Cube and Cube Roots

FUNDAMENTALS

Cube and cube root

- **Cube:-** If y is a non-zero number, then $y \times y \times y$ written as y^3 is called the cube of y or simply y cubed.

e.g.,

(i) $(5)^3 = 5 \times 5 \times 5 = 125$. Thus, Cube of 5 is 125.

(ii) $(9)^3 = 9 \times 9 \times 9 = 729$. Thus,

- **Perfect cube:-** A natural number n is a perfect cube if it is the cube of some natural number.

Or

Natural number n is a perfect cube if there exists a natural number whose cube is n

i.e. $n = x^3$

e.g.,(i) 343 is a perfect cube, because there is a natural number 7 such that

$343 = 7 \times 7 \times 7 = 7^3$

e.g., (ii) $4^3 = 4 \times 4 \times 4 = 64$

$5^3 = 5 \times 5 \times 5 = 125$

$9^3 = 9 \times 9 \times 9 \times = 723$

Properties of perfect cube:

- If 'n' is even, then n^3 is also even.
- If 'n' is odd, then n^3 is also odd.
- If 'm' is even and 'n' is odd, then $m^3 \times n^3$ is even.
- If a number's units place has digit 1, 4, 5, 6, then its Cube also ends in the same digit
- Cube of negative number is negative

  $(-1)^3 = -1, (-9)^3 = -729$

Some Shortcuts to find cubes

- Column method:- Let x = ab (where a is tens digit and b is units digit)

Be a 2 digit natural number.

Then $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

e.g.. Find the cube of 26 by using column method.

Solution:- By Using column method , we have

Column-I a^3	**Column-II** $3 \times a^2 \times 6$	**Column-III** $3 \times a \times b^2$	**Column-IV** b^3
$2^3 = 8$	$3 \times 2^2 \times 6$	$3 \times 2 \times 6^2$	$6^3 = 216$

8	72	216	
9	23	+21	
17	95	237	
17	5	6	6

$\therefore (26)^3 = 17576$

Cube root:- If 'x' is a perfect cube and for some integers y, $x = y^3$, then the number 'y' is called cube root of 'x\ It is denoted by $y = \sqrt[3]{x} or\, x^{\frac{1}{3}}$.

Example:-

$27 = 3^3$ $\quad\therefore \sqrt[3]{27} = 3$

$729 = 9^3$ $\quad\therefore \sqrt[3]{729} = 9$

$-1000 = (-10)^3$ $\quad\therefore \sqrt[3]{1000} = 10$

$0.008 = (0.2)^3$ $\quad\therefore \sqrt[3]{0.008} = 0.2$

$\frac{1}{125} = \left(\frac{1}{5}\right)^3$ $\quad\therefore \sqrt[3]{\frac{1}{125}} = \frac{1}{5}$

Method to find the cube root of a number:

- Prime factorization method:-

  Follow these steps:
- Resolve the given number into its prime factors
- Make triplets of equal factors.
- Take the product of the prime factors, choosing one factor out of every triplet.

e.g.,(i) Find the cube root of 2744

solution:- By prime factorization we get

$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$

$\therefore \sqrt[3]{2744} = 2 \times 7 = 14$

2	2744
2	1372
2	686
7	343
7	49
	7

e.g.,(ii) What is the smallest number by which 3087 must be divided so that the quotient is a perfect cube?

Solution:- Resolving 3087 in to prime factors, we get $3087 = 3 \times 3 \times 7 \times 7 \times 7$. By grouping the factors clearly, if we divide 3087 by $3 \times 3 = 9$ the quotient would be $7 \times 7 \times 7$ which is a perfect cube.

Note:-

- Cube root of a negative number is negative, i.e., $\sqrt[3]{-x^3} = -x$
- Cube root of product of two integers, is product of their cube roots: $\sqrt[3]{x.y} = \sqrt[3]{x}.\sqrt[3]{y}$
- Cube root of the a rational number, is cube root of Numerator divided by cube root of Denominator: $\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x.}}{\sqrt[3]{y}} (y \neq 0)$

Remember these identities

- $(a-b)(a+b) = a^2 - b^2$
- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 + b^3 + c^3 - 3abc \quad = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$