EXERCISE 9.4 (NCERT)

Find the sum to n terms of each of these series in Ex 1 to 7.

Q No. 1 $1\times2+2\times3+3\times4+4\times5 \dots\dots$

Sol. The given series is $1\cdot2+2\cdot3+3\cdot4+\dots\dots$

Let T_n denotes the n^{th} term of the series

Since each term of given series is product of two factors

The first factor of each term in order are $1,2,3,4 \dots$ an AP.

The second factors of each term in order are $2,3,4 \dots$ An AP

$\therefore T_n = (n^{th}$ term of $1,2,3 \dots) \times (n^{th}$ term of $2,3,4 \dots)$

$= [1+(n-1)1][2+(n-1)1]$

$= (1+n-1)(2+n-1)$

$= n(n+1) = n^2+n.$

Put $n=1,2,3 \dots\dots n$ we get

$T_1 = 1^2+1$

$T_2 = 2^2+2$

$T_3 = 3^2+3$

$\dots\dots$

$\dots\dots$

$T_n = n^2+n.$

Adding vertically.

$T_1+T_2+T_3 \dots T_n = (1^2+2^2+3^2+\dots n^2)+(1+2+3+\dots n)$

$\therefore S_n = \Sigma n^2+\Sigma n$

$$= \frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{2n+1}{3}+1\right] = \frac{n(n+1)}{2}\left[\frac{2n+1+3}{3}\right]$$

$$= \frac{n(n+1)}{2}\left(\frac{2n+4}{3}\right) = \frac{n(n+1)\cdot2(n+2)}{2\times3} = \frac{n(n+1)(n+2)}{3}$$

Q No 2 $1\times2\times3+2\times3\times4+3\times4\times5+\cdots$

Sol. Given Series is $1.2.3+2.3.4+3.4.5\cdots$

$\therefore$ n^{th} term of Series $= T_n = (n^{th}$ term of $(1,2,3\cdots))(n^{th}$ term of $(2,3,4\cdots))(n^{th}$ term of $3,4,5\cdots)$

$$= [1+(n-1)1][2+(n-1)1][3+(n-1)1]$$
$$= (1+n-1)(2+n-1)(3+n-1)$$
$$= n(n+1)(n+2) = n(n^2+3n+2) = n^3+3n^2+2n.$$

Putting $n=1,2,3\cdots n.$ in $T_n = n^3+3n^2+2n$ and adding vertically we get

$$S_n = \sum n^3 + 3\sum n^2 + 2\sum n$$

$$= \frac{n^2(n+1)^2}{4} + 3\,\frac{n(n+1)(2n+1)}{6} + 2\cdot\frac{n(n+1)}{2}$$

$$= n(n+1)\left[\frac{n(n+1)}{4} + \frac{2n+1}{2} + 1\right]$$

$$= n(n+1)\left[\frac{n(n+1)+2(2n+1)+4}{4}\right] = \frac{n(n+1)}{4}\left[\frac{n^2+n+4n+2+4}{4}\right]$$

$$= \frac{n(n+1)}{4}(n^2+5n+6) = \frac{n(n+1)(n+2)(n+3)}{4}.$$

Q No 3: $3\times1^2+5\times2^2+7\times3^2\cdots$

Sol. Given Series is $3\times1^2+5\times2^2+7\times3^2\cdots$

Then n^{th} term $T_n = (n^{th}$ term of $3,5,7\cdots)\times(n^{th}$ term of $1,2,3\cdots)^2$

$$= [3+(n-1)2][1+(n-1)1]^2$$
$$= (3+2n-2)(1+n-1)^2 = (2n+1)(n^2) = 2n^3+n^2$$

$$\therefore\ S_n = 2\sum n^3 + \sum n^2$$

$$= 2\,\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{6}\left[\frac{3n(n+1)+(2n+1)}{1}\right] = \frac{n(n+1)}{6}\left[3n^2+3n+2n+1\right]$$

$$= \frac{n(n+1)(3n^2+5n+1)}{6}$$

Q No. 4 $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{2\times3} + \cdots$

Soln Given Series is $\frac{1}{1\cdot2} + \frac{1}{2\cdot3} + \frac{1}{3\cdot4} + \cdots$

$$\therefore T_n = \frac{1}{[1+(n-1)1][2+(n-1)1]} = \frac{1}{n(n+1)}$$

$$= \frac{1+n-n}{n(n+1)} = \frac{(n+1)-n}{n(n+1)} = \frac{n+1}{n(n+1)} - \frac{n}{n(n+1)}$$

$$= \frac{1}{n} - \frac{1}{n+1}$$

Putting $n = 1, 2, 3 \cdots n$ and Adding Vertically we get

$$T_1 = \frac{1}{1} - \frac{1}{2}$$

$$T_2 = \frac{1}{2} - \frac{1}{3}$$

$$T_3 = \frac{1}{3} - \frac{1}{4}$$

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$$T_{n-1} = \frac{1}{n-1} - \frac{1}{n}$$

$$T_n = \frac{1}{n} - \frac{1}{n+1}$$

$$\therefore S_n = 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \frac{n}{n+1}$$

Q No 5: $5^2+6^2+7^2+ \cdots +20^2$

Sol. We know that

$$1^2+2^2+3^2 \cdots +n^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{---(1)}$$

Put $n = 20$ in (1)

$$1^2+2^2+3^2+\cdots 20^2 = \frac{20(20+1)(2\times20+1)}{6}$$

$$= \frac{20\times21\times41}{6} = 2870$$

$$\therefore 1^2+2^2+3^2+\cdots +20^2 = 2870 \quad \text{---(2)}$$

Now put $n=4$ in (1)

$$1^2+2^2+3^2+4^2 = \frac{4(4+1)(2\times4+1)}{6} = \frac{4\times5\times9}{6} = 30 \quad \text{--- (3)}$$

Subtracting (3) from (2) we get:-

$$5^2+6^2+7^2+ \cdots 20^2 = 2870-30 = 2840.$$

Q No 6 $3\times8+6\times11+9\times14+\cdots\cdots$

Sol: Let T_n be the n^{th} term of given Series

Then $T_n = (n^{th}$ term of $3,6,9\cdots) \times (n^{th}$ term of $8,11,14\cdots)$

$$= [3+(n-1)3][8+(n-1)3] = [3+3n-3][8+3n-3]$$

$$= (3n)(3n+5) = 9n^2+15n.$$

$$\therefore S_n = 9\sum n^2 + 15\sum n$$

$$= 9 \cdot \frac{n(n+1)(2n+1)}{6} + 15\frac{n(n+1)}{2}.$$

$$= \frac{3}{2}n(n+1)[2n+1+5]$$

$$= \frac{3}{2}n(n+1)(2n+6) = 3n(n+1)(n+3)$$

Q No 7. $1^2+(1^2+2^2)+(1^2+2^2+3^2)+\cdots\cdots$

Sol. The given series is $1^2+(1^2+2^2)+(1^2+2^2+3^2)+\cdots$

$$\therefore T_n = 1^2+2^2+3^2+\cdots n^2$$

$$= \frac{n(n+1)(2n+1)}{6} = \frac{1}{6}[n(2n^2+n+2n+1)]$$

$$T_n = \frac{1}{6}(2n^3+3n^2+n).$$

Putting $n=1,2,3,\cdots n$ and adding we get

$$S_n = \frac{1}{6}\left[2\sum n^3+3\sum n^2+\sum n\right]$$

$$= \frac{1}{6}\left[2\frac{n^2(n+1)^2}{4}+3\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right]$$

$$= \frac{1}{6} \times \frac{n(n+1)}{2}\left[n(n+1) + (2n+1) + 1\right] = \frac{1}{12} n(n+1)(n^2+n+2n+1+1)$$

$$= \frac{1}{12} n(n+1)(n^2+3n+2) = \frac{1}{12} n(n+1)(n+1)(n+2) = \frac{1}{12} n(n+2)(n+1)^2$$

Find the sum to n terms of the series in Exercise 8 to 10 whose n^{th} term is given by.

Q No. 8 $n(n+1)(n+4)$

Sol. Here $T_n = n(n+1)(n+4) = n(n^2+5n+4) = n^3+5n^2+4n$.

$$\therefore S_n = \sum n^3 + 5\sum n^2 + 4\sum n$$

$$= \frac{n^2(n+1)^2}{4} + 5 \cdot \frac{n(n+1)(2n+1)}{6} + 4\frac{n(n+1)}{2}$$

$$= n(n+1)\left[\frac{n(n+1)}{4} + \frac{5(2n+1)}{6} + 2\right]$$

$$= n(n+1)\left[\frac{3n(n+1) + 10(2n+1) + 24}{12}\right]$$

$$= \frac{n(n+1)(3n^2+3n+20n+24+10)}{12}$$

$$= \frac{n(n+1)(3n^2+23n+34)}{12}$$

Q No 9 $n^2 + 2^n$

Sol: Here $T_n = n^2 + 2^n$

Putting $n = 1, 2, \cdots n$ and adding.

$$S_n = \sum n^2 + (2^1+2^2+2^3+\cdots 2^n)$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{2(2^n-1)}{2-1}$$

$$= \frac{n(n+1)(2n+1)}{6} + 2(2^n-1)$$

Q No 10 $(2n-1)^2$

Sol. Here $T_n = (2n-1)^2$

or $T_n = 4n^2 - 4n + 1$

Putting $n = 1, 2, \ldots n$ and adding

$$S_n = 4\sum n^2 - 4\sum n + n$$

$$= 4\,\frac{n(n+1)(2n+1)}{6} - 4\,\frac{n(n+1)}{2} + n$$

$$= \frac{n}{3}\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$$

$$= \frac{n}{3}\left[4n^2 + 6n + 2 - 6n - 6 + 3\right]$$

$$= \frac{n}{3}\left[4n^2 - 1\right]$$

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