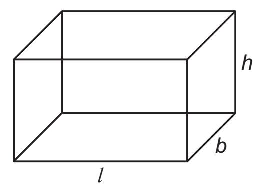
# 11

# Volume and Surface Area of Solids

## (Further Continued from Class IX)

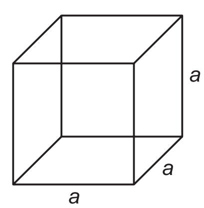
### KEY FACTS

**1. Cuboid:** For a cuboid of length $l$, breadth $b$ and height $h$,



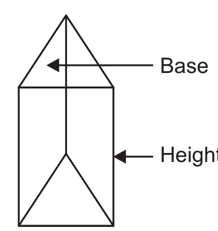

- **Volume of cuboid = $(l \times b \times h)$ cu. units.**
- **Whole surface of cuboid = $2(lb + bh + lh)$ sq. units.**
- **Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$ units.**
- **Area of four walls = $2(l + b)h$ sq. units.**

**2. Cube:** For a cube of edge length $a$,




- **Volume = $a^3$ cu. units.**
- **Whole surface = $6a^2$ sq. units.**
- **Diagonal = $\sqrt{3}a$ units.**

**3. Prism:**




- **Surface area** = 2 × Area of base shape + Perimeter of base shape × Height
- **Volume** = Area of base shape × Height of prism.

In case of a triangular prism,

◆ Area of base (triangle) = $\frac{1}{2} \times b \times h$, if base ($b$) and height ($h$) of triangle are known.

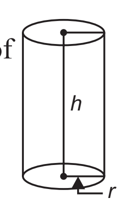
◆ Area of base (scalene triangle) = $\sqrt{s(s-a)(s-b)(s-c)}$, if all the sides $a, b, c$ are known where $s = \frac{a+b+c}{2}$

◆ Area of base (equilateral triangle) = $\frac{\sqrt{3}}{4} a^2$, where '$a$' is each side of the equilateral triangle.

For a hexagonal prism,

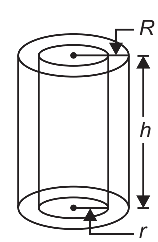
Area of base (regular hexagon) = $\frac{\sqrt{3}}{2} (\text{edge})^2$.

**4.** (*a*) **Right Circular Solid Cylinder:** For a right circular cylinder of radius of base ($r$) and perpendicular height ($h$),




- Curved Surface Area ($CSA$) = $\mathbf{2\pi rh}$ sq. units.
- Total Surface Area ($TSA$) = $\mathbf{2\pi rh + 2\pi r^2 = 2\pi r(h + r)}$ sq. units
- Volume = $\pi r^2 h$ cu. units.

(*b*) **Right Circular Hollow Cylinder:** For a hollow cylinder, whose inner radius = $r$, Outer radius = $R$ and perpendicular height = $h$,

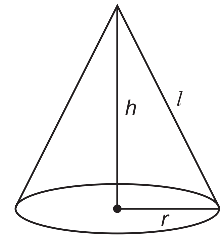



- **Curved Surface Area** = External $CSA$ + Internal $CSA$

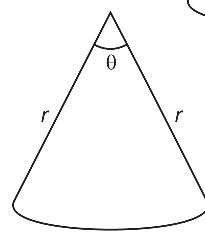
$= 2\pi Rh + 2\pi rh = \mathbf{2\pi h (R + r)}$ **sq. units.**

- **Total Surface Area** = Curved Surface Area + Area of bases
  $= 2\pi h\,(R + r) + 2\pi\,(R^2 - r^2)$
  $= 2\pi\,(R + r)\,(h + R - r)$ sq. units.
- Volume of material used in making the hollow cylinder = $\pi R^2h - \pi r^2h = \pi h\,(R^2 - r^2)$ cu. units.

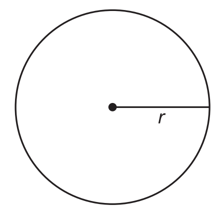
**5. Right Circular Cone:** For a right circular cone of base radius ($r$), perpendicular height ($h$) and slant height ($l$),




- **Curved Surface Area = $\pi rl$** $= \pi r\sqrt{h^2 + r^2}$ sq. units.
- **Total Surface Area = $\pi rl + \pi r^2 = \pi r(l + r)$** $= \pi r\,(\sqrt{h^2 + r^2} + r)$ sq. units
- **Volume = $\frac{1}{3}\pi r^2 h$ cu. units.**
- Curved surface area of a cone, when sector of a circle is converted into a cone = $\frac{\theta}{360^\circ} \times \pi r^2$, where $\theta$ is the sector angle and $r$ the bounding radii




**6. Sphere:** For a sphere with radius **($r$)**,

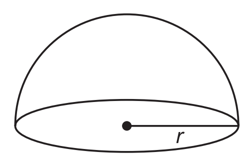



- **Volume = $\frac{4}{3}\pi r^3$ cu. units**
- **Surface area = $4\pi r^2$ sq. units**

For a hollow sphere of external radius **($R$)** and internal radius **($r$)**,

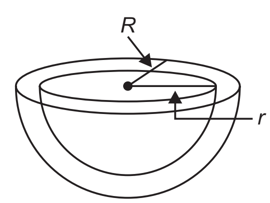
- **Volume = $\frac{4}{3}\pi(R^3 - r^3)$ cu. units**

**7. Hemisphere:** For a hemisphere with radius **($r$)**,




- **Volume = $\frac{2}{3}\pi r^3$ cu. units.**
- **Curved surface area = $2\pi r^2$ sq. units.**
- **Total surface area = $3\pi r^2$ sq. units.**

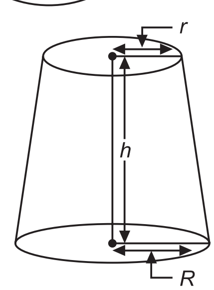
For a hollow hemisphere of external radius **($R$)** and internal radius **($r$)**,




- Volume = $\frac{2}{3}\pi\,(R^3 - r^3)$ cu. units.
- Curved surface area = $2\pi\,(R^2 + r^2)$ sq. units.
- Total surface area = $2\pi\,(R^2 + r^2) + \pi(R^2 - r^2)$ sq. units.

**8. Frustum of a cone:** If a cone is cut by a plane parallel to the base of the cone, then the portion between the plane and base is called the frustum of a cone.

If ***R*** and ***r*** are respectively the radii of the base and top of the frustum and ***h***, the height of the frustum, then




- **Volume of frustum of right circular cone = $\frac{\pi h}{3}$ [$R^2 + r^2 + Rr$] cu.** units.
- **Lateral curved surface area of frustum** $= \pi(R + r)l$ sq. units,
  where, $l$ = slant height of frustum = $\sqrt{h^2 + (R - r)^2}$.
- Total surface area of frustum = Lateral surface area + Area of base + Area of top
  $= \pi(R + r)l + \pi R^2 + \pi r^2 = \pi[R^2 + r^2 + l(R + r)]$ sq. units.
- Total surface area of bucket = $\pi(R + r)l + \pi r^2$ (as a bucket is open at the bigger end).

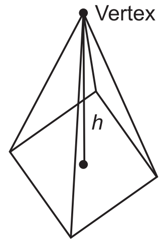
**9. Pyramid:** A solid figure with a polygonal base and triangular faces that meet at a common point (vertex) outside the plane of the base.

**Types of Pyramids**

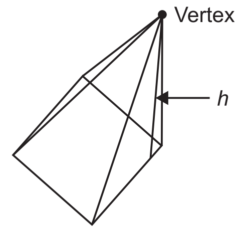
**Regular Pyramid:** The base of the pyramid is a regular polygon and hence its lateral faces are equally sized.

**Irregular Pyramid:** The base of the pyramid is an irregular polygon and hence its lateral faces are not equally sized.

**Right Pyramid:** If the vertex or apex is directly above the centre of the base, it is a right pyramid, otherwise an **oblique pyramid.**



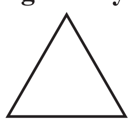
Right Pyramid



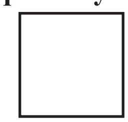
Oblique Pyramid

Pyramids can also be classified according to the shape of their bases :

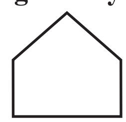
- **Triangular Pyramid**
- **Square Pyramid**
- **Pentagonal Pyramid**
- **Hexagonal Pyramid**



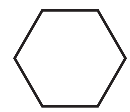
**Base:** Triangle



**Base:** Square



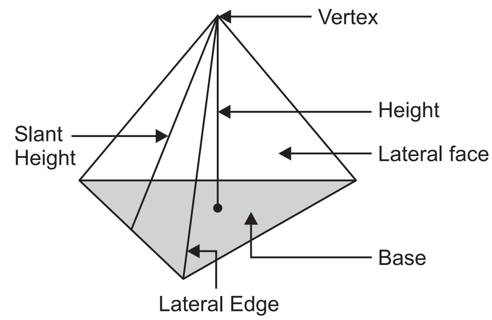
**Base:** Pentagon



**Base:** Hexagon and so on;

**Parts of a Pyramid:**

- **Height:** Perpendicular from vertex to base.
- **Lateral Edge:** The edges through the vertex of a pyramid.
- **Slant Height:** The height of a lateral face of a regular pyramid. It is the line segment joining the vertex to the mid-point of any one of the sides of the base.



Triangular Pyramid

- **Volume of a pyramid =** $\mathbf{\frac{1}{3}}$ **× Base Area × Height.**
- **Surface area of a pyramid when all side faces are same = Base Area +** $\mathbf{\frac{1}{2}}$ **× Perimeter of base × Slant height.**
- **Surface area of a pyramid, when all side faces are different = Base Area + Lateral Area.**
- **For a right pyramid with an equilateral triangle of side '*a*' as base and height '*h*'.**
  - ◆ Lateral edge = $\sqrt{h^2 + \frac{a^2}{3}}$
  - ◆ Slant height = $\sqrt{h^2 + \frac{a^2}{12}}$
  - ◆ Lateral surface area = $\frac{1}{2} \times$ Perimeter of base × Slant height = $\frac{1}{2} \times 3a \times \sqrt{h^2 + \frac{a^2}{12}}$
  - ◆ Total surface area = $\frac{1}{3} \times 3a \times \sqrt{h^2 + \frac{a^2}{12}} + \frac{\sqrt{3}}{4} a^2$
  - ◆ Volume = $\frac{1}{3} \times \frac{\sqrt{3}}{4} a^2 \times h = \frac{\sqrt{3}}{12} a^2 h$
  - ◆ Area of one lateral face = $\frac{1}{2}$ × Length of edge of base × Slant height = $\frac{1}{2} \times a \times \sqrt{h^2 + \frac{a^2}{12}}$
- **For a regular tetrahedron (all edges are equal, all four faces including base are congruent equilateral triangles)**
  - ◆ Height = $\sqrt{\frac{2}{3}}$ edge
  - ◆ Slant height = $\frac{\sqrt{3}}{2}$ edge

◆ Lateral surface area = $\frac{3\sqrt{3}}{4}(\text{edge})^2$

◆ Total surface area = $\sqrt{3}\,(\text{edge})^2$

◆ Volume = $\frac{\sqrt{2}}{12}(\text{edge})^3$.

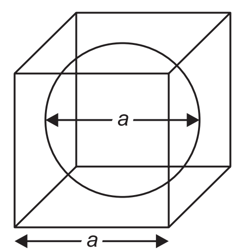
**10. Immersion of Solids:** If a solid or solids of given dimension is/are dropped in a vessel partly filled with water, and is submerged completely, then the rise in water level in the vessel can be calculated using the principle:
Volume of displaced water = Volume of submerged solid, where the height of water displaced will be the required height.

**11.** When a solid is melted and converted to another solid, then volume of both the solids remain the same, assuming there is no wastage in conversions.
Also, **Number of new solids obtained by recasting**

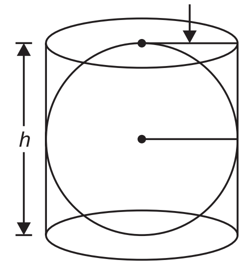
$$= \frac{\text{Volume of the solid that is melted}}{\text{Volume of the solid that is made}}.$$

**12. Combination of Solids:** When calculating the surface area of a combination of solids, you need to consider the visible surface of the component solids, which is normally the total curved surface area of the component solids. For volume, you need to calculate the total volume of the component solids.

**13. Some common solids inscribed in a given solid or solid circumscribing other solid.**

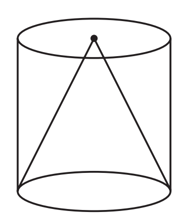



- If a largest possible sphere is circumscribed by a cube of edge '$a$' units, then radius of the sphere = $\frac{a}{2}$ units.




- If a largest possible sphere is inscribed in a cylinder of radius '$a$' units and height "$h$" units, then
  ($i$) For $h > a$, radius of sphere = $a$ units
  ($ii$) For $h < a$, radius = $\frac{h}{2}$ units.

- If a largest possible cube is inscribed in a sphere of radius '$a$' units, then the edge of the cube = $\frac{2a}{\sqrt{3}}$ units.



- If a largest possible cone is inscribed in a cylinder of radius '$a$' units and height '$h$' units, then radius of the cone = '$a$' units, height of the cone = '$h$' units.

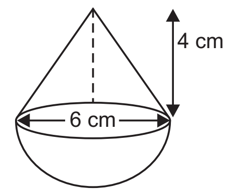
**14. Volume of water that flows out through a pipe** = (Cross-section area × Speed × Time)

## SOLVED EXAMPLES

### Combination of Solids

**Ex. 1. A toy is in the form of a cone mounted on a hemisphere such that the diameter of the base of the cone is equal to that of the hemisphere. If the diameter of the base of the cone is 6 cm and its height is 4 cm, what is the surface area of the toy in sq. cm. (Take π = 3.14)** **(*CDS 2011*)**

**Sol.** For the conical part, $r = \frac{6}{2} = 3$ cm, $h = 4$ cm, $l = \sqrt{h^2 + r^2} = \sqrt{3^2 + 4^2}$ cm $= 5$ cm




Surface area of the conical part $= \pi rl = (3.14 \times 3 \times 5)$ cm$^2$ = 47.1 cm$^2$

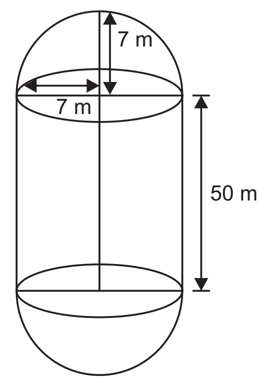
For the hemispherical part, $r = \frac{6}{2} = 3$ cm

Surface area of the hemispherical part $= 2\pi r^2 = 2 \times 3.14 \times 3 \times 3 = 56.52$ cm$^2$

$\therefore$ Surface area of the toy = 47.1 cm$^2$ + 56.52 cm$^2$ = **103.62 cm$^2$.**

**Ex. 2. A storage tank consists of a circular cylinder with a hemisphere adjoined on either side. If the external diameter of the cylinder be 14 m and its length be 50 m, then what will be the cost of painting it at the rate of ₹ 10 per sq. m? (*MAT 2003*)**

**Sol.** Let $r$ (= 7) cm be the radius of the base of the cylinder, hence of hemispheres and $h$, the height of the cylinder




Surface area of the tank

= Curved surface area of cylinder + 2 × Curved surface area of hemisphere

$= 2\pi rh + 2 \times 2\pi r^2$

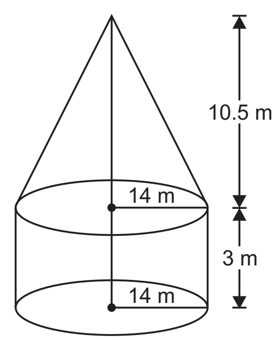
$= 2 \times \frac{22}{7} \times 7 \times 50 + 4 \times \frac{22}{7} \times 7 \times 7$

$= 2200 + 616 = 2816$ m$^2$

$\therefore$ Required cost of painting = 2816 × ₹ 10 = ₹ **28160.**

**Ex. 3. A tent is of the shape of a right circular cylinder upto a height of 3 metres and then becomes a right circular cone with a maximum height of 13.5 metres above the ground. Calculate the cost of painting the inner side of the tent at the rate of ₹ 2 per square metre, if the radius of the base is 14 metres.**

**Sol.** Total area to be painted = Curved Surface Area of cylinder + *CSA* of cone




*CSA* of cylinder $= 2\pi rh = \left(2 \times \frac{22}{7} \times 14 \times 3\right)$m$^2$ = 264 m$^2$

*CSA* of cone $= \pi rl$ where $l$ = slant height

$l = \sqrt{r^2 + h^2} = \sqrt{14^2 + (10.5)^2}$ m

$= \sqrt{196 + 110.25}$ m $= \sqrt{306.25}$ m $= 17.5$ m

$\therefore$ *CSA* of cone $= \left(\frac{22}{7} \times 14 \times 17.5\right)$m $= 770$ m$^2$

$\therefore$ Cost of painting = ₹ 2 × Total *CSA*

= ₹ 2 × (264 + 770) = ₹ [2 × 1034] = ₹ **2068.**

## Immersion of Solids

**Ex. 4. In a cylindrical vessel of diameter 24 cm filled up with sufficient quantity of water, a solid spherical ball of radius 6 cm is completely immersed. What is the increase in height of water level? (*CDS 2012*)**

**Sol.** Let the height of the increased water level be $h$ cm.

Then, Volume of water displaced = Volume of the sphere

$\Rightarrow \pi r^2 h = \frac{4}{3}\pi R^3$, (where, $r$ = radius of cylinder, $R$ = radius of sphere)

$\Rightarrow 12 \times 12 \times h = \frac{4}{3} \times 6 \times 6 \times 6 \Rightarrow h = \frac{4 \times 6 \times 6 \times 6}{3 \times 12 \times 12} =$ **2 cm.**

**Ex. 5. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker containing some water and are fully submerged. The diameter of the beaker is 7 cm. Find how many marbles have been dropped in it if the water rises by 5.6 cm.** (***SSC 2011***)

**Sol.** Number of marbles $= \dfrac{\text{Volume of raised water in the cylindrical beaker}}{\text{Total Volume of marbles}} = \dfrac{\pi r^2 h}{\frac{4}{3}\pi R^3}$ $\left(\begin{array}{r}\text{where } r = \text{radius of beaker,}\\ R = \text{radius of marble}\end{array}\right)$

$$= \frac{\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 5.6}{\frac{4}{3}\times\frac{22}{7}\times 0.7\times 0.7\times 0.7} = \mathbf{150.}$$

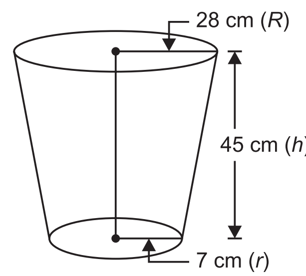
## Frustum of a Cone

**Ex. 6. What is the capacity and surface area of a bucket, the radii of whose circular ends are 28 cm and 7 cm and height is 45 cm?**

**Sol.** Capacity of the bucket = Volume of frustum of a cone $= \dfrac{\pi h}{3}[R^2 + r^2 + Rr]$

$$= \frac{22}{7}\times\frac{45}{3}[(28)^2 + (7)^2 + 28\times 7] = \frac{330}{7}\times[784 + 49 + 196]$$

$$= \frac{330}{7}\times 1029 = 330\times 147 = \mathbf{48510\ cu.\ cm.}$$




Surface area of the bucket = Lateral surface area of the frustum of the cone + Area of the bottom

$= \pi l(R + r) + \pi r^2$

(Slant height) $l = \sqrt{h^2 + (R-r)^2} = \sqrt{45^2 + (28-7)^2} = \sqrt{2025 + 441} = \sqrt{2466} = 49.66$ cm (approx)

$\therefore$ Surface area $= \dfrac{22}{7}\times 49.66\times 35 + \dfrac{22}{7}\times 7^2$

$= (5462.6 + 154)$ sq. cm = **5616.6 sq. cm.**

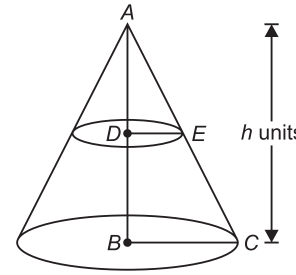
**Ex. 7. A right circular cone is divided into two portions by a plane parallel to the base and passing through a point which is at $\frac{1}{3}$ of the height from the top. The ratio of the volume of the smaller cone to that of the remaining frustum of the cone is:** (***CDS 2001***)

**Sol.** Let the height $AB$ of the cone be $h$ units. Then $AD = \frac{1}{3}h$

As $\Delta ADE \sim \Delta ABC$ (similar $\Delta$s) $\dfrac{AD}{AB} = \dfrac{DE}{BC} = \dfrac{1}{3} \Rightarrow AD = \dfrac{1}{3}AB$ and $DE = \dfrac{1}{3}BC$

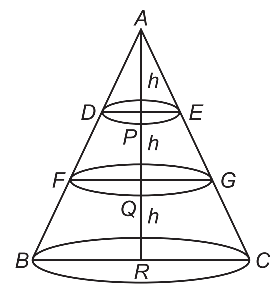
$$\therefore \quad \text{Required ratio} = \frac{\frac{1}{3}\pi(DE)^2\times AD}{\frac{1}{3}\pi(BC)^2\times AB - \frac{1}{3}\pi(DE)^2\times AD} = \frac{(DE)^2\times AD}{BC^2\times AB - DE^2\times AD}$$

$$= \frac{\frac{1}{9}BC^2\times\frac{1}{3}AB}{BC^2\times AB - \frac{1}{9}BC^2\times\frac{1}{3}AB} = \frac{\frac{1}{27}}{1-\frac{1}{27}} = \frac{1/27}{26/27} = \mathbf{1 : 26.}$$



**Ex. 8. A right circular cone is cut by two planes parallel to the base and trisecting the altitude. What is the ratio of the volumes of the three parts; top, middle and bottom respectively? (*CDS 2005*)**

**Sol.** The cone be divided into three parts by the two planes trisecting the cone. Let the height of each of the trisected portion be $h$ units.




It is obvious the $\Delta ADE$, $\Delta AFG$ and $\Delta ABC$ are similar. Let DE = $2x$.

As the three given $\Delta s$ are similar, $\dfrac{FG}{DE} = \dfrac{AQ}{AP} = \dfrac{2h}{h}$

$\Rightarrow FG = 2DE = 4x$

Similarly $BC = 3DE = 6x$

$\therefore$ $PE = x$, $QG = 2x$ and $RC = 3x$

Now, Volume of cone $ABC = \dfrac{1}{3} \times \pi \times (3x)^2 \times (3h) = 9\pi x^2 h$ ...(*i*)

Volume of cone $AFG = \dfrac{1}{3} \times \pi \times (2x)^2 \times (2h) = \dfrac{8}{3}\pi x^2 h$ ...(*ii*)

Volume of cone $ADE = \dfrac{1}{3} \times \pi \times (x)^2 \times h = \dfrac{1}{3}\pi x^2 h$ ...(*iii*)

$\therefore$ Volume of middle portion $DEFG$ = Vol. of cone $AFG$ – Vol. of cone $ADE$ = $\dfrac{8}{3}\pi x^2 h - \dfrac{\pi x^2 h}{3} = \dfrac{7\pi x^2 h}{3}$ ...(*iv*)

Volume of the lowermost portion $FGBC$ = Vol. of cone $ABC$ – Vol. of cone $AFG$

$$= 9\pi x^2 h - \frac{8}{3}\pi x^2 h = \frac{19\pi x^2 h}{3}$$

$\therefore$ Reqd. ratio = $\dfrac{\pi x^2 h}{3} : \dfrac{7\pi x^2 h}{3} : \dfrac{19\pi x^2 h}{3}$ = **1 : 7 : 19.**

**Ex. 9. A container opened from the top and made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container at the rate of ₹ 20 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 8 per 100 cm². (Take $\pi$ = 3.14) (*NCERT*)**

**Sol.** Reqd. cost of milk = Vol. of the container × ₹ 20. The container is in the shape of the frustum of a cone whose, height **($h$)** = 16 cm, radius of lower end **($r_1$)** = 8 cm, radius of upper end **($r_2$)** = 20 cm

$\therefore$ Volume of the container = $\dfrac{\pi}{3} \times h \times (r_1^2 + r_2^2 + r_1 r_2)$ [Using formula for frustum of a cone]

$$= \frac{3.14}{3} \times 16 \times [(8)^2 + (20)^2 + 160]$$

$$= \frac{3.14}{3} \times 16 \times (64 + 400 + 160)$$

$$= \frac{3.14 \times 16 \times 624}{3} \text{ cm}^3 = \frac{3.14 \times 16 \times 624}{3 \times 1000} \text{ L} = 10.45 \text{ L (approx)}$$

$\therefore$ Cost of milk = ₹ (10.45 × 20) = ₹ 209 (approx)

Area of the metal sheet used to make the container

= Lateral surface area of the container + Area of the base = $\pi l\,(r_1 + r_2) + \pi r_1^2$

where, $l = \sqrt{h^2 + (r_2 - r_1)^2} = \sqrt{16^2 + (20-8)^2} = \sqrt{256 + 144} = \sqrt{400} = 20$ cm

$\therefore$ Required area = 3.14 × 20 × 28 + 3.14 × 64 = 1758.4 + 200.96 = 1959.36 cm²

$\therefore$ Cost of metal sheet = ₹ $\left(\dfrac{8 \times 1959.36}{100}\right)$ = **₹ 156.75** (approx).

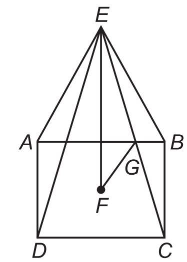
## Pyramids and Tetrahedron

**Ex. 10. A right pyramid 10 cm high has a square base of which the diagonal is 10 cm. What is the whole surface area of the pyramid?** (*CDS 1996*)

**Sol.** Whole surface area of the pyramid

= Lateral surface area + Area of the square base + $(\text{Side})^2$

= 4 × Area of triangular faces




Each side of the square $(a) = \dfrac{\text{diagonal}}{\sqrt{2}} = \dfrac{10}{\sqrt{2}}$ cm, Height $(h)$ = 10 cm

$EG$ = height of a triangular face

$AB$ = base of the triangular face

$\therefore$ Height of the triangle $= \sqrt{(10)^2 + \left(\dfrac{1}{2}\times\dfrac{10}{\sqrt{2}}\right)^2} = \sqrt{100+\dfrac{25}{2}} = \sqrt{112.5} = 10.6$ cm (approx)

$\therefore$ Whole surface area $= 4\times\dfrac{1}{2}\times AB\times EG + (AB)^2 = 4\times\dfrac{1}{2}\times\dfrac{10}{\sqrt{2}}\times 10.6 + \left(\dfrac{10}{\sqrt{2}}\right)^2$

$= 149.91\ \text{cm}^2 + 50\ \text{cm}^2 = \mathbf{200\ cm^2}$ (approx).

**Ex. 11. The base of a right pyramid is an equilateral triangle each side of which is 4 m long. Every slant edge is 5 m long. Find the lateral surface area and volume of the pyramid?**

**Sol.** Let '$h$' be the height of the pyramid and '$a$' the length of each side of the base of an equilateral triangle.

Then, slant edge $= \sqrt{h^2 + \dfrac{a^2}{3}}$

$\Rightarrow 5 = \sqrt{h^2 + \dfrac{16}{3}} \Rightarrow 25 - \dfrac{16}{3} = h^2 \Rightarrow h^2 = \dfrac{59}{3} \Rightarrow h = \sqrt{\dfrac{59}{3}}$ m

Slant height $= \sqrt{h^2 + \dfrac{a^2}{12}} = \sqrt{\dfrac{59}{3}+\dfrac{16}{12}} = \sqrt{21}$ m

$\therefore$ Lateral surface area $= \dfrac{1}{2}$ (Perimeter of base × Slant height) $= \dfrac{1}{2}(4+4+4)\times\sqrt{21}\ \text{m}^2 = \mathbf{6\sqrt{21}\ m^2}$

Volume of the pyramid $= \dfrac{1}{3}$ (Area of base × height) $= \dfrac{1}{3}\times\dfrac{\sqrt{3}}{4}\times 4^2\times\sqrt{\dfrac{59}{3}}\ \text{m}^3 = \mathbf{\dfrac{4\sqrt{59}}{3}\ m^3}$.

**Ex. 12. A right pyramid is on a regular hexagonal base. Each side of the base is 10 m and its height is 30 m. Find the volume of the pyramid ?**

**Sol.** Volume of a pyramid $= \dfrac{1}{3}$ × (Area of the base) × Height

$= \dfrac{1}{3}\times\dfrac{3\sqrt{3}}{2}\times(5)^2\times 30 = 649.52\ \text{m}^3 \approx \mathbf{650\ m^3}$. $\left(\text{Area of a regular hexagon of side '}a\text{'} = \dfrac{3\sqrt{3}}{2}a^2\right)$

**Ex. 13. Find the volume, lateral surface area and total surface area of a regular tetrahedron whose edge is 16 cm.**

**Sol.** Volume of a regular tetrahedron $= \dfrac{\sqrt{2}}{12}(\text{edge})^2 = \dfrac{\sqrt{2}}{12}\times 16^2 = \dfrac{\sqrt{2}\times 256}{12}\ \text{cm}^3 = \mathbf{\dfrac{64\sqrt{2}}{3}}\ \text{cm}^3$

Lateral surface area $= \dfrac{3\sqrt{3}}{4}(\text{edge})^2 = \dfrac{3\sqrt{3}}{4}\times 16^2\ \text{cm}^2 = \mathbf{192\sqrt{3}\ cm^2}$

Total surface area $= \sqrt{3}\,(\text{edge})^2 = \sqrt{3}\times 16^2\ \text{cm}^2 = \mathbf{256\sqrt{3}\ cm^2}$.

**Ex. 14. If '$p$' be the length of the perpendicular drawn from the vertex of a regular tetrahedron to its opposite face and each edge is of length $2a$, show that $3p^2 = 8a^2$.**

**Sol.** Height of tetrahedron = $\sqrt{\frac{2}{3}} \times$ (length of an edge)

$$\Rightarrow p = \sqrt{\frac{2}{3}} \times 2a \Rightarrow p^2 = \frac{8a^2}{3} \Rightarrow \mathbf{3p^2 = 8a^2}.$$

## PRACTICE SHEET–1

**1.** A vertical cone of Volume $V$ with vertex downwards is filled with water upto half its height. The volume of water is:

(*a*) $\frac{V}{16}$ (*b*) $\frac{V}{8}$ (*c*) $\frac{V}{4}$ (*d*) $\frac{V}{2}$

**(*CDS 2001*)**

**2.** If $h$, $C$, $V$ are respectively the height, the curved surface area and volume of a cone, then $3\pi V^3 - C^2h^2 + 9V^2$ is equal to

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

**(*CDS 2003*)**

**3.** What is the semi-vertical angle of a cone whose lateral surface area is double the base area?

(*a*) 30º (*b*) 45º (*c*) 60º (*d*) 15º

**(*CDS 2004*)**

**4.** A square hole of cross-sectional area 4 cm$^2$ is drilled across a cube with its length parallel to a side of the cube. If an edge of the cube measures 5 cm, what is the total surface area of the body so formed?

(*a*) 140 cm$^2$ (*b*) 142 cm$^2$ (*c*) 162 cm$^2$ (*d*) 182 cm$^2$

**(*CDS 2004*)**

**5.** A closed right circular cone contains water upto a height $h/2$ above the base, where $h$ is the height of the cone. To what height does the water rise if the cone is inverted?

(*a*) $h/2$ (*b*) $3h/4$ (*c*) $\left(\frac{7}{8}\right)^{1/2} h$ (*d*) $\left(\frac{7}{8}\right)^{1/3} h$

**(*CDS 2005*)**

**6.** Two concentric spheres $A$ and $B$, have radii $r$ and $2r$ respectively. A cone is inscribed in the latter so as to circumscribe the former. What is the curved surface area of the cone?

(*a*) $2\pi r^2$ (*b*) $4\pi r^2$ (*c*) $6\pi r^2$ (*d*) $8\pi r^2$

**(*CDS 2005*)**

**7.** The radius and height of a right solid circular cone are $r$ and $h$ respectively. A conical cavity of radius $r/2$ and height $h/2$ is cut out of the cone. What is the whole surface of the rest of the portion?

(*a*) $\frac{\pi r}{4}(5\sqrt{r^2+h^2}+3r)$ (*b*) $\frac{5\pi r}{4}(\sqrt{r^2+h^2})$

(*c*) $\frac{3\pi r}{4}(\sqrt{r^2+h^2}+r)$ (*d*) $\frac{3\pi r}{7}(\sqrt{r^2+h^2}+r)$

**(*CDS 2005*)**

**8.** A sphere of radius 13 cm is cut by a plane whose distance from the centre of the sphere is 5 cm. What is the circumference of the plane circular section?

(*a*) 10 π cm (*b*) 12 π cm (*c*) 24 π cm (*d*) 26 π cm

**(*CDS 2005*)**

**9.** A double cone is formed by a complete revolution of the triangle $ABC$ about the side $AB$. The sides $BC$ = 6.5 cm, $CA$ = 2 cm and the perpendicular from $C$ on $AB$ = 1.6 cm. The volume of the double cone is approximately:

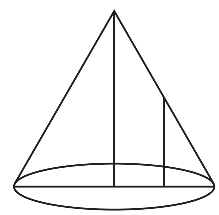
(*a*) 25 cm$^3$ (*b*) 24 cm$^3$ (*c*) 22 cm$^3$ (*d*) 20 cm$^3$

**(*CDS 2001*)**

**10.** A sphere of radius 5 cm exactly fits into a cubical box. The ratio of the surface of the box and the surface of the sphere is:

(*a*) 19 : 9 (*b*) 21 : 11 (*c*) 23 : 13 (*d*) 25 : 13

**11.** The diameter of a right conical tent is 6 metre. If a pole of length 2 metres can be fixed up in the tent at half the distance of the radius from the centre of the base, then the area of canvas required is



(*a*) 10 π (*b*) 12 π

(*c*) 15 π (*d*) 16 π **(*CDS*)**

**12.** A thin walled glass paper weight consists of a hollow cylinder with a hollow cone on top as shown in the given Fig. (*a*). The paper weight contains just enough sand to fill the cylinder. The paper weight is now turned upside down as shown in Fig. (*b*).

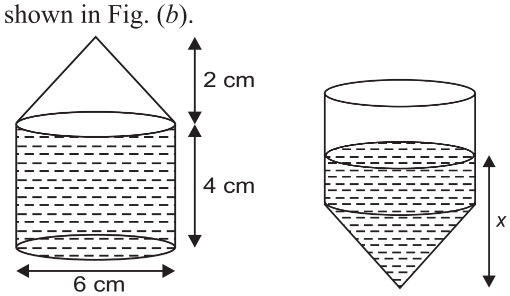
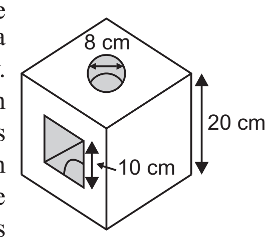



Fig. (*a*) Fig. (*b*)

Calculate the depth of the sand (marked $x$ in the diagram).

(*a*) 4 cm (*b*) 5 cm (*c*) $\frac{16}{3}$ cm (*d*) 3 cm

**13.** A solid cube has a square hole cut through horizontally and a circular hole cut through vertically. Both the holes are cut centrally in appropriate faces. The dimensions of the cube and the hole are shown in the diagram. Calculate the volume remaining after the holes have been cut. (Take π = 3.14)




(*a*) 4995.2 cm$^3$ (*b*) 5497.6 cm$^3$

(*c*) 5748.8 cm$^3$ (*d*) 5994.2 cm$^3$

**14.** Shown below in Fig. (*a*) is a closed box which is a prism of length 40 cm. The cross-section of the box is shown in Fig. (*b*) with all right angles marked.

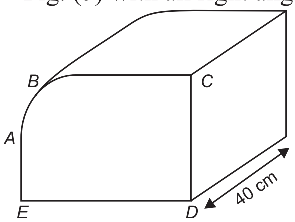



Fig. (*a*)

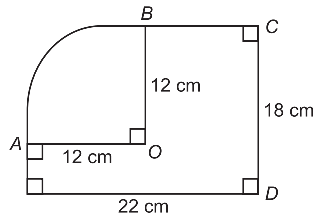



Fig. (*b*)

The approximate volume of the prism is

(*a*) 15200 $m^3$ (*b*) 14600 $m^3$

(*c*) 13650 $m^3$ (*d*) 12500 $m^3$

**15.** A child consumed an ice-cream of inverted right-circular conical shape form the top and left only 12.5% of the cone for her mother. If the height of the ice-cream cone was 8 cm, what was the height of the remaining ice-cream cone?

(*a*) 2.5 cm (*b*) 3 cm (*c*) 3.5 cm (*d*) 4 cm.

**(*JMET 2009*)**

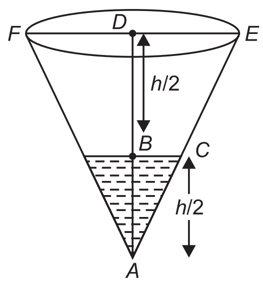
## ANSWERS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** (*b*) | **2.** (*a*) | **3.** (*a*) | **4.** (*d*) | **5.** (*d*) | **6.** (*c*) | **7.** (*a*) | **8.** (*c*) | **9.** (*d*) | **10.** (*b*) |
| **11.** (*c*) | **12.** (*c*) | **13.** (*b*) | **14.** (*b*) | **15.** (*d*) | | | | | |

## HINTS AND SOLUTIONS

**1.** Let the water be filled in *AEF* as shown in the given figure.

Then, $AB = BD = h/2$, where $h$ is the height of the cone.




Let $DE = r$ be the radius of the cone.

Then $V = \frac{1}{3}\pi r^2 h$

$\Delta ABC \sim \Delta ADE$

$\therefore \quad \frac{AB}{AD} = \frac{BC}{DE}$

$\Rightarrow \quad \frac{h/2}{h} = \frac{BC}{r} \Rightarrow BC = \frac{1}{2}r$

$\therefore$ Volume of water = Vol. of cone $ABC = \frac{1}{3}\pi.(BC)^2 \times AB$

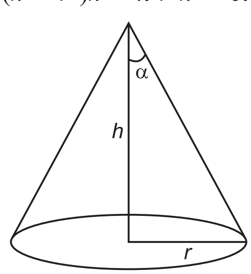
$$= \frac{1}{3}\pi.\left(\frac{1}{2}r\right)^2 \times \frac{1}{2}h = \frac{1}{24}\pi r^2 h = \frac{1}{8}\left(\frac{1}{3}\pi r^2 h\right) = \boldsymbol{\frac{1}{8}V}.$$

**2.** $C = \pi rl = \pi r\sqrt{h^2 + r^2}$ and $V = \frac{1}{3}\pi r^2 h$ where, $r$ and $l$ are respectively the radius of the base and slant height of the cone.

$\therefore\ 3\pi Vh^3 - C^2h^2 + 9V^2 = 3\pi \times \frac{1}{3}\pi r^2 h \times h^3 - \pi^2 r^2(h^2 + r^2)h^2 + 9 \times \frac{1}{9}\pi^2 r^4 h^2$

$$= \pi^2 r^2 h^4 - \pi^2 r^2(h^2 + r^2)h^2 - \pi^2 r^4 h^2 = \mathbf{0}.$$

**3.** Let the semi vertical angle of the cone be $\alpha$, the height $h$, radius of base $r$, and slant height $l$.




Then, Lateral (Curved) surface area of cone = $\pi rl = \pi r\,(r \operatorname{cosec} \alpha)$

Base area of the cone = $\pi r^2$

$\therefore$ Given, $\pi r(r \operatorname{cosec} \alpha) = 2\pi r^2$

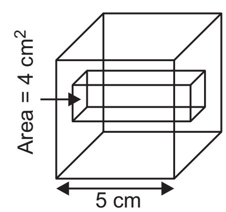
$\Rightarrow$ $\operatorname{cosec} \alpha = 2 = \operatorname{cosec} 30^\circ$

$\therefore$ $\boldsymbol{\alpha = 30^\circ}$.

**4.** Total surface area of the cube having a hole of cross sectional area 4 $cm^2$

= *TSA* of cube – 2 × 4 $cm^2$

= $6(5)^2 - 2 \times 4 = (150 - 8)\ cm^2 = 142\ cm^2$




Now, Area of four walls of the hole made parallel to a side of the cube = $4 \times 5 \times 2 = 40\ cm^2$

$\therefore$ Total surface area of the body = $142\ cm^2 + 40\ cm^2$

= **182 cm²**.

**5.** Let $r$ and $h$ respectively be the radius of the base and height of the cone *AFE* (Fig. (*a*))

Then, Volume of water

= Vol. of cone *AFE* – Vol. of cone *AGC*

$$= \frac{1}{3}\pi r^2 h - \frac{1}{3}\pi\left(\frac{r}{2}\right)^2.\frac{h}{2}$$

$$\left(\because AB = \frac{h}{2} \text{ and } \frac{AB}{AD} = \frac{BC}{DE} \Rightarrow BC = \frac{1}{2}r\right)$$

$$= \frac{1}{3}\pi r^2 h - \frac{\pi r^2 h}{24} = \frac{7}{24}\pi r^2 h.$$

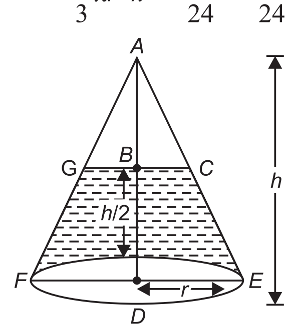



Fig. (*a*)

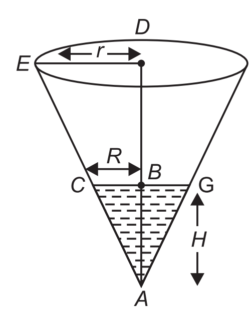



Fig. (*b*)

Now as shown in Fig. (*b*), Let the height of water in the inverted cone be $H$ cm and radius of the surface of water = $R$ cm.

Then, by similarity of Δs, $\frac{R}{H} = \frac{r}{h}$

$\Rightarrow R = \frac{Hr}{h}$

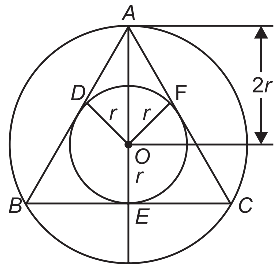
$\therefore$ Volume of water in inverted cone = $\frac{1}{3}\pi R^2 H$

$$= \frac{1}{3}\pi\left(\frac{rH}{h}\right)^2 H = \frac{1}{3}\pi\frac{r^2}{h^2}H^3$$

But given, $\frac{1}{3}\pi\frac{r^2H^3}{h^2}=\frac{7}{24}\pi r^2h$

$\Rightarrow H^3=\frac{7}{8}h^3 \Rightarrow \boldsymbol{H=\left(\frac{7}{8}\right)^{1/3}h.}$

**6.** Let $ABC$ be the cone circumscribing the sphere with centre $O$ and radius $r$ and inscribed in the sphere with centre $O$ and radius $2r$. By the properties of a circle,




$OD \perp AB$ and bisects $AB$,

$OF \perp AC$ and bisects $AC$,

$OE \perp BC$ and bisects $BC$,

Also, $\angle ODA = \angle OFA = \angle OEB = 90°$

Also $AB = BC = AC$

So, $AD^2 + OD^2 = AO^2$ (Pythagoras' Th.)

$\Rightarrow AD^2 = AO^2 - OD^2 = (2r)^2 - r^2 = 4r^2 - r^2 = 3r^2$

$\Rightarrow AD = \sqrt{3}r.$

$\Rightarrow AB = 2 \times AD = 2\sqrt{3}r = l$(slant height of cone) and

$BE=\frac{1}{2}BC=\frac{1}{2}AB=\frac{1}{2}\times 2\sqrt{3}r=\sqrt{3}r.$

$\therefore$ Curved surface area of cone $= \pi rl$

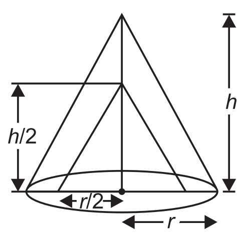
$= \pi \times \sqrt{3}r \times 2\sqrt{3}r = \boldsymbol{6\pi r.}$

**7.** Outer slant height $=\sqrt{h^2+r^2}$

Inner slant height

$=\sqrt{\left(\frac{h}{2}\right)^2+\left(\frac{r}{2}\right)^2}$

$=\frac{1}{2}\sqrt{h^2+r^2}$



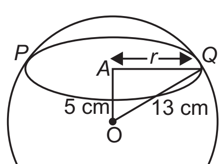

$\therefore$ Total surface area = Outer curved surface area + Inner curved surface area + Area of base

$=\pi r\sqrt{h^2+r^2}+\pi\frac{r}{2}\times\frac{\sqrt{h^2+r^2}}{2}+\pi\left[r^2-\left(\frac{r}{2}\right)^2\right]$

$=\pi r\sqrt{h^2+r^2}+\frac{\pi r}{4}\sqrt{h^2+r^2}+\frac{3\pi r^2}{4}$

$=\frac{5\pi r}{4}\sqrt{h^2+r^2}+\frac{3\pi r^2}{4}=\boldsymbol{\frac{\pi r}{4}[5\sqrt{r^2+h^2}+3r].}$

**8.** Let $O$ be the centre of the sphere and let $A$ be the centre of the circular plane. Then radius ($r$) of the plane



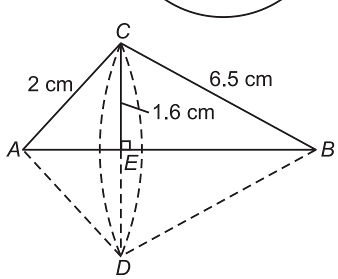

$=\sqrt{13^2-5^2}=\sqrt{169-25}$

$=\sqrt{144}=12$ cm

$\therefore$ Circumference of the plane circular section $= 2\pi r$

$= 2 \times \pi \times 12$ cm $=$ **24 π cm.**

**9.** Here, $CAB$ is the given triangle. Since $\Delta CAB$ revolves about the side $AB$, it forms two cones $CAD$ and $CBD$.




Radius of both the cones $= CE = ED = 1.6$ cm

Height of cone $CAD$

$= AE = \sqrt{2^2-1.6^2}=\sqrt{4-2.56}=\sqrt{1.44}=1.2$ cm

Height of cone $CBD = EB$

$=\sqrt{(6.5)^2-(1.6)^2}=\sqrt{42.25-2.56}=\sqrt{39.69}=6.3$ cm.

$\therefore$ Volume of double cone

= Vol. of cone $CAD$ + Vol. of cone $CBD$

$=\frac{1}{3}\times\frac{22}{7}\times(1.6)^2\times 1.2+\frac{1}{3}\times\frac{22}{7}\times(1.6)^2\times 6.3$

$=\frac{22}{21}\times 2.56\times(1.2+6.3)=\frac{22}{21}\times 2.56\times 7.5$

$= 20.11\text{ cm}^3 \approx \boldsymbol{20\text{ cm}^3.}$

**10.** Radius of sphere = 5 cm

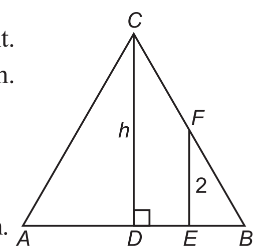
$\therefore$ Each edge of cubical box = 10 cm

$\therefore$ Required ratio $=\frac{\text{Surface area of box}}{\text{Surface area of sphere}}$

$=\frac{6\times(10)^2}{4\times\frac{22}{7}\times(5)^2}=\frac{600\times 7}{4\times 25\times 22}=\frac{21}{11}=\boldsymbol{21:11.}$

**11.** Let $h$ be the height of the conical tent.

Radius ($r$) of the conical tent = 3 cm.




$DE=EB=\frac{1}{2}DB=1.5$ cm.

$\Delta CDB \sim \Delta FEB$

$\Rightarrow \frac{h}{2}=\frac{DB}{EB}\Rightarrow\frac{h}{2}=\frac{3}{1.5}\Rightarrow h=4$ cm.

$\therefore$ Slant height of the cone $(l)=\sqrt{h^2+r^2}=\sqrt{4^2+3^2}=5$ cm.

$\therefore$ Curved surface area of the cone $= \pi rl$

$= \pi \times 3 \times 5\text{ cm}^2$

$= \boldsymbol{15\pi\text{ cm}^2.}$

**12.** Volume of sand in the cylinder $= \pi r^2h$

$= \pi \times (3)^2 \times 4\text{ cm}^3$

$= 36\pi\text{ cm}^3$

Volume of cone $=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\times(3)^2\times 2=6\pi\text{ cm}^3$

Now when the paper weight is turned upside down, vol. of sand in cone $= 6\pi\text{ cm}^3$

Remaining volume of sand in cylinder $= 36\pi\text{ cm}^3 - 6\pi\text{ cm}^3$

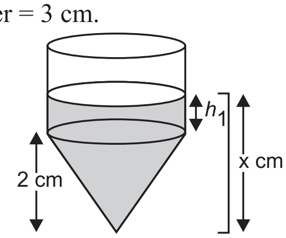
$= 30\pi\text{ cm}^3.$

Let '$h_1$' be the height of the part of the cylinder filled with sand. Radius of base of cylinder = 3 cm.

$\therefore \pi \times (3)^2 \times h_1 = 30\pi$

$\Rightarrow h_1=\frac{30}{9}=\frac{10}{3}$ cm

$\therefore x=\left(2+\frac{10}{3}\right)\text{cm}=\boldsymbol{\frac{16}{3}\text{ cm.}}$




**13.** Remaining volume = Volume of cube

– Volume of cuboid formed by cutting the square hole

– Volume of the cylinder formed by cutting the circular hole

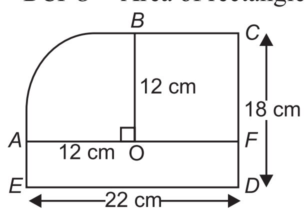
+ Common volume of cuboid and cylinder

$= (20)^3 – (10)^2 \times 20 – \pi \times (4)^2 \times 20 + \pi \times (4)^2 \times 10$
$= 8000 – 2000 – 160\pi = 6000 – 160\pi = 5497.6 \text{ cm}^3.$

**Note:** The common portion is a cylinder of diameter 8 cm and height 10 cm (side of square).

**14.** Volume of prism = Area of base × height.

Area of base = Area of crossection shown in Fig. (b)
= Area of quadrant $AOB$ + Area of rectangle $BCFO$ + Area of rectangle $AFED$



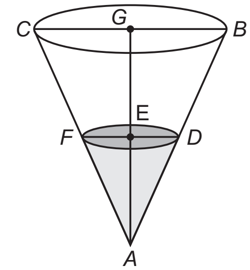

$$= \frac{1}{4}\times\frac{22}{7}\times(12)^2 + (10\times12) + (22\times6)$$

$$= \left(\frac{792}{7}+120+132\right)\text{cm}^2 = 365\frac{1}{7}\text{ cm}^2$$

$$\therefore \text{ Volume of prism} = \left(\frac{2556}{7}\times40\right)\text{cm}^3 = 14605.71 \text{ cm}^3$$

$\approx$ **14600 cm³.**

**15.** $ADF$ is the part of the cone that is filled with remaining ice cream. Now $\Delta AEF \sim \Delta AGC$, so




$$\frac{AE}{EF} = \frac{AG}{GC} \Rightarrow EF = \frac{AE\times GC}{AG}$$

Let $AE = h$, $GC = r$ (radius of the bigger cone), $AG = 8$ cm, $EF = r_1$ (radius of smaller conical part)

Then, $r_1 = \frac{hr}{8}$

So, Volume of bigger cone $= \frac{1}{3}\pi r^2.8$

Volume of smaller conical part filled with icecream

$$= \frac{1}{3}\pi r_1^2.h = \frac{1}{3}\pi\frac{h^2r^2}{64}.h = \frac{1}{3}\times\frac{\pi h^3r^2}{64}$$

Given, $\frac{1}{3}\times\frac{\pi h^3 r^2}{64} = 12.5\% \text{ of } \frac{1}{3}\pi r^2.8$

$$\Rightarrow \quad \frac{h^3}{64} = \frac{125}{1000}\times8$$

$$\Rightarrow \quad h^3 = \frac{125\times8\times64}{1000} \Rightarrow h = \frac{5\times8}{10} = \mathbf{4\ cm.}$$

## PRACTICE SHEET–2

**1.** A container is in the form of a right circular cylinder surmounted by a hemisphere of the same radius 15 cm as the cylinder. If the volume of the container is 32400 $\pi$ cm³, then the height $h$ of the container satisfies which one of the following ?

(*a*) 135 cm < $h$ < 150 cm (*b*) 140 cm < $h$ < 147 cm
(*c*) 145 cm < $h$ < 148 cm (*d*) 139 cm < $h$ < 145 cm

(***CDS 2008***)

**2.** A fountain having the shape of a right circular cone is fitted into a cylindrical tank of volume $V$, so that the base of the tank coincides with the base of the cone and the height of the tank is the same as that of the cone. The volume of water in the tank, when it is completely filled with water from the fountain is

(*a*) $\frac{V}{2}$ (*b*) $\frac{V}{3}$ (*c*) $\frac{2V}{3}$ (*d*) $\frac{V}{4}$

**3.** A vessel is in the form of a hemi-spherical bowl mounted by a hollow cylinder. The diameter of the sphere is 14 cm and the total height of the vessel is 13 cm. Find its capacity. $\left(\text{Take } \pi = \frac{22}{7}\right)$

(*a*) 1426.66 cm³ (*b*) 1264.66 cm³
(*c*) 1642.66 cm³ (*d*) 1624.66 cm³

**4.** A circus tent is cylindrical upto a height of 3 m and conical above it. If the diameter of the base is 105 m and the slant height of the conical part is 53 m, find the total canvas used in the making the tent?

(*a*) 9735 m² (*b*) 9537 m² (*c*) 9537 m² (*d*) 9753 m²

**5.** A cylindrical vessel of base radius 14 cm is filled with water to some height. If a rectangular solid of dimensions 22 cm × 7 cm × 5 cm is immersed in it, what is the rise in the water level?

(*a*) 0.5 cm (*b*) 1.0 cm (*c*) 1.25 cm (*d*) 1.5 cm

(***CDS 2009***)

**6.** Half of a large cylindrical tank open at the top is filled with water and identical heavy spherical balls are to be dropped into the tank without spilling water out. If the radius and the height of the tank are equal and each is four times the radius of a ball, what is the maximum number of balls that can be dropped?

(*a*) 12 (*b*) 24 (*c*) 36 (*d*) 48

(***CDS 2010***)

**7.** A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water, in such a way that the whole solid is submerged in water. If the radius of the cylinder is 5 cm and its height is 10.5 cm, find the volume of water left in the cylindrical tub. $\left(\text{Use } \pi = \frac{22}{7}\right)$

(*a*) 386.83 cm³ (*b*) 836.83 cm³
(*c*) 683.83 cm³ (*d*) 638.83 cm³

**8.** A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and the diameter of the cone is 4 cm. If a right circular cylinder circumscribes the solid, find how much more space will it cover.

(*a*) 6 $\pi$ cm³ (*b*) 4 $\pi$ cm³ (*c*) 8 $\pi$ cm³ (*d*) 10 $\pi$ cm³

**9.** In a rocket shape fire cracker, explosive powder is to be filled up inside the metallic enclosure. The metallic enclosure is made up of a cylindrical base and conical top with a base of radius 8 cm. The ratio of the height of the cylinder and the cone is 5 : 3. A cylindrical hole is drilled through the metal solid with height one-third the height of metal solid. What should be the radius of the hole, so that the volume of the hole (in which the gun powder is filled up) is half of the volume of metal solid after drilling?

(*a*) $4\sqrt{3}$ cm (*b*) 4 cm
(*c*) 3 cm (*d*) None of the above

**(*IIFT 2010*)**

**10.** A spherical iron ball is dropped into a cylindrical vessel of base diameter 14 cm, containing water. The water level is increased by $9\frac{1}{3}$ cm. What is the radius of the ball?

(*a*) 3.5 cm (*b*) 7 cm (*c*) 9 cm (*d*) 12 cm

**(*CDS 2005*)**

**11.** A gulab jamun, contains sugar syrup upto about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns each shaped like a cylinder with two hemispherical ends of length 5 cm and diameter 2.8 cm.

(*a*) 383 $cm^3$ (*b*) 833 $cm^3$ (*c*) 338 $cm^3$ (*d*) 388 $cm^3$

**(*NCERT*)**

**12.** A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ $m^3$ of air. If the internal diameter of the dome is equal to its total height above the floor, find the height of the building?

(*a*) 2 m (*b*) 6 m (*c*) 4 m (*d*) 8 m

**(*NCERT*)**

**13.** A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top which is open is 5 cm. It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

(*a*) 90 (*b*) 150 (*c*) 200 (*d*) 100

**(*NCERT*)**

**14.** A heavy sphere of maximum possible volume is to be completely immersed into a cylindrical jar of radius *a* containing water upto to a height 2*a*. The minimum height of the jar so that no water spills out of it is:

(*a*) $\frac{10a}{3}$ (*b*) $\frac{11a}{3}$ (*c*) $\frac{12a}{3}$ (*d*) $\frac{13a}{3}$

**(*CDS 2003*)**

**15.** A solid is hemispherical at the bottom and conical above it. If the surface areas of the two parts are equal, then the volumes of the two parts are in the ratio:

(*a*) $1 : \sqrt{3}$ (*b*) $2 : \sqrt{3}$ (*c*) $3 : \sqrt{3}$ (*d*) 1 : 1

**16.** The radii of the circular ends of a bucket of height 40 cm are of lengths 35 cm and 14 cm. What is the volume of the bucket?

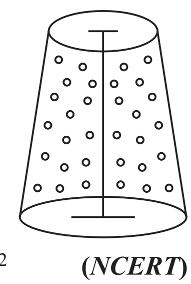
(*a*) 60060 cu. cm (*b*) 70040 cu. cm
(*c*) 80080 cu. cm (*d*) 80160 cu. cm. **(*CDS 2011*)**

**17.** A right circular cone is cut by a plane parallel to its base in such a way that the slant heights of the original and the smaller cone thus obtained are in the ratio 2 : 1. If $V_1$ and $V_2$ are respectively the volumes of the original cone and the new cone, then what is $V_1 : V_2$?

(*a*) 2 : 1 (*b*) 3 : 1 (*c*) 4 : 1 (*d*) 8 : 1

**(*CDS 2008*)**

**18.** A fez, the cap used by the turks is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of the material used for making it.



(*a*) 210 π $cm^2$ (*b*) 226 π $cm^2$
(*c*) 326 π $cm^2$ (*d*) 341 π $cm^2$ **(*NCERT*)**

**19.** A metallic right circular cone 20 cm high and whose vertical angle is 60º is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

(*a*) 6497.44 m (*b*) 4967.44 cm
(*c*) 7964.44 cm (*d*) 6794.44 cm **(*NCERT*)**

**20.** A tent is made in the form of a conic frustum surmounted by a cone. The diameters of the base and top of the frustum are 20 m and 6 m respectively and the height is 24 m. If the height of the tent is 28 m, find the quantity of canvas required.

(*a*) 280 π $m^2$ (*b*) 325 π $m^2$ (*c*) 235 π $m^2$ (*d*) 340 π $m^2$

**21.** A cone 12 cm high is cut 8 cm from the vertex to form a frustum with a volume of 190 cu. cm. Find the radius of the cone.

(*a*) 3.46 cm (*b*) 4.63 cm (*c*) 5 cm (*d*) 3.64 cm

**22.** There is a cone 2 m high which has a volume of 1.5 cu. m. The cone is bisected by a horizontal plane, forming a smaller cone and a frustum of equal values. What is the height of the new cone?

(*a*) $\sqrt[3]{2}$ m (*b*) $\sqrt[3]{3}$ m (*c*) $\sqrt[3]{4}$ m (*d*) $\sqrt[3]{6}$ m

**23.** A right circular cone with radius to height ratio as 12 : 5 is cut parallel to its base to get a smaller cone and a frustum. If the height of the smaller cone to the height of the frustum are in the ratio 3:1, by what percentage is the combined total surface area of the smaller cone and frustum more with respect to the original cone.

(*a*) 22% (*b*) 32% (*c*) 46% (*d*) None of these

**24.** The base of a right pyramid is a square of side 40 cm long. If the volume of the pyramid is 800 $cm^3$, then its height is:

(*a*) 5 cm (*b*) 10 cm (*c*) 15 cm (*d*) 20 cm

**(*SSC 2011*)**

**25.** The base of a right pyramid is a square of side 16 cm long. If its height be 15 cm, then the area of the lateral surface in square centimetre is:

(*a*) 136 (*b*) 544 (*c*) 800 (*d*) 1280

**(*SSC 2011*)**

**26.** If the length of each side of a regular tetrahedron is 12 cm, then the volume of the tetrahedron is

(*a*) $144\sqrt{2}$ cu. cm (*b*) $72\sqrt{2}$ cu.cm
(*c*) $8\sqrt{2}$ cu. cm (*d*) $12\sqrt{2}$ cu. cm

**27.** If the slant height of a right pyramid with square base is 4 metre and the total slant surface of the pyramid is 12 square metre, then the ratio of total slant surface and area of the base is :

(*a*) 16 : 3 (*b*) 24 : 5 (*c*) 32 : 9 (*d*) 12 : 3

(***SSC 2012***)

**28.** A regular pyramid has a square base with side 10 cm and a vertical height of 20 cm. If the height increases by 10% of its original value and the volume is constant, the percentage change in the side of the square base with respect to its original value is approximately

(*a*) +5% (*b*) +10% (*c*) –5% (*d*) –10%

(***JMET 2009***)

**29.** The base of a pyramid is an equilateral triangle of side 1 metre. If the height of the pyramid is 4 metres, then the volume is:

(*a*) 0.550 $m^3$ (*b*) 0.577 $m^3$

(*c*) 0.678 $cm^3$ (*d*) 0.750 $m^3$ (***CDS 1999***)

**30.** A right pyramid stands on an equilateral triangular base of area $36\sqrt{3}$ cm$^2$. If the area of one of the lateral faces is 42 cm$^2$, find the volume of the pyramid.

(*a*) $\frac{360}{\sqrt{3}}$ cm$^3$ (*b*) $12\sqrt{471}$ cm$^3$

(*c*) $12\sqrt{37}$ cm$^3$ (*d*) $12\sqrt{111}$ cm$^3$

**31.** The surface area and volume respectively of a regular tetrahedron of height '*h*' are:

(*a*) $\frac{\sqrt{3}}{2}h^2, \frac{\sqrt{3}}{8}h^3$ (*b*) $\frac{3\sqrt{3}}{2}h^2, \frac{3\sqrt{3}}{8}h^3$

(*c*) $\frac{3\sqrt{3}}{4}h^2, \frac{3\sqrt{3}}{16}h^2$ (*d*) $\frac{3\sqrt{3}}{2}h^2, \frac{\sqrt{3}}{8}h^3$

**32.** A right pyramid has an equilateral triangular base of side 4 units. If the number of square units of its whole surface area be three times the number of cubic units of its volume, find its height.

(*a*) 6 units (*b*) 10 units (*c*) 8 units (*d*) 4 units

**33.** There is a pyramid on a base which is a regular hexagon of side '2*a*' cm. If every slant height of this pyramid is of length $\frac{5a}{2}$ cm, then the volume of this pyramid is

(*a*) $3a^3$ cm$^3$ (*b*) $3\sqrt{2}\,a^3$ cm$^3$

(*c*) $3\sqrt{3}\,a^3$ cm$^3$ (*d*) $6a^3$ cm$^3$ (***SCC 2011***)

**34.** The radius of a cone is $\sqrt{2}$ times the height of the cone. A cube of maximum possible volume is cut from the same cone. What is the ratio of volume of the cone to the volume of the cube?

(*a*) 3.18 π (*b*) 2.25 π (*c*) 2.35 π (*d*) None of these

**35.** Ankit has a right circular cylinder which he inserted completely into a right circular cone of height 20 cm. The vertical angle of the cone is 60º and the diameter of the cylinder is $10\sqrt{3}$ cm. The volume of the cone is

(*a*) $\frac{4000}{7}\pi$ cm$^3$ (*b*) $\frac{8000}{3}\pi$ cm$^3$

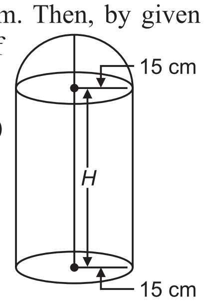
(*c*) $\frac{8000}{9}\pi$ cm$^3$ (*d*) $\frac{3000}{7}\pi$ cm$^3$

## ANSWERS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** (*a*) | **2.** (*c*) | **3.** (*c*) | **4.** (*a*) | **5.** (*c*) | **6.** (*b*) | **7.** (*c*) | **8.** (*c*) | **9.** (*a*) | **10.** (*b*) |
| **11.** (*c*) | **12.** (*c*) | **13.** (*b*) | **14.** (*a*) | **15.** (*b*) | **16.** (*c*) | **17.** (*d*) | **18.** (*b*) | **19.** (*c*) | **20.** (*d*) |
| **21.** (*b*) | **22.** (*c*) | **23.** (*d*) | **24.** (*c*) | **25.** (*b*) | **26.** (*a*) | **27.** (*a*) | **28.** (*c*) | **29.** (*b*) | **30.** (*b*) |
| **31.** (*d*) | **32.** (*c*) | **33.** (*c*) | **34.** (*b*) | **35.** (*c*) | | | | | |

## HINTS AND SOLUTIONS

**1.** Let the height of the cylinder be *H* cm. Then, by given condition, Vol. of hemisphere + Vol. of cylinder = Vol. of container, *i.e*,




$\frac{2}{3}\pi r^3 + \pi r^2 H = 32400\,\pi$ $(\because r = 15)$

$\Rightarrow \frac{2}{3}\times\pi\times 3375 + \pi\times 225\times H = 32400\pi$

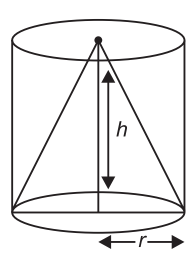
$\Rightarrow 2 \times 1125 + 225\,H = 32400$

$\Rightarrow 10 + H = 144 \Rightarrow H = 134$ cm

$\therefore$ Height of the container = $H$ + 15 = 134 cm + 15 cm = **149 cm**

Hence option (*a*) is the correct answer.

**2.** Volume of the cylindrical tank = $V = \pi r^2 h$



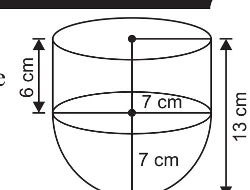

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{V}{3}$

$\therefore$ Volume of water in the tank

$= V - \frac{V}{3} = \mathbf{\frac{2V}{3}}.$

**3.** Total capacity of the bowl

= Vol. of the cylinder + Vol. of the hemisphere



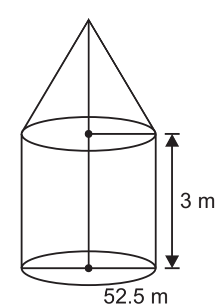

$= \left(\pi r^2 h + \frac{2}{3}\pi r^3\right)$ cm$^3$

$= \pi r^2\left(h + \frac{2}{3}r\right)$ cm$^3$ $= \frac{22}{7}\times 7^2 \times\left(6 + \frac{2}{3}\times 7\right)$ cm$^3$

$= \frac{22}{7}\times 7\times\frac{32}{3}$ cm$^3$ $= \frac{4928}{7}$ cm$^3$ $=$ **1642.66 cm$^3$.**

**4.** Total canvas reqd. in making the tent

= Curved surface area of the conical part + Curved surface area of cylindrical part




= ($\pi rl + 2\pi rh$) m$^2$ = $\pi r$ ($l + 2h$) m$^2$

$= \left(\frac{22}{7}\times 52.5\times(53+6)\right)$ m$^2$

$= \left(\frac{22}{7}\times 52.5\times 59\right)$ m$^2$ = **9735 m$^2$**

**5.** Vol. of the solid = 22 cm × 7 cm × 5 cm = 770 cu. cm

Let the height of water rise in the cylinder be $h$ cm. Then, $\pi r^2 h = 770$

$$\Rightarrow \frac{22}{7}\times 14\times 14\times h = 770 \Rightarrow h = \frac{770\times 7}{14\times 14\times 22} = \frac{5}{4} = \mathbf{1.25\,cm}$$

**6.** Let the radius of a ball = $r$ cm.

$\therefore$ Radius of the base of the cylinder = $4r$

Height of the cylinder = $4r$

Vol. of spherical ball = $\frac{4}{3}\pi r^3$

Now, Vol. of water in the cylindrical tank = $\pi(4r)^2(2r)$ (height of water = $2r$) $= 32\pi r^3$.

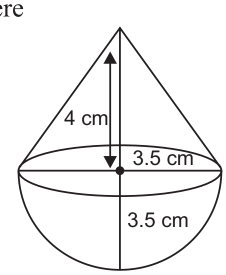
$\therefore$ Volume of remaining portion of the cylindrical tank $= 32\pi r^3$.

Now, if the number of spherical balls = $n$, then

$$n\times\frac{4}{3}\pi r^3 = 32\pi r^3 \Rightarrow n = \mathbf{24}.$$

**7.** In the given solid, radius of hemisphere = radius of base of cone ($r$) = 3.5 cm

Height of cone ($h$) = 4 cm.




$\therefore$ Volume of the solid = Volume of the hemispherical part + Volume of the conical part

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi r^2(2r+h) = \frac{1}{3}\times\frac{22}{7}\times(3.5)^2\times(7+4)$$

$$= \frac{1}{3}\times\frac{22}{7}\times 12.25\times 11 = 141.17\text{ cm}^3$$

When the solid is submerged in the cylindrical tub, the volume of water that flows out of the cylinder is equal to the volume of the solid.

Therefore,

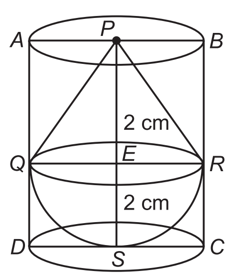
Volume of water left in the cylinder

= Volume of cylinder – Volume of solid

$$= \left(\frac{22}{7}\times 25\times 10.5 - 141.17\right)\text{cm}^3$$

$= (825 - 141.17)$ cm$^3$ = **683.83 cm$^3$.**

**8.** Let $PQR$ be the cone surmounted on the hemisphere $QRS$ circumscribed by the cylinder $ABCD$. Then,




Radius of the hemisphere = radius of the base of the cylinder = radius of the base of the cone = **2 cm.**

$\Rightarrow ER = ES = SC =$ 2 cm

Height of the cone = 2 cm

Height of the cylinder $PE + ES$ = 2 cm + 2 cm = 4 cm.

$\therefore$ Required space

= Volume of the cylinder – Volume of the solid toy

$$= \left[\pi\times 2^2\times 4 - \left(\frac{2}{3}\pi(2)^3 + \frac{1}{3}\pi(2)^2 2\right)\right]\text{cm}^3$$

$= [16\pi - 8\pi]$ cm$^3$ = **8π cm$^3$.**

**9.** Given, the ratio of the height of the cylindrical base to height of the conical top = 5 : 3. Let their heights be 5 $k$ and 3 $k$ respectively. Let $R$ be the radius of the base of the cylinder as well as the cone,

Let $r$ be the radius of the cylindrical hole.

Height of the cylindrical hole

$$= \frac{1}{3}\times\text{Height of the solid} = \frac{8K}{3}.$$

Volume of the solid

= Volume of cylindrical base + Volume of conical top

$$= \pi R^2\times 5K + \frac{1}{3}\pi R^2\times 3K = 6\pi R^2 K.$$

Volume of the drilled cylindrical hole $= \pi r^2\frac{8K}{3} = \frac{8\pi r^2 K}{3}$

Volume of the metal solid left after drilling $= 6\pi R^2 k - \frac{8\pi r^2 k}{3}$

Given, Vol. of cylindrical hole $= \frac{1}{2}\times$ Vol. of metal solid left

$$\Rightarrow \frac{8\pi r^2 k}{3} = \frac{1}{2}\left(6\pi R^2 k - \frac{8\pi r^2 k}{3}\right)$$

$$\Rightarrow \frac{8\pi r^2 k}{3} + \frac{8\pi r^2 k}{6} = 3\pi R^2 k$$

$$\Rightarrow 4\pi r^2 k = 3\pi R^2 k \Rightarrow r^2 = \frac{3R^2}{4} \Rightarrow r = \frac{\sqrt{3}}{2}R$$

Given, $R = 8 \Rightarrow r = \frac{\sqrt{3}}{2}\times 8 = \mathbf{4\sqrt{3}\ cm.}$

**10.** Let $R$ be the radius of the ball. Then,

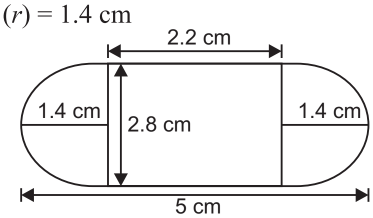
Volume of water displaced = Volume of iron ball

$$\Rightarrow \pi\times\left(\frac{14}{2}\right)^2\times\frac{28}{3} = \frac{4}{3}\times\pi\times R^3$$

$\Rightarrow R^3 = 7^3 \Rightarrow \boldsymbol{R = 7.}$

**11.** The gulab jamun can be considered as the combination of three solids as shown in the given diagram.

Here, radius of the hemispherical ends = radius of the cylinder ($r$) = 1.4 cm




Height of the cylindrical part ($h$) = 5 – (1.4 + 1.4) cm = 2.2 cm.

$\therefore$ Volume of one gulab jamun

= 2 × (Volume of hemisphere) + Volume of cylinder

$$= 2\times\left\{\frac{2}{3}\pi r^3\right\} + \pi r^2 h = \pi r^2\left(\frac{4}{3}r + h\right)$$

$$= \frac{22}{7}\times 1.4\times 1.4\left\{\frac{4}{3}\times 1.4 + 2.2\right\}$$

$$= \frac{22}{7}\times\frac{14}{10}\times\frac{14}{10}\left\{\frac{4}{3}\times\frac{14}{10} + \frac{22}{10}\right\}$$

$= \frac{154}{25} \times \frac{61}{15} = \frac{9394}{375}$ cm$^3$

$\therefore$ Quantity of syrup found in 45 gulab jamuns

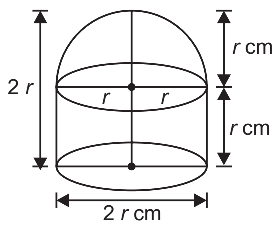
$= 30\%$ of $\left(45 \times \frac{9394}{375}\right)$

$= \frac{3}{10} \times 45 \times \frac{9394}{375} = 338.184 \approx$ **338 cm$^3$.**

**12.** Let the height of the building

= internal diameter of the dome = $2r$ m.

$\therefore$ Radius of the building = radius of dome = $\frac{2r}{2} = r$ m




Height of cylindrical portion

$= 2r - r = r$ m.

Volume of the cylinder

$= \pi r^2(r) = \pi r^3$ m$^3$

Volume of hemispherical dome

$= \frac{2}{3}\pi r^3$ m$^3$

$\therefore$ Total volume of the building

$= \pi r^3 + \frac{2}{3}\pi r^3 = \frac{5}{3}\pi r^3$ m$^3$

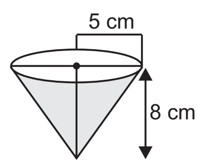
Given, $\frac{5}{3}\pi r^3 = 41\frac{19}{21} = \frac{880}{21} \Rightarrow r^3 = \frac{880 \times 7 \times 3}{5 \times 22 \times 21} = 8$

$\therefore$ $r = \sqrt[3]{8} = 2$ m

Hence, height of the building = $2r$ = **4 m.**

**13.** Volume of water in the vessel = Volume of the inverted cone

$= \frac{1}{3} \times \pi \times (5)^2 \times 8$ cm$^3$




Let the number of lead shots = $n$

Volume of one lead shot = $\frac{4}{3} \times \pi \times (0.5)^3$

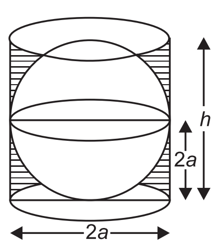
$\therefore$ Total volume of lead shots

= Volume of water flowing out

$\Rightarrow$ $n \times \frac{4}{3} \times \pi \times (0.5)^3 = \frac{1}{4} \times \frac{1}{3} \times \pi \times (5)^2 \times 8$

$\Rightarrow$ $n = \frac{25 \times 8}{16 \times 0.125}$ = **100.**

**14.** Let the maximum height of the cylinder be $h$. Then,

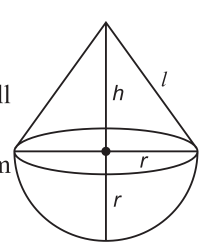



Volume of cylinder of maximum height = Volume of water filled + Volume of sphere

$\Rightarrow \pi a^2.h = \pi.a^2.2a + \frac{4}{3}\pi a^3$

$\Rightarrow \pi a^2 h = \pi a^2\left(2 + \frac{4}{3}\right) \Rightarrow h = \mathbf{\frac{10a}{3}}$

**15.** Let the radius of the hemispherical as well as base of conical portion = $r$ cm




Let the vertical height of the cone = $h$ cm

Let the slant height of the cone = $l$ cm.

Given, $\pi rl = 2\pi r^2$

$\Rightarrow$ $l = 2r$

$\therefore$ $h = \sqrt{l^2 - r^2} = \sqrt{4r^2 - r^2} = \sqrt{3r^2} = r\sqrt{3}$

$\therefore$ Required ratio = Vol. of hemisphere : Vol. of cone

$= \frac{2}{3}\pi r^3 : \frac{1}{3}\pi r^2 . r\sqrt{3} =$ **2 : $\sqrt{3}$.**

**16.** Bucket is a frustum of a right circular cone. Here,

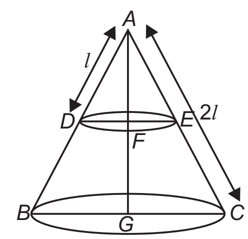
$R = 35$ cm, $r = 14$ cm, $h = 40$ cm.

Volume of frustum of cone = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$

$= \frac{22}{7} \times \frac{40}{3}(35^2 + 14^2 + 35 \times 14)$

$= \frac{880}{21}(1225 + 196 + 490) = \frac{880}{21} \times 1911 =$ **80080 cu. cm.**

**17.** Let $ABC$ be the original cone cut by the plane $DFE$ such that,




$AC : AD = 2 : 1$.

$\because \Delta AFE \sim \Delta AGC$

$\frac{FE}{GC} = \frac{AE}{AC} = \frac{1}{2}$

$\Rightarrow$ Radius of cone $ABC = GC = 2r$ (say), then radius of cone $ADE = FE = r$.

So, required ratio = $\frac{\text{Vol. of cone } ABC}{\text{Vol. of cone } ADE}$

$\Rightarrow$ $\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi \times 4r^2 \times \sqrt{(2l)^2 - (2r)^2}}{\frac{1}{3}\pi \times r^2 \times \sqrt{l^2 - r^2}} = \mathbf{\frac{8}{1}}$**.**

**18.** For the fez, height ($h$) = 15 cm, $r_1$ = 4 cm, $r_2$ = 10 cm

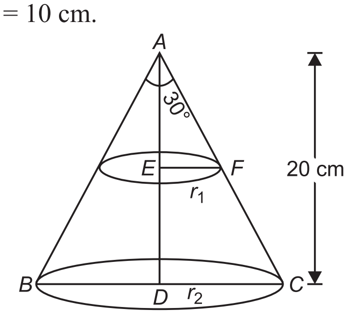
Total curved surface area of the fez = Lateral surface area of the frustum + Area of upper closed end

$= \pi(r_1 + r_2)l + \pi r_1^2 = \pi\{(4 + 10) \times 15\} + \pi.16$

$= (210\pi + 16\pi)$ cm$^2$ = **226 $\pi$ cm$^2$.**

**19.** Given $\angle BAC = 60^\circ \Rightarrow \angle DAC = 30^\circ$.

$AD = 20$ cm $\Rightarrow AE = ED = 10$ cm.




In $\Delta AEF$, $\tan 30^\circ = \frac{EF}{AE}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_1}{10}$

$\Rightarrow r_1 = \frac{10}{\sqrt{3}}$ cm

In $\Delta ADC$, $\tan 30^\circ = \frac{DC}{AD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_2}{20} \Rightarrow r_2 = \frac{20}{\sqrt{3}}$ cm

$\therefore$ Volume of the frustum cut by the plane

$= \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{\pi \times 10}{3}\left(\frac{100}{3} + \frac{400}{3} + \frac{200}{3}\right) = \frac{7000\pi}{9}$ cm$^3$

Let the length of the wire be $h$ cm.

Given, radius of wire = $\frac{1}{32}$ cm.

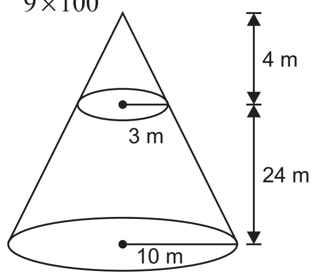
$\therefore$ Volume of wire (cylinder) = $\pi \times \left(\frac{1}{32}\right)^2 \times h$

So, $\frac{7000\pi}{9} = \frac{\pi h}{32 \times 32}$

$\Rightarrow h = \frac{7000 \times 32 \times 32}{9}$ cm $= \frac{7000 \times 32 \times 32}{9 \times 100}$ m = **7964.44 m.**

**20.** Area of canvas required

= Lateral surface area of the conic frustum + Curved surface area of the cone.




Height of conical part

= 28 m – 24 m = 4 m

Slant height ($l_1$) of the conical part = $\sqrt{4^2 + 3^2} = \sqrt{25} = 5$m.

For the furstum, $h$ = 24 m, $R$ = 10 m, $r$ = 3 m

Slant height ($l_2$) of the frustum

$= \sqrt{h^2 + (R - r)^2} = \sqrt{24^2 + (10 - 3)^2}$

$= \sqrt{576 + 49} = \sqrt{625} = 25$ m.

$\therefore$ Area of canvas reqd = $\pi(R + r)l_2 + \pi r l_1$

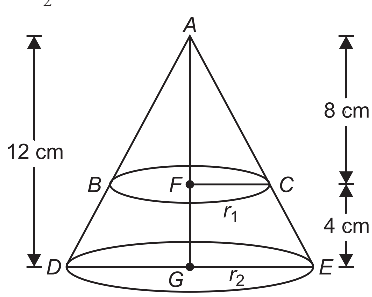
$= \pi \times 13 \times 25 + \pi \times 3 \times 5 = (325\pi + 15\pi)$ m$^2$ = **340π m²**.

**21.** Given, Height of the cone = AG = 12 cm

Height ($h$) of the frustum = $FG$ = 4 cm and $AF$ = 8 cm

$\Delta AFC \sim \Delta AGE$, so

$\frac{AF}{AG} = \frac{r_1}{r_2} \Rightarrow r_1 = \frac{8}{12} r_2 = \frac{2}{3} r_2$




Volume of frustum = $\frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 \times r_2)$

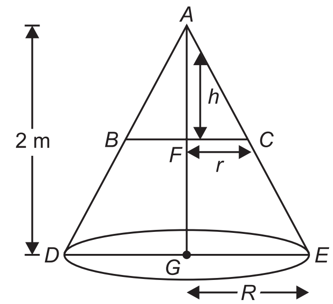
$\Rightarrow\ 190 = \frac{22}{7} \times \frac{4}{3}\left(\frac{4}{9} r_2^2 + r_2^2 + \frac{2}{3} r_2^2\right)$

$\Rightarrow\ 190 = \frac{88}{21}\left(\frac{19}{9} r_2^2\right)$

$\Rightarrow\ r_2^2 = \frac{190 \times 9 \times 21}{88 \times 19} = 21.47$ (approx)

$\Rightarrow\ r_2 = \sqrt{21.47}$ = **4.63 cm approx.**

**22.** Volume of original cone




$V_1 = \frac{\pi}{3} R^2 H = \frac{3}{2}$

$\Rightarrow \frac{\pi}{3} R^2 \times 2 = \frac{3}{2}$

$\Rightarrow R^2 = \frac{9}{4\pi} \Rightarrow R = \frac{3}{2\sqrt{\pi}}$

From similar $\Delta s$ $AFC$ and $AGE$,

we have $\frac{r}{h} = \frac{R}{2} \Rightarrow r = \frac{hR}{2}$

$\Rightarrow r = \frac{h}{2} \times \frac{3}{2\sqrt{\pi}} = \frac{3h}{4\sqrt{\pi}}$

Volume of smaller cone = $\frac{\pi}{3} r^2 h = \frac{3}{4}$

$\Rightarrow \frac{\pi}{3}\left(\frac{3h}{4\sqrt{\pi}}\right)^2 \times h = \frac{3}{4}$

$\Rightarrow \frac{\pi}{3} \frac{9h^2}{16\pi} \times h = \frac{3}{4} \Rightarrow h^3 = 4 \Rightarrow h = \sqrt[3]{4}$ **m.**

**23.** Let the radius and height of the circular cone be 12 $k$ and 5 $k$ respectively.

Given : $\frac{\text{Height } (h_1) \text{ of smaller cone}}{\text{Height } (h_2) \text{ of frustum}} = \frac{3}{1}$

$\therefore\ h_1 = \frac{3}{4} \times 5k$ and $h_2 = \frac{1}{4} \times 5k$

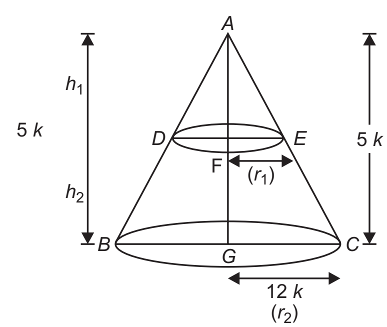
$\Delta AFE \sim \Delta AGC$, so

$\frac{FE}{GC} = \frac{AF}{AG}$

$\Rightarrow\ \frac{r_1}{r_2} = \frac{h_1}{AG}$

$\Rightarrow\ \frac{r_1}{12k} = \frac{\frac{15k}{4}}{5k}$

$\Rightarrow\ r_1 = \frac{3}{4} \times 12k = 9k.$




Total surface area of original cone = $\pi r_2 l + \pi r_2^2$

$= \pi \times 12k \times \sqrt{(12k)^2 + (5k)^2} + \pi \times (12k)^2$

$= \pi \times 12k \times 13k + 144\,\pi k^2$

$= 156\,\pi k^2 + 144\,\pi k^2 = 300\,\pi k^2$

Total surface area of smaller cone

$= \pi \times 9k \times \sqrt{\left(\frac{15k}{4}\right)^2 + (9k)^2} + \pi(9k)^2$

$= \pi \times 9k \times \frac{39}{4} k + 81\pi k^2 = \frac{351\pi k^2}{4} + 81\pi k^2 = \frac{675\pi k^2}{4}$

Total surface area of frustum

$= \pi \times (9k + 12k) \times \sqrt{\left(\frac{5k}{4}\right)^2 + 3k^2} + \pi[(12k)^2 + (9k)^2]$

$= \pi \times 21k \times \frac{13}{4}k + \pi(144k^2 + 81k^2)$

$= \frac{273\pi k^2}{4} + 225\,\pi k^2 = \frac{1173}{4}\,\pi k^2$

$\therefore$ Reqd. difference = Total surface area of (smaller cone + frustum) – Total surface area of original cone

$= \left(\frac{1173}{4}\pi k^2 + \frac{675}{4}\pi k^2\right) - 300\,\pi k^2$

$= \frac{1848}{4}\pi k^2 - 300\pi k^2 = 462\pi k^2 - 300\pi k^2 = 162\,\pi k^2$

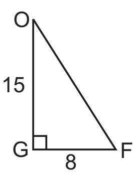
$\therefore$ Reqd. percent = $\left(\frac{162\,\pi k^2}{300\,\pi k^2} \times 100\right)\% =$ **54%.**

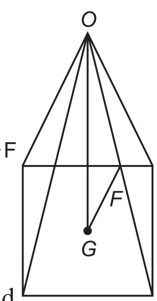
**24.** Area of the base = 40 cm × 40 cm = 1600 sq. cm.

Volume of pyramid = $\frac{1}{3}$ × Area of base × height

$\Rightarrow\ 8000 = \frac{1}{3} \times 1600 \times h \Rightarrow h = \frac{8000 \times 3}{1000} =$ **15 cm.**

**25.** Given, $OG$ = 15 cm




$GF = \frac{1}{2}$ × side of square = 8 cm

$\therefore$ Height of triangle = $OF = \sqrt{15^2 + 8^2}$

$= \sqrt{225 + 64} = \sqrt{289}$ = 17 cm.

$\therefore$ Area of the lateral surface of the pyramid

= 4 × Area of one triangle edge

$= 4 \times \frac{1}{2}$ × base × height $= 4 \times \frac{1}{2} \times 16 \times 17 = 544$ sq. cm.

**26.** Volume of a regular tetrahedron = $\frac{\sqrt{2}}{12}(\text{edge})^3$

$= \frac{\sqrt{2}}{12}(12)^3$ cu. cm = $\mathbf{144\sqrt{2}}$ **cu. cm.**

**27.** Slant surface or Lateral surface area of a pyramid

$= \frac{1}{2}$ × Perimeter of base × Slant height

Given, $12 = \frac{1}{2} \times 4 \times a \times 4$

where, each side of the square = $\boldsymbol{a}$ metres

$\Rightarrow\ a = \frac{24}{16} = \frac{3}{2}$ m.

$\therefore$ Area of base = $\left(\frac{3}{2}\right)^2 \text{m}^2 = \frac{9}{4}\text{m}^2$.

$\therefore$ Reqd. ratio = $12 : \frac{9}{4} =$ **16 : 3.**

**28.** Volume of the pyramid = $\frac{1}{3}$ × Area of base × height

$= \frac{1}{3} \times 100 \times 20 \text{ cm}^3$

New height of the pyramid = 22 cm

Let the new side of the square be $x$ cm. Then,

New volume = $\frac{1}{3} \times x^2 \times 22$

So, $\frac{1}{3} \times x^2 \times 22 = \frac{1}{3} \times 100 \times 20$

$\Rightarrow\ x^2 = \frac{100 \times 20}{22} = 90.90 \Rightarrow x \approx 9.5$

$\therefore$ % change in the side of the square base

$= \left(\frac{10 - 9.5}{10} \times 100\right)\% =$ **5% less.**

**29.** Volume of the pyramid = $\frac{1}{3}$ × Area of base × Height

$= \frac{1}{3} \times \frac{\sqrt{3}}{4} \times (1)^2 \times 4 \text{ m}^2 = \frac{1.732}{3} = \mathbf{0.577\ m^2}$ **(approx)**

**30.** Let the length of each side of the base be $a$ cm. Then, area of the base = $36\sqrt{3}$ cm$^2$

$\Rightarrow\ \frac{\sqrt{3}}{4}a^2 = 36\sqrt{3} \Rightarrow a^2 = 36 \times 4 \Rightarrow a = 12$ cm.

Let $h$ be the height of the pyramid and $l$ be its slant height.

Then, $l = \sqrt{h^2 + \frac{a^2}{12}}$

$\Rightarrow\ l^2 = h^2 + \frac{a^2}{12} \Rightarrow l^2 = h^2 + \frac{144}{12} \Rightarrow l^2 = h^2 + 12$ ...($i$)

Area of a lateral face

$= \frac{1}{2}$(length of a edge of base × slant height)

$= \frac{1}{2}(a \times l)$

$\Rightarrow\ 42 = \frac{1}{2} \times (12 \times l) \Rightarrow l = \frac{42 \times 2}{12} = 7$ cm.

Putting the value of $l$ in ($i$) we have

$h^2 = (7)^2 - 12 = 49 - 12 = 37 \Rightarrow h = \sqrt{37}$ cm

$\therefore$ Volume of pyramid = $\frac{1}{3}$ × Area of base × Height

$= \frac{1}{3} \times 36\sqrt{3} \times \sqrt{37} \text{ cm}^3 = \mathbf{12\sqrt{111}\ cm^3}$.

**31.** If the length of each edge of a regular tetrahedron = $a$ units, then height of the tetrahedron = $\sqrt{\frac{2}{3}}\,a$

$\Rightarrow\ h = \sqrt{\frac{2}{3}}\,a \Rightarrow a = \sqrt{\frac{3}{2}}\,h$

$\therefore$ Surface area of the tetrahedron = $\sqrt{3}\,(\text{edge})^2$

$= \sqrt{3}\left(\sqrt{\frac{3}{2}}h\right)^2 = \frac{3\sqrt{3}}{2}h^2$

Volume of the tetrahedron = $\frac{\sqrt{2}}{12}(\text{edge})^3$

$= \frac{\sqrt{2}}{12}\left(\sqrt{\frac{3}{2}}\,h\right)^3 = \frac{3\sqrt{3}}{24}h^3 = \mathbf{\frac{\sqrt{3}}{8}h^3}.$

**32.** Let $a$ be the length of each side of the base, $h$ be the height and $l$ be the slant height of the pyramid. Here, $a$ = **4 cm.**

$\therefore$ Slant height ($l$) = $\sqrt{h^2+\frac{a^2}{12}} = \sqrt{h^2+\frac{16}{12}} = \sqrt{h^2+\frac{4}{3}}$

According to the given question,

Lateral surface area + Area of the base = 3 (Volume)

$\Rightarrow \frac{1}{2}\times 12\times\sqrt{h^2+\frac{4}{3}}+\frac{\sqrt{3}}{4}\times(4)^2 = 3\times\frac{1}{3}\times\left(\frac{\sqrt{3}}{4}\times 4^2\times h\right)$

$\Rightarrow 6\sqrt{h^2+\frac{4}{3}}+4\sqrt{3} = 4\sqrt{3}\ h$

$\Rightarrow 6\sqrt{h^2+\frac{4}{3}} = 4\sqrt{3}\ (h-1)$

$\Rightarrow 36\left(h^2+\frac{4}{3}\right) = 48\ (h-1)^2$

$\Rightarrow 3\left(h^2+\frac{4}{3}\right) = 4\ (h-1)^2$

$\Rightarrow 3h^2+4 = 4h^2-8h+4$

$\Rightarrow h^2-8h = 0 \Rightarrow h(h-8) = 0 \Rightarrow$ $\boldsymbol{h = 8}$ **as** $\boldsymbol{h \neq 0}$**.**

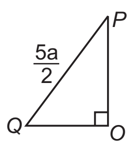
**33.** $ABCDEF$ is the regular hexagon with $AB = BC = CD = DE = EF = FE = a$ cm

Here, Height of pyramid = $OP$, Slant height of pyramid = $PQ$

So in right angled $\Delta\ POQ$, $PQ^2 = PO^2 + OQ^2$.

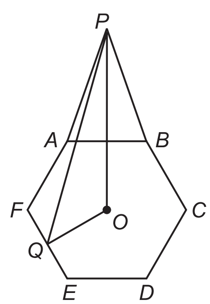
Area of the hexagonal base = $6\times\frac{\sqrt{3}}{4}\times(2a)^2$

$= 6\times\frac{\sqrt{3}}{4}\times 4a^2 = 6\sqrt{3}\ a^2\ \text{cm}^2.$




Height = $\sqrt{\left(\frac{5a}{2}\right)^2-(2a)^2}$

$= \sqrt{\frac{25}{4}a^2-4a^2} = \sqrt{\frac{9a^2}{4}} = \frac{3}{2}a$ cm.




$\therefore$ Volume of pyramid = $\frac{1}{3}$ × area of base × height

$= \frac{1}{3}\times 6\sqrt{3}\ a^2\times\frac{3}{2}a = \boldsymbol{3\sqrt{3}\ a^3\ \text{cm}^3}$**.**

**34.** Let each side of the cube be $a$ cm.

Then $DE$ = diagonal of square face = $\sqrt{2}a$

$\Rightarrow DG = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$ cm.

Let the radius of the cone, *i.e.*, $AF = FB = r$ cm.

Height $FC = h$ cm. Given, $r = h\sqrt{2}$

In similar triangles, $AFC$ and $DGC$

$\frac{AF}{FC} = \frac{DG}{GC} \Rightarrow \frac{r}{h} = \frac{\frac{a}{\sqrt{2}}}{(h-a)}$

$\Rightarrow \frac{a/\sqrt{2}}{h-a} = \sqrt{2} \Rightarrow a = 2(h-a) \Rightarrow h = \frac{3a}{2},\ r = \frac{3a}{2} * \sqrt{2}$

$\therefore$ Volume of cone : Volume of cube = $\dfrac{\frac{1}{3}\pi\times\left(\frac{3a\sqrt{2}}{2}\right)^2\times\frac{3a}{2}}{a^3}$

$= \frac{9}{4}a^3\pi : a^3 = \frac{9}{4}\pi = \boldsymbol{2.25\ \pi.}$

**35.** The cylinder inside the cone is shown in Fig. ($a$).

Here, $AQ = 20$ cm, $PG = QE = \frac{1}{2}DE = 5\sqrt{3}$ cm $\angle CAB = 60°$.

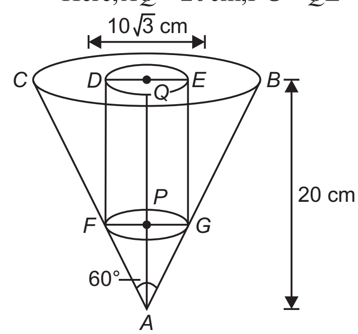



Fig. ($a$)

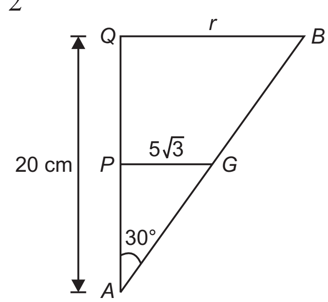



Fig. ($b$)

The cross-section as a plane figure is shown is Fig. ($b$).

$\frac{PG}{AP} = \tan 30° \Rightarrow \frac{5\sqrt{3}}{AP} = \frac{1}{\sqrt{3}} \Rightarrow AP = 15$ cm

Let $QB = r$ be the radius of the cone.

Then, in similar triangles $APG$ and $AQB$,

$\frac{AP}{PG} = \frac{AQ}{QB} \Rightarrow QB = \frac{AQ.PG}{AP} = \frac{20\times 5\sqrt{3}}{15} = \frac{20}{\sqrt{3}}$ cm.

$\therefore$ Radius of cone = $\frac{20}{\sqrt{3}}$ cm, Height of cone = 20 cm.

$\therefore$ Volume of cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\pi\times\left(\frac{20}{\sqrt{3}}\right)^2\times 20 = \boldsymbol{\frac{8000\ \pi}{9}\ \text{cm}^3}$**.**

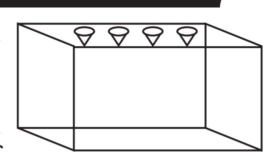
## SELF ASSESSMENT SHEET

**1.** A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1cm$^3$ of iron has approximately 8 g mass. (Use $\pi$ = 3.14)

($a$) 982.62 kg ($b$) 892.26 kg

($c$) 829.26 kg ($d$) 928.62 kg

**2.** A pen stand made of wood is in the shape of a cuboid with conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.



($a$) 532.54 cm$^3$ ($b$) 523.54 cm$^3$

($c$) 534.54 cm$^3$ ($d$) 543.54 cm$^3$

**3.** A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. The answer upto 2 dp is :

(*a*) 1.31 $m^3$ (*b*) 1.13 $m^3$ (*c*) 1.42 $m^3$ (*d*) 1.24 $m^3$

**4.** A solid cube of side 5.5 cm is dropped into a cylindrical vessel partly filled with water. The diameter of the vessel is 11 cm. If the cube is wholly submerged, the level of water will rise by:

(*a*) 3.75 cm (*b*) 0.75 cm (*c*) 1.75 cm (*d*) 2.85 cm.

(***CDS 2003***)

**5.** A water tank is hemispherical at the bottom and cylindrical on top of it. The radius is 12 m. If the total capacity is 3312 $\pi$ $m^3$, then the capacities of the two portions are in the ratio

(*a*) 8 : 9 (*b*) 8 : 11 (*c*) 8 : 13 (*d*) 8 : 15

(***CDS 1996***)

**6.** A right circular cone and a right circular cylinder have equal bases and equal heights.

If the ratio $\frac{\text{The lateral surface of cone}}{\text{The lateral surface of the cylinder}}$ equals $\frac{5}{8}$, then the semi-vertical angle of the right circular cone must be

(*a*) $22\frac{1}{2}^\circ$ (*b*) 30º (*c*) $\tan^{-1}\left(\frac{3}{4}\right)$ (*d*) 45º

**7.** If a right circular cone of a certain height has its upper part cut off by a plane passing through the mid-point of its axis and at right angles to it, then the volume of lower portion of the cone and that of the upper portion of the cone will be in the ratio

(*a*) 7 : 1 (*b*) 1 : 7 (*c*) 1 : 4 (*d*) 4 : 1

**8.** Find the volume of a tetrahedron the sides of whose base are 9 cm, 12 cm and 15 cm and height = 15 cm.

(*a*) 225 $cm^3$ (*b*) 270 $cm^3$ (*c*) 360 $cm^3$ (*d*) 200 $cm^3$

**9.** The base of a right pyramid is an equilateral triangle of side 4 cm. The height of the pyramid is half of its slant height. The volume of the pyramid is :

(*a*) $\frac{8\sqrt{3}}{9}$ $cm^3$ (*b*) $\frac{4\sqrt{3}}{9}$ $cm^3$

(*c*) $\frac{7}{3}$ $cm^3$ (*d*) $\frac{18}{\sqrt{3}}$ $cm^3$

**10.** A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of icecream. The icecream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones that can be filled with icecream.

(*a*) 150 (*b*) 100 (*c*) 10 (*d*) 20 **(*NCERT*)**

## ANSWERS

**1.** (*b*) **2.** (*b*) 3 (*b*) **4.** (*c*) **5.** (*d*) **6.** (*c*) **7.** (*a*) **8.** (*c*) **9.** (*a*) **10.** (*c*)

## HINTS AND SOLUTIONS

**1.** Mass of pole = Volume of pole × 8g.

Volume of pole = Vol. of base cylinder + Vol. of top cylinder

$= \pi \times (12)^2 \times 220 + \pi \times (8)^2 \times 60$

Now calculate.

**2.** Volume of wood in the entire stand

= Volume of cuboidal stand – Volume of 4 conical depressions

$= l \times b \times h - 4 \times \frac{1}{3}\pi r^2 h,$

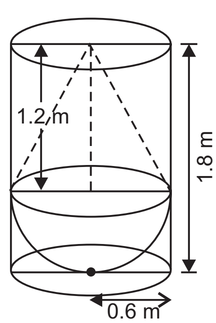
$= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times 1.4.$

**3.** Volume of water left in the cylinder

= Volume of water filled in th right circular cylinder – Volume of the solid

= Vol. of right circular cylinder

– (Vol. of cone + Vol. of hemisphere)



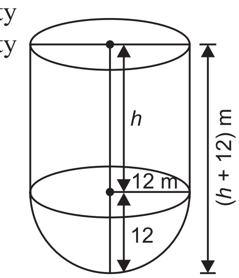

$= \frac{22}{7} \times 0.6 \times 0.6 \times 1.8 - \left(\frac{1}{3} \times \frac{22}{7} \times (0.6)^2 \times 1.2 + \frac{2}{3} \times \frac{22}{7} \times (0.6)^3\right)$

**4.** Let the level of water rise by $h$ cm. Then,

Volume of displaced water = Vol. of cube

$\Rightarrow \frac{22}{7} \times (5.5)^2 \times h = (5.5)^3.$

**5.** Total capacity of the tank = Capacity of hemispherical portion + Capacity of cylindrical portion




$\Rightarrow \frac{2}{3}\pi r^3 + \pi r^2 h = 3312\,\pi$

$\Rightarrow \pi r^2\left(\frac{2}{3}r + h\right) = 3312\,\pi$

$\Rightarrow 144\left(\frac{2}{3} \times 12 + h\right) = 3312$

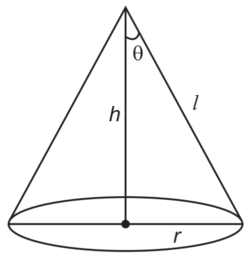
$\Rightarrow 144(8 + h) = 3312 \Rightarrow 8 + h = 23 \Rightarrow h = 15$ cm.

$\therefore$ Reqd. ratio $= \frac{\frac{2}{3}\pi r^3}{\pi r^2 h} = \frac{\frac{2}{3} \times 12}{15} =$ **8 : 15.**

**6.** Let $r$ be the radius of the cone and the cylinder and $h$ be the height of both. Then,

$$\frac{\text{Lateral surface of the cone}}{\text{Lateral surface of the cylinder}} = \frac{5}{8}$$

$$\Rightarrow \frac{\pi r l}{2\pi r h} = \frac{5}{8} \Rightarrow \frac{h}{l} = \frac{4}{5}$$

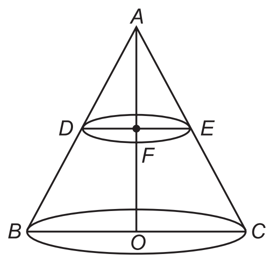



If θ is the semi-vertical angle of the cone, then $\cos\theta = \frac{4}{5}$

$$\Rightarrow \tan\theta = \frac{3}{4} \text{ or } \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

**Note :** $r^2 = \sqrt{l^2 - h^2} = 3$

**7.** Let $OA = h$, $OC = r$. Then, $AF = h/2$ and by similarity of triangles $AFE$ and $AOC$, $FE = r/2$




$$\therefore \frac{\text{Vol. of portion } DECB}{\text{Vol. of cone } ADE}$$

$$= \frac{\text{Vol. of cone } ABC - \text{Vol. of cone } ADE}{\text{Vol. of cone } ADE}$$

$$= \frac{\frac{1}{3}\pi r^2 h - \frac{1}{24}\pi r^2 h}{\frac{1}{24}\pi r^2 h} = \frac{\frac{7}{24}\pi r^2 h}{\frac{1}{24}\pi r^2 h} = \mathbf{7 : 1.}$$

**8.** Let $a = 9$ cm, $b = 12$ cm, $c = 15$ cm. Then,

$$s = \frac{a+b+c}{2} = \frac{9+12+15}{2} = 18 \text{ cm}$$

$$\therefore \text{Area of the base} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{18(18-9)(18-12)(18-15)}$$

$$= \sqrt{18 \times 9 \times 6 \times 3} = 54 \text{ cm}^2$$

∴ Volume of the tetrahedron

$$= \frac{1}{3} \times (\text{Area of the base} \times \text{height})$$

$$= \frac{1}{3} \times 54 \times 20 \text{ cm}^3 = \mathbf{360\ cm^3.}$$

**9.** Let $h$ be the height of the pyramid and $l$ its slant height, $a$ the length of each side of the base.

Given, $h = \frac{l}{2}$ $\qquad \left(\because l = \sqrt{h^2 + \frac{a^2}{12}}\right)$

$$\Rightarrow h = \frac{1}{2}\sqrt{h^2 + \frac{a^2}{12}} \Rightarrow 4h^2 = h^2 + \frac{a^2}{12}$$

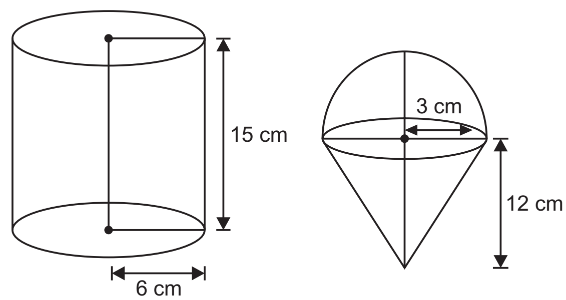
$$\Rightarrow 3h^2 = \frac{a^2}{12} \Rightarrow h^2 = \frac{a^2}{36} = \frac{16}{36} = \frac{4}{9} \Rightarrow h = \frac{2}{3}$$

$$\therefore \text{Volume of the pyramid} = \frac{1}{3} \times \text{Area of the base} \times \text{height}$$

$$= \frac{1}{3} \times \frac{\sqrt{3}}{4} \times (4)^2 \times \frac{2}{3} \text{ cm}^3 = \mathbf{\frac{8\sqrt{3}}{9}\ cm^3}.$$

**10.** Number of cones

$$= \frac{\text{Volume of cylindrical icecream container}}{\text{Volume of one cone filled with icecream}}$$




$$= \frac{\pi r_1^2 h_1}{(\text{Vol. of conical part} + \text{Vol. of hemispherical part})}$$

$$= \frac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3}$$

$$= \frac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 (h_2 + 2r_2)}$$

$$= \frac{3 \times 6 \times 6 \times 15}{3 \times 3 \times (12+6)} = \frac{1620}{162} = \mathbf{10.}$$