Applications of Derivatives

Multiple Choice Questions (1 Marks)

1 Rate of change of perimeter of a square with respect to its side is :
(a)2 (b)1 (c)4 (d)3

2 Radius of a circle is increasing at the rate of $2\ m/s$. Rate of change of its circumference is :
(a)$4\pi\ m/s$ (b)$2\ m/s$ (c)$2\pi\ m/s$ (d)$4\ m/s$

3 Radius of a sphere is increasing at the rate of $5\ m/s$. Rate of change of its surface area, when radius is $4\ m$, is
(a)$120\pi\ m^2/s$ (b) $160\pi\ m^2/s$ (c) $32\pi\ m^2/s$ (d) $80\pi\ m^2/s$

4 $f(x) = \sin x$ is strictly decreasing in the interval :
(a)$\left(\frac{\pi}{2}, \pi\right)$ (b)$\left(\pi, \frac{3\pi}{2}\right)$ (c)$\left(0, \frac{\pi}{2}\right)$ (d)$\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

5 $f(x) = \cos x$ is strictly increasing in the interval :
(a)$\left(\frac{\pi}{2}, \pi\right)$ (b)$\left(\pi, \frac{3\pi}{2}\right)$ (c)$\left(0, \frac{\pi}{2}\right)$ (d)$\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

6 $f(x) = x^2$ strictly increases on :
(a)$(0, \infty)$ (b)$(-\infty, 0)$ (c)$(-7, -3)$ (d)$(-\infty, -3)$

7 If f is differentiable at critical points then the value of derivative of f at critical point is :
(a)1 (b)-1 (c)0 (d)2

8 On the curve $y = f(x)$ if $f'(a) = 0$ then $x = a$ is called a
(a)Practical point on the curve (b)Critical point on the curve
(c)Maximum point on the curve (d)Minimum point on the curve

9 On the curve $y = f(x)$ if $f'(a) = 0$ and $f''(a) < 0$ then $x = a$ is point of
(a)Maxima (b)Minima (c)Inflexion (d)infinity

10 On the curve $y = f(x)$ if $f'(a) = 0$ and $f''(a) > 0$ then $x = a$ is point of
(a)Maxima (b)Minima (c)Inflexion (d)infinity

True/False

1) Function f decreases where $f'(x) > 0$.
2) Function f decreases where $f'(x) < 0$.
3) $f(x) = \sin x$ is strictly decreasing function in $\left[0, \frac{\pi}{2}\right]$.
4) The value of function f is maximum at a if $f'(a) = 0$ and $f''(a) < 0$.
5) Logarithmic function $f(x) = \log x$ is a strictly increasing function.
6) Velocity of a moving particle cannot be expressed as derivative of displacement function of the particle.
7) The value of function f is maximum or minimum at a if $f'(a) = 0$.
8) The value of function f is minimum at a if $f'(a) = 0$ and $f''(a) < 0$.
9) When $f'(a) = 0$ then $x = a$ is called a critical point on the curve $y = f(x)$
10) If a given cylindrical bucket is being filled with water with a given rate then we can evaluate the rate of change of the volume of water cylinder inside the bucket.

2 Marks Questions

1. The volume of spherical balloon is increasing at the rate of 25 c.c./s . Find the rate of change of its surface area at the instant when its radius is 5 cm.
2. The side of square sheet is increasing at the rate of 3 cm/s. At what rate is the area increasing when the side is 10 cm long?
3. The side of square sheet is increasing at the rate of 5 cm/s. At what rate is the perimeter increasing when the side is 7 cm long?
4. The radius of spherical soap bubble is increasing at the rate of 0.2 cm/s. Find the rate of change of its volume when its radius is 4 cm.

5. The radius of spherical soap bubble is increasing at the rate of 0.8 cm/s. Find the rate of change of its surface area when the radius is 5 cm.
6. The edge of a cube is decreasing at the rate of 2 cm/s . Find the rate of change of its volume when the length of edge of the cube is 5 cm.
7. The edge of a cube is decreasing at the rate of 2 cm/s . Find the rate of change of its surface area when the length of edge is 6 cm.
8. Determine the intervals in which the following functions are increasing or decreasing :

(a) $f(x) = x^3 + 2x^2 - 1$ (b) $f(x) = 30 - 24x + 15x^2 - 2x^3$ (c) $f(x) = 20 - 12x + 9x^2 - 2x^3$
(d) $f(x) = 17 - 18x + 12x^2 - 2x^3$ (e) $f(x) = 20 - 9x + 6x^2 - x^3$ (f) $f(x) = 6 + 12x + 3x^2 - 2x^3$
(g) $f(x) = 2x^3 - 15x^2 + 36x + 1$ (h) $f(x) = x^3 - 6x^2 + 9x + 8$ (i) $f(x) = 2x^3 - 12x^2 + 18x + 5$

6 Marks Questions

1. Find the volume of the biggest right circular cone which is inscribed in a sphere of radius $9cm$.
2. Prove that the height of a right circular cylinder of maximum volume, which is inscribed in a sphere of radius R, is $\frac{2R}{\sqrt{3}}$.
3. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.
4. A wire of length 25 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What could be the lengths of the two pieces so that the combined area of the square and circle is minimum?
5. A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. What could be the lengths of the two pieces so that the combined area of the square and equilateral triangle is minimum?
6. Prove that the perimeter of a right angled triangle of given hypotenuse equal to 5 cm is maximum when the triangle is isosceles.
7. Of all rectangles with perimeter 40 cm/100 cm/80 cm find the one having maximum area. Also find the area.
8. Find the volume of the largest cylinder that can be inscribed in a sphere of radius R.
9. Find the volume of largest cone that can be inscribed in a sphere of radius R.
10. Show that height of the cylinder of maximum volume that can be inscribed in a sphere of 30 cm is $\frac{60}{\sqrt{3}}$ cm.
11. A window is in the form of rectangle surmounted by a semi-circle opening. If the perimeter of window is 10 cm/20 cm/ 30 cm, find the dimensions of the window so as to admit maximum possible light through the whole opening.
12. Show that the height of a closed cylinder of given volume and least surface area is equal to its diameter.
13. Find two positive numbers whose sum is 16 and the sum of whose cubes is maximum.
14. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.
15. A square piece of tin of side 18cm is to be made into a box without top, by cutting off small squares from each corner and folding up the flaps to form the box. What should be the side of the small square to be cut off so that the volume of the box is the maximum possible.