CHAPTER 12

INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. For every point $P(x, y, z)$ on the xy-plane,
(a) $x = 0$ (b) $y = 0$
(c) $z = 0$ (d) None of these

2. For every point $P(x, y, z)$ on the x-axis (except the origin),
(a) $x = 0, y = 0, z \neq 0$ (b) $x = 0, z = 0, y \neq 0$
(c) $y = 0, z = 0, x \neq 0$ (d) None of these

3. The distance of the point $P(a, b, c)$ from the x-axis is
(a) $\sqrt{b^2 + c^2}$ (b) $\sqrt{a^2 + c^2}$
(c) $\sqrt{a^2 + b^2}$ (d) None of these

4. Point (–3, 1, 2) lies in
(a) Octant I (b) Octant II
(c) Octant III (d) Octant IV

5. The three vertices of a parallelogram taken in order are (–1, 0), (3, 1) and (2, 2) respectively. The coordinate of the fourth vertex is
(a) (2,1) (b) (–2,1)
(c) (1,2) (d) (1,–2)

6. The point equidistant from the four points (0,0, 0), (3/2, 0, 0), (0,5/2, 0) and (0, 0, 7/2) is:
(a) $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{5}\right)$ (b) $\left(3, 2, \frac{3}{5}\right)$
(c) $\left(\frac{3}{4}, \frac{5}{4}, \frac{7}{4}\right)$ (d) $\left(\frac{1}{2}, 0, -1\right)$

7. The perpendicular distance of the point $P(6, 7, 8)$ from xy-plane is
(a) 8 (b) 7
(c) 6 (d) None of these

8. The ratio in which the join of points (1, –2, 3) and (4, 2, –1) is divided by XOY plane is
(a) 1 : 3 (b) 3 : 1
(c) –1 : 3 (d) None of these

9. The ratio in which the line joining the points (2,4, 5) and (3, 5, – 4) is internally divided by the xy-plane is
(a) 5 : 4 (b) 3 : 4
(c) 1 : 2 (d) 7 : 5

10. L is the foot of the perpendicular drawn from a point $P(6, 7, 8)$ on the xy-plane. The coordinates of point L is
(a) (6, 0, 0) (b) (6, 7, 0)
(c) (6, 0, 8) (d) None of these

11. If the sum of the squares of the distance of the point (x, y, z) from the points (a, 0, 0) and (–a , 0, 0) is $2c^2$, then which one of the following is correct?
(a) $x^2 + a^2 = 2c^2 - y^2 - z^2$ (b) $x^2 + a^2 = c^2 - y^2 - z^2$
(c) $x^2 - a^2 = c^2 - y^2 - z^2$ (d) $x^2 + a^2 = c^2 + y^2 + z^2$

12. The equation of set points P such that $PA^2 + PB^2 = 2K^2$, where A and B are the points (3, 4, 5) and (–1, 3, –7), respectively is
(a) $K^2 - 109$ (b) $2K^2 - 109$
(c) $3K^2 - 109$ (d) $4K^2 - 10$

13. The ratio in which the join of (2, 1, 5) and (3, 4, 3) is divided by the plane $(x + y - z) = \frac{1}{2}$ is:
(a) 3 : 5 (b) 5 : 7
(c) 1 : 3 (d) 4 : 5

14. The octant in which the points (– 3, 1, 2) and (– 3, 1, – 2) lies respectively is
(a) second, fourth (b) sixth, second
(c) fifth, sixth (d) second, sixth

15. Let L, M, N be the feet of the perpendiculars drawn from a point P(7, 9, 4) on the x, y and z-axes respectively. The coordinates of L, M and N respectively are
(a) (7, 0, 0), (0, 9, 0), (0, 0, 4) (b) (7, 0, 0), (0, 0, 9), (0, 4, 0)
(c) (0, 7, 0), (0, 0, 9), (4, 0, 0) (d) (0, 0, 7), (0, 9, 0), (4, 0, 0)

16. If a parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes, then the length of the diagonal is
(a) 7 units (b) 5 units
(c) 8 units (d) 3 units

17. The points A(4, – 2, 1), B(7, – 4, 7), C (2, – 5, 10) and D (– 1, – 3, 4) are the vertices of a
(a) tetrahedron (b) parallelogram
(c) rhombus (d) square

18. x-axis is the intersection of two planes are
(a) xy and xz (b) yz and zx
(c) xy and yz (d) None of these

19. The point (– 2, – 3, – 4) lies in the
(a) first octant (b) seventh octant
(c) second octant (d) eighth octant

20. A plane is parallel to yz-plane, so it is perpendicular to:
(a) x-axis (b) y-axis
(c) z-axis (d) None of these

21. The locus of a point for which x = 0 is
(a) xy-plane (b) yz-plane
(c) zx-plane (d) None of these

22. If L, M and N are the feet of perpendiculars drawn from the point P(3, 4, 5) on the XY, YZ and ZX-planes respectively, then
(a) distance of the point L from the point P is 5 units.
(b) distance of the point M from the point P is 3 units.
(c) distance of the point N from the point P is 4 units.
(d) All of the above.

23. If the point A (3, 2, 2) and B(5, 5, 4) are equidistant from P, which is on x-axis, then the coordinates of P are
(a) $\left(\frac{39}{4}, 2, 0\right)$ (b) $\left(\frac{49}{4}, 2, 0\right)$
(c) $\left(\frac{39}{4}, 0, 0\right)$ (d) $\left(\frac{49}{4}, 0, 0\right)$

24. The points (0, 7, 10), (– 1, 6, 6) and (– 4, 9, 6) form
(a) a right angled isosceles triangle
(b) a scalene triangle
(c) a right angled triangle
(d) an equilateral triangle

25. The point in YZ-plane which is equidistant from three points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1) is
(a) (0, 3, 1) (b) (0, 1, 3)
(c) (1, 3, 0) (d) (3, 1, 0)

26. Perpendicular distance of the point P(3, 5, 6) from y-axis is
(a) $\sqrt{41}$ (b) 6
(c) 7 (d) None of these

27. The coordinates of the point R, which divides the line segment joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in the ratio k : 1, are
(a) $\left(\frac{kx_2 - x_1}{1-k}, \frac{ky_2 - y_1}{1-k}, \frac{kz_2 - z_1}{1-k}\right)$
(b) $\left(\frac{kx_2 + x_1}{1+k}, \frac{ky_2 + y_1}{1+k}, \frac{kz_2 + z_1}{1+k}\right)$
(c) $\left(\frac{kx_2 + x_1}{1-k}, \frac{ky_2 + y_1}{1-k}, \frac{kz_2 + z_1}{1-k}\right)$
(d) None of these

28. The ratio, in which YZ-plane divides the line segment joining the points (4, 8, 10) and (6, 10, – 8), is
(a) 2 : 3 (externally) (b) 2 : 3 (internally)
(c) 1 : 2 (externally) (d) 1 : 2 (internally)

29. The ratio in which YZ-plane divides the line segment formed by joining the points (– 2, 4, 7) and (3, – 5, 8), is
(a) 2 : 3 (externally) (b) 2 : 3 (internally)
(c) 1 : 3 (externally) (d) 1 : 3 (internally)

30. If the origin is the centroid of a ΔABC having vertices A(a, 1, 3), B(– 2, b, – 5) and C(4, 7, c), then
(a) a = – 2 (b) b = 8
(c) c = – 2 (d) None of these

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

31. P(a, b, c); Q (a + 2, b + 2, c – 2) and R (a + 6, b + 6, c – 6) are collinear.
Consider the following statements :
I. R divides PQ internally in the ratio 3 : 2
II. R divides PQ externally in the ratio 3 : 2
III. Q divides PR internally in the ratio 1 : 2
Which of the statements given above is/are correct ?
(a) Only I (b) Only II
(c) I and III (d) II and III

32. Consider the following statements
I. The x-axis and y-axis together determine a plane known as xy-plane.
II. Coordinates of points in xy-plane are of the form $(x_1, y_1, 0)$.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

33. Consider the following statement
I. Any point on X-axis is of the form $(x, 0, 0)$
II. Any point on Y-axis is of the form $(0, y, 0)$
III. Any point on Z-axis is of the form $(0, 0, z)$
Choose the correct option.
(a) Only I and II are true. (b) Only II and III are true.
(c) Only I and III are true. (d) All are true.

34. I. The distance of the point (x, y, z) from the origin is given by $\sqrt{x^2 + y^2 + z^2}$.
II. If a point R divides the line segment joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio $m : n$ externally, then

$$R = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

35. I. The (0, 7, – 10), (1, 6, – 6) and (4, 9, –6) are the vertices of an isosceles triangle.
II. Centroid of the triangle whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

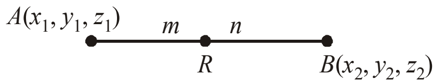
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

36. I. The coordinates of the mid-point of the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

II. If a point R divides the line segment joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio $m : n$ internally, then

$$R = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$



Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

37. Distance between the points (2, 3, 5) and (4, 3, 1) is $a\sqrt{5}$. The value of 'a' is
(a) 2 (b) 3 (c) 9 (d) 5

38. The perpendicular distance of the point $P(6, 7, 8)$ from xy-plane is
(a) 8 (b) 7
(c) 6 (d) None of these

39. The ratio in which the YZ-plane divide the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8) is 2 : m. The value of m is
(a) 2 (b) 3 (c) 4 (d) 1

40. Given that A(3, 2, – 4), B(5, 4 – 6) and C(9, 8, – 10) are collinear. Ratio in which B divides AC is 1 : m. The value of m is
(a) 2 (b) 3 (c) 4 (d) 5

41. If the origin is the centroid of the triangle with vertices A (2a, 2, 6), B (– 4, 3b, –10) and C (8, 14, 2c), then the sum of value of a and c is
(a) 0 (b) 1 (c) 2 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

42. **Assertion:** The coordinates of the point which divides the join of A (2, –1, 4) and B (4, 3, 2) in the ratio 2 : 3 externally is C (–2, –9, 8)
Reason : If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points, and let R be a point on PQ produced dividing it externally in the ratio $m_1 : m_2$. Then the coordinates of R are

$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right)$$

43. **Assertion :** If three vertices of a parallelogram ABCD are A(3, –1, 2), B (1, 2, –4) and C (–1, 1, 2), then the fourth vertex is (1, –2, 8).
Reason : Diagonals of a parallelogram bisect each other and mid-point of AC and BD coincide.

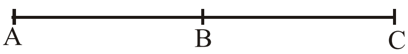
44. **Assertion :** The distance of a point P(x, y, z) from the origin O(0, 0, 0) is given by $OP = \sqrt{x^2 + y^2 + z^2}$.
Reason : A point is on the x-axis. Its y-coordinate and z-coordinate are 0 and 0 respectively.

45. **Assertion :** Coordinates (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.
Reason : Opposite sides of a parallelogram are equal and diagonals are not equal.

46. **Assertion :** If P (x, y, z) is any point in the space, then x, y and z are perpendicular distances from YZ, ZX and XY-planes, respectively.
Reason : If three planes are drawn parallel to YZ, ZX and XY-planes such that they intersect X, Y and Z-axes at (x, 0, 0), (0, y, 0) and (0, 0, z), then the planes meet in space at a point P(x, y, z).

47. **Assertion :** The distance between the points P(1, – 3, 4) and Q(– 4, 1, 2) is $\sqrt{5}$ units.
Reason : $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
where, P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2).

48. **Assertion :** Points (– 2, 3, 5), (1, 2, 3) and (7, 0, – 1) are collinear.
Reason : Three points A, B and C are said to be collinear, if AB + BC = AC (as shown below).



49. **Assertion :** Points (– 4, 6, 10), (2, 4, 6) and (14, 0, – 2) are collinear.
Reason : Point (14, 0, – 2) divides the line segment joining by other two given points in the ratio 3 : 2 internally.

50. **Assertion :** The XY-plane divides the line joining the points (– 1, 3, 4) and (2, – 5, 6) externally in the ratio 2 : 3.
Reason : For a point in XY-plane, its z-coordinate should be zero.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

51. What is the locus of a point which is equidistant from the points (1, 2, 3) and (3, 2, – 1) ?
(a) $x + z = 0$ (b) $x - 3z = 0$
(c) $x - z = 0$ (d) $x - 2z = 0$

52. What is the shortest distance of the point (1, 2, 3) from x-axis ?

(a) 1 (b) $\sqrt{6}$

(c) $\sqrt{13}$ (d) $\sqrt{14}$

53. The equation of locus of a point whose distance from the y-axis is equal to its distance from the point (2, 1, –1) is

(a) $x^2+y^2+z^2=6$ (b) $x^2-4x^2+2z^2+6=0$

(c) $y^2-2y^2-4x^2+2z+6=0$ (d) $x^2+y^2-z^2=0$

54. ABC is a triangle and AD is the median. If the coordinates of A are (4, 7, – 8)and the coordinates of centroid of the triangle ABC are (1, 1, 1), what are the coordinates of D?

(a) $\left(-\frac{1}{2},2,11\right)$ (b) $\left(-\frac{1}{2},-2,\frac{11}{2}\right)$

(c) (–1, 2, 11) (d) (–5, –11, 19)

55. In three dimensional space the path of a point whose distance from the x-axis is 3 times its distance from the yz-plane is:

(a) $y^2+z^2=9x^2$ (b) $x^2+y^2=3z^2$

(c) $x^2+z^2=3y^2$ (d) $y^2-z^2=9x^2$

56. Let (3, 4, – 1) and (– 1, 2, 3) be the end points of a diameter of a sphere. Then, the radius of the sphere is equal to

(a) 2 units (b) 3 units

(c) 6 units (d) 7 units

57. Find the coordinates of the point which is three fifth of the way from (3, 4, 5) to (– 2, – 1, 0).

(a) (1, 0, 2) (b) (2, 0, 1)

(c) (0, 2, 1) (d) (0, 1, 2)

58. The coordinates of the points which trisect the line segment joining the points P(4, 2, – 6) and Q(10, – 16, 6) are

(a) (6, – 4, – 2) and (8, 10, – 2)

(b) (6, – 4, – 2) and (8, – 10, 2)

(c) (–6, 4, 2) and (– 8, 10, 2)

(d) None of these

59. If A(3, 2, 0), B(5, 3, 2) and C(– 9, 6, – 3) are three points forming a triangle and AD, the bisector of ∠BAC, meets BC in D, then the coordinates of the point D are

(a) $\left(\frac{17}{8},\frac{57}{8},\frac{17}{8}\right)$ (b) $\left(\frac{19}{8},\frac{57}{16},\frac{17}{16}\right)$

(c) $\left(\frac{8}{17},\frac{8}{19},\frac{17}{8}\right)$ (d) None of these

60. The mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, – 1), then the vertices are

(a) (7, 2, 5), (3, 12, 17), (– 3, 4, – 7)

(b) (7, 2, 5), (3, 12, 17), (3, 4, 7)

(c) (7, 2, 5), (–3, 11, 15), (3, 4, 8)

(d) None of the above

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(c)** On xy-plane, z-co-ordinate is zero.
2. **(c)** On x-axis, y and z-co-ordinates are zero.
3. **(a)** Let $(a, 0, 0)$ be a point on x-axis.

   Required distance $= \sqrt{(a-a^2)+(b-0)^2+(c-0)^2}$

   $= \sqrt{b^2+c^2}$
4. **(b)** $(-3, 1, 2)$ lies in second octant.
5. **(b)** Let $A(-1, 0)$, $B(3, 1)$, $C(2, 2)$ and $D(x, y)$ be the vertices of a parallelogram ABCD taken in order. Since, the diagonals of a parallelogram bisect each other.

   $\therefore$ Coordinates of the mid point of AC
   = Coordinates of the mid-point of BD

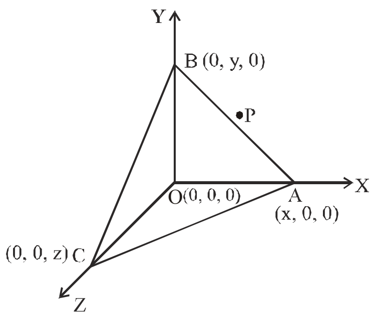
   $\Rightarrow \left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{3+x}{2}, \frac{1+y}{2}\right)$

   $\Rightarrow \left(\frac{1}{2}, 1\right) = \left(\frac{3+x}{2}, \frac{y+1}{2}\right)$

   $\Rightarrow \frac{3+x}{2} = \frac{1}{2}$ and $\frac{y+1}{2} = 1$

   $\Rightarrow$ $x = -2$ and $y = 1$.

   Hence the fourth vertex of the parallelogram is $(-2, 1)$
6. **(c)**



   We know the co-ordinate of P which is equidistant from four points A(x, 0, 0), B (0, y, 0), C (0, 0, z), O (0, 0, 0) is $\frac{1}{2}(x, y, z)$

   $\therefore$ Given: points are $(0, 0, 0)$, $\left(\frac{3}{2}, 0, 0\right)$, $\left(0, \frac{5}{2}, 0\right)$ and $\left(0, 0, \frac{7}{2}\right)$

   $\therefore$ Co-ordinate of point $P = \frac{1}{2}\left(\frac{3}{2}, \frac{5}{2}, \frac{7}{2}\right) = \left(\frac{3}{4}, \frac{5}{4}, \frac{7}{4}\right)$
7. **(a)** Let L be the foot of perpendicular drawn from the point $P(6, 7, 8)$ to the xy-plane and the distance of this foot L from P is z-coordinate of P, i.e., 8 units.
8. **(b)** Let A $(1, -2, 3)$ and B $(4, 2, -1)$. Let the plane XOY meet the line AB in the point C such that C divides AB in the ratio $k : 1$, then $C \equiv \left(\frac{4k+1}{k+1}, \frac{2k-2}{k+1}, \frac{-k+3}{k+1}\right)$.

   Since C lies on the plane XOY i.e. the plane $z = 0$,

   therefore, $\frac{-k+3}{k+1} = 0 \Rightarrow k = 3$.
9. **(a)** Let the line joining the points $(2, 4, 5)$ and $(3, 5, -4)$ is internally divided by the xy - plane in the ratio $k : 1$.

   $\therefore$ For xy plane, $z = 0$

   $\Rightarrow 0 = \frac{-k \times 4 + 5}{k+1} \Rightarrow 4k = 5 \Rightarrow k = \frac{5}{4}$.

   so, ratio is 5 : 4
10. **(b)** Since L is the foot of perpendicular from P on the xy-plane, z-coordinate is zero in the xy-plane. Hence, coordinates of L are $(6, 7, 0)$.
11. **(b)** Let the point be $P(x, y, z)$ and two points, $(a, 0, 0)$ and $(-a, 0, 0)$ be A and B

    As given in the problem,

    $PA^2 + PB^2 = 2c^2$

    so, $(x+a)^2+(y-0)^2+(z-0)^2+(x-a)^2+(y-0)^2+(z-0)^2=2c^2$

    or, $(x+a)^2 + y^2 + z^2 + (x-a)^2 + y^2 + z^2 = 2c^2$

    $\Rightarrow x^2 + 2a + a^2 + y^2 + z^2 + x^2 - 2a + a^2 + y^2 + z^2 = 2c^2$

    $\Rightarrow 2(x^2 + y^2 + z^2 + a^2) = 2c^2$

    $\Rightarrow x^2 + y^2 + z^2 + a^2 = c^2$

    $\Rightarrow x^2 + a^2 = c^2 - y^2 - z^2$
12. **(b)** Let the coordinates of point P be (x, y, z).

    Here, $PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$

    $PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$

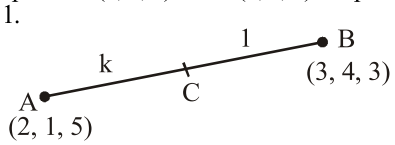
    By the given condition

    $PA^2 + PB^2 = 2K^2$

    We have

    $(x-3)^2+(y-4)^2+(z-5)^2+(x+1)^2+(y-3)^2+(z+7)^2=2K^2$

    i.e. $2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = 2K^2 - 109$
13. **(b)** As given plane $x + y - z = \frac{1}{2}$ divides the line joining the points A$(2, 1, 5)$ and B$(3, 4, 3)$ at a point C in the ratio $k : 1$.



    Then coordinates of C

    $\left(\frac{3k+2}{k+1}, \frac{4k+1}{k+1}, \frac{3k+5}{k+1}\right)$

    Point C lies on the plane,

    $\Rightarrow$ Coordinates of C must satisfy the equation of plane.

So, $\left(\frac{3k+2}{k+1}\right)+\left(\frac{4k+1}{k+1}\right)-\left(\frac{3k+5}{k+1}\right)=\frac{1}{2}$

$\Rightarrow 3k+2+4k+1-3k-5=\frac{1}{2}(k+1)$

$\Rightarrow 4k-2=\frac{1}{2}(k+1)$

$\Rightarrow 8k-4=k+1 \Rightarrow 7k=5$

$\Rightarrow k=\frac{5}{7}$

Ratio is 5 : 7.

14. (d) The point (– 3, 1, 2) lies in second octant and the point (– 3, 1, – 2) lies in sixth octant.

15. (a) Since L is the foot of perpendicular from P on the x-axis, its y and z-coordinates are zero. So, the coordinates of L is (7, 0, 0). Similarly, the coordinates of M and N are (0, 9, 0) and (0, 0, 4), respectively.

16. (a) Length of edges of the parallelopiped are 5 – 2, 9 – 3, 7 – 5 i.e., 3, 6, 2.

$\therefore$ Length of diagonal is $\sqrt{3^2+6^2+2^2}=7$ units.

17. (b) Here, the mid-point of AC is

$\left(\frac{4+2}{2},\frac{-2-5}{2},\frac{1+10}{2}\right)=\left(3,-\frac{7}{2},\frac{11}{2}\right)$

and that of BD is

$\left(\frac{7-1}{2},\frac{-4-3}{2},\frac{7+4}{2}\right)=\left(3,-\frac{7}{2},\frac{11}{2}\right)$

So, the diagonals AC and BD bisect each other.

$\Rightarrow$ ABCD is a parallelogram.

As $|AB|=\sqrt{3^2+2^2+6^2}=7$ and

$|AD|=\sqrt{5^2+1^2+3^2}=\sqrt{35}\neq|AB|$,

Therefore, ABCD is not a rhombus and naturally, it cannot be a square.

18. (a)

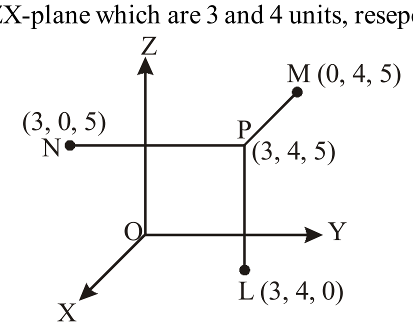
19. (b) The point (– 2, – 3, – 4) lies on negative of x, y and z-axis.

$\therefore$ It lies in seventh octant.

20. (a) A plane is parallel to yz-plane which is always perpendicular to x-axis.

21. (b) For yz-plane x = 0, locus of point for which x = 0 is yz-plane.

22. (d) L is the foot of perpendicular drawn from the point P(3, 4, 5) to the XY-plane. Therefore, the coordinates of the point L is (3, 4, 0). The distance between the point (3, 4, 5) and (3, 4, 0) is 5. Similarly, we can find the lengths of the foot of perpendiculars on YZ and ZX-plane which are 3 and 4 units, resepctively.



23. (d) The point on the x-axis is of the form P(x, 0, 0). Since, the points A and B are equidistant from P. Therefore, $PA^2 = PB^2$,

i.e., $(x-3)^2+(0-2)^2+(0-2)^2$
$=(x-5)^2+(0-5)^2+(0-4)^2$

$\Rightarrow \quad 4x = 25+25+16-17$ i.e., $x=\frac{49}{4}$

Thus, the point P on the x-axis is $\left(\frac{49}{4},0,0\right)$ which is equidistant from A and B

24. (a) Let P(0, 7, 10), Q(– 1, 6, 6) and R(– 4, 9, 6) be the vertices of a triangle

Here, $PQ=\sqrt{1+1+16}=3\sqrt{2}$

$QR=\sqrt{9+9+0}=3\sqrt{2}$

$PR=\sqrt{16+4+16}=6$

Now, $PQ^2+QR^2=\left(3\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2$
$=18+18=36=(PR)^2$

Therefore, ΔPQR is a right angled triangle at Q. Also, OQ = QR, Hence, ΔPQR is a right angled isosceles triangle.

25. (b) Since x-coordinate of every point in YZ-plane is zero. Let P(0, y, z) be a point on the YZ-plane such that PA = PB = PC.

Now, PA = PB

$\Rightarrow \quad (0-2)^2+(y-0)^2+(z-3)^2$
$=(0-0)^2+(y-3)^2+(z-2)^2$,

i.e., $z-3y=0$

and PB = PC

$\Rightarrow \quad y^2+9-6y+z^2+4-4z=y^2+z^2+1-2z$,

i.e., $3y+z=6$

On simplifying the two equations, we get y = 1 and z = 3. Here, the coordinate of the point P are (0, 1, 3).

26. (d) Let M is the foot of perpendicular from P on the y-axis, therefore its x and z-coordinates are zero. The coordinates of M is (0, 5, 0). Therefore, the perpendicular distance of the point P from y-axis $=\sqrt{3^2+6^2}=\sqrt{45}$.

27. (b) The coordinates of the point R which divides PQ in the ratio k : 1 where coordinates of P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2) are obtained by taking $k=\frac{m}{n}$ in the coordinates of the point R which divides PQ internally in the ratio m : n, which are as given below.

$$\left(\frac{kx_2+x_1}{1+k},\frac{ky_2+y_1}{1+k},\frac{kz_2+z_1}{1+k}\right)$$

28. (a) Let YZ-plane divides the line segment joining A (4, 8, 10) and B(6, 10, –8) at P(x, y, z) in the ratio k : 1. Then, the coordinates of P are

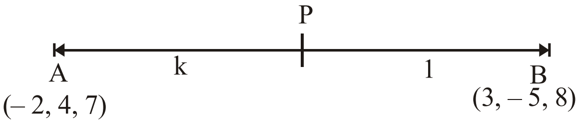
$$\left(\frac{4+6k}{k+1},\frac{8+10k}{k+1},\frac{10-8k}{k+1}\right)$$

Since, P lies on the YZ-plane, its x-coordinates is zero.

i.e., $\frac{4+6k}{k+1}=0$ or $k=-\frac{2}{3}$

Therefore, YZ-plane divides AB externally in the ratio 2 : 3

29. **(b)** The given points are A(– 2, 4, 7) and B(3, – 5, 8).
Let the point P(0, y, z) in YZ-plane divides AB in the ratio k : 1. Then,



x-coordinate of point $P = \dfrac{mx_2 + nx_1}{m+n}$

$\dfrac{k \times 3 + 1 \times (-2)}{k+1} = 0$ ($\because$ x-coordinate of P is zero)

$\Rightarrow \quad 3k - 2 = 0$

$\Rightarrow \quad k = \dfrac{2}{3}$

$\Rightarrow \quad k : 1 = 2 : 3$

$\therefore$ YZ-plane divides the segment internally in the ratio 2 : 3

30. **(a)** For centroid of ΔABC,

$$x = \frac{a-2+4}{3} = \frac{a+2}{3}$$

$$y = \frac{1+b+7}{3} = \frac{b+8}{3}$$

and $$z = \frac{3-5+c}{3} = \frac{c-2}{3}$$

But given centroid is (0, 0, 0).

$\therefore \quad \dfrac{a+2}{3} = 0 \Rightarrow a = -2$

$\dfrac{b+8}{3} = 0 \Rightarrow b = -8$

$\dfrac{c-2}{3} = 0 \Rightarrow c = 2$

STATEMENT TYPE QUESTIONS

31. **(d)** Given that P (a, b, c), Q (a + 2, b + 2, c – 2) and R (a + 6, b + 6, c – 6) are collinear, one point must divide, the other two points externally or internally.
Let R divide P and Q in ratio k : 1 so,
taking on x-coordinates

$\dfrac{k(a+2)+a}{k+1} = a+6$

$\Rightarrow \quad k(a+2) + a = (k+1)(a+6)$

$\Rightarrow \quad ka + 2k + a = ka + 6k + a + 6 \Rightarrow -4k = 6$

or $\quad k = -\dfrac{3}{2}$

Negative sign shows that this is external division in ratio 3 : 2. So, R divides P and Q externally in 3 : 2 ratio.
Putting this value for y - and z - coordinates satisfied :
for y - coordinate

$\dfrac{3(b+2)-2b}{3-2} = 3b + 6 - 2b = b + 6$

and for z-coordinate :

$\dfrac{3(c-2)-2c}{3-2} = \dfrac{3c-6-2c}{1} = c - b$

Statement II is correct.
Also, let Q divide P and R in ratio p : 1 taking an x-coordinate:

$\dfrac{p(a+6)+a}{p+1} = a+2$

$\dfrac{p.a+6p+a}{p+1} = a+2$

$\Rightarrow \quad pa + 6p + a = pa + a + 2p + 2$

$\Rightarrow \quad 4p = 2 \Rightarrow p = \dfrac{1}{2}.$

Positive sign shows that the division is internal and in the ratio 1 : 2
Verifying for y - and z- coordinates, satisfies this results.
For y coordinate,

$\dfrac{(b+6)\times 1 + 2b}{3} = \dfrac{3b+6}{3} = b+2$

and for z-coordinate,

$\dfrac{c-6+2c}{3} = \dfrac{3c-6}{3} = c-2$

values are satisfied.
So, statement III is correct.

32. **(c)** **33.** **(d)**

34. **(c)** Both the statements are true.

35. **(c)** Both the statements are true.

36. **(c)** Both the given statements are true.

INTEGER TYPE QUESTIONS

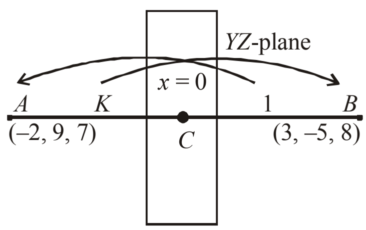
37. **(a)** The given points are (2, 3, 5) and (4, 3, 1).
$\therefore$ Required distance

$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2} = \sqrt{4+0+16}$

$= \sqrt{20} = 2\sqrt{5}$

38. **(a)** Let L be the foot of perpendicular drawn from the point P(6, 7, 8) to the xy-plane and the distance of this foot L from P is z-coordinate of P, i.e., 8 units.

39. **(b)** Let the points be A(–2, 4, 7) and B(3, –5, 8) on YZ-plane, x-coordinate = 0.



Let the ratio be K : 1.
The coordinates of C are

$\left(\dfrac{3K-2}{K+1}, \dfrac{-5K+4}{K+1}, \dfrac{8K+7}{K+1}\right)$

Clearly $\dfrac{3K-2}{K+1} = 0 \Rightarrow 3K = 2 \Rightarrow K = \dfrac{2}{3}$

Hence required ratio is 2 : 3.

40. **(a)** Suppose B divides AC in the ratio λ : 1.

$\therefore \quad B = \left(\dfrac{9\lambda+3}{\lambda+1}, \dfrac{8\lambda+2}{\lambda+1}, \dfrac{-10\lambda-4}{\lambda+1}\right) = (5, 4, -6)$

$\Rightarrow \dfrac{9\lambda+3}{\lambda+1}=5, \dfrac{8\lambda+2}{\lambda+1}=4, \dfrac{-10\lambda-4}{\lambda+1}=-6$

$\Rightarrow \lambda=\dfrac{1}{2}$

So, required ratio is 1 : 2.

41. **(a)** Centriod of Δ ABC are $\left(\dfrac{2a+4}{3}, \dfrac{3b+16}{3}, \dfrac{2c-4}{3}\right)$

Given centroid = (0, 0, 0)

$\therefore \quad 2a+4=0,\ 16+3b=0,\ 2c-4=0$

$\Rightarrow \quad a=-2,\ b=\dfrac{-16}{3},\ c=2$

Hence, a + c = 0

ASSERTION - REASON TYPE QUESTIONS

42. **(a)** **Assertion :**

$x=\dfrac{2\times4-3\times2}{2-3}, y=\dfrac{2\times3-3(-1)}{2-3},$

$z=\dfrac{2\times2-3\times4}{2-3}$

$\Rightarrow x=-2, y=-9, z=8$

43. **(a)** Since diagonals of a parallelogram bisect each other therefore, mid-point of AC and BD coincide.

$\therefore \quad (1,0,2)=\left(\dfrac{x+1}{2}, \dfrac{y+2}{2}, \dfrac{z-4}{2}\right)$

$\Rightarrow \quad \dfrac{x+1}{2}=1, \dfrac{y+2}{2}=0, \dfrac{z-4}{2}=2$

$\Rightarrow \quad x=1, y=-2, z=8$

44. **(b)** Both Assertion and Reason is correct.

45. **(a)** **Assertion:** The given points are $A(-1, 2, 1)$, $B(1, -2, 5)$, $C(4, -7, 8)$ and $D(2, -3, 4)$, then by distance formula

$AB=\sqrt{(1+1)^2+(-2-2)^2+(5-1)^2}$

$=\sqrt{4+16+16}=\sqrt{36}=6$

$BC=\sqrt{(4-1)^2+(-7+2)^2+(8-5)^2}$

$=\sqrt{9+25+9}=\sqrt{43}$

$CD=\sqrt{(2-4)^2+(-3+7)^2+(4-8)^2}$

$=\sqrt{4+16+16}=\sqrt{36}=6$

$DA=\sqrt{(-1-2)^2+(2+3)^2+(1-4)^2}$

$=\sqrt{9+25+9}=\sqrt{43}$

$AC=\sqrt{(4+1)^2+(-7-2)^2+(8-1)^2}$

$=\sqrt{25+81+49}=\sqrt{155}$

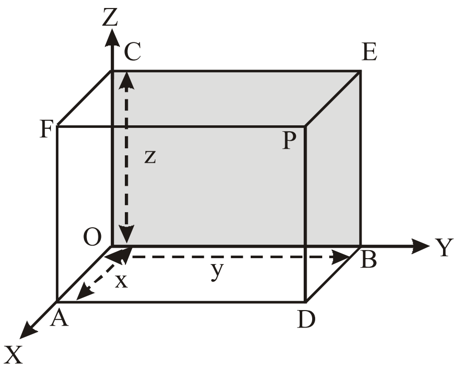
$BD=\sqrt{(2-1)^2+(-3+2)^2+(4-5)^2}$

$=\sqrt{1+1+1}=\sqrt{3}$

Now, since $AB = CD$ and $BC = DA$ i.e., opposite sides are equal and $AC \neq BD$ i.e. the diagonals are not equal. So, points are the vertices of parallelogram.

46. **(b)** **Assertion :** Through, the point P in the space, we draw three planes, parallel to the coordinates planes, meeting the X-axis, Y-axis and Z-axis in the points A, B and C, respectively. We observe that OA = x, OB = y and OC = z. Thus, if P(x, y, z) is any point in the space, then x, y and z are perpendicular distances from YZ, ZX and XY-planes, respectively.

Reason : Given x, y and z, we locate the three points A, B and C on the three coordinate axes. Through the points A, B and C, we draw planes parallel to the YZ-plane, ZX-plane and XY-plane, respectively. The point of intersection of these three planes, namely ADPF, BDPE and CEPF is obviously the point P, corresponding to the ordered triplet (x, y, z).



47. **(d)** The distance PQ between the points P(1, – 3, 4) and Q(– 4, 1, 2) is

$PQ=\sqrt{(-4-1)^2+(1+3)^2+(2-4)^2}$

$=\sqrt{25+16+4}=\sqrt{45}=3\sqrt{5}$ units

48. **(a)** The given points are

A(–2, 3, 5), B(1, 2, 3), C(7, 0, – 1)

Distance between A and B

$AB=\sqrt{(-2-1)^2+(3-2)^2+(5-3)^2}$

$=\sqrt{(-3)^2+(1)^2+(2)^2}=\sqrt{9+1+4}=\sqrt{14}$

Distance between B and C

$BC=\sqrt{(1-7)^2+(2-0)^2+(3+1)^2}$

$=\sqrt{(-6)^2+(2)^2+(4)^2}$

$=\sqrt{36+4+16}=\sqrt{56}=2\sqrt{14}$

Distance between A and C

$AC=\sqrt{(-2-7)^2+(3-0)^2+(5+1)^2}$

$=\sqrt{(-9)^2+(3)^2+(6)^2}$

$=\sqrt{81+9+36}=\sqrt{126}=3\sqrt{14}$

Clearly, AB + BC = AC

Hence, the given points are collinear.

49. **(c)** Let A(– 4, 6, 10), B(2, 4, 6) and C(14, 0, – 2) be the given points. Let the point P divides AB in the ratio k : 1. Then, coordinates of the point P are

$$\frac{2k-4}{k+1}, \frac{4k+6}{k+1}, \frac{6k+10}{k+1}$$

Let us examine whether for some value of k, the point P coincides with point C.

On putting $\frac{2k-4}{k+1}=14$, we get $k=-\frac{3}{2}$

When $k=-\frac{3}{2}$, then $\frac{4k+6}{k+1}=\frac{4\left(-\frac{3}{2}\right)+6}{-\frac{3}{2}+1}=0$

and $\frac{6k+10}{k+1}=\frac{6\left(-\frac{3}{2}\right)+10}{-\frac{3}{2}+1}=-2$

Therefore, C (14, 0, – 2) is a point which divides AB externally in the ratio 3 : 2 and is same as P. Hence A, B, C are collinear.

50. (a) Suppose xy-plane divides the line joining the given points in the ratio $\lambda : 1$. The coordinates of the points of division are $\left[\frac{2\lambda-1}{\lambda+1},\frac{-5\lambda+3}{\lambda+1},\frac{6\lambda+4}{\lambda+1}\right]$. Since, the points lies on the XY-plane.

$\therefore\quad \frac{6\lambda+4}{\lambda+1}=0 \Rightarrow \lambda=\frac{-2}{3}$

CRITICALTHINKING TYPE QUESTIONS

51. (d) Let (h, k, ℓ) be the point which is equidistant from the points (1, 2, 3) and (3, 2, –1)

$\Rightarrow\ \sqrt{(h-1)^2+(k-2)^2+(\ell-3)^2}$

$=\sqrt{(h-3)^2+(k-2)^2+(\ell+1)^2}$

$\Rightarrow\ (h-1)^2+(\ell-3)^2=(h-3)^2+(\ell+1)^2$

$\Rightarrow\ h^2+1-2h+\ell^2-6\ell+9=h^2-6h+9+\ell^2+2\ell+1$

$\Rightarrow\ -2h-6\ell=-6h+2\ell$

$\Rightarrow\ 6h-2h-6\ell-2\ell=0 \Rightarrow 4h-8\ell=0$

$\Rightarrow\ h-2\ell=0$

Putting $h = x$ and $\ell = z$

We get locus of points (h, k, ℓ)

as, $x-2z=0$

52. (c) Any point on x-axis has $y = z = 0$

Distance of the point (1, 2, 3) from x-axis is the distance between point (1, 2, 3) and point (1, 0, 0)

$=\sqrt{(1-1)^2+(2-0)^2+(3-0)^2}=\sqrt{2^2+3^2}$

$=\sqrt{4+9}=\sqrt{13}$

53. (c) The variable point is $P(x, y, z)$.

Its distance from the y-axis $=\sqrt{x^2+z^2}$

Its distance from (2, 1, –1)

$=\sqrt{(x-2)^2+(y-1)^2+(z+1)^2}$

Given

$\sqrt{x^2+z^2}=\sqrt{(x-2)^2+(y-1)^2+(z+1)^2}$

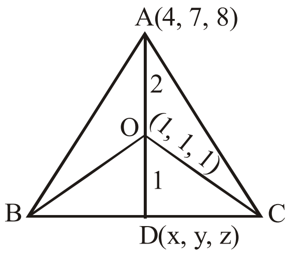
$\Rightarrow y^2-2y-4x+2z+6=0$

54. (b) Let coordinates of D be (x, y, z)

Co-ordinates of centroid is (1, 1, 1), and of A, is (4, 7, 8)

Centroid divides median in 2 : 1 ratio

So, $\frac{AO}{OD}=2:1$



For x :

$1=\frac{2\times x+1\times 4}{1+2}$

$\Rightarrow\ x=-1/2$

For y :

$1=\frac{2y+1\times 7}{1+2}\quad\Rightarrow y=-2$

For z :

$1=\frac{2\times z+1\times -8}{3}\quad\Rightarrow z=+11/2$

$\therefore$ Coordinates of D are (–1/2, –2, 11/2)

55. (a) Let $P(x_1, y_1, z_1)$ be the point.

Then distance of P from x-axis $=\sqrt{y_1^2+z_1^2}$

Given plane is x = 0 (yz-plane)

Distance of $P(x_1, y_1, z_1)$ from yz-plane is $\frac{x_1}{\sqrt{1}}$

From the given condition,distance of P from x-axis = 3 × distance of P from yz-plane

$\sqrt{y_1^2+z_1^2}=3x_1$

Squaring, $y_1^2+z_1^2=9x_1^2$

Thus, path of $P(x_1, y_1, z_1)$ is got by putting x, y, z in place of x_1, y_1, z_1 as $y^2+z^2=9x^2$

56. (b) Let P(3, 4, – 1) and Q(– 1, 2, 3) be the end points of the diameter of a sphere.

$\therefore$ Length of diameter = PQ

$=\sqrt{(-1-3)^2+(2-4)^2+(3+1)^2}$

$=\sqrt{16+4+16}=\sqrt{36}=6$ units

$\therefore$ Radius $=\frac{6}{2}$ **= 3 units**

57. (d) Let A = (3, 4, 5), B = (– 2, – 1, 0) and P(x, y, z) be the required point. As P is three-fifth of the way from A to B, we have

$AP=\frac{3}{5}AB \Rightarrow AP=\frac{3}{5}(AP+PB)$

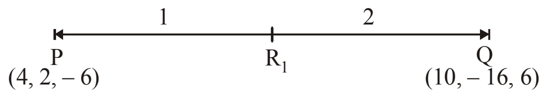
$\Rightarrow\ 5AP=3AP+3PB \Rightarrow \frac{AP}{PB}=\frac{3}{2}$

$\Rightarrow$ P divides [AB] in the ratio 3 : 2

$\therefore\ P\equiv\left(\frac{3\times(-2)+2\times 3}{3+2},\frac{3\times(-1)+2\times 4}{3+2},\frac{3\times 0+2\times 5}{3+2}\right)$

$\Rightarrow\ P=(0, 1, 2)$

58. (b) Let the points R_1 and R_2 trisects the line PQ i.e., R_1 divides the line in the ratio 1 : 2.

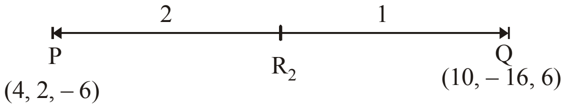


$$\Rightarrow R_1 = \left(\frac{1\times 10+2\times 4}{1+2}, \frac{1\times(-16)+2\times 2}{1+2}, \frac{1\times 6+2\times(-6)}{1+2}\right)$$

$$= \left(\frac{10+8}{3}, \frac{-16+4}{3}, \frac{6-12}{3}\right) = \left(\frac{18}{3}, \frac{-12}{3}, \frac{-6}{3}\right)$$

$$= (6, -4, -2)$$

Again, let the point R_2 divides PQ internally in the ratio 2 : 1. Then.



$$\Rightarrow R_2 = \left(\frac{2\times 10+1\times 4}{2+1}, \frac{2\times(-16)+1\times 2}{2+1}, \frac{2\times 6+1\times(-6)}{2+1}\right)$$

$$= \left(\frac{20+4}{3}, \frac{-32+2}{3}, \frac{12-6}{3}\right) = \left(\frac{24}{3}, \frac{-30}{3}, \frac{6}{3}\right)$$

$$= (8, -10, 2)$$

59. **(b)** $AB = \sqrt{(5-3)^2+(3-2)^2+(2-0)^2} = \sqrt{4+1+4} = 3$

$$AC = \sqrt{(-9-3)^2+(6-2)^2+(-3-0)^2}$$

$$= \sqrt{144+16+9} = 13$$

Since, AD is the bisector of $\angle BAC$, we have

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{3}{13}$$

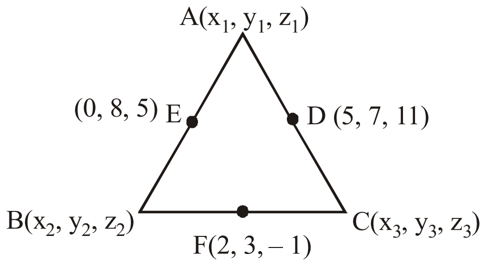
i.e., D divides BC in the ratio 3 : 13.

Hence, the coordinates of D are

$$\left(\frac{3(-9)+13(5)}{3+13}, \frac{3(6)+13(3)}{3+13}, \frac{3(-3)+13(2)}{3+13}\right)$$

$$= \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$$

60. (a) Let the vertices of a triangle be $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$.



Since D, E and F are the mid-points of AC, BC and AB

$$\therefore \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = (0, 8, 5)$$

$$\Rightarrow x_1+x_2 = 0, y_1+y_2 = 16, z_1+z_2 = 10 \quad \ldots\text{(i)}$$

$$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right) = (2, 3, -1)$$

$$\Rightarrow x_2+x_3 = 4, y_2+y_3 = 6, z_2+z_3 = -2 \quad \ldots\text{(ii)}$$

and $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}, \frac{z_1+z_3}{2}\right) = (5, 7, 11)$

$$\Rightarrow x_1+x_3 = 10, y_1+y_3 = 14, z_1+z_3 = 22 \quad \ldots\text{(iii)}$$

On adding eqs. (i), (ii) and (iii), we get

$2(x_1+x_2+x_3) = 14$, $2(y_1+y_2+y_3) = 36$.

$2(z_1+z_2+z_3) = 30$,

$\Rightarrow x_1+x_2+x_3 = 7, y_1+y_2+y_3 = 18,$

$z_1+z_2+z_3 = 15 \quad \ldots\text{(iv)}$

On solving eqs. (i), (ii), (iii) and (iv), we get

$x_3 = 7, x_1 = 3, x_2 = -3$

$y_3 = 2, y_1 = 12, y_2 = 4$

and $z_3 = 5, z_1 = 17, z_2 = -7$

Hence, vertices of a triangle are (7, 2, 5), (3, 12, 17) and (– 3, 4, – 7).