The first measurement has more accuracy (because it is closer to the true value) but less precision (its resolution is only 0.1 cm), while the second measurement is less accurate but more precise.

13. Errors in Measurement

13.1 Systematic Error

The systematic errors are those errors that tend to be in one direction, either positive or negative. Some of the sources of systematic errors are:

(i) Instrumental Error

Instrumental error that arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument, etc.

(ii) Imperfection in Experimental Technique

Systematic errors can be minimised by improving experimental techniques, selecting better instrumental and removing personal bias as far as possible. For a given set-up, these errors may be estimated to a certain extent and the necessary corrections may be applied to the readings.

(iii) Personal Errors

Personal errors that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc.

13.2 Random Errors

The random errors are those errors, which occur irregularly and hence are random with respect to sign and size. These can arise due to random and unpredictable fluctuations in experimental conditions.

13.3 Least Count Error

The smallest value that can be measured by the measuring instrument is called its least count. All the readings or measured values are good only up to this value.

The least count error is the error associated with the resolution of the instrument.

13.4 Permissible Error

Error in measurement due to the limitation (least count) of the instrument, is called permissible error.

From mm scale → we can measure upto 1 mm accuracy (least count). From this we will get measurement like ℓ = 34 mm

Max uncertainty can be 1 mm.

Max permissible error ($\Delta\ell$) = 1 mm.

But if from any other instrument, we get ℓ = 34.5 mm then maximum permissible error ($\Delta\ell$) = 0.1 mm and if from a more accurate instrument, we get ℓ = 34.527 mm then maximum permissible error ($\Delta\ell$) = 0.001 mm = place value of last number

Maximum permissible error in a measured quantity is equal to the place value of the last number.

13.5 Maximum Permissible Error in Result Due to Error in Each Measurable Quantity

Let Result f(x, y) contains two measurable quantity x and y

Let error in x is $\pm \Delta x$ i.e. $x \in (x - \Delta x, x + \Delta x)$

error in y is $\pm \Delta y$ i.e. $y \in (y - \Delta y, y + \Delta y)$

Case : (I)

If $f(x, y) = x + y$

$df = dx + dy$

error in f = $\Delta f = \pm \Delta x \pm \Delta y$

max possible error in f = $(\Delta f)_{max}$

= max of $(\pm \Delta x \pm \Delta y)$

$(\Delta f)_{max} = \Delta x + \Delta y$

Case : (II)

If $f = x - y$

$df = dx - dy$

$(\Delta f) = \pm \Delta x \mp \Delta y$

max possible error in f = $(\Delta f)_{max}$

= max of $(\pm \Delta x \mp \Delta y)$

$\Rightarrow \ (\Delta f)_{max} = \Delta x + \Delta y$

For getting maximum permissible error, sign should be adjusted, so that errors get added up to give maximum effect

i.e. $f = 2x - 3y - z$

$(\Delta f)_{max} = 2\Delta x + 3\Delta y + \Delta z$

14. Absolute Error, Relative Error and Percentage Error

Suppose to measure some quantity, we take several observations, a_1, a_2, a_3.... a_n . To find the absolute error in each measurement and percentage error , we have to follow these steps

- First of all mean of all the observations is calculated : $a_{mean} = (a_1 + a_2 + a_3 + ... + a_n) / n$. The mean of these values is taken as the best possible value of the quantity under the given conditions of measurements..
- **Absolute Error**

  The magnitude of the difference between the best possible or mean value of the quantity and the individual measurement value is called the absolute error of the measurement. The absolute error in an individual measured value is:

  $\Delta a_n = | a_{mean} - a_n |$

  The arithmetic mean of all the absolute errors is taken as the final or mean absolute error.

  $\Delta a_{mean} = (|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ... + |\Delta a_n|)/n$

- **Relative and Percentage Error**

  Relative error is the ratio of the mean absolute error and arithmetic mean .

  Relative error = $\dfrac{\Delta a_{mean}}{a_{mean}}$

  When the relative error is expressed in percent, it is called the percentage error.

  Thus,

  Percentage error = $\dfrac{\Delta a_{mean}}{a_{mean}} \times 100\%$

Example 15:

Following observations were taken with a vernier calipers while measuring the length of a cylinder: 3.29 cm, 3.28 cm, 3.29 cm, 3.31 cm, 3.28 cm, 3.27 cm, 3.29 cm, 3.30cm Then find :

(a) Most accurate length of the cylinder.

(b) Absolute error in each observation.

(c) Mean absolute error

(d) Relative error

(e) Percentage error

Express the result in terms of absolute error and percentage error.

Solution:

(a) Most accurate length of the cylinder will be the mean length = 3.28875 cm = 3.29 cm

(b) Absolute error in the first reading = 3.29 − 3.29 = 0.00 cm

Absolute error in the second reading = 3.29 − 3.28 = 0.01 cm

Absolute error in the third reading = 3.29 − 3.29 = 0.00 cm

Absolute error in the forth reading = 3.39 − 3.31 = −0.02 cm

Absolute error in the fifth reading = 3.29 − 3.28 = 0.01 cm

Absolute error in the sixth reading = 3.29 − 3.27 = 0.02 cm

Absolute error in the seventh reading = 3.29 − 3.29 = 0.00 cm

Absolute error in the last reading

= 3.29 − 3.30 = −0.01 cm

(c) Mean absolute error

$$\overline{\Delta \ell} = \frac{0.00 + 0.01 + 0.00 + 0.02 + 0.01 + 0.02 + 0.00 + 0.01}{8} = 0.01 \text{ cm}$$

(d) Relative error in length = $\dfrac{\overline{\Delta \ell}}{\ell} = \dfrac{0.01}{3.29}$ = 0.0030395 = 0.003

(e) Percentage error = $\dfrac{\overline{\Delta \ell}}{\bar{\ell}} \times 100$ = 0.003 × 100 = 0.3%

So length ℓ = 3.29 cm ± 0.01 cm (in terms of absolute error)

⇒ ℓ = 3.29 cm ± 0.30% (in terms of percentage error)

15. Combination of Errors

In Sum : If Z = A + B, then ΔZ = ΔA + ΔB, maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A + B} + \frac{\Delta B}{A + B}$$

In Difference : If Z = A – B, then maximum absolute error is ΔZ = ΔA + ΔB and maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A - B} + \frac{\Delta B}{A - B}$$

In Product : If Z = AB, then the maximum fractional error,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

where ΔZ/Z is known as fractional error.

In Division : If Z = A/B, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

In Power : If Z = A^n then $\dfrac{\Delta Z}{Z} = n\dfrac{\Delta A}{A}$

In more general form if $Z = \dfrac{A^x B^y}{C^q}$

then the maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = x\frac{\Delta A}{A} + y\frac{\Delta B}{B} + q\frac{\Delta C}{C}$$

Example 16:

Two resistors of resistances R_1 = (100 ± 3) ohm and R_2 = (200 ± 4) ohm are connected (a) in series (b) in parallel. Find the equivalent resistance of the (a) series combination, (b) parallel combination. Use for (a) the relation $R = R_1 + R_2$ and for (b) $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$ and $\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

Solution:

(a) The equivalent resistance of series combination

$R = R_1 + R_2$ = (100 ± 3) ohm + (200 ± 4) ohm = 300 ± 7 ohm.

(b) The equivalent resistance of parallel combination

$$R' = \frac{R_1R_2}{R_1 + R_2} = \frac{200}{3} = 66.7 \text{ ohm}$$

Then, from $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$

We get,

$$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Delta R' = (R'^2)\left[\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right]$$

$$\Delta R' = \left(\frac{66.7}{100}\right)^2 3 + \left(\frac{66.7}{200}\right)^2 4 = 1.8$$

Then, R' = 66.7 ± 1.8 ohm

(Here, ΔR is expressed as 1.8 instead of 2 to keep in conformity with the rules of significant figures)

Example 17:

If measured value of resistance R = 1.05 Ω, wire diameter d = 0.60 mm, and length ℓ = 75.3 cm, then find maximum permissible error in resistivity.

Given $\rho = \frac{R\left(\frac{\pi d^2}{4}\right)}{\ell}$

Solution:

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = \frac{\Delta R}{R} + 2\,\frac{\Delta d}{d} + \frac{\Delta \ell}{\ell}$$

R = 1.05 Ω $\rightarrow$ ΔR = 0.01 Ω (least count)

d = 0.60 mm $\rightarrow$ Δd = 0.01 mm (least count)

ℓ = 75.3 $\rightarrow$ $\Delta\ell$ = 0.1 cm (least count)

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = \left(\frac{0.01\Omega}{1.05\Omega} + 2 \times \frac{0.01mm}{0.60mm} + \frac{0.1cm}{75.3cm}\right) \times 100 = 4\%$$

Example 18:

To find the value of 'g' using simple pendulum. T = 2.00 sec; ℓ = 1.00 m was measured. Estimate maximum permissible error in 'g'. Also find the value of 'g' .

(use $\pi^2 = 10$)

Solution:

$$T = 2\pi\sqrt{\frac{\ell}{g}} \quad \Rightarrow \quad g = \frac{4\pi^2\ell}{T^2}$$

$$\left(\frac{\Delta g}{g}\right)_{max} = \frac{\Delta\ell}{\ell} + 2\,\frac{\Delta T}{T}$$

$$= \left(\frac{0.01}{1.00} + 2 \times \frac{0.01}{2.00}\right) \times 100\ \% = 2\ \%$$

$$\text{value of } g = \frac{4\pi^2\ell}{T^2} = \frac{4 \times 10 \times 1.00}{(2.00)^2} = 10.0\ m/s^2$$

$$\left(\frac{\Delta g}{g}\right)_{max} = \frac{2}{100} \quad \text{so} \quad \frac{\Delta g_{max}}{10.0} = \frac{2}{100}$$

so $(\Delta g)_{max} = 0.2$ = max error in 'g'

so 'g' = (10.0 ± 0.2) m/s^2

Example 19:

Find the relative error in Z, If $Z = A^4B^{1/3}/CD^{3/2}$.

Solution:

The relative error in Z is

$$\frac{\Delta z}{z} = 4\left(\frac{\Delta A}{A}\right) + \frac{1}{3}\left(\frac{\Delta B}{B}\right) + \left(\frac{\Delta C}{C}\right) + \frac{3}{2}\left(\frac{\Delta D}{D}\right)$$

Example 20:

The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. What is the accuracy in the determination of g ?

Solution:

$g = 4\pi^2L/T^2$

Here, $T = \frac{t}{n}$ and $\frac{\Delta T}{T} = \frac{\Delta t}{t}$

The errors in both L and t are the least count errors. Therefore.

$$\frac{\Delta g}{g} = \left(\frac{\Delta L}{L}\right) + 2\left(\frac{\Delta T}{T}\right)$$

$$\frac{\Delta g}{g} = \frac{0.1}{20.0} + 2\left(\frac{1}{90}\right) = 0.027$$

Thus the percentage error in g is

$$\frac{\Delta g}{g} \times 100\% = \left[\left(\frac{\Delta L}{L}\right) + 2\left(\frac{\Delta T}{T}\right)\right] \times 100\% = 2.7\%$$

Example 21:

The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm. Calculate area of the rectangle with error limits.

Solution:

Given ℓ = (5.7 ± 0.1) cm

and b = (3.4 ± 0.2) cm

Area A = ℓ × b = 5.7 × 3.4 = 19.38 cm^2

$$\frac{\Delta A}{A} = \left(\frac{\Delta \ell}{\ell} + \frac{\Delta b}{b}\right)$$

$$= \pm \left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right) = \pm \left(\frac{0.34 + 1.14}{5.7 \times 3.4}\right) = \pm \frac{1.48}{19.38}$$

$$\Rightarrow \Delta A = \pm \frac{1.48}{19.38} \times A$$

$$= \pm \frac{1.48}{19.38} \times 19.38 = 1.48$$

Area = (19.38 ± 1.48) cm^2

Concept Builder-4

Q.1 When measuring the diameter of sphere,. following observation were taken.
5.29 cm, 5.28 cm, 5.29, 5.31, 5.28, 5.27, 5.29, 5.29 cm. then find
(a) Most accurate length of the cylinder. (b) Absolute error in each observation.
(c) Mean absolute error (d) Relative error
(e) Percentage error

Q.2 The initial and final temperatures of water as recorded by an observer are (40.6 ± 0.2)°C and (78.3 ± 0.3)°C. Calculate the rise in temperature.

Q.3 A thin copper wire of length L increase in length by 2% when heated from T_1 to T_2. If a copper cube having side 10 L is heated from T_1 to T_2 what will be the percentage change in
(i) area of one face of the cube
(ii) volume of the cube.

Q.4 The resistance R = V/I where V = (100 ± 5) V and I = (10 ± 0.2)A. Find the percentage error in R.

16. Significant Figures

Every measurement involves errors. Thus, the result of measurement should be reported in a way that indicates the precision of measurement. Normally, the reported result of measurement is a number that includes all digits in the number at are known reliably plus the first digit that is uncertain. The reliable digits plus the first uncertain digit are known as significant digits or significant figures.

Significant figures indicate, as already mentioned, the precision of measurement which depends on the least count of the measuring instrument. A choice of change of different units does not change the number of significant digits or figures in a measurement.

16.1 Common Rules of Counting Significant Figures

Rule 1 :

All non-zero digits are significant

i.e. 123.56 has five S.F.

Rule 2 :

All zeros occurring between two non-zero digits are significant (obviously)

i.e. 1230.05 has six S.F.

Rule 3 :

$$\begin{array}{c|c|c} \ell = 3.5\text{cm} & \ell = 3.50\text{cm} & \ell = 3.500\text{cm} \\ \downarrow & \downarrow \text{closer!} & \downarrow \text{more closer!} \\ 0.1\text{cm} & 0.01\text{cm} & 0.001\text{cm} \\ \text{accuracy} & \text{accuracy} & \text{accuracy} \\ \downarrow & \downarrow & \downarrow \\ \ell \text{ lies between} & \ell \text{ lies between} & \ell \text{ lies between} \\ (3.4 - 3.6) & (3.49 - 3.51) & (3.499 - 3.501) \end{array}$$

So trailing zeroes after decimal place are significant (Shows the further accuracy)

$$\begin{array}{c|c|c} 3.5\text{cm} & 3.50\text{cm} & 3.500\text{cm} \\ \downarrow & \downarrow & \downarrow \\ \text{Two S.F.} & \text{Three S.F.} & \text{Four S.F.} \end{array}$$

Once a measurement is done, significant figures will be decided according to closeness of measurement. Now if we want to display the measurement in some different units, the S.F. shouldn't change (S.F. depends only on accuracy of measurement)

Number of S.F. is always conserved, change of units cannot change S.F.

Suppose measurement was done using mm scale, and we get ℓ = 85 mm (Two S.F.)

If we want to display it in other units.

$$\begin{array}{ccccc} 85\text{mm} & \rightarrow & 8.5\text{cm} & \rightarrow & 0.085\text{m} \\ \downarrow & & & & \downarrow \\ 85000\text{mm} & & & & 0.000085\text{km} \\ = 8.5 \times 10^{4}\text{mm} & & & & = 8.5 \times 10^{-5}\text{km} \end{array}$$

All should have two S.F.

The following rules support the conservation of S.F.

Rule 4:

From the previous example, we have seen that,

$$0.\underbrace{0000}_{\text{Not significant}}85\text{km} \longrightarrow \text{also should has two S.F. 8 and 5}$$

So leading Zeros are not significant.

In the number less than one, all zeros after decimal point and to the left of first non-zero digit are insignificant (arises only due to change of unit)

0.000305 has three S.F.

$\Rightarrow$ 3.05×10^{-4} has three S.F.

Rule 5 :

From the previous example, we have also seen that

85000 μm ⟶ should also has two S.F. 8 and 5

Not significant

So the trailing zeros are also not significant.

The terminal or trailing zeros in a number without a decimal point are not significant. (Also arises only due to change of unit)

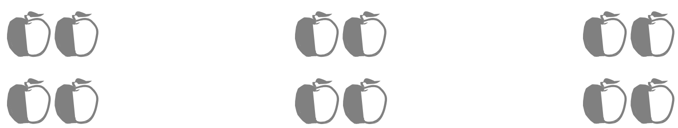
154 m = 15400 cm = 154000 mm

$= 154 \times 10^{9}$ nm

all has only three S.F. all trailing zeros are insignificant

Rule 6 :

There are certain measurement, which are exact i.e.



Number of apples are = 12 (exactly) = 12.000000........... ∞

This type of measurement is infinitely accurate so, it has ∞ S.F.

Numbers of students in class = 125 (exact)

Speed of light in the vacuum = 299,792,458 m/s (exact)

16.2 Rules for Arithmetic Operation with Significant Figures

(i) in multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

(ii) In addition, or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.

17. Rounding off the Uncertain Digits

The result of computation with approximate numbers, which contain more than one uncertain digit, should be rounded off. The rules of rounding off numbers to the appropriate significant figures are obvious in most cases.

Rules of Rounding Off

- If removable digit is less than 5 (50%) ; drop it.

  $\underline{\mathbf{47.8}}33 \xrightarrow[\text{till one decimal place}]{\text{Round off}} 47.8$

- If removable digit is greater than 5(50%), increase the last digit by 1.

  $\underline{\mathbf{47.8}}62 \xrightarrow[\text{till one decimal place}]{\text{Round off}} 47.9$

If removable number is exactly 5(50%)

If last number is even drop 5

$\underline{20.65} \rightarrow 20.6$

If last number is odd, increase the last digit by 1

$\underline{20.75} \rightarrow 20.8$

Example 22:

In ohm’s law exp., reading of voltmeter across the resistor is 12.5 V and reading of current i = 0.20 Amp. Estimate the resistance in correct S.F.

Solution:

$$R = \frac{V}{i} = \frac{12.5 \to 3SF}{0.20 \to 2SF} = 62.5\Omega \xrightarrow[\text{to 2 S.F.}]{\text{round off}} 62\Omega$$

Example 23

Write down the number of significant figures in the following:

(a) 165 (b) 2.05 (c) 34.000 m (d) 0.005

(e) 0.02340 N m^{-1}

Solution:

(a) 3SF (b) 3 SF (c) 5 SF (d) 1 SF (e) 4 SF

Concept Builder-5

Q.1 Round off the following numbers within three significant figures -

(i) 0.03927 kg (ii) 4.085 x 10^8 sec (iii) 5.2354 m (iv) 4.735 x 10^{-6} kg

Q.2 Round off the following numbers to 3 significant digits-

(i) 899.68 (ii) 987.52 (iii) 2.0082 (iv) 336.5 (v) 335.5

Q.3 The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 m respectively. Give the area and volume of the sheet to correct number of significant figures.

Q.4 Write number of significant digits

(i) 62.3 cm (ii) 6.23 × 10^1 cm (iii) 20.000 (iv) 0.02 × 10^{-19}

(v) 500.000 (vi) 0.5210 (vii) 896.80 (viii) 201

(ix) 1200 (x) 1200 N

Q.5 Solve with regards to significant figure

(i) 908 + 2.76 (ii) 999 – 989 (iii) 625 ÷ 125

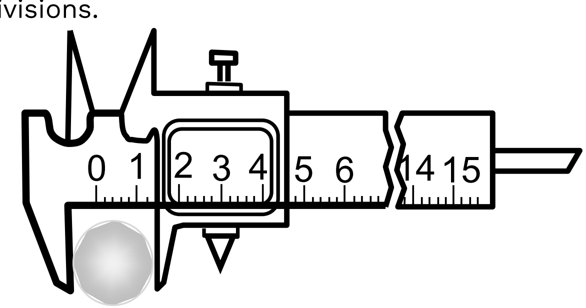
18. Measuring Instruments

18.1 Least Count

The smallest value of a physical quantity which can be measured accurately with an instrument is called the least count (L. C.) of the measuring instrument.

Least Count of Vernier Calipers

Suppose the size of one main scale division (M.S.D.) is M units and that of one vernier scale division (V. S. D.) is V units. Also let the length of 'a' main scale divisions is equal to the length of 'b' vernier scale divisions.



$aM = bV \Rightarrow V = \frac{a}{b}M$

$\therefore\ M - V = M - \frac{a}{b}M$ or $M - V = \left(\frac{b-a}{b}\right)M$

The quantity (M– V) is called vernier constant (V. C.) or least count (L. C.) of the vernier calipers.

$$\boxed{L.C. = M - V = \left(\frac{b-a}{b}\right)M}$$

Example 24:

One cm on the main scale of vernier calipers is divided into ten equal parts. If 20 divisions of vernier scale coincide with 8 small divisions of the main scale. What will be the least count of caliper?

Solution:

20 div. of vernier scale = 8 div. of main scale $\Rightarrow$ 1 V. S .D. = $\left(\frac{8}{20}\right)$ M. S. D. = $\left(\frac{2}{5}\right)$ M. S. D.

Least count = 1 M. S. D. – 1 V. S . D.

= 1 M. S. D. – $\left(\frac{2}{5}\right)$ M. S. D. = $\left(1-\frac{2}{5}\right)$ M. S. D.

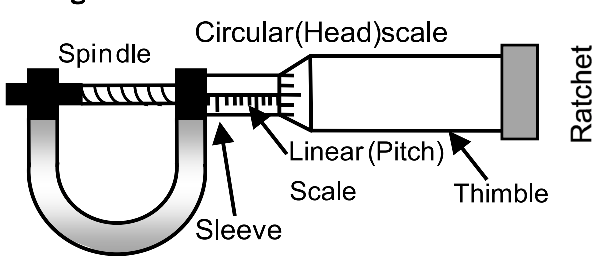
= $\frac{3}{5}$ M. S. D. = $\frac{3}{5} \times 0.1$ cm = 0.06 cm

($\because$1 M. S. D. = $\frac{1}{10}$ cm = 0.1 cm)

Note : for objective questions

L. C. = M – V = $\left(\frac{b-a}{b}\right)$ M = $\left(\frac{20-8}{20}\right)\left(\frac{1}{10}\right)$ cm $\Rightarrow \frac{3}{50}$ cm = 0.06 cm

18.2 Least Count of Screw Gauge



Lease Count = $\frac{\text{Pitch}}{\text{Total no. of division on the circular scale}}$

where pitch is defined as the distance moved by the screw head when the circular scale is given one complete rotation. i.e.

Pitch = $\frac{\text{Distance moved by the screw on the linear scale}}{\text{No. of full rotations given}}$

Note: With the decrease in the least count of the measuring instrument, the accuracy of the measurement increases and the error in the measurement decreases.

Example 25:

The n^{th} division of main scale coincides with $(n + 1)^{th}$ division of vernier scale. Given one main scale division is equal to 'a' units. Find the least count of the vernier calipers.

Solution:

(n + 1) divisions of vernier scale = n divisions of main scale

$\therefore$ 1 vernier division $= \frac{n}{n+1}$ main scale division

Least count = 1 M. S. D. – 1V. S. D.

$= \left(1 - \frac{n}{n+1}\right)$ M.S.D. $= \left(\frac{1}{n+1}\right)$ M. S. D. $= \frac{a}{n+1}$

Example 26:

The least count of a stop watch is $\frac{1}{5}$ second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. How much will be the percentage error in the measurement of time?

Solution:

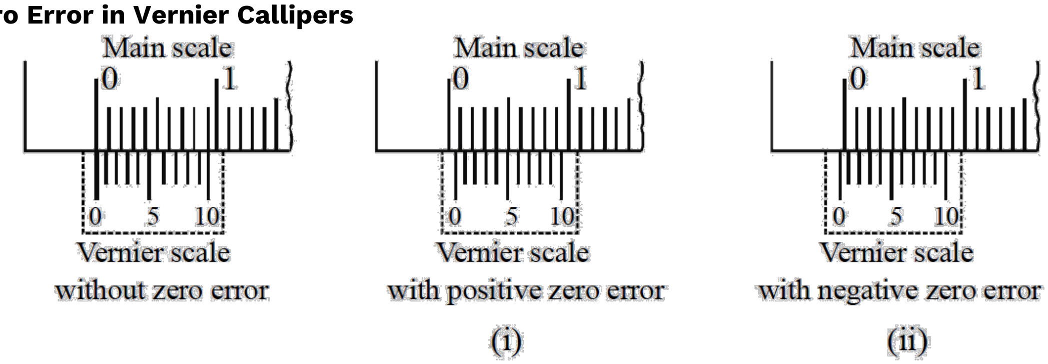
Error in measuring 25 sec.$= \frac{1}{5}$ sec. = 0.2 sec.

$\therefore$ percentage error $= \frac{0.2}{25} \times 100\% = 0.8\%$

Note: The final absolute error in this type of questions is taken to be equal to the least count of the measuring instrument.

19. Zero Error

19.1 Zero Error in Vernier Callipers



Calculation of Zero Error for Vernier Calipers

Positive zero error = (No. of Division of VS coincided with MS).LC

Negative zero error = -(Total division in VS – No. of division of VS coincided with MS).LC

Correct Reading with Zero Error

Reading = MSR + VSR × LC

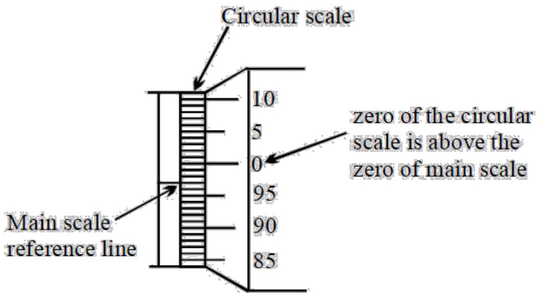
Correct reading = (Reading) – (Zero error)

The zero error is always subtracted with sign from the reading to get the corrected value.

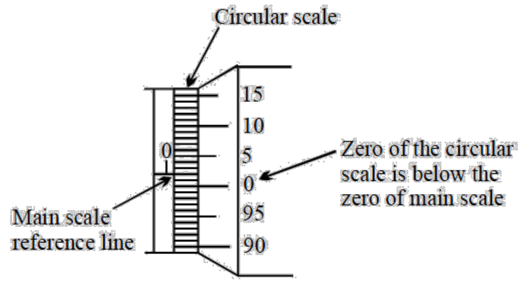
19.2 Zero Error in Screw Gauge

If there is no object between the jaws (i.e. jaws are in contact), the screw gauge should give zero reading. But due to extra material on jaws, even if there is no object, it gives some excess reading. This excess reading is called Zero error.

Negative Zero Error
(3 division error) i.e., –0.003 cm



Positive Zero Error
(2 division error) i.e., +0.002 cm



Calculation of Zero Error for Screw Gauge

Reading = LSR + CSR × LC

Positive zero error = (No. of division of CS on MS).LC

Negative zero error = -(Total division on CS – No. of division of CS on MS).LC

Correct reading = (Reading) – (zero error)

Remember

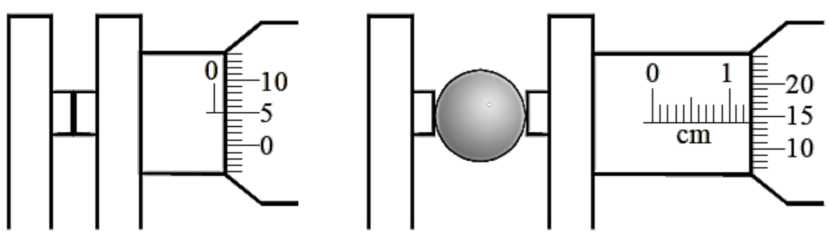
To get correct reading take zero error with their sign.

Positive zero error = + (Numerical value of zero error)

Negative zero error = – (Numerical value of zero error)

Example 27:

What is the diameter of sphere shown in figure. Pitch of screw gauge is 1 mm and number of divisions in circular scale are 50 :



(1) 1.238 cm (2) 1.218 cm (3) 1.228 cm (4) 1.215 cm

Solution:

Zero Error:

Least count of circular scale $= \frac{1}{50} = 0.02$ mm

Reading of main scale = 0.0 cm

Number of division coincied = 5

reading of circular scale = 5 × 0.02

Zero error = 0.10 mm = +0.010 cm

For diameter sphere:

Reading of main scale = 1.2 cm

No. of division coincided = 14

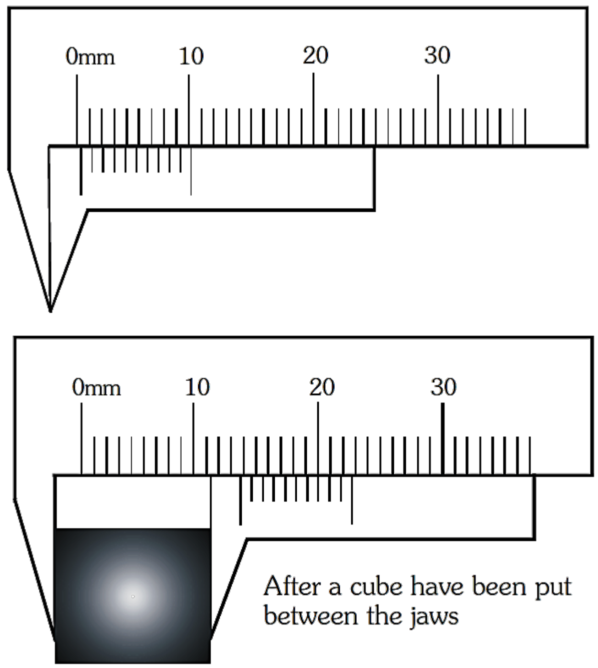
Reading of circular scale = 14 × 0.02 = 0.28 mm = 0.028 cm

Reading = 1.228 cm

Diameter = 1.228 – 0.010 = 1.218 cm

Example 29:

Find the thickness of the cubical object using a defective vernier calliper. Main scale has mm marks and 10 divisions of vernier scale coincide with 9 divisions of main scale.



(1) 13.8 mm (2) 13.4 mm (3) 14.1 mm (4) 13.0 mm

Solution:

Least count of vernier callipers $= \left(1 - \dfrac{9}{10}\right) \times 1\text{ mm} = 0.1\text{ mm}$

Zero error = VSR × LC = 3 × 0.1 = 0.3 mm

Reading = MSR + VSR × LC = 13mm + 7 × 0.1 mm = 13.7 mm

Correct reading = Reading – Zero error = 13.7 – 0.3 mm = 13.4 mm

20. Order of Magnitude

Order of magnitude of a quantity is the power of 10 required to represent that quantity. This power is determined after rounding off the value of the quantity properly. For rounding off, the last digit is simply ignored if it is less than 5 and, is increased by one if it is 5 or more than 5.

- When a number is divided by 10^x (where x is the order of the number) the result will always lie between 0.5 and 5 i.e. $0.5 \le N/10^x < 5$

Example : Order of magnitude of the following values can be determined as follows :

(a)	49	=	$4.9 \times 10^1 \approx 10^1$	$\therefore$	Order of magnitude = 1
(b)	51	=	$5.1 \times 10^1 \approx 10^2$	$\therefore$	Order of magnitude = 2
(c)	0.049	=	$4.9 \times 10^{-2} \approx 10^{-2}$	$\therefore$	Order of magnitude = –2
(d)	0.050	=	$5.0 \times 10^{-2} \approx 10^{-1}$	$\therefore$	Order of magnitude = –1
(e)	0.051	=	$5.1 \times 10^{-2} \approx 10^{-1}$	$\therefore$	Order of magnitude = –1

Concept Builder-6

Q.1 One centimetre on the main scale of vernier callipers is divided into ten equal parts. If 20 divisions of vernier scale coincide with 19 small divisions of the main scale then what will be the least count of the callipers.

Q.2 If the number of divisions on the circular scale is 100 and number of full rotations given to screw is 8 and distance moved by the screw is 4 mm, then what will be least count of the screw gauge.

Q.3 Give the order of the following :

(a) 1 (b) 1000 (c) 499 (d) 500

(e) 501 (f) 1 AU (1.496×10^{11} m) (g) 1 Å (10^{-10} m)

(h) Speed of light (3.00×10^{8} m/s) (i) Gravitational constant (6.67×10^{-11} N – m^2/kg^2)

(j) Avogadro constant (6.02×10^{23} mol^{-1}) (k) Planck's constant (6.63×10^{-34} J-s)

(l) Charge on electron (1.60×10^{-19} C) (m) Radius of H–atom (5.29×10^{-11} m)

(n) Atmospheric pressure (1.01×10^{5} Pa) (o) Mass of earth (5.98 × 1024 kg)

(p) Mean radius of earth (6.37×10^{6} m)

ANSWER KEY FOR CONCEPTS BUILDERS

CONCEPT BUILDER-1

1. No **2.** 0.1 rad

3. (7×10^{-6}) μm **4.** 25 m/s

5. one fourth **6.** 4 times

CONCEPT BUILDER-2

1. (i) $[M^1L^1T^{-2}]$ (ii) $[M^1L^2T^{-2}]$

(iii) $[M^1L^2T^{-2}]$ (iv) $[M^1L^2T^{-3}]$

(v) $[M^1L^{-1}T^{-2}]$ (vi) $[M^0L^0T^{-1}]$

(vii) $[M^0L^0T^{-2}]$

2. $M^{-1}L^3T^{-2}$ **3.** $M^1L^2T^{-1}$

4. $(\alpha) = [M^1L^{-1}T^0]$; $(\beta) = [T^{-1}]$

5. (A, C, D)

CONCEPT BUILDER-3

1. (C,D)

2. (i) $[a] = [ML^4T^{-2}]$; $[b] = [ML^3T^{-2}]$;

(ii) his answer is dimensionally incorrect

3. $f \propto \frac{1}{\ell}\sqrt{\frac{P}{\rho}}$ **4.** $k \propto \frac{P^2}{M}$

5. $2 \times 10^3 \ \frac{kg}{m^3}$

CONCEPT BUILDER-4

1. (a) 5.29, (c) 0.01, (d) 0.00189,

(e) 0.189%

2. (37.7 ± 0.5)°C

3. (i) 4%; (ii) 6% **4.** 7%

CONCEPT BUILDER-5

1. (i) 0.0393 kg (ii) 4.08×10^8 sec

(iii) 5.24 m (iv) 4.74×10^{-6} kg

2. (i) 900, (ii) 988, (iii) 2.01, (iv) 336, (v) 336

3. Area = 4.255 m^2, Volume = 8.55 m^3

4. (i) 3 (ii) 3 (iii) 5 (iv) 1 (v) 6 (vi) 4

(vii) 5 (viii) 3 (ix) 2 (x) 4

5. (i) 911, (ii) 10.0, (iii) 5.00

CONCEPT BUILDER-6

1. 0.005 cm **2.** 0.005 mm

3. (a) 0, (b) 3, (c) 2, (d) 3, (e) 3, (f) 11,

(g) –10, (h) 8, (i) –10, (j) 24,

(k) –33, (l) –19, (m) –10, (n) 5,

(o) 25, (p) 7

Exercise - I

Units & Quantities

1. Which of the following system of units is not based on units of mass, length and time alone

(1) SI (2) MKS
(3) FPS (4) CGS

2. Which of the following quantity is unitless?

(1) Velocity / length
(2) Pressure / length
(3) Displacement / length
(4) Force / length

3. The ratio of one micron to one nanometre is

(1) 10^{3} (2) 10^{-3}
(3) 10^{-6} (4) 10^{-1}

4. Temperature can be expressed as a derived quantity in terms of which of the following.

(1) Length and mass
(2) Mass and time
(3) Length mass and time
(4) In terms of none of these

5. Density of wood is 0.5 gm/cc in the CGS system of units. The corresponding value in MKS units is

(1) 500 (2) 5
(3) 0.5 (4) 5000

6. Which of the following is not the unit of time.

(1) Micro second
(2) leap year
(3) Lunar months
(4) Parallactic second

7. Which of the following is smallest unit

(1) Milimetre (2) Angstrom
(3) Fermi (4) Metre

8. Which relation is wrong

(1) 1 Calorie = 4.18 Joules
(2) 1 Å = 10^{-10} m
(3) 1 MeV = 1.6×10^{-13} Joules
(4) 1 Newton = 10^{-5} Dynes

Dimensions, Dimensional Analysis

9. When a wave travels in a medium, the displacement of a particle located at distance x at time t is given by y = a sin (bt – cx) where a, b and c are constants of the wave. The dimension of b/c are same as that of :

(1) wave velocity (2) wave length
(3) wave amplitude (4) wave frequency

10. The dimensional formula of k in y = sin(kx) is
(if x is the distance)

(1) $M^0L^0T^{-1}$ (2) $M^{-1}L^{-1}T^0$
(3) $M^0L^{-1}T^0$ (4) $M^0L^0T^0$

11. The method of dimensional analysis can be used to derive which of the following relations ?

(1) $N_0e^{-\lambda t}$
(2) $A\sin(\omega t + kx)$
(3) $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
(4) None of the above

12. Which of the following does not have the dimensions of force ?

(1) Potential / length
(2) Energy / length
(3) Weight
(4) Rate of change of momentum

13. Which of the following is incorrect statement

(1) A dimensionally correct equation may be correct
(2) A dimensionally correct equation may be incorrect
(3) A dimensionally incorrect equation may be correct
(4) A dimensionally incorrect equation is incorrect

14. A dimensionless quantity

(1) Never has a unit
(2) Always has a unit
(3) May have a unit
(4) Does not exist

15. A unitless quantity
(1) Does not exist
(2) Always has a nonzero dimension
(3) Never has a nonzero dimension
(4) May have a nonzero dimension

16. Which of the following is incorrect
(1) All derived quantities may be represented dimensionally in terms of the base quantities
(2) A base quantity cannot be represented dimensionally in terms of other base quantities
(3) The dimension of a derived quantity is never zero in any base quantity
(4) The dimension of a base quantity in other base quantities is always zero.

17. Two physical quantities of which one is a vector and the other is a scalar having the same dimensional formula are :
(1) Work and energy
(2) Torque and work
(3) Impulse and momentum
(4) Power and pressure

18. The equation of a wave is given by
$Y = A \sin \omega\left(\frac{x}{v} - k\right)$ where ω is the angular velocity and v is the linear velocity. The dimension of k is
(1) LT (2) T
(3) T^{-1} (4) T^2

19. The time dependence of a physical quantity P is given by $P = P_0 e^{(-\alpha t^2)}$, where α is a constant and t is time. The constant α
(1) dimensionless
(2) has dimensions T^{-2}
(3) has dimensions of P
(4) has dimensions T^2

20. In a particular system the units of length mass and time are chosen to be 10 cm, 10 g and 0.1 s respectively. The unit of force in this system will be equal to
(1) 0.1 N (2) 1 N
(3) 10 N (4) 100 N

21. The dimensional formula of angular velocity is
(1) $M^0L^0T^{-1}$ (2) MLT^{-1}
(3) $M^0L^0T^1$ (4) ML^0T^{-2}

22. Given the force (F) is given $F = Pt^{-1} + Qt$. Here t is time. The unit of P is same as that of :
(1) displacement (2) velocity
(3) acceleration (4) momentum

23. The force is given in terms of time (t) and displacement (x) by the equation :
$F = A \cos Bx + C \sin Dt$
Then, the dimensions of $\frac{D}{B}$ are :
(1) $M^0L^0T^0$ (2) $M^0L^0T^{-1}$
(3) $M^0L^{-1}T^0$ (4) M^0LT^{-1}

24. In the relation $y = r \sin (\omega t - kx)$ the dimensions of $\frac{\omega}{k}$ are :
(1) $M^0L^1T^0$ (2) $M^0L^1T^{-1}$
(3) $M^0L^0T^3$ (4) $M^0L^1T^0$

Error, Signification Digits

25. For a cubical block, error in measurement of sides is $\pm 1\%$ and error in measurement of mass is $\pm 2\%$, then maximum possible error in density is-
(1) 1% (2) 5%
(3) 3% (4) 7%

26. To estimate 'g' (from g = $4\pi^2 \frac{L}{T^2}$), error in measurement of L is $\pm$2% and error in measurement of T is $\pm$3%. The error in estimated 'g' will be -

(1) ± 8% (2) ± 6%
(3) ± 3% (4) ± 5%

27. The least count of a stop watch is 0.2 second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. The percentage error in the time period is

(1) 16% (2) 0.8 %
(3) 1.8 % (4) 8 %

28. The dimensions of a rectangular block measured with a vernier callipers having least count of 0.1 mm is 5 mm × 10 mm × 5 mm. The maximum percentage error in measurement of volume of the block is

(1) 5 % (2) 10 %
(3) 15 % (4) 20 %

29. An experiment measures quantities x, y, z and then t is calculated from the data as t = $\frac{xy^2}{z^3}$. If percentage errors in x, y and z are respectively 1%, 3%, 2%, then percentage error in t is :

(1) 10 % (2) 4 %
(3) 7 % (4) 13 %

30. The external and internal diameters of a hollow cylinder are measured to be (4.23 ± 0.01) cm and (3.89 ± 0.01) cm. The thickness of the wall of the cylinder is

(1) (0.34 ± 0.02) cm (2) (0.17 ± 0.02) cm
(3) (0.17 ± 0.01) cm (4) (0.34 ± 0.01) cm

31. The mass of a ball is 1.76 kg. The mass of 25 such balls is

(1) 0.44×10^3 kg (2) 44.0 kg
(3) 44 kg (4) 44.00 kg

32. Zero error of an instrument introduces

(1) Systematic errors (2) Random errors
(3) Both (4) None

33. What is the fractional error in g calculated from $T = 2\pi\sqrt{\ell / g}$? Given that fractional errors in T and ℓ are ±2 and ±2 respectively.

(1) 4 (2) 0
(3) 6 (4) 2

34. A thin copper wire of length ℓ metre increases in length by 2% when heated through 10°C. What is the percentage increase in area when a square copper sheet of length ℓ metre is heated through 10°C ?

(1) 4% (2) 8%
(3) 16% (4) None of the above

35. The resistance is $R = \frac{V}{I}$ where V = 100 ± 5 Volts and I = 10±0.2 amperes. What is the total error in R ?

(1) 5 % (2) 7 %
(3) 5.2 % (4) $\left(\frac{5}{2}\right)$%

36. The length, breadth and thickness of a strip are (10.0 ± 0.1)cm, (1.00 ± 0.01)cm and (0.100 ± 0.001)cm respectively. The most probable error in its volume will be

(1) ± 0.03 cm^3 (2) ± 0.111 cm^3
(3) ± 0.012 cm^3 (4) None of these

37. If error in measuring diameter of a circle is 4%, the error in circumference of the circle would be :-

(1) 2 % (2) 8 %
(3) 4 % (4) 1 %

38. Percentage error in measuring the radius and mass of a solid sphere are 2% & 1% respectively. Then error in measurement of moment of inertia with respect to its diameter is :- $\left(I = \frac{2}{5}MR^2\right)$

(1) 3 % (2) 6 %
(3) 5 % (4) 4 %

39. The heat generated in a circuit is dependent upon the resistance, current and time for which the current is flown. If the error in measuring the above are as 1%, 2% and 1% respectively, then maximum error in measuring heat will be ($H = I^2Rt$)

(1) 2% (2) 3%

(3) 6% (4) 1%

40. The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimate of kinetic energy obtained by measuring mass and speed ? $\left(k = \frac{1}{2}mv^2\right)$

(1) 11 % (2) 8 %

(3) 5 % (4) 1 %

41. While measuring acceleration due to gravity by a simple pendulum a student makes a positive error of 1% in the length of the pendulum and a negative error of 3% in the value of the time period. His percentage error in the measurement of the value of g will be -

(1) 2 % (2) 5 %

(3) 7 % (4) 10 %

42. If the error in the measurement of radius of a sphere is 2%, then the error in the determination of volume of the sphere will be :

(1) 4% (2) 6%

(3) 8% (4) 2%

43. A student measures the distance traversed in free fall of a body, initially at rest, in a given time. He uses this data to estimate g, the acceleration due to gravity. If the maximum percentage errors in measurement of the distance and the time are e_1 and e_2 respectively, the percentage error in the estimation of g is :

(1) $e_2 - e_1$ (2) $e_1 + 2e_2$

(3) $e_1 + e_2$ (4) $e_1 - 2e_2$

44. In an experiment four quantities a, b, c and d are measured with percentage error 1%, 2%, 3% and 4% respectively. Quantity P is calculated as follows :

$P = \frac{a^3b^2}{cd}$; % error in P is :

(1) 10% (2) 7%

(3) 4% (4) 14%

45. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(1) 5, 1, 2 (2) 5, 1, 5

(3) 5, 5, 2 (4) 4, 4, 2

46. The edge of a cube is $a = 1.2 \times 10^{-2}$ m. Then its volume will be recorded as :

(1) $1.72 \times 10^{-6}\ m^3$ (2) $1.728 \times 10^{-6}\ m^3$

(3) $1.7 \times 10^{-6}\ m^3$ (4) $1.73 \times 10^{-6}\ m^3$

47. When a copper sphere is heated, maximum percentage change will be observed in (on heating radius will increase)

(1) radius (2) area

(3) volume (4) none of these

(Questions 48 to 53)

Find significant figures in the following observations-

48. 0.007 gm

(1) 1 (2) 2

(3) 3 (4) 4

49. 2.64×10^{24} kg

(1) 1 (2) 2

(3) 3 (4) 4

50. 0.2370 gm/cm^3

(1) 1 (2) 2

(3) 3 (4) 4

51. 6.320 J/K

(1) 1 (2) 2

(3) 3 (4) 4

52. 6.032 N/m^2

(1) 1 (2) 2

(3) 3 (4) 4

53. $0.0006032\ K^{-1}$

(1) 1 (2) 2

(3) 3 (4) 4

54. Dimensional formula of a physical quantity is $[M^{-1}L^{3}T^{-2}]$. The errors in measuring quantities M, L and T respectively are 2%, 3% and 4%. The maximum percentage error that occurs in measuring the quantity is

(1) 9% (2) 10%

(3) 14% (4) 19%

55. If length of a rectangle is 2.1 m and width is 1.62 m then its area will be

(1) $3.402\ m^2$ (2) $3.4\ m^2$

(3) $3.40\ m^2$ (4) $3\ m^2$

Measurement

56. A vernier callipers has 20 divisions on the vernier scale which coincide with 19 divisions on the main scale. The least count of the instrument is 0.1 mm. The main scale divisions are of

(1) 0.5 mm (2) 1 mm

(3) 2 mm (4) 1/4 mm

57. One centimeter on the main scale of vernier callipers is divided into ten equal parts. If 10 divisions of vernier scale coincide with 8 small divisions of the main scale, the least count of the callipers is

(1) 0.01 cm (2) 0.02 cm

(3) 0.05 cm (4) 0.005 cm

58. A student measured the diameter of a wire using a screw gauge with least count 0.001 cm and listed the measurements. The correct measurement is –

(1) 5.3 cm (2) 5.32 cm

(3) 5.320 cm (4) 5.3200 cm

59. A pendulum clock, designed to give correct time in planes, gives incorrect time at hill station. Type of error in measurement of time can be :-

(1) Systematic error (2) Random error

(3) Gross error (4) None of these

60. A vernier callipers has 40 divisions on the vernier scale which coincide with 38 divisions on the main scale. The least count of the instrument is 0.1 mm. The main scale divisions are of

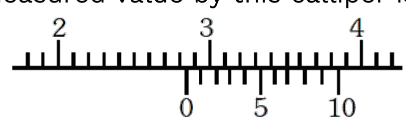
(1) 0.5 mm

(2) 1 mm

(3) 2 mm

(4) 1/4 mm

61. In main scale of a vernier callipers 1 cm divided into 10 equal divisions. The vernier scale has 10 equal divisions that correrspond to 9 main scale divisions. The measured value by this calliper is :



(1) 2.82 cm

(2) 2.83 cm

(3) 2.87 cm

(4) 2.89 cm

62. One full rotation of the cap of a screw gauge is equivalent to 5 mm. The cap has 100 division. Find the least count.

(1) 0.5 m

(2) 0.05 mm

(3) 0.005 m

(4) 5×10^{-4} m

63. Distance moved by a screw in 5 rotation is 5 mm and total number of circular divisions is 100. The least count of the screw gauge is :-

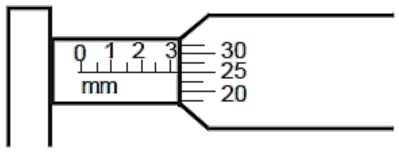
(1) 1 mm

(2) 0.001 cm

(3) 0.0001 cm

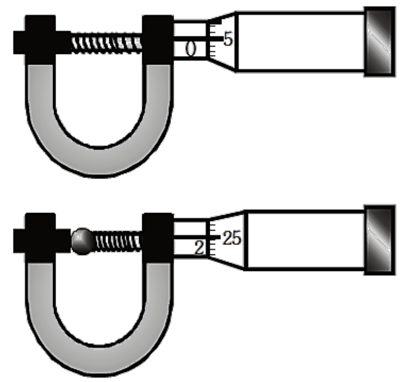
(4) 0.001 mm

64. Find the reading of micrometer ?



(1) 3.25 mm (2) 3.24 mm
(3) 3.30 mm (4) 3.41 mm

65. The number of circular division on the shown screw gauge is 100. It moves 0.1 cm on main scale for one complete rotation and main scale has 1 mm marks. The diameter of the ball is :-



(1) 2.25 mm (2) 2.20 mm
(3) 1.20 mm (4) 1.25 mm

ANSWER KEY																									
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
Ans.	1	3	1	4	1	4	3	4	1	3	4	1	3	3	3	3	2	2	2	1	1	4	4	2	2
Que.	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
Ans.	1	2	1	4	3	2	1	3	1	2	1	3	3	3	2	3	2	2	4	1	3	3	1	3	4
Que.	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**										
Ans.	4	4	4	4	2	3	2	3	1	3	3	2	2	1	2										

Exercise - II

1. The fundamental unit which has same power in the dimensional formula of surface tension and coefficient of viscosity is
(1) Mass
(2) Length
(3) Time
(4) None

2. Match list I with list II and select the correct answer by using the codes given below the lists

	List I (Item)		**List II (Units of length)**
A.	Distance between earth and stars	1.	Micron
B.	Inter atomic distance in a solid	2.	Angstrom
C.	Size of nucleus	3.	Light year
D.	Wavelength of Infrared Laser	4.	Fermi
		5.	Kilometer

Codes :

	A	**B**	**C**	**D**
(1)	5	4	2	1
(2)	3	2	4	1
(3)	5	2	4	3
(4)	3	4	1	2

3. Two resistors $R_1=(24 \pm 0.5)\ \Omega$ and $R_2=(8 \pm 0.3)\ \Omega$ are joined in series. The equivalent resistance is
(1) $32 \pm 0.33\ \Omega$
(2) $32 \pm 0.8\ \Omega$
(3) $32 \pm 0.2\ \Omega$
(4) $32 \pm 0.5\ \Omega$

4. A quantity is represented by $X = M^a L^b T^c$. The percentage error in measurement of M, L and T are $\alpha\%$, $\beta\%$ and $\gamma\%$ respectively. The percentage error in X would be
(1) $(\alpha a + \beta b + \gamma c)$ %
(2) $(\alpha a - \beta b + \gamma c)$ %
(3) $(\alpha a - \beta b - \gamma c)$ %
(4) None of these

5. In a vernier caliper, N divisions of vernier scale coincide with (N – 1) divisions of main scale (in which 1 division represents 1mm). The least count of the instrument in cm. should be
(1) N (2) N – 1
(3) $\frac{1}{10N}$ (4) $\frac{1}{N-1}$

6. The length of a cylinder is measured with a meter rod having least count 0.1 cm. Its diameter is measured with vernier callipers having least count 0.01 cm. Given the length is 5.0 cm. and radius is 2.00 cm. The percentage error in the calculated value of volume will be:
(1) 2% (2) 1%
(3) 3% (4) 4%

7. A physical quantity *X* is related to four variables *a*, *b*, *c* and *d* as follows, $X = \frac{a^2b^3}{c\sqrt{d}}$. Errors in measurement of *a*, *b*, c, *d* are 1%, 3%, 2% and 2% respectively. What is percentage error in quantity *X*?
(1) 12% (2) 8%
(3) 14% (4) 5%

8. If the energy $E = m^a c^b$ where m is the mass and c is the velocity of light, then the values of a and b are, respectively
(1) 1, 2 (2) 2, 1
(3) –1, 2 (4) –2, 1

(Questions 9 to 12)

Using screw gauge, the observation of the diameter of a wire are 1.324, 1.326, 1.334, 1.336 cm respectively.

9. Find the average diameter of wire :
(1) 1.33 (2) 1.32
(3) 1.30 (4) 1.31

10. Find the mean error :

(1) 0.006 (2) 0.005

(3) 0.007 (4) 0.004

11. Find the relative error :

(1) 0.004 (2) 0.002

(3) 0.005 (4) 0.006

12. Find the Percentage error :

(1) 0.4% (2) 0.2%

(3) 0.3% (4) 0.5%

13. Which of the following combination of three quantities P, Q, R having different dimension cannot be meaningful ?

(1) PQ – R (2) PQ – 1

(3) (P – Q)/R (4) $\frac{PR - Q^2}{QR}$

14. Suppose refractive index μ is given as $\mu = A + B/\lambda^2$, were A and B are constants and λ is wavelength then the dimension of B are same as that of :-

(1) wavelength (2) pressure

(3) area (4) volume

15. The equation of the stationary wave is

$$y = 2A\sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$$

Which of the following statements is wrong ?

(1) the unit of ct is same as that of λ

(2) the unit of x is same as that of λ

(3) the unit of $2\pi c/\lambda$ is same as that of $2\pi x/\lambda$ t

(4) the unit of c/λ is same as that of x/λ

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	2	2	1	3	3	3	1	1	2	1	1	3	3	4

Exercise – III (Previous Year Question)

1. The ratio of the dimension of Planck's constant (= energy / frequency) and that of the moment of inertia is the dimension of :- **[AIPMT 2005]**

(1) Velocity
(2) Angular momentum
(3) Time
(4) Frequency

2. The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$ where a, b and c are constants. The dimensions of a, b and c are respectively: **[AIPMT 2006]**

(1) LT^{-2}, L and T (2) L^2, T and LT^2
(3) LT^2, LT and L (4) L, LT and T^2

3. Dimension of electrical resistance is :– **[AIPMT 2007]**

(1) $[ML^2T^{-3}A^{-1}]$ (2) $[ML^2T^{-3}A^{-2}]$
(3) $[ML^3T^{-3}A^{-2}]$ (4) $[M^{-1}L^3T^3A^2]$

4. Which two of the following five physical parameters have the same dimensions ? **[AIPMT 2008]**

(a) energy / volume
(b) refractive index
(c) dielectric constant
(d) Young's modulus
(e) magnetic field

(1) (a) and (d) (2) (a) and (e)
(3) (b) and (d) (4) (c) and (e)

5. If the dimensions of a physical quantity are given by $M^aL^bT^c$, then the physical quantity will be : **[AIPMT 2009]**

(1) Force if a = 0, b = –1, c = – 2
(2) Pressure if a = 1, b = – 1, c = – 2
(3) Velocity if a = 1, b = 0, c = – 1
(4) Acceleration if a = 1, b = 1, c = – 2

6. The dimension of $\frac{1}{2}\varepsilon_o E^2$, where $\in_0$ is permittivity of free space and E is electric field, is :- **[AIPMT 2010]**

(1) $[MLT^{-1}]$ (2) $[ML^2T^{-2}]$
(3) $[ML^{-1}T^{-2}]$ (4) $[ML^2T^{-1}]$

7. The density of a material in CGS system of units is 4 g/cm^3. In a system of units in which unit of length is 10 cm and unit of mass is 100 g, the value of density of material will be :- **[AIPMT 2011]**

(1) 0.04 (2) 0.4
(3) 40 (4) 400

8. The dimensions of $(\mu_0 \in_0)^{-1/2}$ are

(if speed of light = $\frac{1}{\sqrt{\mu_0 \in_0}}$) : **[AIPMT 2011]**

(1) $[L^{1/2}T^{-1/2}]$ (2) $[L^{-1}T]$
(3) $[LT^{-1}]$ (4) $[L^{-1/2}T^{1/2}]$

9. If voltage across a bulb rated 220 Volt 100 Watt drops by 2.5% of its rated value, the percentage of the rated value by which the power would decrease is : **[AIPMT-2012]**

(1) 5% (2) 10%
(3) 20% (4) 2.5%

10. In an experiment four quantities a, b, c and d are measured with percentage errors 1%, 2%, 3% and 4% respectively. Quantity P is calculated as follows $P = \frac{a^3b^2}{cd}$, percentage error in P is: **[NEET-2013]**

(1) 4% (2) 14%
(3) 10% (4) 7%

11. If force (F), velocity (V) and time (T) are taken as fundamental units, then the dimensions of mass are: **[AIPMT-2014]**

(1) $[FVT^{-1}]$ (2) $[FVT^{-2}]$
(3) $[FV^{-1}T^{-1}]$ (4) $[FV^{-1}T]$

12. If energy (E), velocity (V) and time (T) are chosen as the fundamental quantities, the dimensional formula of surface tension will be : **[AIPMT-2015]**

(1) $[EV^{-1}T^{-2}]$ (2) $[EV^{-2}T^{-2}]$
(3) $[E^{-2}V^{-1}T^{-3}]$ (4) $[EV^{-2}T^{-1}]$

13. If dimension of critical velocity v_c, of liquid flowing through a tube is expressed as $(\eta^x \rho^y r^z)$, where η, ρ and r the coefficient of viscosity of liquid, density of liquid and radius of the tube respectively, then the values of x, y and z are given by : **[Re-AIPMT-2015]**

(1) 1, 1, 1 (2) 1, –1, –1
(3) –1, –1, 1 (4) –1, –1, –1

14. Planck's constant (h), speed of light in vacuum (c) and Newton's gravitational constant (G) are three fundamental constants. Which of the following combinations of these has the dimension of length? **[NEET-2016]**

(1) $\sqrt{\frac{hc}{G}}$ (2) $\sqrt{\frac{Gc}{h^{3/2}}}$

(3) $\frac{\sqrt{hG}}{c^{3/2}}$ (4) $\frac{\sqrt{hG}}{c^{5/2}}$

15. A physical quantity of the dimensions of length that can be formed out of c, G and $\frac{e^2}{4\pi\varepsilon_0}$ is [c is velocity of light, G is universal constant of gravitation and e is charge] : **[NEET-2017]**

(1) $c^2\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$ (2) $\frac{1}{c^2}\left[\frac{e^2}{G\,4\pi\varepsilon_0}\right]^{1/2}$

(3) $\frac{1}{c}G\frac{e^2}{4\pi\varepsilon_0}$ (4) $\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$

16. A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and zero of circular scale division coincides with 25 divisions above the reference level. If screw gauge has a zero error of – 0.004 cm, the correct diameter of the ball is: **[NEET-2018]**

(1) 0.521 cm (2) 0.525 cm
(3) 0.053 cm (4) 0.529 cm

17. The unit of thermal conductivity is- **[NEET-2019]**

(1) $W\ m\ K^{-1}$ (2) $W\ m^{-1}\ K^{-1}$
(3) $J\ m\ K^{-1}$ (4) $J\ m^{-1}\ K^{-1}$

18. In an experiment, the percentage of error occurred in the measurement of physical quantities A, B, C and D are 1%, 2%, 3% and 4% respectively. Then the maximum percentage of error in the measurement X, where $X = \frac{A^2B^{1/2}}{C^{1/3}D^3}$, will be : **[NEET-2019]**

(1) –10% (2) 10%

(3) $\left(\frac{3}{13}\right)\%$ (4) 16%

19. The main scale of a vernier caliper has n divisions/cm. n divisions of the vernier scale coincide with (n – 1) divisions of main scale. The least count of the vernier caliper is : **[NEET-2019 (Odisha)]**

(1) $\frac{1}{(n+1)(n-1)}$cm (2) $\frac{1}{n}$cm

(3) $\frac{1}{n^2}$cm (4) $\frac{1}{n(n+1)}$cm

20. Dimensions of stress is : **[NEET-2020]**

(1) $[ML^0T^{-2}]$ (2) $[ML^{-1}T^{-2}]$
(3) $[MLT^{-2}]$ (4) $[ML^2T^{-2}]$

21. A screw gauge has least count of 0.01 mm and there are 50 divisions in its circular scale. The pitch of the screw gauge is : **[NEET-2020]**

(1) 0.5 mm (2) 1.0 mm
(3) 0.01 mm (4) 0.25 mm

22. Taking into account of the significant figures, what is the value of 9.99 m–0.0099 m? **[NEET-2020]**

(1) 9.980 m
(2) 9.9 m
(3) 9.9801 m
(4) 9.98 m

23. Time intervals measured by a clock give the following readings :
1.25 s, 1.24 s, 1.27 s, 1.21 s and 1.28 s.
What is the percentage relative error of the observations ? **[NEET_Covid_2020]**

(1) 2 % (2) 4 %
(3) 16 % (4) 1.6 %

24. If force [F], acceleration [A] and time [T] are chosen as the fundamental physical quantities. Find the dimensions of energy. **[NEET-2021]**

(1) [F] [A] [T] (2) [F] [A] $[T^2]$
(3) [F] [A] $[T^{-1}]$ (4) [F] $[A^{-1}]$ [T]

25. A screw gauge gives the following readings when used to measure the diameter of a wire Main scale reading : 0 mm
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to100 divisions on the circular scale. The diameter of the wire from the above data is : **[NEET-2021]**

(1) 0.52 cm (2) 0.026 cm
(3) 0.26 cm (4) 0.052 cm

26. If E and G respectively denote energy and gravitational constant, then $\frac{E}{G}$ has the dimensions of **[NEET-2021]**

(1) $[M^2][L^{-1}][T^0]$ (2) $[M][L^{-1}][T^{-1}]$
(3) $[M][L^0][T^0]$ (4) $[M^2][L^{-2}][T^{-1}]$

27. Plane angle and solid angle have: **[NEET-2022]**

(1) Units but no dimensions
(2) Dimensions but no units
(3) No units and no dimensions
(4) Both units and dimensions

28. The area of a rectangular field (in m^2) of length 55.3 m and breadth 25m after rounding off the value for correct significant digits is: **[NEET-2022]**

(1) 138×10^1
(2) 1382
(3) 1382.5
(4) 14×10^2

29. The physical quantity that has the same dimensional formula as pressure is: **[Re-NEET-2022]**

(1) Momentum
(2) Young's modulus of elasticity
(3) Coefficient of viscosity
(4) Force

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	4	1	2	1	2	3	3	3	1	2	4	2	2	3	4	4	2	4	3	2	1	4	4	2	4
Que.	26	27	28	29																					
Ans.	1	1	4	2																					