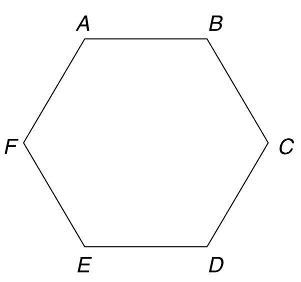
CHAPTER 6

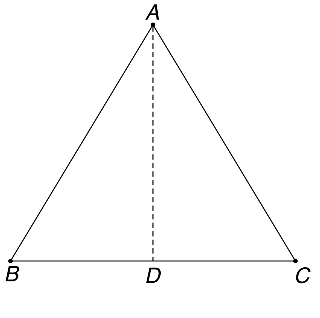
Electrostatics

LEVEL 1

Q. 1: **(a)** Six equal charges have been placed at the vertices of a regular hexagon. Charge at vertex A is moved to the centre of the hexagon and there it experiences a net electrostatic force of magnitude F. Charge at E is also moved to the centre so as to double the magnitude of the charge at the centre. Calculate the magnitude of the electrostatic force that this central charge experiences now.



(b) Three charges of equal magnitude lie on the vertices of an equilateral triangle ABC. All of them are released simultaneously. The charge at A experiences initial acceleration along AD where D is the midpoint of the side BC. Find the direction of initial acceleration of the charge at B.



Q. 2: Two identical small conducting balls have positive charges q_1 and q_2 respectively. The force between the balls when they are placed at a separation is F. The balls are brought together so that they touch and then put back in their original positions. Prove that the force between the balls now, cannot be less than F.

Q. 3: Point charges $-q$, $2q$, $-3q$, q, $-q$, $2q$, $-3q$, q, $-q$, $2q$, $-3q$, and q have been placed at marks 1, 2, 3, 4, 5 .....12 respectively on the circular dial of a clock. Find the electric field intensity at the centre of the dial if distance of each charge from the centre is r.

Q. 4: Let q_1 be a positive charge equal to the magnitude of the total charge on all electrons present in 0.9 mg of pure water and q_2 be the charge on a 6.35 mg copper sphere from which 0.1% of its total electrons have been removed.

Avogadro's number, $N_A = 6 \times 10^{23}$

Molar mass of H_2O = 18 g

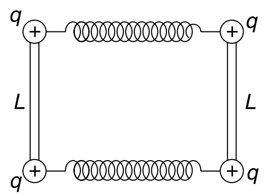
Molar mass of Cu = 63.5 g

(a) Find force between q_1 and q_2 if they are placed at a separation of 1 km.

(b) How does this force compare with the weight of a car having mass of 1200 kg? What conclusion can you draw from the result?

Q. 5: Three point charges $q_1 = q$, $q_2 = q$ and $q_3 = Q$ are placed at points having position vectors $\vec{r}_1$, $\vec{r}_2$ and $(\vec{r}_1 + \vec{r}_2)$ respectively. It is known that $|\vec{r}_1| = |\vec{r}_2| = \vec{r}$. The net electrostatic force on q_3 is $\sqrt{3}$ times the force applied by either of q_1 or q_2 on q_3. Find $\vec{r}_1 \cdot \vec{r}_2$.

Q. 6: Two stiff non conducting rods have length L each and have small balls connected to their ends. The rods are placed parallel to each other and the balls are connected by two identical springs as shown. When each ball is given a charge q, the system stays in equilibrium when it is in the shape of a square. If natural relaxed length of each spring is $L/2$ find the force constant (k) for them.



Q. 7: Five identical charges, q_1 each, are placed at the vertices of a regular pentagon having side length l_1. The net electrostatic force on any of the charges due to other four is F_1. Find the electrostatic force F_2 on any one of the five identical charges, q_2 each, placed at the vertices of a regular pentagon having side length l_2.

Q. 8: A simple pendulum has a bob of mass m which carries a charge q on it. Length of the pendulum is L. There is a uniform electric field E in the region. Calculate the time period of small oscillations for the pendulum about its equilibrium position in following cases:

(i) E is vertically down having magnitude $E = \dfrac{mg}{q}$

(ii) E is vertically up having magnitude $E = \dfrac{2mg}{q}$

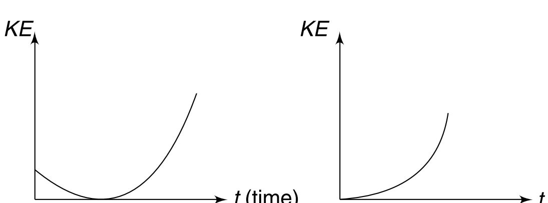
(iii) E is horizontal having magnitude $E = \dfrac{mg}{q}$

(iv) E has magnitude of $E = q$ and is directed upward making an angle of 45° with the horizontal.

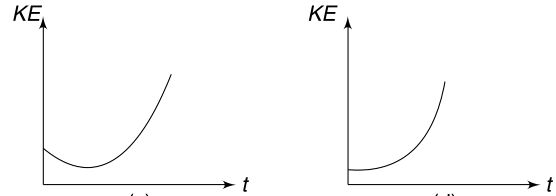
Q. 9: Two identical negative charges are fixed on X axis at equal distances from the origin (O). A particle having positive charge starts at a large distance from O, moves along the Y axis, passes through the origin and moves far away from O in the positive Y direction. Describe qualitatively how its acceleration charges as it moves. Draw a rough graph showing the variation of acceleration (a) vs position of the particle (y). Take the acceleration of the particle to be positive in $+Y$ direction.

Q. 10: An electron is either released from rest or projected with some initial velocity in a uniform electric field. Neglect any other force on the electron apart from electrostatic force. Which of the graphs shown in fig could possibly represent the change in kinetic energy of the electron during its course of motion?

Explain the situation in each case.



(a) (b)

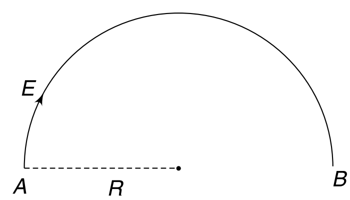


(c) (d)

Q. 11: In a region of space an electric field line is in the shape of a semicircle of radius R. Magnitude of the field at all point is E. A particle of mass m having charge q is constrained to move along this field line. The particle is released from rest at A.

(a) Find its kinetic energy when it reaches point B.

(b) Find the acceleration of the particle when it is at midpoint of the path from A to B.



Q. 12: Electric field $E = -bx + a$ exists in a region parallel to the X direction (a and b are positive constants). A charge particle having charge q and mass m is released from the origin $X = 0$. Find the acceleration of the particle at the instant its speed becomes zero for the first time after release.

Q. 13: A Sphere of radius R_0 carries a volume charge density proportional to the distance (x) from the origin, $\rho = \alpha x$ where α is a positive constant.

(a) Calculate the total charge in the sphere of radius R_0.

(b) The sphere is shaved off so as to reduce its radius. What should be the radius (R) of the remaining sphere so that it contains half the charge of the original sphere?

(c) Find the electric field at a point inside the sphere at a distance $r < R$. Give your answer for original sphere of radius R_0 as well as for the smaller sphere of radius R that was left after shaving off the original sphere.

Q. 14: Draw electric field lines for charge distributions given below–

(a) Two equal point charges placed at a separation.

(b) Two point charges $2q$ and $-q$ placed at a separation. Describe qualitatively how the lines will appear at a very large distance from the two charges?

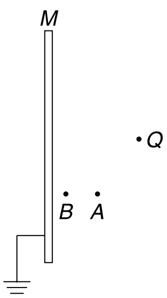
(c) Three point charges, each equal to $+q$ placed at the vertices of an equilateral triangle.

Q. 15: Three conducting concentric spherical shells of radii R, $2R$ and $3R$ carry some charge on them. The potential at the centre is 50 V and that of middle and outer shell is 20 V and 10 V respectively. Find the potential of the inner Shell.

Q. 16: The potential at a point in an electric field is given by $V = \text{R}r$ volt where r is distance of the point from the origin of the co-ordinate system. Find the electric field at a point $r = \left(\hat{i} + 2\hat{j} + 2\sqrt{5}\hat{k}\right)\, m$

Q. 17: Electric potential in a 3 dimensional space is given by $V = \left(\frac{1}{x} + \frac{1}{y} + \frac{2}{z}\right)$ volt where x, y and z are in meter. A particle has charge $q = 10^{-12}$ C and mass $m = 10^{-9}$ g and is constrained to move in xy plane. Find the initial acceleration of the particle if it is released at (1, 1, 1) m.

Q. 18: A metal plate M is grounded. A point charge $+Q$ is placed in front of it. Consider two points A and B as shown in fig. At which point (A or B) is the electric field stronger? At which point is the potential higher?

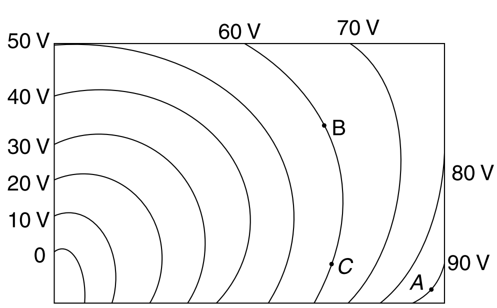


Q. 19: Equipotential lines arising from a static charge distribution has been shown in a certain region of space.

(a) Draw the electric field lines starting from point A

(b) If a charge is released at A, will it move along the field line passing through A?

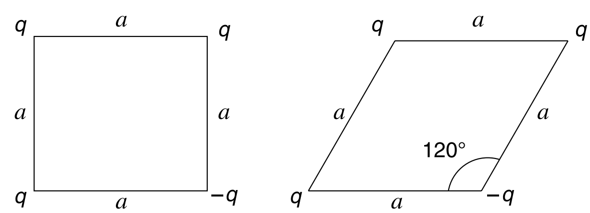
(c) How much work is done by the electric force if a charge moves from B to C? Is the knowledge of path important in this calculation?



Q. 20: (a) The equipotential curves in x, y plane are given by $x^2 - y^2 = V$ when V is potential. Draw the rough sketch of electric field lines in x–y plane.

(b) Repeat the above question if the potential field is given by $V = x^2 + y^2 - 4x + 4$

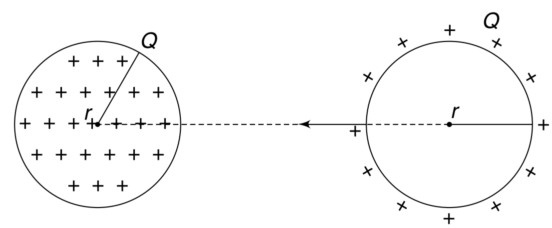
Q. 21: Four point charges q, q, q and $-q$ are placed at the vertices of a square of side length **a**. The configuration is changed and the charge are positioned at the vertices of a rhombus of side length **a** with $-q$ charge at the vertex where angle is 120°. Find the work done by the external agent in changing the configuration.



Q. 22: There is a ball of radius r having uniformly distributed volume charge Q on it and there is a spherical shell of radius r having uniformly distributed surface charge Q on it. The two spheres are far apart.

(a) A point charge q is moved slowly from the centre of the shell (through a small hole in it.) to the centre of the ball. Find work done by the external agent in the process.

(b) The two spheres are brought closer so that their centers are separated by $4r$. Now calculate the amount of work needed in slowly moving a point charge q from the centre of the shell to the centre of the ball. Assume that charge on one ball does not alter the charge distribution of the other. Does your answers in (a) and (b) differ? Why?



Q. 23: A uniformly charged sphere has charge Q. An electron (charge $-e$, mass m) revolves around it in a circular orbit of radius r.

(a) Write the total energy (i.e., sum of its kinetic energy and electrostatic potential energy in the field of the sphere) of the electron.

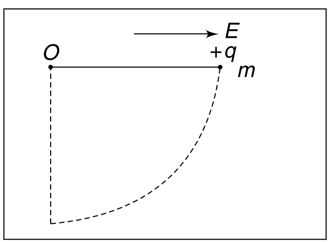
(b) If the time period of revolution of the electron in circular orbit of radius r is T, then find the time period if the orbital radius is made $4r$.

Q. 24: A uniformly charged sphere has radius R and charge Q on it. A negatively charged particle having mass m and charge $-q$ shoots out of the surface of the sphere with speed V. The minimum speed (V_0) for which the particle escapes the attraction of the sphere may be called as escape speed. Will the value of V_0 depend on magnitude of q? [Recall that escape speed of a body from the surface of the earth does not depend on its mass].

Q. 25: There is a uniform horizontal electric field of strength E in a region. A pendulum bob is pulled to make the string horizontal and released. The bob has mass m and charge q.

(a) Find the maximum angle (θ_0) that the bob swings before coming to rest momentarily.

(b) Find E if the bob comes to rest when the string is vertical.



Q. 26: A conducting sphere of radius R carries a charge Q. It is enclosed by another concentric spherical shell of radius $2R$. Charge from the inner sphere is transferred in infinitesimally small installments to the outer sphere. Calculate the work done in transferring the entire charge from the inner sphere to the outer one.

Q. 27: The ratio of energy density in electric field (u_E) to square of potential (V^2) at a point A at a distance x from a static point charge Q is η.

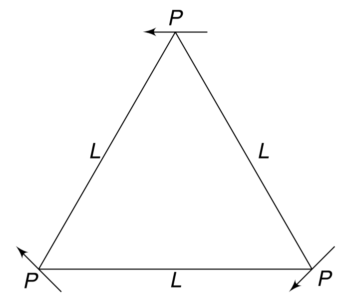
(a) Write the value of the ratio at a distance $2x$ from the point charge Q.

(b) Write the dimensional formula for the ratio η.

Q. 28: (a) Calculate the largest possible electrostatic energy in 1 cm^3 volume of air. The dielectric breakdown of air happens when field exceeds 3×10^6 V/m.

(b) A conducting ball of radius 10 cm is placed in distilled water ($\in_r = 80$) and charged with a charge $Q = 2 \times 10^{-9}$C. Calculate the energy used up in charging the ball.

Q. 29: Three short electric dipoles, each of dipole moment P, are placed at the vertices of an equilateral triangle of side length L. Each dipole has its moment oriented parallel to the opposite side of the triangle as shown in the fig. Find the electric field and potential at the centroid of the triangle.

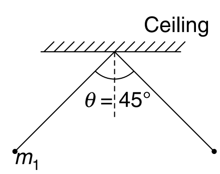


Q. 30: Short electric dipole of dipole moment P is placed at the centre of a ring of radius R having charge Q uniformly distributed on its circumference. The dipole moment vector is along the axis of the ring. Find force on the dipole due to the ring.

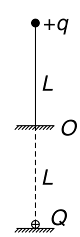
Q. 31: Calculate the electric dipole moment of a system comprising of a charge $+q$ distributed uniformly on a semicircular arc or radius R and a point charge $-q$ kept at its centre.

LEVEL 2

Q. 32: Two unequal masses, $m_1 = 2m$ and $m_2 = m$ have unequal positive charge on them. They are suspended by two mass-less threads of unequal lengths from a common point such that, in equilibrium, both the masses are on same horizontal level. The angle between the two strings is $\theta = 45°$ in this position. Find the Electrostatic force applied by m_1 on m_2 in this position.



Q. 33: A particle of mass m and charge q is attached to a light insulating thread of length L. The other end of the thread is secured at point O. Exactly below point O, there is a small ball having charge Q fixed on an insulating horizontal surface. The particle remains in equilibrium vertically above the ball with the string taut. Distance of the ball from point O is L. Find the minimum value of Q for which the particle will be in a stable equilibrium for any gentle horizontal push given to it.



Q. 34: Four identical charges, Q each, are fixed at the vertices of a square. A free charge q is placed at the centre of the square. Investigate the nature of equilibrium of charge q if it is to be displaced slightly along any of the two diagonals of the square.

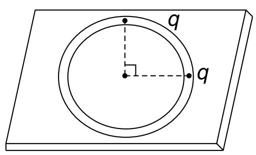
Q. 35: A horizontal circular groove is made in a wooden board. Two positive charges (q each) are placed in the groove at a separation of 90° (see figure). Where shall we place (in the groove) a third charge and what shall be its magnitude such that all three of them remain at rest after they are released. Answer for two cases:

(a) When the third charge is positive.

(b) When the third charge is negative.

Neglect friction and assume that the groove is very thin just wide enough to accommodate the particles.

[Take: $\sin 22.5° = 0.38$; $\cos 22.5° = 0.92$]



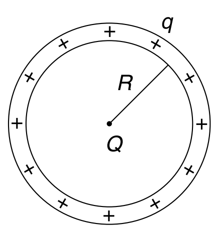
Q. 36: Two identical positive charges Q each, are placed on the x axis at points $(-a, 0)$ and $(a, 0)$. A point charge of magnitude q is placed at the origin. For small displacement along x axis, the charge q executes simple harmonic motion if it is positive and its time period is T_1. If the charge q is negative, it performs oscillations when displaced along y axis. In this case the time period of small oscillations is T_2. Find $\frac{T_1}{T_2}$.

Q. 37: A ring of radius R has uniformly distributed charge q. A point charge Q is placed at the centre of the ring.

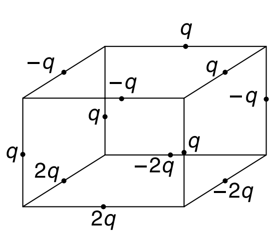
(a) Find the increase in tension in the ring after the point charge is placed at its centre.

(b) Find the increase in force between the two semicircular parts of the ring after the point charge is placed at the centre.

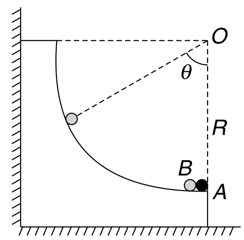
(c) Using the result found in part (b) find the force that the point charge exerts on one half of the ring.



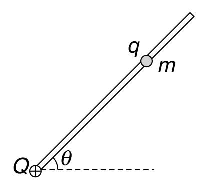
Q. 38: Twelve charges have been placed at the centre of each side of a cube as shown in the figure. Find the magnitude of Electric force acting on a charge Q placed at the centre of the cube. Take the side length of the cube to be r.



Q. 39: A fixed non conducting smooth track is in the shape of a quarter circle of radius R in vertical plane. A small metal ball A is fixed at the bottom of the track. Another identical ball B, which is free to move, is placed in contact with ball A. A charge Q is given to ball A which gets equally shared by the two balls. Ball B gets repelled and ultimately comes to rest in its equilibrium position where its radius vector makes an angle θ ($\theta < 90°$) with vertical. Mass of ball is m. Find charge Q that was given to the balls.

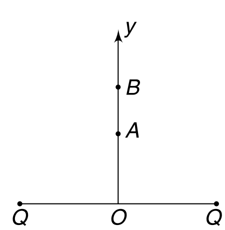


Q. 40: A smooth fixed rod is inclined at an angle θ to the horizontal. At the bottom end of the rod there is a fixed charge $+Q$. There is a bead of mass m having charge q that can slide freely on the rod. The equilibrium separation of the bead from fixed charge Q is x_0. Find the frequency of oscillation of the bead if it is displaced a little from its equilibrium position.



Q. 41: Two charges, Q each, are fixed on a horizontal surface at separation $2a$. Line OY is vertical and is perpendicular bisector of the line joining the two charges. Another particle of mass m and charge q has two equilibrium positions on the line OY, at A and B. The distances OA and OB are in the ratio $1:3\sqrt{3}$

(a) Find the distance of the point on the line OY where the particle will be in stable equilibrium.

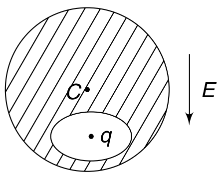


(b) Where will the particle experience maximum electric force – at a point above B or at a point between A and B or somewhere between O and A? Where is the acceleration of particle maximum on y axis from O to B?

Q. 42: A neutral spherical conductor has a cavity. A point charge q is located inside it. It is in equilibrium. An external electric field (E) is switched on that is directed parallel to the line joining the centre of the sphere to the point charge.

(a) What is the direction of acceleration of the charge particle inside the cavity after E is switched on.

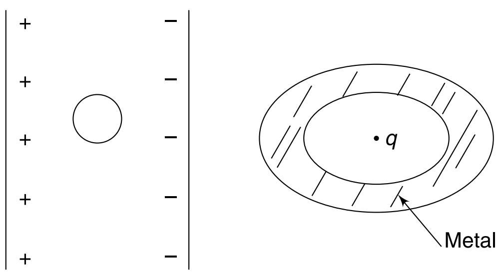
(b) How is the induced charge on the wall of the cavity affected due to the external field.



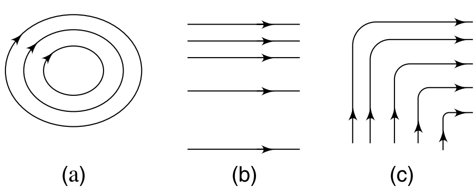
Q. 43: Draw electric field lines in following situations:

(a) A small neutral metal sphere is placed between the plates of an ideal parallel plate capacitor. [see figure]

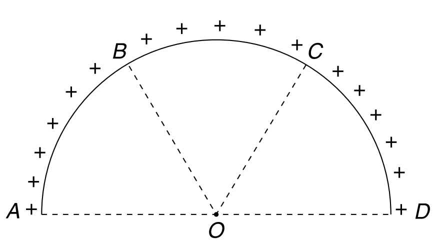
(b) A point charge is trapped inside a cavity in a neutral metal block (see figure)



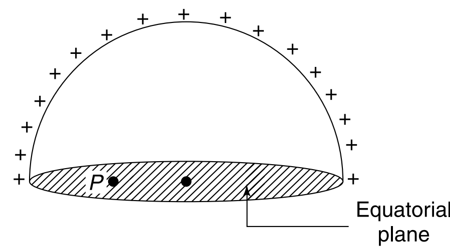
Q. 44: Three configurations of electrostatic field lines have been shown in the figure. Are these configurations possible? Explain your answer.



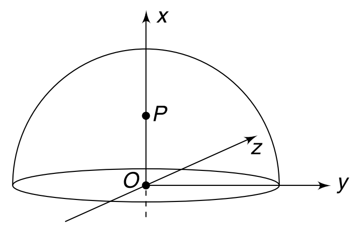
Q. 45: A uniformly charged semicircular ring ($ABCD$) produces an electric field E_0 at the centre O. AB, BC and CD are three equal arcs on the ring. Portion AB and CD are cut from either side and removed. Find the field at O due to remaining part BC.



Q. 46: Consider a uniformly charged hemispherical shell. What can you say about the direction of electric field at points on the equatorial plane. (e.g. point P)

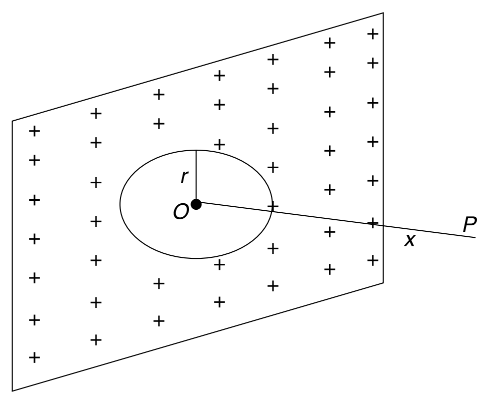


Q. 47: Consider a uniformly charged thin spherical shell as shown in figure. Radius of the shell is R.



The electric field at point $P(x, 0, 0)$ is $\vec{E}$. What is the electric field at a point $Q(-x, 0, 0)$. Given $x < R$.

Q. 48: There is an infinite non conducting sheet of charge having uniform charge density σ. The electric field at a point P at a distance x from the sheet is E_0. Point O is the foot of the perpendicular drawn from point P on the sheet. A circular portion of radius $r << x$ centered at O is removed from the sheet. Now the field at point P becomes $E_0 - \Delta E$. Find ΔE.



Q. 49: A thread having linear charge density λ is in the shape of a circular arc of radius R subtending an angle θ at the centre.

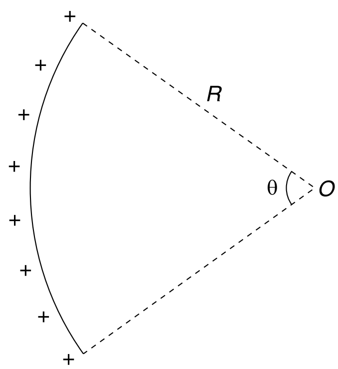
(a) Find the electric field at the centre.

(b) Using the expression obtained in part (a) find the field at the centre if the thread were semicircular

(c) Find the field at centre using the expression obtained in part (a) for the case $\theta \to 0$. Is the result justified?

(d) A thread having total charge Q (uniformly distributed) is in the shape of a circular arc of radius R subtending an angle θ at centre. Write the expression for the field at the center. Obtain the field when $\theta \to 0$.

Make sure you understand the difference in case (c) and (d)

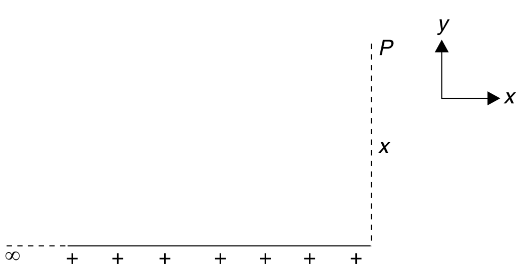


Q. 50: (a) There is an infinitely long thread uniformly charged with linear charge density λ C/m. Using Gauss' law, calculate the electric field (E_0) at a distance x from the thread.

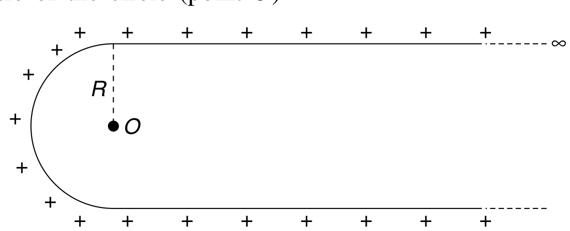
(b) Now consider a semi-infinite uniformly charged thread (linear charge density = λ) as shown in figure. Find the y component of electric field at point P in terms of E_0. Use simple qualitative argument.

(c) For the situation described in (b) calculate the x component of electric field at point P using the method of integration.

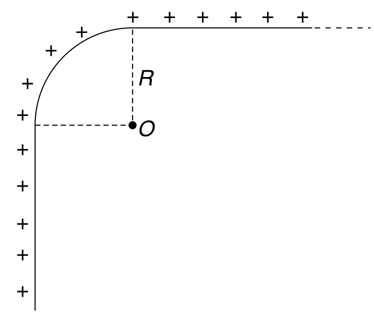
(d) Find the angle that the electric field at P makes with x direction.



Q. 51: An infinitely long line charge is bent in U shape as shown in figure. The semicircular part has radius R and linear charge density is λ C/m. Using the results obtained in last two problems, calculate the electric field intensity at centre of the circle (point O)



Q. 52: Repeat the above problem if the semicircular part is replaced with a quarter circle (see figure).

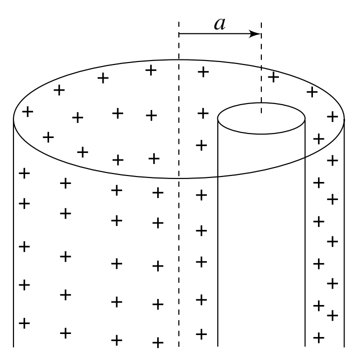


Q. 53: (a) There is a long uniformly charged cylinder having a volume charge density of ρ C/m^3. Radius of the cylinder is R. Find the electric field at a point at a distance x from the axis of the cylinder for following cases

(i) $x < R$ (ii) $x > R$

What is the maximum field produced by the charge distribution at any point?

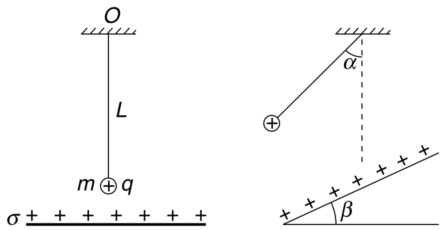
(b) The cylinder described in (a) has a long cylindrical cavity. The axis of cylindrical cavity is at a distance a from the axis of the charged cylinder (see figure). Find electric field inside the cavity.



Q. 54: A pendulum has a bob of mass m carrying a positive charge q. Length of the pendulum string is L. Beneath the pendulum there is a large horizontal dielectric sheet of charge having uniform surface charge density of σ C/m^2. [figure (i)]

(a) Find the time period of small oscillations for the pendulum

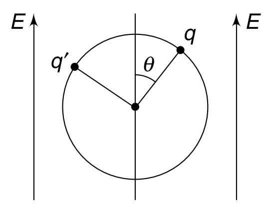
(b) Now the dielectric sheet of charge is tilted so as to make an angle β with horizontal. Find the angle (α) that the thread makes with vertical in equilibrium position. Find time period of small oscillations in this case. [figure (ii)]



Q. 55: A uniform non conducting ring has mass m and radius R. Two point charges q and q' are fixed on its circumference at a separation of $\sqrt{2}R$. The ring remains in equilibrium in air with its plane vertical in a region where exists a uniform vertically upward electric field E. Given $E = \dfrac{4mg}{7q}$.

(a) Find angle θ in equilibrium position (see figure).

(b) The ring is given a small rotation in the plane of the figure and released. Will it perform oscillations?

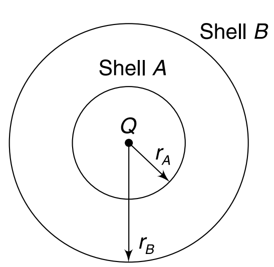


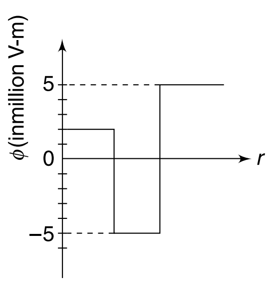
Q. 56: An infinitely long line charge has linear charge density λ C/m. The line charge is along the line $x = 0$, $z = 2$ m. Find the electric field at point (1, 1, 1) m.

Q. 57: A charged particle is placed at the centre of two thin concentric spherical charged shells, made of non-conducting material. Figure A shows cross-section of the arrangement. Figure B gives the net flux ϕ through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere.

(a) Find charge on the central particle and shell A.

(b) In which range of the finite values of r, is the electric field zero?



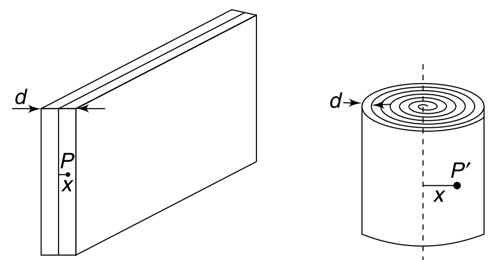


Q. 58: There are two infinite slabs of charge, both of thickness d with the junction lying on the plane $x = 0$. The slab lying in the range $0 < x < d$ has a uniform charge density $+\rho$ and the slab lying in the region $-d < x < 0$ has uniform charge density $-\rho$. Find the Electric field everywhere and plot its variation along the x axis.

Note: This can be used to model the variation of electric field in the depletion layer of a $p{-}n$ junction.

Q. 59: In an insulating medium (dielectric constant = 1) the charge density varies with y Co-ordinate as $\rho = by$, Where b is a positive constant. The electric field is zero at $y = 0$ and everywhere else it is along y direction. Calculate the electric field as a function of y.

Q. 60: A non conducting sheet of thickness d and large surface area contains a uniformly distributed charge of density ρ throughout its volume. The electric field at a point P inside the sheet at a distance x from the central plane is E_1. Now the sheet is rolled to form a large solid cylinder. Field at a point P' inside the cylinder at a distance x from its axis is E_2. Find the ratio $\dfrac{E_1}{E_2}$



Q. 61: A charge distribution generates a radial electric field $\vec{E} = \dfrac{a}{r^2} e^{-\frac{r}{k}} \hat{r}$, where r is distance from the origin, $\hat{r}$ is a unit vector in radial direction away from the origin and a and k are positive constants. The electric field extends around the origin up to a large distance.

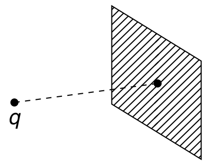
(a) Find the charge (q_0) that must be located at the origin to create such a field

(b) Find the quantity of charge (q) that must be spread around the charge q_0 at origin to create such a field.

Q. 62: A pyramid has four faces, all of them being equilateral triangle of side a. A charge Q is placed at the centre of one of the faces. What is the flux of electric field emerging from any one of the other three faces of the pyramid?

Q. 63: A point charge is placed very close to an infinite plane. What is flux through the plane?

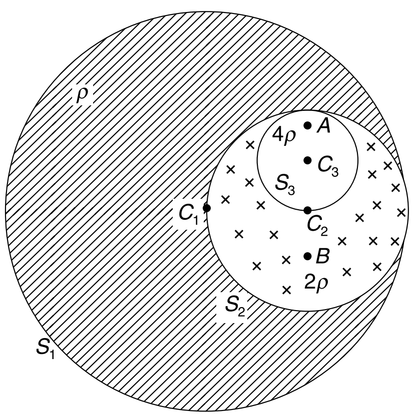
Q. 64: Point charge q is placed at a point on the axis of a square non-conducting surface. The axis is perpendicular to the square surface and is passing through its centre. Flux of Electric field through the square caused due to charge q is ϕ. If the square is given a surface charge of uniform density σ, find the force on the square surface due to point charge q.



Q. 65: In the figure shown, spheres S_1, S_2 and S_3 have radii R, $\frac{R}{2}$ and $\frac{R}{4}$ respectively. C_1, C_2 and C_3 are centers of the three spheres lying in a plane. Angle $\angle C_1C_2C_3$ is right angle. Sphere S_3 has a uniformly spread volume charge density 4ρ. The remaining part of S_2 has uniform charge density of 2ρ and the left over part of S_1 has a uniform charge density of ρ.

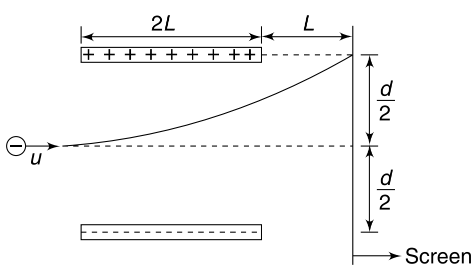
(a) Find electric field at a point A at a distance $\frac{R}{8}$ from C_3 on the line C_2C_3 (see figure)

(b) Find electric field at point B at a distance $\frac{R}{4}$ from C_2 on the line C_3C_2 (see figure)



Q. 66: An electron (charge = e, mass = m) is projected horizontally into a uniform electric field produced between two oppositely charged parallel plates, as shown in figure. The charge density on both plates is $\pm\sigma$ C/m^2 and separation between them is d. You have to assume that only electric force acts on the electron and there is no field outside the plates. Initial velocity of the electron is u, parallel to the plates along the line bisecting the gap between the plates. Length of plates is $2L$ and there is a screen perpendicular to them at a distance L.

(i) Find σ if the electron hits the screen at a point that is at same height as the upper plate.



(ii) Final the angle θ that the velocity of the electron makes with the screen while it strikes it.

Q. 67: A particle is projected at a speed of u = 40 m/s in vertically upward direction in a place where exists a horizontal uniform electric field E_0. The specific charge of the particle is $\frac{4g}{3E_0}$.

(a) Find the time after projection, when speed of the particle will be least.

(b) Find the time (after projection) when displacement of the particle becomes perpendicular to its acceleration.

(c) Assuming that the particle has been projected from a great height and the electric field is present in large region, what angle the velocity of the particle will make with horizontal after a long time?

Q. 68: A charged particle having mass m is projected in a uniform electric field with a kinetic energy K_0. After time t_0 it was observed that the kinetic energy of the particle was $\frac{K_0}{4}$ and its velocity was perpendicular to the field.

(a) How much more time is required for the particle to regain its lost kinetic energy?

(b) Write the impulse of the electric force acting on the particle between the two points where its kinetic energy is K_0.

Neglect all other forces on the particle apart from the electrostatic force.

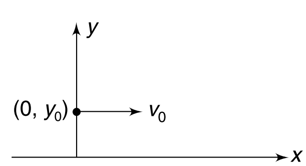
Q. 69: Electric field in xy plane is directed along positive y direction and its magnitude changes with y co-ordinate as

$$E = ay^2$$

A particle having charge q and mass m is projected at point $(0, y_0)$ with velocity

$$\vec{v} = v_0\hat{i}$$

Neglect all other forces on the particle apart from the electric force. Calculate the slope of the trajectory of the particle as a function of y.



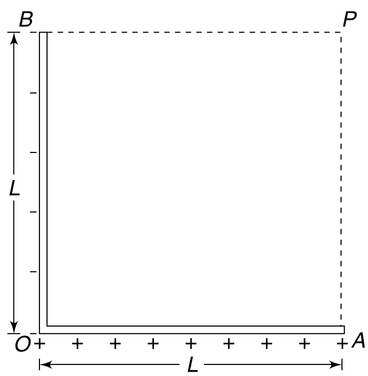
Q. 70: Two point charges $-q$ and $18q$ are placed on x axis at $-x_0$ and x_0 respectively. Draw the variation of potential along x axis assuming $18q$ has been removed. Draw another graph showing variation of potential when only

18q is present. Superimpose the two graphs to obtain the variation of potential along x axis when both charges are present.

Q. 71: L shaped rod has equal and opposite charge ($\pm Q$) spread along its both arms (see figure).

(a) What is direction of electric field at point P?

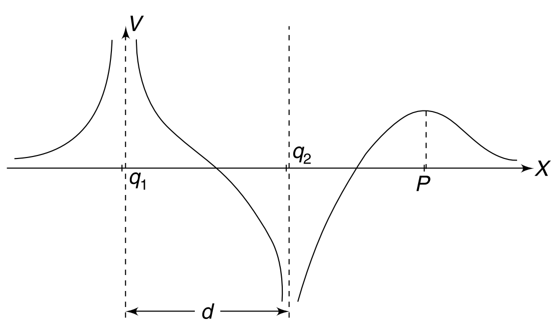
(b) Write electric potential at P.



Q. 72: Two point charge q_1 and q_2 lie on X axis at some separation d. The given figure shows the graphical variation of electric potential due to these two charges along the X axis.

(a) What are signs of $q_1$and q_2? Which charge has larger magnitude?

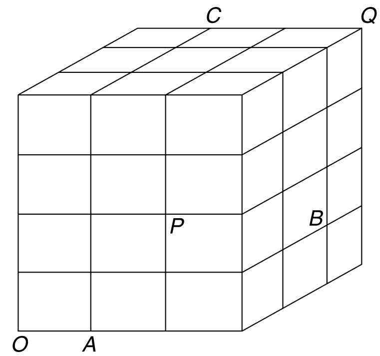
(b) Origin is the point between the charges where potential is zero. Distance of q_2 from origin is $\frac{d}{4}$. Find the distance of point P (marked in figure) from charge q_2.



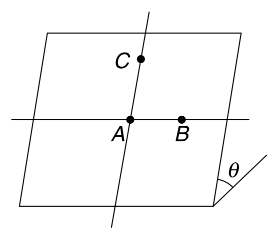
Q. 73: A uniform electric field exists in a region of space. Potential at points O, A, B and C are $V_0 = 0$, and $V_A = -1V$, $V_B = -6V$ and $V_C = -3V$ respectively. All the cubes shown in fig have side length of 1 m.

(a) Find $V_P - V_Q$

(b) Find the smallest distance of a point from O where the potential is $-2V$



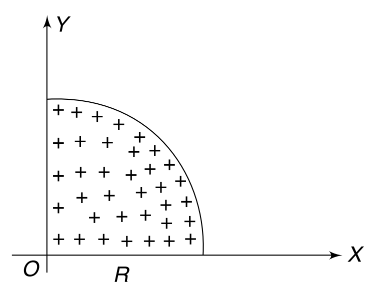
Q. 74: It is known that there exists a uniform electric field in a certain region. Imagine an incline plane (figure) in the region which is inclined at $\theta = 37°$ to the horizontal. When one moves horizontally along the incline from A to B (AB = 1 cm), the electric potential decreases by 10V. Similarly, potential at C (AC = 1 cm) is less than potential at A by 10 V where line AC lies on the incline and is perpendicular to AB. When one moves vertically up from point A to a point D (AD = 1 cm), the potential drops by 10 V again. Find the magnitude of the electric field in the region and the angle that it makes with vertical. $\left[\text{given } \sin 37° = \frac{3}{5}\right]$



Q. 75: The quarter disc of radius R (see figure) has a uniform surface charge density σ.

(a) Find electric potential at a point (O, O, Z)

(b) Find the Z component of electric field at (O, O, Z)



Q. 76: Electric field in a three dimensional space is directed radially towards a fixed point and its magnitude varies with distance (r) from the fixed point as $E = 4r$ V/m

(a) Draw electric field lines to approximately represent such a field.

(b) Calculate the quantity of charge present inside a spherical volume of radius a centered at the fixed point.

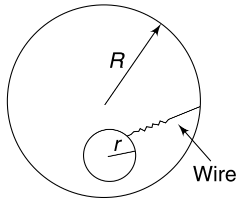
(c) Find potential difference $(V_A - V_B)$ between two points $A(1, 1, \sqrt{2})m$ and $B(0, 3, 4)m$. Take the fixed point to be the origin.

Q. 77: When a conducting disc is made to rotate about its axis, the centrifugal force causes the free electrons to be pushed toward the edge. This causes a sort of polarization and an electric field is induced. The radial movement of free electrons stops when electric force on an electron balance the centrifugal force. Calculate the potential difference developed across the centre and the edge of a disc of radius R rotating with angular speed ω. [This potential difference is sometimes called as sedimentation potential]. Take mass and charge of an electron to be m and e respectively.

Q. 78: Consider a cube having a uniform volume charge density. Find the ratio of electrostatic potential at the centre to the potential at a corner of the cube.

Q. 79: A metallic sphere of radius R has been charged to a potential of $V = 100$ volt. A thin hemispherical conducting shell has dimensions so that it can just fit on the half of the metallic sphere. The shell is originally grounded. Now, using an insulating handle, it is placed on top of the charged sphere so as to perfectly cover its top half. The shell is removed from the sphere and again grounded. After this the shell is again placed on the sphere, removed and then grounded. The process is continued till the potential of the sphere becomes $V' = 6.25$ volt. How many times the shell was placed on the sphere?

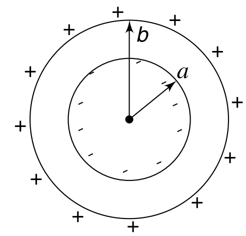
Q. 80: A conducting ball of radius r is charged to a potential V_0. It is enveloped by a thin walled conducting sphere of radius $R(> r)$ and the two spheres are connected by a conducting wire. Find the potential of the outer sphere.



Q. 81: A thick conducting spherical shell of inner radius a and outer radius b is shown in figure. It is observed that the inner face of the shell carries a uniform charge density $-\sigma$. The outer surface also carries a uniform surface charge density $+\sigma$.

(a) Can you confidently say that there must be a charge inside the shell? Find the net charge present on the shell.

(b) Find the potential of the shell.



Q. 82: A conducting bubble of radius a, thickness $t(t << a)$ has potential V. Now the bubble collapses into a droplet. Find the potential of the droplet assuming that there is no leakage of charge.

Q. 83: A point charge q has been placed at a distance x from the centre of a neutral solid conducting sphere of radius $R(x > R)$. Find the potential of the sphere. How will your answer change if the sphere is not solid, rather it is a thin shell of conductor.

Q. 84: There is a hemispherical shell having charge Q uniformly distributed on its surface. Radius of the shell is R. Find electric potential and field at the centre (of the sphere).

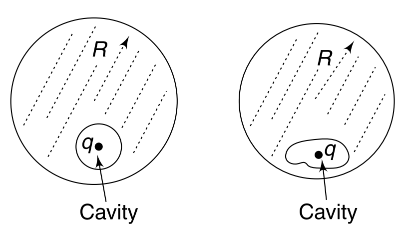
Q. 85: There is a hemisphere of radius R having a uniform volume charge density ρ. Find the electric potential and field at the centre.

Q. 86: Find the potential at a point on the edge of a uniformly charged disc. The surface charge density is σ and radius of the disc is R.

Q. 87: A solid spherical conductor of radius R has a spherical cavity inside it (see figure). A point charge q is placed at the centre of the cavity.

(a) What is the potential of the conductor?

(b) If the charge q is shifted inside the cavity by a distance Δx, how does the potential of the conductor change?

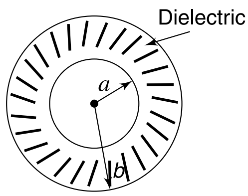


(c) How does your answer to the question (a) and (b) change if the cavity is not spherical and the charge q is placed at any point inside it (see figure)

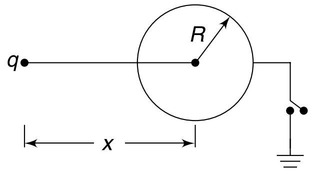
(d) Draw electric field lines in entire space in each case. In which case all field lines are straight lines.

Q. 88: Conducting ball of radius a is surrounded by a layer of dielectric having inner radius a and outer radius b.

The dielectric constant is K. The conducting ball is given a charge Q. Write the magnitude of electric field and electric potential at the outer surface of the dielectric.



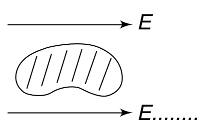
Q. 89: A point charge q is placed at a distance x from the centre of a conducting sphere of radius $R(< x)$.



(a) How much charge will flow through the switch S when it is closed to ground the sphere?

(b) Find the current through the switch S when charge 'q' is moved towards the sphere with velocity V.

Q. 90: A conductor is placed in a uniform external electric field (see figure). Sketch the equipotential surfaces.

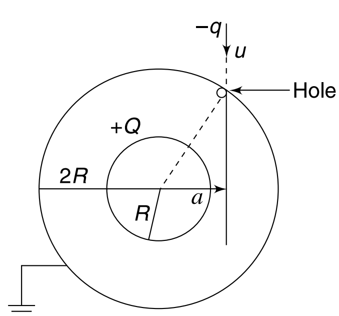


Q. 91: Two concentric spherical shells have radii R and $2R$. The outer shell is grounded and the inner one is given a charge $+Q$. A small particle having mass m and charge $-q$ enters the outer shell through a small hole in it. The speed of the charge entering the shell was u and its initial line of motion was at a distance $a = \sqrt{2}R$ from the centre.

(a) Find the radius of curvature of the path of the particle immediately after it enters the shell.

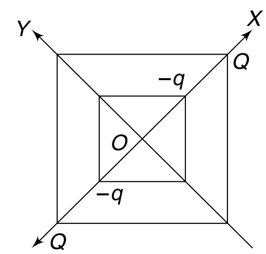
(b) Find the speed with which the particle will hit the inner sphere.

Assume that distribution of charge on the spheres do not change due to presence of the charge particle.



Q. 92: There charges q, $3q$ and $12q$ are to be placed on a straight line AB having 12 cm length. Two of the charges must be placed at end points A and B and the third charge can be placed anywhere between A and B. Find the position of each charge if the potential energy of the system is to be minimum. In the position of minimum potential energy what is the force on the smallest charge?

Q. 93: Two square of sides a and $2a$ are placed in xy plane with their centers at the origin. Two charges, $-q$ each, are fixed at the vertices of smaller square (lying on X axis). Two charges, Q each, are fixed at the vertices of bigger square on the X axis (see figure).



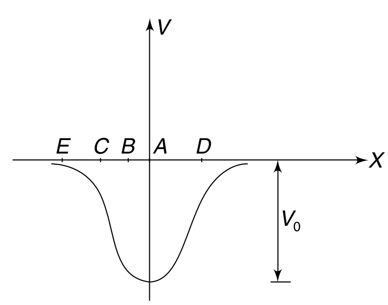
(a) Find work required to slowly move the larger square to infinity from the position shown.

(b) Find work done by the external agent in slowly rotating the inner square by 90° about the Y axis followed by a rotation of 90° about the Z axis.

Q. 94: A certain charge distribution produces electric potential that varies along the X axis as shown in figure. [There is no field in y or z direction]

(a) At which point (amongst A, B, C, D and E) does a negative charge feel the greatest force in positive X direction?

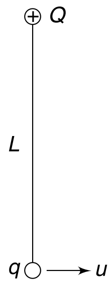
(b) Find the upper limit of the speed that a proton can have, as it passes through the origin, and still remain bound near the origin. Mass and charge of a proton are m and e. How will your answer change for an electron?



Q. 95: A simple pendulum has length L. Bob of the pendulum has mass m and carries a charge q. A point charge Q is fixed at the point of suspension. Find the minimum speed (u) of projection at the lowest point so that the pendulum bob completes the vertical circle. Give your answer for following two cases:

(a) $Q = \dfrac{8\pi\epsilon_0 L^2\ mg}{q}$

(b) $Q = \dfrac{2\pi\epsilon_0 L^2\ mg}{q}$

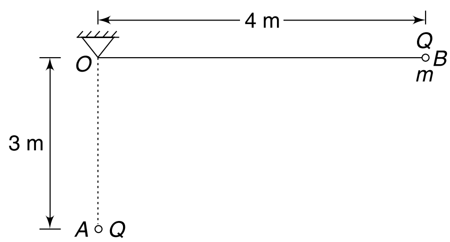


Q. 96: Below the fixed end O of the insulating horizontal thread OB, there is a fixed charge A of $Q = 20\mu c$. At the end B of the thread there is a small mass m carrying charge $Q = 20\mu c$. The mass is released from the position shown and it is found to come to rest when the thread becomes vertical. Assume that the thread does not hit the fixed charge at A. [$g = 10\ \text{m/s}^2$]

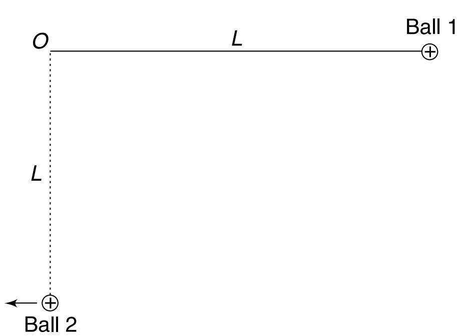
(a) Find mass m.

(b) Find tension in the thread in the equilibrium position when the thread is vertical.

(c) Is the equilibrium mentioned in (b) stable or unstable?



Q. 97: A small positively charged ball of mass m is suspended by an insulating thread of length L. This ball remains in equilibrium with string horizontal when another small charged ball is placed exactly at a distance L below the point of suspension of the first ball. The second ball is slowly moved away from the first ball to a far away point. (The second ball is moved horizontally so that the first ball does not accelerate). As a result the first ball lowers down to the original position of the second ball and the string become vertical. Find the work done by the external agent in removing the second ball.



Q. 98: A particle (A) having charge Q and mass m is at rest and is free to move. Another particle (B) having charge q and mass m is projected from a large distance towards the first particle with speed u.

(a) Calculate the least kinetic energy of the system during the subsequent motion.

(b) Find the final velocity of both the particles.

Consider coulomb force only.

Q. 99: In the last question, the two particles A and B are initially held at a distance $r = \dfrac{qQ}{2\pi\epsilon_0\ mu^2}$ apart. Particle B is projected directly towards A with velocity u and particle A is released simultaneously. Find the velocity of particle A after a long time. Consider coulomb force only.

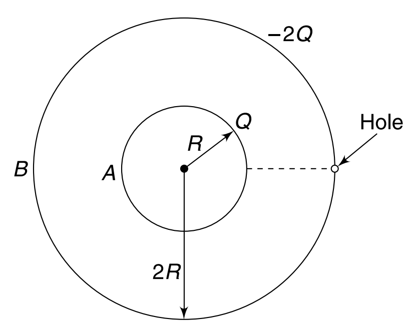
Q. 100: Two positively charged balls having mass m and $2m$ are released simultaneously from a height h with horizontal separation between them equal to x_0. The ball with mass $2m$ strikes the ground making an angle of 45° with the horizontal.

(a) At what angle, with horizontal, the other ball hits the ground?

(b) Find the work done by the electrostatic force during the course of fall of the two balls.

Q. 101: A and B are two concentric spherical shells made of conductor. Their radii are R and $2R$ respectively. The two shells have charge Q and $-2Q$ on them. An electron escapes from the surface of the inner shell A and moves towards a small hole in the outer shell B.

(a) What shall be the minimum kinetic energy of the emitted electron so that it can escape to infinity through the small hole in outer shell?



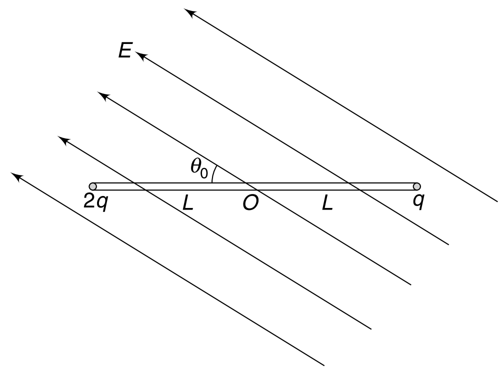
(b) What will be your answer if charge on both the shells were $+Q$? Charge on electron = e.

Q. 102: A thin uniform rod of mass M and length $2L$ is hinged at its centre O so that it can rotate freely in horizontal plane about the vertical axis through O. At its ends the insulating rod has two point charges $2q$ and q (see figure). An electric field E is switched on making an angle $\theta_0 = 60°$

with the initial position of the rod. The field is uniform and horizontal.

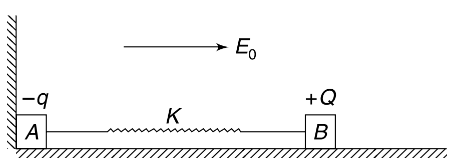
(a) Calculate the maximum angular velocity of the rod during subsequent motion.

(b) Find the maximum angular acceleration of the rod.

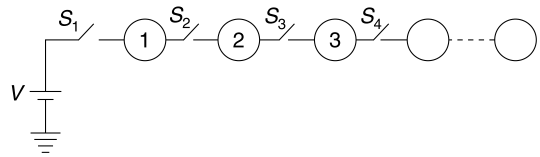


Q. 103: Two charge particles are moving such the distance between them remains constant. The ratio of their masses is 1 : 2 and they always have equal and opposite momentum. The particles interact only though electrostatic force and no other external force is acting on them. The electrostatic interaction energy for the pair of the particles is $-U_0$. Find the kinetic energy of the lighter particle. How does the kinetic energy change with time?

Q. 104: Two blocks A and B are connected by a spring made of a non conducting material. The blocks are placed on a non conducting smooth horizontal surface (see figure). The wall touching A is also non conducting. Block A carries a charge $-q$. There exists a uniform electric field of intensity E_0 in horizontal direction, in the entire region. Find the value of minimum positive charge Q that we must place on block B and release the system so that block A subsequently leaves contact with the wall. Force constant of the spring is k. Neglect interaction between charges on the blocks.



Q. 105: Large number of identical conducting spheres have been laid as shown in figure. Radius of each sphere is R and all of them are uncharged. Switch S_1 is closed to connect sphere 1 to the positive terminal of a V volt cell whose other terminal is grounded. After some time switch S_1 is opened and S_2 is closed. Thereafter, S_2 is opened and S_3 is closed, next S_3 is opened and S_4 is closed. The process is continued till the last switch is closed. Consider the cell and spheres to be your system and calculate the loss in energy of the system in the entire process.



Q. 106: A short electric dipole has dipole moment p. Find the distance of farthest point from the dipole where-

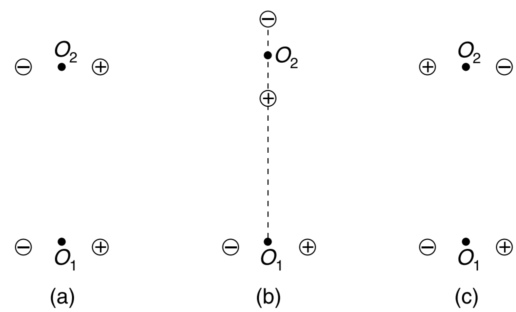
(a) potential due to the dipole is V_0

(b) Electric field due to the dipole is E_0

Q. 107: Two identical electric dipoles are arranged parallel to each other with separation between them large compared to the length of individual dipole. The electrostatic energy of interaction of the two dipoles in this position is U.

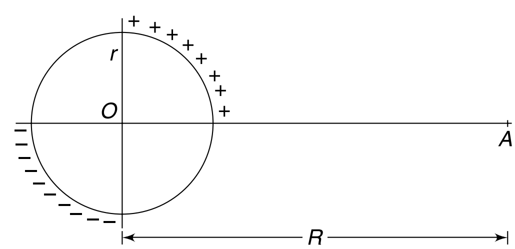
(a) Find work done in slowly rotating one of the dipoles by 90° so as to bring it to position shown in Fig. (b).

(b) Find work done in rotating one of the dipoles by 180° so as to bring it to the position shown in Fig. (c).



O_1 and O_2 are centers of the dipoles.

Q. 108: A ring of radius r has a uniformly spread charge $+q$ on quarter of its circumference. The opposite quarter of the ring carries a charge $-q$ uniformly spread over it. Find the electric potential at a point A shown in the figure. Point A is at a distance $R(>> r)$ from the centre of the ring.



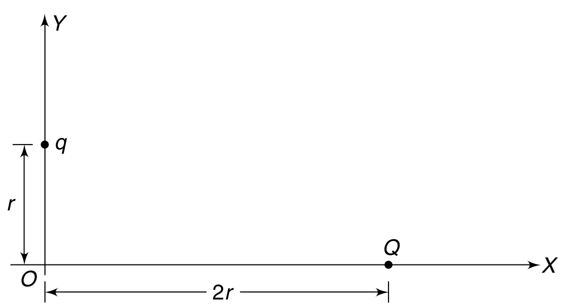
LEVEL 3

Q. 109: Three small equally charged identical conducting balls are suspended from identical insulating threads secured at one point. Length (L) of the threads is large compared to the equilibrium separation (a) between any two balls.

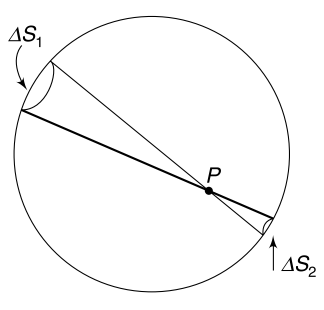
(a) One of the balls is suddenly discharged. Find the separation between the charged balls when equilibrium is restored. Assume that the threads do not interfere and balls do not collide.

(b) If two of the balls are suddenly discharged, how will the balls behave after this? Find the separation between the balls when equilibrium is restored. The threads do not interfere.

Q. 110: Two charged particle of equal mass are constrained to move along X and Y direction. The $X-Y$ plane is horizontal and the tracks are smooth. The particles are released from rest when they were at positions shown in the figure. At the instant distance of q becomes $2r$ from the origin, find the location of charge Q.



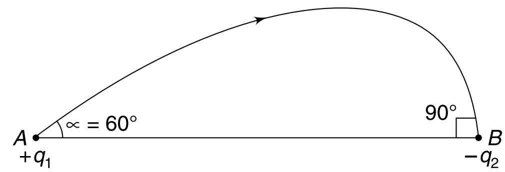
Q. 111: Consider a uniformly charged spherical shell. Two cones having same semi vertical angle, and their common apex at P, intercept the shell. The intercepts have area $\Delta S1$ and $\Delta S2$. For a cone of very small angle, $\Delta S1$ and $\Delta S2$ will be very small and charge on them can be regarded as point charge for the purpose of writing electric field at point P. Prove that the charge on $\Delta S1$ and $\Delta S2$ produce equal and opposite field at P. Hence, argue that field at all points inside the uniformly charged spherical shell is zero.



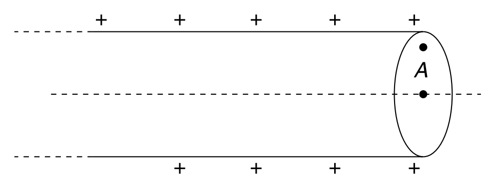
Q. 112: Two point charges $+q_1$ and $-q_2$ are placed at A and B respectively. An electric line of force emerges from q_1 making an angle $\alpha = 60°$ with line AB and terminates at $-q_2$ making an angle of 90° with the line AB.

(a) Find $\left|\frac{q_1}{q_2}\right|$

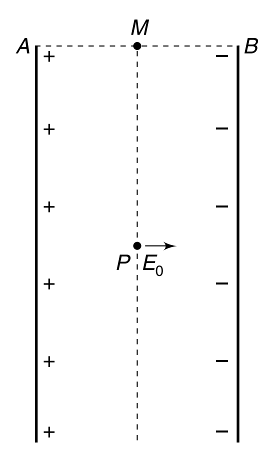
(b) Find the maximum value of angle α at which a line emitted from q_1 terminates on charge q_2.



Q. 113: There is a semi-infinite hollow cylindrical pipe (i.e. one end extends to infinity) with uniform surface charge density. What is the direction of electric field at a point A on the circular end face?

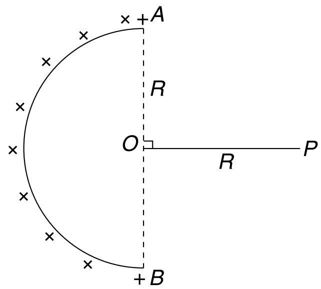


Q. 114: Two equal insulating threads are placed parallel to each other. Separation between the threads (= d) is much smaller than their length. Both the threads have equal and opposite linear charge density on them. The electric field at a point P, equidistant from the threads (in the plane of the threads) and located well within (see figure) is E_0. Calculate the field at mid point (M) of line AB.



Q. 115: If coulomb's law were $F = K\frac{qQ}{r^3}$, calculate the electric field due to a uniformly charged line charge at a distance d from it. The linear charge density on the line charge is λ C/m, and it is of infinite length.

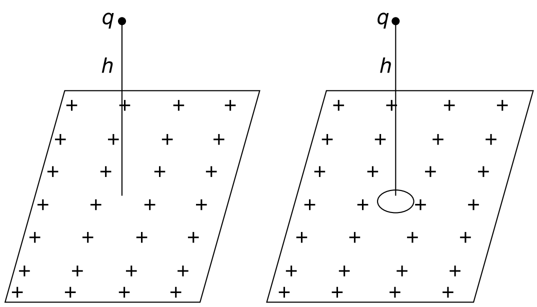
Q. 116: A semicircular ring of radius R carries a uniform linear charge density of λ. P is a point in the plane of the ring at a distance R from centre O. OP is perpendicular to AB. Find electric field intensity at point P.



Q. 117: A small charged ball is in state of equilibrium at a height h above a large horizontal uniformly charged dielectric plate having surface charge density of σ C/m^2.

(a) Find the acceleration of the ball if a disc of radius $r(<< h)$ is removed from the plate directly underneath the ball.

(b) Find the terminal speed (V_0) acquired by the falling ball. Assume that mass of the ball is m, its radius is x and coefficient of viscosity of air is η. Neglect buoyancy and assume that the ball acquires terminal speed within a short distance of its fall.



Q. 118: In a certain region of space the electrostatic field depends only on the coordinates x and y as follows.

$$E = 0 \quad \text{for} \quad \sqrt{x^2 + y^2} < r_0$$

$$E = a(x\hat{i} + y\hat{j})/(x^2 + y^2), \text{ for } \sqrt{x^2 + y^2} > r_0$$

where a is a positive constant, and $\hat{i}$ and $\hat{j}$ are the unit vectors along the X- and Y-axes. Find the charge within a sphere of radius $2r_0$ with the centre at the origin.

Q. 119: A solid sphere of radius R has total charge Q. The charge is distributed in spherically symmetric manner in the sphere. The charge density is zero at the centre and increases linearly with distance from the centre.

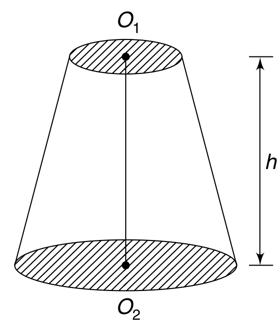
(a) Find the charge density at distance r from the centre of the sphere.

(b) Find the magnitude of electric field at a point 'P' inside the sphere at distance r_1 from the centre.

Q. 120: A ball of radius R carries a positive charge whose volume charge density depends only on the distance r from the ball's centre as: $\rho = \rho_0\left(1 - \frac{r}{R}\right)$

Where r_0 is a constant. Take ε to be permittivity of the ball. Calculate the maximum electric field intensity at a point (inside or outside the ball) due to such a charge distribution.

Q. 121: A frustum is cut from a right circular cone. The two circular faces have radii R and $2R$ and their centers are at O_1 and O_2 respectively. Height of the frustum is $h = 3R$. When a point charge Q is placed at O_1, the flux of electric field through the circular face of radius $2R$ is ϕ_1 and when the charge Q is placed at O_2, the flux through the other circular face is ϕ_2. Find the ratio $\frac{\phi_1}{\phi_2}$.



Q. 122: The electric field in a region of space varies as $E = (x\hat{i} + 2y\hat{j} + 3z\hat{k})$ V/m

(a) Consider an elemental cuboid whose one vertex is at (x, y, z) and the three sides are dx, dy and dz; sides being parallel to the three co-ordinate axes. Calculate the flux of Electric field through the cube.

(b) Using the expression obtained in (a) find the charge enclosed by a spherical surface of radius r, centred at the origin.

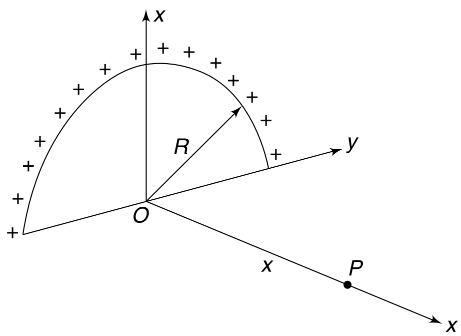
Q. 123: A ball of radius R has a uniformly distributed charge Q. The surrounding space of the ball is filled with a volume charge density $\rho = \frac{b}{r}$, where b is a constant and r is the distance from the centre of the ball.

It was found that the magnitude of electric field outside the ball is independent of distance r.

(a) Find the value of Q.

(b) Find the magnitude of electric field outside the ball.

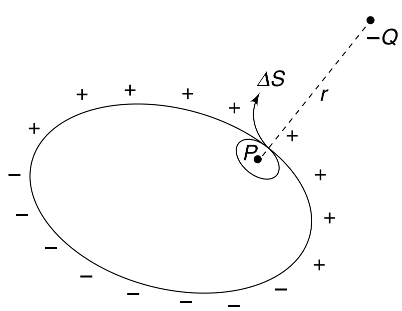
Q. 124: A uniform semicircular ring of radius R is in yz plane with its centre at the origin. The half ring carries a uniform linear charge density of λ.



(a) Find the x, y and z component of Electric field at a point $P(x, 0, 0)$ on the axis of the ring.

(b) Prove that the field at P is directed along a line joining the centre of mass of the half ring to the point P.

Q. 125: A charge $-Q$ is placed at some distance from a neutral conductor. Charge is induced on its surface. In the neighbourhood of a point P on its surface, the charge density is σ C/m^2. Consider a small area ΔS on the surface of the conductor encircling point P. Find the resultant force experienced by the area ΔS due to charge present on the surface elsewhere and the charge $-Q$.



Q. 126: A soap bubble of radius $R = 1$ cm is charged with the maximum charge for which breakdown of air on its surface does not occur. Calculate the electrostatic pressure on the surface of the bubble. It is know that dielectric breakdown of air takes place when electric field becomes larger then $E_0 = 3 \times 10^6$ V/m.

Q. 127: A conducting sphere of radius R is cut into two equal halves which are held pressed together by a stiff spring inside the sphere.

(a) Find the change in tension in the spring if the sphere is given a charge Q.

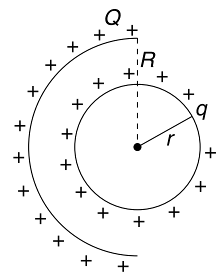
(b) Find the change in tension in the spring corresponding to the maximum charge that can be placed on the sphere if dielectric breakdown strength of air surrounding the sphere is E_0.

Q. 128: Surface tension of a soap solution is T. There is a soap bubble of radius r. Calculate the amount of charge that must be spread uniformly on its surface so that its radius becomes $2r$. Atmospheric pressure is P_0. Assume that air temperature inside the bubble remains constant.

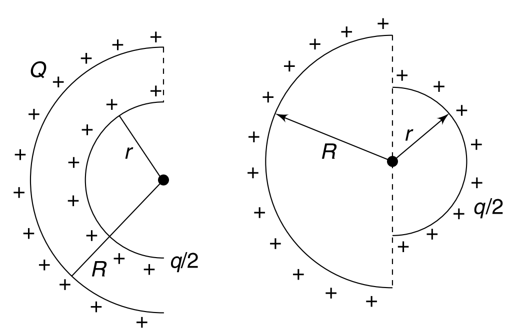
Q. 129: A point charge Q has been placed at a point outside a neutral spherical conductor. The induced charge density at point P on the surface of the conductor is $-\sigma$. The distance of point P from the point charge Q is $2R$ (where R is radius of the conductor).

Find the magnitude and direction of electric field at a point outside the conductor that is very close to its surface near P.

Q. 130: A spherical shell of radius r carries a uniformly distributed surface charge q on it. A hemispherical shell of radius $R(> r)$ is placed covering it with its centre coinciding with that of the sphere of radius r. The hemisphere has a uniform surface charge Q on it. The charge distribution on the sphere and the hemisphere is not affected due to each other. Calculate the force that the sphere will exert on the hemisphere.



Q. 131: In the last question half of the inner sphere is removed along with its charge (i.e., the remaining half has charge $\frac{q}{2}$). Find the force between the bigger and smaller hemispheres in the two cases shown in figure (a) and figure (b).

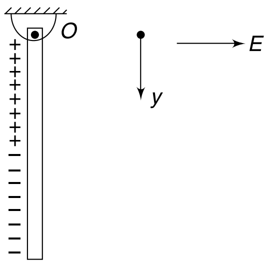


(a) (b)

Q. 132: A charged ball with mass m and charge q is dropped from a height h over a non-conducting smooth horizontal plane. There exists a uniform electric field E_0 in vertically downward direction and the coefficient of restitution between the ball and the plane is e. Find the maximum height attained by the ball after n^{th} collision.

Q. 133: In the last question, the electric field in vertical direction is switched off and a field of same strength (E_0) is switched on in horizontal direction. Find the horizontal velocity of the ball during the n^{th} collision. Also calculate the time interval between n^{th} and $(n + 1)^{th}$ collision.

Q. 134: A thin insulating rod of mass m and length L is hinged at its upper end (O) so that it can freely rotate in vertical plane. The linear charge density on the rod varies with distance (y) measured from upper end as



$$\left.\begin{aligned}\lambda &= ay^2 \quad ; \quad 0 \le y \le \frac{L}{2} \\ &= -by^n \quad ; \quad \frac{L}{2} < y \le L\end{aligned}\right\}$$

Where a and b both are positive constants. When a horizontal electric field E is switched on the rod is found to remain stationary.

(a) Find the value of constant b in terms of a. Also find n.

(b) Find the force applied by the hinge on the rod, if $EaL^3 = 45$ mg.

Q. 135: Two fixed charges $-2Q$ and Q are located at the points with coordinates $(-3a, 0)$ and $(+3a, 0)$ respectively in the $x - y$ plane.

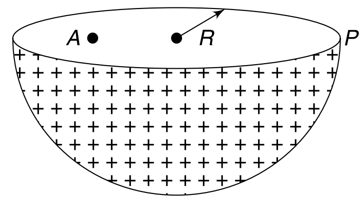
(a) Show that all points in the $x-y$ plane where the electric potential due to the two charges is zero, lie on a circle. Find its radius and the location of its centre.

(b) Give the expression $V(x)$ at a general point on the $x-$axis and sketch the function $V(x)$ on the whole $x-$axis.

(c) If a particle of charge $+q$ starts from rest at the centre of the circle, show by a short quantitative argument that the particle eventually crosses the circle. Find its speed when it does so.

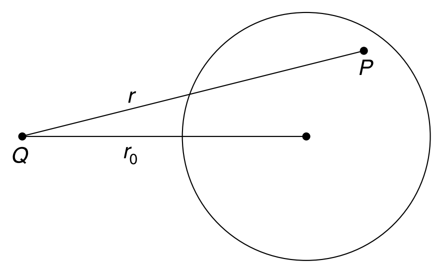
Q. 136: A metallic sphere of radius R has a small bulge of hemispherical shape on its surface. The radius of the bulge is r. If a charge Q is given to the sphere, calculate the quantity of charge on the surface of the bulge. Assume that charge is uniformly distributed on the surface of the bulge (though it is wrong !) and also on the remaining surface of the sphere.

Q. 137: A hemispherical bowl of radius R carries a uniform surface charge density of σ. Find potential at a point P located just outside the rim of the bowl (see figure). Also calculate the potential at a point A located at a distance $R/2$ from the centre on the equatorial plane.

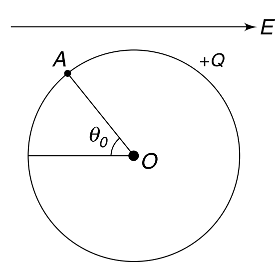


Q. 138: Two infinite line have linear charge densities $-\lambda$ and $+\lambda$. They are parallel to z axis passing through x axis at points $x = -a$ and $x = a$ respectively. Show that the equipotential surface having potential $\frac{\lambda \ln(2)}{4\pi\varepsilon_0}$ is a cylinder having radius $2\sqrt{2}a$.

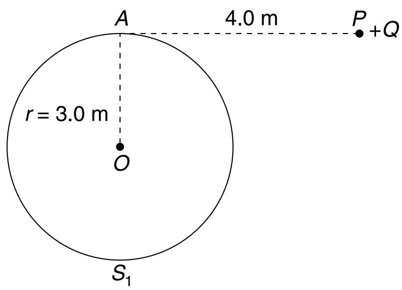
Q. 139: A conducting shell having no charge has radius R. A point charge Q is placed in front of it at a distance r_0 from its centre. Find potential due to charge induced on the surface of the shell at a point P inside the shell. Distance of point P from point charge Q is r.



Q. 140: A conducting sphere of radius R having charge Q is placed in a uniform external field E. O is the centre of the sphere and A is a point on the surface of the sphere such that AO makes an angle of $\theta_0 = 60°$ with the opposite direction of external field. Calculate the potential at point A due to charge on the sphere only.



Q. 141: Consider a solid neutral conducting sphere S_1 of radius $r = 3.0$ m. A point charge $Q = +2\mu C$ is placed at point P such that $AP = 4.0$ m (AP is tangent to the sphere). Charge Q' is induced on the surface of the conducting sphere S_1. Consider another non conducting sphere (S_2) of same radius r. Charge Q' is spread on the surface of S_2 in exactly the same way as it is present on the surface of conducting sphere S_1 [i.e., the distribution of charge on surface of S_2 is exact replica of the induced surface charge on S_1]. There is no other charge in vicinity of S_2. Find the smallest potential (at a point) on the surface of sphere S_2.

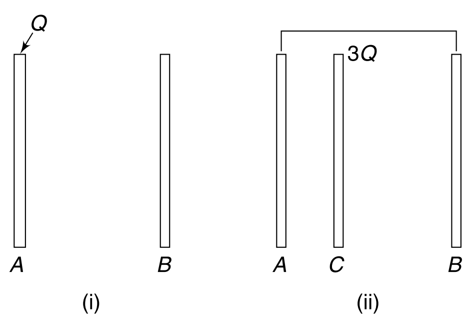


Q. 142: A and B are two large identical thin metal plates placed parallel to each other at a small separation. Plate A is given a charge Q.

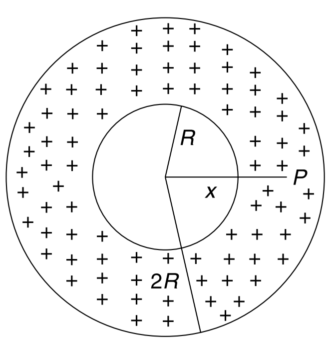
(i) Find the amount of charge on each of the two faces of A and B.

(ii) Another identical plate C having charge $3Q$ is inserted between plate A and B such that distance of C from B is twice its distance from A. Plate A and B is shorted using a conducting wire. Find charge on all six faces of plates A, B and C.

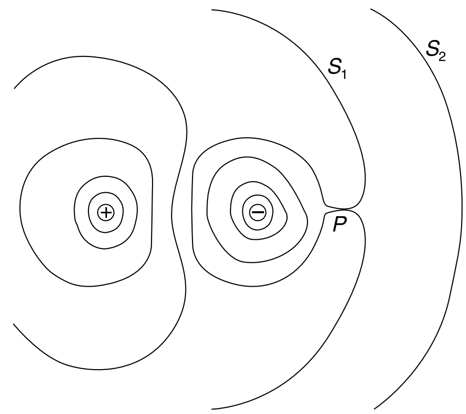
(iii) In the situation described in (ii) the plate A is grounded. Now write the charge on all six faces.



Q. 143: A non conducting shell of inner and outer radii R and $2R$ respectively has a charge Q uniformly distributed in its volume. Find the electric potential at a point P at a distance r_0 from the centre such that $R < r_0 < 2R$.



Q. 144: The figure shows equipotential surfaces due to two point charges $+3Q$ and $-Q$ placed at a separation d.



(a) Will the shape of epuipotential surface be spherical very close to both the point charges? What shape of equipotentials will be seen at very far away points from the pair of charges.

(b) Find the distance of point P from the negative charge.

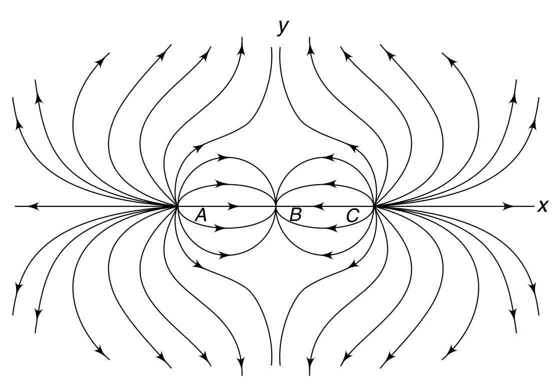
(c) Find the potential of the surface marked as S_1 is the figure.

(d) Consider the surface having zero potential. Write the flux of electric field through this surface.

Q. 145: Three point charge have been placed along the x axis at points A, B and C. Distance $AB = BC$. The field lines generated by the system is shown in figure.

(a) Plot the variation of electric potential along x axis. Show potential on y axis of your graph.

(b) Plot the variation of electric potential along y axis with B as origin. Shows potential on x axis in your graph.



Q. 146: A conducting shell has inner radius R and outer radius $2R$. A charge $+q$ is given to the spherical shell.

(a) Find the electric field at a point which is at a distance x from the centre of the shell. Give your answer for three cases

(i) $x < R$ (ii) $R < x < 2R$ (iii) $x > 2R$

(b) Find the electric potential in all the three cases mentioned in (a)

(c) Find field and potential in all the three cases mentioned in (a) after a point charge $-q$ is introduced at the centre of the shell.

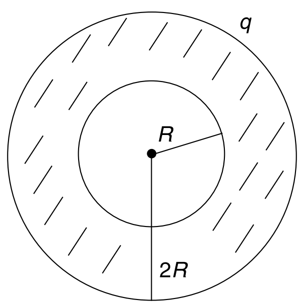
(d) Write the electrical potential energy of the system consisting of the shell and the point charge at its centre.

(e) Find the electrostatic force that the shell exerts on the point charge.

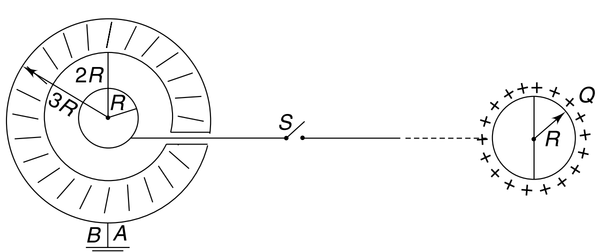
(f) Now, another point charge $+q$ is placed at a distance $4R$ from the centre of the shell. Find electric field and potential in following cases.

(i) $x < R$

(ii) $R < x < 2R$



Q. 147: A solid conducting sphere of radius R is surrounded by a concentric metallic shell of inner radius $2R$ and outer radius $3R$. The shell is earthed. The inner sphere is connected to a switch S by a thin conducting wire through a small hole in the shell. By closing the switch S, the inner sphere is connected to a distant conducting sphere of radius R having charge Q. Find the charge that flows to earth through wire AB.

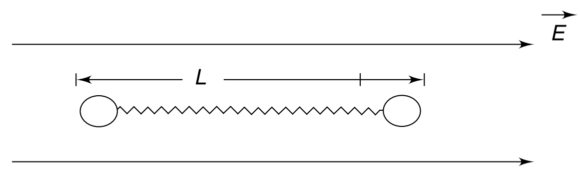


Q. 148: Two small identical conducting balls each of radius r and mass m are placed on a frictionless horizontal table, connected by a light conducting spring of force constant K and un-deformed length $L (L >> r)$. A uniform electric field of strength E is switched on in horizontal direction parallel to the spring.

(a) How much charge will appear on the two balls when they are at separation L.

(b) The system fails to oscillate if $K < K_0$. Find K_0.

(c) Assuming $K = 2K_0$, find the time period of oscillation after the electric field is switched on.



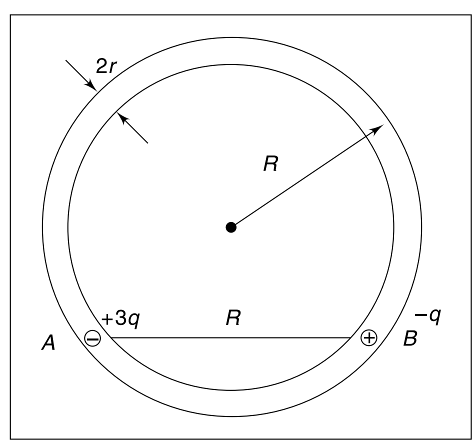
Q. 149: Four charges have equal magnitude. Two of them are positive and remaining two are negative. Charges have been placed on the vertices of a rectangle and the electrostatic potential energy of the system happens to be zero.

(a) Show the arrangement of the charges on the vertices of the rectangle.

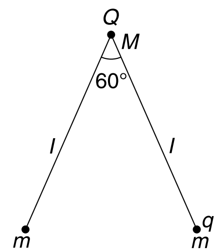
(b) If the smaller side of the rectangle has a length of 1.0 m, show that length of larger side must be less than 2.0 m.

[Actually you need to solve on algebraic polynomial to get the exact length of the larger side. Here I am not exactly interested in making you solve that equation]

Q. 150: On a horizontal table, there is a smooth circular groove of mean radius R. The walls of the groove are non conducting. Two metal balls (each having mass m and radius r) are placed inside the groove with their centers R apart. The balls just fit inside the groove. The two balls are given charge $+3q$ and $-q$ and released from state of rest. Ignore the non-uniformity in charge distribution as the balls come close together and collide. The collision is elastic. Find maximum speed acquired by each ball after they collide for the first time.



Q. 151: A particle having charge Q and mass $M = 2m$ is tied to two identical particles, each having mass m and charge q. The strings are of equal length, l each, and they are inextensible. The system is held at rest on a smooth horizontal surface (with string taut) in a position where the strings make an angle of 60° between them. From this position the system is released.



(a) Find the amplitude of oscillation of M

(b) Find maximum speed acquired by M

(c) Find tension in the string when all the three particles get in one straight line.

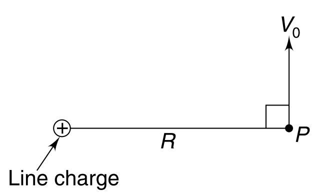
Q. 152: Four point charges $+8\,\mu$C, $-1\,\mu$C, $-1\,\mu$C and, $+8\,\mu$C are fixed at the points $-\sqrt{27/2}\,m$, $-\sqrt{3/2}\,m$, $+\sqrt{3/2}\,m$ and $+\sqrt{27/2}\,m$ respectively on the y-axis. A particle of mass 6×10^{-4} kg and $+0.1\,\mu$C moves along the $x-$direction. Its speed at $x = +\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin in this case. Assume that there is no force apart from electrostatic force.

Q. 153: A dielectric disc of radius R and uniform positive surface charge density σ is placed on the ground with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc from a height H with zero initial velocity. The charge – mass ratio of the particle is $\dfrac{q}{m} = \dfrac{4\varepsilon_0 g}{\sigma}$.

(i) Find the value of H if the particle just reaches the disc.

(ii) Sketch the potential energy of the particle as a function of its height and find its height in equilibrium position

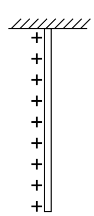
Q. 154: An infinite line charge is perpendicular to the plane of the figure having linear charge density λ. A particle having charge Q and mass m is projected in the field of the line charge from point P. The point P is at a distance R from the line charge and velocity given to the particle is perpendicular to the radial line at P (see figure).



(a) Find the speed of the particle when its distance from the line charge grows to ηR ($\eta > 1$).

(b) Find the velocity component of the particle along the radial line (joining the line charge to the particle) at the instant its distance becomes ηR.

Q. 155: A uniformly charged non conducting rod is suspended vertically at its end. The rod can swing freely in the vertical plane without any friction. The linear charge density on the rod is λ C/m and it has a uniformly distributed mass of M. Length of the rod is L. A uniform horizontal Electric field (E) is switched on in the region.



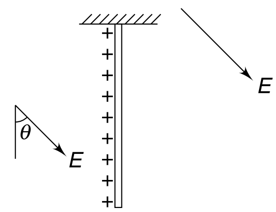
(a) For what value of electric field (call it E_0) the rod just manages to make itself horizontal?

(b) If Electric field E_0 is switched on, what is the maximum angular speed acquired by the rod during its motion?

Q. 156: In the last question let us assume that the uniform electric field makes an angle θ with the vertical in downward direction. With the uniformly charged rod in vertical position the field is switched on. Mass of the rod, its length and charge per unit length is M, L and λ respectively.

(a) Find the strength of field (E) such that the rod can reach the horizontal position if $\theta = 30°$

(b) Find the minimum strength of field (E) such that the rod can reach the horizontal position if $\theta = 60°$



(c) However high the field might be, the rod cannot become horizontal if $\theta < \theta_0$. Find θ_0

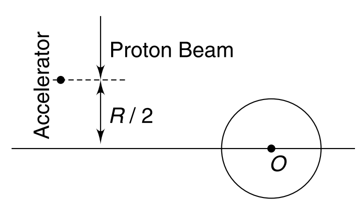
Q. 157: A massless rod of length L has two equal charges (q) tied to its ends. The rod is free to rotate in horizontal plane about a vertical axis passing through a point at a distance $\dfrac{L}{4}$ from one of its ends. A uniform horizontal electric field (E) exists in the region.

(a) Draw diagrams showing the stable and unstable equilibrium positions of the rod in the field.

(b) Calculate the change in electric potential energy of the rod when it is rotated by an angle $\Delta\theta$ from its stable equilibrium position.

(c) Calculate the time period of small oscillations of the rod about its stable equilibrium position. Take the mass of each charge to be m.

Q. 158: A proton accelerator produces a narrow beam of protons, each having an initial speed of v_0. The beam is directed towards an initially uncharged distant metal sphere of radius R. The sphere is fixed and centered at point O. The initial path of the beam is at a distance of (R/2) from the centre, as indicated in the diagram.



The protons in the beam that collide with the sphere get absorbed and cause it to become uniformly charged. The subsequent potential field at the accelerator due to the sphere can be neglected. Assume the mass of the proton as m_P and the charge on it as e.

(a) After a long time, when the potential of the sphere reaches a constant value, sketch the trajectory of proton in the beam.

(b) Once the potential of the sphere has reached its final, constant value, find the minimum speed v of a proton along its trajectory path.

(c) Find the limiting electric potential of the sphere.

Q. 159: Consider a uniformly charged spherical shell of radius R having charge Q. Charge Q can be thought to be made up of number of point charges q_1, q_2, q_3 ... etc. The electrostatic energy of the charged shell is sum of interaction energies of all possible pairs of charges.

$$U = \sum_{\text{all pairs}} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}$$

Where r_{ij} is distance between q_i and q_j. For continuous charge on the shell, the summation has to be carried through integration.

(a) Calculate the electrostatic energy of the shell. We can term this energy as self energy of the shell.

(b) Calculate the work done in assembling a spherical shell of uniform charge Q and radius R by bringing charges in small installments from infinity and putting them on the shell.

Do you find the answers in (a) and (b) to be same?

Q. 160:

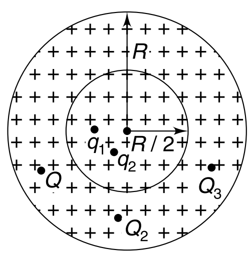
(a) Use the method in part (b) of the previous problem to calculate the electrostatic self energy of a uniformly charged sphere of radius R having charge Q.

(b) Divide the above sphere (mentally) into two regions-spherical concentric part having radius $\frac{R}{2}$ and the remaining annular part $\left(\text{between } \frac{R}{2} \text{ and } R\right)$.

Denote the point charges in sphere of radius $R/2$ by q_1, q_2, q_3 ... etc.

The charges in annular part be denoted by Q_1, Q_2, Q_3 ... etc.

Calculate the electrostatic interaction energy for all pairs like $[(Q_i, Q_j) + (q_i + q_j)]$.



Q. 161: A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged.

S_1 is given a charge Q brought into contact with S_2 and removed. S_1 is recharged such that the charge on it is again Q and it is again brought into contact with S_2 and removed. This procedure is repeated n times.

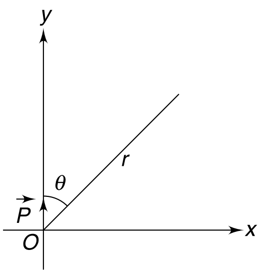
(i) Find the electrostatic energy of S_2 after n such contact with S_1.

(ii) What is the limiting value of this energy as $n \to \infty$?

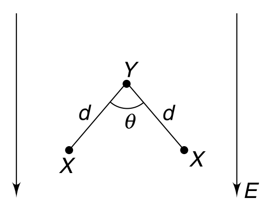
Q. 162: A short electric dipole is placed at the origin of the Co-ordinate system with its dipole moment P along y direction. Give answer to following questions for points which are at large distance r from the origin in $x - y$ plane. [r is large compared to length of the dipole].

(a) Find maximum value of x component of electric field at a point that is at r_0 distance from the origin.

(b) Prove that (for $0 < \theta < 90°$) all the points, where electric field due to the dipole is parallel to x–axis, fall on a straight line. Find the slope of the line.



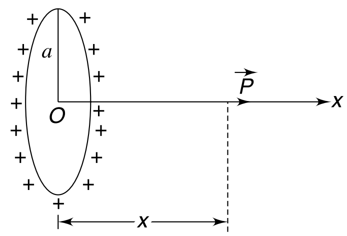
Q. 163: A tri atomic molecule X_2Y has plane structure as shown in figure. Due to difference in electronegativity, charge acquired by each X atom is q and charge on Y atom is $-2q$. The bond length between Y and X is d, and angle between the two bonds is $\theta = 120°$ mass of one atom of X and Y are m and 8m respectively. The molecule is placed in a uniform Electric field E and is making small oscillations about an axis perpendicular to the plane of figure and passing through the centre of mass of the molecule. Find the time period of oscillation.



Q. 164: A short electric dipole (dipole moment P) is placed on the axis of a uniformly charged ring at a distance x from the centre as shown in figure. Radius of the ring is a and charge on it is Q.

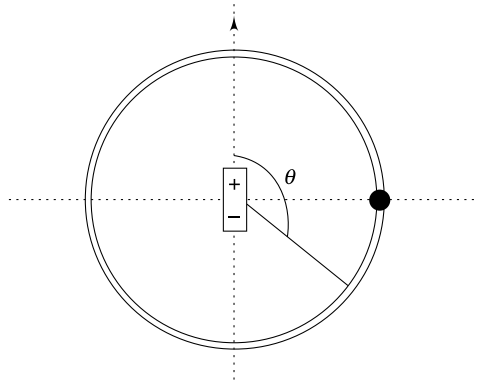
(i) Write the force on the dipole when $x = \frac{a}{2}$ and when $x = a$. Why the direction of force at two points is different?

(ii) Is the force on the dipole zero if $x = 0$? If not, where will you place the dipole so that force on it is zero.



Q. 165: Consider two spheres of the same radius R having uniformly distributed volume charge density of same magnitude but opposite sign ($+\rho$ and $-\rho$). The spheres overlap such that the vector joining the centre of the negative sphere to that of the positive sphere is $\vec{d}$. ($d << R$). Find magnitude of electric field at a point outside the spheres at a distance r in a direction making an angle θ with $\vec{d}$. Distance r is measured with respect to the mid point of the line joining the centers of the two spheres.

Q. 166: A small, electrically charged bead can slide on a circular, frictionless, thin, insulating ring. Charge on the bead is Q and its mass is m . A small electric dipole, having dipole moment P is fixed at the centre of the circle with the dipole's axis lying in the plane of the circle. Initially, the bead is held on the perpendicular bisector of the dipole (see fig.) Ignore gravity and answer the following questions.



(a) Write the speed of the bead when it reaches the position θ shown in the figure.

(b) Find the normal force exerted by the ring on the bead at position θ.

(c) How does the bead move after it is released? Where will the bead first stop after being released?

(d) How would the bead move in the absence of the ring?

Q. 167: A short electric dipole ($\vec{P}$) has been placed in a uniform electric field ($\vec{E}$) with the dipole moment vector ($\vec{P}$) parallel to $\vec{E}$. Show the field lines in the region. Mark the null points (i.e. the points where the field is zero)

ANSWERS

1. (a) $2F$ towards point F

(b) In a direction making an angle of 60° with BA (NOT along BC).

3. Zero.

4. (a) 120000 N

(b) Force is nearly 10 times the weight of the car.

Two conclusions can be drawn from the result-

(i) Electrostatic force is a strong force

(ii) One coulomb is a very large charge.

5. $\frac{r^2}{2}$

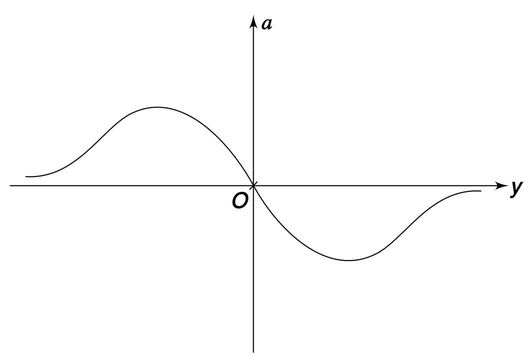
6. $\therefore k = \frac{q^2}{4\pi\epsilon_0 L^3}\left(\frac{2\sqrt{2}+1}{\sqrt{2}}\right)$

7. $F_2 = F_1\left(\frac{q_2^2}{q_1^2}\frac{l_1^2}{l_2^2}\right)$

8. (i) $T = 2\pi\sqrt{\frac{L}{2g}}$ (ii) $T = 2\pi\sqrt{\frac{L}{g}}$

(iii) $T = 2\pi\sqrt{\frac{L}{\sqrt{2}g}}$ (iv) $T = 2\pi\sqrt{\frac{L}{g}}$.

9.



10. All graphs are possible.

(a) electron has been projected in the direction of electric field

(b) electron has been released from rest

(c) electron has been projected making an acute angle with field

(d) electron has been projected in a direction making an obtuse angle or right angle to the direction of field

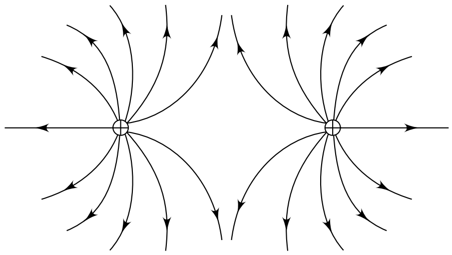
11. (a) $K_B = \pi qER$ (b) $a_t = \frac{qE}{m}\sqrt{1+\pi^2}$

12. $-\frac{qa}{m}$

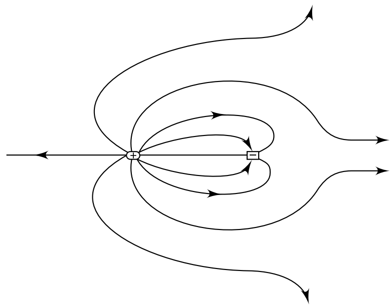
13. (a) $\pi\alpha R_0^4$ (b) $\dfrac{R_0}{(2)^{1/4}}$

(c) $\dfrac{\alpha r^2}{4\in_0}$

14. (a)

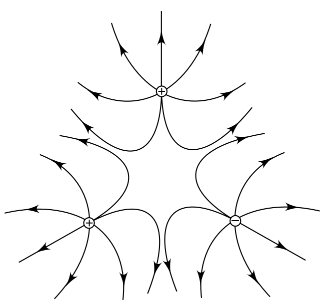


(b)



Field lines will appear to be radial at a large distance.

(c)



15. 50 V

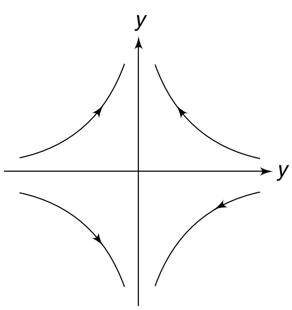
16. $\vec{E} = \left(\hat{i} + 2\hat{j} + 2\sqrt{5}\hat{k}\right)$ V/m

17. $\vec{a} = (\hat{i} + \hat{j})$ m/s^2

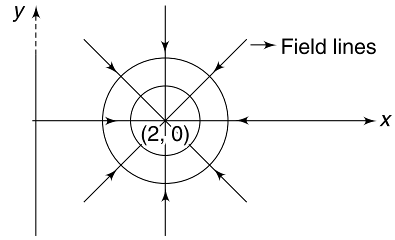
18. $E_A > E_B$ $V_A > V_B$

19. (b) No, (c) Zero, No

20. (a)



(b)



21. $W = -\dfrac{Kq^2}{a}\left(1 - \dfrac{1}{\sqrt{3}}\right)$

22. For both (a) and (b) $W = \dfrac{Qq}{8\pi\in_0 r}$

23. (a) $-\dfrac{Qe}{8\pi\in_0 r}$. (b) $8T$

24. Yes, V_0 depends on q.

$V_0 = \sqrt{\dfrac{2KQq}{mR}}$

25. (a) $\theta_0 = 2\tan^{-1}\left(\dfrac{qE}{mg}\right)$ (b) $E = \dfrac{mg}{q}$

26. $-\dfrac{Q^2}{16\pi\in_0 R}$

27. (a) $\dfrac{\eta}{4}$ (b) $[M^{-1}L^{-5}T^4A^2]$

28. (a) 39.6μJ (b) 2.25×10^{-9}J

29. $E = \dfrac{6\sqrt{3}KP}{L^3}$; $V = 0$

30. $\dfrac{PQ}{4\pi\in_0 R^3}$

31. $P = \dfrac{2Rq}{\pi}$

32. $\left(\dfrac{\sqrt{17} - 3}{2}\right) mg$

33. $\dfrac{32\pi\varepsilon_0 L^2 mg}{q}$

34. Stable equilibrium.

35. (a) $Q = +3.14q$ (b) $Q = -0.22q$

36. $\dfrac{1}{\sqrt{2}}$

37. (a) $\Delta T = \dfrac{KQq}{2\pi R^2}$ (b) $\dfrac{KQq}{\pi R^2}$ (c) $\dfrac{KQq}{\pi R^2}$

38. $\dfrac{8KQq}{r^2}$

39. $Q = 8R\sin\left(\dfrac{\theta}{2}\right)\sqrt{2\pi\varepsilon_0 mg\sin\left(\dfrac{\theta}{2}\right)}$

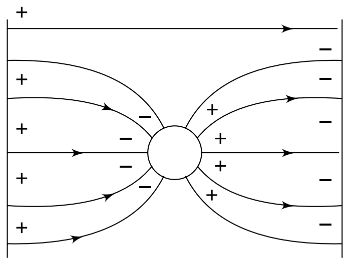
40. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{2g\sin\theta}{x_0}}$

41. (a) $\dfrac{3}{2}a$

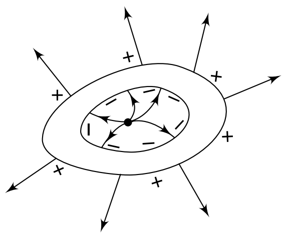
(b) Maximum electric force is between A and B. Maximum acceleration is at O

42. (a) No acceleration (b) Not affected

43. (a)



(b)



44. None of the configurations are possible.

45. $\dfrac{E_0}{2}$ perpendicular to AD

46. Perpendicular to the equatorial plane.

47. $\vec{E}$

48. $\dfrac{\sigma}{4\varepsilon_0}\dfrac{r^2}{x^2}$

49. (a) $E = \dfrac{2K\lambda}{R}\sin\left(\dfrac{\theta}{2}\right)$ (b) $E = \dfrac{2K\lambda}{R}$

(c) $E = 0$ (d) $E = \dfrac{KQ}{R^2}$

50. (a) $E_0 = \dfrac{\lambda}{2\pi\varepsilon_0 x}$ (b) $E_y = \dfrac{E_0}{2}$

(c) $E_x = \dfrac{E_0}{2} = \dfrac{\lambda}{4\pi\varepsilon_0 x}$ (d) 45°

51. Zero

52. $\dfrac{\lambda}{2\sqrt{2}\pi\varepsilon_0 R}$

53. (a) (i) $\dfrac{\rho x}{2\varepsilon_0}$

(ii) $\dfrac{\rho R^2}{2\varepsilon_0 x}$ Field at the surface is maximum

$E_{max} = \dfrac{\rho R}{2\varepsilon_0}$

(b) $\vec{E} = \dfrac{\rho\vec{a}}{2\varepsilon_0}$

54. (a) $T = 2\pi\sqrt{\dfrac{L}{g - \dfrac{q\sigma}{2m\varepsilon_0}}}$

(b) $T = 2\pi\sqrt{\dfrac{L}{\sqrt{g^2 + \left(\dfrac{qE}{m}\right)^2 - 2g\dfrac{qE}{m}\cos\beta}}}$

55. (a) $\theta = \tan^{-1}\left(\dfrac{3}{4}\right)$ (b) yes

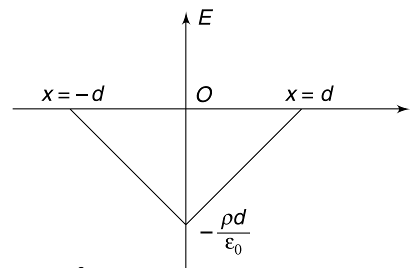
56. $\vec{E} = \dfrac{\lambda}{4\pi\varepsilon_0}(\hat{i} - \hat{k})$

57. (a) 17.7 μC, -53.1μC (b) No where

58. $E = -\dfrac{\rho}{\varepsilon_0}(d + x)$ $\quad -d < x \le 0$

$= -\dfrac{\rho}{\varepsilon_0}(d - x)$ $\quad 0 \le x < d$

$= 0$ $\quad |x| > d$



59. $E = \dfrac{by^2}{2\varepsilon_0}$

60. 2 : 1

61. (a) $q_0 = 4\pi\varepsilon_0 a$ (b) $q = -4\pi\varepsilon_0 a$

62. $\dfrac{Q}{6\varepsilon_0}$

63. $\dfrac{q}{2\varepsilon_0}$

64. $\sigma\phi$

65. (a) $\dfrac{\sqrt{5}}{6}\dfrac{\rho R}{\varepsilon_0}$ (b) $\dfrac{\sqrt{41}}{24}\dfrac{\rho R}{\varepsilon_0}$

66. (i) $\sigma = \dfrac{\varepsilon_0 m d u^2}{8eL^2}$ (ii) $\theta = \tan^{-1}\left(\dfrac{\varepsilon_0 m u^2}{2Le\cdot\sigma}\right)$

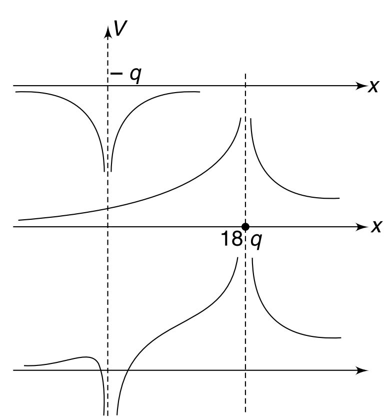
67. (a) $\dfrac{36}{25}$ s (b) $\dfrac{72}{25}$ s

(c) $\tan^{-1}\left(\dfrac{3}{4}\right)$

68. (a) t_0 (b) $\sqrt{6mK_0}$

69. $\sqrt{\dfrac{2qa}{3mV_0^2}(y^3 - y_0^2)}$

70.



71. (a) Parallel to $\overrightarrow{AB}$ (b) Zero.

72. (a) $q_1 \to +\text{ve}$; $q_2 \to -\text{ve}$
q_1 has larger magnitude
(b) $\dfrac{d}{\sqrt{3}-1}$

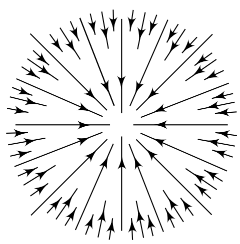
73. (a) 5 V (b) $\sqrt{\dfrac{2}{3}}$ m

74. $E = 15$ V/cm; $\cos^{-1}\left(\dfrac{2}{3}\right)$

75. (a) $V = \dfrac{\sigma}{8\in_0}\left[\sqrt{R^2+Z^2} - Z\right]$

(b) $E_Z = \dfrac{\sigma}{8\in_0}\left[1 - \dfrac{Z}{\sqrt{R^2+Z^2}}\right]$

76. (a)



(b) $-16\pi\in_0 a^3$ (c) -42 Volt

77. $\dfrac{mR^2\omega^2}{2e}$

78. 2 : 1

79. 4

80. $\dfrac{Vr}{R}$

81. (a) Yes. $4\pi\sigma(b^2 - a^2)$ (b) $\dfrac{\sigma b}{\in_0}$

82. $V' = V\left(\dfrac{a}{3t}\right)^{1/3}$

83. $\dfrac{1}{4\pi\in_0}\dfrac{q}{x}$ for both cases.

84. $E = \dfrac{q}{8\pi\in_0 R^2}$; $\dfrac{q}{4\pi\in_0 R}$

85. $E = \dfrac{\rho R}{4\in_0}$; $V = \dfrac{\rho R^2}{4\in_0}$

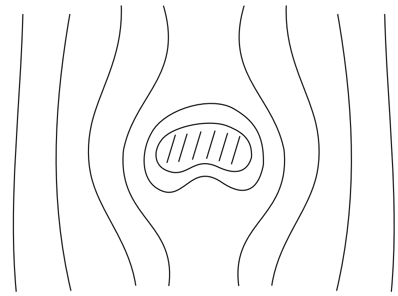
86. $\dfrac{\sigma R}{\pi\in_0}$

87. (a) $\dfrac{1}{4\pi\in_0}\dfrac{q}{R}$ (b) No change
(c) No change.

88. $E = \dfrac{1}{4\pi\in_0}\dfrac{Q}{b^2}$; $V = \dfrac{1}{4\pi\in_0}\dfrac{Q}{b}$

89. (a) $-\dfrac{R}{x}q$ (b) $\dfrac{Rq}{x^2}v$

90.



91. (a) $r = \dfrac{16\sqrt{2}\pi\in_0 R^2 mu^2}{Qq}$ (b) $V = \sqrt{u^2 + \dfrac{Qq}{4\pi\in_0 mR}}$

92. $12q$ and $3q$ at the two ends and q at a distance of 8 cm from $2q$. Force on q is zero.

93. (a) $\dfrac{8\sqrt{2}}{3}\dfrac{kqQ}{a}$ (b) $\left(\dfrac{8\sqrt{2}}{3} - \dfrac{4\sqrt{2}}{\sqrt{5}}\right)\dfrac{kqQ}{a}$

94. (a) D (b) $\sqrt{\dfrac{2eV_0}{m}}$

Electron cannot remain bound for any speed.

95. (a) $\sqrt{4gL}$ (b) $\sqrt{4.5gL}$

96. (a) 72 g (b) 4.32 N
(c) unstable

97. $W_{ext} = -3$ mgL

98. (a) $\dfrac{1}{4}$ mu^2
(b) A moves to right with velocity u. B is at rest.

99. $\left(\dfrac{1+\sqrt{3}}{2}\right)u$

100. (a) $\theta = \tan^{-1}\left(\dfrac{1}{2}\right)$ (b) 6 mgh

101. (a) $\dfrac{KQe}{2R}$ (b) $\dfrac{3}{2}\dfrac{KQe}{R}$

102. (a) $\sqrt{\dfrac{3qE}{ML}}$ (b) $\dfrac{3\sqrt{3}}{2}\dfrac{qE}{ML^2}$

103. $\dfrac{U_0}{3}$

104. $Q = \dfrac{q}{2}$

105. $\dfrac{10}{3}\pi\in_0 RV^2$

106. (a) $\sqrt{\dfrac{KP}{V_0}}$ (b) $\left(\dfrac{2KP}{E_0}\right)^{1/3}$

where $K = \dfrac{1}{4\pi\in_0}$

107. (a) $-\dfrac{KP^2}{r^3}$ (b) $-\dfrac{2KP^2}{r^3}$

108. $V = \dfrac{qr}{\pi^2 \epsilon_0 R^2}$

109. (a) $\left(\dfrac{2}{3}\right)^{\frac{1}{3}} a$ (b) $\left(\dfrac{1}{9}\right)^{\frac{1}{3}} a$

110. Q is at a distance of $4r$ from O.

112. (a) $\left|\dfrac{q_1}{q_2}\right| = \dfrac{2}{1}$ (b) $\alpha_{max} = 90°$

113. Parallel to the axis of the cylinder.

114. $\dfrac{E_0}{2}$

115. $E = \dfrac{K\lambda\pi}{2d^2}$

116. $\dfrac{1}{4\pi\epsilon_0} \dfrac{\lambda}{R} \ln(\sqrt{2} + 1)$

117. (a) $\dfrac{gr^2}{2h^2}$ (b) $V_0 = \dfrac{mgr^2}{12\pi\eta x h^2}$

118. $q = 16\pi\epsilon_0 a r_0$

119. (a) $\rho(r) = \dfrac{Q}{\pi R^4} r$ (b) $\dfrac{Q r_1^2}{4\pi\epsilon_0 R^4}$

120. $E_{max} = \dfrac{\rho_0 E}{9\varepsilon}$

121. $\left(\dfrac{\sqrt{13} - 3}{\sqrt{10} - 3}\right)\sqrt{\dfrac{10}{13}}$.

122. (a) $6\,dx\,dy\,dz$ (b) $8\varepsilon_0 \pi r^3$

123. (a) $Q = 2\pi b R^2$ (b) $E = \dfrac{b}{2\varepsilon_0}$

124. (a) $E_x = \dfrac{K\lambda R\pi \cdot x}{(R^2 + x^2)^{3/2}}$ (b) $E_y = 0$

(c) $E_z = -\dfrac{2K\lambda R^2}{(R^2 + x^2)^{3/2}}$

125. $\dfrac{\sigma^2}{2\varepsilon_0} \Delta s$

126. $P = \dfrac{1}{2} \varepsilon_0 E_0^2 = 39.6 \text{ N/m}^2$

127. (a) $F = \dfrac{Q^2}{32\pi\varepsilon_0 R^2}$ (b) $F = \dfrac{\pi\varepsilon_0 E_0^2 R^2}{2}$

128. $8\pi r^2 \left[\varepsilon_0 \left(7P_0 + \dfrac{12T}{r}\right)\right]^{1/2}$

129. $\dfrac{\sigma}{\varepsilon_0}$ in radially inward direction.

130. $\dfrac{Qq}{4\pi\varepsilon_0 R^2}\left(1 - \dfrac{1}{\sqrt{2}}\right)$

131. $\dfrac{Qq}{8\pi\varepsilon_0 R^2}\left(1 - \dfrac{1}{\sqrt{2}}\right)$ in both cases

132. he^{2n}

133. $V_{xn} = \dfrac{qE}{m}\sqrt{\dfrac{2h}{g}}\left[1 + 2e\left(\dfrac{1 - e^{n-1}}{1 - e}\right)\right]$ and $T = 2e^n \sqrt{\dfrac{2h}{g}}$

134. (a) $b = \dfrac{a}{15}$; $n = 2$ (b) $\sqrt{2}$ mg

135. (a) radius $4a$, centre at $(5a, 0)$

(b) $V = \dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{|(x - 3a)|} - \dfrac{2}{|(x + 3a)|}\right]$

(c) At $x = 9a$ where $V = 0$, the charged particle eventually crosses the circle.

$v = \sqrt{\dfrac{Qq}{8\pi\varepsilon_0 ma}}$

136. $\dfrac{Qr}{2R}$

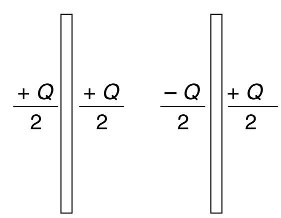
137. At both P and A the potential is $\dfrac{\sigma R}{2\varepsilon_0}$

139. $\dfrac{KQ}{r_0} - \dfrac{KQ}{r}$

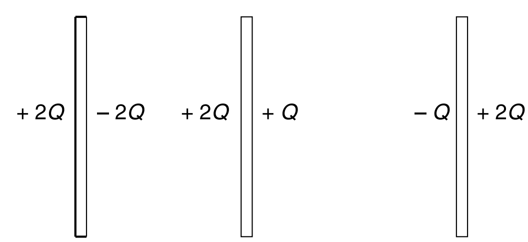
140. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R} - \dfrac{ER}{2}$

141. 5400 V.

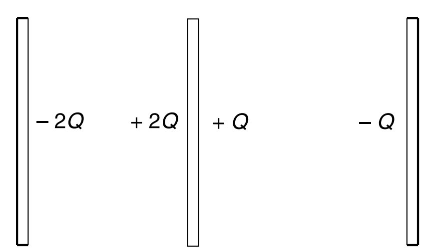
142. (i)



(ii)



(iii)

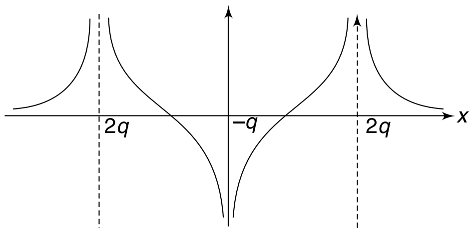


143. $\dfrac{6}{7}\dfrac{KQ}{R} - \dfrac{KQ}{7R^3}\left(\dfrac{r_0^2}{2} - \dfrac{R^3}{r_0}\right)$

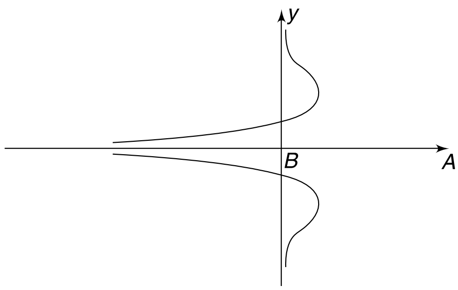
144. (a) Yes, spherical (b) $\dfrac{d}{\sqrt{3} - 1}$

(c) $\frac{2KQ}{d}(2-\sqrt{3})$ (d) $\frac{-Q}{\epsilon_0}$

145 (a)



(b)



146. (a) (i) $E = 0$ (ii) $E = 0$ (iii) $E = \frac{Kq}{x^2}$

(b) (i) $V = \frac{Kq}{2R}$; (ii) $V = \frac{Kq}{2R}$ (iii) $V = \frac{Kq}{x}$

(c) (i) $V = \frac{Kq}{x^2}$; $V = Kq\left(\frac{1}{R} - \frac{1}{x}\right)$

(ii) $E = 0$; $V = 0$

(iii) $E = 0$; $V = 0$

(d) $-\frac{Kq^2}{2R}$

(e) zero

(f) (i) $E = \frac{Kq}{x^2}$; $V = \frac{Kq}{4R} - \frac{Kq}{x} + \frac{Kq}{R}$

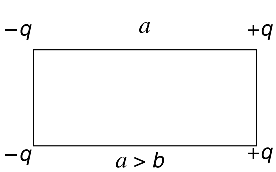
(ii) $E = 0$; $V = \frac{Kq}{4R}$

147. $\frac{2Q}{3}$

148. (a) $\pm 2\pi\epsilon_0 ErL$ (b) $k_0 = 2\pi\epsilon_0 rE^2$

(c) $\frac{1}{E}\sqrt{\frac{\pi m}{\epsilon_0 r}}$

149.



150. $V_0 = \sqrt{\frac{Kq^2(4R-7r)}{2mrR}}$

151. (a) $\frac{\sqrt{3}}{4}l$ (b) $\frac{q}{4}\sqrt{\frac{1}{\pi\varepsilon_0 ml}}$

(c) $\frac{q}{16\pi\varepsilon_0 l^2}(5q + 4Q)$

152. $(v_0)_{\min} = 3$ m/s, $K = 3 \times 10^{-4}$ J

153. (i) $H = \frac{4}{3}R$ (ii) $H = \frac{R}{\sqrt{3}}$

154. (a) $V = \sqrt{V_0^2 + \frac{\lambda Q}{\pi\varepsilon_0 m}\ln\eta}$

(b) $V_r = \sqrt{V_0^2\left(1 - \frac{1}{\eta^2}\right) + \frac{\lambda Q}{\pi\varepsilon_0 m}\ln\eta}$

155. (a) $E_0 = \frac{Mg}{\lambda L}$ (b) $\omega = \sqrt{3(\sqrt{2}-1)\frac{g}{L}}$

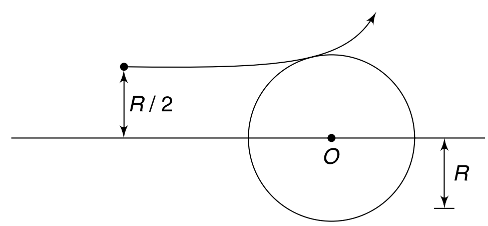
156. (a) Not possible (b) $E = \left(\frac{2}{\sqrt{3}-1}\right)\frac{Mg}{\lambda L}$

(c) $\theta_0 = 45°$

157. (b) $\Delta U = \frac{EqL}{2}(1 - \cos\Delta\theta)$

(c) $T = \pi\sqrt{\frac{5mL}{Eq}}$

158. (a)



(b) $v_0/2$ (c) $\frac{3m_P v_0^2}{8e}$

159. (a) $\frac{Q^2}{8\pi\epsilon_0 R}$ (b) same as in (a)

160. (a) $\frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 R}$ (b) $\frac{147}{320}\frac{Q^2}{4\pi\varepsilon_0 R}$

161. (i) $U_n = \frac{q_n^2}{8\pi\varepsilon_0 R}$ Here $q_n = \frac{QR}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right]$

(ii) $U_\infty = \frac{Q^2 R}{8\pi\varepsilon_0 r^2}$

162. (a) $\frac{3}{2}\frac{KP}{r_0^3}$ (b) $\frac{1}{\sqrt{2}}$

163. $T = \frac{8\pi}{3}\sqrt{\frac{md}{qE}}$

164. (i) at $x = \frac{a}{2}$; $F = \frac{16}{5^{3/2}}\frac{KQP}{a^3}$

at $x = a$; $F = -\dfrac{KQP}{2^{5/2}a^3}$

(ii) No; Force is zero at $x = \pm\dfrac{a}{\sqrt{2}}$

165. $\dfrac{\rho R^3 d}{3\varepsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$

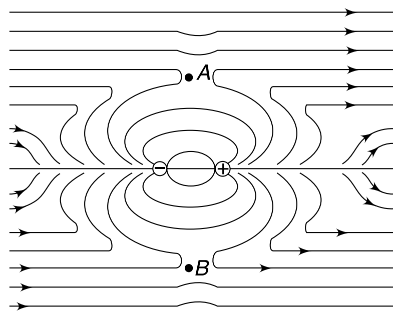
166. (a) $v = \sqrt{-2\dfrac{QKP\cos\theta}{mr^2}}$,

(b) Zero

(c) The bead oscillates. It stops at a point diametrically opposite to its starting point.

(d) Exactly same as it would move in the presence of the ring

167.



SOLUTIONS

1. (a) When there is q charge at the centre and vertices B, C, D, E, F, the net force on the central charge will be towards A (due to charge at D). The force due to charge at B and E cancel out. Similarly, force due to charge at F and C cancel out

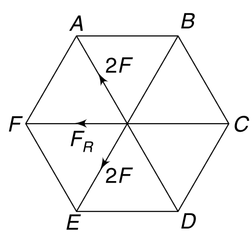
$$\frac{Kq \cdot q}{y^2} = F$$

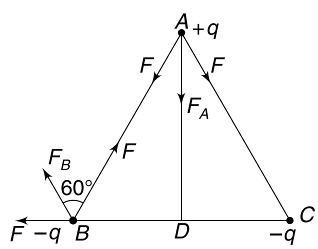
Where y = distance of centre from any vertex.

When charge at centre is $2q$ with no charge at A and E, the force due to charge at C and F cancel out. Force due to charge at B and D each will be

$$\frac{Kq(2q)}{y^2} = 2F$$

$\therefore$ Resultant is $F_R = \sqrt{(2F)^2 + (2F)^2 + 2\cdot(2F)(2F)\cdot\cos 120^\circ} = 2F$ towards point F





(b) Since all charges have same magnitude, the force between any two of them will have equal magnitude. Charge at B and C must attract the charge at A for its acceleration to be along AD.

Hence, charge at B and C must have same sign. They will repeal each other.

Thus charge at B will experience two equal forces at 120° as shown. Resultant is force F_B.

Hence charge at B gets accelerated in a direction making an angle of 60° with BA.

2. The force between two charges is proportional to the product of the charges. $F \propto q_1 q_2$

After touching, the charge on each particle will be $q = \dfrac{q_1 + q_2}{2}$

$$F_{\text{new}} \propto \left(\frac{q_1 + q_2}{2}\right)^2$$

Since arithmetic mean is greater than or equal to geometric mean for two numbers hence

$$\frac{q_1 + q_2}{2} \geq \sqrt{q_1 q_2}$$

$$\left(\frac{q_1 + q_2}{2}\right)^2 \geq q_1 q_2 \Rightarrow F_{new} \geq F$$

3. There are three $-q$ charges which are kept at angular separation of 120° from each other. The resultant of three co planar vectors of equal magnitude at separation of 120° from each other is zero. Hence, these 3 charges produce zero field. Similarly, three number of $+q$ charges, three number of $2q$ and three number of $-3q$ charges produce zero field at centre.

4. q_1 = positive charge equal to the magnitude of the total charge on all electrons present in 0.9 mg of pure water

= charge on electrons present in electrons of $\frac{0.9 \times 10^{-3}}{18} = 5 \times 10^{-5}$ mol water

= charge on $5 \times 10^{-5} \times N_A \times 10$ electrons (since each molecule has 10 electrons)

= charge on 3×10^{20} electrons

= $3 \times 10^{20} \times 1.6 \times 10^{-19}$ C = 48 C

q_2 = charge on a 6.35 mg copper sphere from which 0.1% of its total electrons have been removed.

= charge on 0.1% of electrons on 10^{-4} mol Cu

= charge on $0.001 \times 10^{-4} \times N_A \times 29$ electrons (since each Cu atom has 29 electrons)

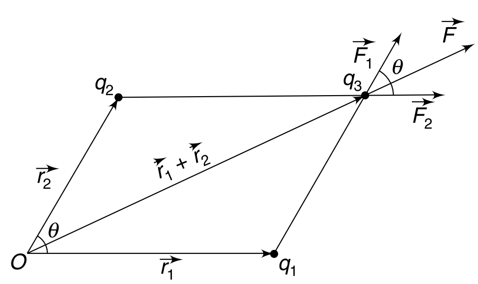
= charge on 1.74×10^{18} electrons

= $1.74 \times 10^{18} \times 1.6 \times 10^{-19}$ C = 0.27 C

Force between q_1 and q_2 if they are placed at a separation of 1 km is

$$F = 9 \times 10^9 \frac{q_1 q_2}{r^2} = 120000 \text{ N}$$

5. The situation is shown in Fig



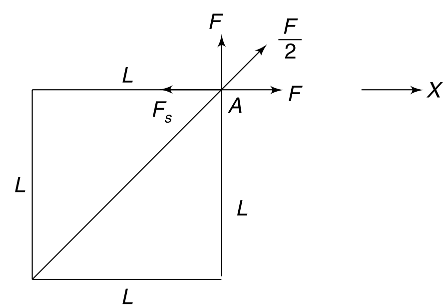
$$|\vec{F}_1| = |\vec{F}_2| = F_0 \text{ (say). Given } F = \sqrt{3}\, F_0$$

$$\sqrt{F_0^2 + F_0^2 + 2F_0^2 \cdot \cos\theta} = \sqrt{3}F_0 \Rightarrow \cos\theta = \frac{1}{2}; \text{ i.e, } \theta = 60°$$

$$\therefore \quad \vec{r}_1 \cdot \vec{r}_2 = r \cdot r \cdot \cos 60° = \frac{r^2}{2}$$

6. Consider the equilibrium of charge at A

F = force due to other charge at distance $L = \frac{kq^2}{L^2}$



Force due to diagonally opposite charge $\frac{kq^2}{(\sqrt{2}L)^2} = \frac{F}{2}$

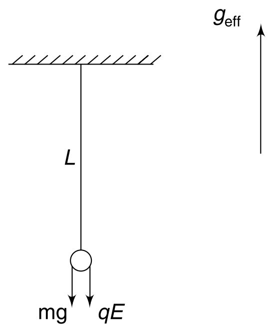
The resultant force in X direction $F_x = F + \frac{F}{2}\cos 45° = F\left(1 + \frac{1}{2\sqrt{2}}\right)$

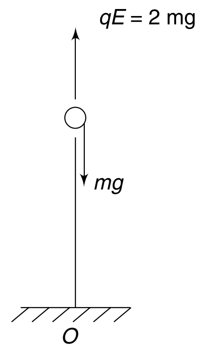
This gets balanced by the spring force. $k \cdot \frac{L}{2} = F\left(1 + \frac{1}{2\sqrt{2}}\right)$

$$kL = \frac{q^2}{4\pi\epsilon_0 L^2}\left(\frac{2\sqrt{2}+1}{\sqrt{2}}\right) \qquad \therefore \quad k = \frac{q^2}{4\pi\epsilon_0 L^3}\left(\frac{2\sqrt{2}+1}{\sqrt{2}}\right)$$

7. From dimensional analysis $\quad F_1 \,\alpha\, \frac{q_1^2}{l_1^2}$ and $F_2 \,\alpha\, \frac{q_2^2}{l_2^2}$

$$\Rightarrow \quad \frac{F_2}{F_1} = \left(\frac{q_2}{q_1}\frac{l_1}{l_2}\right)^2 \quad \Rightarrow \quad F_2 = F_1\left(\frac{q_2^2}{q_1^2}\frac{l_1^2}{l_2^2}\right)$$





8. (i) In this case effective value of g can be taken as

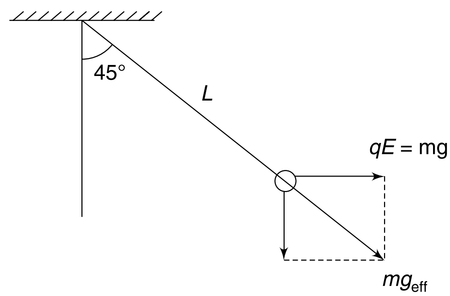
$$g_{eff} = \frac{mg + gE}{m} = 2g$$

$$\therefore \quad T = 2\pi\sqrt{\frac{L}{2g}}$$

(ii) In this case equilibrium position is as shown in second figure. Effective acceleration is vertically upward given by–

$$g_{eff} = \frac{qE - mg}{m} = g$$

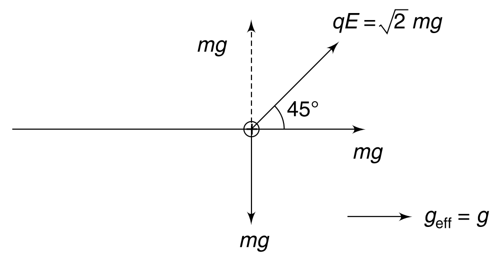
$$T = 2\pi\sqrt{\frac{L}{g}}$$



(iii) Equilibrium position is shown

$$g_{eff} = \sqrt{2}g$$

$$T = 2\pi\sqrt{\frac{L}{\sqrt{2}\,g}}$$



(iv) Equilibrium is shown (thread is horizontal)

$$\therefore \quad T = 2\pi\sqrt{\frac{L}{g}}$$

11. (a) Work done by the electric force on the particle

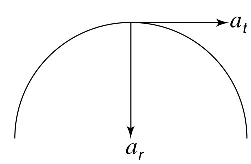
$$W = \int_A^B q\vec{E}\cdot\vec{dl} = qE\cdot\pi R$$

$$\therefore \quad K_B - K_A = qE\pi R$$

$$\therefore \quad K_B = \pi qER \qquad [\because \; K_A = 0]$$

(b) At mid point let the speed of the particle be V.

$$\frac{1}{2}\text{mV}^2 = qE\,\frac{\pi R}{2} \quad \Rightarrow \quad \frac{V^2}{R} = \frac{\pi qE}{m} = a_r$$



This is radial acceleration of the particle. It arises due to constraining forces.

Tangential acceleration $\quad a_t = \dfrac{qE}{m}$

$\therefore$ Resultant acceleration

$$a = \sqrt{a_r^2 + a_t^2} = \frac{qE}{m}\sqrt{1+\pi^2}$$

12.

$$F = qE$$

$$ma_0 = q(-bx + a) \qquad [a_0 = \text{accleration}]$$

$$V\frac{dv}{dx} = -\frac{bq}{m}x + \frac{aq}{m}$$

$$\int_0^V Vdv = -\frac{bq}{m}\int_0^x xdx + \frac{aq}{m}\int_0^x dx$$

$$\frac{V^2}{2} = -\frac{bqx^2}{2m} + \frac{aqx}{m}$$

$$V = 0 \quad \text{when} \quad \frac{bqx^2}{2m} = \frac{aqx}{m} \quad \Rightarrow x = \frac{2a}{b}$$

Now $$a_0 = \frac{q}{m}(-bx + a)$$

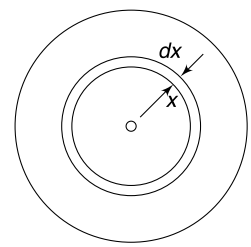
at $$X = \frac{2a}{b}$$

$$\text{acceleration} = \frac{q}{m}(-2a + a) = -\frac{qa}{m}$$

13. (a) Consider an elemental shell of thickness dx as shown. Charge in the shell is $dq = \rho 4\pi x^2 dx = 4\pi\alpha x^3 dx$

$\therefore$ charge in sphere of radius R_0 will be

$$Q_0 = \int dq = 4\pi\alpha \int_0^{R_0} x^3 dx = \pi\alpha R_0^4$$



(b) Charge in sphere of radius R will be $Q = \pi\alpha R^4$

As per question $$Q = \frac{Q_0}{2} \quad \therefore R^4 = \frac{R_0^4}{2} \quad \Rightarrow \quad R = \frac{R_0}{(2)^{1/4}}$$

(c) Field at a distance $r < R$ will depend on the inner charge only. Charge lying outside r will not contribute to the field. Hence, the answer for both cases will be same

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\pi \cdot \alpha r^4}{r^2} = \frac{\alpha r^2}{4\epsilon_0}$$

15. Electric field inside the inner shell is zero. Potential of the inner shell and all points inside it will be constant.

16. $$E = -\frac{dV}{dr} = \frac{25}{r^2}$$

The direction of field is along $\vec{r}$ at any point (why?)

At $$\vec{r} = \hat{i} + 2\hat{j} + 2\sqrt{5}\hat{k}$$

$$r = \sqrt{1^2 + 2^2 + (2\sqrt{5})^2} = 5 \text{ m.}$$

$\therefore$ $$E = \frac{125}{25} = 5 \text{ V/m} \qquad \therefore \vec{E} = 5\left(\frac{\vec{r}}{r}\right) = \hat{i} + 2\hat{j} + 2\sqrt{5}\,\hat{k}$$

17. $$V = \frac{1}{x} + \frac{1}{y} + \frac{2}{z}$$

$$\vec{E} = -\frac{\partial V}{\partial X}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} = \frac{1}{x^2}\hat{i} + \frac{1}{y^2}\hat{j} + \frac{2}{z^2}\hat{k}$$

At (1, 1, 1) m

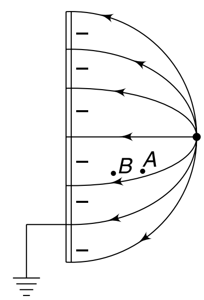
$$\vec{E} = \hat{i} + \hat{j} + 2\hat{k}$$

Resultant force on the particle in XY plane is

$$\vec{F} = q(\hat{i} + \hat{j})$$

$\therefore$ $$\vec{a} = \frac{q}{m}(\hat{i} + \hat{j}) = \frac{10^{-12}\text{C}}{10^{-12}\text{kg}}(\hat{i} + \hat{j}) = (\hat{i} + \hat{j}) \text{ m/s}^2.$$

18. Negative charge gets induced on the metal surface, so as to make its potential zero. Field lines are as shown.



Density of lines is higher at A

$$\therefore \quad E_A > E_B$$

While moving is direction of the field, the potential drops

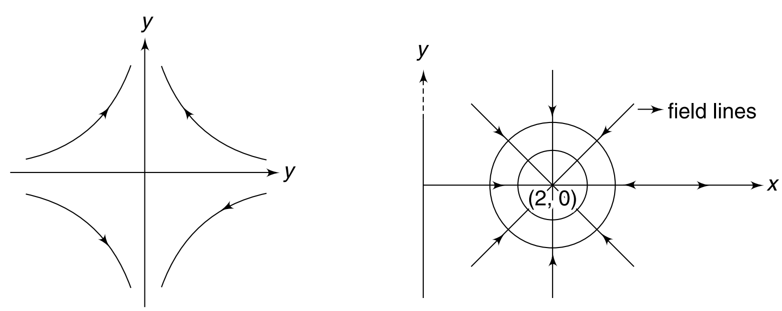
$$\therefore \quad V_A > V_B$$

20. (a)
$$E_x = -\frac{\partial V}{\partial x} = -2x$$
$$E_y = -\frac{\partial V}{\partial y} = 2y$$

(b)
$$V = (x-2)^2 + y^2$$

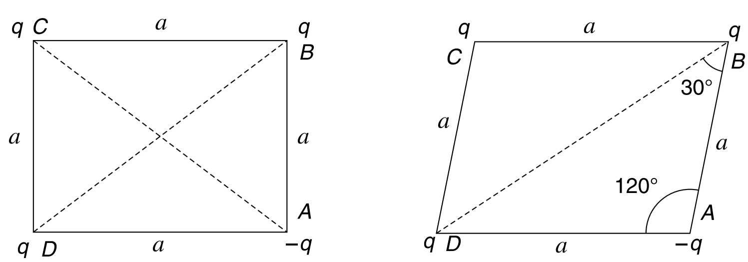
∴ Equipotentials are circles with centre at (2, 0)

∴ Field lines are radial directed towards the centre of the circle.



21. On square: There will be six terms in the expression of potential energy for the system, as there are six pairs of interactions.

U_{AB} and U_{AD} are negative whereas U_{BC} and U_{CD} are positive terms of same magnitude. Similarly U_{BD} and U_{AC} cancel out. Hence PE is zero.



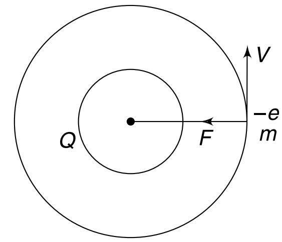
On Rhombus: U_{AB} and U_{AD} cancel out with U_{BC} and U_{CD}

Length $BD = 2a\cos 30° = \sqrt{3}a$

$AC = 2a\cos 60° = a$

$$\therefore \quad U = U_{BD} + U_{AC} = K\frac{q^2}{\sqrt{3}a} - K\frac{q^2}{a} = -\frac{Kq^2}{a}\left(1 - \frac{1}{\sqrt{3}}\right)$$

$$\therefore \quad W_{\text{extAgt}} = U_{\text{Rhombus}} - U_{\text{Square}} = -\frac{Kq^2}{a}\left(1 - \frac{1}{\sqrt{3}}\right)$$



22. (a) $W_{ext} = qV_{ball\,centre} - qV_{shell\ centre}$

$$= q\left[\frac{3}{2}\frac{1}{4\pi\epsilon_0}\frac{Q}{r} - \frac{1}{4\pi\epsilon_0}\frac{Q}{r}\right] = \frac{Qq}{8\pi\epsilon_0 r}$$

(b) $V_{ball\,centre} = \frac{3}{2}\cdot\frac{1}{4\pi\epsilon_0}\frac{Q}{r} + \frac{1}{4\pi\epsilon_0}\frac{Q}{4r} = \frac{7Q}{16\pi\epsilon_0 r}$

$$V_{shell\ centre} = \frac{1}{4\pi\epsilon_0 r}\frac{Q}{r} + \frac{1}{4\pi\epsilon_0}\frac{Q}{4r} = \frac{5Q}{16\pi\epsilon_0 r}$$

$$\therefore \quad W_{ext} = q\left[\frac{7Q}{16\pi\epsilon_0 r} - \frac{5Q}{16\pi\epsilon_0 r}\right] = \frac{Qq}{8\pi\epsilon_0 r}$$

23. (a) Let orbital speed be V. Electrostatic attraction provides the centripetal force.

$$\frac{mV^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{Q(e)}{r^2}$$

$$\therefore \quad V = \sqrt{\frac{Qe}{4\pi\epsilon_0 mr}}$$

$$\therefore \quad E = K + U = \frac{1}{2}\,\mathrm{mV}^2 + \frac{1}{4\pi\epsilon_0}\frac{Q(-e)}{r}$$

$$= \frac{1}{2}\cdot\frac{1}{4\pi\epsilon_0}\frac{Qe}{r} + \frac{1}{4\pi\epsilon_0}\frac{Q(-e)}{r}$$

$$= -\frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{Qe}{r} = -\frac{Qe}{8\pi\epsilon_0}$$

(b) $$T = \frac{2\pi r}{V} \Rightarrow T^2 = \frac{4\pi^2 r^2}{V^2} = \frac{4\pi^2 r^2}{Qe}\cdot 4\pi\epsilon_0\, mr = \frac{16\pi^3\epsilon_0 m}{Qe}\cdot r^3$$

$$\therefore \quad T^2 \propto r^3 \quad \therefore \quad \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$$

$$\left(\frac{T_2}{T}\right)^2 = 4^3 \quad \therefore \quad T_2 = 8T$$

24. The particle will escape if

$$\frac{1}{2}mV_0^2 + K\frac{Q(-q)}{R} = 0$$

$$V_0 = \sqrt{\frac{2KQq}{mR}}$$

$$V_0 \propto \sqrt{q}$$

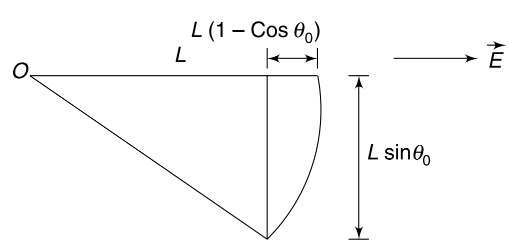
25. (a) let the bob come to rest in the position shown. Using conservation of energy we have

Loss in gravitational PE = Gain in electrostatic PE [$\because$ change in kE = 0]

$$\therefore \quad mg\,L\sin\theta_0 = qEL(1 - \cos\theta_0)$$

$$mg\left(2\sin\frac{\theta_0}{2}\cos\frac{\theta_0}{2}\right) = qE\,2\cdot\sin^2\frac{\theta_0}{2}$$

$$\Rightarrow \quad \tan\frac{\theta_0}{2} = \frac{qE}{mg} \Rightarrow \theta_0 = 2\tan^{-1}\left(\frac{qE}{mg}\right)$$

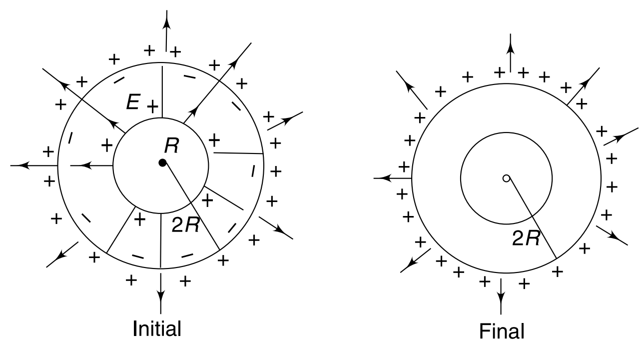


(b) For $\theta_0 = \pi/2$

$$\tan\frac{\pi}{4} = \frac{qE}{mg}$$

$$E = \frac{mg}{q}$$

26. The electric field in initial and final configuration is as shown below:



In initial configuration there is electric field in all region $r > R$. The field varies as $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$. In final configuration there is no field in the region $R < r < 2R$. Elsewhere the field is similar to that in initial configuration. Thus initial configuration has more energy stored in electric field given by

$$\Delta E = \int_R^{2R} \frac{1}{2}\epsilon_0 E^2 dV = \frac{1}{2}\epsilon_0 \int_R^{2R} \left(\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\right)^2 4\pi r^2 dr$$

$$= \frac{Q^2}{8\pi\epsilon_0}\int_R^{2R}\frac{dr}{r^2} = -\frac{Q^2}{8\pi\epsilon_0}\left[\frac{1}{2R} - \frac{1}{R}\right] = \frac{Q^2}{16\pi\epsilon_0 R}$$

$$W_{\text{ext}} = U_f - U_i = -\frac{Q^2}{16\pi\epsilon_0 R}$$

27. (a)

$$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \frac{K^2Q^2}{x^4}$$

$$V = K\frac{Q}{x}$$

$$\therefore \quad \frac{u_E}{V^2} = \frac{\frac{1}{2}\epsilon_0 \frac{K^2Q^2}{x^4}}{\frac{K^2Q^2}{x^2}} = \frac{\epsilon_0}{2x^2}$$

Given $\quad \frac{\epsilon_0}{2x^2} = \eta$

At a distance $2x$, obviously the value of this ratio will be $\frac{\eta}{4}$

(b)

$$[\eta] = \frac{[\epsilon_0]}{[L^2]} = \frac{[M^{-1}L^{-3}T^4A^2]}{[L^2]} = [M^{-1}L^{-5}T^4A^2]$$

28. (a)

$$\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\times 8.8\times 10^{-12}\times(3\times 10^6)^2 \text{ J/m}^3$$

$$= 39.6 \text{ J/m}^3 = 39.6\ \mu\text{J/cm}^3$$

(b) Energy consumed to charge the ball = Electrostatic self energy of the ball

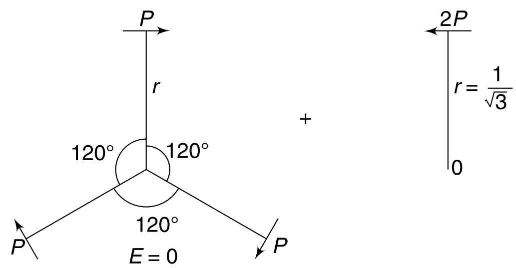
$$= \frac{Q^2}{8\pi\epsilon_0 \epsilon_r R} = \frac{4 \times 10^{-18}}{2 \times 80 \times 0.1} \times 9 \times 10^9 = 2.25 \times 10^{-9}\text{J}$$

29. System is equivalent to the superposition of two systems shown in figure:

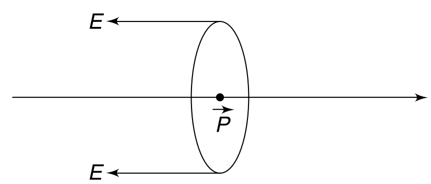
$$E = K\frac{2P}{r^3} \text{ Opposite to } 2\vec{P}$$

$$\therefore \qquad E = 2\frac{KP}{r^3} = \frac{2KP}{\left(\frac{L}{\sqrt{3}}\right)^3} = 6\sqrt{3}\,\frac{KP}{L^3}$$

Potential = 0



30. Each point on the circumference of the ring lies on the perpendicular bisector of the dipole. Field due to dipole at these points is $E = \frac{1}{4\pi\epsilon_0}\frac{P}{R^3}$ directed opposite to $\vec{P}$.



$\therefore$ Force on ring

$$F = QE$$

$$= \frac{1}{4\pi\epsilon_0}\frac{PQ}{R^3}$$

(directed opposite to $\vec{P}$)

From Newton's third law, the ring will exert equal and opposite force on the dipole.

31. A charge $+dq$ on the ring and a charge $-dq$ at O forms a dipole having dipole moment

$$dp = Rdq \qquad \text{(direction as shown in figure)}$$

Summing up all such dipole moment will give the net dipole moment of the system. From symmetry the resultant of all such $\overrightarrow{dp}$ vectors will be along X direction.

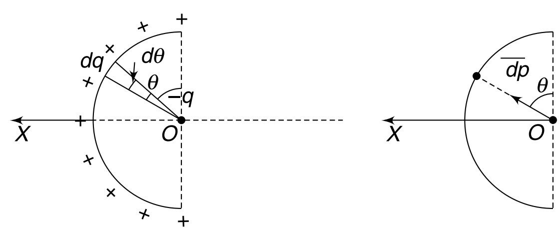
$$\therefore \qquad \vec{P} = \int_{0=0}^{\pi} dp \cdot \sin\theta = R\int_0^{\pi} dq \sin\theta$$

But

$$dq = \frac{q}{\pi R}\cdot Rd\theta = \frac{qd\theta}{\pi}$$

$$\therefore \qquad \vec{P} = \frac{Rq}{\pi}\int_0^{\pi}\sin\theta\, d\theta = -\frac{Rq}{\pi}[\cos\theta]_0^{\pi}$$

$$= -\frac{Rq}{\pi}[-1-1] = 2\frac{Rq}{\pi} \text{ along } OX$$

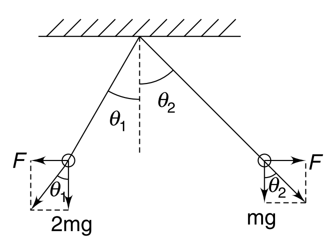


32. F = Electrostatic force.

For equilibrium

$$\tan\theta_1 = \frac{F}{2mg} \quad \text{and} \quad \tan\theta_2 = \frac{F}{mg}$$

$$\tan(\theta_1 + \theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$$



$$\tan 45^\circ = \frac{\dfrac{F}{2\,mg} + \dfrac{F}{mg}}{1 - \dfrac{F}{2\,mg}\cdot\dfrac{F}{mg}}$$

$\Rightarrow$ $$2m^2g^2 - F^2 = 3mgF \Rightarrow F^2 + 3mgF - 2m^2g^2 = 0$$

$\therefore$ $$F = \frac{-3\,mg \pm \sqrt{9m^2g^2 + 8m^2g^2}}{2}$$

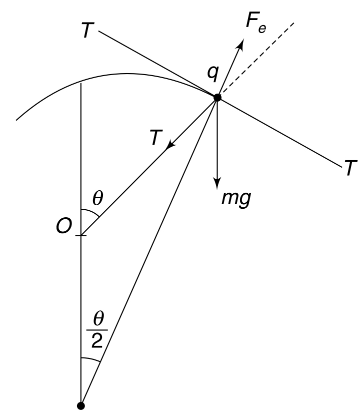
Negative sign of F is not acceptable

$\therefore$ $$F = \left(\frac{\sqrt{17} - 3}{2}\right)\text{mg}$$

33. In equilibrium $\dfrac{KqQ}{(2L)^2} > mg \Rightarrow Q > \dfrac{4L^2 mg}{Kq}$

Now consider the particle in a slightly displaced position

Equilibrium will be stable if tangential component (along TT) of electrostatic repulsion is greater than the tangential component of mg.



$\Rightarrow$ $$F_e \sin\left(\frac{\theta}{2}\right) > mg\sin\theta$$

For small displacement, $\sin\dfrac{\theta}{2} \simeq \dfrac{\theta}{2}$

$\therefore$ $$K\frac{qQ}{(2L)^2}\cdot\frac{\theta}{2} > mg\theta$$

$\therefore$ $$Q > \frac{8L^2 mg}{Kq} = \frac{32\pi\varepsilon_0 L^2 mg}{q}$$

35. For charges to remain at rest, tangential force on each of them must be zero.

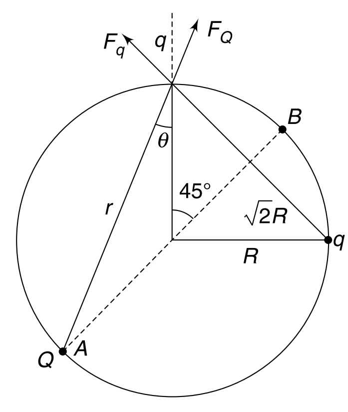
(a) Due to symmetry, the third charge has to be located at A or B.

When it is positive its location is A. For tangential force to be zero on one of the equal charges-

$$F_q\cos 45^\circ = F_Q\cdot\sin\theta$$

$$\theta = \frac{180 - 135}{2} = 22.5^\circ$$

And $$r = 2R\cos\theta$$



$\therefore$ $$\frac{qq}{(\sqrt{2}R)^2}\cdot\frac{1}{\sqrt{2}} = \frac{qQ}{(2R\cos\theta)^2}\cdot\sin\theta$$

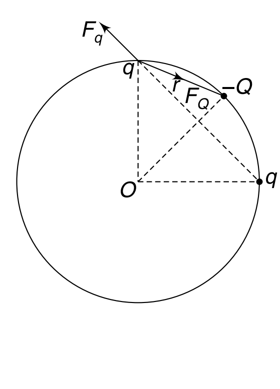
$\therefore$ $$\frac{Q}{q} = \sqrt{2}\,\frac{\cos^2\theta}{\sin\theta}$$

$$Q = \left[1.41 \times \frac{(0.92)^2}{0.38}\right]q = (3.14)\,q$$

(b) $$r = 2R\sin(22.5^\circ)$$

$$\frac{F_q}{(\sqrt{2}R)^2}\cdot\frac{1}{\sqrt{2}} = F_Q\cdot\sin(90 - 22.5^\circ)$$

$$\frac{qq}{2\sqrt{2}R^2} = \frac{qQ}{(2R\sin 22.5^\circ)^2}\cos(22.5^\circ)$$



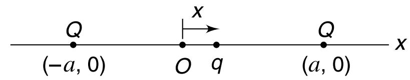
$$Q = \sqrt{2}\,\frac{\sin^2(22.5^\circ)}{\cos\,(22.5^\circ)}q$$

$$= \frac{1.41 \times (0.38)^2}{(0.92)}q$$

$$= 0.22q$$

36. SHM along x

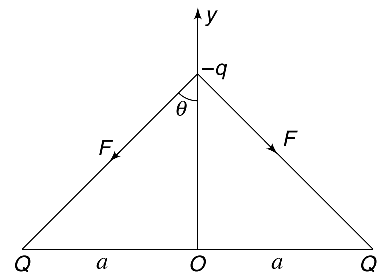
Net force acting on q when it is displaced by x is



$$F = \frac{KQq}{(a+x)^2} - \frac{KQq}{(a-x)^2}$$

$$F = -\frac{4KQqax}{(a^2-x^2)^2} \simeq -\frac{4KQqax}{a^4} \qquad (x << a)$$

$$= -\frac{4KQq}{a^3}x$$



$$\therefore \quad \text{acceleration} = -\frac{4KQq}{ma^3}x$$

$$\therefore \quad T_1 = 2\pi\sqrt{\frac{ma^3}{4KQq}}$$

SHM along y

Resultant force when $-q$ is displaced by y is

$$F_R = -2F\cos\theta = -2\frac{KQq}{(a^2+y^2)}\cdot\frac{y}{\sqrt{a^2+y^2}} = -\frac{2KQqy}{(a^2+y^2)^{3/2}} \simeq -\frac{2KQqy}{a^3}$$

$$\therefore \quad \text{acceleration} = -\frac{2KQq}{ma^3}y$$

$$\therefore \quad T_2 = 2\pi\sqrt{\frac{ma^3}{2KQq}} \qquad \therefore \quad \frac{T_1}{T_2} = \frac{1}{\sqrt{2}}$$

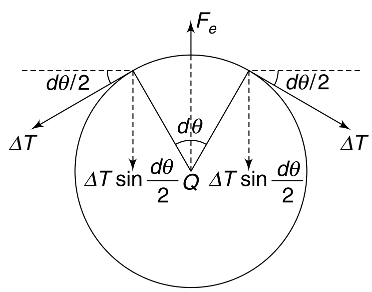
37. With no charge at the centre, there is some tension in the ring due to repulsion of charge present on the ring.

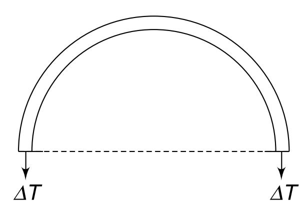
(a) When Q is placed at the centre the tension increases by ΔT. Consider an infinitesimally small element on the ring having angular width $d\theta$ as shown.

$$2\Delta T\sin\left(\frac{d\theta}{2}\right) = F_e$$

$$2\Delta T\cdot\frac{d\theta}{2} = \frac{KQ\lambda R d\theta}{R^2}$$

$$\Delta T = \frac{KQ\lambda}{R} = \frac{KQq}{2\pi R^2}$$





(b) Clearly the answer is $2\Delta T = \dfrac{KQq}{\pi R^2}$

(c) Answer to (b) must be the answer to (c) also.

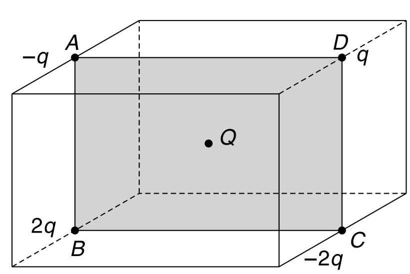
38. Consider 4 charges on the square $ABCD$. Force due to these 4 charges

$$F_1 = 2\sqrt{2}\frac{KQq}{r^2} \text{ parallel to } BC$$

Similarly, force due to 4 charges on square $EFGH$ is

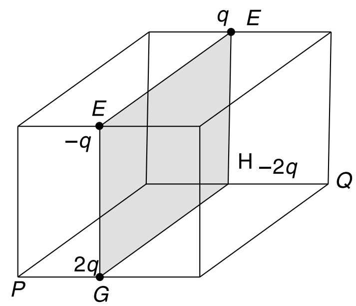
$$F_2 = 2\sqrt{2}\frac{KQq}{r^2} \text{ parallel to } GH$$

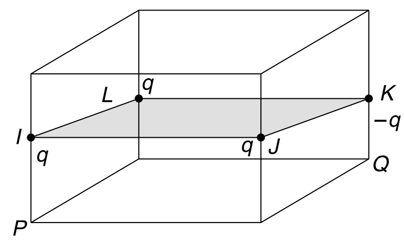
Resultant of $\vec{F}_1$ and $\vec{F}_2$ will be $\dfrac{4KQq}{r^2}$ parallel to PQ.



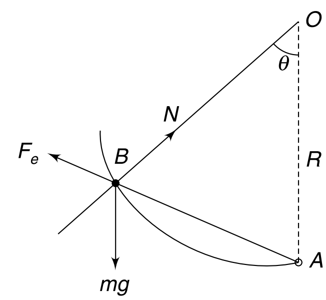
Force due to 4 charges at I, J, K, L is also parallel to PQ and has magnitude $F = \dfrac{4KQq}{r^2}$

Hence, resultant is $\dfrac{8KQq}{r^2}$ along PQ





39 Let charge on both A and B be q



$$AB = 2R\sin\left(\frac{\theta}{2}\right)$$

$$F_e = \frac{Kq^2}{4R^2\sin^2\left(\frac{\theta}{2}\right)}$$

For net tangential force to be zero

$$mg\sin\theta = F_e\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$mg\sin\theta = \frac{Kq^2}{4R^2\sin^2\left(\frac{\theta}{2}\right)}\cdot\cos\left(\frac{\theta}{2}\right)$$

$$32\pi\varepsilon_0\, mgR^2\sin^3\left(\frac{\theta}{2}\right) = q^2 \qquad \left[\because\ K = \frac{1}{4\pi\varepsilon_0}\right]$$

$$\therefore \qquad q = 4R\sin\left(\frac{\theta}{2}\right)\sqrt{2\pi\varepsilon_0\, mg\sin\left(\frac{\theta}{2}\right)}$$

$$\text{Total charge given} = 2q$$

$$Q = 8R\sin\left(\frac{\theta}{2}\right)\sqrt{2\pi\varepsilon_0\, mg\sin\left(\frac{\theta}{2}\right)}$$

40. In equilibrium

$$\frac{KQq}{x_0^2} = mg\sin\theta \qquad \text{...(1)}$$

If the charge is displaced by $x(<< x_0)$

$$ma = \frac{KQq}{(x_0 + x)^2} - mg\sin\theta$$

$$= \frac{KQq}{x_0^2\left(1 + \frac{x}{x_0}\right)^2} - mg\sin\theta$$

$$= \frac{KQq}{x_0^2}\left(1 + \frac{x}{x_0}\right)^{-2} - mg\sin\theta$$

Using binomial expansion and neglecting higher order terms-

$$ma = \frac{KQq}{x_0^2}\left[1 - \frac{2x}{x_0}\right] - mg\sin\theta$$

$$ma = -\frac{2KQq}{x_0^2}\cdot x \qquad \text{[using 1]}$$

$$ma = -\left(2\frac{mg\sin\theta}{x_0}\right)\cdot x \qquad \text{[again using 1]}$$

$$\therefore \quad a = -\left(\frac{2g\sin\theta}{x_0}\right)x \qquad \therefore \quad \omega = \sqrt{\frac{2g\sin\theta}{x_0}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{2g\sin\theta}{x_0}}$$

41. (a) Field due to two $+Q$ charges has a maxima at a distance $y_0 = \frac{a}{\sqrt{2}}$ on the line OY. Equilibrium will be stable only if charged particle is placed above $y_0 = \frac{a}{\sqrt{2}}$ (i.e., at B). Field due to two $+Q$ charges at a distance y on the line OY is-

$$E = \frac{2KQy}{(a^2 + y^2)^{3/2}}$$

At y and $3\sqrt{3}y$ the electric force balances mg.

$$\therefore \quad qE_y = qE_{3\sqrt{3}y} = mg$$

$$\therefore \quad \frac{q\cdot 2KQy}{(a^2+y^2)^{3/2}} = \frac{q\cdot 2KQ\, 3\sqrt{3}y}{[a^2 + (3\sqrt{3}y)^2]^{3/2}} \Rightarrow a^2 + 27y^2 = 3(a^2 + y^2) \Rightarrow 24y^2 = 2a^2$$

$$y = \frac{a}{\sqrt{12}} \qquad \therefore \quad y_B = 3\sqrt{3}y = \frac{3}{2}a$$

(b) Maximum electric force on the particle is

$$F_0 = \frac{2KQq\frac{a}{\sqrt{2}}}{\left(a^2 + \frac{a^2}{2}\right)^{3/2}} = \frac{4KQq}{3\sqrt{3}a^2} = 0.76\frac{KQq}{a^2} \qquad \text{...(i)}$$

Weight of the particle is equal to the electric force at $y = \frac{a}{\sqrt{12}}$

$$mg = \frac{2KQq\frac{a}{\sqrt{12}}}{\left(a^2 + \frac{a^2}{12}\right)^{3/2}} = \frac{24KQq}{13\sqrt{13}a^2} = 0.51\frac{KQq}{a^2} \qquad \text{...(ii)}$$

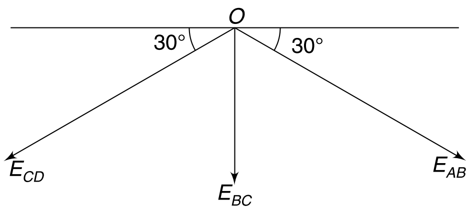
Net force will be maximum at O where electric force is zero and mg remains unbalanced. Particle will have maximum acceleration at O.

44. Electrostatic field is conservative. Field lines cannot form closed loops otherwise the field will perform a net work if a charge is moved in a closed path along the field line. But a conservative force must perform zero work in a closed path. For an electrostatic field $\oint \vec{E}\cdot\vec{dl} = 0$

In Fig. (b) and (c) you can imagine a rectangular path along which $\oint \vec{E}\cdot\vec{dl} \neq 0$

Hence none of the field configuration is possible

45. Field at O due to three segments will have same magnitude (say E) and will have directions as shown.



$$E = |E_{AB}| = |E_{BC}| = |E_{CD}|$$

$\therefore$ $$E_0 = E_{BC} + E_{AB}\sin 30° + E_{CD}\sin 30°$$

$\Rightarrow$ $$E_0 = E + \frac{E}{2} + \frac{E}{2} \qquad \therefore \quad E = \frac{E_0}{2}$$

46. Assume that two perpendicular components of field at P due to the hemisphere are-

E_1 towards centre (say towards right in given Fig.) and E_2 perpendicular to the circular equatorial plane (downward in given Fig.).

Now imagine another hemisphere so as to complete a sphere. This sphere will have its own field at P whose components will be-

E_1 towards centre (towards right in given Fig.) and E_2 perpendicular to the circular equatorial plane(up in given Fig.). Since field inside uniformly charged sphere is zero hence E_1 must be zero.

47. *Hint:* If you consider the complete sphere, field at P as well as Q will be zero.

$\Rightarrow$ $$\vec{E}^{P}_{\text{Hemisphere 1}} + \vec{E}^{P}_{\text{Hemisphere 2}} = 0$$

48. If $r << x$, charge on the removed disc is like a point charge equal to $q = \pi r^2 \cdot \sigma$

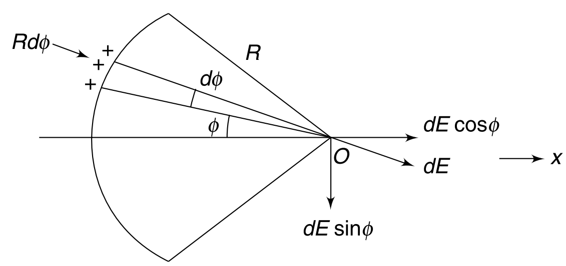
$\therefore$ $$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{\pi r^2\sigma}{x^2} = \frac{\sigma}{4\varepsilon_0}\frac{r^2}{x^2}$$

49. (a) Consider an element of angular width $d\phi$.

Width of element = $Rd\phi$

Charge on element = $\lambda R d\phi$

Field at O due to this element



$$dE = \frac{K\lambda R d\phi}{R^2}$$

Due to symmetry field is along x only. We need to add all $dE\cos\phi$ components.

$$E = \int_{-\theta/2}^{\theta/2} \frac{K\lambda}{R}\cos\phi\, d\phi = \frac{2K\lambda}{R}\sin\left(\frac{\theta}{2}\right)$$

(b) For semicircular thread $\theta = 180°$

$\therefore$ $$E = \frac{2K\lambda}{R}$$

(c) As $\theta \to 0$

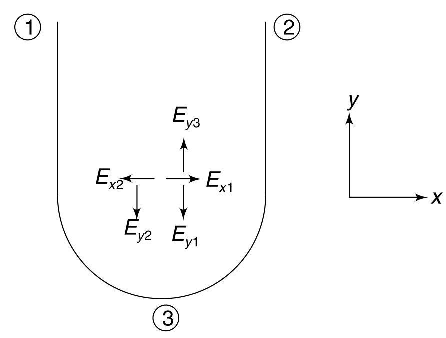
$$\sin\left(\frac{\theta}{2}\right) \to 0 \qquad \therefore \quad E = 0$$

(d) $\lambda = \dfrac{Q}{R\theta}$

$\therefore$ $$E = \frac{2KQ}{R^2}\frac{\sin\left(\frac{\theta}{2}\right)}{\theta} = \frac{KQ}{R^2}\left(\frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}}\right)$$

When $\theta \to 0$, $\dfrac{\sin\frac{\theta}{2}}{\frac{\theta}{2}} \to 1 \qquad \therefore \quad E = \dfrac{KQ}{R^2}$

51. The x and y components of field due to the three components 1, 2 and 3 have been shown.



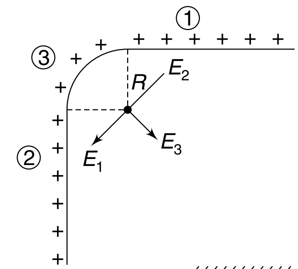
E_{x1} and E_{x2} cancel out.

$$E_{y1} = E_{y2} = \frac{K\lambda}{R}$$

$$E_{y3} = \frac{2K\lambda}{R}$$

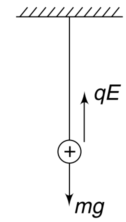
Hence, resultant in y direction in also zero.

52. Resultant is E_3.



$$E_3 = \frac{2K\left(\lambda\,\frac{\pi R}{2}\right)\sin\left(\frac{\pi}{4}\right)}{\frac{\pi}{2}R^2} = \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 R}$$

54. (a)

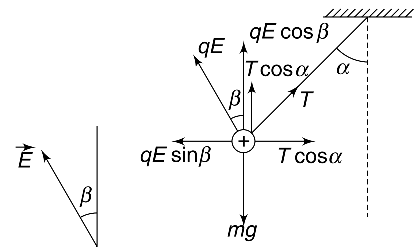


$$g_{\text{eff}} = \frac{mg - qE}{m}$$

$$= g - \frac{q\sigma}{2m\varepsilon_0}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{L}{g - \frac{q\sigma}{2m\varepsilon_0}}}$$

(b) In this case the electric field makes an angle β with vertical. For equilibrium



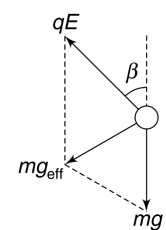
$$T\cos\alpha + qE\cos\beta = mg$$

$$\Rightarrow \quad T\cos\alpha = mg - qE\cos\beta \qquad \ldots(1)$$

$$\text{And} \quad T\sin\alpha = qE\sin\beta \qquad \ldots(2)$$

From (1) and (2)

$$\tan\alpha = \frac{qE\sin\beta}{mg - qE\cos\beta} \quad \text{where} \quad E = \frac{\sigma}{2\varepsilon_0}$$

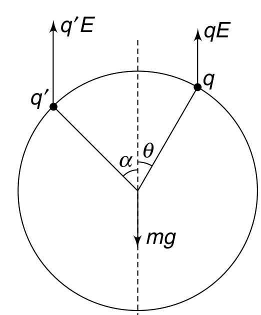


$$g_{\text{eff}}^2 = g^2 + \left(\frac{qE}{m}\right)^2 + 2g\,\frac{qE}{m}\cdot\cos(\pi - \beta)$$

$$\therefore \quad T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2 - 2g\,\frac{qE}{m}\cos\beta}}}$$

55. (a) From geometry of the figure $\alpha + \theta = 90°$

Translational equilibrium



$$q'E + qE = mg$$

$$q'E + \frac{4mg}{7} = mg \quad \left[\because \; E = \frac{4mg}{7q}\right]$$

$$q' = \frac{3mg}{7E} \qquad \ldots(1)$$

Rotational equilibrium

$$qE\,R\sin\theta = q'ER\sin\alpha$$

$$\frac{4}{7}\sin\theta = \frac{3}{7}\cos\theta \quad [\because \alpha = 90 - \theta]$$

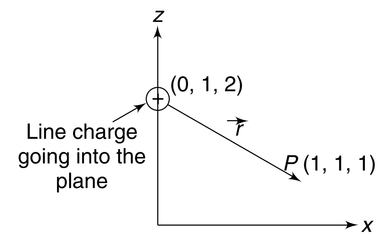
$$\therefore \quad \tan\theta = \frac{3}{4}$$

(b) Suppose the ring is given a small clockwise rotation.

This will cause the anticlockwise torque due to qE to increase whereas the clockwise torque due to $q'E$ will decrease. There will be a net restoring torque.

Hence, there will be oscillations.

56. Foot of the perpendicular from P on the line charge has co-ordinates (0, 1, 2).



$$\therefore \quad \vec{r} = 1\hat{i} + 0\hat{j} - 1\hat{k}$$

$$\hat{r} = \frac{1}{\sqrt{2}}(\hat{i} - \hat{k})$$

$\therefore$ Field at P

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{1}{2\pi\varepsilon_0\sqrt{2}}\frac{1}{\sqrt{2}}(\hat{i} - \hat{k})$$

$$\therefore \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}(\hat{i} - \hat{k})$$

57. (a) The flux through a spherical surface of radius less than that of the inner sphere is equal to $\frac{q}{\varepsilon_0}$. Where q is the charge on the central particle.

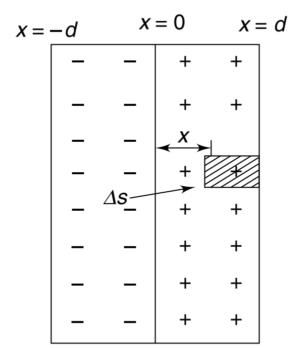
$$\frac{q}{\epsilon_0} = 2 \times 10^6 \text{ V-m}$$

$$\Rightarrow \quad q = 8.85 \times 10^{-12} \times 2 \times 10^6 = 17.7\ \mu\text{C}$$

(b) The flux becomes negative after radius becomes greater than the radius of the inner sphere. It means there is a negative charge on the inner sphere which is larger inmagnitude than the charge on the central particle. The outer shell has still larger positive charge. One can figure out that the electric field exists everywhere.

58. The field on the surface of the composite slab will be zero.

Consider a cylindrical Gaussian surface of cross sectional area ΔS as shown in the figure. One of the circular face of the cylinder is at a distance x from the central plane and the other face is on the surface of the slab.



Let the field at distance x from central plane be E.

From Gauss' Law

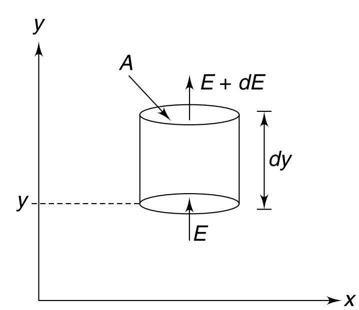
$$E\Delta S = \rho\Delta S(d - x)/\varepsilon_0$$

$$\Rightarrow \quad E = \rho(d - x)/\varepsilon_0$$

The direction will be towards left.

Similarly, one can write field in the region to the left of $x = 0$

59. Consider a cylindrical Gaussian surface as shown.



$$(E + dE)A - EA = \frac{\rho A dy}{\varepsilon_0}$$

$$dE = \frac{by\,dy}{\varepsilon_0}$$

$$\therefore \quad \int_0^E dE = \frac{b}{\varepsilon_0}\int_0^y y\,dy$$

$$E = \frac{by^2}{2\varepsilon_0}$$

60. Field on the central plane of the sheet = 0

Take a cylindrical Gaussian surface as shown.

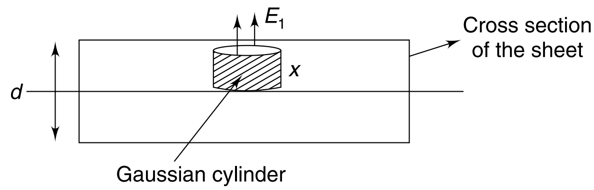
$$E_1\Delta S = \frac{\rho \cdot \Delta S \cdot x}{\varepsilon_0}$$

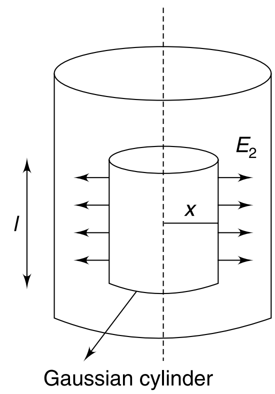
$$E_1 = \frac{\rho x}{\varepsilon_0}$$

By rolling the sheet we have just created a cylinder having uniform volume charge density ρ. Field inside such cylinder can be calculated by considering a co-axial cylindrical Gaussian surface.

$$E_2 \cdot 2\pi x \cdot L = \frac{\rho \pi x^2 L}{\varepsilon_0}$$

$$E_2 = \frac{\rho x}{2\varepsilon_0} \qquad \therefore \quad \frac{E_1}{E_2} = \frac{2}{1}$$





61. Consider a Gaussian surface in the shape of a sphere centered at the origin.
Radius of the sphere is r.
Flux through the sphere

$$\phi = E \cdot 4\pi r^2 = 4\pi a e^{-\frac{r}{k}}$$

Charge inside the Gaussian sphere

$$q = \varepsilon_0 \phi = 4\pi\varepsilon_0 a e^{-\frac{r}{k}}$$

(a) When $r \to 0$

$q \to 4\pi\varepsilon_0 a$

Hence, charge at the origin is $4\pi\varepsilon_0 a$

(b) When $r \to \infty$

$q \to 0$

$\therefore$ Total charge in entire space is zero.

$\therefore$ Charge spread around q_0 is $q = -q_0$.

64. $\phi = \int \vec{E} \cdot \vec{d}s$

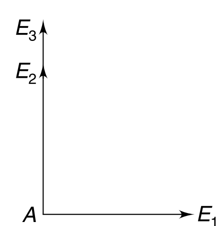
Force $F = \int \sigma \vec{E} \cdot \vec{d}s = \sigma\phi$

65. (a) Field at A:

Due to 'ρ' is $E_1 = \dfrac{\rho}{3\varepsilon_0}\left(\dfrac{R}{2}\right)$ parallel to C_1C_2

Due to '2ρ' is $E_2 = \dfrac{2\rho}{3\varepsilon_0}\left(\dfrac{R}{4}\right)$ parallel to $C_2C_3 = \dfrac{\rho}{3\varepsilon_0}\dfrac{R}{2}$

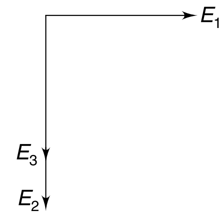
Due to '3ρ' is $E_3 = \dfrac{4\rho}{3\varepsilon_0} \cdot \dfrac{R}{8}$ parallel to $C_3A = \dfrac{\rho}{3\varepsilon_0}\dfrac{R}{2}$



$\therefore$ $E_A = \sqrt{E_1^2 + (E_2 + E_3)^2} = \dfrac{\sqrt{5}}{6}\dfrac{\rho R}{\varepsilon_0}$

(b) Field at B

Due to 'ρ' is $E_1 = \dfrac{\rho}{3\varepsilon_0}\dfrac{R}{2}$ parallel to C_1C_2



Due to '2ρ' in entire sphere S_2 is

$$E_2 = \frac{2\rho}{3\varepsilon_0}\cdot\frac{R}{4} \text{ in direction } C_2B$$

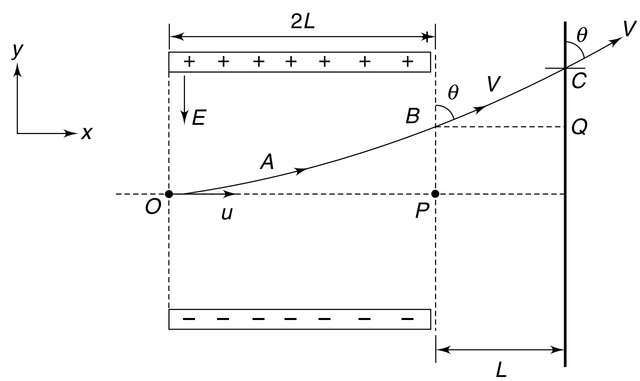
Due to '2ρ' in S_3 is

$$E_3 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi\left(\frac{R}{4}\right)^3\cdot 2\rho}{\left(\frac{R}{2}\right)^2} \text{ in direction } C_3B$$

$$= \frac{\rho}{3\varepsilon_0}\frac{R}{8}$$

$\therefore$ $$E_B = \sqrt{E_1^2 + (E_2 + E_3)^2}$$

$$= \frac{\sqrt{41}}{24}\frac{\rho R}{\varepsilon_0}$$



66. (a) Acceleration of the electron

$$a_y = \frac{eE}{m} = \frac{e\sigma}{m\varepsilon_0} (\uparrow)$$

Path of the electron from O to A to B (i.e. between the plates) is parabolic.

The electron reaches O to B in time $t_1 = \frac{2L}{u}$

At B $$V_x = u$$

$$V_y = 0 + a_y t_1 = \frac{e\sigma}{m\varepsilon_0}\frac{2L}{u}$$

Also, $$PB = \frac{1}{2}a_y t_1^2 = \frac{1}{2}\frac{e\sigma}{m\varepsilon_0}\left(\frac{2L}{u}\right)^2$$

$$= 2\frac{e\sigma}{m\varepsilon_0}\frac{L^2}{u^2}$$

Time to travel from B to C

$$t_2 = \frac{L}{u}$$

V_y does not change after the electron crosses the plates

$\therefore$ $$QC = V_y t_2 = \frac{e\sigma}{m\varepsilon_0}\frac{2L}{u}\frac{L}{u} = \frac{2e\sigma}{m\varepsilon_0}\frac{L^2}{u^2}$$

According to the problem

$$PB + QC = \frac{d}{2} \quad \therefore \quad \frac{4e\sigma}{m\varepsilon_0}\frac{L^2}{u^2} = \frac{d}{2}$$

$\therefore$ $$\sigma = \frac{md\varepsilon_0 u^2}{8eL^2}$$

(ii) $$\tan\theta = \frac{V_x}{V_y}$$

$$= \frac{u(m\varepsilon_0 u)}{2Le\sigma} = \frac{\varepsilon_0 mu^2}{2Le\sigma}$$

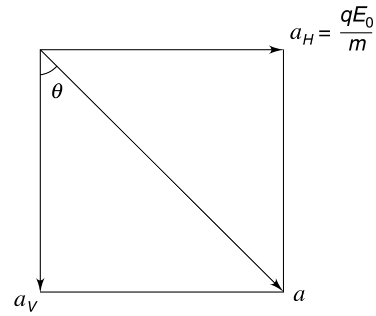
67. Acceleration of the particle

Vertical acceleration $a_v = g$

Horizontal acceleration $a_H = \frac{qE_0}{m} = \frac{4gE_0}{3E_0} = \frac{4g}{3}$

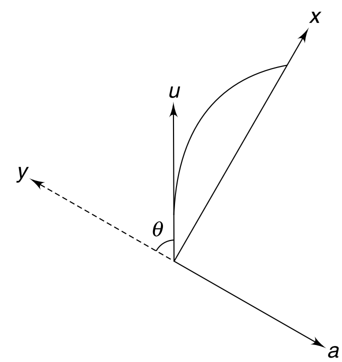
$$a = \sqrt{g^2 + \left(\frac{4g}{3}\right)^2} = \frac{5}{3}\text{ g}$$

$$\tan\theta = \frac{4}{3}$$



Consider, x as direction perpendicular to acceleration and y as direction opposite to a.

Now, the situation is like projectile motion.



(a) $t = \frac{u_y}{a} = \frac{40\cos\theta}{\frac{5g}{3}} = \frac{40 \times 3 \times 3}{5 \times 10 \times 5} = \frac{36}{25}$ s

(b) $2t = \frac{72}{25}$ s

(c) Velocity will get almost parallel to acceleration.

$\therefore \quad \alpha = \tan^{-1}\left(\frac{3}{4}\right)$

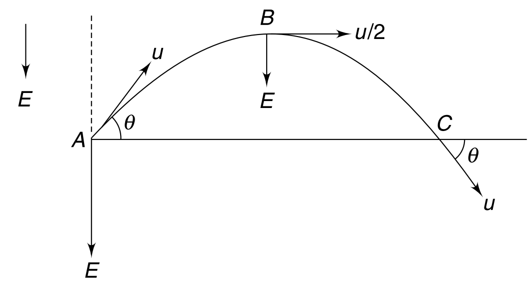
68. (a) Situation is shown in figure

Particle is projected at A. At time t_0 it is at B moving $\perp_r$ to the direction of the field.

At B speed $= \frac{u}{2} \Rightarrow \theta = 60°$

Particle will acquire its initial speed when at C.

$\therefore$ Answer to (a) is t_0



(b) Impulse $= \Delta P = 2mu\sin\theta$

$$= 2\sqrt{2mK_0}\cdot\sin 60° = \sqrt{6mK_0}$$

69. Acceleration $a_y = \frac{qE}{m} = \frac{qay^2}{m}$

$$V_y\frac{dV_y}{dy} = \frac{qay^2}{m}$$

$$\int_0^{V_y} V_y dV_y = \frac{qa}{m}\int_{y_0}^{y} y^2 dy$$

$$\frac{V_y^2}{2} = \frac{qa}{3m}(y^3 - y_0^3) \qquad \therefore \quad V_y = \sqrt{\frac{2qa}{3m}(y^3 - y_0^3)}$$

$$\text{Required slope} = \frac{V_y}{V_x} = \frac{V_y}{V_0} = \sqrt{\frac{2qa}{3mV_0^2}(y^3 - y_0^2)}$$

71. One easy way of thinking can be to select one identical element on part OA and OB each. Point P is on perpendicular bisector of the line joining this pair of charges. It is easy to see that field due to such a pair is parallel to AB. All such pairs contribute in same direction. Hence, resultant field is parallel to AB. The pair of charges selected above will produce zero potential at P.

72. (a) Very close to a positive charge the potential will be positive irrespective of other charge in the surrounding. Similarly, potential will be negative at a point very close to a negative charge. Hence, q_1 is positive and q_2 is negative.

Since $V = 0$ at a point that is close to q_2 hence q_2 has smaller magnitude than q_1.

(b) According to the question

$V_{\text{origin}} = 0$

$$K\frac{q_2}{d/4} + K\frac{q_1}{3d/4} = 0 \Rightarrow q_1 = -3q_2$$

If $\quad q_2 = -q;\quad$ then $\quad q_1 = +3q$

At P $\quad \frac{dV}{dx} = 0 \Rightarrow E_p = 0$

Let distance of P from q_2 be x.

$$K\frac{|q_2|}{x^2} = K\frac{|q_1|}{(d+x)^2} \Rightarrow (d+x)^2 = 3x^2$$

$$d + x = \pm\sqrt{3}x$$

$$\Rightarrow \quad \frac{d}{\sqrt{3}-1} = x$$ [–ve sign is not acceptable]

73. (a) Let x, y and z direction be as shown.

$$V_A - V_0 = -E_x(1\,\text{m})$$

$$\therefore \quad E_x = 1\,\text{V/m} \quad ...(1)$$

$$V_B - V_0 = -E_x(3\,\text{m}) - E_y(2\,\text{m}) - E_z(1\,\text{m})$$

$$-6 = -3E_x - 2E_y - E_z$$

$$2E_y + E_z = 3 \quad ...(2)$$

And $\quad V_c - V_o = -E_x(1\,\text{m}) - E_y(3\,\text{m}) - E_z(4\,\text{m})$

$$-3 = -1 - 3E_y - 4E_z \Rightarrow 3E_y + 4E_z = 2 \quad ...(3)$$

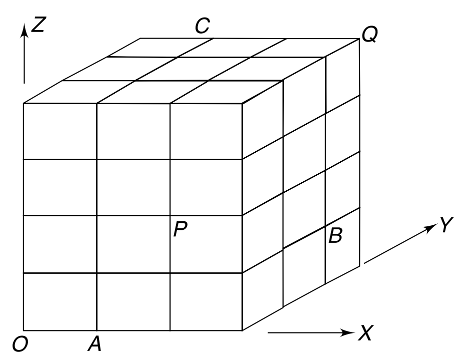
Solving (2) and (3)

$$E_y = 2\,\text{V/m}$$

$$E_z = -1\,\text{V/m}$$

$$V_Q - V_p = -E_x(1\,\text{m}) - E_y(3\,\text{m}) - E_z(2\,\text{m})$$

$$V_Q - V_p = -1 - 6 + 2 \Rightarrow V_P - V_Q = 5\,\text{V}$$



(b) $E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$ V/m

The potential will change at fastest rate when we move in the direction of field.

$$d = \frac{2\text{ Volt}}{\sqrt{6}\text{ Volt/m}} = \sqrt{\frac{2}{3}}\text{ m}$$

74. Let direction along AB be X and that along AC be Y. Perpendicular to the incline is Z direction.

$$E_x = \frac{\Delta V}{\Delta X} = \frac{10\text{ V}}{1\text{ cm}} = 10\frac{\text{V}}{\text{cm}}$$

Field is directed along the decreasing potential.

Similarly, $\quad E_y = \frac{\Delta V}{\Delta y} = 10\frac{\text{V}}{\text{cm}}$

Similarly, $\quad E_V = 10\frac{\text{V}}{\text{cm}}$

But $$E_V = E_Z \cos 37° + E_y \sin 37°$$

$$10 = \frac{4}{5}E_z + 10 \times \frac{3}{5} \quad \Rightarrow E_z = 5 \text{ V/cm}$$

$\therefore$ $$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{10^2 + 10^2 + 5^2} = 15 \text{ V/cm}$$

$$\vec{E} = (10\hat{i} + 10\hat{j} + 5\hat{k}) \text{ V/cm}$$

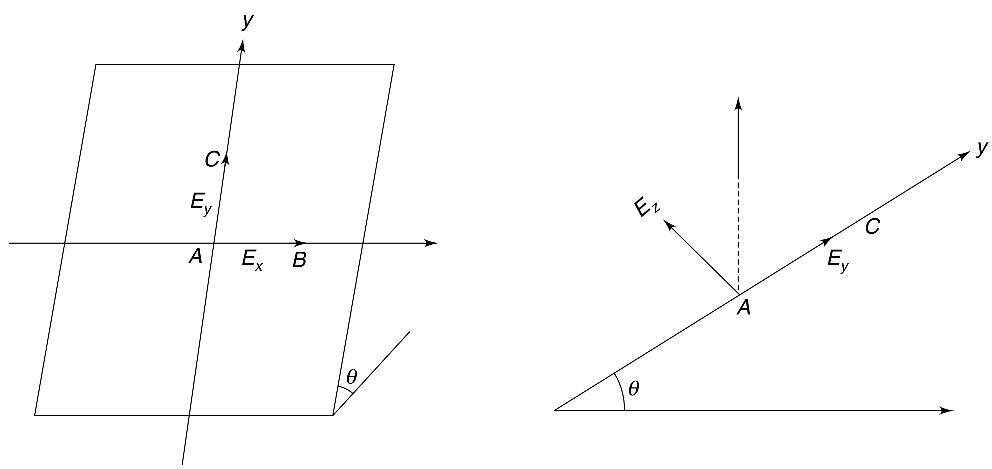
Unit vector along vertical $\hat{n} = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$

Angle between $\vec{E}$ and $\hat{n}$ is given by

$$\cos\alpha = \frac{\vec{E}\cdot\hat{n}}{E}$$

$$\cos\alpha = \frac{10}{15}$$

$$\alpha = \cos^{-1}\left(\frac{2}{3}\right)$$



75. (a) Potential will be given by

$V = \left(\frac{1}{4}\right)^{th}$ the potential due to complete disc having charge density σ

$$V = \frac{\sigma}{8\epsilon_0}[\sqrt{R^2 + Z^2} - Z]$$

(b) $$E_z = -\frac{\partial V}{\partial Z} = \frac{\sigma}{8\epsilon_0}\left[1 - \frac{Z}{\sqrt{R^2 + Z^2}}\right]$$

76. (a) The space has negative charge distribution. Radial field directed towards the fixed point terminate on these negative charges so that density of lines go on decreasing as one moves towards the fixed point

(b) Flux through sphere of radius a is

$$\phi = -(4a)(4\pi a^2) = -16\pi a^3$$

$\therefore$ $$\frac{q}{\epsilon_0} = -16\pi a^3$$

$$q = -16\pi\epsilon_0 a^3$$

(c) $r_A = \sqrt{1^2 + 1^2 + (\sqrt{2})^2} = 2\,\text{m}$

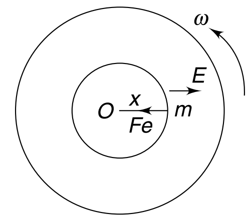
$r_B = \sqrt{0 + 3^2 + 4^2} = 5\,\text{m}$

[All points at a distance of 2 m from origin are equipotential. Similarly, all points at a distance 5 m are at same potential]

$$\therefore \quad V_B - V_A = -\int_{r_A}^{r_B} \vec{E}\cdot\vec{dr} = -\int_{2}^{5} (-4r)dr$$

$$= \frac{4}{2}[r^2]_2^5 = 2 \times 21 = 42 \text{ Volt}$$

$$\therefore \quad V_A - V_B = 42 \text{ Volt}$$



77. For an electron at a distance x from the centre (when it is in equilibrium in the reference frame of the disc)

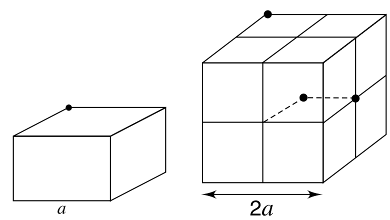
Electric force $(F_e) = m\omega^2 x$

$$eE = m\omega^2 x \qquad \therefore E = \frac{m\omega^2}{e}x$$

$\therefore$ Potential difference between centre (O) and circumference (C) is

$$V_C - V_o = -\int_{x=o}^{R} E\,dx \qquad \therefore \quad V_o - V_c = \frac{m\omega^2}{e}\int_o^R x\,dx = \frac{m\omega^2 R^2}{2e}$$

78. Consider a uniformly charged cube of side length a, and charge density ρ.



From dimensional analysis

$$V_{\text{corner}} \propto \frac{\rho a^3}{a}$$

$$V_{\text{corner}} \propto \rho a^2$$

Now consider a big cube of side length $2a$

$$\frac{V_{\text{corner}}^{\text{Big}}}{V_{\text{corner}}^{\text{Small}}} = \left(\frac{2a}{a}\right)^2 = \frac{4}{1} \qquad \text{...(1)}$$

But it is clear from the figure that

$$V_{\text{centre}}^{\text{Big}} = 8V_{\text{Corner}}^{\text{Small}}$$

$$\therefore \quad \text{from (1)} \quad \frac{V_{\text{Corner}}^{\text{Big}}}{\frac{1}{8}V_{\text{Centre}}^{\text{Big}}} = \frac{4}{1} \Rightarrow \frac{V_{\text{Conrner}}}{V_{\text{Centre}}} = \frac{1}{2}$$

79. Charge on a conductor will always be on its outer surface. Shell is thin and fits perfectly over the sphere. Half the charge on sphere will reside on the shell. As the shell is removed, the charge on it also goes away with it. When grounded, the shell will give away all its charge.

Every time shell is mounted and removed, the charge on sphere become half (or its potential become half)

$$\therefore \quad \frac{100}{2^n} = 6.25 \quad \Rightarrow n = 4$$

80. *Hint:* the entire charge of the inner sphere will get transferred to the outer ball. On the outer sphere the charge will be distributed uniformly on its surface.

81. (a) If there is no charge inside the shell, its entire charge will reside on its outer surface. But the question says that there is charge on the inner wall as well. Hence, there must be a charge inside it.

There is positive charge inside the shell that is equal in magnitude to the charge on the inner wall.

Net charge on the shell = charge on outer surface + charge on inner surface

$$q_{\text{shell}} = 4\pi b^2\sigma - 4\pi a^2\sigma$$

(b) Potential of the shell $V = \frac{1}{4\pi\varepsilon_0}\frac{4\pi b^2\sigma}{b} = \frac{b\sigma}{\varepsilon_0}$

82. Given $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$

Volume of liquid does not change when the bubble collapses to form a drop.

$$4\pi a^2 t = \frac{4}{3}\pi r^3 \text{ where '}r\text{' is the radius of the drop}$$

$$r = (3a^2 t)^{1/3}$$

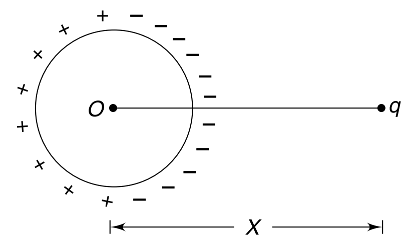
Charge remains conserved. Hence new potential is

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = V\left(\frac{a}{3t}\right)^{1/3}$$

83. Charge is induced on the surface of the sphere but net charge on the surface is zero. Due to induced charge, the potential at the centre of the sphere is zero.

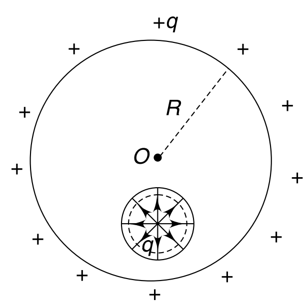
$\therefore$ Potential at O

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{x}$$



The whole conductor is at a single potential. Even if the sphere is hollow, the charge distribution on the surface will remain identical (there is no reason for it to be different !). Hence answer remains same.

87. In all cases, a charge $-q$ is induced on the inner wall of the cavity and a charge $+q$ is induced on the outer surface. [why exactly $-q$ charge is induced on the cavity wall? why not $-2q$? Explain]. The charge on the outer surface is uniformly distributed in all cases.



The electric field produced by the point charge and the charge on the cavity wall is limited to the space inside the cavity only.

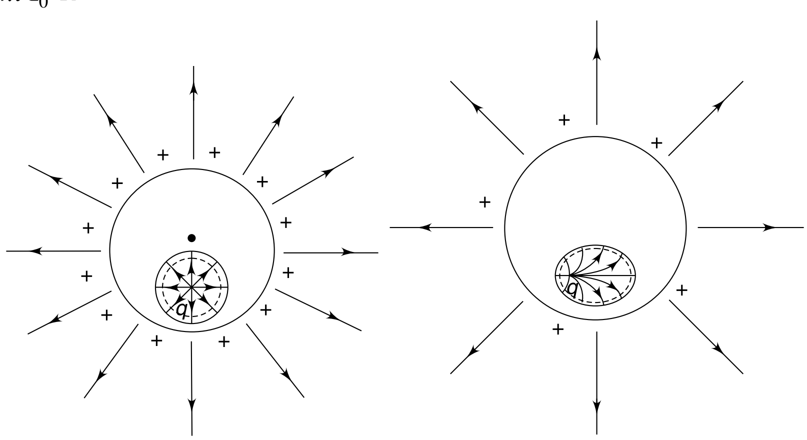
Resultant field due to these two charges is zero at all points outside the cavity!! Field lines starting at the point charge $+q$ and terminating on the cavity walls ($-q$ charge) tells us that no field is created due to this pair of charges outside the cavity.

(a) Field outside the sphere is due to $+q$ charge on its outer surface only

$\therefore$ Outside field resembles a field due to a point charge placed at the centre O of the sphere.

$\therefore$ Potential = work done in slowly moving a unit positive test charge from infinity on to the conductor

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{R}$$



(b) No change

(c) No change

(d) Field line are normal to the conductor surface. These lines are all straight when charge is at the centre of the spherical cavity

88. *Hint:* The net charge inside the ball is Q though charge will get induced on the dielectric surface.

89. Point charge q induces a charge on the surface of the conducting sphere.

Potential of the sphere = potential at the centre

[whole sphere is at one potential]

The induced charge will cause no potential at the centre.

$$\therefore \quad V_{\text{sphere}} = V_{\text{Centre}} = \frac{Kq}{x}$$

Now, when the sphere is earthed, its potential (i.e, potential at its centre) must become zero. For this the earth supplies a charge q_0.

$$V_{\text{sphere}} = V_{\text{centre}} + V_{\substack{\text{induced}\\ \text{charge}}} + V_{q_0}$$

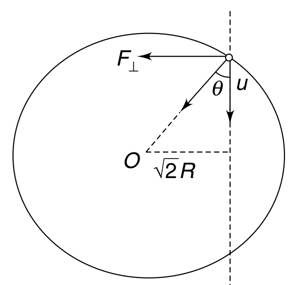
$$O = \frac{Kq}{x} + 0 + \frac{Kq_0}{\text{R}} \quad \Rightarrow q_0 = -\frac{qR}{x}$$

(b) $i = \left|\frac{dq_0}{dt}\right| = \frac{qR}{x^2}\frac{dx}{dt} = \frac{qR}{x^2}\upsilon$

91. Charge $-Q$ is induced on the inner surface of the outer shell. There is no charge on the outer surface of the outer shell as it is grounded.

An electric field exists in the space between the two shells.

$$E = K\frac{Q}{x^2} \quad \text{for } R < x < 2R$$



Just after the charge enters, it experience a force due to this electric field directed towards the centre.

$$F = K\frac{Qq}{(2R)^2}$$

Component of this force perpendicular to the direction of instantaneous velocity u is

$$F_\perp = F\sin\theta = F\frac{\sqrt{2}R}{2R} = \frac{F}{\sqrt{2}}$$

$$= K\frac{Qq}{4\sqrt{2}R^2}$$

If radius of curvature of the path is r

$$\frac{mu^2}{r} = F_\perp$$

$$\therefore \quad \frac{mu^2}{r} = \frac{KqQ}{4\sqrt{2}R^2}$$

$$\therefore \quad r = \frac{16\sqrt{2}\pi \in_0 R^2mu^2}{Qq}$$

(b) The potential difference between the two spheres is

$$V_{\text{inner}} - V_{\text{outer}} = \left(K\frac{Q}{R} - \frac{KQ}{2R}\right) - \left(\frac{KQ}{2R} - \frac{KQ}{2R}\right)$$

$$= \frac{KQ}{2R}$$

Energy conservation for the charge entering the shell gives:

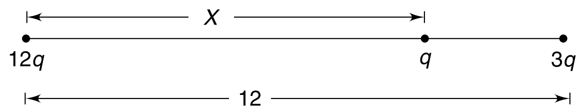
$$\frac{1}{2}mv^2 + (-q)V_{\text{inner}} = \frac{1}{2}mu^2 + (-q)V_{\text{outer}}$$

$$\therefore \quad \frac{1}{2}mv^2 = \frac{1}{2}mu^2 + q(V_{\text{inner}} - V_{\text{outer}})$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \frac{Qq}{8\pi\epsilon_0 R}$$

$$\therefore \quad v = \sqrt{u^2 + \frac{Qq}{4\pi\epsilon_0 mR}}$$

92. The largest charge $12q$ must be placed at one of the corners, and the smallest charge between the two charges.



Let the position be as shown in figure [The trick is to understand that PE of a pair of charges $\propto \frac{qQ}{r^2}$. Therefore, the pair having largest value of qQ must be farthest]

$$U = Kq^2\left[\frac{36}{12} + \frac{12}{X} + \frac{3}{12 - X}\right]$$

U will be minimum when $\frac{dU}{dX} = 0$

$$\Rightarrow \quad \frac{12}{X^2} = \frac{3}{(12-X)^2} \quad \Rightarrow \left(\frac{12 - X}{X}\right)^2 = \frac{1}{4} \quad \Rightarrow 24 - 2X = X \quad \Rightarrow X = 8\,\text{cm}$$

In this position force on q is $= K\frac{12q^2}{8^2} - K\frac{3q^2}{4^2} = 0$

93. Sum of interaction energy of each $-q$ charge with $+Q$ charge in original position is

$$U_0 = \left[-\frac{KqQ}{\frac{a}{\sqrt{2}}} - \frac{KqQ}{\frac{3a}{\sqrt{2}}}\right] \times 2 = -\frac{8\sqrt{2}}{3}\frac{KqQ}{a}$$

(a) When larger square is shifted to infinity, this interaction energy will become zero.

$$\therefore \quad W_{\text{ext}} = 0 - U_0 = \frac{8\sqrt{2}}{3}\frac{KqQ}{a}$$

(b) First rotation brings the $-q$ charges on z axis at co-ordinates $z = \frac{a}{\sqrt{2}}$ and $z = -\frac{a}{\sqrt{2}}$

The second rotation does not charge the position of two charges.

$\therefore$ distance of each $-q$ charge from every Q charge is

$$r = \sqrt{\left(\frac{a}{\sqrt{2}}\right)^2 + (\sqrt{2}a)^2} = \sqrt{\frac{5}{2}}\cdot a$$

$\therefore$ Interaction energy for $-q$, Q pairs is

$$U = \frac{-KqQ}{\sqrt{\frac{5}{2}}\cdot a} \times 4 = \frac{4\sqrt{2}}{\sqrt{5}}\frac{KqQ}{a}$$

$$\therefore \quad W_{\text{ext}} = U - U_0 = \left[\frac{8\sqrt{2}}{3} - \frac{4\sqrt{2}}{\sqrt{5}}\right]\frac{KqQ}{a}$$

94. (a) The electric field has maximum strength where the magnitude of slope of the *V–x* graph is maximum. Among marked points, we have large slopes at *C* and *D*. Positive slope at *D* means that field is in negative *X* direction at that point. At this point a negative charge will feel largest force in positive *X* direction.

(b) Total energy of the proton shall be less than zero for it to remain bound.

At origin total energy is

$$\frac{1}{2}mv^2 + (-V_0)e \qquad \therefore \quad \frac{1}{2}mv^2 + (-V_0)e \le 0$$

$$v \le \sqrt{\frac{2eV_0}{m}}$$

An electron cannot remain bound as it will always experience a force away from the origin.

95. The tension decreases as the pendulum rises. In case (a) the electrostatic force between the charges is

$$F_0 = \frac{1}{4\pi\epsilon_0}\frac{qQ}{L^2} = 2mg$$

In this case the tension cannot become zero anywhere. Even if the bob rises to the top and has zero speed there, tension is the string will be *mg*.

Therefore, to complete the circle we need to ensure that the bob has enough speed to rise to the top point.

i.e. $$u = \sqrt{4gL}$$

In case (b)

$$F_e = \frac{mg}{2}$$

If v is speed at the top

$$T + mg - F_e = \frac{mv^2}{L}$$

$$T + \frac{mg}{2} = \frac{mv^2}{L}$$

$\therefore$ for $T \ge 0$

$$v \ge \sqrt{\frac{gL}{2}}$$

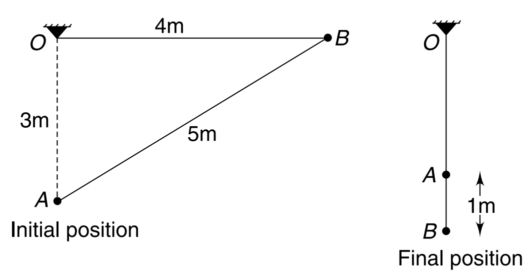
Energy conservation gives

$$u \ge \sqrt{4.5gL}$$

96. (a) **Energy conservation**

$$K\frac{Q\cdot Q}{5} + mg(4) = K\frac{Q\cdot Q}{1}$$

$$\Rightarrow \qquad m = \frac{K}{g}\frac{QQ}{5} = \frac{9\times10^9\times(20\times10^{-6})^2}{10\times5} = 7.2\times10^{-2}\text{ kg} = 72\text{ g}$$



(b) $T = mg + K\dfrac{Q\cdot Q}{1^2} = 7.2\times10^{-2}\times10 + 9\times10^9\times(20\times10^{-6})^2 = 4.32\text{ N}$

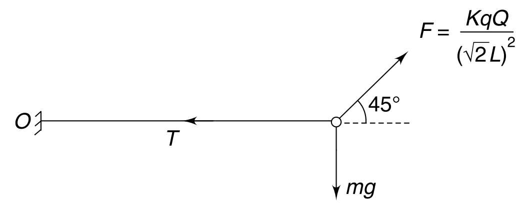
(c) Unstable.

When released, the mass accelerates and then retards to stop when just below A. It means, if disturbed from equilibrium, it will experience a force away from equilibrium.

97. Equilibrium of ball 1.

$$\frac{F}{\sqrt{2}} = mg$$

$$\frac{KqQ}{2\sqrt{2}L^2} = mg \quad ...(1)$$



$$W_{ext} = U_f - U_i$$

$$= 0 - \left[mgL + \frac{KqQ}{\sqrt{2}L}\right] = -[mgL + 2mgL]$$

$$= -3mgL$$

98. (a) KE will be least when PE of the system is maximum, i.e. when charges are closest to each other. In this situation both charges will have same velocity (v).

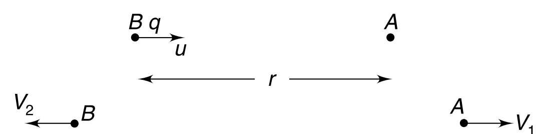
$$mv + mv = mu$$

$$\Rightarrow \quad v = \frac{u}{2}$$

$$\therefore \quad KE_{min} = \frac{1}{2}m\left(\frac{u}{2}\right)^2 + \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{1}{4}mu^2$$

(b) A will move to right with velocity u and B will be at rest.

99. Let the final velocity (at ∞ separation) be V_1 and V_2 as shown.



Momentum Conservation

$$mV_1 - mV_2 = mu$$

$$V_1 - V_2 = u \quad ...(1)$$

Energy Conservation

$$\frac{1}{2}mV_1^2 + \frac{1}{2}mV_2^2 = \frac{qQ}{4\pi\epsilon_0 \cdot r} + \frac{1}{2}mu^2$$

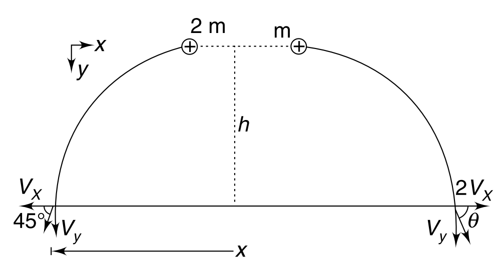
$$\Rightarrow \quad mV_1^2 + m(V_1 - u)^2 = 2mu^2 \quad \left[\because \; r = \frac{qQ}{2\pi\epsilon_0 mu^2}\right]$$

$$\Rightarrow \quad 2V_1^2 - 2uV_1 - u^2 = 0 \quad \Rightarrow V_1 = \left(\frac{1+\sqrt{3}}{2}\right)u$$

100. (a) The y component of velocity of two balls will be same.

$$V_y = \sqrt{2gh}$$

For momentum to remain conserved along horizontal direction, the X component of velocity of the lighter ball will be twice that of the heavier ball (in opposite direction).



$$\text{Given} \quad \tan 45° = \frac{V_y}{V_x} \Rightarrow V_y = V_x = \sqrt{2gh}$$

$$\therefore \quad \tan\theta = \frac{V_y}{2V_x} = \frac{1}{2}$$

(b) **Energy Conservation** $\frac{1}{2}2m(V_x^2 + V_y^2) + \frac{1}{2}m(V_y^2 + (2V_x)^2) + U_f$

$$= U_i + 3\,\text{mgh}$$

Where U_i and U_f are initial and final electrostatic potential energy of the charge pair

$$V_x = V_y = \sqrt{2gh}$$

$\therefore$ $$4{\cdot}5m\,(2gh) + U_f = U_i + 3\,\text{mgh}$$

$\therefore$ $$6\,\text{mgh} = U_i - U_f$$

$$W_{\text{elect}} = -(U_f - U_i) = U_i - U_f$$

$$= 6\,\text{mgh}$$

101. (a) The electric field in the region between the two shells is radially outward and outside the larger shell it is radially inwards. Once the electron crosses the hole it will be automatically pushed to infinity.

The KE of the electron shall be sufficient enough to carry it to the outer shell.

$$K_{\min} + (-e)V_A = (-e)V_B$$

$$K_{\min} = e(V_A - V_B)$$

$K_{\min} = eKQ\left(\frac{1}{R} - \frac{1}{2R}\right)$ (the potential difference depends only on the charge on the inner shell)

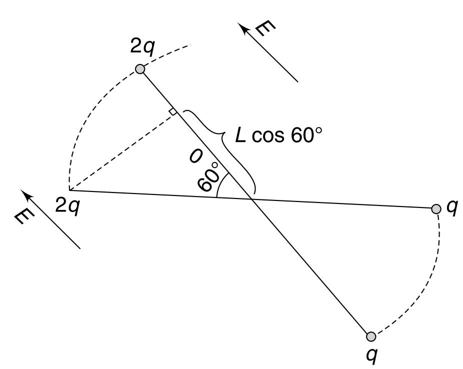
$K_{\min} = KeQ\frac{1}{2R}$

(b) In this case the field is radially outwards throughout. The KE must be large enough to carry the electron to infinity.

$$K_{\min} + (-e)V_A = 0$$

$$K_{\min} = eV_A = eK\left(\frac{Q}{2R} + \frac{2Q}{2R}\right) = \frac{3KQe}{2R}$$

102. (a) ω is maximum when angular acceleration α is zero. This happens when rod becomes parallel to the electric field.



Loss in electrostatic PE

$$\Delta U = 2q\cdot E[L - L\cos 60°] - qE[L - L\cos 60°]$$

$$= \frac{qEL}{2}$$

$$\therefore \qquad \frac{1}{2} I\omega^2 = \frac{qEL}{2}$$

$$\frac{1}{2}\frac{M(2L)^2}{12}\cdot\omega^2 = \frac{qEL}{2}$$

$$\omega = \sqrt{\frac{3qE}{ML}}$$

(b) The rod will perform oscillation with an amplitude of 60° on either side of the direction of $\vec{E}$.

Maximum torque

$$\tau_{\max} = 2qE\sin 60° - qE\sin 60°$$

$$= \frac{\sqrt{3}}{2} qE$$

$$\therefore \qquad \frac{M(2L)^2}{12}\cdot\alpha = \frac{\sqrt{3}}{2} qE$$

$$\alpha = \frac{3\sqrt{3}}{2}\frac{qE}{ML^2}$$

103. Hint: Particles are going in circles about their common centre of mass.

104. A will leave contact with the wall when

$$kx_0 = qE_0$$

$$x_0 = \frac{qE_0}{K} = \text{minimum extension in the spring.}$$

After being released, block B must at least move through a distance x_0 before it comes to rest.

Energy conservation for motion of *B*

Gain in spring potential energy = loss in electrostatic potential energy of B

$$\Rightarrow \qquad \frac{1}{2}kx_0^2 = Q\cdot E_0 x_0 \quad \Rightarrow \quad x_0 = \frac{2QE_0}{k}$$

But x_0 must be at least $\frac{qE_0}{k}$

$$\therefore \qquad \frac{2QE_0}{k} = \frac{qE_0}{k} \quad \Rightarrow \quad Q = \frac{q}{2}$$

105. Charge on sphere 1 after S_1 is closed is

$$Q = 4\pi\in_0 RV$$

Now S_1 is opened and S_2 is closed. Charge is shared between sphere 1 & 2.

Charge on 1 & 2 both becomes $\frac{Q}{2}$

Thereafter, S_2 is opened & S_3 closed. Charge on 2 and 3 both will become $\frac{Q}{4}$

This way we can see that final charges on the spheres will be $\frac{Q}{2}, \frac{Q}{4}, \frac{Q}{8}, \frac{Q}{16}$ ...

The final potential of spheres will be $\frac{V}{2}, \frac{V}{4}, \frac{V}{8}, \frac{V}{16}$ ...

Final energy

$$U = \frac{1}{2} 4\pi \in_0 R \left[\left(\frac{V}{2}\right)^2 + \left(\frac{V}{4}\right)^2 + \left(\frac{V}{8}\right)^2 + \left(\frac{V}{16}\right)^2 + ...\right]$$

$$= \frac{1}{2} 4\pi \in_0 RV^2 \left(\frac{1/4}{1 - 1/4}\right) = \frac{1}{6} 4\pi \in_0 RV^2$$

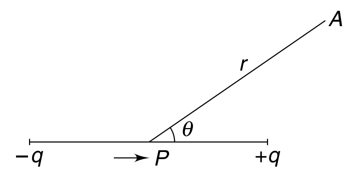
Energy lost by the cell = Work done by the cell

$$= QV = 4\pi \in_0 RV^2$$

Hence, loss in energy of the system

$$\Delta U = 4\pi \in_0 RV^2 - \frac{1}{6} \cdot 4\pi \in_0 RV^2 = \frac{5}{6} 4\pi \in_0 RV^2 = \frac{10}{3} \pi \in_0 RV^2$$

106. (a) Potential at position (r, θ) is



$$V = \frac{KP\cos\theta}{r^2}$$

At a point where potential is V_0

$$r^2 = \frac{KP}{V_0} \cos\theta \quad [\Rightarrow \quad r^2 \propto \cos\theta]$$

Maximum value of $\cos\theta = 1$ (for $\theta = 0°$)

$$\therefore \quad r_{max} = \sqrt{\frac{KP}{V_0}}$$

(b) Field at (r, θ) is

$$E = \frac{KP}{r^3} \sqrt{1 + 3\cos^2\theta}$$

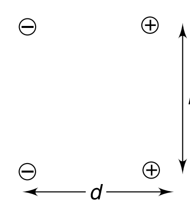
When $E = E_0$

$$r^3 = \frac{KP}{E_0} (1 + 3\cos^2\theta)^{1/2}$$

r is maximum when $\cos\theta = 1$

$$\therefore \quad r_{max} = \left(2\frac{KP}{E_0}\right)^{1/3}$$

107.



$$U = K \cdot \frac{q \cdot q}{r} + K\frac{(-q)(-q)}{r} + K\frac{q(-q)}{\sqrt{r^2 + d^2}} + K\frac{(-q)(q)}{\sqrt{r^2 + d^2}}$$

$$= 2Kq^2 \left[\frac{1}{r} - \frac{1}{(r^2 + d^2)^{1/2}}\right]$$

$$= \frac{2Kq^2}{r} \left[1 - \left(1 + \frac{d^2}{r^2}\right)^{-1/2}\right]$$

$$= \frac{2Kq^2}{r} \left[1 - 1 + \frac{1}{2}\frac{d^2}{r^2}\right]$$

$$= \frac{Kq^2d^2}{r^3} = \frac{KP^2}{r^3} \quad [P = \text{dipole moment of each dipole}]$$

When one of the dipoles is rotated by 90°, interaction energy become zero. It can be easily seen as the potential due to a dipole on its perpendicular bisector is zero.

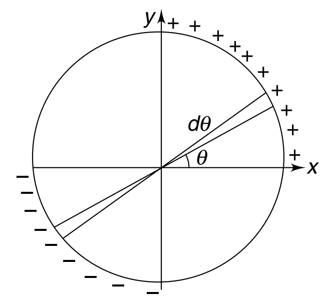
(a) $W = U_{\text{final}} - U_{\text{initial}} = 0 - \dfrac{KP^2}{r^3} = -\dfrac{KP^2}{r^3}$

(b) It can be shown, as above, that potential energy in position shown in figure(c) is

$$U = -\frac{KP^2}{r^3}$$

$$\therefore \quad W = U_{\text{final}} - U_{\text{initial}} = -\frac{KP^2}{r^3} - \frac{KP^2}{r^3} = -\frac{2KP^2}{r^3}$$

108. Consider a segment of ring of angular width $d\theta$. Consider another segment at diametrically opposite end. The two small elements make a dipole. Its dipole moment is



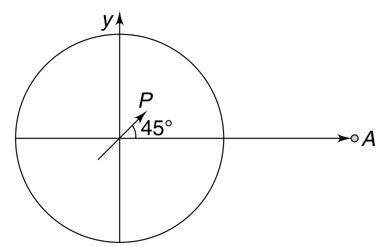
$$dP = (\lambda r d\theta)(2r) = 2\lambda r^2 \, d\theta$$

$$\left[\lambda = \text{linear charge density} = \frac{2q}{\pi r}\right]$$

X component of dipole moment is $dP_x = 2\lambda r^2 \cos\theta d\theta$

Y component is $dP_y = 2\lambda r^2 \sin\theta d\theta$

Adding contributions from all such dipoles



$$P_x = 2\lambda r^2 \int_0^{\pi/2} \cos\theta d\theta = 2\lambda r^2$$

$$P_y = 2\lambda r^2 \int_0^{\pi/2} \sin\theta d\theta = 2\lambda r^2$$

The charge system can be treated like a single dipole of dipole moment

$$P = 2\sqrt{2}\lambda r^2 = \frac{4\sqrt{2}\, q r}{\pi}$$

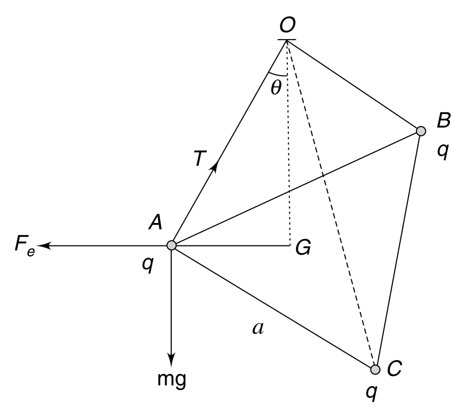
The resultant dipole moment vector make an angle 45° with the x-axis

$\therefore$ Potential at A is

$$V = \frac{1}{4\pi \in_0} \frac{P\cos 45°}{R^2}$$

$$V = \frac{qr}{\pi^2 \in_0 R^2}$$

109. In equilibrium, the three balls are on the vertices of an equilateral triangle in horizontal plane. Resultant electrostatic force on charge A is



$$F_e = K\frac{q^2}{a^2}\cos 30° \times 2 \text{ (Horizontal)}$$

$$= \sqrt{3}\frac{Kq^2}{a^2}$$

$$\therefore \quad \tan\theta = \frac{F_e}{mg} = \frac{\sqrt{3}\, Kq^2}{mga^2}$$

But $\quad \tan\theta \approx \sin\theta = \dfrac{AG}{L} = \dfrac{a}{\sqrt{3}\, L}$

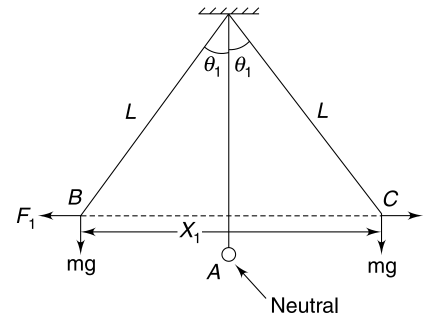
Using above two equations we get

$$a^3 = \frac{3Kq^2L}{mg} \quad \text{...(1)}$$

(a) When one ball gets discharged, it will experience no electric force. It will be in equilibrium with string vertical. The other two balls will be in equilibrium in position shown.

$$\tan\theta_1 = \frac{F_1}{mg}$$

$$\frac{X_1}{2L} = \frac{Kq^2}{mg \cdot X_1^2} \quad [X_1 = \text{separation between } B \ \& \ C] \qquad ...(2)$$



(2) ÷ (1)

$$\left(\frac{X_1}{a}\right)^3 = \frac{2}{3}$$

$$\therefore \quad X_1 = \left(\frac{2}{3}\right)^{\frac{1}{3}} a$$

(b) If B and C are discharged, all the balls will collide at G.

Each of them will acquire charge $\frac{q}{3}$ and then repel each other.

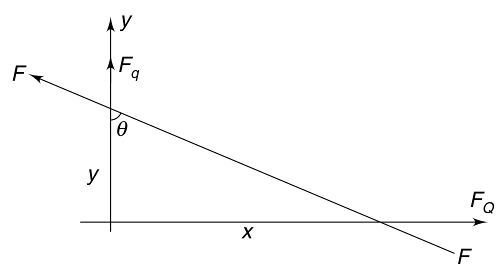
Equilibrium separation (X_2) can be obtained from (1)

$$X_2^3 = \frac{3K\left(\frac{q}{3}\right)^2 L}{mg}$$

$$\therefore \quad \frac{X_2^3}{a^3} = \frac{1}{9}$$

$$\therefore \quad X_2 = \left(\frac{1}{9}\right)^{\frac{1}{3}} \cdot a$$

110. Force on q along y is $\quad F_q = \frac{KqQ}{(x^2 + y^2)} \cdot \cos\theta = \frac{KqQy}{(x^2 + y^2)^{\frac{3}{2}}}$

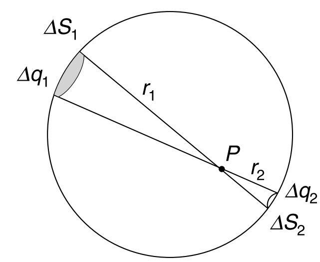


Force on Q along x axis is $\quad F_Q = \frac{KqQ}{(x^2 + y^2)} \sin\theta = \frac{KqQx}{(x^2 + y^2)^{\frac{3}{2}}}$

$$\therefore \quad \frac{F_q}{F_Q} = \frac{y}{x}$$

∴ acceleration of the particles is in the ratio of their respective distance from origin.
In the interval q moves a distance r, Q will move through $2r$.

111. The solid angle at the apex of the cone is



$$\Delta\Omega = \frac{\Delta S_1}{r_1^2} = \frac{\Delta S_2}{r_2^2}$$

If σ is surface charge density $\quad \frac{\sigma \Delta S_1}{r_1^2} = \frac{\sigma \Delta S_2}{r_2^2}$

$$\frac{\Delta q_1}{r_1^2} = \frac{\Delta q_2}{r_2^2}$$

$$K\frac{\Delta q_1}{r_1^2} = K\frac{\Delta q_2}{r_2^2}$$

$$\therefore \quad E_1 = E_2$$

The entire spherical surface can be divided into many such pair of small area which produce zero resultant at p.

112. (a) Solid angle formed by a cone having semi vertical angle α is

$$\Omega = 2\pi(1 - \cos\alpha) = 2\pi(1 - \cos 60°) = \pi\ Sr$$

Therefore, one fourth of the total number of lines emitted from q_1 = Half the number of lines terminating on q_2

$$\therefore \quad N_1 = 2N_2$$

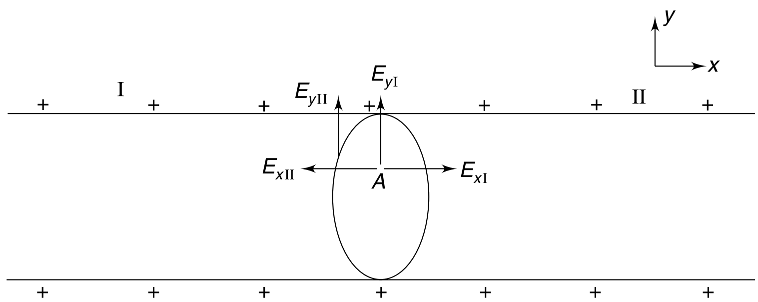
$$\therefore \quad |q_1| = 2|q_2|$$

(b) Half the lines emitted from q_1 do terminate on q_2. Remaining half will go to ∞.

$$\therefore \quad \alpha_{max} = 90°$$

113. Consider the cylindrical shell to be of infinite extent on both sides.

We can imagine it to be made of two identical part I and II.



X & Y components of electric field at A due to part I may have directions as shown. Due to symmetry, the field at A due to part II will have components as shown (E_{XII} & E_{YII}).

Using Gauss law we can shown that field at A is zero.

$$\Rightarrow \quad E_{XI} = E_{XII}$$

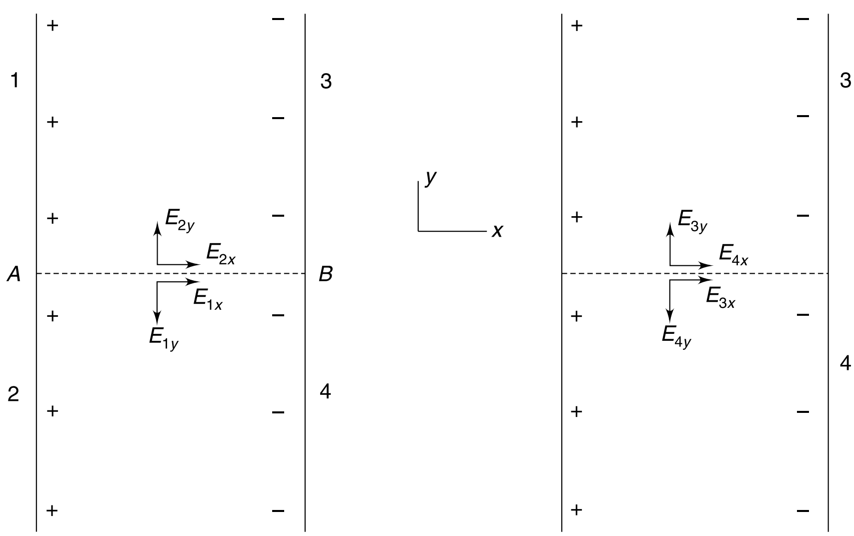
and $$E_{YI} + E_{YII} = 0 \quad \Rightarrow \quad E_{YI} = E_{YII} = 0$$

Hence, field at A due to part I has X component only.

114. At p

$$E_0 = \frac{\lambda}{2\pi\epsilon_0 r} + \frac{\lambda}{2\pi\epsilon_0 r} = \frac{\lambda}{\pi\epsilon_0 \frac{d}{2}} \qquad \left[\because\ r = \frac{d}{2}\right]$$

$$E_0 = \frac{2\lambda}{\pi\epsilon_0 d} \qquad \text{...(1)}$$



Had the threads been very long on both sides of AB, the field at M would have been E_0

$$|E_{1Y}| = |E_{2Y}| = |E_{3Y}| = |E_{4Y}|$$

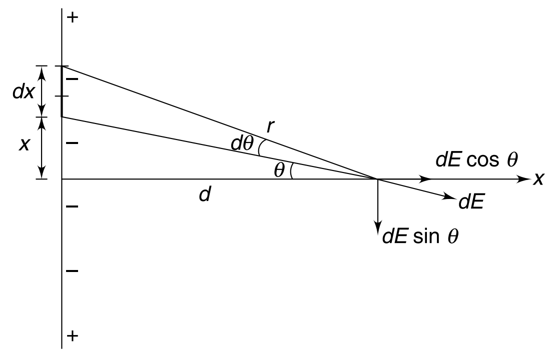
$$|E_{1X}| = |E_{2X}| = |E_{3X}| = |E_{4X}| = \frac{E_0}{4}$$

In the given situation, E_{2Y} and E_{4Y} cancel out and resultant is along AB given by

$$E = E_{2X} + E_{4X} = \frac{E_0}{2}$$

115. ***Note:*** gauss law is not applicable as F is not proportional to $\frac{1}{r^2}$.

Consider an elemental charge as shown in figure.



$$x = d\tan\theta \quad \Rightarrow dx = d\sec^2\theta d\theta$$

$\therefore$ $$dq = \lambda dx = \lambda d\sec^2\theta d\theta$$

Field $$dE = \frac{Kdq}{r^3}$$

Due to symmetry field is along X. Hence resultant field is

$$E = \int dE\cos\theta = \int \frac{Kdq}{r^3}\cdot\frac{d}{r} = K\lambda d^2\int\frac{\sec^2\theta d\theta}{r^4} = K\lambda d^2\int\frac{\sec^2\theta d\theta}{(d\sec\theta)^4}$$

$$= \frac{K\lambda}{d^2}\int\cos^2\theta d\theta$$

For infinite line charge, limit charges from $\theta = -\frac{\pi}{2}$ to $\theta = \frac{\pi}{2}$

$\therefore$ $$E = \frac{K\lambda}{d^2}\int_{-\pi/2}^{\pi/2}\cos^2\theta d\theta = \frac{K\lambda}{2d^2}\int_{-\pi/2}^{\pi/2}(1+\cos 2\theta)d\theta = \frac{K\lambda\pi}{2d^2}$$

116. Consider an element of angular width $d\phi$ as shown in figure.

Charge on the element $dq = \lambda R d\phi$

Distance of the element from point $P = 2R\cos\theta$

Field at P due to the element is $dE = K\frac{dq}{r^2} = K\frac{\lambda R d\phi}{(2R\cos\theta)^2}$

Due to symmetry, field will be along X and we need to add $dE\cos\theta$ for all elements on the ring.

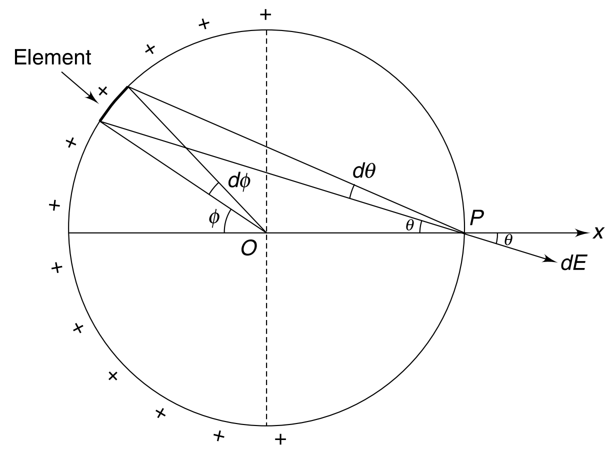
$\therefore$ $$\text{Field} = \int dE\cos\theta = \frac{K\lambda}{4R}\int\sec\theta d\phi$$

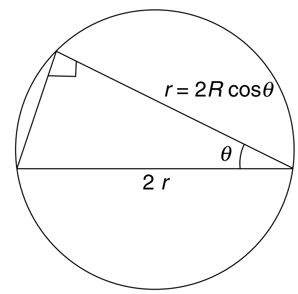
$$= \frac{K\lambda}{2R}\int\sec\theta d\theta \quad [\because \phi = 2\theta,\ d\phi = 2d\theta]$$

θ Changes from $-\frac{\pi}{4}$ to $+\frac{\pi}{4}$ or else, we can integrate from $\theta = 0$ to $\theta = \frac{\pi}{4}$ and double the result.

$$\text{Field} = 2\frac{K\lambda}{2R}\int_0^{\pi/4} \sec\theta d\theta = \frac{K\lambda}{R}[\ln\sec\theta + \tan\theta]_0^{\pi/4}$$

$$= \frac{K\lambda}{R}\left[\ln(\sqrt{2}+1) - \ln 1\right] = \frac{K\lambda}{R}\ln\left(\sqrt{2}+1\right) \text{ along } X$$





117. (a) In equilibrium,

$$mg = qE \quad \left[E = \frac{\sigma}{2\in_0}\right] \qquad ...(1)$$

The removed disc is like a point charge = $\sigma\pi r^2$

$\therefore$ Unbalanced force created by removal of disc

$$= \frac{1}{4\pi\in_0}\frac{\sigma\pi r^2}{h^2}\cdot q = \frac{\sigma r^2}{4\in_0 h^2}\cdot q = \frac{mgr^2}{2h^2} \text{ [where we have used (1)}]$$

$\therefore$ acceleration $a = g\dfrac{r^2}{2h^2}$

(b) When terminal speed is acquired

$$6\pi\eta x V_0 = \frac{mgr^2}{2h^2}$$

$$V_0 = \frac{mgr^2}{12\pi\eta x h^2}$$

118. Notice that the field is radial.

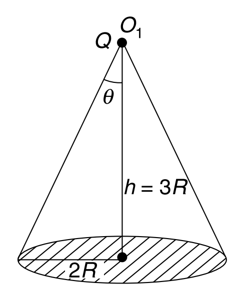
Apply Guass law on a sphere of radius $2r_0$

121. When charge is at O_1 the flux through the face having radius $2R$ is

$$\phi_1 = \frac{q}{\varepsilon_0}\cdot\frac{2\pi(1-\cos\theta)}{4\pi}$$

$$= \frac{q}{2\varepsilon_0}\cdot(1-\cos\theta)$$

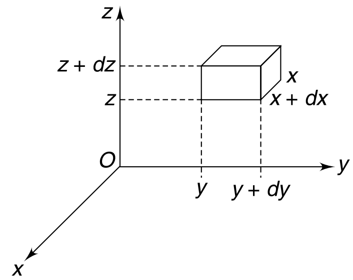
$$= \frac{q}{2\varepsilon_0}\left[1 - \frac{3R}{\sqrt{(2R)^2 + (3R)^2}}\right] = \frac{q}{2\varepsilon_0}\left[1 - \frac{3}{\sqrt{13}}\right]$$



Similarly,
$$\phi_2 = \frac{q}{2\varepsilon_0}\left[1 - \frac{3R}{\sqrt{R^2 + (3R)^2}}\right] = \frac{q}{2\varepsilon_0}\left[1 - \frac{3}{\sqrt{10}}\right]$$

$$\therefore \qquad \frac{\phi_1}{\phi_2} = \left(\frac{\sqrt{13}-3}{\sqrt{10}-3}\right)\cdot\sqrt{\frac{10}{13}}$$

122. (a) Flux through two faces parallel to $y-z$ plane is



$$= \left[(E_x)_{\text{at}(x+dx)} - (E_x)_{\text{at}\,x}\right]\cdot dy\cdot dz$$
$$= [x + dx - x]\,dy\cdot dz = dx\,dy\,dz$$
$$= dV = Vol^m \text{ of cuboid.}$$

Similarly, for faces parallel to xz plane, flux is $= 2dV$

And for faces parallel to xy plane, flux is $= 3dV$

$$\therefore \qquad d\phi_{\text{cuboid}} = (1+2+3)dV = 6dV = 6dx\,dy\,dz$$

(b) Charge inside the elemental cuboid

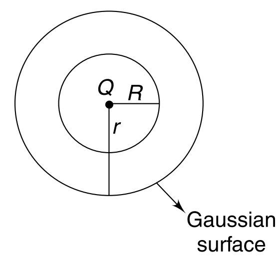
$$dq = 6\varepsilon_0 dV$$

Charge per unit volume $\dfrac{dq}{dV} = 6\varepsilon_0$

$\therefore$ charge inside the sphere

$$q = \frac{4}{3}\pi r^3\cdot 6\varepsilon_0 = 8\varepsilon_0\pi r^3$$

123. The quantity of charge in the space $R < x < r$ is given by



$$q = \int_R^r \rho 4\pi x^2 dx = 4\pi b\int_R^r \frac{x^2}{x}dx$$

$$= 4\pi b\left[\frac{x^2}{2}\right]_R^r = 2\pi b[r^2 - R^2]$$

Let strength of field at a distance r from the centre be E.
Applying Gauss law over spherical surface of radius r we get

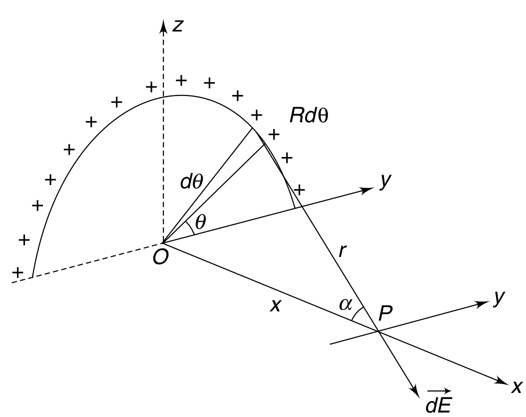
$$E\cdot 4\pi r^2 = \left[Q + 2\pi b[r^2 - R^2]\right]/\varepsilon_0$$

$$E = \left(\frac{Q - 2\pi bR^2}{4\pi\varepsilon_0}\right)\frac{1}{r^2} + \frac{2\pi b}{4\pi\varepsilon_0}$$

For E to not depend on r it is necessary that $Q = 2\pi bR^2$

In this case $\qquad E = \dfrac{2\pi b}{4\pi\varepsilon_0} = \dfrac{b}{2\varepsilon_0}$

124. Consider an element as shown.
Field at P due to this element is

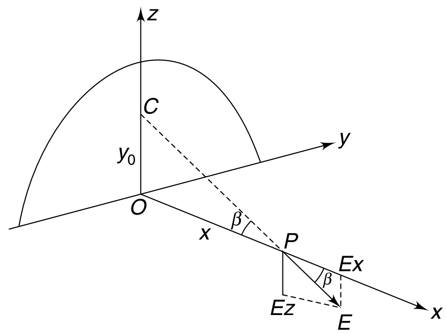


$$dE = K\frac{\lambda R\,d\theta}{r^2} \qquad \left[r = \sqrt{R^2 + x^2}\right]$$

$$dE_x = dE\cos\alpha = \frac{K\lambda R\,d\theta}{r^2}\cdot\frac{x}{r}$$

$$E_x = \int dE_x = \frac{K\lambda R\pi\cdot x}{r^3}$$

$$dE_z = -dE\sin\alpha\cdot\sin\theta = -\frac{K\lambda R}{r^2}\,\frac{R}{r}\,\sin\theta\cdot d\theta$$



$$E_z = -\frac{K\lambda R^2}{r^3}\int_0^{\pi}\sin\,\theta d\theta = -\frac{2K\lambda R^2}{r^3}$$

From symmetry $E_y = 0$

The resultant field is in xz plane making an angle β with the x axis.

$$\tan\beta = \frac{E_z}{E_x} = \left(\frac{2R}{\pi}\right)\frac{1}{x}$$

Clearly the direction of E is along $\overrightarrow{CP}$

When $OC = y_0 = \frac{2R}{\pi}$

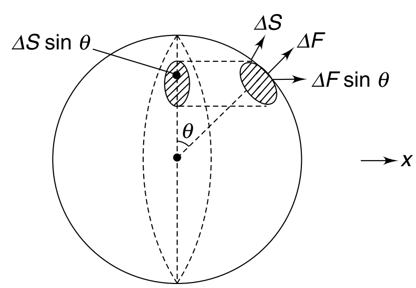
$\therefore$ C is COM of the ring.

126. $P = \frac{{\sigma_{max}}^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E_0^2 \quad \left[\because \frac{\sigma_{max}}{\varepsilon_0} = E_0\right]$

$$= \frac{1}{2}\times 8.8\times 10^{-12}\times(3\times 10^6)^2 = 39.6\,\text{N/m}^2$$

127. (a) Electrostatic pressure on the surface $P = \frac{\sigma^2}{2\varepsilon_0}$

[charge on a small area $\Delta s = \sigma\Delta s$]



Radial force on the area Δs is $\Delta F = P\Delta s = \frac{\sigma^2\Delta s}{2\varepsilon_0}$.

Due to symmetry, the resultant force on one half is along x direction and is obtained by summing up $\Delta F\sin\theta$

$$\therefore \quad F = \sum\Delta F\sin\theta = \frac{\sigma^2}{2\varepsilon_0}\sum\Delta s\sin\theta$$

$$= \frac{\sigma^2}{2\varepsilon_0}\cdot\pi R^2 = \frac{Q^2}{32\pi\varepsilon_0 R^2} \quad \left[\because \sigma = \frac{Q}{4\pi R^2}\right]$$

Note: Electrostatic force ΔF on a charge $\sigma\Delta s$ is actually force due to repulsion from all the charges on the sphere. It includes interaction with charges present on the half of the sphere on which we are calculating the force. But such internal interaction cancels out in pair and the force F obtained above is the force due to one half of the sphere on the other half.

(b) $$Q_{max} = \sigma_{max}\,4\pi R^2 = (\varepsilon_0 E_0)\,4\pi R^2 = 4\pi\varepsilon_0 E_0 R^2$$

$$\therefore \quad F = \frac{\pi\varepsilon_0 E_0^2 R^2}{2}$$

128. Pressure inside a soap bubble of radius r is

$$P_1 = P_0 + \frac{4T}{r}$$

After expansion, the pressure becomes

$$P_2 = \frac{P_1}{8}\,[\because \text{ volume becomes 8 times and temperature is constant}]$$

$$= \frac{P_0}{8} + \frac{T}{2r}$$

Now, Excess pressure is $\Delta P = P_2 - P_0 = \frac{T}{2r} - \frac{7P_0}{8}$

The electrostatic pressure is $\frac{\sigma^2}{2\varepsilon_0}$

$$\therefore \quad \frac{\sigma^2}{2\varepsilon_0} + \Delta P = \frac{4T}{2r}$$

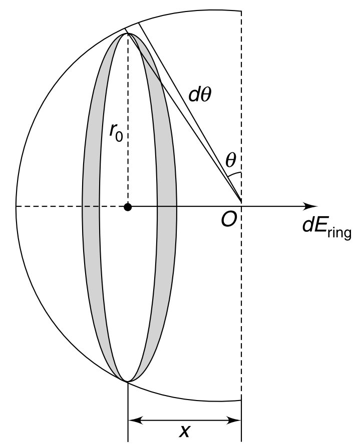
$$\frac{\sigma^2}{2\varepsilon_0} + \frac{T}{2r} - \frac{7P_0}{8} = \frac{4T}{2r}$$

$$\frac{\sigma^2}{\varepsilon_0} = \frac{3T}{r} + \frac{7P_0}{4}$$

$$\sigma = \left[\varepsilon_0\left(\frac{3T}{r} + \frac{7P_0}{4}\right)\right]^{1/2}$$

$$\therefore \quad Q = \sigma_0 4\pi(2r)^2 = 16\pi r^2 \cdot \sigma = 8\pi r^2\left[\varepsilon_0\left(\frac{12T}{r} + 7P_0\right)\right]^{1/2}$$

130. The sphere can be replaced with a point charge (q) at its centre. We can either calculate the force applied by q on each ring element of the hemisphere and add it to get the resultant force or we can calculate electric field produced due to the hemisphere at its centre (by considering similar ring elements). Let's proceed by the second method.



Recall that field due to a charged ring on its axis is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2 + x^2)^{3/2}}$$

Consider a ring element on the hemisphere as shown.

$$r_0 = R\cos\theta$$
$$x = R\sin\theta$$

Charge on the ring $= \left(\frac{Q}{2\pi R^2}\right)(2\pi r_0)(Rd\theta)$

$= Q\cos\theta d\theta$

$$\therefore \quad dE_{\text{ring}} = \frac{1}{4\pi\varepsilon_0}\frac{(Q\cdot\cos\theta d\theta)(R\sin\theta)}{[R^2 + R^2\sin^2\theta]^{3/2}} = \frac{Q}{4\pi\varepsilon_0 R^2}\frac{\cos\theta\sin\theta d\theta}{(1+\sin^2\theta)^{3/2}}$$

$$\therefore \quad E = \frac{Q}{4\pi\varepsilon_0 R^2}\int_{\theta=0}^{\pi/2}\frac{\cos\theta\sin\theta d\theta}{(1+\sin^2\theta)^{3/2}}$$

Put $t = 1 + \sin^2\theta$

$$\frac{dt}{d\theta} = 2\sin\theta\cos\theta \Rightarrow \frac{1}{2}dt = \sin\theta\cos\theta d\theta$$

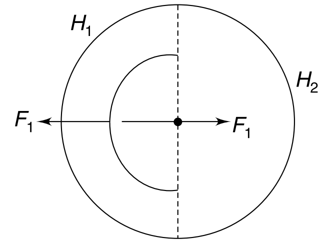
$$\therefore \quad E = \frac{Q}{8\pi\varepsilon_0 R^2}\left[\frac{dt}{t^{3/2}}\right]_{t=1}^{2}$$

$$E = -\frac{Q}{8\pi\varepsilon_0 R^2}\left[\frac{1}{\sqrt{t}}\right]_1^2 = \frac{Q}{4\pi\varepsilon_0 R^2}\left[1 - \frac{1}{\sqrt{2}}\right]$$

$\therefore$ Required force

$$F = E\cdot q = \frac{Qq}{4\pi\varepsilon_0 R^2}\left(1 - \frac{1}{\sqrt{2}}\right)$$

131. Complete the outer sphere.



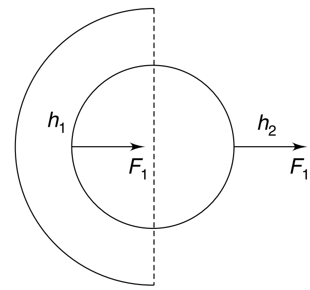
Now this sphere will exert no force on the inner hemisphere.

This means the two hemispheres in the outer sphere exert equal and opposite force on the inner hemisphere.

Call the forces on inner sphere due to two halves H_1 and H_2 of outer sphere as F_1 and F_2.

$$F_1 = F_2 = F \text{ (say)}$$

Now complete the inner sphere with outer sphere as hemispherical



Now, force between one half of inner sphere (h1) and the outer sphere is very much like that between H1 and inner hemisphere in previous drawing.

Force between h_2 and outer hemisphere is like F_2 in previous drawing.

$\therefore$ Resultant force on inner ball due to outer hemisphere is $F_1 + F_2 = 2F\ (\rightarrow)$

This force is already known to us from the previous question.

$\therefore$ Force between the outer and the inner hemispheres = half the answer obtained in last question

$$= \frac{Q}{8\pi\varepsilon_0 R^2}\left(1 - \frac{1}{\sqrt{2}}\right)$$

132. $g_{\text{eff}} = g + \dfrac{qE}{m}$

Velocity just before 1st collision $V_0 = \sqrt{2g_{\text{eff}} \cdot h}$

Velocity just after 1st collision $V_1 = eV_0$

Velocity just before 2nd collision $= V_1$

Velocity just after 2nd collision $= eV_1 = e^2 V_0$

Hence, Velocity just after n^{th} collision $V_n = e^n V_0 = e^n \sqrt{2g_{\text{eff}} \cdot h}$

$\therefore$ Height attained after n^{th} collision $h_n = \dfrac{V_n^2}{2g_{\text{eff}}} = \dfrac{e^{2n} \cdot 2g_{\text{eff}} \cdot h}{2g_{\text{eff}}} = e^{2n} \cdot h$

133. As done in last Q., the vertical velocity after n^{th} collision

$$V_{yn} = e^n V_0 = e^n \sqrt{2gh}$$

The time of flight between n^{th} and $(n+1)^{\text{th}}$ collision is

$$T = \frac{2V_{yn}}{g} = \frac{2e^n\sqrt{2gh}}{g} = 2e^n\sqrt{\frac{2h}{g}}$$

The total time that has passed from start upto n^{th} collision is

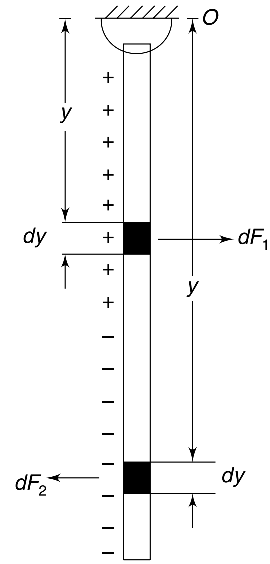
$$T_n = \sqrt{\frac{2h}{g}} + 2\cdot\frac{e^1\sqrt{2gh}}{g} + 2\cdot\frac{e^2\sqrt{2gh}}{g} + 2\cdot\frac{e^3\sqrt{2gh}}{g} + \ldots\ldots + 2\cdot\frac{e^{n-1}\sqrt{2gh}}{g}$$

$$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h}{g}}\left[e + e^2 + e^3 + \ldots\ldots + e^{n-1}\right] = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h}{g}}\left[e\cdot\left(\frac{1-e^{n-1}}{1-e}\right)\right]$$

$\therefore$ Horizontal velocity at the moment of n^{th} collision is

$$V_{xn} = a_x T_n = \frac{qE}{m}\cdot\sqrt{\frac{2h}{g}}\left[1 + 2e\left(\frac{1-e^{n-1}}{1-e}\right)\right]$$

134. (a) From $y = 0$ to $y = \frac{L}{2}$ the rod has positive charge. Force on it (F_1) is towards right due to Electric field.



The lower half has negative charge and electric force (F_2) on it is towards left.

The torque due to F_1 and F_2 must balance.

$$\tau_{F1} = \tau_{F2}$$

$$\int_0^{L/2} E(ay^2 dy)y = \int_{L/2}^{L} E(by^n dy)\ y$$

$$\frac{aL^4}{2^6} = \frac{b}{n+2}\left(\frac{2^{n+2}-1}{2^{n+2}}\right)L^{n+2}$$

Comparing the powers of L we get $n = 2$.

$$\therefore \quad \frac{a}{2^6} = \frac{b}{4}\cdot\left(\frac{2^4-1}{2^4}\right) \Rightarrow b = \frac{a}{15}$$

(b)
$$F_1 = Ea\cdot\int_0^{L/2} y^2 dy = \frac{EaL^3}{24}$$

and
$$F_2 = Eb\cdot\int_{L/2}^{L} y^2 dy = \frac{Ea}{15}\frac{7L^3}{24} = \frac{7}{15}\frac{EaL^3}{24}$$

$\therefore$ Net horizontal force

$$F_x = F_1 - F_2 = \frac{EaL^3}{45}(\rightarrow) = \frac{45mg}{45} = mg(\rightarrow)$$

Weight of the rod = $mg(\downarrow)$

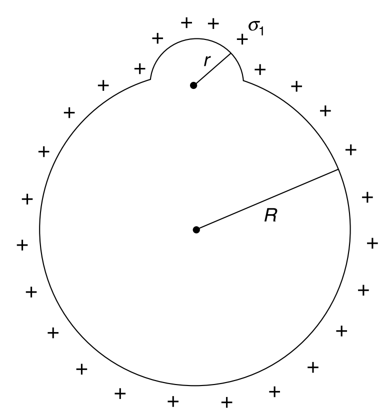
$\therefore$ Force by hinge

$$F_x = mg(\leftarrow)$$

$$F_y = mg\ (\uparrow)$$

$$F_{hinge} = \sqrt{F_x^2 + F_y^2} = \sqrt{2}\,mg$$

136. The bulge will acquire a small charge. Therefore, charge on remaining sphere can be assumed to be $\simeq Q$.



Charge density will be inversely proportional to the radius of curvature of the surface.

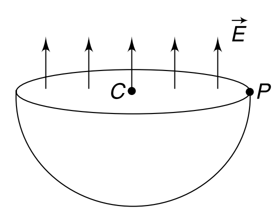
$$\therefore \quad \sigma_1 r = \sigma_2 R$$

$$\sigma_1\cdot r = \frac{Q}{4\pi R^2}\cdot R$$

$$\sigma_1 = \frac{Q}{4\pi R r}$$

$$\therefore \quad q = \sigma_1\cdot 2\pi r^2 = \frac{Q}{4\pi R^2}\cdot 2\pi r^2 = \frac{Qr}{2R}$$

137. Potential at centre

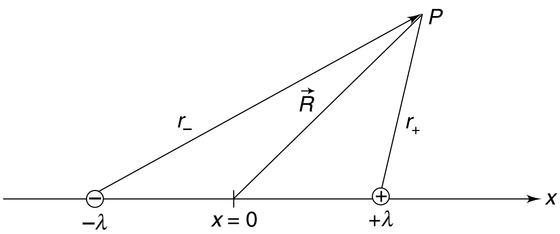


$$V_C = \frac{KQ}{R} = \frac{1}{4\pi\varepsilon_0}\frac{2\pi R^2\cdot\sigma}{R} = \frac{\sigma R}{2\varepsilon_0}$$

Potential at point P and A will also be same because the electric field at all points of the circular base is perpendicular to the circular surface. When one moves from C to P, he is travelling on an equipotential surface.

138. Electric field at a distance r from an infinite line charge of charge density λ is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



Taking potential to be zero at $r = r_0$, we have

$$\int_0^V dV = -\int_{r_0}^{r} E\,dr$$

$$V = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_0}^{r}\frac{dr}{r}$$

$$V = -\frac{\lambda}{2\pi\varepsilon_0}\ln r + \frac{\lambda}{2\pi\varepsilon_0}\ln r_0$$

$$V = -\frac{\lambda}{2\pi\varepsilon_0}\ln r + C$$

If the line charge has negative charge density the constant will be $-C$ [$\because$ it depends on λ]

Now consider a point P as shown. The position vector of point P is $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\therefore \quad V_P = V_+ + V_-$$

$$= \left[-\frac{\lambda}{2\pi\varepsilon_0}\ln(r_+) + C\right] + \left[\frac{\lambda}{2\pi\varepsilon_0}\ln(r_-) - C\right]$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_-}{r_+}\right) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left[\sqrt{\frac{(x+a)^2+y^2}{(x-a)^2+y^2}}\right] = \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\sqrt{\frac{(x+a)^2+y^2}{(x-a)^2+y^2}}\right]$$

For $$V_P = \frac{\lambda\ln(2)}{4\pi\varepsilon_0}$$

$$(x+a)^2 + y^2 = 2(x-a)^2 + 2y^2 \quad \Rightarrow x^2 + y^2 - 6ax + a^2 = 0$$

$$\Rightarrow \quad (x-3a)^2 + y^2 - 8a^2 = 0$$

$$(x-3a)^2 + y^2 = 8a^2$$

Radius of this circle is $2\sqrt{2}\,a$

139. Potential at centre $V_C = V_Q + V_{\text{in}} = K\dfrac{Q}{r_0} + 0 = \dfrac{KQ}{r_0}$

Potential at centre = potential at all the points of the shell

$$\therefore \quad V_P = \frac{KQ}{r_0}$$

$$V_{\text{in}}^P + V_Q^P = \frac{KQ}{r_0} \quad \therefore \quad V_{\text{in}}^P = \frac{KQ}{r_0} - \frac{KQ}{r}$$

140. Potential at A = potential at O

$$V_A = V_0 \qquad [\because \text{ sphere is conducting}]$$

$$V_{AE} + V_{AQ} = V_{OE} + V_{OQ}$$

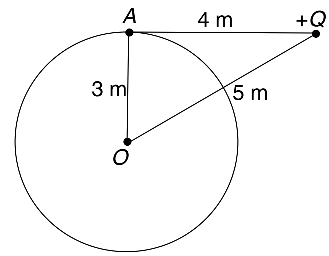
$$(V_{AE} - V_{OE}) + V_{AQ} = V_{OQ}$$

$$ER\cos\theta_0 + V_{AQ} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

$$V_{AQ} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} - \frac{ER}{2}$$

141. For sphere S1

Induced charge will produce zero potential at the centre O.

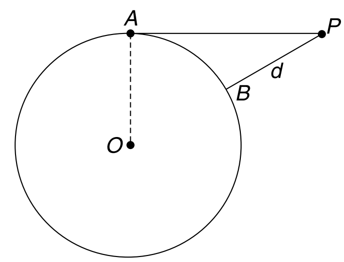


$\therefore$ potential at $O = \frac{KQ}{5} = 9 \times 10^9 \times \frac{2 \times 10^{-6}}{5} = 3600$ V

Because it is a conducting sphere, potential at all points in the sphere will be same. Potential at a point B on the surface of the sphere will be

$$V_B = V_Q + V_{\text{inducded}}$$

$$\therefore \quad V_{\text{induced}} = 3600 - \frac{9 \times 10^9 \times 2 \times 10^{-6}}{d}$$



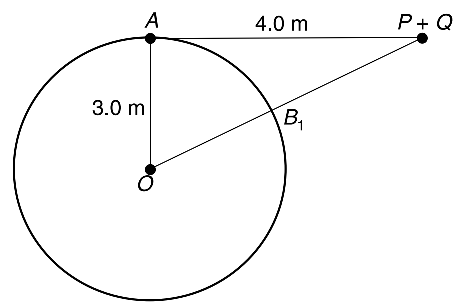
Where d is distance of point B from P.

Potential due to induced charge will be minimum at a point where d is minimum. Such point will lie on straight line OP

Potential due to induced charge will be least at point B_1

$$d = B_1P = 2.0 \text{ m}$$

$$\therefore \quad (V_{\text{induced}}) \text{ at } B_1 = 3600 - \frac{18 \times 10^3}{2} = 5400 \text{ V}$$



Because charge distribution of surface of S_2 is identical to that on S_1, hence 5400 V is least potential on the surface of S_2.

142. The inner faces must have equal and opposite charge. Let it be $\pm x$

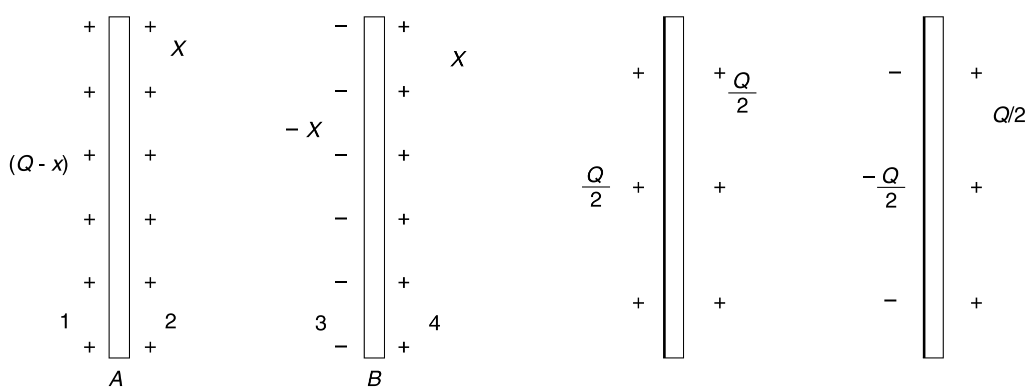
Field inside any of the plate (say B) must be zero.

$$\frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} - \frac{\sigma_3}{2\epsilon_0} - \frac{\sigma_4}{2\epsilon_0} = 0$$

$$Q - x + x - x - x = 0$$

$$\therefore \quad x = \frac{Q}{2}$$

Note: In general if charge Q_1 and Q_2 is given to two plates, charge on both outer surface is same equal to $\frac{Q_1 + Q_2}{2}$ and charge on inner faces is $\pm\left(\frac{Q_1 - Q_2}{2}\right)$



(ii) The inner faces facing each other must have equal and opposite charge. And the outer walls of A and B must have equal charge to ensure zero field inside the conductor.

Hence, the charge distribution can be as shown in figure below.

$$2y - (3Q - x) - x = Q$$

$$2y = 4Q \quad \Rightarrow y = 2Q \qquad \ldots\text{(i)}$$

$$\because \quad V_A = V_B \quad \therefore \quad V_C - V_A = V_C - V_B$$

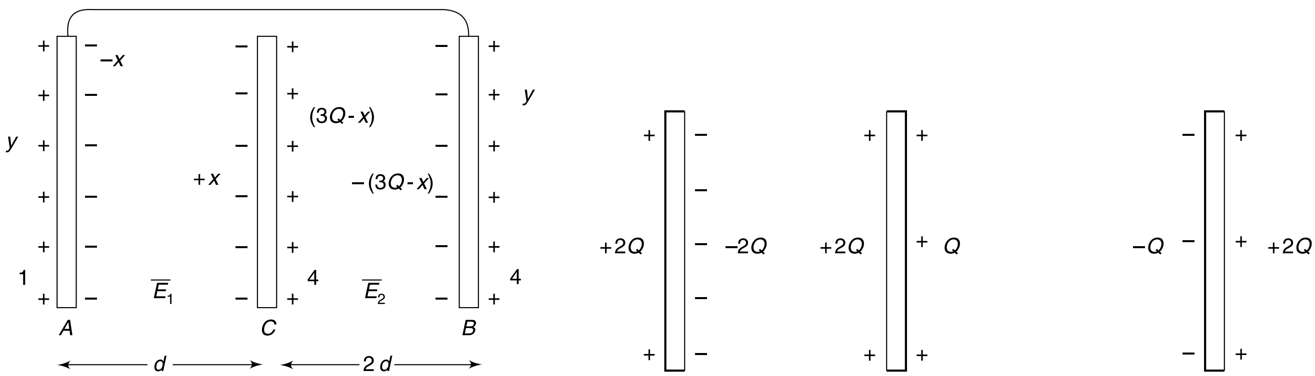
$$E_1 d = E_2 (2d)$$

$$\frac{x}{A \cdot \in_0} d = \frac{(3Q - x)2d}{A \cdot \in_0}$$

$$3x = 6Q \Rightarrow x = 2Q$$

From (1) $\quad y = 2Q$

$\therefore$



(i) When A is grounded, the charge on outer walls of A and B will become zero (so as to make their potentials zero)

For $\quad V_A = V_B$

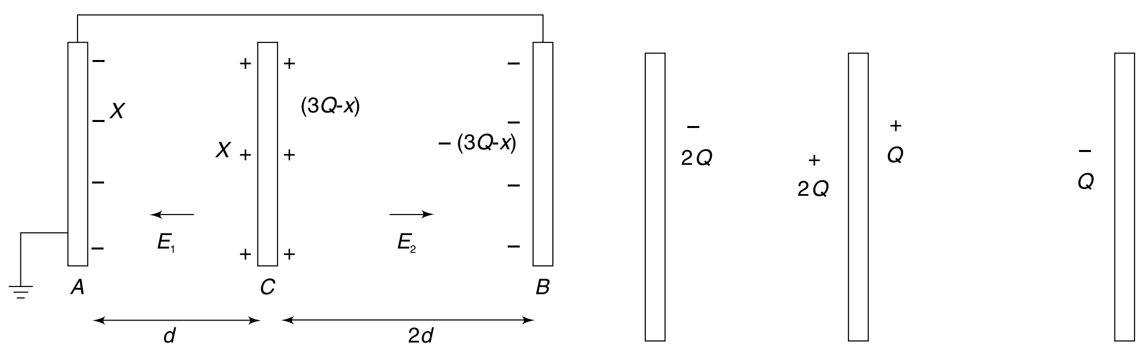
$$E_1 d = E_2 2d$$

$$E_1 = 2E_2$$

$$\therefore \quad x = 2(3Q - x)$$

$$\therefore \quad 3x = 6Q$$

$$x = 2Q$$



143. Charge density $\rho = \dfrac{Q}{\frac{4}{3}\pi(8R^3 - R^3)} = \dfrac{3Q}{28\pi R^3}$

Charge in the region $R < r < x$ $(x < 2R)$ is

$$q = \rho \frac{4}{3}\pi (x^3 - R^3) = \frac{Q}{7R^3}(x^3 - R^3)$$

$\therefore$ Electric field at distance x from the centre is

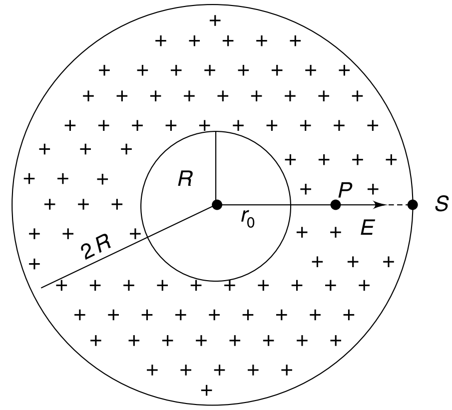
$$E = \frac{Kq}{x^2} = \frac{KQ}{7R^3}\left(x - \frac{R^3}{x^2}\right)$$

$$\int_{V_P}^{V_S} dV = -\int_{r_0}^{2R} E\,dx$$

$$V_S - V_P = -\frac{KQ}{7R^3}\left[\int_{r_0}^{2R} x\,dx - R^3\int_{r_0}^{2R}\frac{dx}{x^2}\right]$$

$$\frac{KQ}{2R} - V_P = -\frac{KQ}{7R^3}\left[2R^2 - \frac{r_0^2}{2} + R^3\left(\frac{1}{2R} - \frac{1}{r_0}\right)\right]$$

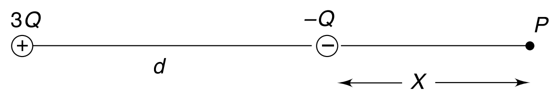
$$V_P = \frac{6KQ}{7R} - \frac{KQ}{7R^3}\left[\frac{r_0^2}{2} - \frac{R^3}{r_0}\right]$$



144. (a) yes. Spherical

(b) At p electric field is zero.

$$\therefore \quad K\frac{Q}{x^2} = K\frac{3Q}{(d+x)^2} \quad \Rightarrow \quad \frac{d+x}{x} = \sqrt{3} \quad \Rightarrow x = \frac{d}{\sqrt{3}-1}$$



(c) Potential of surface S_1 = potential at P

$$V = -\frac{KQ}{x} + K\frac{3Q}{d+x} = \frac{2KQ}{d}(2-\sqrt{3})$$

(d) One can easily show that potential is zero at a point at a distance $\frac{d}{2}$ to the right of $-Q$ charge. Now, $\frac{d}{2} < \frac{d}{\sqrt{3}-1}$.

Hence, this point is to the left of point P. Looking at the given diagram, we can say that zero potential surface will enclose $-Q$ charge only.

$$\therefore \quad \text{Required flux} = -\frac{Q}{\epsilon_0}$$

145. *Hint:* Number of field lines originating at A = no. of lines originating at C = 2 times number of lines terminating at B

$\therefore$ If $\quad q_B = -q$

Then $\quad q_A = q_C = 2q$

For graphs see the answer

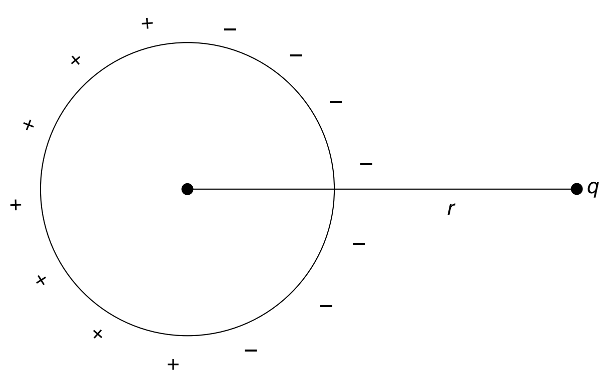
146. Solution for (f)

If a charge q is placed in front of a conducting sphere, the induced charges plus the charge q creates a potential

$V = \frac{Kq}{r}$ at all points of the sphere.

In our case, the point charge $-q$ at the centre of the shell induces $+q$ charge on the inner surface and the charge induced on the outer surface will be just as shown in fig (with $r = 4R$). The system of charge shown in fig above will cause a potential

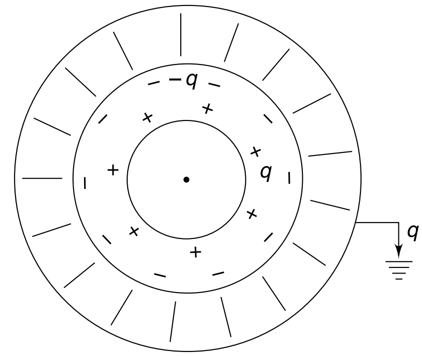
$V = \frac{Kq}{4R}$ at all points $0 \le x \le 2R$.



147. Two sphere of equal size will share charge till both of them acquire same potential. Let charge on inner sphere be q.

A charge $-q$ is induced on the inner surface of the shell.

Potential of the inner sphere = potential of distant sphere



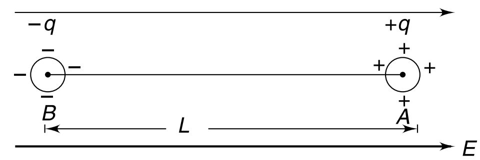
$$= K\frac{q}{R} + K\frac{-q}{2R} = K\frac{Q-q}{R} \quad \Rightarrow \quad q = 2Q - 2q$$

$$\Rightarrow \quad q = \frac{2Q}{3}$$

$\therefore$ charge induced on the inner surface of the shell = $-q = \frac{-2Q}{3}$

$\therefore$ charge that flows to earth = $+q = \frac{2Q}{3}$

148. (a) Let ball A acquire charge $+q$ and B acquire charge $-q$ after the field is switched on



$$V_B = V_0 - \frac{Kq}{r} + \frac{Kq}{L}$$

$$V_A = V_0 - EL + \frac{Kq}{r} - \frac{Kq}{L}$$

Since balls are connected by conducting spring.

(V_0 = potential of point B in electric field E if our system of balls were not there)

$$V_A = V_B$$

$$\Rightarrow \qquad -EL + \frac{Kq}{r} - \frac{Kq}{L} = -\frac{Kq}{r} + \frac{Kq}{L}$$

$$\Rightarrow \qquad 2Kq\left[\frac{1}{r} - \frac{1}{L}\right] = EL \quad \Rightarrow \quad \frac{q}{2\pi\epsilon_0}\frac{1}{r} = EL \quad \left[\because \frac{1}{r} >> \frac{1}{L}\right]$$

$$\therefore \qquad q = 2\pi\epsilon_0 ErL$$

(b) The electric force on the two balls stretches the spring. As the balls move out, induced charge (q) on them increases (as it is directly proportional to separation L between them)

This increases the electric force stretching the balls. The system can oscillate only if spring force increases at a much faster pace. When separation between the balls increase by x, charge on them change by

$$\Delta q = 2\pi\epsilon_0 rE\cdot x$$

Electric force increases by

$$\Delta qE = 2\pi\epsilon_0 rE^2\cdot x$$

For oscillation

$$kx > 2\pi\epsilon_0 rE^2\cdot x$$

$$k > 2\pi\epsilon_0 rE^2$$

For no oscillation $k < 2\pi\epsilon_0 rE^2$

$$\therefore \qquad k_0 = 2\pi\epsilon_0 rE^2$$

(c) Consider the motion of ball A. Let it be in equilibrium when length of spring is l_0

$$k(l_0 - L) = 2\pi\epsilon_0 l_0 rE^2 \qquad ...(1)$$

Consider A to be further displace by x (in the same time B also move by x and spring length increases by $2x$)

$$ma = 2\pi\epsilon_0 rE^2(l_0 + 2x) - k(l_0 + 2x - L)$$

Using (1)

$$ma = -x[2k - 4\pi\epsilon_0 rE^2]$$

$$a = -\left(\frac{2k - 4\pi\epsilon_0 rE^2}{m}\right)x$$

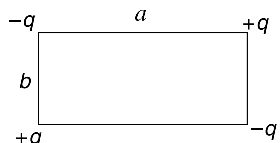
$$= -\left(\frac{8\pi\in_0 rE^2 - 4\pi\in_0 rE^2}{m}\right)x \qquad [\because\ k = 2k_0]$$

$$= -\left(\frac{4\pi\in_0 rE^2}{m}\right)x \qquad [SHM]$$

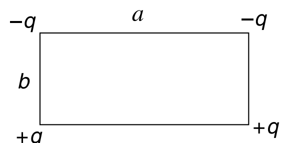
$$\therefore \qquad \omega = \sqrt{\frac{4\pi\in_0 rE^2}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{4\pi\in_0 rE^2}} = \left(\sqrt{\frac{\pi m}{\in_0 \cdot r}}\right)\cdot\frac{1}{E}$$

149. (a) There are ${}^4C_2 = 6$ terms in expression of potential energy and two of them are positive; remaining 4 being negative. The arrangement shown in Fig. (a) and (b) cannot result in zero potential energy because the negative term is less then positive term.

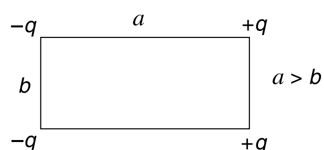


[a > b]
(a)



[a > b]
(b)

An arrangement of kind shown in Fig. (c) can result in zero potential energy



(c)

(b) $$U = K\left[\frac{(-q)(-q)}{b} + \frac{q\cdot q}{b} + \frac{(-q)(q)}{a} + \frac{(-q)(q)}{a} + \frac{(-q)(q)}{\sqrt{a^2+b^2}} + \frac{(-q)(q)}{\sqrt{a^2+b^2}}\right]$$

$$\therefore \quad \frac{2}{b} - \frac{2}{a} - \frac{2}{\sqrt{a^2+b^2}} = 0$$

With $b = 1.0$ m

$$\frac{a-1}{a} = \frac{1}{\sqrt{1+a^2}}$$

$$[a^2 + 1 - 2a][a^2 + 1] = a^2$$

$$a^4 + a^2 - 2a^3 + a^2 + 1 - 2a = a^2$$

$$a^3(a-2) + (a-1)^2 = 0$$

The second term is positive.

First term can be negative only if $a < 2$

$$\therefore \qquad 1 < a < 2$$

150. Let speed of both be V just before collision

Energy Conservation

$$\frac{1}{2}mV^2 \times 2 + \frac{K(3q)(-q)}{2r} = \frac{K(3q)(-q)}{R}$$

$$\therefore \qquad mV^2 = 3Kq^2\left[\frac{1}{2r} - \frac{1}{R}\right] \qquad \text{...(i)}$$

In elastic collision, exchange of velocity will take place and charge on both balls will now be $\frac{3q - q}{2} = q$. Now, the two balls repel each other. Speed will be maximum when separation between the centers of the balls become $2R$.

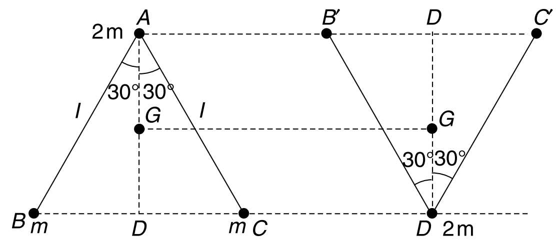
$$\frac{1}{2}mV_0^2 \times 2 + \frac{Kq^2}{2R} = \frac{1}{2}mV^2 \times 2 + \frac{Kq^2}{2r}$$

$$mV_0^2 = 3Kq^2\left[\frac{1}{2r} - \frac{1}{R}\right] + \frac{Kq^2}{2}\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$= \frac{Kq^2}{2rR}[3R - 6r + R - r] = \frac{Kq^2}{2rR}(4R - 7r) \quad \therefore V_0 = \sqrt{\frac{Kq^2}{2mrR}(4R - 7r)}$$

Note: Energy loss takes place when charge redistribution takes place during collision.

151. (a) In original position, the COM of the system is located at centre of line AD (i.e. at a distance of $AG = \frac{\sqrt{3}}{4} l$ from A). The particles move such that the COM does not get displaced. When all three come to rest again, particle of mass 2 m will be at D [This particle experiences net force along AD and moves on this line itself] and the other two will be symmetrically placed at B' and C'.



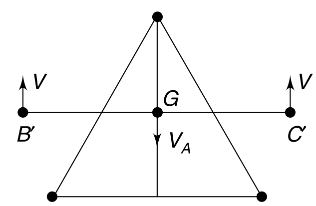
Initial position

Position when all three are at rest again

$\therefore$ A and D are extreme positions of oscillation of particle of mass 2 m.

$\therefore$ Amplitude $= \frac{AD}{2} = \frac{\sqrt{3}}{4}l$

(b) Speed of the particle of mass $2m$ will be maximum when all three fall in a straight line passing through COM(G)



Momentum conservation $MV_A = mV + mV$

$$2mV_A = 2mV \Rightarrow V_A = V$$

Energy Conservation

$$2.\ K\frac{Qq}{l} + K\frac{q \cdot q}{2l} + \frac{1}{2}mV^2 + \frac{1}{2}mV^2 + \frac{1}{2}(2m)V^2 = K\frac{qq}{l} + 2\cdot\frac{KQq}{l}$$

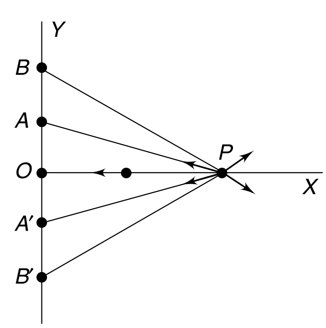
$$\Rightarrow \qquad 2mV^2 = \frac{Kq^2}{2l} \Rightarrow V = \sqrt{\frac{Kq^2}{4ml}} = \frac{q}{4}\sqrt{\frac{1}{\pi\epsilon_0 ml}} \qquad \text{...(i)}$$

(c) Particle of mass m may be viewed to be rotating about particle of mass M with a velocity of $2V$ (for the moment, when all three are in straight line, the central particle is un accelerated. Frame attached to it is inertial)

$$\therefore \qquad \frac{m(2V)^2}{l} = T - \frac{Kq^2}{(2l)^2} - \frac{KqQ}{l^2}$$

$$\therefore \qquad T = \frac{Kq^2}{l^2} + \frac{Kq^2}{4l^2} + \frac{KqQ}{l^2} \qquad \text{[using (i)]}$$

$$= \frac{q}{16\pi\varepsilon_0 l^2}(5q + 4Q)$$



152. $\vec{E}_{P(AA')}$ = Electric field at P due to charges at A and A' $= 2\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_A x_P}{(y_A^2 + x_P^2)^{3/2}}(-\hat{i})$

Similarly, $\qquad \vec{E}_{P(BB')} = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_B x_P}{(y_B^2 + x_P^2)^{3/2}}(\hat{i})$

For $\qquad \vec{E}_P = 0$

$$2\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_A x_P}{(y_A^2 + x_P^2)^{3/2}} = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_B x_P}{(y_B^2 + x_P^2)^{3/2}} \Rightarrow x = \sqrt{2.5} \text{ m}$$

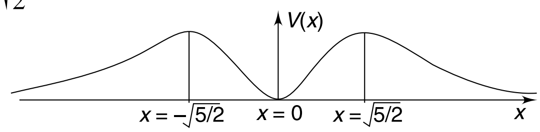
To the left of point P the field is towards left and to the right of point P it is towards right. Hence, v_0 should be just enough to enable the particle to reach P.

[Alternately,

$$V(x) = \frac{2}{4\pi\varepsilon_0}\left(\frac{q_B}{(y_B^2 + x^2)^{1/2}} + \frac{q_A}{(y_A^2 + x^2)^{1/2}}\right)$$

Field will be zero at point where

$$\frac{dV(x)}{dx} = 0 \Rightarrow x = 0 \text{ and } x = \pm\sqrt{\frac{5}{2}}$$



V at x equal to infinity is zero.

And at $x = \sqrt{\frac{5}{2}}$, $V(x)$ is positive $\Rightarrow V(x)$ is maximum at $x = \sqrt{\frac{5}{2}}$

So, if the particle is able to cross $x = \sqrt{\frac{5}{2}}$ then it would automatically reach the origin.]

$$\Rightarrow \quad \frac{1}{2}mv_0^2 = \frac{2q}{4\pi\varepsilon_0}\left[\frac{q_A}{\sqrt{y_A^2 + x_P^2}} + \frac{q_B}{\sqrt{y_B^2 + x_P^2}}\right]$$

Solving we get, $v_0 = 3$ m/s

Kinetic energy at origin = loss of $PE = (PE)_P - (PE)_{\text{origin}} = 3 \times 10^{-4}$ J.

153. *Hint:* Apply conservation of energy. Remember there is gravitational PE apart from the electrostatic PE.

154. (a) Electric field due to the line charge at a distance r is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Potential difference between two points at distance ηR and R is

$$\Delta V = -\int_R^{\eta R} E\,dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_R^{\eta R}\frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\eta$$

Gain in KE of charge = loss in its electrostatic PE

$$\Rightarrow \quad \frac{1}{2}mV^2 - \frac{1}{2}mV_0^2 = \frac{\lambda Q}{2\pi\varepsilon_0}\ln\eta \Rightarrow V = \sqrt{V_0^2 + \frac{\lambda Q}{\pi\varepsilon_0 m}\ln\eta}$$

(b) If $V_\perp$ is the velocity component $\perp_r$ to radius, angular momentum conservation about the line charge says

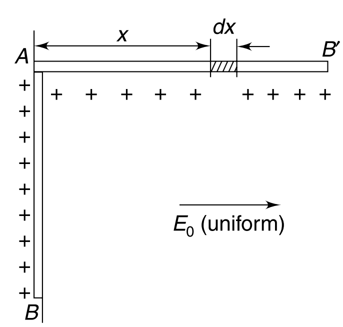
$$mV_\perp \cdot \eta R = mV_0 R \quad \therefore \quad V_\perp = \frac{V_0}{\eta}$$

$$V_\perp^2 + V_r^2 = V^2$$

$$V_r^2 = V^2 - V_\perp^2 = V_0^2 + \frac{\lambda Q}{\pi\varepsilon_0 m}\ln\eta - \frac{V_0^2}{\eta^2}$$

$$\therefore \quad V_r = \sqrt{V_0^2\left(1 - \frac{1}{\eta^2}\right) + \frac{\lambda Q}{\pi\varepsilon_0 m}\ln\eta}$$

155. The torque of electric force on the rod is initially larger than the gravitational force torque. This provides an angular acceleration to the rod and it speeds up. Beyond a certain point, the gravitational force's torque becomes dominant and slows it down to eventually bring the rod at rest when it becomes horizontal.



(a) E_0 can be found by applying law of conservation of energy.

Let potential at line AB be V_0

Electrostatic potential energy of the charge on the rod when it is vertical is $U_i = \lambda L \cdot V_0$

When rod becomes horizontal, we can write its Electrostatic potential energy as

$$U_f = \int_0^L V_x \lambda dx \; [V_x = \text{potential at distance } x \text{ from end } A = V_0 - E_0 x]$$

$$= \int_0^L \lambda (V_0 - E_0 x) dx = \lambda V_0 L - \lambda E_0 \frac{L^2}{2}$$

Loss in Electrostatic potential energy

$$U_i - U_f = \lambda E_0 \frac{L^2}{2}$$

This is equal to gain in gravitational potential energy

$$\lambda E_0 \frac{L^2}{2} = \text{Mg}\frac{L}{2} \quad \Rightarrow E_0 = \frac{\text{Mg}}{\lambda L}$$

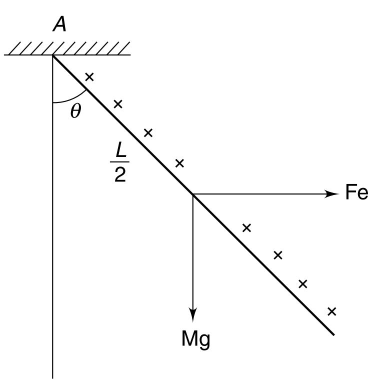
(b) The electric force on the rod is $F_e = \lambda L E_0$. Force can be considered at centre for writing the torque. In equilibrium,

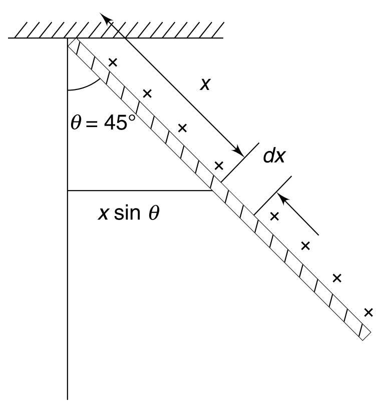
$$\tau_{\text{Fe}} = \tau_{\text{mg}} \text{ [torque about } A]$$

$$\lambda L E_0 \frac{L}{2} \cos\theta = \text{Mg}\frac{L}{2} \sin\theta$$

Putting $E_0 = \dfrac{\text{Mg}}{\lambda L}$ we get $\theta = 45°$

The rod has maximum KE at this position.





KE = (loss in Electrostatic PE) – (gain in gravitational PE)

$$= \int_0^L E_0 x \sin\theta \lambda dx - \text{Mg}\frac{L}{2}(1 - \cos\theta)$$

$$= \frac{E_0 \lambda}{\sqrt{2}} \frac{L^2}{2} - \text{Mg}\frac{L}{2}\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{\text{Mg}L}{2\sqrt{2}} - \text{Mg}\,\frac{L}{2}\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\therefore \qquad \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2 = MgL\left(\frac{\sqrt{2}-1}{2}\right) \quad \Rightarrow \omega = \sqrt{3(\sqrt{2}-1)\frac{g}{L}}$$

156. A small charge on the rod located at A will reach A' when the rod gets horizontal.

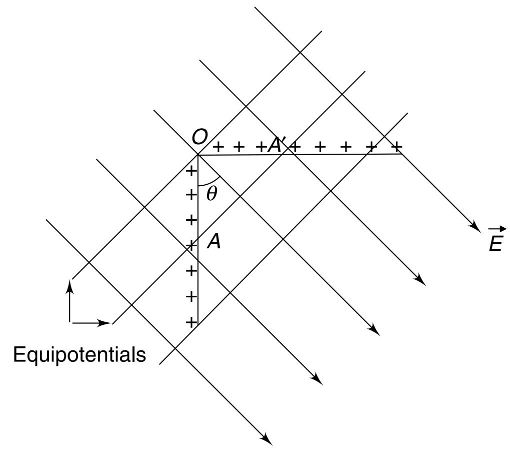
If $\theta < 45°$ then A' will lie on an equipotential having higher potential as compared to point A.

Therefore, the electrostatic potential energy of rod in horizontal position will be more than that in its vertical position. At the same time the gravitational potential energy also increases. This is not possible.

It means rod cannot become horizontal if $\theta = 45°$, however high the field might be.

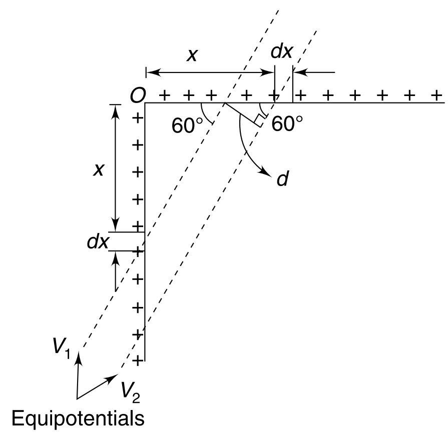
$\therefore$ Ans. to (a) is that it is not possible for rod to become horizontal.

Ans. to (c) is $\theta_0 = 45°$



(b) With $\theta = 60°$, let's calculate the loss in electrostatic PE when the rod becomes horizontal.

A charge element (λdx) at distance x from O will be on two different equipotentials when the rod is vertical and when it is horizontal. Distance between the equipotentials will be



$$d = \left(x - \frac{x}{\sqrt{3}}\right) \cdot \sin 60° = \left(\frac{\sqrt{3}-1}{2}\right)x$$

$\therefore$ loss in PE of the elemental charge will be

$$dU = \lambda dx Ed = \left(\frac{\sqrt{3}-1}{2}\right)\lambda E x\, dx$$

$\therefore$ Total loss in electrostatic PE of the rod

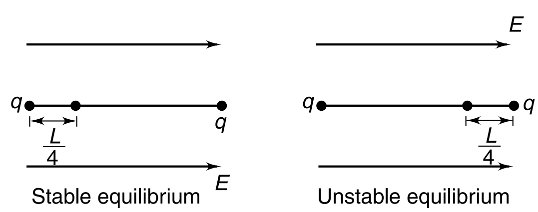
$$\Delta U = \left(\frac{\sqrt{3}-1}{2}\right)\lambda E \int_0^L x\, dx = \left(\frac{\sqrt{3}-1}{4}\right)\lambda E L^2$$

This must be equal to gain in gravitational PE of the rod.

$$\therefore \quad Mg\frac{L}{2} = \left(\frac{\sqrt{3}-1}{4}\right)\lambda E L^2$$

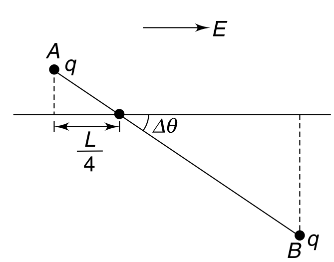
$$E = \left(\frac{2}{\sqrt{3}-1}\right)\frac{Mg}{\lambda L}$$

157. (a)



(b) When the rod is rotated by $\Delta\theta$, charge at the end A moves $\frac{L}{4}(1 - \cos\Delta\theta)$ in the direction of field and the charge at end B moves $\frac{3L}{4}(1 - \cos\Delta\theta)$ opposite to the field.

Charge at B gains PE (as it moves to a location of higher potential) and the charge at A loses PE.



$$\therefore \quad \Delta U = q \cdot E\frac{3L}{4}(1 - \cos\Delta\theta) - q \cdot E\frac{L}{4}(1 - \cos\Delta\theta)$$

$$= E\frac{qL}{4}[3 - 3\cos\Delta\theta - 1 + \cos\Delta\theta]$$

$$\Delta U = \frac{EqL}{4}[2 - 2\cos\Delta\theta] = \frac{EqL}{2}(1 - \cos\Delta\theta) \quad ...(1)$$

(c) Let the angular speed of the rod be ω when it is at small angular displacement θ with respect to its stable position.

$$KE + PE = a \text{ constant}$$

Taking PE to be zero in position of stable equilibrium

We can write PE at position θ using (1)

$$U = \frac{EqL}{2}(1 - \cos\theta)$$

$$KE = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{mL^2}{16} + \frac{9mL^2}{16}\right)\omega^2 = \frac{5mL^2}{16}\omega^2$$

$$\therefore \quad \frac{5mL^2}{16}\omega^2 + \frac{EqL}{2}(1 - \cos\theta) = a \text{ constant.}$$

$$\frac{5mL^2}{16}2\omega\frac{d\omega}{dt} + \frac{EqL}{2}\sin\theta\frac{d\theta}{dt} = 0 \qquad \left[\frac{d\theta}{dt} = \omega \text{ and } \frac{d\omega}{dt} = \alpha\right]$$

$$\therefore \quad \alpha = -\left(\frac{4}{5}\frac{Eq}{mL}\right)\theta \qquad [\sin\theta \simeq \theta]$$

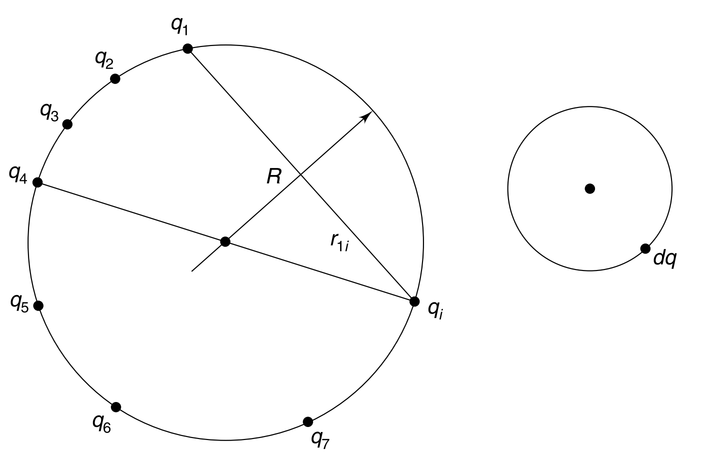
$$\therefore \quad T = 2\pi\sqrt{\frac{5mL}{4Eq}} = \pi\sqrt{\frac{5mL}{Eq}}$$

159. (a) Just remove a small point charge q_i from the shell. This will hardly change the potential of the shell which is equal to

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{1i}} + \frac{q_2}{r_{2i}} + \frac{q_3}{r_{3i}} + \frac{q_4}{r_{4i}} + ...\right] \; [q_i \text{ is not there}]$$

= potential at the location of q_i due to all other charges



Interaction energy of q_i with all other charges

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_{1i}} + \frac{q_2}{r_{2i}} + \frac{q_3}{\gamma_{3i}} + ...\right]q_i$$

If we continue like this interaction of each pair will be counted twice. Hence, the interaction energy shall be written as

$$U = \left[\frac{1}{4\pi\varepsilon_0} \sum_{\substack{i=1,2,3...\\ j=1,2,3...}} \frac{q_i q_j}{r_{ij}}\right] \times \frac{1}{2}$$

The summation will be performed using integration.

Interaction energy of dq with all other charges.

$$= \left[\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\right] dq$$

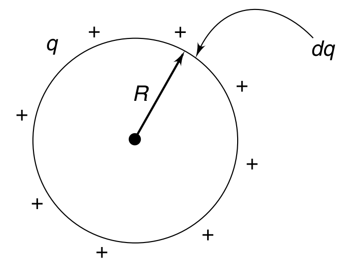
Term inside the bracket is the potential at location of dq due to all other charges

$$\therefore \qquad U = \int_0^Q \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\, dq \times \frac{1}{2}$$

$$= \frac{Q}{8\pi\varepsilon_0 R}\int_0^Q dq = \frac{Q^2}{8\pi\varepsilon_0 R}$$

(b) Consider that charge 'q' has already been gathered. Potential of the shell is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$



Work done in bringing next installment of a small charge dq from infinity

$$dW = dq \cdot V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}\, dq$$

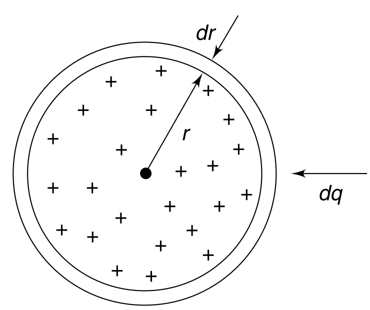
$$\therefore \qquad W = \int dW = \frac{1}{4\pi\varepsilon_0 R}\int_0^Q q\, dq = \frac{Q^2}{8\pi\varepsilon_0 R}$$

The two methods calculate the same thing – self energy of a uniformly charged shell.

160. (a) Imagine that we assemble the sphere by piling up a succession of thin spherical layers of charge of infinitesimal thickness. At each stage, we gather a small amount of charge and put it in a thin layer from r to $r + dr$. We continue till we arrive at a final radius R.

Let q = charge on sphere of radius r

$$= \rho \cdot \frac{4}{3}\pi r^3$$



dq = charge on layer having thickness dr

$$= \rho \cdot 4\pi r^2 dr$$

Work done in bringing charge dq from ∞ to the sphere of radius r is

$$dU = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\cdot dq = \frac{4\pi\rho^2 r^4 dr}{3\varepsilon_0}$$

Total energy required to construct sphere of radius R is

$$U = \frac{4\pi\rho^2}{3\varepsilon_0}\int_0^R r^4 dr = \frac{4\pi\rho^2}{15\varepsilon_0}R^5$$

Put $\qquad \rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$

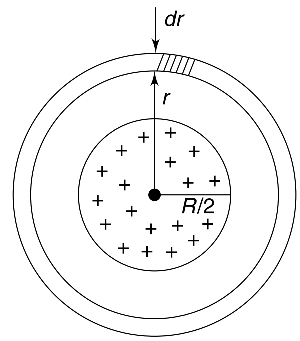
$$U = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 R} \quad ...(A)$$

(b) The desired interaction energy is

$$U^1 = U - U_{qQ}$$

When U = interaction energy of all pairs possible inside the sphere $= \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 R}$

U_{qQ} = interaction energy of the sphere of radius $\frac{R}{2}$ (having charge 'q') with the charge in the annular part (i.e., Q_1, Q_2)....



$$q = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \cdot \rho$$

Potential due to this charge at radius $r(\geq R/2)$

$$V_r = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{R^3\rho}{24\varepsilon_0}\cdot\frac{1}{r}$$

Energy of charge in the layer of thickness dr, in the electrostatic field of q is

$$dU_{qQ} = (\rho 4\pi r^2 dr)\cdot V_r = \frac{\pi R^3\rho^2}{6\varepsilon_0} r\,dr$$

$$\therefore \quad U_{qQ} = \frac{\pi R^3\rho^2}{6\varepsilon_0}\int_{R/2}^{R} r\,dr = \frac{\pi R^5 r^2}{16\varepsilon_0}$$

$$= \frac{9}{256\pi\varepsilon_0}\frac{Q^2}{R} \quad \left[\because \quad \rho = \frac{Q}{\frac{4}{3}\pi R^3}\right]$$

$$\therefore \quad U^1 = U - U_{qQ}$$

$$= \frac{Q^2}{4\pi\varepsilon_0 R}\left[\frac{3}{5} - \frac{9}{64}\right] = \frac{147}{320}\frac{Q^2}{4\pi\varepsilon_0 R}$$

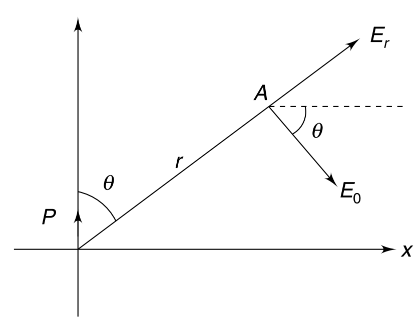
162. We know that field at point A has components (radial and tangential) given by

$$E_r = \frac{K2P\cos\theta}{r^3}; \quad E_\theta = \frac{KP\sin\theta}{r^3}$$

$$\therefore \quad E_x = E_r\sin\theta + E_\theta\cos\theta = K\frac{3P\sin\theta\cos\theta}{r^3}$$

$$E_y = E_r\sin\theta - E_\theta\cos\theta = \frac{KP}{r^3}(2\cos^2\theta - \sin^2\theta)$$

$$= \frac{KP}{r^3}(3\cos^2\theta - 1)$$



(a) For a given $r = r_0$, E_x will be maximum when $\theta = 45°$

$$E_{x\max} = \frac{K3P}{r_0^3}\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \frac{3}{2}\frac{KP}{r_0^3}$$

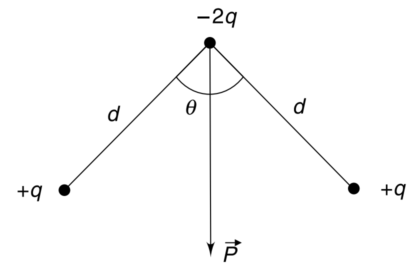
(c) Field is parallel to x axis if $E_y = 0 \Rightarrow 3\cos^2\theta - 1 = 0$

$$\cos\theta = \frac{1}{\sqrt{3}}$$

E_y will be zero at all points on the line defined by $\cos\theta = \frac{1}{\sqrt{3}}$

Slope of line $= \tan(90 - \theta) = \cot\theta = \frac{1}{\sqrt{2}}$.

163. The resultant dipole moment of the molecule is

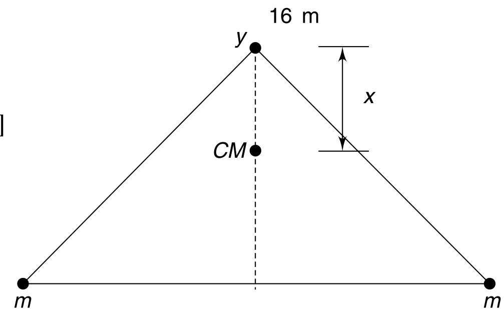


$$P = 2q \cdot d \cdot \cos\left(\frac{\theta}{2}\right) = dq \quad [\because \quad \theta = 120°]$$

Position of COM is as shown.

$$x = \frac{2}{9}d \quad \cos\left(\frac{\theta}{2}\right) = \frac{d}{9}$$

Moment of inertia



$$I_{cm} = I_y - 18m \cdot x^2$$

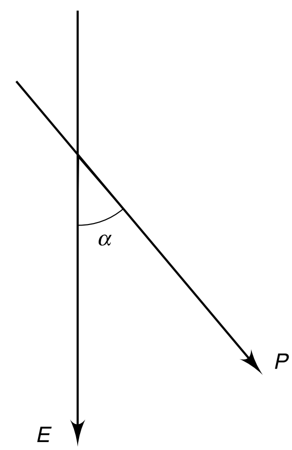
$$= 2md^2 - 18m \cdot \frac{d^2}{81} = \frac{16}{9}md^2$$

Torque when dipole is at an angle α to the field is

$$\tau = PE\sin\alpha \simeq PE\alpha \text{ [for small } \alpha]$$

$$I_{cm} \cdot \frac{d^2\alpha}{dt^2} = -PE \cdot \alpha$$

$$\frac{d^2\alpha}{dt^2} = -\frac{1}{16/9}\frac{qdE}{md^2}\alpha = -\frac{9}{16}\frac{qE}{md} \cdot \alpha$$



$$\omega = \sqrt{\frac{9qE}{16md}} = \frac{3}{4}\sqrt{\frac{qE}{md}}$$

$$\frac{2\pi}{T} = \frac{3}{4}\sqrt{\frac{qE}{md}}$$

$$T = \frac{8\pi}{3}\sqrt{\frac{md}{qE}}$$

164. Electric field at distance x from the centre is

$$E = KQ\frac{x}{(a^2 + x^3)^{3/2}}$$

Potential energy of dipole placed in this electric field is

$$U = -PE\cos 0^0 = -KQP\frac{x}{(a^2 + x^2)^{3/2}}$$

Force on the dipole is

$$F = -\frac{dU}{dx} = KQP\left[\frac{(a^2 + x^2)^{3/2} - \frac{3}{2}x(a^2 + x^2)^{1/2}(2x)}{(a^2 + x^2)^3}\right]$$

$$= KQP\left[\frac{a^2 + x^2 - 3x^2}{(a^2 + x^2)^{5/2}}\right] = KQP\frac{a^2 - 2x^2}{(a^2 + x^2)^{5/2}}$$

For $x = \frac{a}{2}$

$$F = \frac{16}{5^{3/2}}\frac{KQP}{a^3}$$

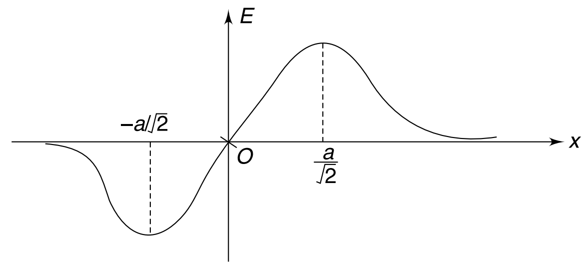
For $x = a$

$$F = -\frac{KQP}{2^{5/2}a^3}$$

Force on the dipole basically depends on $\frac{dE}{dx}$.

(ii) No force is not zero at $x = 0$, because $\dfrac{dE}{dx} \neq 0$

$F = 0$ at $x = \dfrac{a}{\sqrt{2}}$ where $\dfrac{dE}{dx} = 0$



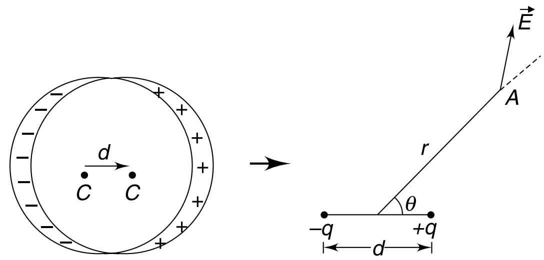
165. For writing field at an outside point, we can consider the charge on each sphere to be located at respective centre.Thus we have a dipole of dipole moment

$$\vec{p} = q\vec{d}$$

Where $$q = \rho \cdot \frac{4}{3}\pi R^3$$

Field at A

$$E = \frac{\rho}{4\pi\varepsilon_0 r^3}\sqrt{3\cos^2\theta + 1} = \frac{\rho}{3\varepsilon_0}\frac{R^3 d}{r^3}\sqrt{3\cos^2\theta + 1}$$

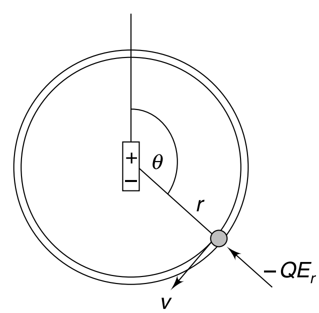


166. (a) Let us apply the law of conservation of energy for the bead having mass m and charge Q:

$$\frac{1}{2}mv^2 + QK\frac{P\cos\theta}{r^2} = \frac{1}{2}mv_0^2 + \frac{QKP\cos(\pi/2)}{r^2} = 0.$$

We can then express the velocity of the bead at angle θ as

$$v = \sqrt{-2\frac{QKP\cos\theta}{mr^2}}, \left(\frac{\pi}{2} \leq \theta \leq \pi\right). \quad ...(1)$$



(b) The circular motion needs a radial force equal to mv^2/r.

The radial component of electric field due to the dipole $E_r = 2\,\dfrac{KP\cos\theta}{r^3}$.

Using the expression of the velocity (equation 1), we notice that the radial force on the bead $(= QE_r)$ is just equal to $-\,mv^2/r$, the required centripetal force. Thus the ring need not exert any force on the bead to sustain circular motion.

(a) The bead would move along a circular path until it reached the point opposite its starting position. The bead would stop there, and then go back to its starting point. It will repeatedly retrace its path executing a periodic motion.

(b) Since the ring does not apply a force on the bead, its absence will make no difference to the motion.