DPP - Daily Practice Problems

Date : ______ **Start Time :** ______ **End Time :** ______

MATHEMATICS CM16

SYLLABUS : Relations and Functions

Max. Marks : 120 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 30 MCQ's. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. Let $R = \{(x, y) : x, y \in N \text{ and } x^2 - 4xy + 3y^2 = 0\}$, where N is the set of all natural numbers. Then the relation R is :
(a) reflexive but neither symmetric nor transitive.
(b) symmetric and transitive.
(c) reflexive and symmetric.
(d) reflexive and transitive.

2. Let $P = \{(x, y) : |x^2 + y^2| = 1, x, y \in R\}$. Then P is
(a) Reflexive (b) Symmetric
(c) Transitive (d) Anti-symmetric

3. Let $f : \{x, y, z\} \to \{1, 2, 3\}$ be a one-one mapping such that only one of the following three statements is true and remaining two are false : $f(x) \neq 2, f(y) = 2, f(z) \neq 1$, then
(a) $f(x) > f(y) > f(z)$ (b) $f(x) < f(y) < f(z)$
(c) $f(y) < f(x) < f(z)$ (d) $f(y) < f(z) < f(x)$

4. Domain of definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real valued x, is
(a) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

5. If R be a relation < from A = {1, 2, 3, 4} to B = {1, 3, 5} i.e., (a, b) $\in R \Leftrightarrow a < b$, then RoR^{-1} is
(a) {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)}
(b) {(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)}
(c) {(3, 3), (3, 5), (5, 3), (5, 5)}
(d) {(3, 3), (3, 4), (4, 5)}

6. If $f(x) = \begin{cases} 2x + a & ; x \geq -1 \\ bx^2 + 3 ; & x < -1 \end{cases}$ and

$g(x) = \begin{cases} x + 4 & ; 0 \leq x \leq 4 \\ -3x - 2 ; & -2 < x < 0 \end{cases}$

If domain of g (f (x)) is [–1, 4], then –
(a) $a = 0, b > 5$ (b) $a = 2, b > 7$
(c) $a = 2, b > 10$ (d) $a = 0, b \in R$

7. Let S be the set of all straight lines in a plane. A relation R is defined on S by $aRb \Leftrightarrow a \perp b$ then R is :
(a) reflexive but neither symmetric nor transitive
(b) symmetric but neither reflexive nor transitive
(c) transitive but neither reflexive nor symmetric
(d) an equivalence relation

8. A function whose graph is symmetrical about the y-axis is given by
(a) $f(x) = \sin[\log(x + \sqrt{x^2 + 1})]$
(b) $f(x) = \dfrac{\sec^4 x + \operatorname{cosec}^4 x}{x^3 + x^4 \cot x}$
(c) $f(x + y) = f(x) + f(y) \ \forall \ x, y \in R$
(d) None of these

9. Let R be a reflexive relation on a finite set A having n-elements, and let there be *m* ordered pairs in R. Then
(a) $m \geq n$ (b) $m \leq n$
(c) $m = n$ (d) None of these

10. If $f : R \to R$, $f(x) = \begin{cases} x|x| - 4, & x \in Q \\ x|x| - \sqrt{3} & x \notin Q \end{cases}$, then f(x) is
(a) one to one and onto (b) many to one and onto
(c) one to one and into (d) many to one and into

11. If f : B → A is defined by $f(x) = \dfrac{3x + 4}{5x - 7}$ and g : A → B is defined by $g(x) = \dfrac{7x + 4}{5x - 3}$, where $A = R - \left\{\dfrac{3}{5}\right\}$ and $B = R - \left\{\dfrac{7}{5}\right\}$ and I_A is an identity function on A and I_B is identity function on B, then
(a) $fog = I_A$ and $gof = I_A$ (b) $fog = I_A$ and $gof = I_B$
(c) $fog = I_B$ and $gof = I_B$ (d) $fog = I_B$ and $gof = I_A$

12. Let f be a real valued function with domain R satisfying $0 \leq f(x) \leq \dfrac{1}{2}$ and for some fixed a > 0,
$f(x + a) = \dfrac{1}{2} - \sqrt{f(x) - (f(x))^2} \ \forall \ x \in R$,
then the period of the function f(x) is
(a) a (b) 2a
(c) non-periodic (d) None of these

13. Let $f(x) = [x]^2 + [x + 1] - 3$ where [x] = the greatest integer function. Then
(a) f(x) is a many-one and into function
(b) f(x) = 0 for infinite number of values of x
(c) f(x) = 0 for only two real values
(d) Both (a) and (b)

14. $f(x) = |x - 1|$, $f : R^+ \to R$ and $g(x) = e^x$, $g : [-1, \infty) \to R$. If the function fog (x) is defined, then its domain and range respectively are
(a) $(0, \infty)$ and $[0, \infty)$ (b) $[-1, \infty)$ and $[0, \infty)$
(c) $[-1, \infty)$ and $\left[1 - \dfrac{1}{e}, \infty\right)$ (d) $[-1, \infty)$ and $\left[\dfrac{1}{e} - 1, \infty\right)$

RESPONSE GRID					
	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ

15. If $X = \{x_1, x_2, x_3\}$ and $Y = \{x_1, x_2, x_3, x_4, x_5\}$ then find which is a reflexive relation of the following ?

(a) $R_1 : \{(x_1, x_1), (x_2, x_2)\}$
(b) $R_1 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3)\}$
(c) $R_3 : \{(x_1, x_1), (x_2, x_2), (x_1, x_3), (x_2, x_4)\}$
(d) $R_3 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_4, x_4)\}$

16. A binary operation * on the set {0, 1, 2, 3, 4, 5} is defined as

$$a * b = \begin{cases} a+b & , \text{ if } a+b < 6 \\ a+b-6 & , \text{ if } a+b \geq 6 \end{cases}$$

the identity element is

(a) 0 (b) 1 (c) 2 (d) 3

17. Let $R = \{(1,3), (2,2), (3,2)\}$ and $S = \{(2,1), (3,2), (2,3)\}$ be two relations on set $A = \{1, 2, 3\}$. Then RoS =

(a) $\{(1,3), (2,2), (3,2), (2,1), (2,3)\}$
(b) $\{(3,2), (1,3)\}$
(c) $\{(2,3), (3,2), (2,2)\}$
(d) $\{(2,3), (3,2)\}$

18. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin \sqrt{x})^2$, then

(a) $f(x) = \sin^2 x, g(x) = \sqrt{x}$
(b) $f(x) = \sin x, g(x) = |x|$
(c) $f(x) = x^2, g(x) = \sin \sqrt{x}$
(d) f and g cannot be determined.

19. Let $f : R \to R$ be a function defined by $f(x) = \dfrac{x-m}{x-n}$, where $m \neq n$, then

(a) f is one-one onto (b) f is one-one into
(c) f is many-one onto (d) f is many-one into

20. If f(x) is defined on (0, 1), then the domain of definition of $f(e^x) + f(\ln|x|)$ is

(a) $(-e, -1)$ (b) $(-e, -1) \cup (1, e)$
(c) $(-\infty, -1) \cup (1, \infty)$ (d) $(-e, e)$

21. If $f(x) = \dfrac{x}{x-1}$, then $\underbrace{(fofo........of)}_{19 \text{ times}}(x)$ is equal to :

(a) $\dfrac{x}{x-1}$ (b) $\left(\dfrac{x}{x-1}\right)^{19}$
(c) $\dfrac{19x}{x-1}$ (d) x

22. Which of the function defined below is one–one?

(a) $f : (0, \infty) \to R, f(x) = x^2 - 4x + 3$
(b) $f : [0, \infty) \to R, f(x) = x^2 + 4x - 5$
(c) $f : R \to R, f(x) = e^x + \dfrac{1}{e^x}$
(d) $f : R \to R, f(x) = \ell n(x^2 + x + 1)$

23. The inverse of $f(x) = \dfrac{2}{3}\,\dfrac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is

(a) $\dfrac{1}{3}\log_{10}\dfrac{1+x}{1-x}$ (b) $\dfrac{1}{2}\log_{10}\dfrac{2+3x}{2-3x}$
(c) $\dfrac{1}{3}\log_{10}\dfrac{2+3x}{2-3x}$ (d) $\dfrac{1}{6}\log_{10}\dfrac{2-3x}{2+3x}$

24. A function f from the set of natural numbers to integers defined by $f(n) = \begin{cases} \dfrac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\dfrac{n}{2}, & \text{when } n \text{ is even} \end{cases}$ is

(a) neither one-one nor onto (b) one-one but not onto
(c) onto but not one-one (d) one-one and onto both

25. If $f(x) = \sin^2 x + \sin^2\left(x + \dfrac{\pi}{3}\right) + \cos x \cos\left(x + \dfrac{\pi}{3}\right)$ and $g\left(\dfrac{5}{4}\right) = 1$, then go f(x) =

(a) 1 (b) 0
(c) sin x (d) None of these

RESPONSE GRID	
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	20. ⓐⓑⓒⓓ 21. ⓐⓑⓒⓓ 22. ⓐⓑⓒⓓ 23. ⓐⓑⓒⓓ 24. ⓐⓑⓒⓓ
	25. ⓐⓑⓒⓓ

26. The domain of the function $f(x) = \sin^{-1}\left\{\log_2\left(\frac{1}{2}x^2\right)\right\}$ is

(a) $[-2, -1) \cup (1, 2]$
(b) $(-2, -1] \cup [1, 2]$
(c) $[-2, -1] \cup [1, 2]$
(d) $(-2, -1) \cup (1, 2)$

27. Let $f(x) = \frac{ax+b}{cx+d}$, then $fof(x) = x$, provided that:

(a) $d = a$
(b) $a = b = c = d = 1$
(c) $a = b = 1$
(d) $d = -a$

28. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined by $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$.

Then
(a) f is both one-one and onto
(b) f is one-one but not onto
(c) f is onto but not one-one
(d) f is neither one-one nor onto.

29. **Statement-1 :** If $f : R \to R$ and $g : R \to R$ be two mappings such that $f(x) = \sin x$ and $g(x) = x^2$, then $fog \neq gof$.

Statement-2 : $(fog)x = f(x)g(x) = (gof)x$

(a) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1.
(b) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1.
(c) Statement -1 is false, Statement-2 is true.
(d) Statement -1 is true, Statement-2 is false.

30. **Statement-1 :** If $f(x) = |x-1| + |x-2| + |x-3|$ where $2 < x < 3$ is an identity function.

Statement-2 : $f : A \to A$ defined by $f(x) = x$ is an identity function.

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

RESPONSE GRID	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ

DAILY PRACTICE PROBLEM DPP CHAPTERWISE 16 - MATHEMATICS			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	35	Qualifying Score	50
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

1. **(d)** $R = \{(x, y) : x, y \in N \text{ and } x^2 - 4xy + 3y^2 = 0\}$

Now, $x^2 - 4xy + 3y^2 = 0 \Rightarrow (x - y)(x - 3y) = 0$

$\therefore\ x = y$ or $x = 3y$

$\therefore\ R = \{(1, 1), (3, 1), (2, 2), (6, 2), (3, 3), (9, 3), ...\}$

Since (1, 1), (2, 2), (3, 3),...... are present in the relation, therefore R is reflexive.

Since (3, 1) is an element of R but (1, 3) is not the element of R, therefore R is not symmetric

Here $(3, 1) \in R$ and $(1, 1) \in R$

$\Rightarrow\ (3, 1) \in R$

$(6, 2) \in R$ and $(2, 2) \in R \Rightarrow (6, 2) \in R$

For all such $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow\ (a, c) \in R$

Hence R is transitive.

2. **(b)** Obviously, the relation is not reflexive and transitive but it is symmetric, because $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$

3. **(c)** Let $f(x) \neq 2$ be true and $f(y) = 2$, $f(z) \neq 1$ are false

$\Rightarrow\ f(x) \neq 2, f(y) \neq 2, f(z) = 1$

$\Rightarrow\ f(x) = 3, f(y) = 3, f(z) = 1$ but then function is many one, similarly two other cases.

4. **(a)** $-\frac{\pi}{6} \le \sin^{-1}(2x) \le \frac{\pi}{2} \Rightarrow -\frac{1}{2} \le 2x \le 1 \Rightarrow x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$

5. **(c)** We have, $R = \{(1, 3); (1, 5); (2, 3); (2, 5); (3, 5); (4,5)\}$

$R^{-1} = \{(3, 1); (5, 1); (3, 2); (5, 2); (5, 3); (5,4)\}$

Hence $RoR^{-1} = \{(3, 3); (3, 5); (5, 3); (5, 5)\}$

6. **(a)** $f(4) = g(4) \Rightarrow 8 + a = 8 \Rightarrow a = 0$

$f(-1) = -2$ for $a = 0$

$f(-1) > f(4)$

$b + 3 > 8 \Rightarrow b > 5$

7. **(b)** We have to test the equivalency of relation R on S.

(1) **Reflexivity :**

In a plane any line be parallel to itself not perpendicular. Hence $a \not R b$, R is not reflexive.

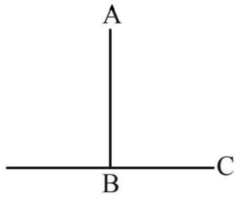
(2) **Symmetry :**

In a plane if a line AB is perpendicular to the other line BC, then BC is also perpendicular to AB, i.e.,

$aRb \Rightarrow AB \perp BC$

And $bRa \Rightarrow BC \perp AB$

Hence R is symmetric.



(3) **Transitivity :**

In a plane, let AB, BC and CA be three lines, such that

$AB \perp BC$ and $BC \perp CD$

$\Rightarrow AB \parallel CD \Rightarrow a \not R b$, R is not transitive.

Hence, R is symmetric but neither reflexive nor transitive.

8. **(d)** A function whose graph is symmetrical about the y-axis must be even

Since sin x and $\log(x + \sqrt{x^2 + 1})$ are odd function therefore $\sin(\log(x + \sqrt{x^2 + 1}))$ must be odd.

Also, $\frac{\sec^4 x + \operatorname{cosec}^4 x}{x^3 + x^4 \cot x}$ is an odd function.

Now, let $f(x + y) = f(x) + f(y)\ \forall x, y \in R$

$\therefore\ f(0 + 0) = f(0) + f(0)\ \therefore f(0) = 0$

$f(x - x) = f(x) + f(-x)$ or $0 = f(x) + f(-x)$

i.e $f(-x) = -f(x)$ $\quad\therefore$ f(x) is odd

9. **(a)** Since R is reflexive relation on A, therefore $(a,a) \in R$ for all $a \in A$.

The minimum number of ordered pairs in R is n.

Hence, $m \ge n$.

10. **(d)** $f(2) = f\left(3^{1/4}\right) \Rightarrow$ many to one function

and $f(x) \neq -\sqrt{3}\ \ \forall x \in R \Rightarrow$ into function

11. **(b)** We have, $gof(x) = g\left(\frac{3x + 4}{5x - 7}\right) = \frac{7\left(\frac{(3x+4)}{(5x-7)}\right) + 4}{5\left(\frac{(3x+4)}{(5x-7)}\right) - 3}$

$= \frac{21x + 28 + 20x - 28}{15x + 20 - 15x + 21} = \frac{41x}{41} = x$

Similarly, $fog(x) = f\left(\frac{7x+4}{5x-3}\right)$

$$= \frac{3\left(\frac{(7x+4)}{(5x-3)}\right)+4}{5\left(\frac{(7x+4)}{(5x-3)}\right)-7}$$

$$= \frac{21x+12+20x-12}{35x+20-35x+21} = \frac{41x}{41} = x$$

Thus, $gof(x) = x,\ \forall\ x \in B$ and $fog(x) = x,\ \forall\ x \in A$, which implies that $gof = I_B$ and $fog = I_A$.

12. (b) $f(x+a) = \frac{1}{2} - \sqrt{f(x) - f(x)^2} = \frac{1}{2} - \sqrt{\frac{1}{4} \left\{\frac{1}{2} - f(x)\right\}^2}$

$$\Rightarrow f(x+2a) = \frac{1}{2} - \sqrt{\frac{1}{4} - \left\{\frac{1}{2} - f(x+a)\right\}^2}$$

$$= \frac{1}{2} - \sqrt{\frac{1}{4} - \left\{\frac{1}{2} - \frac{1}{2} + \sqrt{f(x) - (f(x))^2}\right\}^2}$$

$$= \frac{1}{2} - \sqrt{\frac{1}{4} - f(x) + (f(x))^2} = \frac{1}{2} - \left|\frac{1}{2} - f(x)\right| = f(x)$$

13. (d) $f(x) = [x]^2 + [x+1] - 3 = \{[x]+2\}\ \{[x]-1\}$

So, $x = 1, 1.1, 1.2, \ldots\ldots\ldots \Rightarrow f(x) = 0$

$\therefore$ f (x) is many one.

only integral values will be attained.

$\therefore$ f(x) is into.

14. (b) $f(x) = |x-1| = \begin{cases} 1-x, & 0 < x < 1 \\ x-1, & x \ge 1 \end{cases}$

$g(x) = e^x, x \ge -1$

$$(fog)(x) = \begin{cases} 1-g(x), & 0 < g(x) < 1 \text{ i.e. } -1 \le x < 0 \\ g(x)-1, & g(x) \ge 1 \text{ i.e. } 0 \le x \end{cases}$$

$$= \begin{cases} 1-e^x, & -1 \le x < 0 \\ e^x - 1, & x \ge 0 \end{cases}$$

$\therefore$ domain $= [-1, \infty)$

fog is decreasing in $[-1, 0)$ and increasing in $[0, \infty)$

$fog(-1) = 1 - \frac{1}{e}$ and $fog(0) = 0$

As $x \to \infty$, $fog(x) \to \infty$,

$\therefore$ range $= [0, \infty)$

$\therefore \quad x = \frac{1}{2}\log_e\left(\frac{y}{2-y}\right)$

15. (b) (a) Non-reflexive because $(x_3, x_3) \notin R_1$

(b) Reflexive

(c) Non-Reflexive

(d) Non-reflexive because $x_4 \notin X$

16. (a) The operation table for * is given as

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

From the table, we note that

$a * 0 = 0 * a = a,\ \forall\ a \in \{0,1,2,3,4,5\}$

Hence, 0 is the identity for operation.

17. (c) Here $R = \{(1,3), (2,2); (3,2)\}$, $S = \{(2,1); (3,2); (2,3)\}$

Then $RoS = \{(2,3), (3,2); (2,2)\}$

18. (a) $g(f(x)) = |\sin x|$ indicates that possibly $f(x) = \sin x$, $g(x) = |x|$ Assuming it correct, $f(g(x)) = f(|x|) = \sin|x|$, which is not correct.

$f(g(x)) = \left(\sin\sqrt{x}\right)^2$ indicates that possibly

$g(x) = \sqrt{x}\ \ f(x) = \sin^2 x$ or

$g(x) = \sin\sqrt{x},\ \ f(x) = x^2$

Then $g(f(x)) = g(\sin^2 x) = \sqrt{\sin x} = |\sin x|$

(for the first combination), which is given.

Hence $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$

[Students may try by checking the options one by one]

19. (b) Let $f : R \to R$ be a function defined by

$$f(x) = \frac{x-m}{x-n}$$

For any $(x, y) \in R$

Let $f(x) = f(y)$

$$\Rightarrow \frac{x-m}{x-n} = \frac{y-m}{y-n} \Rightarrow x = y$$

$\therefore$ f is one – one

Let $\alpha \in R$ such that $f(x) = \alpha$

$$\Rightarrow \alpha = \frac{x-m}{x-n} \quad \Rightarrow \quad (x-n)\alpha = x-m$$

$$\Rightarrow x\alpha - n\alpha = x - m \qquad \Rightarrow x\alpha - x = n\alpha - m$$

$$\Rightarrow x(\alpha - 1) = n\alpha - m$$

$\Rightarrow x = \frac{n\alpha - m}{\alpha - 1}$. for $\alpha = 1$, $x \notin R$

So, f is not onto.

20. (a) Since the domain of f is (0, 1),

$\therefore\ 0 < e^x < 1$ and $0 < \ln|x| < 1$

$\Rightarrow \log 0 < x < \log 1$ and $e^0 < |x| < e^1$

$\Rightarrow -\infty < x < 0$ and $1 < |x| < e$

$\Rightarrow x \in (-\infty, 0)$ and $x \in (-e, -1) \cup (1, e)$

$\Rightarrow x \in (-e, -1)$

21. (a) Given $f(x) = \dfrac{x}{x-1}$

$\therefore (fof)(x) = f\{f(x)\} = f\left(\dfrac{x}{x-1}\right)$

$$= \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{\frac{x}{x-1}}{\frac{x-x+1}{x-1}} = \frac{\frac{x}{x-1}}{\frac{1}{x-1}} = x.$$

$\Rightarrow (f \circ f \circ f)(x) = f(f \circ f)(x) = f(x) = \dfrac{x}{x-1}$

$\Rightarrow \underbrace{(f \circ f \circ f.....\circ f)}_{19 \text{ times}}(x) = f(f \circ f)(x) = f(x) = \dfrac{x}{x-1}$

22. (b) By definition only $f(x) = x^2 + 4x - 5$ with domain $[0, \infty)$ is one to one.

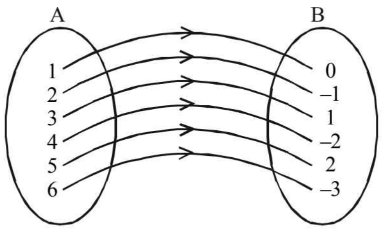
23. (b) If $y = \dfrac{2}{3}\dfrac{10^x - 10^{-x}}{10^x + 10^{-x}}$, $10^{2x} = \dfrac{3y+2}{2-3y}$

or $x = \dfrac{1}{2}\log_{10}\dfrac{2+3y}{2-3y}$ $\therefore f^{-1}(x) = \dfrac{1}{2}\log_{10}\dfrac{2+3x}{2-3x}$.

24. (d) $f : N \to I$

$f(1) = 0, f(2) = -1, f(3) = 1, f(4) = -2,$

$f(5) = 2,$ and $f(6) = -3$ so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B. Hence f is one-one and onto function.

25. (a) We have

$f(x) = \sin^2 x + \sin^2(x + \pi/3) + \cos x \cos(x + \pi/3)$

$= \dfrac{1-\cos 2x}{2} + \dfrac{1-\cos(2x + 2\pi/3)}{2}$

$+ \dfrac{1}{2}\{2\cos x \cos(x + \pi/3)\}$

$= \dfrac{1}{2}\left[\dfrac{5}{2} - \left\{\cos 2x + \cos\left(2x + \dfrac{2\pi}{3}\right)\right\} + \cos\left(2x + \dfrac{\pi}{3}\right)\right]$

$= \dfrac{1}{2}\left[\dfrac{5}{2} - 2\cos\left(2x + \dfrac{\pi}{3}\right)\cos\dfrac{\pi}{3} + \cos\left(2x + \dfrac{\pi}{3}\right)\right]$

$= \dfrac{5}{4}$ for all x.

$gof(x) = g(f(x)) = g\left(\dfrac{5}{4}\right) = 1$ $\quad \left[\because g\left(\dfrac{5}{4}\right) = 1\right.$ (given)] Hence, $gof(x) = 1$, for all x.

26. (c) For f(x) to be defined, we must have

$-1 \le \log_2\left(\dfrac{1}{2}x^2\right) \le 1 \Rightarrow 2^{-1} \le \dfrac{1}{2}x^2 \le 2^1$ [$\because$ the base $= 2 > 1$]

$\Rightarrow 1 \le x^2 \le 4$ ...(1)

Now, $1 \le x^2 \Rightarrow x^2 - 1 \ge 0$ i.e $(x-1)(x+1) \ge 0$

$\Rightarrow x \le -1$ or $x \ge 1$ ...(2)

Also, $x^2 \le 4 \Rightarrow x^2 - 4 \le 0$ i.e $(x-2)(x+2) \le 0$

$\Rightarrow -2 \le x \le 2$ ...(3)

From (2) and (3), we get the domain of f

$= ((-\infty, -1] \cup [1, \infty)) \cap [-2, 2] = [-2, -1] \cup [1, 2]$

27. (d) $f(x) = \dfrac{ax+b}{cx+d}$

$fof(x) = \dfrac{a\left\{\dfrac{ax+b}{cx+d}\right\} + b}{c\left\{\dfrac{ax+b}{cx+d}\right\} + d}$

$\Rightarrow \dfrac{a^2x + ab + bcx + bd}{acx + bc + cdx + d^2} = x$

$\Rightarrow (ac + dc)x^2 + (bc + d^2 - bc - a^2)x$
$- ab - bd = 0, \ \forall x \in R$

$\Rightarrow (a+d)c = 0, \ d^2 - a^2 = 0$

and $(a+d)b = 0$

$\Rightarrow a + d = 0 \Rightarrow d = -a$

28. (d) We have, If $x < 0$ $|x| = -x$

$\therefore f(x) = \dfrac{e^{-x} - e^{-x}}{e^x + e^{-x}} = 0$ $\qquad \therefore f(x) = 0 \ \forall \ x < 0$

$\therefore f(x)$ is not one-one

Next if $x \ge 0$, $|x| = x$ $\qquad \therefore f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Let $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} \Rightarrow y = \dfrac{e^{2x} - 1}{e^{2x} + 1}$ $\qquad \therefore e^{2x} = \dfrac{1+y}{1-y}$

For $x \ge 0$, $e^{2x} \ge 1$ $\qquad \therefore \dfrac{1+y}{1-y} \ge 1 \Rightarrow \dfrac{2y}{1-y} \ge 0$

$\Rightarrow y(y-1) \le 0, \ y \ne 1 \Rightarrow 0 \le y < 1$

$\therefore$ Range of $f(x) = [0 \ \ 1)$ $\qquad \therefore f(x)$ is not onto

29. (d) Since, $(fog)\,x = f\{g(x)\} = f(x^2) = \sin x^2$

and $(gof)\,x = g\,\{f(x)\} = g(\sin x) = \sin^2 x$

$\Rightarrow fog \neq gof$

30. (b) $2 < x < 3 \Rightarrow\ x - 1 > 0$

$x - 2 > 0$

$x - 3 < 0$

$\Rightarrow\ f(x) = x - 1 + x - 2 + 3 - x = x$

$\Rightarrow$ f is an identity function